In: 2022 19th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE), 9-11 November 2022, Mexico City, Mexico.
DOI: 10.1109/CCE56709.2022.9975847

# Mechanical Advantage Assurance Control of Quick-return Mechanisms in Task Space

Jocabed Mendoza
*ESIME-IPN*
*Instituto Politécnico Nacional*
Mexico city, Mexico
jmendozam1903@alumno.ipn.mx

Adolfo Perrusquía
*School of Aerospace, Transport and Manufacturing*
*Cranfield University*
Bedford, UK
Adolfo.Perrusquia-Guzman@cranfield.ac.uk

Juan Alejandro Flores-Campos
*UPIITA-IPN.*
*Instituto Politécnico Nacional*
Mexico city, Mexico
jaflores@ipn.mx

***Abstract*—Quick-return mechanisms are usually controlled by joint-space controllers to avoid instability in the transition between cutting and return phases. These controllers cannot exploit the mechanical advantage associated to the natural mechanism's movement in the task space. It is crucial to guarantee mechanical advantage exploitation to reduce operation time and high-quality cutting finishes. In view of the above, this paper reports the design of a mechanical advantage assurance controller based on: i) a slider physics model that captures all the information associated to the main mechanism's task, and ii) a Jacobian compensator that avoids the controllability problems from the transition between the cutting and return phases. Simulation studies are carried out to verify each component of the proposed controller. A constant cutting velocity task is used as a case of study to demonstrate the mechanical advantage exploitation.**

## I. Introduction

Quick-return mechanisms are a class of mechanic devices mainly applied in manufacturing sectors for cutting processes. They are characterized by having a rotational input crank and a linear output link associated to the cutting tool or slider. Particularly, the output linear velocity of the slider is enhanced by the high-mechanical advantage that these mechanisms exhibit at the beginning and at the end of the slider workspace. Here, the mechanical advantage refers to a measure of force amplification that generates a momentum in a short period of time [1] and, in consequence, an increase of the linear velocity which is essential to reduce operation time and high-quality cutting finishes.

Classic approaches use joint space controllers [2], [3] to ensure mechanical advantage exploitation by indirectly controlling the output link. There exist different methodologies to establish a desired performance in the output link, whilst maintaining the mechanical properties, e.g., the reciprocating and mechanism synthesis methods where the geometry of the mechanism is optimized [4] to guarantee a desired linear velocity profile for a constant input angular velocity. However, it has been demonstrated that this approach is not flexible and robust for similar mechanisms with different kinematic parameters [5].

Task space controllers [6] are implemented as alternative tools to inject a desired linear velocity profile in the slider without applying the reciprocating or synthesis methods. However, these controllers fail to exploit the mechanical advantage due to controllability loss when the Jacobian becomes singular. In addition, access to the input position measurement is not usually available for task-space control applications. Thus, it is crucial to ensure mechanical advantage exploitation whilst guaranteeing a desired performance in the slider to guarantee manufacturing standards [7].

In view of the above, this paper reports a mechanical advantage assurance control that combine the merits of joint and task space controllers. The proposed approach is based on a family of task space controllers constructed from a slider physics model. A Jacobian compensator is designed for mechanical advantage assurance based on geometric properties of quick return mechanisms. Simulation studies are carried out to test each element of the proposed technique under a constant cutting velocity case of study. The contributions of this paper are:

1) The mechanical advantage is exploited without using joint space controllers under reciprocating and mechanism's synthesis methods.
2) A general methodology for task-space control design of any quick-return mechanism is given.
3) A Jacobian compensator that overcomes the controllability problem of standard task space controllers.

## II. Problem Statement

Fig. 1 shows a standard phase diagram of a quick-return mechanism controlled in joint space. The points $x_{\max}$ and $x_{\min}$ denote the maximum and minimum points of the slider workspace and equivalently, they define the singularity points. In these points the transition between the two natural phases of the mechanism movement is performed: the cutting and return phases. Whilst the cutting phase is slow and is associated to the main task of the mechanism, the return phase is fast and it is where the mechanism's mechanical advantage [8] is exploited. However, classical control approaches [5] do not exploit the mechanical advantage of this type of mechanisms. In joint space, both the cutting and return phases are controlled (see Fig. 1) and hence, the return phase does not exhibit its fast return movement. On the other hand, task space controllers cannot be applied because they lose controllability at the singularity points [9] of the slider tool. In this paper, the mechanical advantage of quick-return mechanisms is exploited using task space controllers without compromising the controllability and stability of the closed-loop trajectories.

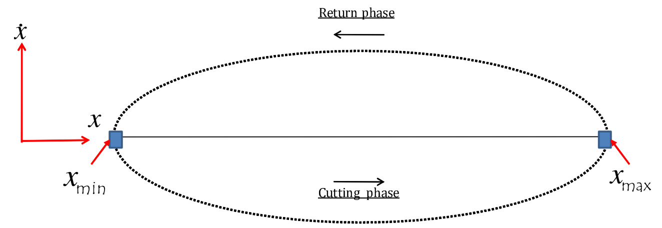



Fig. 1. General phase diagram of a Quick-return mechanism controlled in joint space

## III. Quick-return mechanisms physics

The joint space physics of a 1-degree of freedom (DOF) quick-return mechanism [10] is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \quad (1)$$

where $M(q) \in \mathbb{R}$ denotes the inertia, $C(q,\dot{q}) \in \mathbb{R}$ defines the Coriolis and centripetal forces term, $G(q) \in \mathbb{R}$ stands to the gravitational terms, $\tau \in \mathbb{R}$ is the input torque applied to the crank, and $q \in \mathbb{R}$ is the angular position of the mechanism's crank. The model (1) is high non-linear and it is difficult to extract useful properties associated to the mechanism physics. For this purpose, the extended dynamic model formulation [11] is used to decompose the high-nonlinear physics (1) into a relatively low-nonlinear physics model in terms of the generalized coordinate $q$ and all the secondary variables $\mathbf{s} \in \mathbb{R}^s$ within the mechanism structure as

$$\mathbf{M}'(\mathbf{q}')\ddot{\mathbf{q}}' + \mathbf{C}'(\mathbf{q}',\dot{\mathbf{q}}')\dot{\mathbf{q}}' + \mathbf{G}'(\mathbf{q}') = \rho^{-\top}(\mathbf{q}')\tau \quad (2)$$

where $\mathbf{M}' \in \mathbb{R}^{N\times N}, \mathbf{C}' \in \mathbb{R}^{N\times N}$, and $\mathbf{G} \in \mathbb{R}^N$ define the matrices of the extended model, and $\mathbf{q}' \in \mathbb{R}^N$ is the extended coordinate vector with $N = s+1$. The functions $\sigma(\cdot)$ and $\rho(\mathbf{q}') \in \mathbb{R}^N$ define the mappings between the generalized coordinate to the extended coordinates, i.e.,

$$\begin{array}{ll} \dot{\mathbf{q}}' & = \rho(\mathbf{q}')\dot{q} \\ \mathbf{q}' & = \sigma(q) \end{array}. \quad (3)$$

Is evident that $\sigma(\cdot)$ defines the solution of the forward kinematics of each secondary variable in $\mathbf{q}'$. This formulation is extremely useful because one of the secondary variables in $\mathbf{q}'$ is the slider position $x$ which is directly related to the return and cutting phases of the mechanism's natural movement.

One interesting finding [12] of the model (2) is that we can extract the slider physics without applying any mapping from joint space to task space. In other words, we have that the extended physics can be expressed as

$$\mathbf{M}'(\mathbf{q}') = \begin{bmatrix} \mathbf{M}_{s\times s} & \mathbf{0}_s \\ \mathbf{0}_{1\times s} & m \end{bmatrix} \in \mathbb{R}^{N\times N}$$
$$\mathbf{C}'(\mathbf{q}',\dot{\mathbf{q}}') = \begin{bmatrix} \mathbf{C}_{s\times s} & \mathbf{0}_s \\ \mathbf{0}_{1\times s} & 0 \end{bmatrix} \in \mathbb{R}^{N\times N}$$
$$\mathbf{G}'(\mathbf{q}') = \begin{bmatrix} \mathbf{G}_s \\ g_x \end{bmatrix} \in \mathbb{R}^{N}$$
$$\rho(\mathbf{q}') = \begin{bmatrix} \rho_s(\mathbf{q}') \\ \rho_x(\mathbf{q}') \end{bmatrix} \in \mathbb{R}^{N} \quad (4)$$

where $m$ is the slider mass, $g_x$ is the slider's gravity force component, and $\rho_x(\mathbf{q}')$ is the slider's Jacobian. Therefore, the slider physics can be extracted and is governed by

$$m\ddot{x} + g_x = \left(\sum_{i=1}^{N}\rho_i^2(\mathbf{q}')\right)^{-1}\rho_x(\mathbf{q}')\tau = u. \quad (5)$$

The inverse of the summation within (5) never is zero due to the mechanism configuration. The slider´s model (5) only needs knowledge of the slider's mass instead of the complete mechanism physics. The above model can be expressed as a perturbed second-order linear system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}}_{\mathbf{A}}\mathbf{x}(t) + \underbrace{\begin{bmatrix}0 \\ b\end{bmatrix}}_{\mathbf{B}}(u(t)+d(t)) \quad (6)$$

where $b = 1/m$ and $d = -g_x$. One of the major advantages of this formulation is that we are able to design any linear or discontinuous controller with a predefined closed-loop performance. In the next section, we will prove this statement in simulation studies.

### A. Simulations

We compare different controllers to test the reliability of the slider model. The controllers are designed for both the slider model and a linearised version [13] of the mechanism's physics in task space [14]. The controllers used for comparison are: i) feedback-feedforward control (FFC), ii) linear quadratic tracking (LQT), iii) sliding mode control (SMC), iv) optimal sliding mode control (OSMC), and v) PID control. The Whitworth mechanism [15] of Fig. 2 is used as case of study.

The slider physics model is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}\mathbf{x}(t) + \begin{bmatrix}0 \\ \frac{1}{m_6}\end{bmatrix}u(t)$$

where $m_6$ is the slider mass and the slider's Jacobian is $\rho_x(\mathbf{q}') = -\frac{r_2 r_4}{r_3}\cos(q-\theta_4)\sin(\theta_4-\theta_5)\sec(\theta_5)$. The linear dynamics of the Whitworth mechanism in task space is

$$\dot{\mathbf{x}}(t) = \begin{bmatrix}0 & 1 \\ -0.412 & 0\end{bmatrix}\mathbf{x}(t) + \begin{bmatrix}0 \\ 0.434\end{bmatrix}u(t).$$

The main control objective is to track the following time-varying reference

$$x_d(t) = -0.2 - 0.2\sin(\pi t).$$

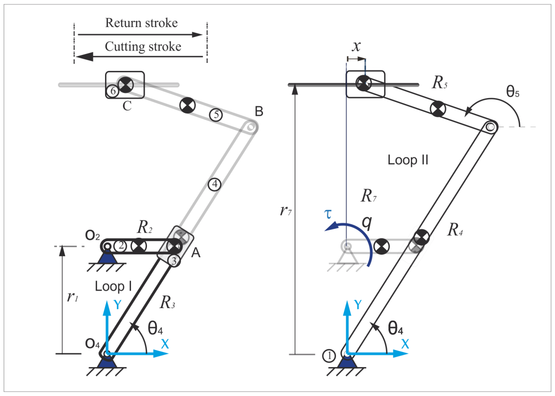



Fig. 2. Whitworth mechanism diagram

The slider mass is set to $m_6 = 1$ kg. The Ackerman formula [16] is used for the FFC design with $\omega_n^2 = 40$ and $\xi = 1$. The weights of the Ricatti equation of the LQT control [14] are proposed as: $\mathbf{Q} = \mathbf{I}$ and $R = 0.1$. The Ackermann-Utkin formula [17] is used for the SMC's hyperplane design with a desired pole of $\lambda = -10$. The OSMC uses the following weight matrix $\mathbf{Q} = \text{diag}\{1, 0.1\}$. The control gains for each physic model are summarized in Table I.

The PID control gains are tuned with the Matlab/Simulink® control toolbox. The gains for the linearised physic model are set to: $K_p = 928.044$, $K_i = 2023.674$, $K_v = 104.506$ and $N = 254.040$. For the slider physics, the gains are: $K_p = 26.56$, $K_i = 14.54$, $K_v = 9.4745$ and $N = 27.065$. In addition, the PID gains are designed to satisfy

$$\frac{y(s)}{y_d(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} = 1$$

where $G(s)$ is the transfer function of the slider physics and $C(s)$ is the transfer function of the PID controller.

Fig. 3 shows the tracking results of each control law using the linearised and slider physics models. The FFC results of Fig. 3(a) shows acceptable tracking results with a small tracking error due to the oversimplified model that have the linear and slider models. This problem is evident in the LQT tracking results of Fig. 3(b) where more tracking error is presented due to the feedforward control; in addition, this controller is sensitive to the time derivative of the desired reference $\mathbf{x}_d$. On the other hand, both SMC and OSMC have good tracking results despite having an oversimplified model. The PID control results show good tracking results with small tracking error due to the tuning procedure [18] using the oversimplified models, nevertheless the controller is reliable.

The mean-squared error (MSE)

$$\bar{e}(t) = \frac{1}{n}\sum_{i=1}^{n}(Le_i)^2,$$

is used as performance metric to numerically evaluate the performance of each controller under different physics models. Here, $L$ is a scaling factor. The MSE results are summarized in Table II with $L = 100$. We can observe that the controllers based on the sliders physics have small MSE in comparison to the linearised model results. Worth noting that PID gains of the linear model are very large in comparison to the PID gains of the slider model, this difference is reflected in the MSE results. In addition, the MSE results are affected by the error of the first 0.5 seconds of the transient time.

## IV. Jacobian compensator

In the previous section, the slider physics model is used to design any linear task space controller that can be effectively applied in the cutting phase. However, the previous controllers lose controllability at the singularity points and hence, the mechanism is not able to exploit the high mechanical advantage of the return phase. To overcome this issue, we exploit an interesting property of quick-return mechanisms that allows to compensate the Jacobian term [19].

The main purpose of the Jacobian $\rho_x(q)$ is to map from joint velocities $\dot{q}$ to task velocities $\dot{x}$ [20]. This mapping is defined by two terms: a change of sign and a state-dependent gain factor. The sign of the Jacobian is proportional to the sign of the input crank and output slider velocities, that is, $\text{sign}(\dot{x}\dot{q})$. The gain factor of the Jacobian is determined by the mechanism velocity kinematics [21].

In view of the above, we can estimate the Jacobian using a constant compensator gain $K_\rho > 0$ which can be set as the magnitude of the real Jacobian's maximum value or any positive scalar. The modelling error of the Jacobian can be attenuated by the controller gains. Then, the Jacobian compensator has the following structure

$$\widehat{\rho}_x = K_\rho \text{s}\left(\dot{x}\dot{q}\right), \tag{7}$$

where

$$\text{s}(\dot{x}\dot{q}) = \begin{cases} \text{sgn}(\dot{x}\dot{q}) & \text{in the cutting phase,} \\ -\text{sgn}(\dot{x}\dot{q}) & \text{otherwise.} \end{cases} \tag{8}$$

The sign function is obtained off-line by observing the directions of the input crank and the slider in the cutting phase. By incorporating the Jacobian compensator, we are able to exploit the mechanism mechanical advantage without any controllability issue.

### A. Simulations

Consider the Whitworth mechanism of Fig. 1. Assume that we do not have measurements of the crank angular position $q$. Here, we want to verify the reliability of the Jacobian compensator in comparison to the real one in a simple regulation task. The FFC control law of Table I using the slider physics is used in this case of study. The FFC under known $q$ is given by (9).

$$\tau = \rho_x^{-1}(q)(\mathbf{K}\mathbf{e}(t) - \mathbf{B}^{\dagger}\mathbf{A}\mathbf{x_d}). \tag{9}$$

The FFC in terms of the Jacobian compensator is given by

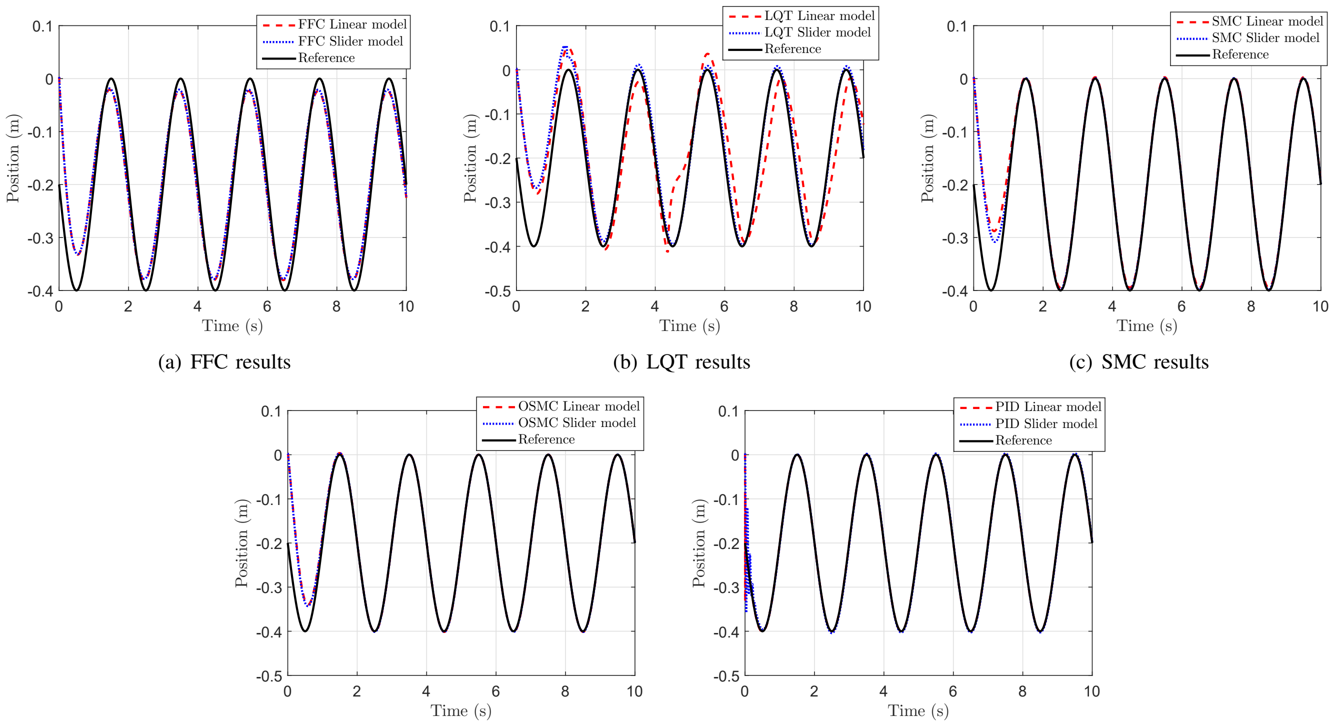
$$\tau = \widehat{\rho}_x^{-1}(\mathbf{K}\mathbf{e}(t) - \mathbf{B}^{\dagger}\mathbf{A}\mathbf{x_d}). \tag{10}$$

The other controllers can also be used to test the Jacobian compensator by using the space transformation mapping [22]

$$\tau = \widehat{\rho}_x^{-1}u(t) \tag{11}$$

TABLE I
CONTROLLER GAINS

| Dynamics | FFC | LQT | SMC | Optimal SMC |
|---|---|---|---|---|
| $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -0.412 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0.434 \end{bmatrix} u(t)$ | $\mathbf{K} = \begin{bmatrix} 91.22 \\ 29.1454 \end{bmatrix}^\top$ | $\mathbf{K} = \begin{bmatrix} 2.3524 \\ 4.5651 \end{bmatrix}^\top$ | $\mathbf{C} = \begin{bmatrix} 23.0415 \\ 2.3041 \end{bmatrix}^\top, \quad K = 4$ | $\overline{\mathbf{C}} = \begin{bmatrix} 3.1623 \\ 1 \end{bmatrix}^\top, \quad K = 1$ |
| $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ | $\mathbf{K} = \begin{bmatrix} 40 \\ 12.6491 \end{bmatrix}^\top$ | $\mathbf{K} = \begin{bmatrix} 3.1623 \\ 4.0404 \end{bmatrix}^\top$ | $\mathbf{C} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}^\top, \quad K = 2$ | $\overline{\mathbf{C}} = \begin{bmatrix} 3.1623 \\ 1 \end{bmatrix}^\top, \quad K = 1$ |




(a) FFC results (b) LQT results (c) SMC results

(d) OSMC results (e) PID results

Fig. 3. Tracking results

TABLE II
MEAN SQUARED ERROR RESULTS.

| Model | FFC | LQT | SMC | OSMC | PID |
|---|---|---|---|---|---|
| **Linear model** | 5.3528 | 42.1356 | 0.0978 | 0.0082 | $1.2786 \times 10^{-4}$ |
| **Slider model** | 5.2106 | 1.0335 | 0.0191 | 0.0089 | 0.0771 |

where $u$ is any task space control law. The desired position is proposed to be within the cutting phase and is given by $x_d = -0.4$ m. Fig. 4 shows the slider's Jacobian time-evolution which will be used to extract the Jacobian upper bound for the compensator's gain.

## V. CONCLUSIONS

This paper reports a mechanical advantage assurance control of quick-return mechanisms. The controller exploits some unique properties of quick-return mechanisms associated to their geometric configuration. The task space controllers are designed in terms of a slider physics model that captures all the relevant information of the task that the mechanism performs naturally. A Jacobian compensator is designed to ensure full controllability and mechanical advantage exploitation at the return phase. The overall approach solves the controllability issue of task space controllers applied to quick return mechanisms and guarantee the mechanical advantage exploitation at the singularity points. Simulation studies are

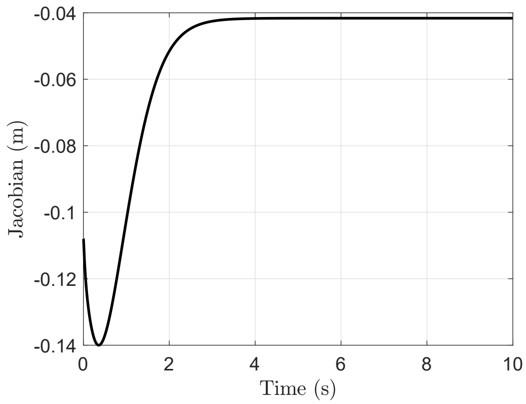



Fig. 4. Whitworth mechanism Jacobian $\rho_x(q)$

carried out to verify the reliability of: i) the slider physics model, ii) the Jacobian compensator and iii) the mechanical advantage exploitation in a constant cutting velocity case of study. Further work will focus on seeking useful properties for other class of mechanisms that can improve the closed-loop performance by exploiting their hidden properties.

## REFERENCES


[1] B. Salamon and A. Midha, "An introduction to mechanical advantage in compliant mechanisms," in *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, vol. 97720, pp. 47–51, American Society of Mechanical Engineers, 1998.

[2] R. F. Fung and K. W. Chen, "Constant speed control of the quick return mechanism driven by a dc motor," *JSME International Journal Series C*, vol. 40, no. 3, pp. 454–461, 1997.

[3] A. Perrusquía, J. A. Flores-Campos, and W. Yu, "Optimal sliding mode control for cutting tasks of quick-return mechanisms," *ISA transactions*, vol. 122, pp. 88–95, 2022.

[4] M. Y. Wang, "Mechanical and geometric advantages in compliant mechanism optimization," *Frontiers of Mechanical Engineering in China*, vol. 4, no. 3, pp. 229–241, 2009.

[5] E. Zheng and X. Zhou, "Modeling and simulation of flexible slider-crank mechanism with clearance for a closed high speed press system," *Mechanism and Machine theory*, vol. 74, pp. 10–30, 2014.

[6] W. Yu and A. Perrusquía, "Simplified stable admittance control using end-effector orientations," *International Journal of Social Robotics*, vol. 12, no. 5, pp. 1061–1073, 2020.

[7] F. Wu, W. Zhang, Q. Li, and P. Ouyang, "Integrated design and pd control of high-speed closed-loop mechanisms," *J. Dyn. Sys., Meas., Control*, vol. 124, no. 4, pp. 522–528, 2002.

[8] W. H. Hsieh and C. H. Tsai, "A study on a novel quick retun mechanism," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 33, no. 3, 2009.

[9] A. Perrusquía and W. Yu, "Task space human-robot interaction using angular velocity jacobian," in *2019 international symposium on medical robotics (ISMR)*, pp. 1–7, IEEE, 2019.

[10] R. Kelly and V. Santibáñez, *Movement Control of Robot Manipulators*. Ribera del Loira, España: Pearson Prentice Hall, 2003.

[11] F. Ghorbel, O. Chételat, R. Gunawardana, and R. Longchamp, "Modeling and set point control of closed-chain mechanisms: Theory and experiment," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 5, pp. 801–815, 2000.

[12] A. Perrusquia, J. A. Flores-Campos, C. R. Torres-Sanmiguel, and N. Gonzalez, "Task space position control of slider-crank mechanisms using simple tuning techniques without linearization methods," *IEEE Access*, vol. 8, pp. 58435–58442, 2020.

[13] A. Perrusquía, "Robust state/output feedback linearization of direct drive robot manipulators: A controllability and observability analysis," *European Journal of Control*, 2022.

[14] A. Perrusquía, "Solution of the linear quadratic regulator problem of black box linear systems using reinforcement learning," *Information Sciences*, vol. 595, pp. 364–377, 2022.

[15] J. A. Flores-Campos, A. Perrusquía, L. H. Hernández-Gómez, N. González, and A. Armenta-Molina, "Constant speed control of slider-crank mechanisms: A joint-task space hybrid control approach," *IEEE Access*, vol. 9, pp. 65676–65687, 2021.

[16] J. Ackermann and V. Utkin, "Sliding mode control design based on ackermann's formula," *IEEE Transactions on Automatic Control*, vol. 43, pp. 234–237, 1998.

[17] M. Salamci and S. Banks, "Optimal sliding surface design for a class of nonlinear systems," *Research Report. ACES Research Report 697. Department of Control and Systems Engineering*, 2008.

[18] A. Perrusquia, J. A. Flores-Campos, and C. R. Torres-San-Miguel, "A novel tuning method of pd with gravity compensation controller for robot manipulators," *IEEE Access*, vol. 8, pp. 114773–114783, 2020.

[19] A. Perrusquía and W. Yu, "Human-in-the-loop control using euler angles," *Journal of Intelligent & Robotic Systems*, vol. 97, no. 2, pp. 271–285, 2020.

[20] M. Mehdi Fateh and H. Farhangfard, "On the Transforming of Control Space by Manipulator Jacobian," *International Journal of Control, Automation, and Systems*, vol. 6, pp. 101–108, February 2008.

[21] J. A. Flores Campos and A. Perrusquía, "Slider position control for slider-crank mechanisms with jacobian compensator," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 233, no. 10, pp. 1413–1421, 2019.

[22] A. Perrusquía, W. Yu, and A. Soria, "Position/force control of robot manipulators using reinforcement learning," *Industrial Robot: the international journal of robotics research and application*, vol. 46, no. 2, pp. 267–280, 2019.

# Mechanical advantage assurance control of quick-return mechanisms in task space

Mendoza, Jocabed

2022-12-15



Mendoza J, Perrusquía A, Flores-Campos JA. (2022) Mechanical advantage assurance control of quick-return mechanisms in task space. In: 2022 19th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE), 9-11 November 2022, Mexico City, Mexico

https://doi.org/10.1109/CCE56709.2022.9975847