Cranfield

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Initial Review of Research into the Application of Modified Stepwise Regression for the Estimation of Aircraft Stability and Control Parameters

M V Cook & H A Hinds

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CONTENTS

INTRODUCTION

A programme of research has recently begun within the college of Aeronautics under a Research Agreement set up with the RAE, Reference 1. Technical supervision is being provided by Dr. A.J. Ross on behalf of the RAE at Farnborough. At Cranfield Mr. M.V. Cook will supervise the research which is to investigate the estimation of aircraft stability and control parameters using a modified stepwise regression technique.

Previous aeronautical applications of the modified stepwise regression method have concentrated on the accurate identification of complex aircraft model structures involving aircraft with six degrees of freedom. Such applications may mean that the computational complexities of the method are overshadowed by the complexities of the aircraft model under investigation. At Cranfield, by applying modified stepwise regression to a simple four degrees of freedom model it is hoped to confirm that the method works equally as well as when it is applied to more complex models. Further, through a simple aircraft model it may be possible to provide enhanced visibility of the analytical techniques involved in the computation of parameters. The research aims to thus improve the understanding of the modified stepwise regression technique and its application to aircraft in general.

1.0 SYSTEM IDENTIFICATION

It is now standard practice to estimate aircraft stability and control parameters in flight conditions where aerodynamic characteristics can be described in linear terms only and where no significant external disturbances are present. In system identification advanced statistical methods, such as those cited in reference 2, have been applied to many multiple-input, multiple-output systems. As an alternative to these estimation methods modified stepwise regression has been developed over the last fifteen years. The procedure is based on ordinary stepwise regression which has been modified by adding a constraint to the parameter selection and a prediction sum of squares (PRESS) criterion for the model structure determination.

Much of the early development of the method was carried out in the U.S.A. by V. Klein, J.G. Batterson and P.C. Murphy. In their work the aircraft equations of motion are in general form, with the aerodynamic force and moment coefficients expressed as polynomials in response and input variables. The modified stepwise regression is constructed to force a linear model for the aerodynamic coefficient first, then it adds significant non linear terms and deletes non-significant terms from the model. The statistical criteria in the stepwise regression for the selection of an adequate model are complemented by the prediction sum of squares criterion and by the analysis of residuals.

Taken from reference 3, there now follows a brief summary of the modified stepwise regression technique and an example of its application.

2.0 MODIFIED STEPWISE REGRESSION

Linear regression is employed to estimate a functional relationship of a dependent variable to one or more independent variables. It is assumed that the dependent variable can be closely approximated as a linear combination of the independent variables. For the system identification of an aircraft operating at low angles of attack, the mathematical model structure for aerodynamic forces and moments is linear and may be written in the form

$$y(t) = \theta_0 + \theta_1 x_1(t) + \theta_2 x_2(t) + \dots + \theta_{n-1} x_{n-1}(t) \qquad (1)$$

Where: $y(t)$ represents the resultant coefficient of aerodynamic force or moment $(C_x, C_y, C_z, C_m, C_l, C_n)$ at time t. These are the dependent variables.

$\theta_1, \theta_2, \dots, \theta_{n-1}$ are the stability and control derivatives; and θ_0 is the value of any particular coefficient corresponding to the initial steady flight conditions.

$x_1, x_2, \dots x_{n-1}$ are the independent aircraft state and control variables $(\alpha, q, \beta, p, r, \eta, \xi, \zeta)$ and may also include combinations of these variables at time t.

When a sequence of N observations on both y and x has been made at times $t_1, t_2, \dots, t_N$, then the measured data can be related by the following set of N linear equations:

$$y(i) = \theta_0 + \theta_1 x_1(i) + \dots + \theta_{n-1} x_{n-1}(i) + \varepsilon(i) \qquad i = 1,2,\dots,N \quad (2)$$

Because Eqn.(1) is only an approximation of the actual aerodynamic relations, the right-hand side of Eqn.(2) includes an additional term, $\varepsilon(i)$, often referred to as the equation error. For $N>n$, the unknown parameters can be estimated from the measurements by the least-squares technique.

The stepwise regression is a procedure which inserts independent variables into the regression model until the regression equation is satisfactory. The order of insertion is determined by using the partial correlation coefficient as a measure of the importance of variables not yet in the regression equation.

At every step of the regression the variables incorporated into the model in previous stages and a new variable entering the model are re-examined using the F statistic. A variable may be taken out of the model depending on the value of this partial F_p statistic which is given by

$$F_p = \hat{\theta}_j{}^2/s^2(\hat{\theta}_j) \qquad j = 1,2,\ldots n \tag{3}$$

where $\hat{\theta}_j$ is the estimate of the parameter θ_j, and $s^2(\hat{\theta}_j)$ is the variance of estimate $\hat{\theta}_j$.

The process of selecting and checking variables continues until no more variables will be admitted to the equation and no more are rejected. The complete computing scheme for the stepwise regression can be found in reference 4.

The computing scheme found in reference 4 is changed slightly in that a constraint is applied to the stepwise regression technique. Hence the name modified stepwise regression in which the linear terms in the model are examined first. The linear terms enter the regression according to their partial correlation coefficients and are kept in the model regardless of the value of F_p. This means that during this part of the procedure no hypothesis testing is applied to reject a term from the model. When all linear terms are included, the non linear terms postulated are searched and the null hypothesis concerning their significance and the significance of all terms already included in the model is tested. Because of the particular constraint applied, the modified stepwise regression (MSR) provides the information about the performance of a linear model.

The MSR method has been used many times with both simulated data and real data. One such example of its use, as described in reference 5, is now given.

2.1 AN APPLICATION OF MSR

In this example, a simulated data set was created using a fourth-order Runge-Kutta integration computer program with a stepsize of 0.0001s. The aerodynamic model in the integration was that estimated by applying the MSR to a high-angle-of-attack lateral manoeuvre which exhibited longitudinal oscillations due to coupling effects. When applied to the simulated data, the MSR selected the correct model structure and parameter estimates, thus verifying the MSR in a noise-free environment. Next, zero mean Gaussian noise was added to the lateral dependent variables C_y, C_l, C_n. The standard deviation for this measurement noise was that estimated from real flight data.

In the side force equation, the selection of a model consisting of the linear terms plus the $p\alpha$ term is based on the maximum F value after the MSR was allowed to consider all candidate variables. This model also corresponds to that indicated by the minimum PRESS value. The $r\alpha^2$ and α^2terms were two of the next terms to enter the model. None of the parameter estimates was statistically different from its true value.

As a measure of the robustness of the MSR, it was also applied to two cases in which both the dependent variables C_y, C_l, C_n and the linear model variables β, p, r were corrupted by zero mean Gaussian noise. The standard deviation of the model variable noise in the first case was that estimated from the ground calibration of the instrumentation system. In the second case, five times higher noise levels were applied to the same model variables. (One would normally expect the possibility of biased parameter estimates from an equation error method).

With the lower level of noise, the MSR reaches a maximum F value at six variables for the side force equation. However, the PRESS selects two additional variables that were not in the simulation model. This emphasises a third piece of information available to the MSR user. The user can examine the F_p's for each of the variables on the regression at a given point. If newly added variables have significantly lower F_p's than those already in the model, one should pick the less complex model equal to or greater than that corresponding to the maximum F value.

The modification which constrains the MSR to first fit the linear model is an important feature. For the cases in which noise was added to the model variables, an unconstrained stepwise regression was inconsistent between the PRESS and maximum F criteria as to the best model structure. Also, terms that were not in the simulated model were accepted in certain best models for an unconstrained stepwise regression. Therefore, this example substantiates the use of the MSR rather than the stepwise regression without constraint.

3.0 LITERATURE SURVEY

In October/November 1988 a literature survey was carried out at Cranfield using an on-line data base system. This survey was used to compliment literature already held on the subject of parameter identification. As a result of this, it is hoped that the list of references found to date is fairly comprehensive and constitutes a good basis for research into modified stepwise regression at Cranfield.

The majority of the literature obtained from these references relates to work done by V. Klein and his colleagues in the U.S.A. Much of the pioneering work in applying modified stepwise regression to identify aircraft stability and control parameters was performed by these people in the late 1970's. Later applications of MSR have been concerned with the identification of the stability and control derivatives of a large scale free flying fighter aircraft model. This work was carried out by the RAE and used flight test data obtained from their High Incidence Research Model (HIRM) aircraft which was flown in the U.S.A. with NASA assistance, reference 6. Within the U.K. further work is being done on HIRM aircraft by the RAE.

Interest in post stall and spin flights has created a need to extend parameter estimation into flight areas where non-linear aerodynamic effects become more pronounced. This introduces the problem of determining how complex the model should be. Although a more complex model can be justified for proper description of aircraft motion, it has not been clear in the past which relationship between model complexity and measurement information would be the best. If too many parameters are sought from a limited amount of data, a reduced accuracy in evaluated parameters can be expected due to large covariance or unrealistic values of some parameters. Alternatively, attempts to identify all parameters might fail.

Problems relating to the model complexity has led to the College of Aeronautics present involvement. Previous work at Cranfield has been supported by the RAE in the field of parameter identification using a simple dynamic data rig (reference 7 and 8). Using a system with only four degrees of freedom greatly reduces the complexity of the mathematical model required.

4.0 THE EXPERIMENTAL FACILITY

It is proposed to use the experimental facility as depicted by the functional block diagram in Fig.1. The components comprising the facility are described in reference 9 and are outlined below.

4.1 WIND TUNNEL

The wind tunnel would normally determine the scope of the facility by constraining model size and by limiting the maximum wind speed available. However, in this case the recently refurbished 'Weybridge' open jet wind tunnel together with the dynamic wind tunnel test rig and supporting equipment will be used. This tunnel was used in previous parameter identification work and is a low-speed open-section wind tunnel. It has an open working section measuring 1.5m by 1.1m diameter, a closed return and a maximum wind velocity of 40m/s. The model size is limited to a maximum wing span of about 0.9m and the flight envelope depends on the tunnel speed and scaling law requirements.

4.2 MODEL SUSPENSION SYSTEM

The suspension system consists of a vertical rod mounted in bearings at its upper and lower ends, so that it may rotate about its vertical axis. The rod is supported, by means of its bearing mounting plates, in a large transportable Dexion framework to which it is rigidly attached by wire bracing. The whole assembly, complete with model, can be removed from the wind tunnel as a unit. A sleeve is keyed to the vertical rod so that it may slide freely in a vertical sense but is constrained to rotate with the rod. The sleeve then forms part of the suspension gimbal which is mounted in the model. The model is thus free to rotate in pitch and roll about the sleeve. Rotation in yaw is about the vertical axis of the rod and vertical translation involves the sleeve sliding on the rod. Angular motion in each axis is sensed by means of potentiometers and is limited to $\pm 30^\circ$. Vertical motion is possible over approximately 0.75m. The test rig is currently the subject of further development. Following completion of this work it will be possible to sense the vertical position (h) and velocity ($\dot{h}$) of the model and to extend the scale of the flight envelope.

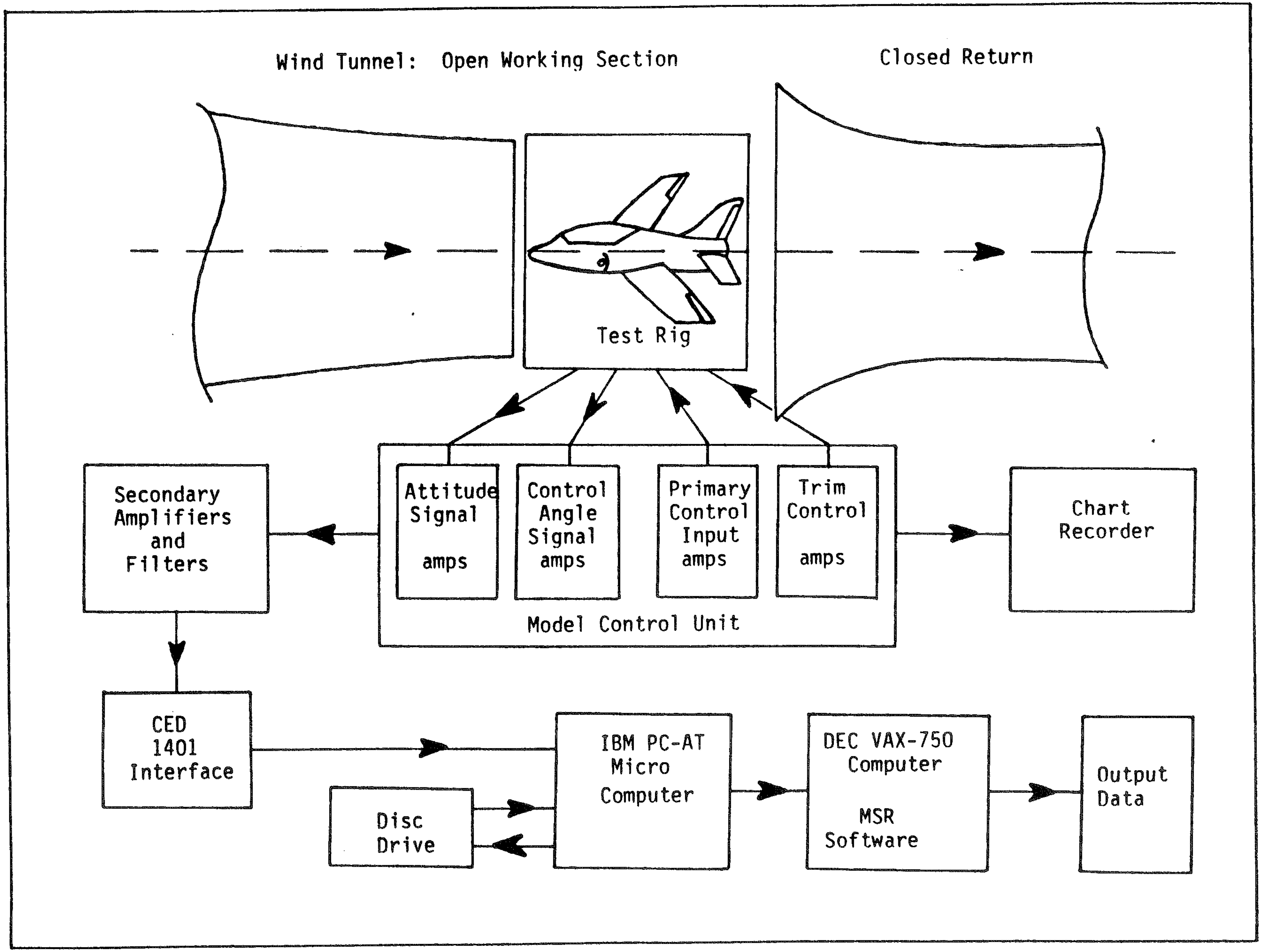

FIGURE 1: BLOCK DIAGRAM REPRESENTING THE EXPERIMENTAL PROCESS OF OPEN-LOOP DYNAMIC TESTS

4.3 ELECTRONIC CONTROL UNIT

The electronic control unit was designed and built as a small, self-contained, transportable console which, for simplicity, employs analogue circuitry throughout. Construction of the control unit is highly modular, to facilitate functional changes, and it provides the following facilities;

(i) electrical power supplies;
(ii) input and output interfaces with the model;
(iii) primary control of the model;
(iv) programmable analogue computer elements for feedback purposes;
(v) output signal interfaces for recording and display;
(vi) input and output interfaces to an external computer.

4.4 AIRCRAFT MODEL

During the initial development of the experimental facility, the aircraft model used was based on the BAe Hawk, reference 7. When suitably scaled, the model had sufficient internal volume for the suspension and control equipment and a reasonable amount of performance data was available. The model was scaled in the ratio 1:12 and had a scale flying weight of 3kg. More recently, the facility has been used to evaluate a combat aircraft configuration with forward swept wing, reference 8.

For both models it was necessary to use a light-weight structure to allow for the weight of the enclosed equipment and ensure that dynamic scaling requirements were met. The models were constructed using aeromodelling techniques and materials which proved adequate for their application in parameter identification work. The models were controlled by tailplane or foreplane and ailerons and rudder, driven by small precision servo-actuators. Control signals to and from the model, together with power-supply cables, were grouped together to form a trailing umbilical connection to the control unit.

The test rig development is the subject of a current MSc project at Cranfield. For the MSR research the BAe Hawk aircraft is to be used and the model has just undergone a complete refurbishment.

4.5 DATA ANALYSIS

In earlier work involving the use of the Hawk model, all data analysis was based on the use of recorded response time histories, since the computer based data acquisition facility was not available. A six-channel pen recorder was used. However, subsequent development of the facility for use with the forward swept wing aircraft model necessitated the addition of a digital data-acquisition system. This basically consists of a signal processor of some kind to convert or digitise analogue data into the form needed for a digital computer link up.

It is proposed to use a digital computer data-acquisition system once again for present work with the Hawk model. The system will comprise a CED 1401 analogue-digital interface linked to an IBM PC-AT microcomputer. The CED 1401 is an intelligent peripheral which can be used to generate and, more importantly, receive waveform, digital and timing signals. Using its own processors, clocks and memory the 1401 may be programmed through a host computer in a variety of languages. In this case the driving computer is the IBM PC which will use PASCAL to communicate with the 1401 interface. There is already some software available for this purpose, under the name of WATERFALL, which claims to provide a powerful tool for use in the investigation of any analogue system. It is hoped to adapt this software to help establish the data acquisition interface with the experimental rig's electronic control unit.

Other College of Aeronautics computing facilities which may be utilised as necessary include BBC Model B and R.M. Nimbus microcomputers as well as a DEC VAX II 750 mainframe computer.

5.0 THEORETICAL DEVELOPMENT

Initially particular reference will be made to the work by Klein et al, the related work of the RAE Aerodynamics group and previous work involving the College of Aeronautics dynamic wind tunnel facility (references 9 and 10). Theoretical developments in the current research programme will largely be concerned with establishing a suitable basis for the various computer programs required. Thus, the main objectives of this work are as follows:

(i) The modified stepwise regression theory for parameter estimation will be developed as far as is necessary for the proposed application.

(ii) Data processing algorithms will be developed as required to convert the recorded experimental data into a format suitable for the parameter estimation program.

(iii) A mathematical model representative of the dynamic wind tunnel model aircraft will be developed. There is some evidence to suggest that the form of the model used in a previous estimation program was not the most suitable, reference 10. The main requirement will be to revise the existing models to fit the application more precisely.

The mathematical model is required for two reasons. Firstly, it forms the basis for the regression equations and secondly, it will be necessary to establish a computer simulation of the aircraft for comparative evaluations. The simulation will be written in the Advanced Continuous Simulation Language (ACSL).

5.1 AIRCRAFT DYNAMICS

The equations describing the longitudinal motion of an aircraft will be considered first, followed by the equations describing lateral motion. References 11 and 12 contain the derivation of the relevant equations which are considered to be "standard theory".

(i) The general dimensional equations of longitudinal symmetric motion for small disturbances (when referred to body axes) may be written as

$$m\dot{u} - \overset{\circ}{X}_u.u - \overset{\circ}{X}_w.w - \overset{\circ}{X}_{\dot{w}}.\dot{w} + (mWe-\overset{\circ}{X}_q).q + mg_1.\theta = \overset{\circ}{X}(t) \qquad (3a)$$

$$-\overset{\circ}{Z}_u.u - \overset{\circ}{Z}_w.w + (m-\overset{\circ}{Z}_{\dot{w}}).\dot{w} - (mU_e+\overset{\circ}{Z}_q).q + mg_2.\theta = \overset{\circ}{Z}(t) \qquad (3b)$$

$$-\overset{\circ}{M}_u.u - \overset{\circ}{M}_w.w \quad - \overset{\circ}{M}_{\dot{w}}.\dot{w} \quad -\overset{\circ}{M}_q.q + I_y.\dot{q} = \overset{\circ}{M}(t) \qquad (3c)$$

where "∘" denotes a dimensional coefficient;

$g_1 = g\cos\theta_e$ $\qquad$ $U_e = V\cos\alpha_e$

$g_2 = g\sin\theta_e$ $\qquad$ $W_e = V\sin\alpha_e$

and since small perturbations are assumed $\dot{\theta} = q$.

Equations (3) are for free flight aircraft. However, with the dynamic rig at Cranfield the Hawk model in the wind tunnel has longitudinal translation suppressed which means Eqn (3a) may be removed completely. Further, when considering wind axes (rather than body axes) and assuming the tunnel speed remains constant (u=0) the following conditions apply:

$\alpha_e = W_e = 0$ $\qquad$ $u = 0$

$U_e = V\cos 0 = V$ $\qquad$ $\dot{u} = 0$

In the case of horizontal steady flight we also have $\theta_e=\alpha_e=W_e=0$ giving $g_1=g$ and $g_2=0$.

Thus the dimensional equations for semi-free flight are

$$-\mathring{Z}_w . w + (m-\mathring{Z}_{\dot w}).\dot w - (mU_e+\mathring{Z}_q).q = \mathring{Z}(t) \tag{4a}$$

$$-\mathring{M}_w . w - \mathring{M}_{\dot w}.\dot w \qquad - \mathring{M}_q . q + I_y . \dot q = \mathring{M}(t) \tag{4b}$$

These equations may be rearranged to give

$$(m-\mathring{Z}_{\dot w})\dot w \qquad = \mathring{Z}_w . w + (mU_e + Z_q).q + \mathring{Z}_\eta . \eta \tag{5a}$$

$$-\mathring{M}_{\dot w}.\dot w + I_y . \dot q = \mathring{M}_w . w + \mathring{M}_q . q \qquad + \mathring{M}_\eta . \eta \tag{5b}$$

It is often convenient to reduce these equations to a more manageable form by dividing the force equation (5a) through by the aircraft mass m and the moment equation (5b) through by the pitch inertia I_y as follows to give

$$(1-z_{\dot w})\dot w \qquad = \mathring{z}_w . w + (U_e+\mathring{z}_q)q + \mathring{z}_\eta . \eta \tag{6a}$$

$$-\mathring{m}_{\dot w}\dot w + \dot q \qquad = \mathring{m}_w . w + \mathring{m}_q . q \qquad + \mathring{m}_\eta . \eta \tag{6b}$$

where $\mathring{z}_{\dot w} = \dfrac{\mathring{Z}_{\dot w}}{m}$; $\quad \mathring{z}_w = \dfrac{\mathring{Z}_w}{m}$; $\quad$ etc.

and $\mathring{m}_{\dot w} = \dfrac{\mathring{M}_{\dot w}}{I_y}$; $\quad \mathring{m}_w = \dfrac{\mathring{M}_w}{I_y}$; $\quad$ etc.

or in matrix form

$$\begin{pmatrix} (1-\mathring{z}_{\dot w}) & 0 & 0 \\ -\mathring{m}_w & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \dot w \\ \dot q \\ \dot \theta \end{pmatrix} = \begin{pmatrix} \mathring{z}_w & (u_e+\mathring{z}_q) & 0 \\ \mathring{m}_w & \mathring{m}_q & 0 \\ 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} \mathring{z}_\eta \\ \mathring{m}_\eta \\ 0 \end{pmatrix} . (\eta) \tag{7}$$

The mass matrix $M = \begin{pmatrix} (1-\mathring{z}_{\dot{w}}) & 0 & 0 \\ -\mathring{m}_{\dot{w}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ has an inverse which is given by

$$M^{-1} = \begin{pmatrix} \dfrac{1}{(1-\mathring{z}_{\dot{w}})} & 0 & 0 \\ \dfrac{\mathring{m}_{\dot{w}}}{(1-\mathring{z}_{\dot{w}})} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Premultiplying equation (7) by this inverse mass matrix leads to the standard state variable form of the equations of longitudinal motion

$$\begin{pmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} z_w & z_q & 0 \\ m_w & m_q & 0 \\ 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} z_\eta \\ m_\eta \\ 0 \end{pmatrix} . (\eta) \qquad (8)$$

where $z_w = \left(\dfrac{\mathring{z}_w}{1-\mathring{z}_{\dot{w}}}\right)$; $\quad z_q = \left(\dfrac{u_e+\mathring{z}_q}{1-\mathring{z}_{\dot{w}}}\right)$; $\quad z_\eta = \left(\dfrac{\mathring{z}_\eta}{1-\mathring{z}_{\dot{w}}}\right)$

$$m_w = \left(\frac{\mathring{m}_{\dot{w}} \mathring{z}_w}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_w\right); \quad m_q = \left(\frac{(u_e+\mathring{z}_q)\mathring{m}_{\dot{w}}}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_q\right); \quad m_\eta = \left(\frac{\mathring{m}_{\dot{w}} \mathring{z}_\eta}{(1-\mathring{z}_{\dot{w}})} + \mathring{m}_\eta\right)$$

Note that equation (8) is of the form $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ which has the output equation $\underline{y} = C\underline{x} + D\underline{u}$. In this case $\underline{y}$ is given by

$$\underline{y} = \begin{pmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} . (\eta) \qquad (9)$$

(ii) The general dimensional equations of <u>lateral</u> <u>asymmetric</u> <u>motion</u>, referred to body axes, for small disturbances may be written as

$$m_{\dot{v}} - \mathring{Y}_v.v - (mW_e+Y_p)p + (mU_e-\mathring{y}_r)r - mg_1\Phi - mg_2\Psi = \mathring{Y}(t) \qquad (10a)$$

$$-\mathring{L}_v.v + I_x.\ \dot{p} - \mathring{L}_p.p \ - I_{xz}.\dot{r} - \mathring{L}_r.r = \mathring{L}(t) \qquad (10b)$$

$$-\mathring{N}_v.v + I_{xz}.\dot{p} - \mathring{N}_p.p + \ I_z.\dot{r} - \mathring{N}_r.r = \mathring{N}(t) \qquad (10c)$$

where $g_1 = g\cos\theta_e$;
$g_2 = g\sin\theta_e$;

and since small perturbations are assumed $\dot{\Phi} = p$; $\dot{\Psi} = r$.

It should be noted that more generally the following relationship applies.

$$\begin{pmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\Phi\tan\theta & \cos\Phi\tan\theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi\sec\theta & \cos\Phi\sec\theta \end{pmatrix} . \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

However, when the perturbations about the aircraft body fixed axes are small, Φ, θ and Ψ are small and the above relationships reduce to

$$\dot{\Phi} = p$$
$$\dot{\theta} = q$$
$$\dot{\Psi} = r$$

In the case of semi-free flight in the wind tunnel equation (10a) is removed as lateral translation of the aircraft model is suppressed. Thus, referred to wind axes we are left with only

$$-\mathring{L}_v.v + I_x.\dot{p} - \mathring{L}_p.p - I_{xz}.\dot{r} - \mathring{L}_r.r = \mathring{L}(t) \qquad (11a)$$

$$-\mathring{N}_v.v + I_{xz}\dot{p} - \mathring{N}_p.p - I_z.\ \dot{r} - \mathring{N}_r.r = \mathring{N}(t) \qquad (11b)$$

$-\mathring{L}_v.v$ and $-\mathring{N}_v.v$ have been retained as in experimental work we will take the sideslip angle and yaw angle to be equivalent.

Equations (11) may be rearranged as follows

$$I_x\dot{p} - I_{xz}\dot{r} = \mathring{L}_v v + \mathring{L}_p p + \mathring{L}_r r + \mathring{L}_\xi \xi + \mathring{L}_\zeta \zeta \tag{12a}$$

$$I_{xz}\dot{p} + I_z\dot{r} = \mathring{N}_v v + \mathring{N}_p p + \mathring{N}_r r + \mathring{N}_\zeta \zeta + \mathring{N}_\xi . \xi \tag{12b}$$

To reduce these equations still further we may divide (12a) through by I_x and (12b) through by I_z to get

$$\dot{p} - e_x\dot{r} = \mathring{l}_v v + \mathring{l}_p p + \mathring{l}_r r + \mathring{l}_\xi \xi + \mathring{l}_\zeta \zeta \tag{13a}$$

$$e_z\dot{p} + \dot{r} = \mathring{n}_v v + \mathring{n}_p p + \mathring{n}_r r + \mathring{n}_\xi \xi + \mathring{n}_\zeta \zeta \tag{13b}$$

where $e_x = I_{xz}/I_x$; $\mathring{l}_v = \mathring{L}_v/I_x$; $\mathring{l}_p = \mathring{L}_p/I_x$; etc

and $e_z = I_{xz}/I_z$; $\mathring{n}_v = \mathring{N}_v/I_z$; $\mathring{n}_p = \mathring{N}_p/I_x$; etc.

The dynamic model aircraft is free to rotate in yaw but there is no translation in the y-direction so that the aerodynamic sideforces and gravity components are balanced by the support system. However, the fact that dy/dt = 0 implies that the lateral acceleration,

$$\dot{v} - pW_e + rU_e = 0 \tag{14}$$

Therefore, the lateral equations of motion, (incorporating equations 13 and 14), with respect to wind tunnel simulations for steady horizontal datum flight may be expressed in matrix form as

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -e_x & 0 & 0 \\ 0 & e_z & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} 0 & W_e & -U_e & 0 & 0 \\ \mathring{l}_v & \mathring{l}_p & \mathring{l}_r & 0 & 0 \\ \mathring{n}_v & \mathring{n}_p & \mathring{n}_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} v \\ p \\ r \\ \phi \\ \Psi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \mathring{l}_\xi & \mathring{l}_\zeta \\ \mathring{n}_\xi & \mathring{n}_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \tag{15}$$

Equation (15) is of the form $M\underline{\dot{x}} = A\underline{x} + B\underline{u}$, where

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -e_x & 0 & 0 \\ 0 & e_z & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1/(1+e_x e_z) & e_z/(1+e_x e_z) & 0 & 0 \\ 0 & -e_x/(1+e_x e_z) & 1/(1+e_x e_z) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Premultiplying equation (15) by M^{-1} once again yields equations of motion which are in the standard state variable form

$$\underline{\dot{x}} = A\underline{x} + B\underline{u}$$

$$\text{i.e. } \begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} 0 & w_e & -u_e & 0 & 0 \\ l_v & l_p & l_r & 0 & 0 \\ n_v & n_p & n_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} v \\ p \\ r \\ \phi \\ \Psi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \qquad (16)$$

where $E_{xz} = 1 + e_x e_z$

$$l_v = \left\{\frac{\mathring{l}_v}{E_{xz}} + \frac{e_x \mathring{n}_v}{E_{xz}}\right\}; \quad l_p = \left\{\frac{\mathring{l}_p}{E_{xz}} + \frac{e_x \mathring{l}_p}{E_{xz}}\right\}; \quad l_r = \left\{\frac{\mathring{l}_r}{E_{xz}} + \frac{e_x \mathring{l}_r}{E_{xz}}\right\};$$

$$n_v = \left\{\frac{-e_z \mathring{l}_v}{E_{xz}} + \frac{\mathring{n}_v}{E_{xz}}\right\}; \quad n_p = \left\{\frac{-e_z \mathring{l}_p}{E_{xz}} + \frac{\mathring{n}_p}{E_{xz}}\right\}; \quad n_r = \left\{\frac{-e_z \mathring{l}_r}{E_{xz}} + \frac{\mathring{n}_r}{E_{xz}}\right\};$$

$$\text{and } l_\xi = \left\{\frac{\mathring{l}_\xi}{E_{xz}} + \frac{e_x \mathring{n}_\xi}{E_{xz}}\right\}; \quad l_\zeta = \left\{\frac{\mathring{l}_\zeta}{E_{xz}} + \frac{e_x \mathring{n}_\zeta}{E_{xz}}\right\}$$

$$n_z = \left\{\frac{-e_z \mathring{l}_\xi}{E_{xz}} + \frac{\mathring{n}_\xi}{E_{xz}}\right\}; \quad n_\zeta = \left\{\frac{-e_z \mathring{l}_\zeta}{E_{xz}} + \frac{\mathring{n}_\zeta}{E_{xz}}\right\}$$

The corresponding output equation $\underline{y} = C\underline{x} + D\underline{u}$ is given by

$$\underline{y} = \begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \Psi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \xi \\ \zeta \end{pmatrix}$$

6.0 ASSESSMENT OF THE PRESENT RESEARCH STAGE

(i) An on-line literature search of relevant data bases has been performed and a good proportion of the references found have been obtained for evaluation.

(ii) The Hawk Model has been refurbished by the workshop at the College of Aeronautics.

(iii) The servo-mechanism with which it is hoped to measure h and $\dot{h}$ is in development and good progress has been made to date.

(iv) The entire electrical and electronic wiring of the test facility has recently been revised and brought up to date as part of a MSc. project. The result of this exercise is that the system is now fully documented. Development of the vertical motion sensor continues.

(v) Representative small perturbation equations of motion for the semi-free flight model aircraft have been established.

6.1 SHORT TERM GOALS

There are two main short term goals at present. The first of these is to develop the interface software on the IBM-PC so that the CED1401 analogue-to-digital converter may be used. Secondly, it is intended to completely recalibrate the Hawk model and the control system in the near future.

It is also planned to start the development of two separate computer programs. These being the aircraft simulation using the Advanced Continuous Simulation Language (ACSL) and the parameter identification software. The development of these programs are long term goals which will run throughout the first year of this research project.

7.0 CONCLUSIONS

The final mathematical model produced by running the MSR parameter software may be verified in many ways. Checks on the accuracy of estimated parameters and the prediction qualities, through the use of the ACSL simulation, are considered as the most convenient way to verify the model. The parameter estimates will also be checked against theoretical predictions and known data already published on the BAe Hawk stability and control derivatives It is further hoped to perform comparative studies with previous CoA work and possibly to use other estimation software programs if these can be made available.

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