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The Analysis of Reinforced Circular and Elliptical Cutouts under Various Loading Conditions

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SUMMARY

The effect of reinforced cutouts in a plane sheet under various loading conditions is considered, and a number of experimental results are given for circular and elliptical cutouts with a uniform plate reinforcement, subjected to various systems of biaxial tension and pure shear.

These experiments were conducted using a plane loading frame, and the results are compared with the theoretical plane stress solution. For the circular cutout the effect of neglecting the bending stiffness of the reinforcement is considered.

Some additional experiments were carried out on a 60 in. diameter pressurised cylinder containing an elliptical hole reinforced according to Mansfield's neutral hole theory. The strains in the sheet in the region of the neutral hole are compared with the corresponding strains in the uncut sheet.

The experimental results obtained generally show a good agreement with the theory.

CONTENTS

NOTATION

ϕ	Airy stress function
r, θ	polar co-ordinates
σ_r	radial stress
σ_θ	tangential stress
$\tau_{r\theta}$	shear stress
ϵ_r	radial strain
ϵ_θ	tangential strain
a	inner radius of reinforcement for circular cutout
b	outer radius of reinforcement for circular cutout
m	$= a/b$
t	thickness of sheet
n	ratio of reinforcement thickness to sheet thickness
f_t	tensile stress at infinity
f_s	shear stress at infinity
$A_1, A_2, C_2, c_1, d_1, B_1, F_1$	constants in solution for reinforced circular cutout
x, y	rectangular co-ordinates
p, q, c	arbitrary constants
f	tensile stress
A	cross-sectional area of reinforcement
T	tension in reinforcement
R	$= \frac{a_o + b_o}{2}$ for elliptical cutout, or radius of circular cutout
a_o	semi-major axis of ellipse
b_o	semi-minor axis of ellipse

Notation (Continued)

Q	critical stress combination
β	angle in plane of ellipse
ν	Poisson's ratio
λ	stress concentration factor

1. Introduction

An exact solution for a circular cutout having a plate reinforcement of uniform cross-section in an infinite plane sheet has been given by Gurney[1] who made use of the real stress function. For other shapes of cutout the method of conformal transformation may be employed, by making use of the complex stress function, developed by Muskhelishvili[6]. In a previous paper[4] this method has been applied to the solution of unreinforced cutouts of various shapes, and a number of experimental results have been obtained which show good agreement with the theory.

Wittrick[2,7] has extended the method to the solution of reinforced elliptical cutouts and cutouts of other shapes for which the transformation function can be expressed as a simple polynomial function. The assumption is made that the cutout has a compact reinforcing member concentrated at its edge, and the bending stiffness of the reinforcement is neglected.

Mansfield[3] has extended the use of the real stress function to investigate the problem of the neutral hole, that is, a hole of such a shape and distribution of reinforcement that the stresses in the sheet are unaffected by the presence of the hole. Again the assumption is made that the reinforcement has negligible bending stiffness.

In nearly all of the theoretical work, it is assumed that the problem can be reduced to that of a cutout in an infinite flat plate. When the cutout is in a flat panel it may be assumed that the effect of the boundary of the panel is negligible provided that the ratio of cutout dimension to panel dimension is not greater than $\frac{1}{4}$. The plane stress solution may be applied to problems of cutouts in curved shells provided that the dimensions of the cutout are small compared with the radius of curvature of the shell. The results may therefore be applied for example to cutouts for windows or doors in an aircraft pressure cabin, or to problems of cutouts in nuclear reactors.

However, there is little experimental evidence recorded to support the theoretical solutions for reinforced cutouts, although both Manfield[3] and Richards[8] have carried out some experimental work using plastic material.

In this paper, some experimental results[5] are given for circular and elliptical cutouts having a symmetrical plate reinforcement of uniform cross-section. The tests were carried out in a plane loading frame, using aluminium alloy panels, which enabled various systems of biaxial tension and pure shear to be applied. The experimental results are compared with the infinite flat plate theory, and for the reinforced circular cutout a comparison is made with both the theoretical solutions of Gurney and Wittrick in order to investigate the effect of the bending stiffness of the reinforcement on the stresses in the sheet.

Some additional tests were carried out on a 60 in. diameter pressurised cylinder containing an elliptical cutout which was reinforced in accordance with Mansfield's netral hole theory. The strains in the sheet in the region of the neutral hole are compared with the strains in the uncut sheet.

2. Experimental Work

Some preliminary experimental work was carried out on reinforced circular and elliptical cutouts in a pressurised cylinder. This had previously been used for tests on unreinforced cutouts, and is identical to that described in Ref. 4.

The cylinder was 60 in. diameter and 72 in. long, constructed from 18 s.w.g. (0.048 in.) L72 aluminium alloy sheet. The cylinder was pressurised using air to give a nominal hoop stress of the order of 10,000 lb/in^2, and was mounted on a trolley at one end to permit free longitudinal expansion under pressure to take place. The air pressure was contained by a plate, rolled to the contour of the cylinder, and shaped to fit accurately into the cutout. This plate was supported externally so that the transverse pressure loading would not be reacted at the boundary of the cutout.

Flat cast iron bulkheads were used for the ends of the cylinder, and these were designed to withstand the pressure load and in addition to act as suitable loading points for combined pressure/torsion and pressure/axial tension tests which it had been originally intended to carry out.

A number of tests were carried out on reinforced circular and elliptical cutouts in the pressure cylinder, having maximum dimensions not greater than about 8 in. However, because of the considerable experimental difficulties arising, it was decided to use the preliminary results obtained on the pressure cylinder to confirm the use of flat plate theory, and to carry out the subsequent work (with the exception of the neutral hole tests) using a plane loading frame (Figs. 11 and 12). This enabled the use of stress ratios other than the 2:1 biaxial pressure stresses, and in addition the effect of combined shear and direct loading could be examined if required.

The plane loading frame had also previously been used for tests on unreinforced cutouts, and is fully described in Ref. 4. The panels used in the plane loading frame were 28 in. square, and made from 16 s.w.g. (0.064 in.) L72 aluminium alloy sheet. It was found that provided the maximum dimension of the cutout in the panel did not exceed about 5 in. the effect of the panel boundary would be negligible, and in this way reasonable comparison should be obtained with the infinite flat plate theory.

The panel was loaded through heavy edge members by means of two 3000 lb. turnbuckles, used in conjunction with calibrated dynamometers. The maximum direct stress in the panel was 1000 lb/in^2, but the shear stress was restricted by panel buckling to 700 lb/in^2.

Tests were carried out, using the plane loading frame, on a circular cutout and a $\sqrt{2:1}$ elliptical cutout, having a constant section reinforcement. In the case of the circular cutout, the reinforcement was chosen so that the weight of the reinforcement added was equal to the weight of the sheet removed by the cutout. For the elliptical cutout, the cross-sectional area of reinforcement added corresponded to the minimum area of reinforcement (i.e. at the minor axis) for a $\sqrt{2:1}$ elliptical neutral hole of the same dimensions.

The cutouts were reinforced by doubler plates of uniform width symmetrically placed each side of the sheet, and attached to the sheet by Araldite 901 cold setting adhesive. Under applied loading this proved to be an adequate joint.

The tangential strains in the sheet were measured around the cutout at the junction of the reinforcement, under various systems of biaxial tension and pure shear. The experimental results are compared with the theoretical plane stress solutions in Figs. 1 - 4. In addition the tangential strains in the sheet were measured along radial lines at certain points around the cutout, to show the reduction in tangential strain away from the junction of the reinforcement.

Tinsley type 6H electrical resistance strain gauges were used in conjunction with a Savage and Parsons recorder. To obtain strain measurements as close as possible to the junction of the reinforcement, the backing paper of the strain gauge was trimmed, and it was found that measurements could effectively be made at a distance of 0.15 in. from the junction. The dimensions of the circular and elliptical cutouts and the strain gauge positions are shown in Figs. 13 and 14 respectively.

Some further experimental work was carried out on a neutral hole, using the 60 in. diameter cylinder which was pressurised by air to a nominal stress of 10,000 lb/in^2. A $\sqrt{2}:1$ elliptical cutout was made in the cylinder, and this was reinforced by plates placed symmetrically each side of the sheet, and attached to the sheet by 6 B.A. bolts placed at 1.2 in. pitch. The width of the reinforcing plate was varied so that the distribution of reinforcement around the cutout was in accordance with Mansfield's neutral hole theory.

Measurements of radial and tangential strains were made on radial lines at certain points around the cutout, and these strains were compared with the strains previously measured at corresponding points on the sheet before the cutout was made, and compared also with the theoretical strains. The dimensions of the neutral hole and the strain gauge positions are shown in Fig. 15. Tinsley type 6H strain gauges were again used, with the Savage and Parsons recorder.

3. Theoretical Considerations

3.1. Reinforced Circular Cutouts (Fig. 16)

The solution to the problem of a circular cutout with a uniform plate reinforcement in an infinite plane sheet under various loading conditions has been given by Gurney[1]. It is assumed that the reinforcement is symmetrically placed each side of the sheet, and the effect of the abrupt change in thickness at the junction of the reinforcement and sheet has been neglected. Apart from this, if it is assumed that a linear stress-strain law exists for the material, then the solution given is exact.

Gurney uses the real stress function ϕ of the plane theory of elasticity satisfying, in the absence of body forces, the biharmonic equation

$$\nabla^4 \phi = 0 .$$

A general form of solution is obtained, and this is used to satisfy the boundary conditions at the inner and outer edges of the reinforcement and at infinity.

The solution gives the stress distribution in the sheet and in the reinforcement when the cutout is subjected to either equal biaxial tensions or pure shear, and by super-position of these two cases any system of biaxial tension and shear may be obtained.

Gurney has assumed a value for Poisson's ratio $\nu = \frac{1}{4}$ and uses this value in the numerical work. The solution may be obtained in a more general form for any value of ν, as follows:

(a) Solution for equal biaxial tension f_t.

The stresses in the plate are given by

$$\sigma_r = f_t \left(\frac{A_1}{r^2} + 1 \right) ,$$

$$\sigma_\theta = f_t \left(- \frac{A_1}{r^2} + 1 \right) ,$$

and $$\tau_{r\theta} = 0 ,$$

where $$A_1 = \frac{(1 - \nu)(1 - m^2)n - (1 + \nu)m^2 - (1 - \nu)}{(1 + \nu)(1 - m^2)n + (1 + \nu)m^2 + (1 - \nu)} \cdot b^2 ,$$

and a is the radius of the cutout (inner radius of reinforcement), b is the outer radius of the reinforcement, n is the ratio of total thickness of reinforcement to thickness of sheet, and m is defined by

$$m = \frac{a}{b} .$$

The stresses in the reinforcing ring are given by

$$\sigma_r = f_t \left(\frac{A_2}{r^2} + 2 C_2 \right) ,$$

$$\sigma_\theta = f_t \left(- \frac{A_2}{r^2} + 2 C_2 \right) ,$$

and $$\tau_{r\theta} = 0 ,$$

where $$A_2 = \frac{- 2 m^2}{(1 + \nu)(1 - m^2)n + (1 + \nu)m^2 + (1 - \nu)} \cdot b^2 ,$$

and $$C_2 = \frac{1}{(1 + \nu)(1 - m^2)n + (1 + \nu)m^2 + (1 - \nu)}$$

The values of the constants A_1, A_2 and C_2 have been calculated in Table 1 for Poisson's ratio $\nu = \frac{1}{3}$ for a range of values of m and n.