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Multiobjective Optimisation of Restricted Complexity Controllers

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Abstract

Restricted complexity controller design for the active suspension benchmark problem [1] is considered. The control design specifications of the benchmark is firstly recast into a mixed H_∞/l_1 optimisation problem, which is solved via a convex optimisation approach. A globally optimal Pareto curve is produced via the optimisation. It presents the limits of performance and is served as a reference for restricted complexity controller design. Then, controllers with different complexity are designed via a direct optimisation approach. The performance indices of these controllers are compared with the global Pareto curve. Based on the comparison, the controller with three parameters is determined as the one achieving acceptable performance with lowest complexity. Experimental results on the real system confirm the satisfactory performance achieved by the controller.

1 Introduction

In recent decades, numerous control synthesis approaches have been proposed in the literature. Sophisticated theories supporting these approaches are well developed. Many approaches have been successfully

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applied to different engineering systems. In spite of the progress, design and optimisation of restricted complexity controller, *i.e.* controller with less complexity than the plant model), remains a challenging problem for control community. In a wider sense, restricted complexity controllers could include fixed-order controllers, such as PID controllers, decentralized controllers and static output feedback controllers. Such controllers are preferred in most practical systems due to their low cost (economically and computationally), flexibility, robustness, easy maintenance and transparency for understanding.

Most control specifications, such as H_∞, H_2 and l_1 norms and their combinations, are convex in terms of closed-loop transfer functions [2]. Therefore, modern control synthesis approaches are convex optimisation based and lead to globally optimal solutions, which determine the limits of performance achievable with control systems. However, these convex optimisation approaches usually lead to complexity of controllers higher than that of plant models. To overcome this deficiency, two routes can be taken: reducing the model complexity (model reduction) before control design and/or reducing the controller complexity (controller reduction) after it being designed. However, both approaches have their own problems. The model reduction approach will introduce more uncertainties into the model, hence, usually leads to results more conservative. In the second route, the objective to performing controller reduction is totally different from the one used for controller design. Therefore, the global optimality gained at design stage could completely disappear even with sophisticated closed-loop performance preserving techniques being applied.

Apart from the above two pathes to get restricted complexity controllers, it can also be done via direct optimisation. This approach has been known in the literature for many decades. In 1953, Graham had introduced several performance criteria for control system design and presented a way to use an analog computer to get optimal closed-loop systems in standard forms [5]. Due to the recently rapid development of computing techniques, nonlinear optimisation has found wide applications to control synthesis problems, for examples, the PID optimal tuning for nonlinear systems [7] and commercial software for nonlinear control design using optimisation [6]. The main advantage of a direct optimisation approach is its wide applicability. Almost any control synthesis problem can be recast in a nonlinear optimisation one. Also, for any synthesis approach, if the global optimality of the results is not guaranteed, direct optimisation can always be applied at the end to improve the design. Moreover, the resulting controller complexity is independent from the complexity of plant model in a direct optimisation approach. Hence, more accurate model could be considered to reduce model uncertainty so that the design conservativeness could be reduced as well. It is true that the direct optimisation approach is generally not convex. Therefore, the solutions obtained might only be local optima. However, even the local optima can also provide certain information about the limits of performance, *i.e.* they are the lower bound of the achievable performance limits, particularly for restricted complexity controllers. For any controller if it can achieve better performance than the local optima, then it is well designed. Otherwise, it has a space for improvement.

In this work, a direct optimization approach to design parameters of a fixed-order controller is adopted for the active suspension benchmark problem. To compromise the drawback of non-convexity of the direct optimisation, a global optimization approach is firstly developed to produce a Pareto curve, which reveals the performance limits for any controller. Then a direct optimisation approach is developed and solved for several controllers with different complexity. The multiobjective performance achievable by these controllers is checked with the global Pareto curve. Based on this comparison, the lowest complexity controller which achieves acceptable performance is determined. Experimental results from the real system confirm the succusses of the design. The paper is organized as follows. A brief description of the active suspension system is provided in section 2. Section 3 presents a convex optimisation approach, which leads to the globally optimal Pareto curve for determination of performance limits. In section 4, a direct optimisation approach is described and several restricted complexity controllers have been obtained. The lowest-order controller is determined by performance comparison with the Pareto curve. The lowest-order controller is evaluated in section 5 and the paper closes with some concluding remarks in section 6.

2 Active suspension problem

The active suspension system is modelled in two discrete-time transfer functions, the primary path transfer function, $G_p(q) = \frac{C(q)}{D(q)}$, which is the transfer function from the primary force (disturbance), u_p to the residual force (output), y and the secondary path transfer function, $G(q) = \frac{B(q)}{A(q)}$, which represents the secondary path from the control input, u to the output, y. Here and in the rest of the paper, $q = z^{-1}$ and $f_s = 800$ [Hz] denotes the sampling frequency of the system. The block-diagram of the system is shown in Figure 1. For details of the system, see [1].

The overall goal is to design a linear discrete-time controller with restricted complexity, $K(q) = \frac{R(q)}{S(q)}$ from the residual force, y to the control input, u so that the effect of disturbance on the residual force is attenuated. More precisely, given the closed-loop block diagram shown in Figure 1, define closed-loop output and input sensitivity functions, $S_{yp}(q)$ and $S_{up}(q)$, respectively:

$$S_{yp}(q) = \frac{1}{1+KG} = \frac{A(q)S(q)}{P(q)} \tag{1}$$

$$S_{up}(q) = -\frac{K}{1+KG} = -\frac{A(q)R(q)}{P(q)} \tag{2}$$

where, $P(q) = A(q)S(q) + B(q)R(q)$.

The control objective can be stated as follows:

1. The controller should have a zero at $z = -1$ so that the gain is equal to zero at the frequency of $f_s/2$, where f_s is the sampling frequency.

2. The input and output sensitivity transfer functions should satisfy the constraints, shown in Figures 2(a) and (b), i.e. $|S_{up}| \leq C_{up}$ and $|S_{yp}| \leq C_{yp}$ at all frequencies.

Apart from the above constraints, the system also may potentially have an input saturation problem. Since this requirement is not clearly stated in the benchmark specification, it is treated as a soft requirement, i.e. to satisfy the sensitivity constraints at the same time to minimise the input magnitude. The sensitivity specifications of the benchmark problem can be converted into standard H_∞ norm constraints:

$$\|S_y(\omega)\|_\infty = \max_\omega \left| \frac{S_{yp}(e^{-j\omega/f_s})}{C_{yp}(\omega)} \right| \leq 1 \tag{3}$$

$$\|S_u(\omega)\|_\infty = \max_\omega \left| \frac{S_{up}(e^{-j\omega/f_s})}{C_{up}(\omega)} \right| \leq 1 \tag{4}$$

Define $T_{u,up}$ the closed-loop transfer function from the disturbance (input of the primary path) to the input,

$$T_{u,up}(q) = S_{up}(q)G_p(q) = \frac{A(q)R(q)C(q)}{P(q)D(q)} \tag{5}$$

Then input magnitude is bounded by the l_1 norm of $T_{u,ud}$, $|u| \leq \|T_{u,up}\|_1 |u_p|$. Hence, the benchmark can be recast as a l_1 optimisation problem with H_∞ constraints.

$$\begin{aligned} J &= \min_{K(q)} \|T_{u,up}\|_1 \qquad (6) \\ \text{s.t.} \quad & \|S_y(\omega)\|_\infty \leq 1 \\ & \|S_u(\omega)\|_\infty \leq 1 \\ & K(-1) = 0 \end{aligned}$$

3 Convex optimisation approach

In the light of standard l_1 design techniques [4], the l_1 minimisation with H_∞ constraints problem can be solved via a convex approach. Let the impulse response model of $T_{u,up}$ be

$$T_{u,up}(q) = \sum_{k=0}^{\infty} h_k q^k = \sum_{k=0}^{\infty} (h_{pk} - h_{nk}) q^k \tag{7}$$

where $h_{pk} \geq 0$ and $h_{nk} \geq 0$ are the positive and negative branches of h_k, respectively. Then the l_1 norm of $T_{u,up}$ can be represented as:

$$\|T_{u,up}\|_1 = \sum_{k=0}^{\infty} |h_k| = \sum_{k=0}^{\infty} (h_{pk} + h_{nk}) \tag{8}$$

Numerically, the upper bound of the infinite summation can be replaced by a sufficient large number, N. Moreover, the l_1 norm minimisation is equivalent to the following problem:

$$\begin{aligned} \min_{\alpha, h_{pk}, h_{nk}} \quad & \alpha \\ \text{s.t.} \quad & \sum_{k=0}^{N} (h_{pk} + h_{nk}) \leq \alpha \end{aligned} \tag{9}$$

Once $T_{u,up}$ is determined, closed-loop sensitivities can be derived as follows:

$$S_{up} = T_{u,up}/G_p \tag{10}$$

$$S_{yp} = 1 - S_{up}G \tag{11}$$

Hence the sensitivity constraints can be evaluated accordingly. To ensure closed-loop stability, any nonminimum zero of G_p should be included in $T_{u,up}$, *i.e.* for any z satisfies $G_p(z) = 0$ and $|z| > 1$,

$$T_{u,up}(z^{-1}) = \sum_{k=0}^{N} (h_{pk} - h_{nk}) z^{-k} = 0 \tag{12}$$

According to the first specification, $K(-1) = 0$, so as $S_{up}(-1) = 0$ and

$$T_{u,up}(-1) = \sum_{k=0}^{N} (h_{pk} - h_{nk})(-1)^k = 0 \tag{13}$$

For the benchmark problem, the impulse response length of $T_{u,up}$ is set to $N = 50$. The global optimum is obtained and the minimal l_1 norm is 0.8864. Furthermore, by changing the maximal violation level of the output sensitivity constraint, several minimal l_1 norms of $T_{u,up}$ are obtained by solving the convex optimisation. The Pareto curve based on these results is shown in Figure 3. As described in [2, 3], the Pareto curve determines the achievable limits of performance trade-off and will be used in the restricted complexity controller design in the following section.

4 Direct optimisation approach

Since the input saturation is not an explicit constraint of the benchmark problem, it is ignored for low-order controller design to benefit the sensitivity performance. However, the controller designed will be compared with the Pareto curve to ensure both sensitivity and l_1 norm are satisfied.

The objective function for low-order controller design is the total violation of both sensitivity constraints. This cost is to be minimised subject to closed-loop stability, *i.e.* all closed-loop poles should have their moduli less than one. Let

$$K(q) = \frac{R(q)}{S(q)} = \frac{(r_0 + r_1 q + \cdots r_{m-1} q^{m-1})(1+q)}{1 + s_1 q + \cdots s_n q^n} \tag{14}$$

Then the direct optimisation problem is setup as follows.

$$\begin{aligned} \min_{r_0,\ldots,r_{m-1},s_1,\ldots,s_n} \quad & \sum_{k=1}^{400} (J_1(k) + J_2(k)) \\ \text{s.t.} \quad & |p| < 1, \forall p \in \{p | P(p^{-1}) = 0\} \end{aligned} \tag{15}$$

where

$$\begin{aligned} J_1(k) &= \begin{cases} |S_{up}(q_k)| - C_{up}(k), & \text{if } |S_{up}(q_k)| > C_{up}(k) \\ 0 & \text{otherwise} \end{cases} \\ J_2(k) &= \begin{cases} |S_{yp}(q_k)| - C_{yp}(k), & \text{if } |S_{yp}(q_k)| > C_{yp}(k) \\ 0 & \text{otherwise} \end{cases} \\ q_k &= e^{-j2\pi k/f_s} \end{aligned}$$

The above constrained nonlinear optimisation problem is solved for $2 \leq m+n \leq 8$. Controllers with the same complexity (number of parameters) are compared to determined the one which has the minimal cost. The determined controllers are compared with the Pareto curve for the maximal output sensitivity violation (all controllers have nearly zero input sensitivity violation) against the l_1 norm of $T_{u,up}$. The results shown in Figure 3 indicate that controllers with complexity ($n+m$) of 2, 4 and 5 are too far away from the limits, whilst controllers with complexity of 6, 7 and 8 are close to the end where $\|T_{u,up}\|$ is too large. Therefore, the one with $m = 1$ and $n = 2$ is selected as the low complexity controller with acceptable performance. The controller designed is as follows.

$$K(q) = \frac{0.0202(1+q)}{1 - 0.6747q - 0.3164q^2} \tag{16}$$

5 Controller evaluation

The controller is evaluated in two ways. Firstly, it is evaluated via simulation using the nominal models, G and G_p provided by the benchmark problem. The simulation system is connected as shown in Figure 1. The disturbance signal u_p is the same as the one provided in the benchmark package. Then both sensitivity functions are estimated via the spectral analysis approach *i.e.* by applying the utility function provided in the benchmark package to the simulation data recorded.

The second evaluation is done by performing five experimental tests on the real system. The sensitivity functions are estimated using the experimental data in the same way as described above. The sensitivity functions estimated from the simulation and from one of the five experimental tests are shown in Figures 4 (a) and (b) respectively. From the figures it can be seen that the input sensitivity constraint is perfectly satisfied (only very small violation around frequency 230 [Hz] for the experimental results). The output sensitivity performance has only minor violation, which is mainly around frequency 225 [Hz] for the experimental results, and a minor shift at low frequencies for both simulation and experimental results. Hence, The overall performance is satisfactory. The total violation of the simulation results and all experimental results are summarised in Table 1.

6 Conclusions

The active suspension benchmark problem has been examined using multiobjective optimisation approaches. A convex optimisation problem has been formulated to provide a global Pareto curve to the benchmark problem. The Pareto curve provides a reference for further low complexity controller design and also reveals the limits of performance trade-off between control effort and sensitivity constraints. Low complexity controllers are designed using a direct optimization approach. The best low-order controller is determined via comparison with the Pareto curve. The satisfactory performance of the controller is confirmed with simulation and experimental results.

References

[1] Call for papers EJC special issue on *design and optimisation of restracted complexity controllers*. web site `http://www-ejc.ensieg.inpg.fr/`, 2001.

[2] S. P. Boyd and C. H. Barratt. *Linear Controller Design, Limits of Performance*. Prentice Hall Information and System Sciences Series. Prentice Hall, Englewood Cliffs, NJ, 1991.

[3] Y. Cao and Z.J. Yang. Multiobjective controllability analysis. In J. Grievink and J. van Schijndel, editors, *European Symposium on Computer Aided Process Engineering – 12*, volume 10 of *Computer-Aided Chemical Engineering*, pages 451–456, The Hague, The Netherlands, May 2002. Elsevier.

[4] M. A. Dahleh and I. J. Diaz-Bobillo. *Control of Uncertain Systems, A Linear Programming Approach*. Prentice Hall, Englewood Cliffs, New Jersey, 1995.

[5] D. Graham and R.C. Lathrop. The synthesis of optimum transient response: Criteria and standard forms. *AIEE Transactions*, 72(2):278–288, 1953.

[6] The MathWorks Inc. *Nonlinear Control Design Blockset for Use with Simulink – User's Guide*. 3 Apple Hill Drive, Natick MA 01760-2098, 1.1.6 edition, July 2002.

[7] G.P. Liu and S. Daley. Optimal-tuning of nonlinear PID control of hydralic systems. *Control Engineering Practice*, 8:1045–1053, 2000.

Table 1: Total sensitivity violation

Test	Violation of S_{up}	Violation of S_{yp}	Total Violation
Simulation	1.0587	25.328	26.2866
Experiment 1	7.1911	23.6811	30.8792
Experiment 2	6.4670	22.5461	29.0131
Experiment 3	6.1765	22.22	28.3965
Experiment 4	5.5755	21.5918	27.1673
Experiment 5	7.1004	22.7406	29.8410

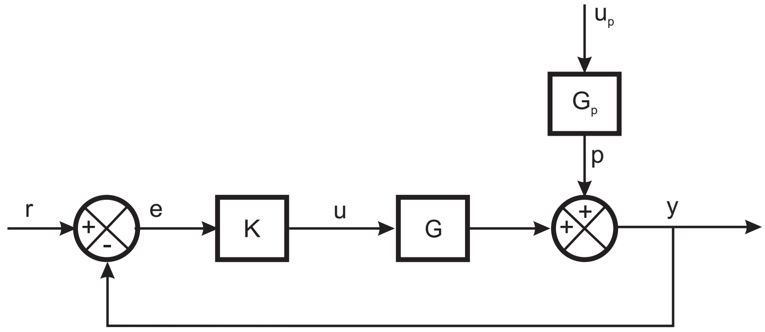

Figure 1: Active suspension control diagram

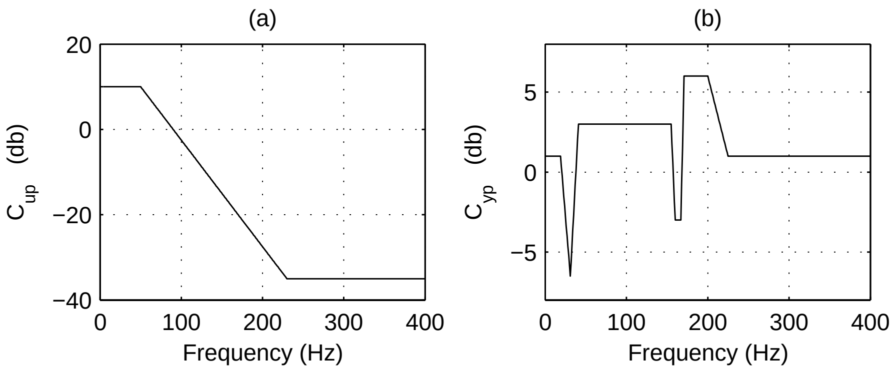

Figure 2: Constraints on closed-loop (a) input sensitivity function, (b) output sensitivity function.

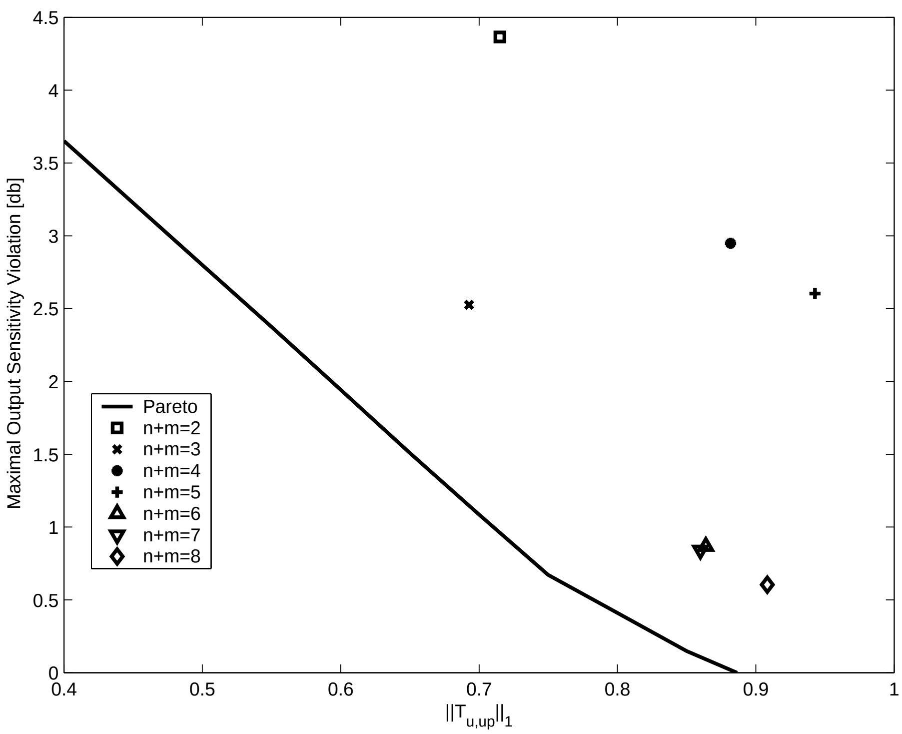

Figure 3: Globally optimal Pareto curve and performance of low-complexity controllers

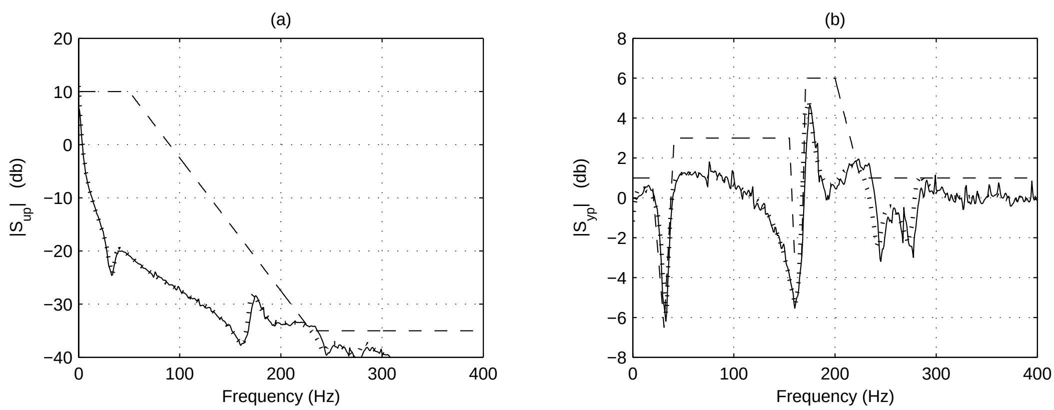

Figure 4: Controller evaluation (a) input sensitivity function, (b) output sensitivity function. In both figures, solid lines are experimental results, dotted lines are simulation results and dashed lines are specifications.