THE DETECTION & QUANTIFICATION OF CHAOS IN SUPPLY CHAINS

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ABSTRACT

In recent years it has become accepted that Logistics and Supply Chain systems are susceptible to uncertainty by the generation of deterministic chaos [Wilding, 1998a; Levy, 1994; Mosekilde & Larsen, 1988]. In this paper an explanation of a methodology for detecting and quantifying deterministic chaos within measured supply chain data is discussed. The paper describes the use of Lyapunov exponents [Peitgen, Jurgens, & Saupe, 1992; Wolf, 1986] and how these can be used to determine the average predictability horizon of a chaotic system [Wilding, 1997b]. This can then be used as a method of quantifying the amount of uncertainty from chaos within a system. The magnitude of the Lyapunov exponent gives a reflection of the time scale over which the dynamics of the system are predictable, so the exponent can be used to approximate the average prediction horizon of a system [Wolf et al., 1985; Shaw, 1981]. After this prediction horizon has been reached the future dynamics of the system become unforecastable. This occurs because any cause and effect relationship between current data and previous data becomes increasingly blurred and is eventually lost.

INTRODUCTION

When detecting chaos within measured data one is looking for the key characteristics of the following definition.   The definition used in this work is adapted from that proposed by Kaplan and Glass [Kaplan & Glass, 1995 p.27] and Abarbanel [Abarbanel, 1996 p.15]:

Chaos is defined as aperiodic, bounded dynamics in a deterministic system with sensitivity dependence on initial conditions, and has structure in phase space.

The key terms can be defined as follows:
- **Aperiodic;** the same state in never repeated twice.
- **Bounded;** on successive iterations the state stays in a finite range and does not approach plus or minus infinity.
- **Deterministic;** there is a definite rule with no random terms governing the dynamics.

- **Sensitivity to initial conditions;** two points that are initially close will drift apart as time proceeds.
- **Structure in Phase Space;** Nonlinear systems are described by multidimensional vectors. The space in which these vectors lie is called phase space (or state space). The dimension of phase space is an integer [Abarbanel, 1996]. Chaotic systems display discernible patterns when viewed [Stacey, 1993a; Stewart, 1989].

CHAOS ANALYSIS METHODOLOGY

Figure 1 depicts the key stages of the analysis methodology used to detect and quantify deterministic chaos present within time series data generated within supply chains. The following paragraphs will give an overview of each stage.

Step 1 - Define the null hypothesis
Proving definitively the existence of chaos from observed data would require an infinite amount of data. This therefore requires a stochastic technique that will make use of an appropriate null hypothesis. This procedure means that one does not set out to prove the existence of chaos but to reject some other null hypothesis that implies chaos is not present. The procedure of hypothesis testing is widely used in statistical analysis and a similar approach can be used for determining whether chaos is present in data. Kanji [Kanji, 1993] describes a five step method for hypothesis testing.

Wilding [Wilding, 1997b] used a deterministic simulation to ensure that no uncertainty was present from external sources. However, a cautious approach was taken in the research and the possibility of unattributed "computer noise" becoming a major influence on the results is also to be investigated. This resulted in the following null hypothesis:

> *"The dynamics are linear with Gaussian white noise random inputs, or the dynamics are linear exhibiting periodic behaviour."*

Step 2 - Identify discriminating statistic to test the null hypothesis.
To demonstrate that any data is inconsistent with the null hypothesis a discriminating statistic needs to be selected. This quantity can be calculated for the measured data and also for a set of data that is known to be consistent with the null hypothesis. There are many discriminating statistics that can potentially be used in the analysis of chaotic systems however one of the most robust and commonly used is the Lyapunov exponent [Abarbanel, 1996; Kaplan & Glass, 1995; Peitgen, Jurgens, & Saupe, 1992; Sprott & Rowlands, 1995]. This quantifies sensitivity to initial conditions and can also differentiate between random, periodic and stable behaviour [Sprott & Rowlands, 1995]. The Lyapunov exponent enables a clear measure of "sensitivity to initial conditions" to be made which can be used for the calculation of average prediction horizons [Wilding, 1997a]. The average prediction horizon can then be used to quantify the uncertainty generated within the system. The magnitude of the exponent also differentiates between aperiodic data and stable or periodic data.

If the maximum exponent is negative the system is stable or periodic. If the value is zero the system is stable or periodic but may be close to bifurcation, i.e. the system is marginally stable [Peitgen, Jurgens, & Saupe, 1992; Wolf et al., 1985].

If the system has a positive Lyapunov exponent prediction may still be possible in the short term. The exponent can be used to give an indication how stable the system is and over what period of time a small error is magnified to a level that makes distinguishing it from the

original signal impossible.

Step 3 - Reconstruct the attractor
To reconstruct the attractor to which the measured data proceeds both geometric and algorithm based approaches are used.

The measured data from the simulation required the initial transients to be removed so the steady state data was used for reconstruction. Plotting the measured data against time and noting on the graph where any fluctuations settled down achieves this. A further check on the data to ensure no initial transients are present was the test for boundedness outlined in step 5 of the methodology.

Return maps [Kaplan & Glass, 1995 p.303; Gleick, 1987 p.143] were then produced, this gave a view of the overall structure of the attractor. This made it possible to distinguish between random, periodic and possible chaotic behaviour.

The time series was then reconstructed using time lag embedding techniques [Kaplan & Glass, 1995 p.308; Takens, 1981; Mane, 1981]. This was done to obtain an accurate embedding dimension that could then be used to obtain the discriminating statistic. Three techniques were used, enabling cross-checking and promoting confidence in the embedding dimension figure. The capacity and correlation dimension were calculated and also plotted against embedding dimensions [Grassberger & Procaccia, 1983; Sprott & Rowlands, 1995]. Using the Takens embedding theorem [Takens, 1981] values of embedding dimension were also calculated from the capacity and correlation dimensions. Percentage false nearest neighbours against embedding dimension graphs [Abarbanel, 1996 p.260] were also used to obtain values for embedding dimensions. Using these three techniques a robust approximation of the embedding dimension could be obtained despite only finite amounts of measured data being available.

Step 4 - Calculate the discriminating statistic.
Once the embedding dimension has been calculated the Lyapunov exponent can be calculated for the measured data. This gives a measure of sensitivity to initial conditions and also distinguishes between aperiodic, stable or periodic data.

Step 5 - Ensure "boundedness" of data.
Dynamics can be said to be bounded if the data stays within a finite range and does not approach infinity as time increases. However, to test for bounded stability, one would in theory, need to wait until time is equal to infinity. A related concept for assessing boundedness is that of "stationarity" [Kaplan & Glass, 1995 p.314]. A time series can be described as stationary when it displays "similar behaviour" throughout its duration. "Similar behaviour" is defined if the mean and standard deviation remain the same throughout the time series. Ensuring that the mean and standard deviation in one third of the time series is equal to that in the remaining two thirds can assess this. (One could use quarters or tenths if so desired.)

An alternative approach, which is more applicable to analysis of chaotic data, is splitting the measured data into two {for convenience these will be called data sets 2 and 3. Data set 1 is the original data with initial transients removed} and calculating the discriminating statistics for these data sets. If these values are of the same magnitude as each other and also data set 1 then one can be confident that the data is bounded.

If the data was found not to be bounded an investigation of whether all the initial transients had

been removed from the initial data was carried out.

Step 6 - Generate and analyse "surrogate" data.
"Surrogate data" is the method used for the generation of data consistent with the null hypothesis. The technique involves performing a Fourier transformation on the measured data, the phase of each Fourier component is set to a random value between 0 and 2 π, and then an inverse Fourier transformation is undertaken. This technique removes any deterministic relationships within the data but preserves the power spectrum and correlation function [Kaplan & Glass, 1995 pp.343 344; Sprott & Rowlands, 1995]. It therefore generates data consistent with the null hypothesis.

Once again, by calculating the discriminating statistic for the surrogate data sets one can generate a range of values for data consistent with the null hypothesis. Then by seeing if the discriminating statistic for the measured data falls within this range further evidence is provided of whether the measured data is consistent with the null hypothesis.

The discriminating statistic was then calculated for the surrogate data and compared with the value for data set 1. If these are of the same magnitude it indicates the data generated conforms to the null hypothesis.

Step 7 - Review evidence from steps 1 - 6.
By reviewing the evidence obtained from steps 1 to 6 it is possible to demonstrate whether the data fails to adhere to the null hypothesis and thus exhibits the properties of deterministic chaos i.e.: *"Chaos is defined as aperiodic, bounded dynamics in a deterministic system with sensitivity dependence on initial conditions, and has structure in phase space".*

If the systems investigated are shown to exhibit chaotic behaviour this will provide evidence that uncertainty within the supply chain may result from the internal processes used to control the system.

CALCULATING THE PREDICTION HORIZON

The magnitude of the Lyapunov exponent gives a reflection of the time scale over which the dynamics of the system are predictable, so the exponent can be used to approximate the average prediction horizon of a system [Wolf et al., 1985; Shaw, 1981]. After this prediction horizon has been reached the future dynamics of the system become unforecastable. This occurs because any cause and effect relationship between current data and previous data becomes increasingly blurred and is eventually lost. For a discrete system the Lyapunov exponents are measured in bits/iterations (where bits = binary digits).

If the Lyapunov exponent is known, an approximation of how far ahead the future behaviour is predictable can be made. This can be calculated by dividing the relative accuracy with which the two nearby points are specified by the Lyapunov Exponent [Wolf et al., 1985].

For example, if the system has a positive exponent of 0.75 and an initial point is specified with an accuracy of $1x10^6$ or 20 bits (To describe $1x10^6$ in binary code 20 bits are required i.e. 11110100001001000000 = $1x10^6$), then future behaviour would be unpredictable after 26 iterations (20/0.75). After this time the small initial uncertainty will cover the whole attractor requiring a new measurement of the system to describe its behaviour.

For example, if the inventory within a warehouse can be specified with an accuracy of $1x10^3$ i.e. 1000 +/- 1 units of inventory then this equates to an accuracy 1111101000 (10 bits), and the inventory control system behaves chaotically with a Lyapunov exponent of 0.15 bits/iteration.

Then time until accuracy can only be specified with accuracy of 1 bit or 1 unit of inventory is $10/0.15 = 67$ days. After this period of time the exponential increase in the error renders any methods of prediction invalid.

CONCLUSION

This paper has presented a robust methodology for the detection and quantification of chaos. The methodology has been successfully applied to the detection of chaos from simulated supply chains and has enabled the development of new management guidelines for supply chain re-engineering [Wilding, 1998a; Wilding, 1998b]. A more detailed discussion of the methods outlined in the paper can be found in [Wilding, 1997b].

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