The Displacement Fffect of Pitot Tubes in Narrow Wakes at Subsonic and Supersonic Speeds

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## SUATARY

The apporent displaconent of the effective centro of a circular pitot tube from its geometric centre whon placed in nerron vakes has been measured at sub- ond supersonic speeds. Similar effects were found at all spoeds. If the tube diometer was small compared with the wake width, the displacomont was towards tho reesion of higher velocity, and was propontional to the tube outside dimeter. For largor tubes the displacenent was roduced, and was reversed in direction whon the tube dianeter exceeded about trmee tines the wake rilith.

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16. Iist of Symbols
a. radius of cylinder (see hppendix)

D outside dianeter of pitot tube
a inside dianeter of pitot tube
$\mathrm{H}_{\mathrm{p}} \quad$ pitot pressure (neasured) in wake
$H_{0} \quad r e f o r e n c e ~ p i t o t ~ p r e s s u r e ~(m e a s u r e d) ~ o u t s i d e ~ w a k e ~$
1I liach number
$p_{s} \quad$ stagnation pressure ahead of pitot tube shock wave
$p_{0} \quad$ calculated freestrean stagnation pressure downstrean of trailing edge shock wave
$r, \theta \quad$ cylindrical polor coordinates
s transverse gradient of total head ratio $\frac{\partial H / H_{0}}{\partial y}$
uo freestrean velocity along cylinder axis (see ippenaix)
v apparent wake width
$w_{0}$ undisturbed wake width
W. maximum undisturbed wake width
$(x, y)$ rectangular cartesion coordinates
6 displacement of stagnation streanline
そ vorticity

It has been shom by Young and himas (1) that at low subsouic speeds the offective centre of a pitot tube in a transvorse gradient of total head is displaced fron the gometric centre of tho tube towards the region of higher velocity. Thoir results showed that over a fairly wide range of conditions this displacment was approximately equel to 0.18 D , where $D$ is the outside diameter of the pitot tube. It follows that the displaconent effect is smoll if the pitot tube diameter is small compared with the width of the region of varying total head. In genoral, thererore, in experimonts in wakes or boundary layors at subsonic speeds it is usual to select a pitot tube dianeter small compered with the wake width or boundary layer thickness.

At supersonic speeds, however, it may often be desirable to use a pitot tube whose size is large compared with the dimensions of the wake or boundary layer which it is traversing. This may be due either to the small size of model being used, or in intermittent wind tunnels where the long time lag associated with a small tube must be avoided. It is therefore important to know the magnitude, if any, of the pitot tube displacement effect in flows at supersonic speeds. IVeasurements with pitot tubes in the wake of an aerofoil at a liach nurber of 1.96 have been reported by Johanoson and Mair (2). These tests were conducted in the wake of a 0.75 in , span single vodge acrofoil, 2.5 in , downstream of the trailing edge. The outside diameter of the tubes used varioa from $0,036 i n$, to $0,720 \mathrm{in}$., but no displacement orifoct as found by Young and Taas could be detected.

The experiments described in this report were intended to investigate the displacement offect over a wide range of liach numbers and tube dianeters. For completeness, some tests having similar ratios of pitot tuoc diameter to mole winth were also carried out at a low subsonic speed. Chese lattor tosts extended the ronce of the Youns and Kaas tests to the case where the pitot fube diameters was of the same order ats the wase width.

The experimental results at supersonic speeds were obtained between 1953-1955 by Brown A.C. and Olds T. ( $\mathrm{H}=1.75$ ),
 ( $\mathrm{N}=3.19$ ), as a part fulfilment of the requirements for the Diploma of the College of Aeronautics.
4. Description of tests and apparatus
(i) Supersonic

These tests were conducted in the College of Aeronautics $2 \frac{1}{2}$ in. x $2 \frac{1}{2} i n$. supersonic intermittent wind tunnel at Mach numbers of $1.75,2.36$ and 3.19, In normal operation, air flows from a dry air bag at atmospheric pressure through the working section of the tunnel into vacuum tanks which are exhausted by continuous pumping. The normal available munning time varies from about 30 secs, to 90 secs, according to the Hach number. The stagnation pressure of the tunnel was approximately equal to atmospheric pressure, the small difference being measured by a Chattock gauge. The freestrean liach number was found by measuring the static prescure on the surface of a calibrating cone placed in the working section.

The dimensions of the pitot tubes testod at each Mach number are given in table I. The tubes were first tested in the empty working section to check that they $a l l$ gave the same reading. A two-dimensional symmetrical double wedge aerofoil of $\frac{1}{2} \mathrm{in}$. chord and thiclmess chord ratio ten per cent was then fitted in the tumel, and traverses wore made through its wake, $1 / 4 i n$, behind the trailing adge, The traverses were made in a direction perpendicular to the plane of the wing, by mounting each pitot tube on a sliding support moved by a microneter screw, A static pressure tube was also traversed behind the wing at the some position. At II $=3.19$, a similar wing of 1.75 in, chord was also used, in order to obtain a wider walce,

The pitot pressures were measured on a vertical mercury manometer, to an accuracy of about 0.01 in . of mercury. Corresponding readings of static pressure were taken from tappings in the top liner of the working section, A number of Schlioren photographs were taken at each llach number, and some of these at $\mathrm{M}=3.19$ are show in $\mathrm{figs} .15-18$.

## (ii) Iow smeed

Two sets of low speed tests were made, the first being at a wind speed of 59 f.p.s., using the pitot tubes whose dimensions are given in table II。 Traverses were made in the wake of a 2 in, chord symmetrical 10 per cent thick acrofoil, with the pitot mouth 1 in , from the trailing edge. The total head pressure was measured on an inclined tube manometor, and the traverses vere repeated several times in one direction only, to avoid errors due to backlash in the lead screw of the traversing gear. In these tests all the pitot tubes oxcept that of 32 mm , diameter were very small
comparod with the wake wiath.
The sucond sot of tests was made using very namror wakos, given by $1 / 16 \mathrm{in}$, and $1 / 32 \mathrm{in}$. thick flat plates of 1 in . chord at zero incidence. The wind speed was 50 f.p.s. and pitot tuives 2, 3, 4 and 9 (table II) were used.
5. Results

## (i) Sunorsonic

At each Mach number, static tube traverses were made across the working section in the wake of the aerofoil, and the static pressure was found to be constant within about $\pm 2$ per cent. The pitot tubes wore then tested in the empty working section, and found to give similar readings of total head, indicating that no errors due to viscous effects wore present. This might be expectod, as Shermon (3) gives $R e=200$ as the Reynolds number below which viscous effects on pitot tubes are important. In all these tests, the Roynolds mumber, based on conditions in front of the pitot tube shock wave, was preater than 1000 por m. The readings of total head obtained from the pitot tube traverses were expressed as a fraction $H_{p} / p_{0}$, where $H_{p}$ is the total head reading, and $p_{0}$ is the calculated stagnation pressure behind the trailing edge shook wave in an inviscid flow. Typical total head profilos across the woke at II $=3.19 \mathrm{are}$ shom in fig. 1. It will be seen that the apporent wake width at ony given value of $H_{\rho} / P_{0}$ increases as the tube diamotor is increased. Thus the effect of pitot size in this case is to displace the effective contre of the tube towards the region of lower velocity - an effect opposite to that found by Young and Maas at subsonic speeds,


As will be seen from the diagran above, this rise in total head occurs in the region behind the fan-shaped compression waves near the trailing edge. Since it is probable that the total head loss across this comprossion fon is slightly less than that across the single oblique shock wave further out, this would explain the higher measured totol head reading. This effect was not observed in the casc of the larger wing, where boundary layor separation, induced by the upstrean influence of the pitot tubes, occurred at about 60 per cent chord.

The results of the traverses behind the 1.75 in. chord wing at $\mathbf{H}=3.19$ are show in fig. 2. It will be scen that the displacement effect is in tho opposite direction from that found when using the smaller wing.

In figs. 5 to 8, the apparent rake width w , at vorious values of $\mathrm{H} / \mathrm{p}_{\mathrm{o}}$, is plotted against the oxtermal Aianeter of the pitot tubes the intornal dianeter boing conetent. The undisturbed wake widith, wo, at each valuc of total head, is given by extrapolating these curves back to zoro dianeter, and the displacement of the effective tube centre fron its gcometric centre, $\delta$, is equal to $\frac{i}{2}\left(w-w_{0}\right)$. On each of these graphs is also shown the estinated undisturbed maximum wale width, "Wo This is taken as the moximum width of the linear portion of the profile given by the smallest tube used. It may not be an accurate estimate of the interference free maximun wake width but is useful as a reforence longth in analysing the results.


The variation of displacenent with internal diametor was not investigated as such. In the results obtained for thioae of constant ratio $\mathrm{d} / \mathrm{D}$ of internal to external dianeter, nozt of the displaccment was accounted for by the veriation of axtomal dianeter, and the effect of internal dianeter for valuos of this ratio less than 0.5 was found to be negligible. For $\frac{d}{D}>0.5$, a slightly smallor displaccnent was round.

## (ii) Iow specd

411 the readings were expressca as ratios of indioatod total head to free strean total head, and typical profiles with different wakes are given in figs. 3 and 4 . Plots o? the apparent weke width against dianeter are shown in ligs. 9 and 10, together with an approximate value for the undisturbed moximun wake width $W_{0}$ (soo dofinition above). Tho enfsct of wouthe the intuzmi dianctor, with constant oursioe dismetor, is show in ties. 12, where wale wilths obtained in the first set of experinents with pitot tubes of 32m. outside diancter and inside dianeters of 1.7 mm . to 20m. are showm. It will be seen that as in the supersonic case, the intermal dianater hos negligible effect for $0.1<d / D<0.5$. For valuos of $d / D$ greater than 0.5 , the displacenent effect was slightly reduced. For vory snall inside dianeters, the displacenent effect was slightly increased.
6. Discussion
(i) Nature of the flow pattern

To clarify the physical picture, let us consider a pitot tube placed in an inviscid shear flow, such that the change in velocity across its diametor is small compared with the mean strean velocity. The vorticity present in the froestrean will give rise to a curvature of the streanlines in the vicinity of the tube and in particular to the stagnation

streanline which intersects the nouth of the tube. This stagnation streanline, therefore, doos not cone from a position for upstrean coaxial with the pitot tube, but fron a position displacod tomards the region of higher volocity.

Since no flow is possible dow the pitot tube a free stremine will exist across the nouth of tho tube. This must be approxinately 2 . Iine (or surface, in the threo-dimensional case) of constant prossure end volocity: the pressure is oqual to the total hoad of the stagnation strocmline, end the volocity is zero. Thus the flow past the pitot tube will be similar to that past a solid body of the sane outside dinensions. The inside dianeter therefore should have littlo effect on the external flow.

Then the pitot tube spans the contre of a wake, and is cotel wpon by regions of opposite vorticity, the probtion is obviousije thre complicated. In atterpt was nade to ootain a two-aimensional picture of such a flow in a smoke tunnel, but ony measureable displaconent of the stagnation stromline was masked by the rapid spreading of the flow near the tube nouth.

## (ii) Direction of Displacement

The low speed experinents of Young and Haas, and also the earlicr low speed tests made at Cranfield, showed that the displacoment effect was torvards the region of higher velocity, and equal in practically all cases to 0.18 D . It
appeared to be independent of total head and velocity gradient. On the other hand almost all the supersonic tests showed a displacement in the opposite direction, which varied considerably with total head and Mach numbor. (Except for the larger wing at $\mathrm{II}=3.19$, only one wake was used at each Mach number).
/It was ...

## 今

Since on theorotical grounds the displacement effect is a function of the sign of the vorticity ahead of the pitot tube it was suggested that the negative displacement effect at supersonic speeds might be associated with a change in sign of the vorticity of the flow in crossing the bow shock wave of the pitot tube. In fact it is found that for a plane shock wave, upstream of which the supersonic flow is of constant vorticity and static temperature, a reversal in sign of the vorticity downstream of the shock wave occurs for Wach numbers between 1 and 2. In cases where the totel energy, rather than the static temperature, is constant upstrean of tho shock wave, the vorticity is reduced in sign at all supersonic Mach numbers This might therefore explain the change in direction of the displacemont in the supersonic experinents. However, there is no evidence as to whether eithor the temperature oc total onergy is constant across the flow, and it is difficult to establish whether this revorsal in vortioity would take place across the curved bow wave. In any case, as the latcr oxperiments showed, a. reversal of displacement can also occur at subsonic speeds.

It was noted, however, that the earlior low speed. tests werc all made using tubes of smaller dioneter then the maxinum wake width, whereas the supersonic tests giving a reversed displacenent used very narrow wakes, and the tube diancters were then at least twice the wale width. The only large supersonic wake used, thet fron the $1.3 / 4 \mathrm{in}$. wing at $15=3.19$, gave a displacement in the same dircotion as the ecrlier low speed tests. It may be noted that boundery laver separation occurred on the larger wine at ii $=3.19$. No displacenont offect was apperent in the relatively flat central region of the wake but a positive effect was found in the regions of shear flow, (We shall coll this direction positive). Some further experinents were made at low speeds using very namow wakes and large tubes, and a nogative displacenent was found. The direction of displacenent appears therefore to be controlled, in our experinents, not by the strecm liach mumber, but by the size of tube relative to the walke viditho In figo 13 , the value of $\delta / D$ given by experjinent is plotted as a function of $D / T_{0}$, where $W_{0}$ is the undistuxbed naximun wake width. A logarithmic scalo has been used for $D_{i} /$ No $_{0}$, as those volues vory fron about 0,01 (Young and iiaas 0,014 in, tube) to 10 (Cranficld 6.4me tubc). It will bo seen that both tho subsonic and supersonic rosults, togethor with those of Young and Zioas, Iio roughly on a straight lino. Tubes for which tho valuo of $D / T_{0}$ is greater then about 2 give a nogative displacoment offect. In the case of Johomneson and liair, it was fomerly assumed that no dieplaconont was show by their results. However, for their tubes $B$ and $C$, $D / 1$ is of the order of 1 or 2 , so wh should oxpect the displacement to be vory mall. Plotting the wake wiaths as in fig, 11, we see that the moon displacenent for tube $D$ is soout 0.06 D , and for this tube $\mathrm{D} / \mathrm{H}=3.6$. This result thervioce agroes qualetetively with ours, although the totel head profilc given by tube $D$ is considorably distorted. The other large tube, F, which has a very mall intomal diancter, gives a negligible displacement. This may bo due to the small intomol dianeter cuusing on increased positive displecoment as found in our experiments, and so reducing the negative displacement which would othorwise have been obteined for this tube.

As mentioned in the previous section, it was not possible to obtain a satispactory flow pattern in the smoke tunnel for tho case of a very wide pitot tube. It is evident, howevor, that as the tube size incroases, the upstrearn effect of the tube on the wake dovelopment will become greator, and there will be considerable distortion of the wake profile and cxternal flow strearlines. An increase of the actual wake width will of course have a similur effect on the total head readings as a negative displacemont effect. The real displacement effect, which is always positive, will be masked by the growth of the wake, and it is perhaps surprising that such little distortion of the true wake prorile occurs when
a tube of fairly large dianetor is used.

## (iii) Magnitudo of the Uj.splacomont Effect

Having discussed qualitativcly the effect of tube sizc, we can consider in more detail the behaviour of the displacement $\delta$ whon the tube is much straller than the wake width. Under these conditions the tube may be considered as in a unidircetional constant shear flow. The experinental rosults then shom $\delta / D$ to be equel to 0.18 D under a wide range of conditions, (Only one such resulf was obtoined in the supersonic tosts, howover, owing to the small size of wake possibile in the tumel). is pointed out in ref. $1, \delta / \mathrm{D}$ should be some function of $D / u_{0} \zeta_{0}$, where $\zeta_{5}$ and $u_{0}$ are the vorticity and mean velocity of the flow noar the tube mouth. The results thorefore imply that $\mathrm{F}\left[\mathrm{D} / \mathrm{u}_{0} \mathrm{H}\right]$ is equal to
 negative values, with a discontinuity as $D / u_{0} \zeta_{0}$ passes through zero. Physically, this is difficult to accept, and it is more likely that the function decreases rapidly for valuos close to zero, as would occur at the centre of a wake on with a very small tube。 (The displaconent given by the 10.. tube in the low speed tests does, in fact appear to be wacl loss then $0,18 D$ ).

Whe Xlor oround a three dinonsional body in sheer Plov, such as a pitot tube, has not been treated theoretically, but some light may be shed on the problen by considering the case of a two dimensional cylinder (see Appendix). The displacement of the stagnation stremiline from the body axis for upstrean is found to be

$$
\frac{\delta}{a}=\frac{u_{0}}{\zeta a}\left[1 \pm \sqrt{1+\frac{\zeta^{2} a^{2}}{2 u_{0}^{2}}}\right]
$$

(using tho presont sign convention)


Where $a$ is the radius of the cylindor. The fom of this function is as shown."


It will be scen that as $\frac{\text { Ya }}{u_{0}}$ becomes large compered to unity, $\frac{\delta}{a}$ tends torrards a constant value of $\frac{1}{\sqrt{2}}$, Prosumably, therefore, the displaconont effoct on the pitot tube will behave in roughly tho same manner when $\zeta D / u_{0}$ becones large with respect to a certain value. This value must be considerably less than 0.1 , however, for the experinental results show that $\frac{\delta}{D}$ remains constant for values of $\frac{\zeta_{0}}{u_{0}}$ dow vo 0.1 . To eromine experimentally the behaviour of $\frac{S^{\mathrm{D}}}{}{ }^{u_{0}}$. in the region Whene $\frac{6}{u_{0}}<0.1$, a semius of extremeay moll pitot tubes in a wide woke would be needed.

Coning now to the case where the tube spans the wake, and the displacement is reversed, the flow is clearly too complex to find a theoretical relation for $\delta$. Examination of figs. 5 to 11 shows that $\delta$ increases rapidly with tube

[^0]dianeter, and appeors to be almost proportional to $\frac{H_{p}}{H}$, where $H_{0}$ is the pitotreading outside the wele. It also increases with increasing velocity gradient (although the veriation of the Inttor vas suall in the supersonic experiments). By flotting values of
$\log \frac{\delta}{\mathrm{H}_{\mathrm{P}} / \mathrm{H}_{\mathrm{O}} \cdot \mathrm{S}}$ (where S is the totel head ratio gradient
$\frac{\partial \mathrm{H}_{\mathrm{p}} / \mathrm{H}_{\mathrm{O}}}{\partial \mathrm{y}}$ )against $\log \mathrm{D}($ fig. 14), it is found that the aisplaccment verios approximately as $D^{2}$, which gives the dinensionelly correct formula $\delta=k \frac{H}{H_{0}} \cdot s D^{2}$. The present results give values for $k$ of 0.012 to 0.023 for the supersonic tests, and about 0.033 for the low speed test. The widest tube used in Johannesen and Tair's experinents gives a velue of 0.035 , Thus the order of magnitude of the displaconent effect due to distortion of the wake by a very wide twhe ney be taken as very approximately $0.02 \mathrm{H} / \mathrm{I}_{0}$ s $\mathrm{D}^{2}$.

This is only applicable where the tube is at least twice tho wake width, whereas the rolation $\delta=0.18 D$ seens to epply only for tube dieactors less then one third of the whe vidth. In tho intomodiate region, the true displaconent cffect is clearly rodifiod by the effects of distortion of tho wato, Although no simple relation can be suggested for the valuo of $\delta$ in this rogion, sone iden of the megritude of tho erfoct may be obtainod fron fig. 14. It will be noted that when $D / T_{0}$ is approxinately 2 , the resultant displaconent is practically zero.
(iv) focuracy of the total hoad rondings

In the shem rogion of the wake profiles, any Ammeuraey of total hoed reading has been taken as pert of tho Uisplacenont offect. Thus, if a correction for the aisplacemont is applied to the tube position, the reading of the tube will be the corroct total head at the new position.

Noar the wake centre, where the total head gradient folls to zoro, there should be no error in the total head readings provided viscous effects are negligible. This eppears to be substentiated by experiment, since the variations in totol hoad reading at the wake centre ore Icss than 2 per cent for tubes swallor then the wake width. In the case of the widor tubes, the spreading of the woke vill of course alter the true total head at its centre.

Por the widest tubes used, the total hend at the wake centre appears to be alnost doubled due to the mesence of the tube,

## Acknorrledronents

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## 7. Conclusions

Experinents have been performed to measure the displacement of the effective centre of a pitot tube from its geonetric centre when placed in wakes behind various thin two-dinensional wings. The measurements were nacie at low subsonic speeds and also at supersonic Miach numbers of 1.75, 2.45 and 3.19 , using pitot tubes whose dianoters veried fron 0.5 m . to 31 m. . The following conclusions were reachod.
(i) Then the pitot tube is in a wole whose width is o.t lenst theee times the tube dinneter $D$, or in a unidirectionn? velocity crodient, the atisplacment is towards the region of higher velocity, and is cqual to 0.18D. Theoretical considerations, howover, indicote thet the displecenent decreases Pron this veluo when the total hed gradient or tube dianetor become very amall.
(ii) If the noximun wake wiath $W_{0}$ is less then the tube diametor, the apporent displacement effect decreases to zoro, and then becones negative (i.e. in the direction of docronsing velocity) as the ratio $D / T_{0}$ increases.
(iii) For valuas of $D / T_{0}$ greater than about 3 , the resulting displacement in the diruction of decreasing velocity is mporimaty proportioncl to $D^{2}$, 5/G0 (the ratio of total head measured at a point in the wake to that outside the wake), and $s$ (the transverso gradient of $\mathrm{H} / \mathrm{Ii}_{0}$ arross the tube face).
(iv) Tho displecement is proctically independent of intornal tubo dianetor, $d$, provided this is less thon hale the external dianeter. For values of the ratio $d / D$ greater thon this, the displacement is slightly reduced.
(v) IVo arpreciable errors due to tube size were appacent in the pitot readings at tho centro and outer odges of the wake (i.e. where the total head gradient wis zero). However, When the wake itself was distortod by a very large tube, an increased total head reading was obtained at the centre.

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## ATPEMDIX

The displacement erfect for a tro-dinensiond circular cylinder

## in a shear flor

Consider a two-dinensional shear flow with constant vorticity $\zeta$ past a circular cylinder of radius $a$ o The equation for the strearr function, $\psi$, is

$$
\begin{equation*}
\nabla^{2} w=-\zeta \tag{1}
\end{equation*}
$$

with the velocity components $u$ and $v$ siven by
and

$$
\begin{aligned}
& u=\frac{\partial \psi}{\partial y}, \quad v=-\frac{\partial \psi}{\partial x} \\
& \check{\zeta}=\frac{\partial v}{\partial z}-\frac{\partial u}{\partial y} .
\end{aligned}
$$

If the axis of the cylinder is taken as the origin of the rectangular cartesion coordinates ( $x, y$ ) and polor coordinates $(r, \theta)$ and $u_{0}$ is the freestrear velocity at $j=0 \quad(x= \pm \infty)$ thon it can be shom that the solution to equation (1), satisfying the boundary conditions in the freestroan and on the cylinder, is

$$
\begin{equation*}
\psi=-\frac{1}{4} r r^{2}+u_{0}\left(r-\frac{a^{2}}{r}\right) \sin \theta+\frac{2}{4}\left(r^{2}-\frac{a^{4}}{r^{2}}\right) \cos 20 \tag{2}
\end{equation*}
$$

and in the freestream $(x * \infty)$

$$
\begin{equation*}
\psi=-\frac{c}{2} y^{2}+u_{0} y \tag{3}
\end{equation*}
$$

Tho strean function $\dot{\psi}_{\mathrm{s}}$ for the stagnation streanline, and its coordinates, are found from equation (2) when $r=a$ giving

$$
\begin{aligned}
\psi_{S}\left(r^{\prime}, \theta^{\prime}\right)=-\frac{\zeta a^{2}}{4}=-\frac{\zeta}{4} x^{\prime 2} & +\left(x^{\prime}-\frac{a^{2}}{r^{1}}\right) u_{0} \sin \theta \\
& +\frac{\zeta}{4}\left(x^{\prime 2}-\frac{a^{4}}{r^{\prime}}\right) \cos 2 \theta^{\prime}
\end{aligned}
$$

and $\operatorname{since} r^{\prime} \sin \theta^{\prime}=y^{\prime}$

$$
\begin{equation*}
\frac{\zeta}{2}\left(\frac{a^{2}}{r^{\prime}{ }^{2}}+1\right) y^{\prime^{2}}-u_{0} y^{\prime}-\frac{\zeta_{a}^{2}}{4}=0 \tag{4}
\end{equation*}
$$

The stagnation streamline meets the cylinder at the point $P\left(a, \theta_{s}\right)$. From equation (4)
$\sin \theta_{S}=\frac{u_{0}}{2 \zeta a}+1 \pm \sqrt{1+\frac{\zeta^{2} a^{2}}{u_{0}^{2}}}$
For values of $\frac{\zeta_{a}}{u_{0}}<\frac{4}{3}$ equation (5) gives one front and one rear stagnation point. For values of $\frac{\mathrm{y}_{\mathrm{a}}}{u_{0}} \Rightarrow \frac{4}{3}$ there are two front and two rear stagnation points. As $\frac{\zeta a}{u_{0}} \rightarrow \infty\left(u_{0} \rightarrow 0\right)$ the stagnation points are at $\left(30^{\circ}, 150^{\circ}\right.$ and $-30^{\circ}, 210^{\circ}$ ). The displaconent ( $\delta$ ) of the stagnation streamline for upstroan from the cylinder axis is from equation (4), since $\mathrm{r}^{\prime}=\infty, \delta=-\mathrm{y}^{\prime}$,

$$
\frac{\delta}{a}=-\frac{u_{0}}{\zeta_{a}}\left(1 \pm \sqrt{\left.1+\frac{\zeta_{0}^{2} a^{2}}{2 u_{0}^{2}}\right)}\right.
$$

For values of $\frac{\zeta_{a}}{u_{0}} \ll 1 \frac{\delta}{a} \tilde{\sigma}-\frac{\bar{Y}_{a}}{4 u_{0}}$ but for larger values of $\frac{\zeta_{a}}{u_{0}} \frac{\delta}{a}$ becomes more independent of $\frac{Y_{a}}{u_{0}}$, and approaches the limiting values of $\frac{1}{\sqrt{2}}$.

## TABIF I

Tubes used in Supersonic Tests

| Tube | Outside Dian <br> $D\left(\mathrm{~m}_{0} \mathrm{mo}\right)$ | Inside Diam。 <br> d $\left(\mathrm{m}_{0} \mathrm{mo}\right)$ | $\mathrm{d} / \mathrm{D}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.50 | 0.25 | 0.50 |
| 2 | 1.01 | 0.50 | 0.49 |
| 3 | 2.00 | 1.40 | 0.70 |
| 4 | 2.39 | 2.00 | 0.84 |
| 5 | 3.18 | 0.50 | 0.16 |
| 6 | 6.38 | 0.50 | 0.08 |

## TABTE II

Tubes used in 1 ow speed tests

| Tube | Outside Diam, <br> $D(m, 2$, | $\left.\begin{array}{c}\text { Inside Dian, } \\ d(m, m,\end{array}\right)$ | $d / D$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.0 | 0.50 | 0.50 |
| 2 | 3.18 | 0.50 | 0.16 |
| 3 | 4.76 | 0.50 | 0.10 |
| 4 | 6.35 | 0.50 | 0.78 |
| 5 | 31.00 | 0.50 | 0.016 |
| 6 | 31.00 | 3.18 | 0.102 |
| 7 | 31.00 | 4.76 | 0.153 |
| 8 | 31.00 | 6.35 | 0.205 |
| 9 | 31.00 | 19.00 | 0.615 |



FIG. I WAKE TRAVERSES BEHIND $1 / 2$ CHORD WING AT $\mathrm{M} 3 \cdot 19$


FIG. 2. WAKE TRAVERSES BEHIND $1.75^{\prime \prime}$ CHORD WING AT $\mathrm{M}=3.19$


FIG. 3. WAKE TRAVERSES BEHIND $2^{\prime \prime} \mathrm{CHORD}$ WING AT $59 \mathrm{~F} / \mathrm{S}$


FIG. 4. WAKE TRAVERSES BEHIND $1 " X 1 / 32$ FLAT PLATE AT $50 \mathrm{~F} / \mathrm{S}$.


FIG.5. VARIATION OF APPARENT WAKE WIDTH WITH OUTSIDE DIA OF TUBE ( $/ 2 / 2$ CHORD WING AT $M=1.75$ )


FIG. 6. VARIATION OF APPARENT WAKE WIDTH WITH EXTERNAL DIA OF OF TUBE ( $1 / 2$ CHORD WING AT Mu2.36)


FIG. 7. VARIATION OF APPARENT WAKE WIDTH WITH OUTSIDE DIA. OF TUBE ( $1 / 2$ CHORD WING AT M $\mathrm{M} 3 \cdot 19$ )


FIG 8 VARIATION OF APPARENT WAKE WIDTH WITH OUTSIDE DIA. OF TUBE ( $3 / 4 / 4 \mathrm{CHORD}$ WING AT $\mathrm{M}=3.19$ )


FIG. 9. VARIATION OF APPARENT WAKE WIDTH WITH OUTSIDE DIA. OF TUBE ( $2^{\prime \prime}$ CHORD WING AT 59F/3)


FIG.II. VARIATION OF APPARENT WAKE WIDTH WITH OUTSIDE DIA. OF TUBE (JOHANNESEN \& MAIR)


FIG.10. VARIATION OF APPARENT WAKE WIDTH WITH OUTGDE DIA. OF TUBE ('CHORD FLAT PLATE AT SOF/S)


FIG. 12 VARIATION OF APPARENT WAKE WIDTH WITH INSIDE DIA. OF TUNT (2"CHORD WING AT $59 \mathrm{~F} / \mathrm{S}$ )


FIG. I3. VARIATION OF DISPLACEMENT WITH RATIO OF TUBE DIAMETER TO WAKE WIDTH


FIG. I4. VARIATION OF DISPLACEMENT FOR VERY labge diameter tubes


FIG. 15. 1 mm TUBE BEHIND $\frac{1}{2}^{\prime \prime}$ CHORD WING AT $\mathrm{M}=3.19$ ( 2 mm OFFSET).


FIG. 16. 1 mm TUBE BEHIND $\frac{1}{2}$ " CHORD WING AT $\mathrm{M}=3.19(0.5 \mathrm{~mm}$ OFFSET).


IG. 17. 6.4mm TUBE BEHIND $\frac{1}{2} "$ CHORD WING $A T$ M $=3.19$ ( 0.2 mm OFFSET) .

18. 6.4 mm TUBE BEHIND $\frac{1^{\prime}}{}{ }^{\prime \prime}$

CHORD WING AT $\mathrm{M}=3.19(0.5 \mathrm{~mm}$ OFFSET).


[^0]:    * A more general result for two-dimensional cylinders of elliptical cross-section has been given by liitchell and Surray. (4)

