Nonlinear Vibration Analysis of a Complex Aerodynamic Structure

Samson.B.Cooper
University of Bristol, Department of Mechanical Engineering, Queen's Building, University Walk, Bristol, United Kingdom.
E-mail: **sc14784@bristol.ac.uk**

Abstract
Complex shaped aerodynamic structures such missiles are prone to exhibit some level of nonlinear phenomena due to their aerodynamically tailored design and application. Aside from the aerodynamic and aeroelastic challenges experienced by a missile, an important but fundamental challenge encountered by a deployable missile is the inevitable concentrated structural nonlinearities which are observed around the hinge of its fins. Due to the current design and manufacturing process, the hinge of the fin of a missile often consist of complex configurations, joints and other nonlinear features that leads to concentrated structural nonlinearities. Some of the nonlinearities encountered includes off sets, piecewise linear, bilinear nonlinearity, hysteresis, coulomb friction and damping nonlinearities. These nonlinearities are frequently triggered at large vibration amplitudes caused by high pressure loads during operational flight. Activation of these nonlinearities often affect the dynamic response of the missile and in some cases lead to structural failures in the air vehicle. In this context, identifying and predicting the vibration response of aerodynamic structures with nonlinearities will be of great advantage to the present aerospace industries. In this paper, the nonlinear dynamic behaviour of a prototype missile is examined experimentally. The first step involved using acquired input and output data from random and sine sweep vibration test to derive a nonlinear experimental model of the structure, the nonlinear experimental model was developed using the white box identification process (Detection, Characterisation and Parameter Estimation). In addition, Force controlled stepped sine experiments at several excitation levels were conducted to gain useful insight into the amplitude dependant behaviour of the missile and also characterise the dynamic response of the missile in the existence of structural nonlinearities.

Keywords: Experimental Test, Missile, Structural Nonlinearities and Nonlinear Identification

1.Introduction

Nonlinearities often originate from different sources in engineering structures most especially in an industrial application, a large majority of these nonlinearities are narrowed down to the design of the structure, nature of the joints, material and geometric properties. Research on bolted joints and other types of nonlinear features have been proven to introduce large uncertainties in the stiffness and damping properties of a structure which can often render the response of the structure nonlinear, identifying and predicting the effect of these nonlinearities at operational conditions is of current challenge to present structural engineers dealing with complex nonlinear structures. In this context the integration of experimental nonlinear identification and finite element modelling of engineering structures would be of great advantage to the present structural dynamics society. Experimental nonlinear identification is important in many structural dynamic applications, for example in complex aerospace and mechanical structures[1], micromechanical systems with magnetic or friction forces [2], machineries with rubber isolation mounts and assembled structures with bolted interfaces [3]. In most engineering design, the base line structure is often linear, but the vibration testing and operational performance of some of these structures exhibit a level of nonlinear phenomena which can no longer be ignored or assumed as linear [4]. Hence, the accurate representation of these nonlinear behaviour in the finite element model of the structure or built up assembly would be of extreme benefit in obtaining better response prediction at the forcing range of interest.

Examples on the real life application of some of these developed nonlinear identification methods are also available in the literature where the identification of weak nonlinearities was studied on a more complex aerospace structure in [5] where a strategy for non-linear modal identification of weak nonlinear effects on a large aircraft was presented. An aluminium plate attached with two stores used to illustrate the behaviour of a wing and an engine suspended by a means of nonlinear pylon also displayed the presence of weak nonlinearities during a vibration test, the results obtained illustrated some hardening characteristics as show in [6]. Similar study was also carried out on a large helicopter with the identification of weak nonlinear softening behaviour on one of the vibration modes as shown in [7]. Other examples of case studies where nonlinearity have been noticed in aerospace structures can be found in [8] where nonlinearity was also detected at the elastomeric mounts supporting the four turboprop engines of the aircraft during the Ground Vibration Test (GVT) of the Airbus A400M aircraft designed for military purpose. The F-16 fighter aircraft also showed a nonlinear behaviour at wing-to-payload mounting interface of the aircraft when a similar GVT was conducted [9]. Nonlinearities were also detected on the Cassini spacecraft due to the presence of gaps in the support of the Huygens probe [10]. More case studies on the presences of nonlinearities in engineering structures can be found in the literature, it is therefore possible to conclude that the development of identification techniques which are capable of producing satisfactory results when linear identification techniques fail is an active area of study in today's structural dynamics society. In the real world application nonlinearity is ever-present and as engineers push to designing lighter, more flexible and more efficient structures, the design are shifting towards non-linear regime which also shows that there is a need for developing strategies for understanding the nonlinear response of these structures. Hence this paper addresses the nonlinear experimental identification, and the force controlled experimental test conducted on a missile. This involves the use of established and robust identification techniques to identify the type of nonlinearity present in the assembled missile, the complete identification process i.e. (Detection, Characterisation and initial Parameter estimation) was achieved based on experimental data. Measured time series and frequency data driven by Sine-sweep test and random excitation were exploited to gain an initial insight to the dynamic behaviour and properties of the assembly. The structure of the paper is as follows, section 2 describes the first case experimental study conducted on the missile followed by the linear identification based on measured data from low level random excitation. Section 3 includes the nonlinear identification based on measured data and white box identification process (Detection, Characterisation and Parameter Estimation). Section 4 addresses the final sets of results obtained from the force controlled stepped sine test. The conclusion of the study and the collective use of different analysis techniques in this research are finally summarised in section 5.

1.1. Description of the Test Structure

For the purposes of this research, a Missile Test Structure (MTS) has been designed and manufactured and can be seen in Figure 1. MTS, while not an exact scale replica of any particular missile system (for ease of manufacture and classification), has structural features that are typical of a full-size system e.g. multiple body sections, bolted joints, hanger and launch rail assembly.

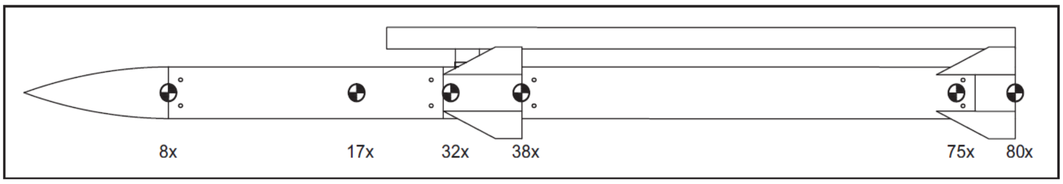

Figure 1: Missile Test Structure

MTS is made up of 4 sections of aluminium tubing, a nylon nose cone, aluminium fins and hangers and a detachable aluminium launch rail. The approximate dimensions are: overall length 1200 mm, outside diameter 60 mm, overall mass 3 kg. Since most of the nonlinear phenomena experienced in mechanical vibrations are attributed to joints, friction and geometric nonlinearities, the test structure was designed to understand the effect of nonlinearities caused by bolted joints and assembled multibody sections at high levels of vibration test.

2. Experimental Test and Linear Identification

To subject the MTS to a representative vibration environment the MTS was hanged on a test frame using fishing lines, springs and light connection ropes as shown in figure 2, the test setup was designed to replicate the usual boundary condition experienced by such device when attached to an air vehicle. The black frame was bolted on a seismic table to disallow any form of movement to the frame during

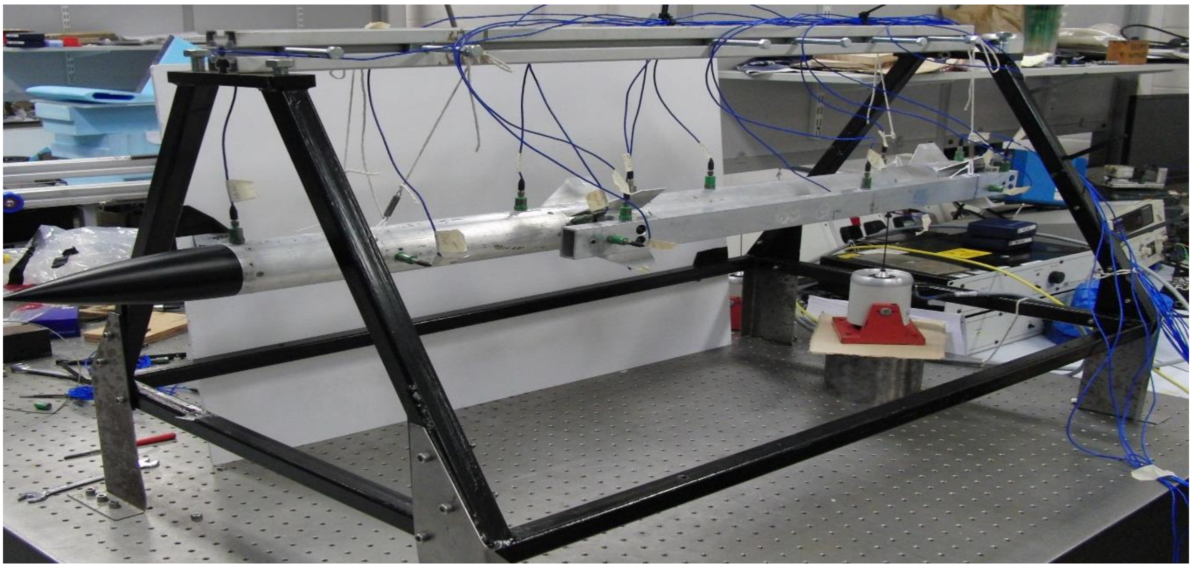

Figure 2: Missile Test Structure Experimental Set-up

the vibration test, the complete assembly was tested and examined to ensure that the level of vibration transferred to the missile test structure is reduced to a minimum or zero value. The assembled missile and hanger was instrumented with 15 accelerometers and a force transducer, the connection areas between the hanger and the missile were instrumented appropriately to capture any nonlinear phenomena exhibited by the bolted connection.

2.1. Low Level Test Campaign

The first measurements obtained from the experimental test comprised of several low random data which were acquired based on broadband excitation, the choice of broadband excitation was made based on its conventional use in modal testing. The use of broadband excitation also provides some early information on the behaviour of the structure and experimental configuration, the low level random test was performed using the Spectral Test module in LMS Test Lab [8], the test structure was excited close to the hanger attachment at the tail section as shown in figure 2. The structure was excited using burst random excitation filtered in 10-1000Hz. The FRFs and associated coherence functions obtained from the test was exploited to identify the linear modal properties of the test structure, the shape of the FRFs and ordinary coherence plots were also used as an indication to determine if the assembly was behaving linearly at the specified excitation level. A selection of the Frequency Response Functions (FRF) and coherence functions obtained from the low level random excitation are plotted below in figures 3a to 3d

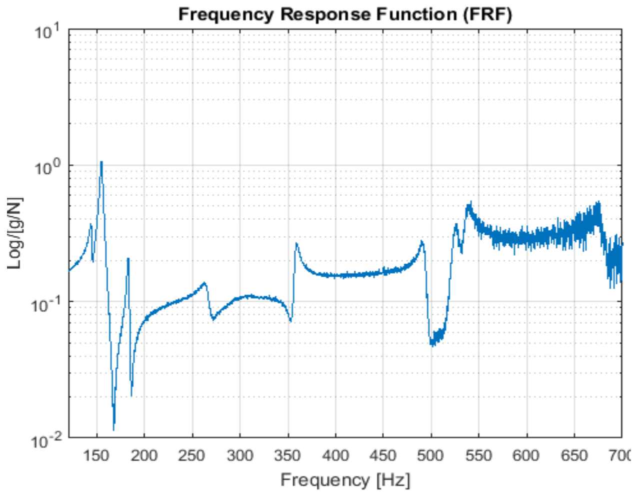

Figure 3a

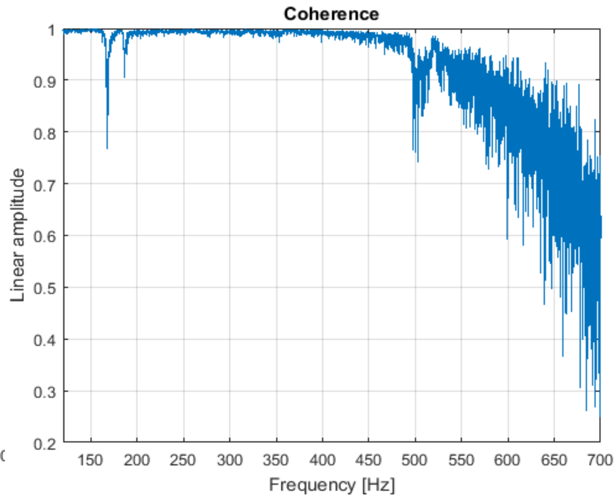

Figure 3b

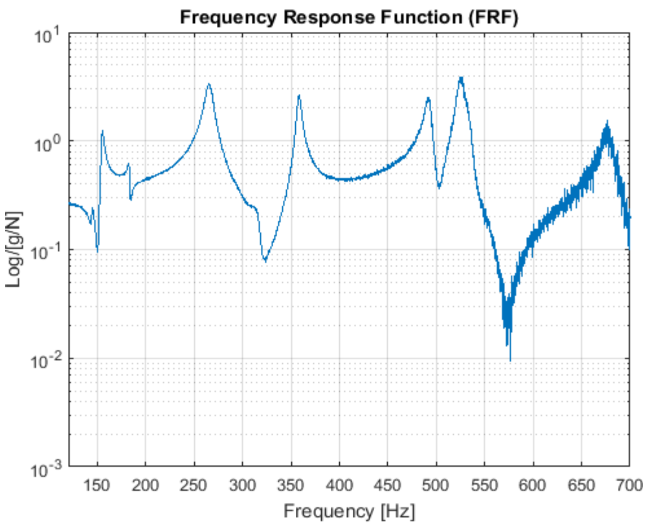

Figure 3c

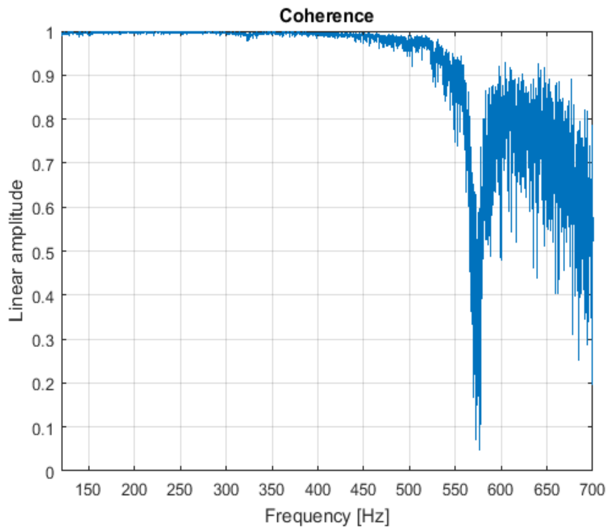

Figure 3d

Figure 3: FRFs and Coherence responses obtained from low-level broadband excitation. Figure 3a &3b (Centre tube of the Missile), Figure 3c & 3d (Far-end of the Hanger)

2.2. Linear Identification

In structural dynamics, the theoretical and experimental aspects of linear system identification have received early attention since the early seventies, this has prompted the design of mature analytical, computational and testing methods such as modal analysis. Currently, modal analysis can be classified as the most popular method of carrying out linear system identification for vibrating structures and has successfully been applied to a range of complex engineering structures over the last few years. A comprehensive text book which covers the theory, practice and application of modal analysis was introduced by Ewins [11], Maia and Silva[12] also proposed a review of techniques developed to implement modal analysis tools. Examples of current techniques used in modal analysis are the polyreference least-squares complex frequency-domain method in [13], the subspace deterministic-stochastic algorithm[14] , the eigen-system realisation algorithm[15], A unified matrix polynomial approach to modal identification was also introduced in [16] and many more methods which can be found in the literature.

For this research the linear resonance frequencies and damping ratios were estimated using the frequency-domain subspace identification algorithm presented in [13], the PolyMAX method uses measured Frequency Response Functions (FRFs) as principal data. In PolyMAX identification, the measured FRF is assumed as the right matrix-fraction model which represents a pair of inputs and output matrices where n is the matrix numerator and m is the denominator:

$$[H(\omega)] = \sum_{r=0}^{P} Z^r[\beta_r].\left[\sum_{r=0}^{P} Z^r[\propto_r]\right]^{-1} \tag{1}$$

Where $[H(\omega)] \in \mathbb{C}^{m\times n}$ is the matrix containing the FRFs between all n inputs and m outputs; $[\beta_r] \in \mathbb{R}^{m\times n}$ are the numerator matrix polynomial coefficients; $[\propto_r] \in \mathbb{R}^{m\times n}$ are the denominator matrix polynomial coefficients and P is the model order. The frequency domain model Z which is derived from discrete-time model is computed using the expression:

$$Z = e^{-j\omega\Delta t} \tag{2}$$

Where Δt is the sampling time. Eq.1 is then written for all values of the frequency axis of the FRF data to retrieve the unknown polynomial coefficients using the least square solutions of these equations. Once the denominator coefficients have been calculated, the poles and modal participation factors are retrieved as the eigenvalues and eigen-vectors of their companion matrix. The resonance frequencies and damping ratios are then calculated as follows:

$$\lambda_i, \lambda_i^* = \xi_i\omega_i \pm j\sqrt{1-\xi_i^2\omega_i} \tag{3}$$

The corresponding resonance frequencies and damping ratio identified using the low level random data obtained from the missile test are presented in table 2. The resonance peaks of the FRFs indicates that the structure is lightly damped across the selected bandwidths, the coherence function corresponding to each FRF or measured position are all close to unity for the whole excited frequency range.

Mode	**Frequency (Hz)**	**Damping ratio (%)**
1	144.38	0.75
2	154.90	0.94
3	183.04	0.52
4	253.57	1.63
5	292.17	1.72
6	315.41	1.34
7	358.05	0.69
8	492.44	0.72
9	524.85	0.75
10	536.18	0.69
11	675.52	0.77

Table 1: Estimated linear resonance frequencies and damping ratios based on low-level random data

To measure the accuracy of the linear identified modal parameters, the measured FRFs are correlated with the synthesised FRFs obtained from the identified modal model. Figures 4a and 4b presents the comparison of both results for selected measured points on the missile, the results obtained from the correlation shows that the PolyMAX method is able to accurately model the measured data obtained from the missile test. Even at higher modes of vibration where the measured data obtained from such structure is expected to be nonlinear, the synthesised FRF still shows a good correlation fir the measured FRF as shown in figure 4.

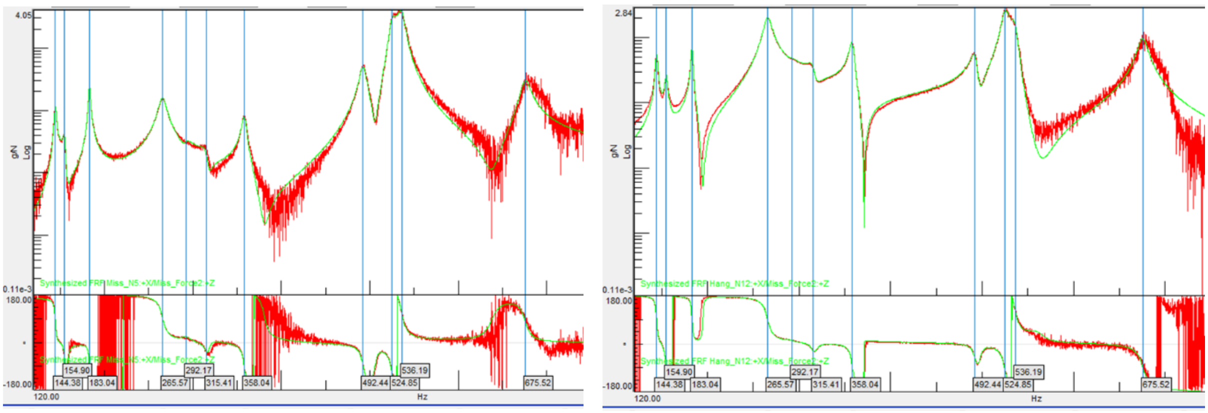

Figure 4: Comparison of the measured FRFs (red) with FRFs synthesised from the identified modal model (green). (Left) Sensor at the centre of the missile; (Right) Sensor on the hanger

3. Nonlinear Experimental Identification

Nonlinearity is an extensive term which could have various meaning in the mathematical and engineering discipline, from the context of a structural dynamicist nonlinearity occurs when a system violates the homogeneity principle i.e the absence of the superposition principle. This means that for any combination of loads applied simultaneously to a system does not yield the same response as the sum of the individual responses to each of the loads acting separately. Currently the superposition principle is said to be the benchmark of linear vibration which also provides an explanation to the failure of current linear identification tools when exposed to nonlinearity. Another important feature of a nonlinear system is the fact that their natural frequencies and mode shapes can vary with respect to the excitation amplitude, these nonlinearities can easily invalidate results based on linear simulations. The dynamic response or behaviour of a strongly nonlinear system are usually significantly different from the response of a linear system as shown in[17],[18],either through simulation or by conducting an experiment. The core nonlinearity identification procedures are performed in this section. Here, new measured response function data are acquired under more closely-controlled excitation conditions, chosen to ensure that the structure is exercised at vibration amplitudes representative of those anticipated in service. The overall objective of section is to be able to characterise the type of nonlinear behaviour exhibited by the missile test structure. Special care and attention are taken into account when obtaining the measurement at this stage to ensure that it is made clear exactly what form of 'response function' is obtained. Strictly, the specific excitation signal used are specified and carefully selected when deriving response functions for the test structure.

3.1. Nonlinearity Detection and Characterisation

Detection indicates that some effect attributed to nonlinearity is observed, and it is deemed that the standard linear model cannot adequately represent the system response. There several techniques of detecting nonlinear behaviour from measured data, this however depends on the type of excitation signal used during the test campaign. Stepped sine and Sine-Sweep excitations are predominantly suitable in determining if a structure has a nonlinear behaviour at higher excitation level, if linear, the structure would produce a pure sine wave in the output and if nonlinear, distortions is easily detected by visualizing the output envelop of the sine wave.

3.1.1. Time Series Inspection

Visualization of raw time series obtained from Sine-Sweep excitations can often reveal some level of nonlinear behaviour in the structure, any form of nonlinear distortions observed in the time response envelop is sufficient to prove the presence of nonlinearity in the structure. In this paper, Sine-Sweep test was conducted on the second, third and fourth mode of the assembly. Accelerations at every sensor locations on the MTS were measured between 0.5N and 15N. The Sine-Sweep test was conduct using the Multi-input Multi-output (MIMO) Sine-Sweep module in LMS Test Lab package , the input excitation force was uncontrolled however parameters such has the start and end frequencies, sweep type and sweep time were specified. Given the knowledge of these parameters, the sweep rate and instantaneous sweep frequencies were calculated based on equations 4 and 5.

$$K = \frac{f_{end} - f_{start}}{T} \qquad (4)$$

Where K=Sweep rate and T= the total sweep time in seconds

$$f_{inst} = f_{start} + Kt \qquad (5)$$

Where f_{inst}=instantaneous frequency and t=time vector

The time responses obtained from the Sine-Sweep test were plotted in terms of the measured acceleration against the calculated sweep frequency, figures 5a-5d shows some illustrations of selected measured response from the MST.

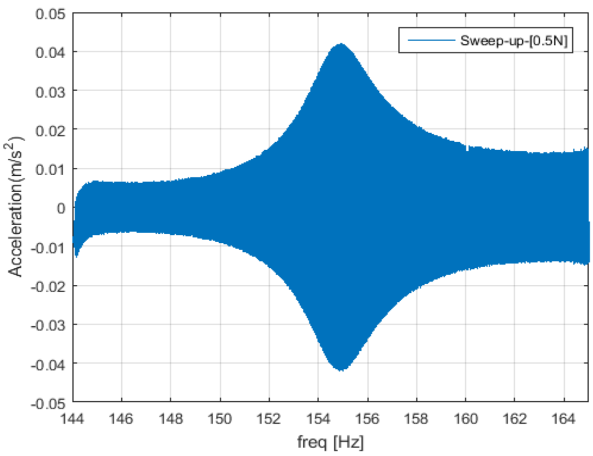

Figure 5a: Response envelop Mode2 @ 0.5N

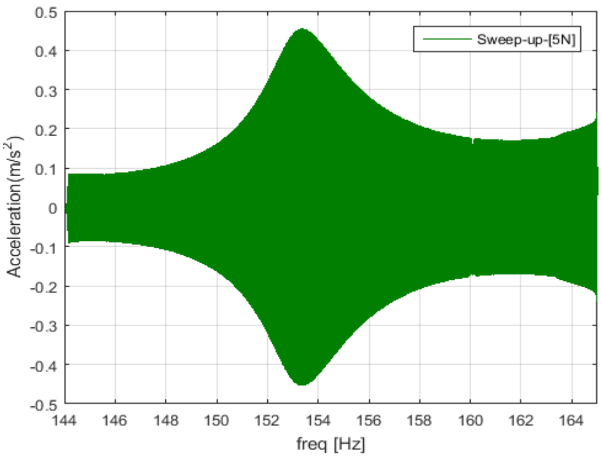

Figure 5b: Response envelop Mode2 @ 5N

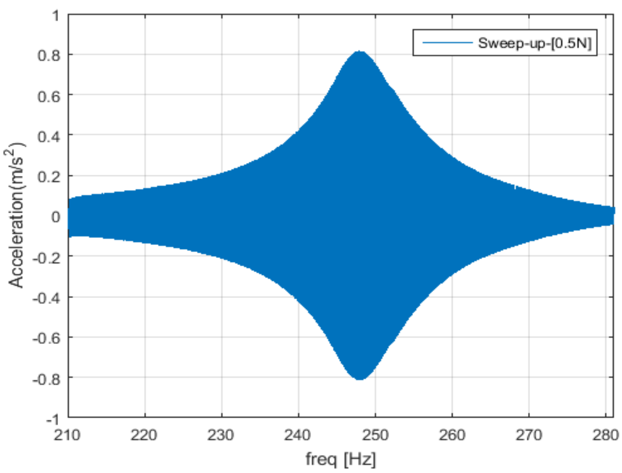

Figure 5c: Response envelop Mode4 @ 0.5N

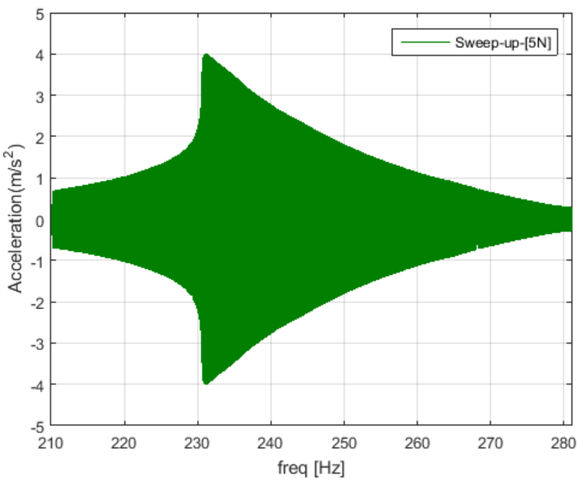

Figure 5d: Response envelop Mode4 @ 5N

Figure 5 Nonlinearity detection based on envelope time series inspection

To detect any form of nonlinear behaviour, the first observation is shown in figure 5c and 5d where an absence of proportionality is noticed between the time responses from low to high excitation level. This indicates the breakdown of superposition principle, secondly, figures 5(d) sweep-up shows clear skewness in the time responses as the excitation level increases from 0.5N to 5N. The skewness in the envelope of oscillation in figure 5(d) where a sudden transition from low to high amplitude of vibration is observed can also be described as a jump phenomenon, this is also a useful technique for detecting nonlinear behaviour in the structure. The final and most important observation from the time response envelope is the shift in the resonance frequency, a negative drop can be seen in the location of the resonance frequency between figure 5(c and d), where the resonance frequency has shifted / reduced from 250Hz to 230.2Hz due to the increase in the excitation level from 0.5N to 5N. Other form of nonlinear behaviours which are observed in the time response envelope are peak distortion, non-smoothness and discontinuity of the sweep response at 5N compared to the response at 0.5N excitation level, indication of multiple solutions and bifurcation points are also observed around the resonance frequency for the response at 0.5N, all these observations are sufficient enough to detect the presence of nonlinearity in the MTS assembly. Another simplified method of detecting nonlinearity in a structure from experimental measurement is based on the assumption that the FRF of a linear system is independent of the input amplitude. This assumption is often used by most researchers in the field of structural dynamics, it serves as a basic initial step in nonlinear identification and it associated with the homogeneity of the system[19]. For a nonlinear structure the FRF is dependent on the magnitude of the input force applied during the experimental test. The FRF cross correlation method presented in [20] is a simple method of quantifying nonlinearity and it has been adopted in this section of the research to detect some nonlinearities in the multiple beam test structure. As stated in [20] the cross correlation of two different functions $x(t) and\ y(t)$ in the time domain is defined as:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt \qquad (6)$$

The representation of equation (34) in the frequency domain can be written in the form:

$$R_{H^H H^L}(\Delta\omega) = \int_{-\infty}^{\infty} H^H(\omega)H^L(\omega+\Delta\omega)d\omega \qquad (7)$$

Where $H^H(\omega)\ and\ H^L(\omega)$ are high and low level FRFs obtained from a vibration test, the cross correlation is founded on a formulation that measures the correlation between two FRFs as a function of its frequency shift. The normalised correlation coefficient of two FRFs is used to measure the amount of variance between the two functions.

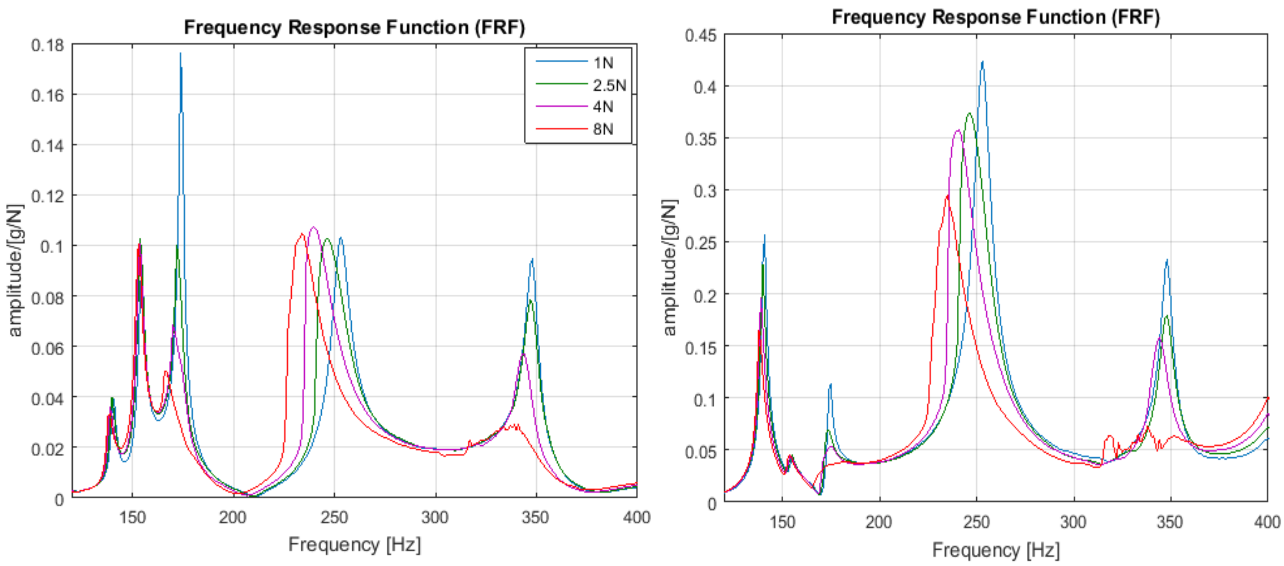

Figure 6: Nonlinearity detection based on envelope time FRF inspection. Left (MTS Nose), right (Hanger)

In this section of the paper, measure time response functions obtained from the Sine-Sweep test were post-processed into the frequency domain to measure the degree of nonlinearity exhibited by the MTS.

The first and most noticeable indicator of nonlinear behaviour is a lack of homogeneity in frequency response functions over different force inputs. By looking at Figure 6 it is immediately obvious that the structure behaves differently for different input forces, in contrast with established linear theory. The shifts in frequency and amplitude of the FRFs were here deemed not to be safely negligible, and we require a full identification of the nonlinearities. A significant number of shifts was observed in the resonance frequency and response amplitude as shown in figure 6a and 6b. The characteristics observed from the Sine-Sweep excitation FRFs shows that the assembly has a softening behaviour within the range of the input excitation levels.

4. Control Stepped-Sine Test

Stepped-sine excitation is the simplest excitation that can be input to a nonlinear system and, by maintaining the input level for all excited frequencies, amplitude-dependent nonlinearities can be emphasised. Although Step-sine excitation test is time consuming and also sometimes challenge due to the number of control parameters and strategies implement in this type of test particularly whilst maintaining a nominal input or output level. However results obtained from such test can be used to provide early characterisation to the type of nonlinearities in the test structure. For this research, Stepped-Sine FRF data were acquired using the (MIMO) Stepped-Sine module in LMS Test Lab package. The amplitude of the sinusoid sent to the shaker was updated to achieve the desired input force levels to within ±5% of the nominal desired input level, only the fundamental frequency of interest was controlled, neglecting harmonic content fed back to the shaker and present at accelerometers. This is as a result of the control strategy implemented in LMS Test Lab, the stepped sine test conducted to include sweeps in both upward and downward frequency direction for the selected bandwidth with forcing amplitude ranging from 1N to 10N. Figure shows a selection of FRFs for controlled test around regions of modes 2 and 3 for the MTS.

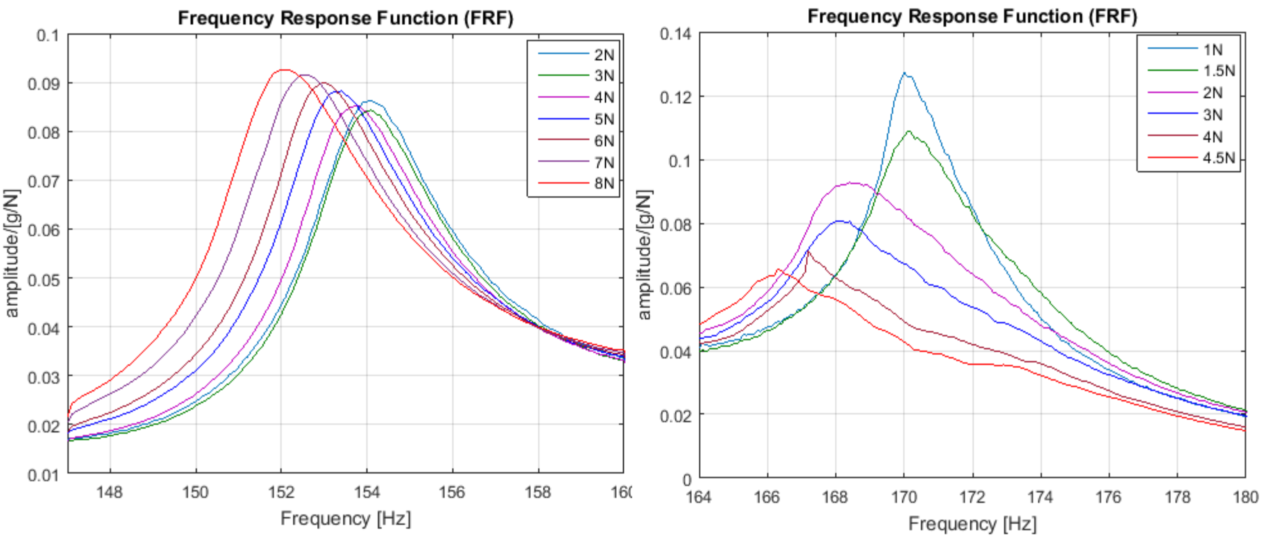

Figure 7 Force Controlled Stepped Sine Test Response Missile Nose (Mode 2). (left), Mode3 (right)

The Stepped-sine FRFs presented in figure 7, each focused around a resonance frequency of interest, and at the lowest and highest input levels that could be achieved (limited by signals acquisition at low input levels and fixturing of the test-rig at high input levels). These step-sine FRFs neglect all higher-order content (on both input and output), and are effectively a targeted (single frequency) Discrete Fourier Transform (DFT)), A softening behaviour is observed for both modes 2 and 3 of interest which also corresponds to the results obtained from the Sine-Sweep test. A sudden transition (jump up) to a higher energy state is observed as the frequency increases and a sudden transition (jump down) to a lower energy state as the frequency reduces. This behaviour is often referred to as the jump phenomena, although the stepped sine test is a different type of test this result also matches with the results plotted in figure 5d in the envelop time response inspection. The Step-Sine FRFs can also provide some characterisation of the nonlinearities over the initial homogeneity check, for the two modes tested a decrease in resonance frequency is observed as the excitation amplitude increases. While the peaks of the FRFs for mode 3 decrease, the peaks for FRFs of mode 2 increases as the input excitation increases the common decreases in resonance frequencies for both modes can classify the response of MST as a softening behaviour for those particular vibrating modes. In addition the FRF response for mode 3 appears to be strongly nonlinear compared to mode 2, mode 3 has a frequency shift of approximately 4Hz over an excitation range between 1N and 4.5N while mode 2 has a frequency shift of approximately 2 Hz over an excitation range between 2N and 8N.

5. Nonlinear Characterisation

Aside from identifying the aspects that drives the nonlinear behaviour (i.e. displacement, velocity), the selection of appropriate functional forms to represent the nonlinearities in the structure is mainly achieved in this step. Nonlinear characterisation also helps in determining the type of nonlinearity in the structure and in addition seeks to provide answers to some major questions that arise when dealing with nonlinear system. Some of the typical questions that arises are listed below:

a) What is the strength of the nonlinearity? i.e is it weak or strong nonlinearity

b) What is the source of the nonlinearity? i.e. is it stiffness or damping nonlinearity or both

c) What is the nonlinear stiffness characteristic? i.e. is it hardening or softening

d) What is the characteristic of the restoring force? i.e. is it symmetric or asymmetric

Using the results obtained from the Stepped-Sine test, the relationship between the force and the acceleration for specific modes was derived using a linearization method for selecting the peak amplitude of each measured FRFs. A polynomial order of best fit was then selected to visualise the nonlinear behaviour for the nonlinear FRFs measured in the stepped sine test. Figure shows the nonlinear characterised trends for modes 2 and 3.

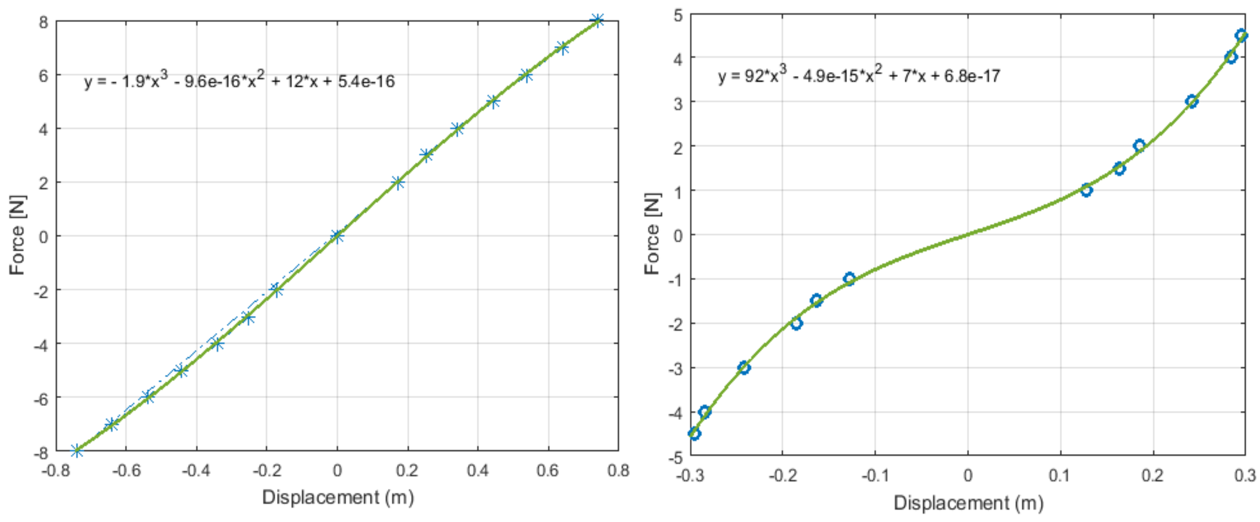

Figure 8 Measured Vs Polynomial Nonlinear Characterisation curve Modes 2 and 3

The stiffness curves show the symmetric nature of the nonlinearities in the MTS for modes 2 and 3 within the excitation range. The stiffness curves for both modes have also revealed that an accurate representation of the nonlinear behaviour in the structure should account for continuous and symmetric effects, the nature of the nonlinearities in the system also indicates that the nonlinear stiffness can be modelled using odd functions or polynomial with odd powers.

6. Conclusion

This paper has presented a case study on nonlinear identification of a missile structure designed to understand the side effects of nonlinearities caused by bolted joints and multibody assemblies. The overall aim of the paper was to demonstrate the application of a selected number of techniques for experimental identification of the missile test structures with nonlinear features incorporated in the design. The aim was achieved by three different types of experimental test, the type of test included Random excitation test which was used for the linear identification. The second test was based on Sine-Sweep excitation test, results obtained from this test were used to detect and ascertain the existence of nonlinearity in the measured time response envelop. The third test was the forced controlled stepped sine test which involved controlling input force that was used to excite the test structure. The overall results obtained from this investigation has demonstrated the presence of nonlinearity in the structure and it is therefore important to include such nonlinear phenomena in the finite element model of the structure.

7. Acknowledgement

Author Samson.B.Cooper would like to acknowledge the financial support of EPSRC and AWE for sponsoring this research.

References

1. Noël, J.P., L. Renson, and G. Kerschen, *Complex dynamics of a nonlinear aerospace structure: Experimental identification and modal interactions.* Journal of Sound and Vibration, 2014. **333**(12): p. 2588-2607.

2. D.A.Czaplewski,C.W.D.,H.Sumali,J.E.Massad,J.D.Kuppers,L.Reines,W.D.Cowan,C. P.Tigges, *A soft landing waveform for actuation of a single pole single throw Ohmic RF MEMs switch.* Journal of Micromechanical Systems 2006. **15**: p. 1586-1594.

3. D.J.Segalman, D.L.G., M.J.Starr,B.R.Resor,M.D.Jew,J.P.Lauffer,N.M.Ames, *Handbook on Dynamics of Jointed Structures* 2009, Sandia National Laboratories National Technical Information Service. p. 1-532.

4. T.Dossogne,J.P.N.,C.Grappasonni,G.Kerschen,B.Peeters,J.Debille,M.Vaes,J.Schoukens. *Nonlinear Ground Vibration Identification of an F-16 Aircraft - Part II Understanding Nonlinear Behaviour in Aerospace Structures Using Sine-sweep Testing* in *International Forum on Aeroelasticity and Structural Dynamics*. 2015. Bristol.

5. U.Fuellekrug, D.G., *Identification of weak non-linearities within complex aerospace structures.* Aerospace Science and Technology, 2012. **23**(1): p. 53-62.

6. M. F.Platten, J.R.W., J. E.Cooper, G.Dimitriadis, *Identification of a Nonlinear Wing Structure Using an Extended Modal Model.* Journal of Aircraft, 2009. **46**(5): p. 1614-1626.

7. Carrella, A. and D.J. Ewins, *Identifying and quantifying structural nonlinearities in engineering applications from measured frequency response functions.* Mechanical Systems and Signal Processing, 2011. **25**(3): p. 1011-1027.
8. C.Croufer, A.S. *Modern Solutions for Ground Vibration and Modal Survey Testing_LMS*. in *LMS Ground Vibration Test Workshop*. 2010. Spain.

9. J.P.Noel, L.R., G.Kerschen,B.peeters,S.Manzato and J.Debille. *Nonlinear dynamic analysis of an F-16 aircraft using GVT data*. in *In Proceedings of the International Forum on Aeroelasticity and Structural Dynamics (IFASD)*. 2013. Bristol.

10. K. Carney, I.Y., K. Smith, and C.Y. Peng. *Nonlinear-Dynamic-Behavior-Cassini-Spacecraft-Modal-Survey*. in *In Proceedings of the 15th International Modal Analysis Conference (IMAC)*. 1997. Orlando,FL.

11. D.J.Ewins, *Modal Testing. Theory, Practice and Application* ed. J.B.Roberts. 2000, Hertfordshire, England, SG7 6AE.: Research Studies Press Ltd.

12. Maia, S., He,Lieven,Lin,Skingle,To,Urgueira, *Theoretical and Experimental Modal Analysis*, ed. J.M.M.S. Nuno M.M.Maia. 1997, Taunton, Somerset: Research Studies Press.

13. B.Peeters, H.V.d.A., P.Guillaume, and J.Leuridan, *The PolyMAX frequency-domain method: A new standard for modal parameter estimation.* Shock and Vibration, 2004. **11**(3-4): p. 395-409.

14. P.Van Overschee, B.D.M., *N4SID:Subspace algorithms for the identification of combined deterministic-stochastic systems.* Automatica, 1994. **30**: p. 75-93.

15. J.S.Juang, R.S.P., *An eigensystem realization algorithm for modal parameter identification and model reduction.* Journal of Guidance, Control and Dynamics,, 1985. **12**: p. 620-627.

16. R.J.Allemang, D.L.B., *A unified matrix polynomial approach to modal identification.* Sound and Vibration, 1998. **221**: p. 301-322.

17. D.Goege, *Fast Identification and Characterization of Nonlinearities in Experimental Modal Analysis of Large Aircraft.* Journal of Aircraft, 2007. **44**(2): p. 399-409.

18. E.Petrov, D.J.E., *State-of-the-art dynamic analysis for nonlinear gas turbine structures.* Journal of Aerospace Engineering,Proc.of the IMech E, 2004. **218**: p. 199-211.

19. Hot, A., et al., *Detection and quantification of non-linear structural behavior using principal component analysis.* Mechanical Systems and Signal Processing, 2012. **26**: p. 104-116.

20. K.A.Kragh, J.J.T., D.Tcherniak. *Experimental Detection and Quantification of Structural Nonlinearity Using Homogeneity and Hilbert Transform Methods*. in *Proceedings of ISMA*. 2010. Belgium.