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ARTICLE

The full employment interest rate implicit in classical economic theory

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Abstract
By including the rate of normal profit in a simple model of the macro-economy, the full employment interest rate is deduced to be 4½% at which Labor is not exploited by Capital. Criticisms by Marx and Keynes of the free-market economy were misdirected at Classical theory instead of the manipulation of interest rates by Central Banks to favour Capital over Labor.

Keywords Rate of normal profit · Full employment interest rate · Classical theory · Central Banks

JEL Classification B0 · E0

1 Introduction

This paper provides a Theorem which proves 3 interrelated facts. Firstly, that in a free-market economy there is a unique full employment interest rate of 4½% at which Labor is not exploited by Capital. Secondly, that the failure of orthodox theory to detect this optimum is due to its disregard of the rate of normal profit necessary to induce savers to invest rather than just earn interest. Thirdly, that the attacks by both Marx (1867) and Keynes (1936) on the free-market economy were misdirected at Classical economics rather than on the manipulation of interest rates, to protect Capital at the expense of Labor, which was commenced in the mid nineteenth century.

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2 Background

The main idea of the Classical school (Smith 1776; Ricardo 1817, etc.) was that markets work best when they are left alone, and that there is nothing but the smallest role for government. The approach is firmly one of laissez-faire and a strong belief in the efficiency of free markets to generate economic development. Markets should be left to work because the price mechanism acts as a powerful 'invisible hand' to allocate resources to where they are best employed. General gluts causing unemployment were impossible because of Say's Law that supply always creates its own demand (Say 1803).

In terms of the macro-economy, the Classical economists assumed that the economy would always return to the full employment level of real output through an automatic self-adjustment mechanism. The interest rate was determined where the downward-sloping investment demand schedule intersected with the supposed upward-sloping saving supply schedule, and by implication, this would be the full employment interest rate.

Therefore, it was implied that the market rate of interest would tend towards a unique rate consistent with full employment and equitable rewards for both Labor and Capital. In Book II, Chapter IV of the Wealth of Nations, Smith (1776) suggested this rate should be about 5%, whilst defending Usury Laws to prevent the interest rate rising higher. On the other hand, Bentham (1787) and others disagreed with Smith, arguing that interest rates should be free for the market to determine.

Keynesian economists are sceptical that, if left alone, free markets will inevitably move towards a full employment equilibrium. The Keynesian approach is interventionist, coming from a belief that the self-interest which governs micro-economic behaviour does not always lead to long run macro-economic development or short run macro-economic stability.

However, in the General Theory (1936), Keynes seemed to agree with Classical theory about there being a full employment interest rate:

> "*In my Treatise on Money I defined what purported to be a unique rate of interest, which I called the natural rate of interest. I believed this to be a development and clarification of Wicksell's 'natural rate of interest*'. (Wicksell, 1898).
> *I had, however, overlooked the fact that in any given society there is, on this definition, a different natural rate of interest for each hypothetical level of employment. And, similarly, for every rate of interest there is a level of employment for which that rate is the 'natural' rate, in the sense that the system will be in equilibrium with that rate of interest and that level of employment. Thus it was a mistake to speak of the natural rate of interest or to suggest that the above definition would yield a unique value for the rate of interest irrespective of the level of employment. I had not then understood that, in certain conditions, the system could be in equilibrium with less than full employment.*
> *I am no longer of the opinion that the concept of a 'natural' rate of interest, which previously seemed to me a most promising idea, has anything very useful or significant to contribute to our analysis. It is merely the rate of interest*

which will preserve the status quo; and, in general we have no predominant interest in the status quo as such.
If there is any such rate of interest, which is unique and significant, it must be the rate which we might term the 'neutral' rate of interest, namely, the natural rate in the above sense which is consistent with full employment, given the other parameters of the system; though this rate might be better described, perhaps, as the optimum rate."

But Keynes never deduced the value of this 'neutral' or 'optimum' interest rate, consistent with full employment. This omission is because he did not incorporate a rate of normal profit (which induces savers to take the risk of investing) as an element in the costs of production, being a deduction from and not a cause of profits.

Central Banks nowadays set interest rates to meet inflation targets with sometimes deleterious effects on income, unemployment, wage levels and wealth inequality. The solution to solving this longstanding problem is to deduce a relationship, drawing upon *both* Classical and Keynesian ideas, between income and the interest rate such that one determines the other and vice versa. It transpires that there is a unique full employment interest rate which would ensure that Labor is not exploited by Capital, as justifiably claimed by opponents of Capitalism such as Marxism, under the current system of variable interest rates.

3 The inventory-based model

Equilibrium occurs when aggregate expenditure (demand) equals aggregate output (supply). If demand exceeds supply, inventories are depleted so that firms will increase production until realized inventories match planned inventories. If supply exceeds demand, inventories are over-stocked, and firms will decrease production until realized inventories match planned inventories.

Households can hold money not spent on consumption as cash or in non-interest current accounts, or earn interest, notated i, in saving accounts. This unspent income is liquidity preference. Alternatively, they can be induced to invest if they can at least receive a minimum additional return above interest, namely a rate of normal profit notated n, such that the return on saving invested is $[i+n]$. Without the prospect of normal profit n, they will not invest but just hold money.

Income is notated Y which equals output Y. Saving from income is the proportion s, such that saving is sY. Investment is expenditure by firms on fixed capital and physical increases in inventories. The following interpretation assumes perfect competition in a simple model with no government or overseas trade, such that cost minimisation entails profit maximization and that all saving is invested, $I=sY$, being induced by the rate normal profit, n, added to interest i, giving a return of $[i+n]$. Equilibrium requires realized inventories to equal *planned* inventories.

4 Planned inventories

Mathematics in unavoidable but is rationed in the text with full details at the end.

There is no need for a production function. Of course, output/income $Y=f$ (labor, capital), but the inventories of raw materials, part-finished and finished consumer and producer goods that permeate production and distribution, are subject to implicit financial costs.

Supply chains are interlinked and interdependent across the economy, but firms minimize costs. Planned inventories are planned unsold output and applying the micro-economic EOQ model (originated by Ford Whitman Harris, 1913) in macro to minimize costs, the planned inventories of firms (as an investment decision) are calculated:

$$\text{Total cost of inventories (TC)} = g\frac{Y}{Q} + [i+n]\frac{Q}{2}(\text{renewal cost} + \text{finance cost}),$$

where Y is the output, Q is the variable order quantity, Y/Q is the number of times inventories are renewed, $Q/2$ is the average inventories held (from Q to 0 in micro), $[i+n]$ is the cost of financing inventories (their opportunity cost) and g is the cost of one complete macro inventory renewal, an exogenous variable representing the prevailing technological, institutional, and demographic conditions.

It is accepted that g is obscure and indeterminate, but this does not affect the result, and doubts about the validity of this application of the EOQ in macro should be tempered by the existence of the economy of scale represented by the square root, which is the key factor in the changes to planned inventories. Setting the first differential of (TC) with respect to Q equal to zero:

$$\text{Planned inventories } \frac{Q^*}{2} = \sqrt{\frac{gY}{2[i+n]}}, \tag{1}$$

$$\text{Minimised Total Cost of inventories (TC)}^* = \sqrt{2gY[i+n]}. \tag{1a}$$

5 Realized inventories/unspent income/liquidity preference

It is self-evident that realized inventories represent the unspent income of households which is their liquidity preference i.e.

Liquidity preference = unspent income = unsold output = realized inventories.

It is assumed that investment by profit maximizing firms is expanded until the marginal efficiency of capital is matched with the interest rate set by the Central Bank which controls the supply of money. Keynes (General Theory, 1936, Chapter 11) defined the marginal efficiency of capital (MEC) as that rate of discount

which would make the present value of the series of annuities given by the returns expected from a capital asset during its life just equal to its supply price.

Therefore with saving continually being invested and renewed from income, the amount of unspent income supplied (liquidity preference L) must be the present value of the recurring return on saving invested to infinity of $[i+n]sY$, discounted by the interest rate which is equal to the MEC.

$$\text{Stock of unspent income} = \text{realised inventories} = L = \sum_{t=1}^{t=\infty} \frac{[i+n]sY}{[1+i]^t},$$

$$\text{Realised inventories} = \text{Liquidity preference} = L = sY + \frac{nsY}{i}. \tag{2}$$

This stock of unspent income, the same as liquidity preference and realized inventories, comprises a stock equivalent to the current saving flow sY, plus a stock of idle stock money nsY/i accumulated from previous postponed consumption. Because the MEC follows the interest rate set by the monetary authorities, the present value of the recurring future returns on saving, $(sY+nsY/i)$, which has been discounted by the [MEC], is consistent with the rate of return on current investment I, and saving is aligned with investment. Saving and investing in the present involves a trade-off between current and future consumption, but saving is not the same as unspent income.

Saving sY is both a flow over time and a stock held during time that is used for the transactions of current consumption, including precautionary balances. Idle money nsY/i from accumulated unspent income not invested is a stock which acts as the base for the creation of credit money as household borrowing for mortgages, credit cards, etc., which forms autonomous consumption.

Money equals liquidity preference equals unspent income equals unsold output equals realized inventories. Hence Banks cannot create money because as Marx correctly asserted, money is a measure of value and a means of circulation. However, the private banking system creates credit money when issuing loans using fractional-reserve banking. Self-evidently, in an economy with no commodities, there would be no money, just credit balances as accounting.

This function for unspent income/realized inventories in (2) appears similar to the liquidity preference hypothesized by Keynes (General Theory, 1936, Chapters 13 and 15) where the stock of current saving sY would act as transactions and precautionary balances and an idle stock of money nsY/i would represent speculative balances. But Keynes's explanation was psychological, and not logical.

It also has a similarity to Marx's (1867) 'hoard' of money held by capitalists which is thrown into production as investment ($sY=I$) and idle money (nsY/i) used for autonomous consumption spending by the wealthy capitalists, both returned as profit. But Marx believed that 'hoarding' was an inevitable defect of the capitalist economy when the rate of profit was falling and never considered the effect of interest rates on the decline in effective demand.

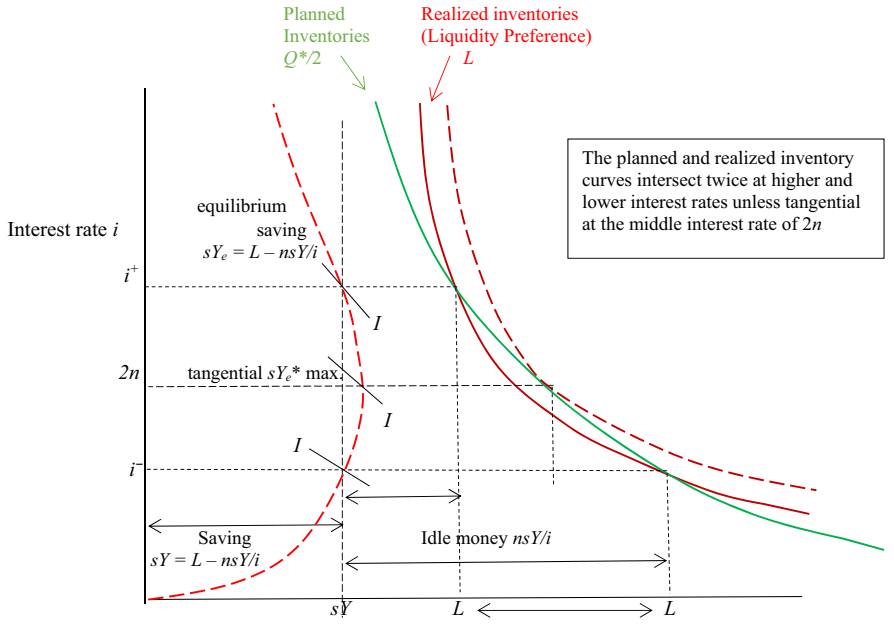

Fig. 1 Planned and realized inventories intersection

6 Equilibrium income

Equilibrium requires realized inventories L in (2) to equal planned inventories $Q^*/2$ in (1). Both curves are downward sloping and intersect twice unless they are tangential (Fig. 1).

$$\text{Equilibrium when } L = \frac{Q^*}{2} \text{ equation (2) = equation (1),}$$

$$\text{Equilibrium when } sY + \frac{nsY}{i} = \sqrt{\frac{gY}{2[i+n]}}.$$

The adjustment by firms to bring about equilibrium income is an iterative process over time, complicated by the square root in the planned inventory function, as changes in output/income works its way through saving, consumption, investment, and idle money such that the result is like turning a glove inside out. The result for output/income is in relative, not absolute values.

$$\text{Equilibrium income } Y = \frac{g}{2s^2}\frac{i^2}{[i+n]^3}, \tag{3}$$

$$\text{Equilibrium saving } sY = \frac{g}{2s}\frac{i^2}{[i+n]^3}. \tag{3a}$$

Equilibrium income is a backward-bending curve, reaching zero on the lower leg and infinity on the upper. Maximum saving/investment and maximum income/output Y^* are found by setting the first derivative of Y with respect to i equal to zero and occurs when $i=2n$.

Whenever Y is not at its maximum Y^*, and under the same technological institutional and demographic conditions represented by the exogenous variable g, income/output Y could be increased if firms employed more labor, whether in the capital or consumer goods industries. In other words, unless $i=2n$, then involuntary unemployment must exist. The optimum combination of labor and capital envisaged by Classical theory for full employment has not been reached. Unless $i=2n$, labor is being 'exploited' by capital as Marx asserted by creating a reserve army of the unemployed and suppressing wages if $i<2n$.

However, if $i>2n$, involuntary unemployment exists even though wages for those employed are higher than they would be at full employment. It is an interest rate either greater than $2n$ or less than $2n$ that is the cause of this market failure, where aggregate demand is less than that required for full employment although there are multiple equilibria situations corresponding to different interest rates, where demand equals supply.

Substituting equilibrium income from (3) into (1) or (2):

$$\text{Equilibrium liquidity preference } L_e = \frac{gi}{2s[i+n]^2}. \tag{2a}$$

Equilibrium income into $L_e - sY$:

$$\text{Equilibrium idle money } \frac{nsY}{i} = \frac{gni}{2s[i+n]^3}. \tag{2b}$$

Equilibrium income into (1a):

$$\text{Equilibrium cost of inventories } (\text{TC})_e^* = \frac{gi}{s[i+n]}. \tag{1b}$$

7 The rate of normal profit

Keynes (1930) and Knight (1921) considered risk and uncertainty and what minimum return on investment might be required to induce savers to invest, but neither introduced a rate of normal profit as an addition to interest as a cost of finance to firms, neither did the Classical or neoclassical schools of thought include the rate of normal profit.

von Thunen (1850) distinguished between 'risk' and 'uncertainty'. Many uncertainties have an objective probability which can be calculated, and such

risks become an element in the costs of production, a deduction from and not a cause of profits. One such risk is that contemplated by savers when they decide to invest for profit rather than just earn interest in a deposit account. This is an uncertainty with an objective probability which can be calculated endogenously to provide a minimum expected return as the incentive to invest, the *rate* of normal profit being n.

Under the assumption of perfect competition, the only profit derived is 'normal' profit (Marshall 1890) which is defined as when total revenue-less total cost (explicit and implicit) is zero. Hence:

$$\text{Total revenue} - \text{total cost} = 0,$$

or in macro where revenue is aggregate income:

$$(\text{Consumption} + \text{saving}) - (\text{wages} + \text{investment} + \text{inventory cost}) = 0.$$

Then with all saving invested and allowing that wages equal induced consumption

$$(\text{Autonomous consumption}) - (\text{inventory cost}) = 0.$$

Assuming that autonomous consumption is the spending of idle money, normal profit is when:

$$\text{equilibrium idle money} - \text{equilibrium inventory costs} = 0.$$

Substituting Eqs. (2b) and (1b)

$$\text{Normal profit when} \frac{gin}{2s[i+n]^3} - \frac{gi}{s[i+n]} = 0.$$

Then the interest rate $i^{\ddagger}$ which coincides with normal profit is a function of the rate of normal profit n

$$i^{\ddagger} = \sqrt{\frac{n}{2}} - n. \tag{4}$$

However, the *rate* of normal profit sufficient to induce savers to invest under all interest rates must be that which coincides not only with normal profit, but also with where the change in saving/investment is minimized with respect to changes in the interest rate. The rate of normal profit is then an endogenous constant consistent with equimarginal risk of gain or loss.

$$\text{Rate of normal profit when } \frac{\mathrm{d}^2(sY)}{\mathrm{d}i^2} = 0 \quad sY \text{ from equation (3a)},$$

which is with an interest rate $i^{\ddagger}$ that is again a function of the rate of normal profit

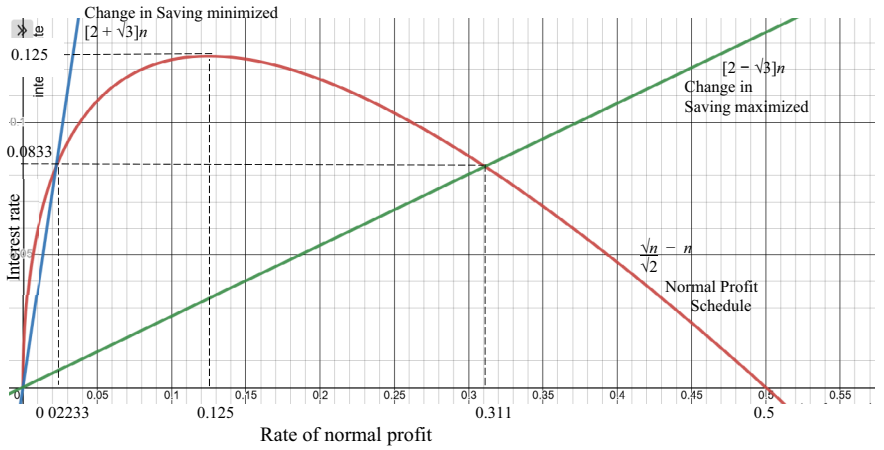

Fig. 2 Rate of normal profit optimization

$$i^{\ddagger} = \left(2 \pm \sqrt{3}\right)n. \tag{4a}$$

There are simultaneous Eqs. (4) and (4a), drawn in Fig. 2 to be solved to deduce the rate of normal profit:

$$\text{The rate of normal profit } n = \frac{\left(2 \pm \sqrt{3}\right)}{12} \approx 0.02233 \text{ or } 0.311. \tag{4b}$$

There are two results, the first of $n = 0.02233$ from minimized change in saving, the second of $n = 0.311$ from maximized change in saving, so the latter can be discarded. The rate of normal is therefore 0.02233 or approximately $n \approx 2¼\%$, endogenously determined.

8 Interest rate relationships

The full employment interest rate of $2n$ is then 4½%. The equilibrium equations for saving/investment in (3a), equilibrium liquidity preference in (2a), equilibrium idle money in (2b), and equilibrium cost of inventories in (1b), all functions of the interest rate set by the Central Bank, are all graphed in Fig. 3. Because the exogenous variables g and s are common to all the functions, they can be omitted to graph the functions.

With interest rates rising above 4½%, all of saving/investment, liquidity preference, and idle money balances decrease. Saving/investment, and therefore income/output Y, is maximized at 4½%, but with interest rates falling below 4½%, idle

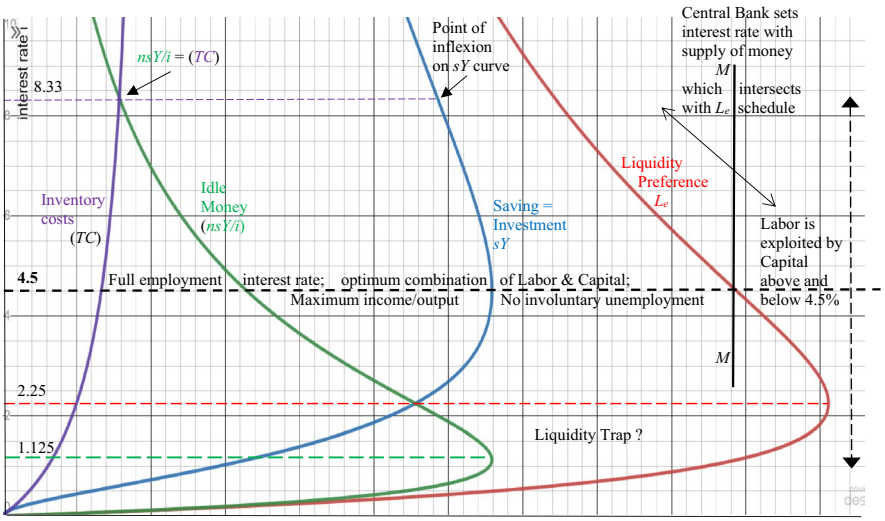

Fig. 3 Equilibrium functions in relationship. *Le* Liquidity preference, *sY* Saving/invesment, *nsY/i* Idle money, *TC* Inventory Costs

money balances increase faster than liquidity preference such that saving/investment decreases.

At an interest rate of 2¼%, liquidity preference reaches a maximum whilst saving/investment and idle money balances are equal amounts. With interest rates falling below 2¼%, liquidity preference itself decreases as well as saving/investment but idle money balances continue to increase until they maximize at an interest rate of $1^{1}/8\%$.

It can be recognized that the liquidity trap hypothesized by Keynes is with interest rates below 2¼%, although equilibrium liquidity never becomes perfectly elastic with respect to the interest rate. On the contrary, it reverses. Of course, to avoid the repeat of mass unemployment with interest rates below 2¼%, the economy can be kept in disequilibrium, with saving not equal to investment, by unconventional monetary policy. But the use of quantitative easing (based upon a fallacy) causes a distortion in asset prices and markets encouraging dubious corporate practices and eventually ends up with escalating inflation unless interest rates are rapidly raised.

On the other hand, Keynes was correct that in a situation with severe involuntary unemployment (25% as predicted by this Theorem) such as the 1930s, lowering interest rate did not help. It is government borrowing and spending that is required to boost aggregate demand. But Keynes did not seem to realize that this policy only worked by raising the interest rate up to 4½%, or that using the same policy with interest rate above 4½% would be counter-productive and cause rampant inflation (as in the 1970s).

Hence, the rise of Monetarism led by Friedman (1963) dampened the excesses of Keynesian policies by controlling the money supply. This entailed raising interest rates well above 4½% with the accompanying increase in unemployment (12% as predicted by this Theorem).

This result has been deduced assuming perfect competition, with all saving invested, and with equilibrium between supply and demand. Without perfect competition, and without the saving/investment equilibrium, the relationship found between income/output and the interest rate does not hold, although it remains as the hidden central spine. Nevertheless, the full employment interest rate of 4½% is invariable, with or without equilibrium, and is the 'optimum' interest rate that Keynes envisaged but never deduced. It is also the case that considering that the economy is invariably in disequilibrium, empirical research would never discover the full employment interest rate.

9 The undetected remedy

It was Marx's contention that the capitalist system would regularly create involuntary unemployment because this reserve army of the unemployed would allow capitalists to reduce wages and make greater profits. A falling rate of profit (and interest rates) and the consequent overproduction of commodities, to which Marx (1867) referred to, are exactly what happen with interest rates falling below 4½%.

Both Marx and Keynes were correct in their contention that the automatic process of the free market to create full employment, as envisaged by Classical economics, was a fallacy. But this was because the free market would not necessarily bring about the interest rate of 4½%, equivalent to the marginal efficiency of capital that is required to eradicate involuntary unemployment.

The solution to the problem of involuntary unemployment is to assist the free market in finding the full employment interest rate, and this requires the Central Bank to fix the interest rate at 4½%. The money supply must be kept consistent with the interest rate of 4½% to avoid inflation or deflation, whilst the government can use fiscal policy to control aggregate demand, tightening policy if the markets are trying to push the MEC up, and loosening policy if they are trying to pull it down. Even Friedman (1968) recommended that deliberate monetary policy should be taken out of the hands of Central Bankers, because of the damage they caused with unemployment and inflation. Instead, he recommended that an autopilot regime should be installed (although not, of course, with a fixed interest rate of 4½%!).

In this case, the government should set the interest rate at 4½%, such that income would always be at its maximum of Y^* with the optimum combination of labor and capital, equitable wages, and no involuntary unemployment. Maximum income Y^* is a function only of the exogenous variable g, the cost of one complete inventory renewal representing the existing technological, institutional, and demographic conditions.

10 Macro-economic policy

The Classical economists assumed that the economy would always return to the full employment level of real output through an automatic self-adjustment mechanism. This would (as here deduced) require the free market to steer the interest rate to

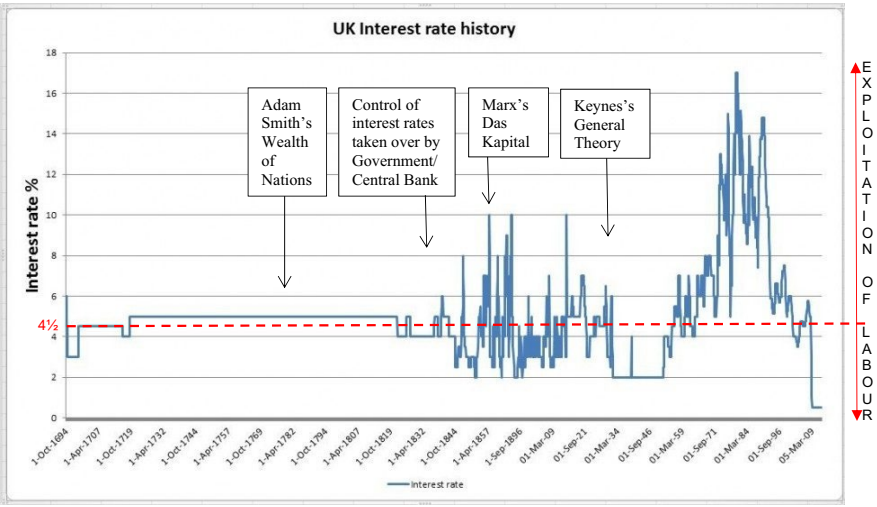

Fig. 4 The UK interest rate history. Source: ONS—Bank of England

4½% and for it to remain there. But after 1833, governments through Central Banks manipulated interest rates so that 4½% was not maintained and unemployment became a regular feature.

There must be a Central Bank to control the currency (as originally argued by Ricardo (1824), but this does not necessarily include the manipulation of interest rates. It was only because governments after the Second World War, in control of both fiscal policy and interest rates, had a tendency to cause inflation by following erroneous macro-economic policies. Hence, control of interest rates was passed to independent Central Banks.

If the interest rate were to be fixed at the full employment rate of 4½%, then the charge that Capital was exploiting Labor would not be true, and the free-market Capitalist System would be superior to Marxist alternatives with central planning because it allows freedom of the individual consistent with democracy. In a just society, only fiscal policy should be used to control the economy, fiscal tightening to dampen overheating, and fiscal loosening to boost activity should that be required.

A glance at the UK interest rate and unemployment history is revealed in Figs. 4 and 5. For a hundred years from the start of the eighteenth century, the interest rate was 4½−5%. But after the Bank Charter Acts of 1833 and 1844, Governments or Central Banks manipulated interest rates to pursue macro-economic policies that would favour Capital at the expense of Labor, thereby 'baking' inequality into the current economic model.

Unemployment levels began to fluctuate when interest rates were moved above or below 4½% which led Marx (1867) and then Keynes (1936) to blame the free market economy rather than the new regime of interest rate manipulation. They were both attacking the wrong cause of the problem.

Fig. 5 The UK unemployment history. Source: ONS—historical unemployment from MGSX

11 Conclusion

This Theorem has proved 3 interrelated facts. Firstly, in a free-market economy there is a unique full employment interest rate of 4½%, implicit in Classical economic theory, at which Labor is not exploited by Capital. The solution to the free-market mechanism taking too long to reach this optimum is to assist the process by fixing the interest rate at 4½%.

Secondly, the failure of orthodox theory to detect this optimum is due to its disregard of the rate of normal profit necessary to induce savers to invest rather than just earn interest. It is an element in the costs of production, a deduction from and not a cause of profits. It may not be possible to empirically identify the rate of normal profit as separate from other profit—but we know it must exist as the inducement to invest, and its existence could be logically contradicted by an empirical test and proved to be false (Popper 1934).

Thirdly, the attacks by both Marx (that Labor is always being exploited by Capital) and Keynes (the tendency to promote unemployment) on the free-market economy were misdirected at Classical economics rather than on the manipulation of interest rates, to protect Capital at the expense of Labor, which was commenced in the mid nineteenth century.

By assuming that the free market would tend towards the full employment interest rate, Classical theory did not contemplate that idle or speculative money would ever cause unemployment. But Fig. 3 displays the interchange between saving/investment and idle money as the interest rate varies, with involuntary unemployment apparent at interest rates both above and below 4½%

Neither Marx nor Keynes identified the full employment interest rate of 4½%, although Keynes suggested that this 'optimum' rate existed. But both Marx and Keynes understood that, as the interest rate declined (below 4½%), idle or speculative money could build up and cause aggregate demand to fall below supply leaving unsold output which caused firms to reduce production and lay off labor. Only with the interest rate fixed at 4½% is Labor not exploited by Capital.

The main divergence of this paper from orthodox theory has been to include a rate of normal profit necessary to induce savers to invest rather than just earn interest.

The other differences are firstly that planned inventories in (1) include an economy of scale through the square root function that minimizes costs; secondly, that idle money as speculative balances in (2), although inversely related to the rate of interest, is also positively related to income and thereby to saving; and thirdly that this model determines income/output in relative, and not absolute values. It is only interested with identifying the cause of involuntary unemployment which so concerned both Marx and Keynes, but neither of whom, it transpires, identified it correctly.

These findings broadly agree with those of Thomas Piketty (2014) which showed how inequality is baked into our current economic model. In a free-market economy, he argues, inequality inevitably rises faster than growth. As the incomes of the rich become reliant more on asset wealth than salaries, the old forms of redistribution, based on income tax and corporation tax, cease to work. The world's 2,153 billionaires control more wealth than the bottom 4.6 billion people (60% of the planet's population) because of the worldwide exploitation of Labor by Capital.

12 Detailed mathematics

12.1 Key notation

Y = Output/income, F(labor, capital, interestratei), endogenous variable.

i = interest rate set by the Central Bank, endogenous variable.

Q = inventory renewal quantity, endogenous variable.

g = cost of one aggregate inventory renewal, representing the existing technological, institutional, and demographic conditions, exogenous variable.

s = proportion of income saved by households, exogenous variable.

n = rate of normal profit necessary to induce savers to invest, endogenous constant.

L = Liquidity preference.

12.2 Axioms

$$\text{Saving} = \text{investment}.$$

$$\text{Liquidity preference} = \text{unspent income} = \text{unsold output} = \text{realized inventories}.$$

*E*quilibrium income when realized inventories = planned inventories.

12.3 Planned inventories

Total costs of inventories are the sum of renewal and financial costs; *Y/Q* is the number of inventory renewals costing *g* each; *Q*/2 is average planned inventories held as half the renewal quantity, and finance cost is $[i+n]$ as the opportunity cost.

$$(TC) = g\frac{Y}{Q} + [i+n]\frac{Q}{2},$$

$$\frac{\mathrm{d}(TC)}{\mathrm{d}Q} = -g\frac{Y}{Q^2} + \frac{[i+n]}{2} = 0,$$

with minimized costs of inventories, such that optimal renewal quantity is

$$Q^* = \sqrt{\frac{2gY}{[i+n]}},$$

so that average inventories consistent with minimized costs

$$\text{Planned inventories} = \frac{Q^*}{2} = \sqrt{\frac{gY}{2[i+n]}} \tag{1}$$

and minimized costs

$$(\text{TC})^* = \sqrt{2gY[i+n]}. \tag{1a}$$

13 Liquidity preference

Savers can choose between earning interest i in deposit accounts or interest *and* normal profit $[i+n]$ when the saving is invested, with the hope of additional profit above normal. Liquidity preference is the present value of the recurring future returns on saving invested $sY[i+n]$ and reflects the marginal efficiency of capital (i=MEC) so that saving and investing in the present involves a trade-off between current and future consumption.

$$\text{Liquidity preference } L = \sum_{t=1}^{t=\infty} \frac{[i+n]sY}{[1+i]^t},$$

$$L = \frac{[i+n]sY}{[1+i]} + \frac{[i+n]sY}{[1+i]^2} + \frac{[i+n]sY}{[1+i]^3} + \ldots\ldots\ldots \frac{[i+n]sY}{[1+i]^{\infty}}.$$

Multiply both sides by $[1+i]$:

$$[1+i]L = [i+n]sY + \frac{[i+n]sY}{[1+i]} + \frac{[i+n]sY}{[1+i]^2} + \frac{[i+n]sY}{[1+i]^3} \ldots\ldots\ldots \frac{[i+n]sY}{[1+i]^{\infty}},$$

$$[1+i]L = [i+n]sY + L,$$

$$L = \frac{[i+n]sY}{i}.$$

$$\text{Liquidity preference } L = sY + \frac{nsY}{i}. \tag{2}$$

14 Equilibrium income

Given an interest rate set by the Central Bank, firms adjust output and income Y to bring realized inventories (2) into line with planned inventories (1), and hence expenditure into line with income for equilibrium

$$\frac{[i+n]sY}{i} = \sqrt{\frac{gY}{2[i+n]}}.$$

Any change in income/output shifts both the planned and realized inventory curves, but not by the same amounts. After an iterative process:

Equilibrium income

$$Y_e = \frac{g}{2s^2}\frac{i^2}{[i+n]^3}. \tag{3}$$

This equation for equilibrium income as a function of the interest rate is a backward-bending curve with an upper leg rising gradually to infinity and a lower leg falling sharply to zero.

The apex of the curve is found by setting the first differential of Y_e with respect to i, equal to zero

$$\frac{\mathrm{d}Y_e}{\mathrm{d}i} = \frac{g}{2s^2}\left[\frac{2i}{[i+n]^3} - \frac{3i^2}{[i+n]^4}\right] = 0,$$

when output/income is at its maximum for all values of g and s, and

$$2i^2 + 2in - 3i^2 = 0,$$

$$i^* = 2n.$$

The full employment interest rate at which there is no involuntary unemployment and wages are at a maximum through the optimum level of investment by firms. Hence Labor is not exploited by Capital if the interest rate is fixed at $i=2n$, but Labor is exploited by Capital at higher or lower interest rates either through unemployment or low wages.

$$Y^* = \frac{2g}{27ns^2}.$$

Maximum income/output in terms of *g, s and n,* when $i=2n$: saving is Y_e in (3) multiplied by s

$$(sY) = \frac{gi^2}{2s[i+n]^3}. \tag{3a}$$

Substituting Y_e from (3) into (2), equilibrium liquidity preference

$$L = \frac{gi}{2s[i+n]^2}, \tag{2a}$$

such that idle money

$$L - (sY) = \frac{gni}{2s[i+n]^3}, \tag{2b}$$

and equilibrium total costs of inventories

$$(\mathrm{TC})_e = \frac{gi}{s[i+n]}. \tag{1b}$$

15 Rate of normal profit

Normal profit is where total revenue minus total costs is zero; or simplified in macro where idle money minus the cost of inventories is zero, using equilibrium conditions. The minimum rate of normal profit required to induce savers to invest is then found where the change in saving with respect to changes in the interest rate is at a minimum (increase or decrease in saving and investment equally likely) so that the rate of normal profit is stable.

$$\text{Idle money} - \text{total cost of inventories} = 0,$$

$$\frac{gni}{2s[i+n]^3} - \frac{gi}{s[i+n]} = 0 \quad \text{equations (2b)} - \text{(1b)} = 0,$$

$$\frac{n}{2} = [i+n]^2,$$

$$i^{\ddagger} = \sqrt{\frac{n}{2}} - n. \tag{4}$$

Then the second differential of saving sY with respect to the interest rate is set to zero:

$$\frac{\mathrm{d}^2(sY)}{\mathrm{d}i^2} = \frac{g}{2s}\left[\frac{2}{[i+n]^3} - \frac{12i}{[i+n]^4} + \frac{12i^2}{[i+n]^5}\right]$$
$$= 0 \text{ at minimum,}$$

$$i^2 - 4in + n^2 = 0,$$

$$i^{\ddagger} = \left[2 \pm \sqrt{3}\right]n. \tag{4a}$$

Satisfying both conditions (4) and (4a) simultaneously with the $i^{\ddagger}$'s equal

$$\left[2 \pm \sqrt{3}\right]n = \sqrt{\frac{n}{2}} - n,$$

$$n = \frac{\left[2 \pm \sqrt{3}\right]}{12}, \tag{4b}$$

$$n \approx 2\tfrac{1}{4}\% \text{ or } 31\%.$$

The *minimum* rate of normal profit is 2¼% and the full employment interest rate, $2n$, is 4½%.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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