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$\mathscr{L}_1$ Adaptive Output Feedback Control of Small Unmanned Aerial Vehicles

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An approach for output feedback $\mathcal{L}_1$ adaptive control of small Unmanned Aerial Vehicles (UAVs) is presented in this paper. The design is based on a state observer instead of the state predictor. The main advantage is that a full state measurement can be avoided, and the design and implementation of the controller are simplified. Furthermore, since the state space description is maintained, the system dynamics including uncertainties can be specified with physical insight, which simplifies practical applications. The adaptation law borrows insights from the sliding mode control to estimate the unknown bounds of external disturbances. Flight test results for the control of a small UAV show the robustness of the $\mathcal{L}_1$ adaptive controller to large uncertainties and disturbances.

Keywords: $\mathcal{L}_1$ Adaptive Control, Unmanned Aerial Vehicles, Fault-tolerant Control, Output feedback Control

1. Introduction

Small fixed-wing UAVs that is, with wingspans less than 2 metres and payload smaller than 2 kg are gaining growing interest because of their low cost, high manoeuvrability, and simple maintenance. They are used for a wide range of military and civilian tasks [1, 2]. Despite the advantages of small UAVs, their control is still a challenging problem because of their small size, light weight, relatively low speeds, mass/payload variations, reduced payload capacity, unknown dynamics, limited sensors suite and insufficient onboard computing [3–6].

A solution can be provided through the use of adaptive control [7, 8]. Adaptive control was introduced to meet the challenge of automatically adjusting the controller parameters in the presence of uncertain and time varying aircraft dynamics . It is a suitable method for small UAV because it is robust against disturbances, automatically reconfigures its parameters without a fault detection scheme, does not need an accurate model of the UAV and it is relatively easy for implementation [9]. The researches in [10–16] have presented successful applications of adaptive control for fixed-wing UAVs.

However, in low-cost UAV control the state vector is not always available through measurements by sensors. This is why it is necessary to design output-feedback approaches that take into account this property of small UAVs. Many methods dealing with adaptive output feedback control are described in [17–25] to cite a few.

A crucial aspect in applying adaptive control techniques to real-world systems is the transient response guarantee, in the absence of which, overly poor tracking behaviour can occur before ideal asymptotic convergence takes place [26–28]. Most adaptive control methods focus on the asymptotic performance, providing no transient performance guarantee without resorting to high-gain feedback [29].

A solution to this issue is based on $\mathcal{L}_1$ adaptive control [30]. The $\mathcal{L}_1$ adaptive control architecture decouples the estimation loop from the control loop through the introduction of a low-pass filter. As a result, arbitrarily fast adaptation can be used without sacrificing system robustness. These characteristics make it suitable for fixed-wing UAVs control in the presence of faults and external disturbances [31–38].

Relatively few approaches are described for output feedback $\mathcal{L}_1$ adaptive control. In [39] was designed an $\mathcal{L}_1$ adaptive output-feedback controller for unknown dimensional systems with unmodeled dynamics and time-varying uncertainties. In [40] was designed an $\mathcal{L}_1$ adaptive output feedback controller for non-strictly-positive-real reference systems. Missile longitudinal autopilot design was used to illustrate the theoretical results. In [41] was presented the application of $\mathcal{L}_1$ adaptive output-feedback control to two different fields of engineering: feedback control of human anaesthesia, and ascent control of a NASA crew launch vehicle. In [42] the $\mathcal{L}_1$ adaptive output feedback controller was considered to accommodate the disturbance entering at the system output. Experimental results have shown successful application to managed pressure drilling. In [43] was developed an $\mathcal{L}_1$ adaptive output feedback controller for a class of underactuated multi-input multi-output (MIMO) systems. The proposed approach has been applied as an augmentation of an existing three-loop autopilot. A control methodology based on $\mathcal{L}_1$ adaptive output feedback control has been applied to temperature control [44]. The work of [45] demonstrated the design of the $\mathcal{L}_1$ adaptive output-feedback control combined with the model predictive control for multivariable nonlinear systems subject to constraints. In [46] $\mathcal{L}_1$ adaptive output feedback control was applied for general Partial Differential Equation (PDE) systems. The design presented in [38] addressed uncertain nonlinear systems in the presence of unmodeled dynamics and actuator faults.

The common drawback of the proposed approaches is that they are based on transfer function formulation which makes the interpretation of uncertainties and the definition of reference models less intuitive compared to a formulation with a state-space model [9,47]. In fact, the state variables can more comprehensively reflect the internal characteristics of a system [38]. Furthermore, the previous approaches for $\mathcal{L}_1$ output feedback adaptive control were formulated under the assumption that the bounds of external disturbances are known. However, these bounds are hard to quantify in practice. Therefore, if an external disturbance goes beyond its supposed bound this will result in poor performance of the controller. Moreover, most of the previous $\mathcal{L}_1$ adaptive output-feedback designs are particularly complex, which makes difficult their practical implementation on the resource-limited onboard computers of small UAVs.

In this paper, based on [9], the state-feedback $\mathcal{L}_1$ adaptive controller for systems with disturbances of unknown bounds [48] is extended to output-feedback. The proposed solution uses a state observer instead of the state predictor, characteristic of the $\mathcal{L}_1$ adaptive controller. When compared with the observer, a state predictor is designed based on the assumption that the full system state is measurable [30], whereas the state observer is based on the measured output of the system. The main advantage of this scheme is that only the output measurements are required, while a state space representation can be maintained, which makes the design and the analysis of the controller similar to the state feedback approach. As a consequence, the system dynamics including uncertainties can be specified with physical insight, which simplifies controller design and analysis. The proposed solution was initially introduced in [47], where was presented an output feedback $\mathcal{L}_1$ adaptive controller that combines the use of an estimated state feedback with a switching adaptation law. However, it was assumed that the bounds of external disturbances are known. Furthermore, only testbed tests were presented.

The contributions of this paper are:

- Development of a method for output feedback $\mathcal{L}_1$ adaptive control based on the Luenberger observer instead of the state predictor, so as to maintain a state-space formulation of the controller design that reflects more comprehensively the internal characteristics of the system.
- Improving the robustness of the output-feedback $\mathcal{L}_1$ adaptive controller in the presence of external disturbances by using the sliding surface structure from sliding mode control. This approach relaxes what is commonly assumed in $\mathcal{L}_1$ adaptive output-feedback control that external disturbances bounds are known.
- Design and flight demonstration of an output feedback $\mathcal{L}_1$ adaptive fault-tolerant controller for fixed-wing UAVs. Only [33,49] presented flight test results for the output feedback $\mathcal{L}_1$ adaptive controller designed in [39].

2. Observer-based $\mathcal{L}_1$ Adaptive Control

Given a class of Single-Input Single-Output systems defined by

$$\begin{aligned}\dot{x}(t) &= A_m x(t) + b\big(\omega u(t) + \theta^\top x(t) + \eta_m(t)\big) + \eta_u(t,x),\\ y(t) &= c^\top x(t), \qquad x(0) = x_0.\end{aligned} \tag{1}$$

where $A_m \in \mathbb{R}^{n\times n}$ is a known Hurwitz matrix that defines the desired dynamics of the system; b, $c \in \mathbb{R}^n$ are known constant vectors; $x(t) \in \mathbb{R}^n$ is the state vector which is assumed available through measurement; $u(t) \in \mathbb{R}$ is the control input; $y(t) \in \mathbb{R}$ is the system output; $\omega \in \mathbb{R}$ is an unknown constant with known sign representing the model input uncertainties; $\theta \in \mathbb{R}^n$ is a vector of constant unknown parameters representing model uncertainties; $\eta_m(t) \in \mathbb{R}$ is an unknown matched disturbance; and $\eta_u(t,x) \in \mathbb{R}^n$ is an unknown unmatched disturbance.

Assumption 1. The pair (A_m, b) is controllable and the pair (A_m, c) is observable.

Assumption 2. The non-linear functions $\eta_m(t)$ and $\eta_u(t,x)$ are uniformly bounded, i.e., there exist unknown real constants $L_m > 0$ and $L_u > 0$, such that for all $t \geq 0$ the following bounds hold:

$$|\eta_m(t)| \leq L_m \text{ and } \|\eta_u(t,x)\| \leq L_u.$$

Assumption 3. The unknown model parameters are bounded, i.e., $\theta \in \Theta$, where Θ is a known compact convex set and $0 < \omega_l \leq \omega \leq \omega_u$.

Remark 1. Assumptions 2 and 3 are conventionally acceptable for real systems, given that a superior bound of disturbances and unknown parameters, which the system may hold without being broken, is usually known from technical specifications or engineering insights.

Assumption 4. The system is minimum phase of relative degree one, i.,e, $c^\top b \neq 0$. This assumption is quite common in adaptive output feedback control, because direct adaptive controllers employ high gain feedback that can drive the system to instability [50].

For this class of systems, the state feedback $\mathcal{L}_1$ adaptive control architecture is composed of a state predictor, a control law, and an adaptation mechanism [30]. When the full state measurement is not available, it is proposed here to use a sate observer instead of the state predictor in order to maintain a state-space representation.

2.1. *Controller Design*

The state observer is defined as follows

$$\begin{aligned}\dot{\hat{x}}(t) =& A_m\hat{x}(t) + b\big(\hat{\omega}(t)u(t) + \hat{\theta}^\top(t)\hat{x}(t) + \hat{\eta}_m(t)\big)\\ &+ \hat{\eta}_u(t) - L_v\tilde{y}(t),\\ \hat{y}(t) =& c^\top\hat{x}(t), \qquad \hat{x}(0) = \hat{x}_0,\end{aligned} \tag{2}$$

where $\hat{x}$ is the predicted state and, $\hat{\theta}(t)$, $\hat{\omega}(t)$, $\hat{\eta}_m(t)$, and $\hat{\eta}_u(t)$ are the estimates of the unknown system parameters and disturbances, $L_v \in \mathbb{R}^n$ is chosen such that $A_c = A_m - L_v c^\top$ is Hurwitz, $\tilde{y}(t) = \hat{y}(t) - y(t)$ is the output estimation error, and $\hat{x}(0)$ is initialized arbitrarily.

Remark 2. A particular case is the use of an open-loop observer by choosing $L_v = 0$ [47].

The sliding surface is given by

$$\sigma(t) = \lambda\tilde{y}(t), \tag{3}$$

where $\lambda \in \mathbb{R}^*$ is an arbitrary real.

The estimation of the matched disturbance $\eta_m(t)$ is defined by

$$\hat{\eta}_m(t) = \begin{cases} -(\lambda c^\top b)^{-1}\alpha\frac{\sigma(t)}{|\sigma(t)|} - \hat{L}_m(t)\frac{\lambda c^\top b\sigma(t)}{|\lambda c^\top b\sigma(t)|} & \text{if}\, \sigma(t) \neq 0,\\ 0 & \text{otherwise},\end{cases} \tag{4}$$

where $\alpha \in \mathbb{R}^+$ is arbitrary, and the estimated bound $\hat{L}_m(t)$ of the unmatched disturbance $\eta_m(t)$ is given by

$$\dot{\hat{L}}_m(t) = \Gamma|\lambda c^\top b\sigma(t)|,\ L_{m0} = \hat{L}_m(0), \tag{5}$$

where $\Gamma \in \mathbb{R}^+$ is the adaptation rate.

The estimation of the unmatched disturbance $\eta_u(t,x)$ is given by

$$\hat{\eta}_u(t) = \begin{cases} -\hat{L}_u(t)\frac{\left(\lambda c^\top\sigma(t)\right)^\top}{\|\lambda c^\top\sigma(t)\|} & \text{if}\, \sigma(t) \neq 0,\\ 0 & \text{otherwise},\end{cases} \tag{6}$$

where the estimated bound $\hat{L}_u(t)$ of the unmatched disturbance $\eta_u(t,x)$ is computed by

$$\dot{\hat{L}}_u(t) = \Gamma\|\lambda c^\top\sigma(t)\|,\ L_{u0} = \hat{L}_u(0). \tag{7}$$

The estimation of the unknown parameter θ and the input gain ω are defined by

$$\begin{aligned}\dot{\hat{\theta}}(t) &= -\Gamma\ \mathrm{Proj}\big(\hat{\theta}(t), \lambda c^\top b\,\sigma(t)\hat{x}(t)\big),\\ \dot{\hat{\omega}}(t) &= -\Gamma\ \mathrm{Proj}\big(\hat{\omega}(t), \lambda c^\top b\,\sigma(t)u(t)\big).\end{aligned} \tag{8}$$

The control law is given by

$$u(s) = kD(s)\Big(k_g\ r(s) - \hat{\nu}(s) - \phi(s)\hat{\eta}_u(s)\Big), \tag{9}$$

where $k > 0$ is arbitrary, $D(s)$ is a transfer function that leads to a strictly proper stable filter $C(s) = \omega kD(s)/(1+\omega kD(s))$ with $C(0) = 1$, the static gain is chosen as $k_g = -1/(c^\top A_m^{-1}b)$, $\hat{\nu}(s)$ is the Laplace transformation of the term $\hat{\theta}^\top(t)\hat{x}(t) + \hat{\omega}(t)u(t) + \hat{\eta}_m(t)$, $\phi(s) = c^\top(sI - A_m)^{-1}/H_m(s)$, $H_m(s) = c^\top(s\mathbb{I} - A_m)^{-1}b$, and $\hat{\eta}_u(s)$ is the Laplace transform of $\hat{\eta}_u(t)$.

Remark 3. The adaptation laws of the external disturbances in equations (4) and (6) use the estimated bounds from equations (5) and (7). This relaxes the assumption that the bounds of the external disturbances are known, which is required in $\mathcal{L}_1$ adaptive control based on projection-type adaptive laws [39].

2.2. *Controller analysis*

In this section, the performance of the $\mathcal{L}_1$ adaptive controller is analysed. More specifically it is shown that:

- The reference model resulting from perfect knowledge of the uncertainties and a corresponding non-adaptive controller is stable, subject to some conditions involving the filter $C(s)$.
- The prediction error, i.e. the errors between the states of the plant and those of the state predictors, is bounded.
- The difference between the states/input of the system and those of the reference system is proportional to the prediction error

Let

$$L = \max_{\theta\in\Theta}\|\theta\|_1, H(s) = (s\mathbb{I} - A_m)^{-1}b, G(s) = \big(1 - C(s)\big)H(s).$$

The $\mathcal{L}_1$ adaptive controller is defined via equations (2) to (9), and is subject to the $\mathcal{L}_1$-norm condition

$$\|G(s)\|_{\mathcal{L}_1}L < 1. \tag{10}$$

Moreover, the design of k and $D(s)$ needs to ensure that

$$G_u(s) = (s\mathbb{I} - A_m)^{-1} - C(s)H(s)\phi(s), \tag{11}$$

is proper and stable.

Closed-Loop Reference System

The reference system in this case is the same as in all previous $\mathcal{L}_1$ adaptive control architectures. The reference system is defined by

$$\begin{aligned}\dot{x}_r(t) =& A_c x_r(t) + b\big(\omega u_r(t) + \theta^\top x_r(t) + \eta_m(t)\big) + \eta_u(t, x_r),\\ y_r(t) =& c^\top x_r(t), \qquad x_r(0) = x_0.\end{aligned} \tag{12}$$

The reference control law is given by

$$u_r(s) = \frac{C(s)}{\omega}\Big(k_g r(s) - \theta^\top x_r(s) - \eta_m(s) - \phi(s)\eta_u(s)\Big). \tag{13}$$

Lemma 1 If the filter $C(s)$ is designed such that it verifies the $\mathcal{L}_1$-norm condition in (10) and the requirement in (11), then the closed-loop reference system in (12) and (13) is Bounded-Input Bounded-Stable (BIBS) stable with respect to the reference input and initial conditions.

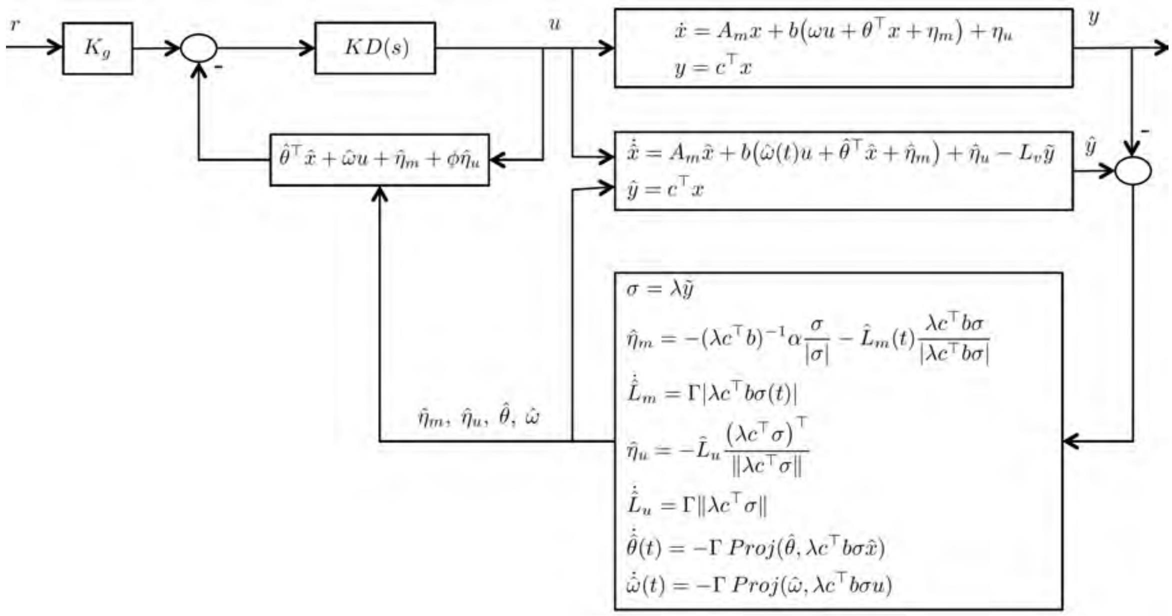


Fig. 1. Block diagram of the observer-based $\mathcal{L}_1$ adaptive controller.

Proof. The closed-loop reference system (12) and (13) can be written

$$\begin{aligned} x_r(s) =& H(s)C(s)K_g r(s) + G(s)\theta^\top x_r(s) \\ &+ G(s)\eta_m(s) + G_u(s)\eta_u(s) + x_{in}(s), \end{aligned} \tag{14}$$

where $x_{in}(s) = (s\mathbb{I} - A_m)^{-1} x_0$.

Then, for all $t \in [0, \tau]$ we have

$$\begin{aligned} \|x_{r_\tau}\|_{\mathcal{L}_\infty} \le& \|C(s)H(s)\|_{\mathcal{L}_1} K_g \|r_\tau\|_{\mathcal{L}_\infty} + \|G(s)\|_{\mathcal{L}_1} L \|x_{r_\tau}\|_{\mathcal{L}_\infty} \\ &+ \|G(s)\|_{\mathcal{L}_1} \|\eta_{m_\tau}\|_{\mathcal{L}_\infty} + \|G_u(s)\|_{\mathcal{L}_1} \|\eta_{u_\tau}\|_{\mathcal{L}_\infty} \\ &+ \|x_{in_\tau}\|_{\mathcal{L}_\infty}, \end{aligned} \tag{15}$$

where $\|\cdot\|_{\mathcal{L}_\infty}$ denotes for the $\mathcal{L}_\infty$ norm and $\|(\cdot)_\tau\|_{\mathcal{L}_\infty}$ denotes for the truncated $\mathcal{L}_\infty$ norm at the time instant τ.

Substituting the upper bounds of η_m and η_u and solving for $\|x_{r_\tau}\|_{\mathcal{L}_\infty}$ in the equation above to obtain the following bound

$$\begin{aligned} \|x_{r_\tau}\|_{\mathcal{L}_\infty} \le& \frac{\|C(s)H(s)\|_{\mathcal{L}_1} K_g \|r_\tau\|_{\mathcal{L}_\infty} + \|G(s)\|_{\mathcal{L}_1} L_m}{1 - \|G(s)\|_{\mathcal{L}_1} L} \\ &+ \frac{\|G_u(s)\|_{\mathcal{L}_1} L_u + \|x_{in}\|_{\mathcal{L}_\infty}}{1 - \|G(s)\|_{\mathcal{L}_1} L}. \end{aligned} \tag{16}$$

If the $\mathcal{L}_1$ norm condition in (10) is verified then $\|x_{r_\tau}\|_{\mathcal{L}_\infty}$ is uniformly bounded for all $\tau > 0$, and the proof is complete. □

Transient and Steady-State Performance

In the following Lemma, it is stated that the prediction error $\tilde{x}(t) = \hat{x}(t) - x(t)$ and the estimation errors of the unknown parameters are bounded.

Lemma 2 The following uniform bound holds for the prediction error

$$\|\tilde{x}\|_{\mathcal{L}_\infty} < \rho = \frac{\alpha}{|\lambda| \|c\| \big(\|A_c\| + \|b\|\, \theta_m\big)},$$

where $\theta_m = \max_{\theta \in \Theta} \|\theta\|$.

Proof. From (1) and (2), the estimation error dynamics can be written as follows

$$\dot{\tilde{x}} = A_c \tilde{x} + b\big(\tilde{\omega} u + \theta^\top \tilde{x} + \tilde{\theta}^\top \hat{x} + \tilde{\eta}_m\big) + \tilde{\eta}_u, \tag{17}$$

where $\tilde{\theta} = \hat{\theta} - \theta$, $\tilde{\omega} = \hat{\omega} - \omega$, $\tilde{\eta}_m = \hat{\eta}_m - \eta_m$ and $\tilde{\eta}_u = \hat{\eta}_u - \eta_u$. We define also $\tilde{L}_m = \hat{L}_m - L_m$ and $\tilde{L}_u = \hat{L}_u - L_u$.

Consider the Lyapunov function candidate

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\Gamma^{-1}\big(\tilde{\theta}^\top \tilde{\theta} + \tilde{\omega}^2 + \tilde{L}_m^2 + \tilde{L}_u^2\big). \tag{18}$$

The derivative of the Lyapunov function is given by

$$\dot{V} = \sigma\dot{\sigma} + \Gamma^{-1}\big(\tilde{\theta}^\top \dot{\hat{\theta}} + \tilde{\omega}\dot{\hat{\omega}} + \tilde{L}_m \dot{\hat{L}}_m + \tilde{L}_u \dot{\hat{L}}_u\big). \tag{19}$$

From (3), the derivative of the sliding surface can be written as follows

$$\dot{\sigma} = \lambda c^\top A_c \tilde{x} + \lambda c^\top b\big(\theta^\top \tilde{x} + \tilde{\theta}^\top \hat{x} + \tilde{\omega} u + \tilde{\eta}_m\big) + \lambda c^\top \tilde{\eta}_u. \tag{20}$$

Replacing (20) in (19), it follows that

$$\begin{aligned}\dot{V} =&\sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\big(\theta^\top\tilde{x} + \tilde{\theta}^\top\hat{x} + \tilde{\omega}u\big)\\ &+ \lambda c^\top b\big(\hat{\eta}_m - \eta_m\big) + \sigma\lambda c^\top\big(\hat{\eta}_u - \eta_u\big)\\ &+ \Gamma^{-1}\big(\tilde{\theta}^\top\dot{\hat{\theta}} + \tilde{\omega}\dot{\hat{\omega}} + \tilde{L}_m\dot{\hat{L}}_m + \tilde{L}_u\dot{\hat{L}}_u\big)\\ =&\sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\theta^\top\tilde{x} + \sigma\lambda c^\top b\hat{\eta}_m\\ &- \sigma\lambda c^\top b\eta_m + \sigma\lambda c^\top\hat{\eta}_u - \sigma\lambda c^\top\eta_u\\ &+ \tilde{\theta}^\top\hat{x}\sigma\lambda c^\top b + \tilde{\omega}u\sigma\lambda c^\top b\\ &+ \Gamma^{-1}\big(\tilde{\theta}^\top\dot{\hat{\theta}} + \tilde{\omega}\dot{\hat{\omega}} + \tilde{L}_m\dot{\hat{L}}_m + \tilde{L}_u\dot{\hat{L}}_u\big).\end{aligned} \tag{21}$$

Given $\hat{\eta}_m$ and $\hat{\eta}_u$ from (4) and (6) and the adaptation law (8) it can be written

$$\begin{aligned}\dot{V} =& -\alpha|\sigma| + \sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\theta^\top\tilde{x}\\ &- \sigma\lambda c^\top b\eta_m - \sigma\lambda c^\top\eta_u\\ &- |\sigma\lambda c^\top b|\hat{L}_m - \|\sigma\lambda c^\top\|\hat{L}_u\\ &+ \Gamma^{-1}\big(\tilde{L}_m\dot{\hat{L}}_m + \tilde{L}_u\dot{\hat{L}}_u\big).\end{aligned} \tag{22}$$

Hence, the following upper bound can be derived

$$\begin{aligned}\dot{V} \leq& -\alpha|\sigma| + \sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\theta^\top\tilde{x}\\ &+ |\lambda c^\top b\sigma|\,|\eta_m| + \|\lambda c^\top\sigma\|\|\eta_u\|\\ &- |\sigma\lambda c^\top b|\hat{L}_m - \|\sigma\lambda c^\top\|\hat{L}_u\\ &+ \Gamma^{-1}\big(\tilde{L}_m\dot{\hat{L}}_m + \tilde{L}_u\dot{\hat{L}}_u\big).\end{aligned} \tag{23}$$

From assumption 2 and 3, it can be written

$$\begin{aligned}\dot{V} \leq& -\alpha|\sigma| + \sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\theta^\top\tilde{x}\\ &- |\sigma\lambda c^\top b|\tilde{L}_m - \sigma\|\lambda c^\top\|\tilde{L}_u\\ &+ \Gamma^{-1}\big(\tilde{L}_m\dot{\hat{L}}_m + \tilde{L}_u\dot{\hat{L}}_u\big).\end{aligned} \tag{24}$$

Considering the adaptation laws from (5) and (7), it follows that

$$\dot{V} \leq -\alpha|\sigma| + \sigma\lambda c^\top A_c\tilde{x} + \sigma\lambda c^\top b\theta^\top\tilde{x}. \tag{25}$$

Given that $\sigma \leq |\sigma|$, $\lambda c^\top A_c\tilde{x} \leq |\lambda|\|c\|\|A_c\|\|\tilde{x}\|$, $\lambda c^\top b\theta^\top\tilde{x} \leq |\lambda|\|c\|\|b\|\theta\|\|\tilde{x}\|$ and since the projection law insures $\|\theta\| \leq \theta_m$, then the following bound holds for (25)

$$\dot{V} \leq |\sigma|\Big(-\alpha + |\lambda|\|c\|\big(\|A_c\| + \|b\|\,\theta_m\big)\|\tilde{x}\|\Big). \tag{26}$$

If α is chosen arbitrarily large so that it verifies $\forall\, t > 0$

$$\alpha > |\lambda|\|c\|\big(\|A_c\| + \|b\|\,\theta_m\big)\|\tilde{x}\|, \tag{27}$$

then

$$\dot{V} < 0. \tag{28}$$

It follows that the sliding surfacer σ, the estimation errors of $\tilde{\theta}$, $\tilde{\omega}$, $\tilde{L}_m$ and $\tilde{L}_u$ are uniformly bounded,

From (27), the following bound holds for $\tilde{x}$

$$\|\tilde{x}\| < \rho = \frac{\alpha}{|\lambda|\|c\|\big(\|A_c\| + \|b\|\,\theta_m\big)}. \tag{29}$$

Recalling, that $\|\cdot\|_{\mathcal{L}_\infty} \leq \|\cdot\|$ this completes the proof. □

Remark 4. We notice that the upper bound of the prediction error in (42) is inverse proportional to the the sliding surface coefficient defined in (3), implying that one can arbitrarily improve the prediction error by increasing the sliding surface coefficient λ.

Next, in the following theorem, the performance bounds of the $\mathcal{L}_1$ adaptive controller are shown.

Theorem Given the system (1), the reference system (12), (13) and the $\mathcal{L}_1$ adaptive controller (2), (5), (8) and (9) we have

$$\|x_r - x\|_{\mathcal{L}_\infty} \leq \gamma_1, \tag{30}$$

and

$$\|u_r - u\|_{\mathcal{L}_\infty} \leq \gamma_2, \tag{31}$$

where

$$\begin{aligned}\gamma_1 =&2\frac{\|G(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1}L}L_m + 2\frac{\|G_u(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1}L}L_u\\ &+ \frac{\|H(s)C(s)H_m^{-1}(s)c^\top\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1}L}\rho,\end{aligned}$$

and

$$\begin{aligned}\gamma_2 =&\|\frac{C(s)}{\omega}\|_{\mathcal{L}_1}\Big(L\gamma_1 + 2\big(L_m + \|\phi(s)\|_{\mathcal{L}_1}L_u\big)\Big)\\ &+ \|\frac{C(s)}{\omega}H_m^{-1}(s)c^\top\|_{\mathcal{L}_1}\rho.\end{aligned}$$

Proof. The control law in (9) can be written as

$$\begin{aligned}u(s) =&\frac{C(s)}{\omega}\Big(K_g r(s) - \theta^\top x(s) - \eta_m(s)\Big)\\ &- \frac{C(s)}{\omega}\Big(\phi(s)\big(\eta_u(s) + \tilde{\eta}_u(s)\big) - \tilde{\nu}(s)\Big),\end{aligned} \tag{32}$$

where $\tilde{\nu}(s)$ is the Laplace transformation of $\tilde{\nu}(t) = \tilde{\omega}u(t) + \tilde{\theta}^\top x(t) + \tilde{\eta}_m(t)$ and $\tilde{\eta}_u(s)$ is the Laplace transformation of $\tilde{\eta}_u(t)$.

Hence, the Laplace transformation of the closed loop system (1) and (32) can be written

$$\begin{aligned}x(s) =&H(s)C(s)K_g r(s)\\ &+ G(s)\big(\theta^\top x(s) + \eta_m(s)\big) + G_u(s)\eta_u(s)\\ &- H(s)C(s)\big(\tilde{\nu}(s) + \phi(s)\tilde{\eta}_u(s)\big) + x_{in}(s).\end{aligned} \tag{33}$$

Taking the difference of (14) and (33) it follows that

$$\begin{aligned}x_r(s) - x(s) =&G(s)\Big(\theta^\top\big(x_r(s) - x(s)\big) + \eta_m(s) - \eta_{mr}(s)\Big)\\ &- G_u(s)\big(\eta_u(s) - \eta_{ur}(s)\big)\\ &+ H(s)C(s)\big(\tilde{\nu}(s) + \phi(s)\tilde{\eta}_u(s)\big).\end{aligned} \tag{34}$$

From (17) the Laplace transformation of the prediction error dynamics can be written

$$\tilde{x}(s) = H(s)\tilde{\nu}(s) + (sI - A_m)^{-1}\tilde{\eta}_u(s). \tag{35}$$

Multiplying both terms of (35) by $H_m^{-1}(s)c^\top$ one obtains

$$H_m^{-1}(s)c^\top \tilde{x}(s) = \tilde{\nu}(s) + \phi(s)\tilde{\eta}_u(s). \tag{36}$$

Substituting in (34) it follows that

$$\begin{aligned} x_r(s) - x(s) =& G(s)\theta^\top \big(x_r(s) - x(s)\big) \\ &- G(s)\big(\eta_m(s) - \eta_{mr}(s)\big) \\ &- G_u(s)\big(\eta_u(s) - \eta_{ur}(s)\big) \\ &+ H(s)C(s)H_m^{-1}(s)c^\top \tilde{x}(s). \end{aligned} \tag{37}$$

Solving for $x_r(s) - x(s)$, the following bound holds for $t \in [0, \tau]$

$$\begin{aligned} \|(x_r - x)_\tau\|_{\mathcal{L}_\infty} \le& \frac{\|G(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} \|(\eta_{m_\tau} - \eta_{mr})_\tau\|_{\mathcal{L}_\infty} \\ &+ \frac{\|G_u(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} \|(\eta_u - \eta_{ur})_\tau\|_{\mathcal{L}_\infty} \\ &+ \frac{\|H(s)C(s)H_m^{-1}(s)c^\top\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} \|\tilde{x}_\tau\|_{\mathcal{L}_\infty}. \end{aligned} \tag{38}$$

Given the upper bound of $\tilde{x}(t)$ from Lemma 2, and the disturbance bounds from assumption 2, it follows that

$$\begin{aligned} \|(x_r - x)_\tau\|_{\mathcal{L}_\infty} \le& 2\frac{\|G(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} L_m \\ &+ 2\frac{\|G_u(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} L_u \\ &+ \frac{\|H(s)C(s)H_m^{-1}(s)c^\top\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1} L} \delta, \end{aligned} \tag{39}$$

which leads to the bound in (30).

To show the second bound in (31), by taking the difference of (9) and (32), one can derive

$$\begin{aligned} u_r(s) - u(s) =& -\frac{C(s)}{\omega}\theta^\top \Big(\big(x_r(s) - x(s)\big)\Big) \\ &+ \frac{C(s)}{\omega}\big(\eta_m(s) - \eta_{mr}(s)\big) \\ &+ \frac{C(s)}{\omega}\phi(s)\big(\eta_u(s) - \eta_{ur}(s)\big) \\ &+ \frac{C(s)}{\omega}\big(\phi(s)\tilde{\eta}_u(s) + \tilde{\nu}(s)\big). \end{aligned} \tag{40}$$

Hence

$$\begin{aligned} u_r(s) - u(s) =& -\frac{C(s)}{\omega}\theta^\top \Big(\big(x_r(s) - x(s)\big)\Big) \\ &+ \frac{C(s)}{\omega}\big(\eta_m(s) - \eta_{mr}(s)\big) \\ &+ \frac{C(s)}{\omega}\phi(s)\big(\eta_u(s) - \eta_{ur}(s)\big) \\ &+ \frac{C(s)}{\omega}H_m^{-1}(s)c^\top \tilde{x}(s). \end{aligned} \tag{41}$$

Consequently, the following bound holds for $t \in [0, \tau]$

$$\begin{aligned} \|(u_r - u)_\tau\|_{\mathcal{L}_\infty} \le& \|\frac{C(s)}{\omega}\|_{\mathcal{L}_1} L \|(x_r - x)_\tau\|_{\mathcal{L}_\infty} \\ &+ 2\|\frac{C(s)}{\omega}\|_{\mathcal{L}_1}(L_m + \|\phi(s)\|_{\mathcal{L}_1} L_u) \\ &+ \|\frac{C(s)}{\omega}H_m^{-1}(s)c^\top\|_{\mathcal{L}_1}\|\tilde{x}_\tau\|_{\mathcal{L}_\infty}, \end{aligned} \tag{42}$$

which holds uniformly for all $\tau \ge 0$, leading to the bound in (31). □

Implementation Issues

In practical applications, the sliding surface $\sigma(t)$ does not go to zero due to sampled computation, noisy measurements or other uncertainties. This results in a persistent increase of the estimated bounds of (5) and (7). A solution to this problem is the dead-zone modification. Hence, equations (5) and (7) are modified to be:

$$\dot{\hat{L}}_m(t) = \begin{cases} \Gamma|\lambda c^\top b\sigma(t)| & \text{if } |\sigma(t)| > \epsilon_m, \\ 0 & \text{if not,} \end{cases} \tag{43}$$

and

$$\dot{\hat{L}}_u(t) = \begin{cases} \Gamma\|\lambda c^\top \sigma(t)\| & \text{if } |\sigma(t)| > \epsilon_u, \\ 0 & \text{if not} \end{cases} \tag{44}$$

where $\epsilon_m \in \mathbb{R}^+$ and $\epsilon_u \in \mathbb{R}^+$ are real constants.

Furthermore, in order to eliminate the chattering, the discontinuous components in equations (4) and (6) are replaced by a smooth sliding mode component to yield

$$\hat{\eta}_m(t) = -(\lambda c^\top b)^{-1}\alpha\frac{\sigma(t)}{|\sigma(t)| + \epsilon} - \hat{L}_m(t)\frac{\lambda c^\top b\sigma(t)}{|\lambda c^\top b\sigma(t)| + \epsilon}, \tag{45}$$

and

$$\hat{\eta}_u(t) = -\hat{L}_u(t)\frac{\big(\lambda c^\top \sigma(t)\big)^\top}{\|\lambda c^\top \sigma(t)\| + \epsilon} \tag{46}$$

where $\epsilon > 0$ is an arbitrarily small constant. This formulation creates a boundary layer about the switching surface in which the system trajectory will remain. Therefore, the chattering problem can be reduced significantly [51].

3. Flight Test Results

Flight experiments were conducted on the Twinstar-II small fixed-wing UAV airframe Fig. 2. The Twinstar-II has a wingspan of 1.4 *m* and a weight of about 1.2 *kg*. It is a popularly small UAV airframe that has been used in several research projects [11, 52]. The UAV can be monitored and commanded by a PC based ground station which is connected via RF link. The sensor suite of the UAV platform consists of a low cost Inertial Measurement Unit (IMU), magnetometer, barometric and differential pressure sensors and a GPS receiver. The Twinstar-II was equipped with an on-board computer which consists of a Gumstix Overo SBC and an FPGA [53]. Fig. 3 shows a scheme of the on-board computer system and the applied sensors.

Fig. 2. Twinstar II Small UAV.

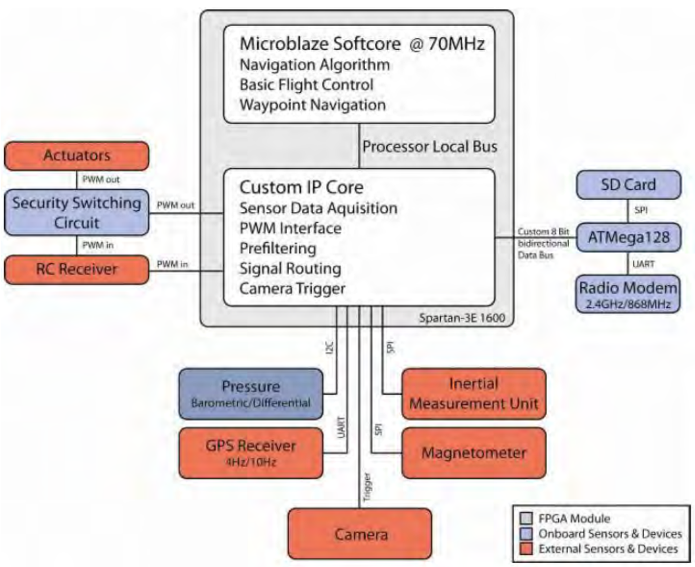


Fig. 3. On-board computer system and applied sensors, actuators and communication interfaces [54].

The control architecture is based on the augmentation of the existing baseline linear controller by the adaptive architecture, as the common approach in aerospace systems [55]. The baseline controller is developed assuming a nominal system and the $\mathcal{L}_1$ adaptive controller is added to compensate for unknown parameters and disturbances that affect the system (Fig. 4). The objective is to design a control input δ_e to enable tracking of the pitch rate command. The total deflection of the elevator

$$\delta_e(t) = u_b(t) + u(t) \tag{47}$$

is the sum of the commands from the baseline linear controller $u_b(t)$ and the adaptive controller $u(t)$.

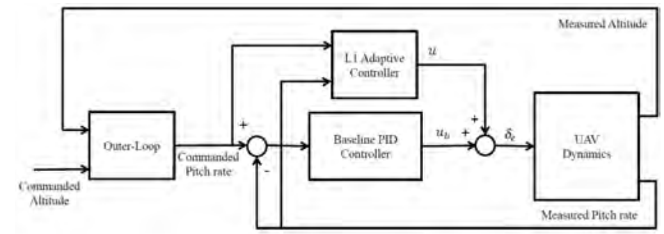


Fig. 4. The augmented controller.

The design is based on the short-period dynamics model of fixed-wing aircraft defined by [56]

$$\underbrace{\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} \frac{Z_\alpha}{V_a} & 1 + \frac{Z_q}{V_a} \\ M_\alpha & M_q \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \alpha \\ q \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{Z_{\delta_e}}{V_a} \\ M_{\delta_e} \end{bmatrix}}_{b} \underbrace{\delta_e}, \tag{48}$$

where q is the pitch rate, α is the angle of attack, V_a is the trimmed airspeed, $(Z_\alpha, Z_q, Z_{\delta_e})$ and $(M_\alpha, M_q, M_{\delta_e})$ are fixed-wing aircraft stability derivatives. It should be noted that the stability derivatives can not be measured, and they vary depending on flight conditions [56].

Taking into account the model uncertainties and the external disturbances, the system in (48) can be extended as follows

$$\dot{x}(t) = A_p x(t) + b_p \delta_e(t) + f(t, x), \tag{49}$$

where $A_p = A + \Delta A$, ΔA is an unknown matrix of model uncertainties, $b_p = b\,\omega$, ω is an unknown factor of the control input uncertainties, and $f(t, x)$ is a vector of unknown non-linear functions.

As mentioned above, the total deflection of the elevator from (47), the resulting system can be written as follows

$$\dot{x}(t) = A_m x(t) + b\omega u(t) + \tilde{f}(t, x), \tag{50}$$

where $A_m = Ax + u_b$ is a known Hurwitz matrix that defines the desired dynamics of the system in the nominal case and $\tilde{f}(t, x) = \Delta A x(t) + (\omega - 1)u_b + f(t, x)$.

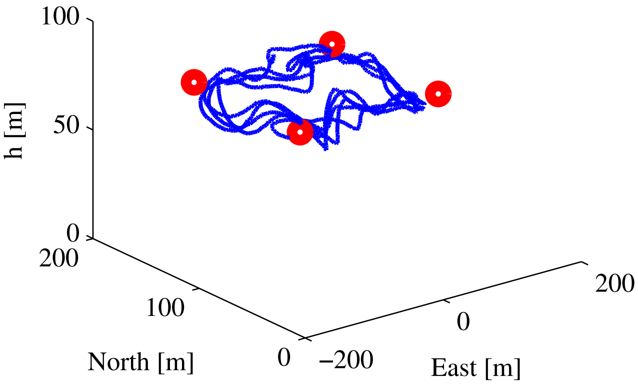
For control design the following approximation can be made

$$\tilde{f}(t, x) = B\big(\theta^\top x(t) + \eta_m(t)\big) + \eta_u(t, x). \tag{51}$$

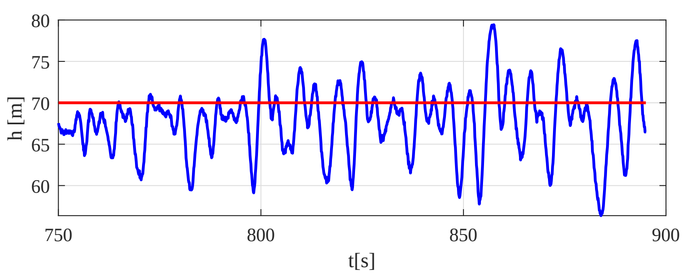
Therefore, the system in (50) can be parametrized as follows

$$\dot{x}(t) = A_m x(t) + B\big(\omega u(t) + \theta^\top x(t) + \eta_m(t)\big) + \eta_u(t, x), \tag{52}$$

where θ is a vector of constant unknown parameters representing model uncertainties, $\eta_m(t)$ is an unknown matched disturbance, and $\eta_u(t, x)$ is an unknown unmatched disturbance. The resulting model is similar to (1), which makes straightforward application of the $\mathcal{L}_1$ adaptive controller (2)-(9).



(a) 3D trajectory



(b) Altitude

Fig. 5. Performance of the adaptive controller under large uncertainties.

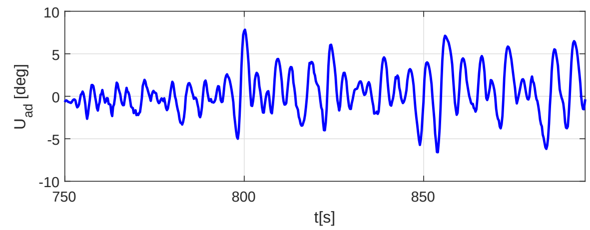
For the controller implementation, the desired eigenvalues of the system were chosen $\lambda_{1,2} = -5.6 \pm 4.2j$, i. e., a pulsation $\omega_n = 7rad/s$ and a damping $\xi = 0.8$. The controller was designed to be robust against model uncertainties within the compact sets $\Theta = [-1,\ 1]$ and $\Omega = [0.25,\ 1.25]$. The $\mathcal{L}_1$ adaptive controller parameters were set $\Gamma = 500$, $D(s) = 1/s$, $k = 150$.

The proposed scenario was that the UAV follows, autonomously, a path defined by four waypoints at a fixed altitude h= 70m. The robustness of the designed controller was tested in the presence of the following faults:

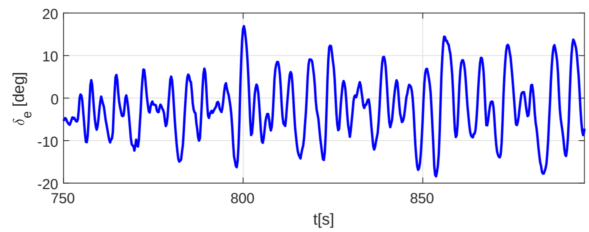
- A loss of actuator effectiveness $\omega = 0.5$;
- A constant control bias $\eta_m(t) = -0.10$ rad.

The onboard control system is not aware of the faults, and the UAV is expected to continue to follow the desired path.

As it is shown in Fig. 5, the adaptive controller, without any retuning, maintains the altitude of the UAV under those uncertainties, introduced at flight time t= $792s$. Note the presence of peaks due to the rolling motion of the UAV when turning. The frequency and amplitude of the peaks are reasonable for this type of UAV and faults conditions. It can be seen in Fig. 6 that the input command of the $\mathcal{L}_1$ adaptive controller and the total elevator command are smooth and do not present saturations.

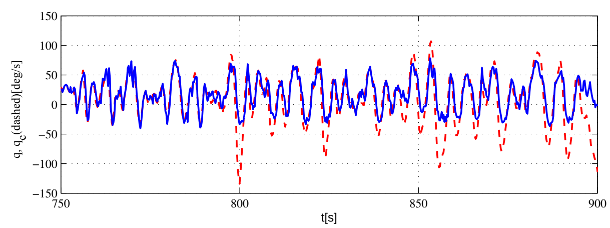


(a) Adaptive command

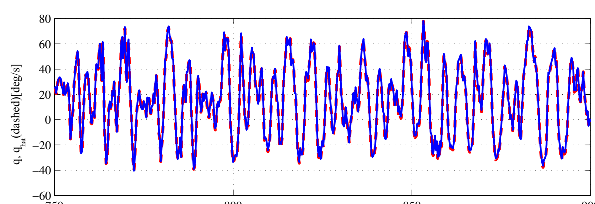


(b) Total elevator command

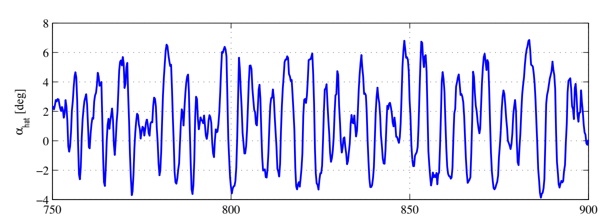
Fig. 6. Output commands with the adaptive controller.



(a) Tracking performance



(b) Pitch rate estimation



(c) Angle of attack estimation

Fig. 7. Performance of the adaptive controller under large uncertainties.

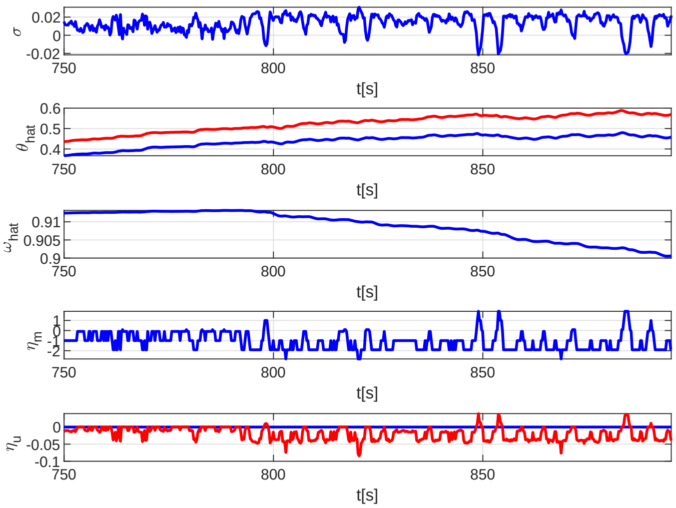
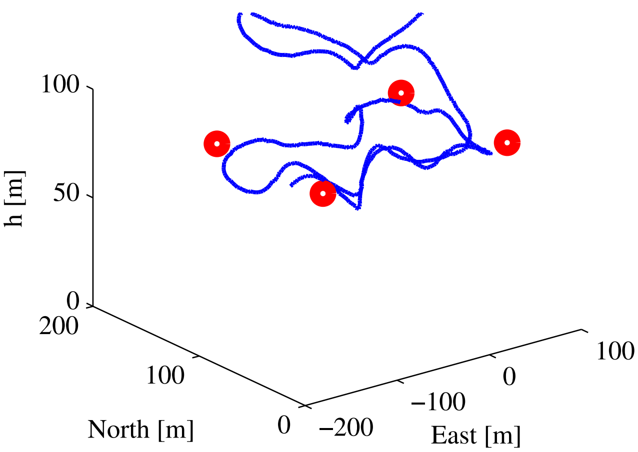
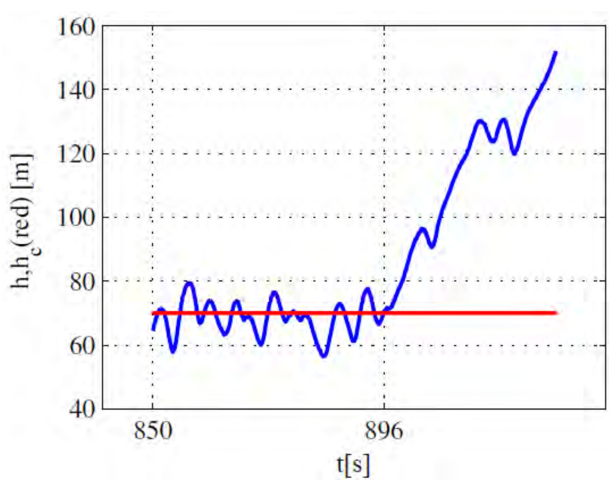


Fig. 8. Parameters of the adaptive controller.



(a) 3D trajectory



(b) Altitude

Fig. 9. Performance of the adaptive baseline under large uncertainties.

Additionally, good tracking and estimation performances for the pitch rate q are demonstrated in Fig. 7. It can, also, be noted that the estimated angle of attack α is within a reasonable interval. Oscillations in the pitch rate and the angle of attack are observed. These oscillations are acceptable given the bias and the loss of effectiveness of the elevator command.

It is also shown in Fig. 8 that the parameters of the adaptive controller are within their corresponding bounds. It should be noted that these parameters do not converge to their true values because this needs persistence of excitation [57]. $\mathcal{L}_1$ adaptive control has guaranteed transient performance and guaranteed robustness without introducing or enforcing persistence of excitation [30].

In contrast, as it appears in Fig. 9, when the adaptive controller was turned off, at time t= 896s, the baseline PI controller was not able to hold the altitude of the UAV under the same uncertainties.

These flight tests conclude that the output feedback $\mathcal{L}_1$ adaptive controller outperforms the baseline controller, and shows good tracking and estimation performance in the presence of faults and uncertainties.

4. Summary

It is shown in this paper that the $\mathcal{L}_1$ adaptive controller can be formulated on the basis of a state space formulation, while using only output measurements rather than full state knowledge. The key idea is to replace the state predictor with a Luenberger observer. Through the use of a state space model, a physical representation of the system parameters can be maintained, which supports uncertainty specification and the definition of the reference dynamics. The adaptation law is based on the sliding surface that provides better robustness under disturbances with unknown bounds. The controller was successfully tested in a real flight, along with the presence of disturbances and actuator loss of effectiveness. This approach offers an alternative design for systems that are not fully instrumented with sensors, such as small UAVs. From this point of view, the use of the presented method is justified.

Future directions may include the use of the proposed method in system monitoring and fault detection based on estimated states. Another more realistic approach to the problem of $\mathcal{L}_1$ adaptive control is to consider that in the real world, external disturbances are completely unpredictable, i.e., they are stochastic phenomenon. Such an approach needs to borrow tools from stochastic analysis.

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