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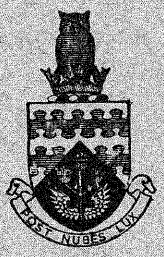

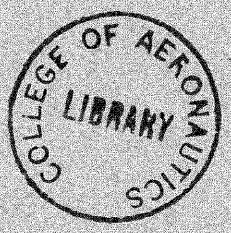

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THE PRESSURE DISTRIBUTION OVER THE FLAT END SURFACES OF COMPRESSED SOLID RUBBER CYLINDERS

by

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The pressure distribution over the flat end surfaces of compressed solid rubber cylinders

- by -

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SUMMARY

The shape of the pressure distributions over the flat end surfaces of compressed solid rubber cylinders have been determined. The cylinders were compressed between metal end plates. The pressure distribution for compressive strains of less than 3% is approximately parabolic. It is unaffected by the strains set up in the rubber due to the differential thermal contraction of the rubber and bonded metal end plates. A method of extending these measurements to large compressive strains, and a possible future programme of work, is outlined.

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1. Introduction

The use of rubber in suspension and support systems, as compression and shear mountings or torque couplings, is well established. In designing these rubber springs it is obviously necessary to be able to predict their load-deformation behaviour in service. Unfortunately, however, they rarely undergo the simple modes of deformation which are well understood and characterised in detail by the kinetic theory of rubberlike elasticity. The deviations from modes of deformation such as pure compression and simple shear are discussed in detail by Davey and Payne (1964). Some aspects of these deviations will now be outlined in order to explain the relevance of the pressure distribution measurements detailed in this report.

Consider the practical methods of compressing a rubber block. The compressive load may be applied directly to the rubber by placing the block between the platens of a press, or by first sandwiching the rubber between rigid end plates and then loading one of these plates. The high friction between the rubber and the platens, or the end plates, minimises or prevents interfacial slip, and, as rubber is almost incompressible, the axial deformation leads to bulging over the free curved surfaces. This is in contrast with pure compression which is a homogeneous deformation and can be characterised by the kinetic theory of rubberlike elasticity (Treloar, 1958). The kinetic theory therefore only provides the basis for the semi-empirical approach which has been adopted to develop a quantitative description of the observed load-deformation behaviour of rubber units in engineering applications.

Rubber compression and shear mountings usually have bonded metal end plates. Interfacial slip is therefore zero and there are very considerable deviations from the load-deformation characteristics predicted by the kinetic theory. The stiffness of a rubber unit is then very dependent upon its shape. Indeed the effective compression modulus of a solid cylinder or rectangular prism of rubber can be increased by a factor of the order of 1000 above the value of the (small strain) Young's modulus of the rubber, simply by altering the dimensions of the unit. (Gent and Lindley, 1959). This extra design parameter has considerably extended the useful engineering applications for rubber compression springs. They are now used for example as bearings or vibration isolators under heavy structures such as bridges (Torr, 1967) or buildings (Reed, 1967).

The load-compression relationships for bonded rubber units assume a strain independent value for the Young's modulus of the rubber, and have been reviewed by Davey and Payne (1964). If the compression is limited to about 10% of the original height of the unit then the compressive load is essentially directly proportional to the deformation, and E_c the effective compression modulus of the unit can be calculated. Measurements on bonded rubber units with both circular and rectangular cross-sections show that E_c is a function of E, the Young's modulus of the rubber, and the shape of the unit. According to Gent and Lindley (1959)

$$E_c = E(1 + kS^2) \qquad (1)$$

where S is the shape factor defined as the ratio of the cross-sectional area to the total force-free area. The constant k is less than unity and decreases with increasing rubber hardness. Values of k have been tabulated.

For strains greater than 10%, i.e. outside the range of linear behaviour, the observed non-linear relationship between compressive stress and strain can be described reasonably well by replacing E with E_c in the relationship predicted by the theory of rubber-like elasticity (Gent and Lindley, 1959).

Then

$$\frac{\sigma}{E} = \frac{1}{3}(1 + kS^2)\left(\frac{1}{\lambda^2} - \frac{1}{\lambda}\right) \qquad (2)$$

where σ is the nominal stress and λ is the deformation ratio.

The observed behaviour at large strains has also been reasonably well described by Lindley (1966). He modified Hooke's Law by using a strain dependent shape factor, S, and therefore a strain dependent 'constant' of proportionality E_c.

Then

$$\frac{\sigma}{E} = \ell n\left(\frac{1}{\lambda}\right) + kS^2\left(\frac{1}{\lambda^2} - 1\right) \qquad (3)$$

A knowledge of the distribution of stresses through a bonded rubber block would obviously lead to an understanding of the role of the shape factor in determining the load-deformation relationships. Furthermore, it could lead to an understanding of the important engineering problem of instability or buckling in rubber mountings. A technique has been established for measuring the distribution of normal stresses (or pressures) across the flat metal end plates of solid rubber cylinders which are compressed by a small (< 4%) axial deformation. Pressure distributions have been determined for two unfilled rubbers. It will be seen that this technique is not limited to units with circular cross-sections.

2. The measurement of the pressure distributions

The method is based upon the observation that if a block of rubber is compressed against a rigid flat plate which contains a small circular hole, then the amount of rubber bulges into the hole is directly proportional to the pressure at the centre of the hole (Hall, 1968).

This proportionality between pressure and bulge height was observed during investigations into the behaviour of rubber in torsion. When a solid rubber cylinder with bonded metal end plates is maintained in a state of torsion and at a constant length there is a distribution of pressures over

the end plates. For the measurement of these pressures a calibrated dynamometer was developed which was essentially a spring-loaded plunger. The principle of its action is simple. A number of small circular holes were cut through the top bonded metal end plate of the rubber unit, and, because of the pressures which develop during torsion, the rubber attempted to bulge into these holes. Bulging was prevented by the circular dynamometer plunger placed at the centre of the hole, and the restricting force measured by the calibrated compression spring. The pressure at the centre of the hole was then calculated by dividing the measured force by the effective cross-sectional area of the dynamometer plunger.

The dimensions of the rubber units, and the size and displacement of the holes in the top plate are shown in Figure 1. The holes were distributed so that there was minimal stress interference between them.

The dynamometer was then removed and the rubber allowed to bulge freely into the holes. The height of the bulges were measured using a dial gauge with a position sensitive plunger. The relationship between the free bulge height in a particular hole, and the pressure acting at the centre of that hole, was therefore established (figure 2).

The effective cross-sectional area of the plunger (0.079 ins^2) was greater than the actual area (0.077 ins^2) because of its effect in almost eliminating bulging in the 0.033 ins^2 annular space between the plunger and the edge of the hole. The method of determination of the effective cross-sectional area is given elsewhere (Hall, 1968). The pressure at the centre of the hole is only exactly equal to the ratio of the restricting force to effective area if the pressure is constant over the area of the plunger. The error involved if the pressure distribution is parabolic, (see section 5), is estimated in the Appendix.

The linear relationship between the calculated pressures and measured free bulges into each of the five holes in the end plate of a unit is shown in figure 2 It is particularly interesting to note that the same straight line is obtained from measurements at each hole. The rubber was bonded to the metal during the vulcanisation process, and hence figure 2 suggests that the stresses set up in the rubber due to the differential thermal contraction of rubber and metal do not affect the pressure measurements. The linearity between pressure and the bulge height was confirmed by measurements on a number of gum rubbers (figure 3).

The distribution of pressures which occur when a rubber unit is compressed can clearly therefore be examined by measuring the rubber bulges into a number of holes disposed across the end plate. The numerical value of the pressures can be determined either by calibrating the height of the bulge in terms of the pressure by using a dynamometer, or by numerically integrating the bulge height distribution over the whole surface and equating it to the compressive load.

3. Preparation of the rubber units

Four rubber compression units were prepared. The dimensions of the complete units, and the size and disposition of the holes in the top metal end plates, were as indicated in figure 1.

Two gum rubbers were used. The details (parts by weight) of their constituent parts are given in Table 1.

Table 1. The recipe for the rubbers

	Units 1, 2 and 3	Unit 4
Natural rubber	100	-
Butyl rubber (Esso grade 218)	-	100
Sulphur	2	2
Zinc oxide	5	5
Stearic acid	1	3
Antioxidant	1	-
Accelerator	1	1.5

The solid rubber cylinders were vulcanised under pressure in a suitable mould for 30 minutes at 150°C.

Unit 1 was bonded to the metal end plates during vulcanisation. The holes in the end plate were plugged and machine ground to a good surface finish, and the plate coated (Chemlock 220, Durham Chemical Co.) in such a manner as to ensure bonding over the plate but not the plug surfaces. The plugs were removed after vulcanisation to expose the free rubber surface. Unit 2 was not bonded to the end plates. Units 3 and 4 were bonded after vulcanisation and cooling, using Eastman 910 adhesive. The same rubber cylinder was used for units 2 and 3. The unit was tested before and after bonding with Eastman 910.

4. Experimental procedure

Each rubber unit was compressed using an Instron testing machine, the load being applied in increments and the axial compression determined. After each load increment the height of rubber bulging into the holes in the top plate was determined by a dial gauge mounted on the plate. The dial gauge was protected by a steel tube (outside diameter 4 inches) which also transmitted the load from the upper platen of the press to the rubber unit.

The units were taken through 5 load-compression cycles so that the bulges could be measured at each of the 5 holes.

The measured bulge heights at each hole, for various loads (and therefore axial compressions) are given in Table 2, and represented graphically in figures 4 and 5.

5. Discussion of the results

Over the limited strain range examined there is a linear relationship between load and compression (figure 4). The error at the origin, a small but finite strain for zero load, is due to slight out-of-alignment after moulding of the flat end faces of the rubber cylinders.

The effective stiffness of each unit has been calculated from the gradient of the straight lines. The hot and cold bonded units have essentially the same stiffness which is significantly greater than that for the unbonded rubber. The properties of the cured natural rubber in unit 1 and unit 2 (or 3) must be very similar. This therefore suggests that the method of bonding does not affect the small strain behaviour, but the friction between rubber and the unbonded end plates was inadequate to prevent interfacial slip.

At each of the five holes the bulge height is directly proportional to the compressive load (Figure 5). The non-zero intersection of some of these straight lines with the ordinate is due to the out of alignment of the flat rubber surfaces after moulding. It can be seen that for a given load, the bulge (or pressure) decreases with increasing displacement from the axis of the unit.

Since the compressive load is proportional to the axial strain, then the bulge height is also proportional to the strain. The distribution of bulge heights, (and therefore pressures), over the end surfaces is shown in figure 6 for a deformation equal to 3% of the original height, (i.e. an axial strain of 3%). It is not possible to discriminate between the distributions for the three bonded rubber units, (Nos. 1, 3 and 4). The strains set up in the rubber due to the differential thermal contraction of rubber and metal after hot bonding do not therefore affect the pressure distribution. The pressure at the edge, (by extrapolation), was 31% of the pressure on the axis. The unbonded unit, No. 2, had a significantly larger pressure fall off from the axis to the edge. The pressure at the edge was only 19% of the pressure on the axis. This is at variance with the postulate that interfacial slip was the cause of the relatively low stiffness of the unbonded unit. The limiting case of complete slip, or zero friction, would give a pure homogeneous compression and an even pressure over the end plates.

The limited number of measurements described in this Note suggest that the difference in stiffness of bonded and unbonded natural rubber units may be attributed to effects other than to interfacial slip. It is possible that the end effects produced by using a bonding agent alter

the effective height of the unit. The semi-empirical expression for the compression modulus (equation 1) has been used for bonded units with considerable success. The shape factor S has always been calculated using the actual height of the unit and the parameter k was found to be dependent upon the stiffness of the rubber. It may be that variations in k are simply a measure of the errors involved in using the actual height of a unit of a particular rubber as its effective height. It should be emphasized that further tests are necessary upon units with a range of rubber hardnesses before any firm conclusuons can be reached.

Load-compression relationships for bonded rubber cylinders have been developed by Gent and Lindley (1959) to describe small strain behaviour. They assumed a linear relationship between compressive load and deformation, where the deformation consisted of two superimposed components:-

(i) the pure homogeneous deformation of one end plates moving towards the other;

(ii) an outward displacement of any point within the rubber so that the free rubber surfaces would take up a parabolic shape.

Assuming classical elasticity the pressure P_r acting on the bonded end plate at a distance r from the axis is then given by

$$P_r = \frac{Ed}{h}\left(1 + \frac{a^2-r^2}{h^2}\right) \qquad (4)$$

where E is Young's modulus of the rubber, and d the axial compression of a unit of height h and radius a. Since the pressure P_r at a point is directly proportional to the height of a bulge b_r, measured at the same radial displacement r, equation (4) suggests a parabolic bulge height distribution, or a linear relationship between b_r and r^2. The distribution of bulge heights shown in Figure 6 have been replotted in figure 7 as b_r against r^2. There are small but systematic deviations from linearity.

At a given radial distance equation (4) suggests that P_r is proportional to the axial compression d. This is in agreement with Figure 5 since the load is proportional to d. According to equation (4) the ratio of the pressure at the edge of the unit to the axial pressure is given by

$$\frac{P_{r=a}}{P_{r=o}} = \left(1 + \frac{a^2}{h^2}\right)^{-1} \qquad (5)$$

and is therefore independent of compressive strain. For the units examined a/h = 3.85 and hence equation (5) predicts that

$$\frac{P_{r=a}}{P_{r=o}} \simeq 0.06 \qquad (6)$$

This is to compare with the experimentally determined values for this ratio of 0.19 and 0.31 for the unbonded and bonded units respectively. The observed proportionality between pressure, bulge height and compression ensures that the observed values for the ratio are independent of compressive strain.

6. Future work

The measurement of pressure distributions appears to be a sensitive indicator of the effects of bonding end plates to a rubber back. The work could be usefully extended for a range of rubbers in both shear and compression, and may well provide an insight into the onset of instabilities such as buckling.

It would be particularly valuable to obtain measurements over a wide range of deformations. The method of measuring bulge height is inadequate for the determination of pressure distributions at large strains. There would be a non-linear relationship between pressure and bulge height at any particular radial position. However, the pressure could be determined using rods of rigid photoelastic material in contact with the free rubber surface in the holes which prevented the bulges developing. The fringe pattern observed in each rod under crossed polaroids would be calibrated in terms of a compressive load. It may be possible to measure pressure distributions when the unit is undergoing dynamic as well as static deformations.

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4.	Lindley	J. Strain Anal. 1,190, 1966.
5.	Reed	Use of Rubber in Engineering Applications NRPRA Conference Proceedings. Maclaren and Sons, 1967.
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7.	Treloar	The Physics of Rubber Elasticity. Clarendon Press, 1958.

Appendix

The error involved in the method of calculation of the pressure at the centre of a hole

The pressure at the centre of a hole in the end plate of a sample is calculated as the force exerted by the dynamometer plunger to restrain bulging, divided by the effective cross-sectional area of that plunger. This assumption is only accurate if the pressure is constant over the area of the plunger.

The error has been calculated for the parabolic bulge height distribution

$$b_r(\text{inches}) = 5.55\times10^{-3}(a^2-r^2) + 10\times10^{-3} \qquad (A.1)$$

This is the relationship between b_r and r^2 which is seen to describe the bulge height distribution at 3% compressive strain (see figures 6 and 7).

Consider a plunger of effective radius c at a radial distance b from the axis of the compression unit (Figure 8).

The force δF acting on the element of area ABCD is given by

$$\delta F = P_r \,.\, r\delta\theta \,.\, \delta r \qquad (A.2)$$

where δr is an infinitesimal increment in r, $\delta\theta$ is given by

$$\delta\theta = 2\cos^{-1}\left(\frac{b^2+r^2-c^2}{2br}\right) \qquad (A.3)$$

and the pressure p_r at a radial distance r is directly proportional to the bulge height b_r.

i.e. $$p_r = kb_r \qquad (A.4)$$

Then the total force F acting on the plunger is given by

$$F = \int_{b-c}^{b+c} kb_r . 2r\cos^{-1}\left(\frac{b^2+r^2-c^2}{2br}\right) dr \qquad (A.5)$$

If b_r is given by (A.1) then evaluation of the subsequent integral gives

$$F = 5.55 \times 10^{-3} \pi e^2 k \, (b^2 + c^2/2 - a^2 - 1.8) \qquad (A.6)$$

The pressure is calculated as $F/\pi c^2$, which is equivalent to a bulge b'_r given by

$$b'_r = \frac{F}{\pi c^2 k} \qquad (A.7)$$

If the true bulge height b_r is given by (A.1) (when r=b), then the error due to this method of calculation is given by

$$b'_r - b_r = 5.55 \times 10^{-3} \, c^2 \text{ inches} \qquad (A.8)$$

and is therefore independent of the position of the hole in the end plate.

The effective area of the plunger was 0.079 ins^2, and therefore $c^2 = 0.025$ ins^2 and $b'_r - b_r = 7 \times 10^{-5}$ inches. The smallest bulge measured at 3% compressive strain was 10.10^{-3} inches at the edge of the unit. Errors are therefore less than 1% and can be ignored.

Table 2 The variation of bulge height with compressive load and axial strain for each unit. Measurements were obtained at 5 different positions over the end plate of each unit.

Key: L = load (lbs); d/h = axial stress (%); b = bulge height (insx10^{-3}).

Unit 1

r = 0 ins.			r = 0.97 ins.			r = 1.20 ins.			r = 1.45 ins.			r = 1.70 ins.		
L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b
16	0.28	2.8	50	0.98	5.8	39	0.77	7.0	38	0.60	3.8	50	0.98	5.8
33	0.61	6.4	82	1.52	8.92	73	1.45	12.6	83	1.28	8.5	82	1.52	8.9
52	0.98	9.7	133	2.20	12.92	107	2.01	17.7	106	1.79	12.9	133	2.20	12.9
77	1.47	14.5	163	2.86	17.6	147	2.64	24.0	145	2.26	16.8	163	2.86	17.6
110	2.03	21.2	199	3.29	20.2	198	3.46	32.1	171	2.62	19.4	199	3.29	20.2
144	2.58	27.0							197	3.05	22.9			
194	3.41	36.9												

Unit 2

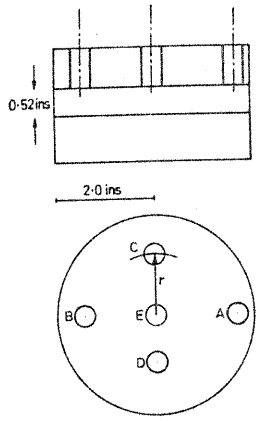
r = 0 ins.			r = 0.88 ins.			r = 1.06 ins.			r = 1.28 ins.			r = 1.52 ins.		
L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b
40	1.16	9.5	40	1.16	6.8	41	1.08		40	1.05		40	1.08	
80	2.02	17.9	80	1.95	13.6	80	1.97	12.5	79	1.78		80	1.86	
120	2.79	25.9	120	2.71	20.8	120	2.78	19.5	120	2.51	13.7	120	2.59	
160	3.52	34.1	160	3.44	27.0	159	3.48	25.3	160	3.25	18.5	160	3.33	17.5
200	4.26	41.2	199	4.14	33.1	187	3.99	29.5	200	3.99	23.7	200	4.10	22.0

Unit 3

r = 0 ins.			r = 0.97 ins.			r = 1.20 ins.			r = 1.45 ins.			r = 1.70 ins.		
L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b
40	0.70	7.8	38	0.74	6.1	40	0.78	5.1	40	0.78	4.8	41	0.78	4.9
80	1.43	14.9	80	1.47	13.6	80	1.47	11.2	80	1.43	8.8	80	1.43	9.1
120	2.09	23.5	120	2.13	20.0	121	2.13	17.0	120	2.13	13.4	120	2.13	13.5
160	2.71	30.0	160	2.79	25.8	160	2.82	23.0	160	2.79	18.5	160	2.75	16.7
200	3.33	37.2	200	3.49	32.1	199	3.37	27.7	200	3.49	23.7	200	3.37	20.7

Unit 4

r = 0 ins.			r = 0.97 ins.			r = 1.20 ins.			r = 1.45 ins.			r = 1.70 ins.		
L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b	L	d/h	b
45	0.78	6.6	43	0.74	5.1	40	0.74	4.8	42	0.74	4.6	36	0.66	4.1
80	1.28	13.1	81	1.40	12.4	80	1.40	12.1	80	1.33	8.3	80	1.40	7.8
118	1.81	18.2	120	1.97	17.5	124	2.02	15.9	120	1.90	12.7	119	1.94	11.2
159	2.48	27.3	160	2.65	22.9	160	2.64	22.1	160	2.51	16.7	159	2.60	15.0
199	3.13	33.0	199	3.27	29.3	199	3.26	26.8	200	3.13	22.1	200	3.22	18.2



HOLE	r, inches	
	UNITS 1,3,4	UNIT 2
A	1·70	1·52
B	1·45	1·28
C	1·20	1·06
D	0·97	0·88
E	0·0	0·0

HOLE DIAMETERS 0·375 ins.

FIG. 1 THE DIMENSIONS OF THE RUBBER UNITS

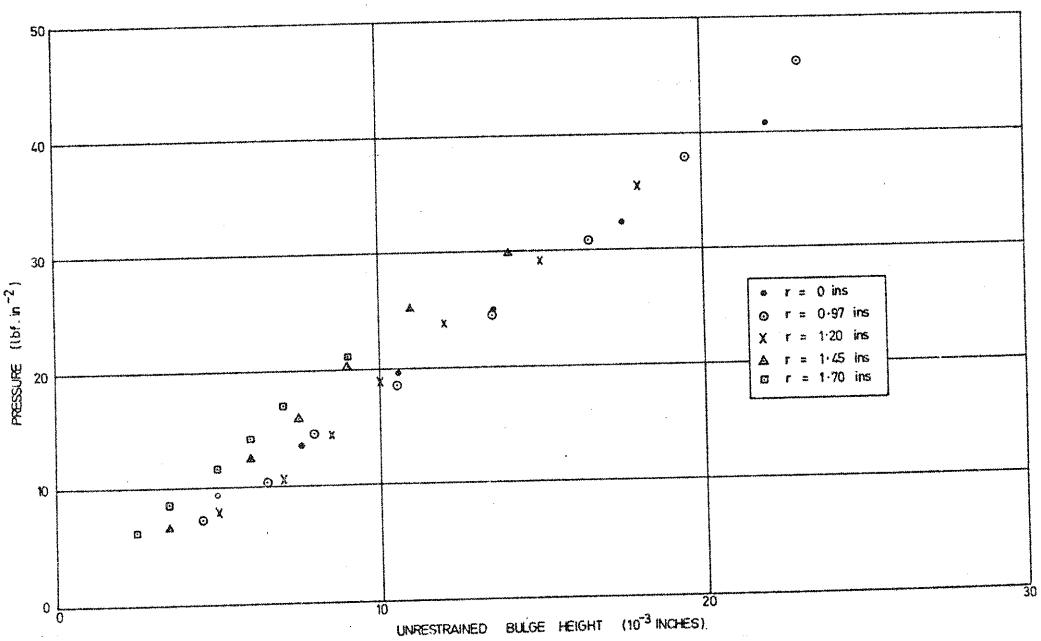


FIG. 2 THE RELATIONSHIP BETWEEN PRESSURE AND FREE BULGE HEIGHT FOR A NATURAL RUBBER UNIT WITH BONDED METAL END PLATES

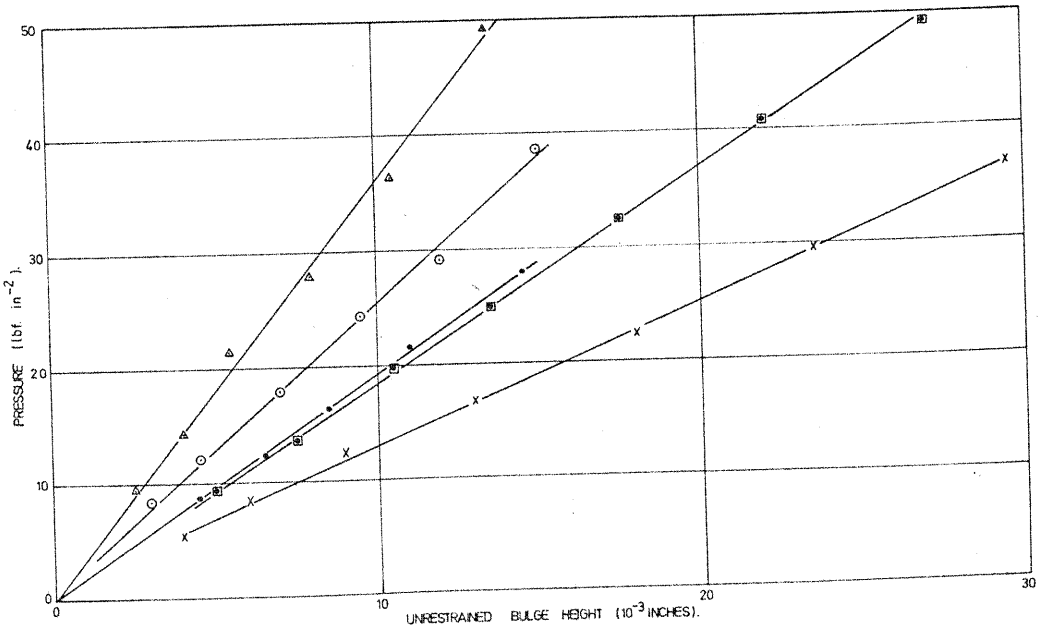


FIG. 3 THE RELATIONSHIP BETWEEN PRESSURE AND FREE BULGE HEIGHT ON THE AXIS OF VARIOUS GUM RUBBERS (FROM HALL 1968)

FIG. 8 THE DIAGRAM FOR THE APPENDIX CALCULATIONS