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Fuzzy Finite Element Model Updating of the DLR AIRMOD test structure

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Abstract

This article presents the application of finite-element fuzzy model updating to the DLR AIRMOD structure. The statement of the problem is well explained by the use of a mass-spring system with three degrees of freedom. Considering the effect of the assembly process on variability measurements, modal tests were carried out for the repeatedly disassembled and reassembled DLR AIRMOD structure. The histograms of the measured data attributed to the uncertainty of the structural components in terms of mass and stiffness are utilised to obtain the membership functions of the chosen fuzzy outputs and to determine the updated membership functions of the uncertain input parameters represented by fuzzy variables. In this regard, a fuzzy parameter is introduced to represent a set of interval parameters through the membership function, and a meta model (kriging, in this work) is used to speed up the updating. The use of non-probabilistic models, i.e. interval and fuzzy models, for updating models with uncertainties is often more practical when the large quantities of test data that are necessary for probabilistic model updating are unavailable.

Keywords: Fuzzy variable, Model Updating, AIRMOD structure

1 Introduction

Considerable attention has been devoted in recent years toward investigating how the uncertainty of measured modal parameters affects the process of model updating. In order to address this problem, stochastic model updating has been introduced. Uncertainty inevitably exists in the measured modal data. It can arise from many sources, such as measurement noise, manufacturing tolerances in structures, and environmental erosion. In the literature of statistical analysis [1,2], two classes of uncertainties may be defined which are known as epistemic and aleatory uncertainties. Epistemic uncertainty is reducible, while aleatory uncertainty is not. Distinguishing between these two types of uncertainty in a real experiment is not trivial. However, by carrying out a controlled experiment such as those shown in [3], such a distinction may be made. The authors of reference [3] demonstrated that among different uncertainty sources, the manufacturing variability related to the assembling process is the most dominant. Manufacturing variability is not reducible, and therefore, it can be considered to belong to the class of aleatory uncertainty. The review paper by Simoen et al. [4] and references therein show that this subject has already garnered a great deal of attention.

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Different methods have been proposed to represent uncertainty in the context of stochastic model updating. These methods may be generally classified as either probabilistic or non-probabilistic. Emphasis has been given to the use of probabilistic models and methods in stochastic model updating. In this regard, several methods were developed. Examples of probabilistic methods dealing with irreducible uncertainty are briefly explained here. Fonseca et al. [5] proposed the use of the maximum likelihood method for the stochastic model updating of a cantilever beam with a point mass with location variability. Hua et al. [6] and Khodaparast et al. [7] considered the uncertainty arising from manufacturing tolerances in multiple test pieces, all built identically with the same materials, by using perturbation methods. Khodaparast et al. [7] showed that the computation of the Hessian matrix is unnecessary. The perturbation-based updating method was subsequently tested in applications to welded structures [8] and composite structures [9]. Govers and Link [10] used the analytical output covariance matrix for the problem of stochastic model updating. Covariance formulas produced by Khodaparast et al. [7] and Govers and Link [10] were shown to be identical by Silva [11], who developed a technique for parameter selection based on the decomposition of the output covariance matrix. Recently, Govers et al. compared the covariance and interval updating techniques [12] by using data obtained by repeated disassembly and reassembly of the DRL AIRMOD structure [3]. Jacquelin et al. [13] used random matrix theory and derived closed-form expressions for the mean and the covariance matrix of the updated stiffness matrix. Bayesian model updating was initially developed by Beck and Katafygiotis ([14, 15]), and their work led to the current high level of interest in the technique (e.g. [16-21]). One problem with the Bayesian approach is the large computational resource required for sampling when using Markov chain Monte Carlo (MCMC) algorithms. Stochastic response surfaces (such as polynomial chaos expansion; PCE [22]) appear to be the most promising tools to accelerate the Bayesian updating approach. The use of stochastic response surface methods such as PCE, traditional response surface methods, and the kriging predictor for efficient stochastic model updating has been frequently reported in literature (e.g. [23-28]).

The probabilistic model updating techniques need large volumes of data. However, in most of the real applications, obtaining such large amounts of data is expensive. In contrast, the non-probabilistic model updating methods, such as interval and fuzzy methods, do not require large quantities of test data and may be considered better approaches in these cases. Interval model updating was proposed by Khodaparast et al. [23]; in their method, the interval models were used to represent irreducible uncertain measured data, and the upper and lower bounds of the updating parameters were updated using a kriging meta model. Haag et al. [28] proposed an inverse approach capable of identifying the fuzzy-valued parameters of a model with epistemic (reducible) uncertainties. The application of fuzzy model selection and identification to the modelling of a brake pad was demonstrated in [29]. In a more recent study by Erdogan and Bakir [30], fuzzy models were incorporated to model the uncertainty due to measurement noise. In this study, the genetic algorithm was used to update the membership functions of uncertain parameters by minimising an objective function. Erdogan et al. [31] later showed the application of fuzzy model updating to the problem of damage detection. In the same context, a method was proposed by Liu and Duan [32] for fuzzy finite element (FE) model updating. In this method, fuzzy models are used to represent the uncertainty in the parameters of the model, and fuzzy model updating is carried out by fuzzifying a traditional objective function. Subsequently, a method was developed to determine the fuzziness of updating parameters by using various degrees of fuzziness of the objective function. An FE model of a bridge was used to demonstrate the application of the aforementioned proposed method. So far, the proposed fuzzy updating methods have dealt with reducible uncertainty in the problem of model updating in structural dynamics. As already mentioned, these are known as epistemic uncertainties, for example the existing measurement noise in the test data. However, when irreducible uncertain measured data are available, these techniques cannot be used as they cannot predict the physical membership functions of the uncertain parameters.

This paper proposes a method for fuzzy FE model updating when the uncertainty in the measured data is classified as aleatory uncertainty and hence is irreducible. The application of the proposed method for the DLR AIRMOD structure with uncertainties due to manufacturing tolerances is demonstrated. In order to represent this source of uncertainty in a set of identical structures, it was decided to assemble and reassemble the DLR AIRMOD structure 130 times [3, 12]. This paper will extend the theory of interval model updating, developed in [23], to fuzzy model updating. The proposed fuzzy model updating starts with a method to determine the so-called "measured fuzzy membership functions." In this study, the measured data are natural frequencies and mode shapes of the DLR AIRMOD structures. The histograms of the measured data are used to determine these membership functions. It is evident that there is no unique way of determining the measured fuzzy membership functions. This could cause additional uncertainty, but it is beyond the scope of this study. We refer the reader to references [34, 35] for more details about the calculation of uncertain measured data fuzzy membership functions. Once the measured data membership functions are known, the membership functions of the updating parameters can be calculated by the method proposed in this paper. A set of interval variables at different levels of membership functions, known as α-cuts, are used to describe the measured fuzzy membership functions. Once the measured fuzzy membership functions are determined, the problem of fuzzy FE model updating can be solved using the solution of a number of interval FE model updating problems. The interval model updating techniques, proposed by the first author in [23], can then be exploited to update the upper and lower bounds of the uncertain parameters at different levels of membership functions. Once the updated bounds are determined, the fuzzy membership functions of the updating parameters will be computed. A mass-spring model with three degrees of freedom is used to demonstrate the application of the proposed method numerically. Finally, the measured modal data containing irreducible uncertainty due to the assembling and reassembling of the DLR AIRMOD structure (mimicking manufacturing variability) are used, and the method is tested with these data.

2 Theory

The objective is to develop a framework that can be used for solving the problem of fuzzy FE model updating. To this end, the aim is to update the membership functions of the uncertain parameters of FE models based on the measured membership functions. This inverse problem has to be solved when it is not possible to directly measure the variability of the uncertain parameters. For example, stiffness and damping terms in structural joints or variability in the properties of the materials cannot be directly measured. Instead, the variability in the dynamic responses of the structure, such as natural frequencies, mode-shapes, or frequency response functions, can be measured. In this application, the fuzzy membership function is the dynamic response - typically, the natural frequencies and mode shapes. Figure 1 shows the procedure of fuzzy FE model updating for a function of two triangular fuzzy variables with five α-levels. As shown in the figure, the method starts with an initial estimate of the membership functions of the input parameters. The fuzzy membership functions of the outputs are then used, and several-interval model updating exercises at different levels of membership functions are performed to update the initial fuzzy membership of input parameters.

When multiple sets of experimental data exist, we first need to determine the fuzzy membership functions of these measured data. We propose to use the histograms fitted to experimental data to compute these fuzzy membership functions. The method uses the observations in a histogram plot and partitions them into bins using a small number of intervals. Therefore, each bin corresponds to a specific number of observations of that output (natural frequencies and mode shapes here). Then, as shown in Figure 2, one point at the middle of each bin is selected to construct the fuzzy membership functions. As indicated in the figure, for the zero level of membership functions, point A, which is the beginning point of the first bin on the left, and point C, which is the last point in the histogram on the right, are chosen. These points define the maximum ranges of variation of the measured data, which is defined by the zero-level cut in the membership function. The x-coordinate (the midpoint of the bins) and y-coordinate (the number of observations) of the selected remaining points are extracted from the histogram. In Figure 2, these points are shown by black circles and the bins by gray rectangles. Once the positions of these points are known, we compute the fuzzy membership functions by drawing straight lines that pass through the aforementioned points. This process is demonstrated in Figure 2. We normalize the membership functions (the alpha values vary from zero to one), and therefore, the fuzzy variables are called normal fuzzy variables. In the construction of the membership functions, expert knowledge can also be included to modify the ranges of variations at different levels of membership functions [30, 31].

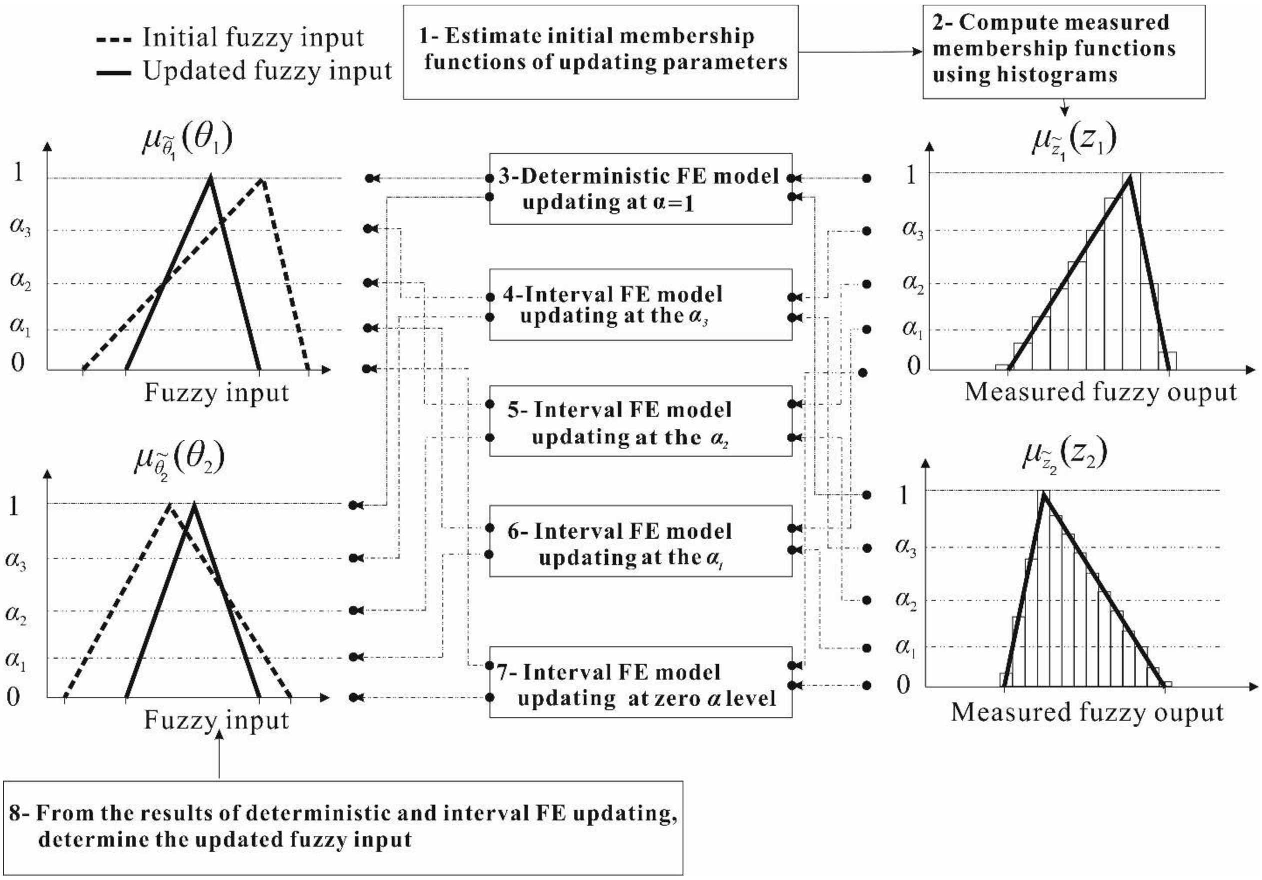


Figure 1: Flowchart for fuzzy finite element model updating using the α-cuts strategy.

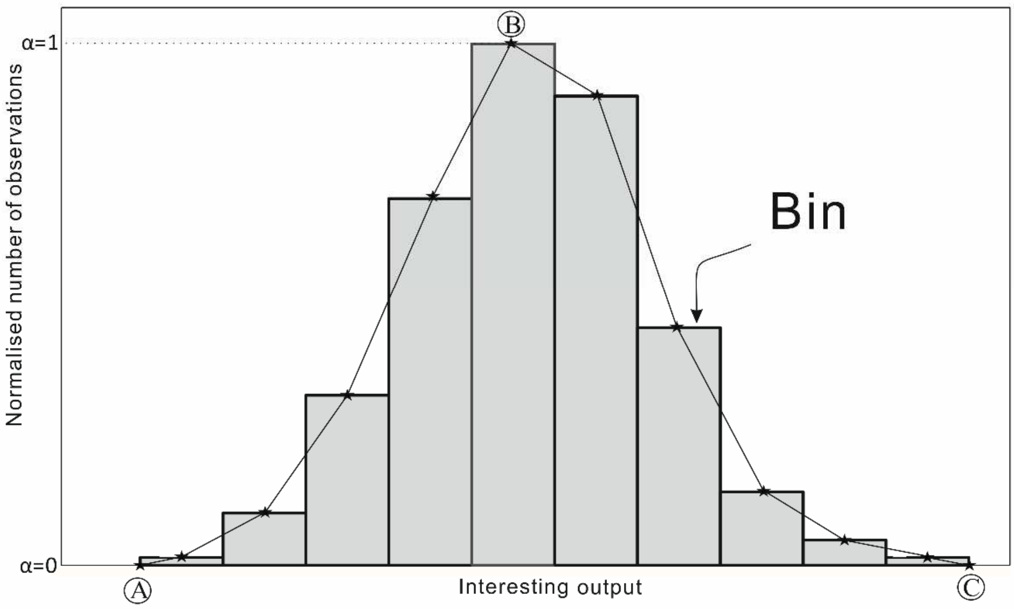


Figure 2: Computing the fuzzy membership function of the measured data using a histogram

As already mentioned in this paper, we use a number of interval variables at different α-cuts to represent a fuzzy variable. We normalize the bounds of interval variables in such a way that the minimum values are mapped to -1, while the maximum values are mapped to 1 [36, 37]. This is useful as it normalizes the updating parameters and output data. As illustrated in Figure 1, the membership functions of the measured data can be iteratively used in the fuzzy model updating method to determine the updating parameters fuzzy membership functions. This can be done by performing a deterministic model updating and a number of interval model updating exercises. The deterministic model updating is carried out at $\alpha = 1$, while interval model updating will be performed at lower levels of membership functions.

One may describe the FE fuzzy model updating problem using the following recursive equation:

$$\widetilde{\boldsymbol{\theta}}_{j+1} = \widetilde{\boldsymbol{\theta}}_j + \widetilde{\boldsymbol{T}}_j(\tilde{\boldsymbol{z}}_m - \tilde{\boldsymbol{z}}_j) \tag{1}$$

where,

$$\tilde{\boldsymbol{z}}_m = \left[\widetilde{\omega}_1^{2(m)} \ \widetilde{\omega}_2^{2(m)} \ \dots \ \widetilde{\omega}_{r_1}^{2(m)} \ \widetilde{\boldsymbol{\phi}}_1^{T(m)} \ \widetilde{\boldsymbol{\phi}}_2^{T(m)} \ \dots \ \widetilde{\boldsymbol{\phi}}_{r_2}^{T(m)}\right] \in \mathbb{R}^{n\times 1} \tag{2}$$

is the vector of fuzzy measured data and

$$\tilde{\boldsymbol{z}}_j = \left[\widetilde{\omega}_1^{2(a)} \ \widetilde{\omega}_2^{2(a)} \ \dots \ \widetilde{\omega}_{r_1}^{2(a)} \ \widetilde{\boldsymbol{\phi}}_1^{T(a)} \ \widetilde{\boldsymbol{\phi}}_2^{T(a)} \ \dots \ \widetilde{\boldsymbol{\phi}}_{r_2}^{T(a)}\right] \in \mathbb{R}^{n\times 1} \tag{3}$$

is the vector of the fuzzy predicted output data; $\widetilde{\boldsymbol{\theta}}_j \in \mathbb{R}^{p\times 1}$ is the vector of fuzzy updating parameters at iteration j; $\widetilde{\boldsymbol{T}}_j \in \mathbb{R}^{p\times n}$ is a fuzzy transformation matrix; $\widetilde{\omega}_i^2$ is the ith fuzzy eigenvalue of the structure ($i = 1, \dots, r_1$ and $r_1$is the number of retained natural frequencies); $\widetilde{\boldsymbol{\phi}}_i^T$ is the ith r_3-dimensional fuzzy eigenvector of the dynamic system ($i = 1, \dots, r_2$ and r_2 is the number of the retained eigenvectors); p is the number of updating parameters, and $n = r_1 + r_2 r_3$ is the number of output data. Interested readers are referred to well-known texts [38-40] for more details about the definition of fuzzy variables, vectors, and matrices. Each element of a fuzzy vector or matrix in the above equations is represented using a fuzzy variable. For example, consider a fuzzy element $\tilde{z} \in \mathbb{R}$ of vector $\tilde{\boldsymbol{z}} \in \mathbb{R}^{n\times 1}$. This element may be described as a pair, including the element $z \in \mathbb{R}$, and its membership function value $\mu_{\tilde{z}}(z)$:

$$\tilde{z} = \left\{(z, \mu_{\tilde{z}}(z)) \middle| z \in \mathbb{R}, \mu_{\tilde{z}}(z) \in [0,1]\right\} \tag{4}$$

In this paper, we describe a fuzzy variable with a set of interval variables. Each interval variable is linked with a specific value of membership function. It is evident that the interval variable with the widest range of variation is associated with the zero values of membership function, $\mu_{\tilde{z}}(z)$, while the membership function of one indicates the interval variable with the lowest range of variation or a deterministic value. As shown in Figure 2, the midpoint of the bin that has the maximum number of observations (normalized number of observation here) is given a membership function of 1. As just mentioned, we assign a zero membership function to the widest possible range of variations shown by points A and C in Figure 2. For the membership functions between zero and 1, $0 \leq \mu_{\tilde{z}}(z) \leq 1$, the membership function is gradually increased based on the points defined in the histograms. The crisp set $\tilde{z}_\alpha^I$ is defined according to the α-cut of $\tilde{z}$ as follows:

$$\tilde{z}_\alpha^I = \{z | \mu_{\tilde{z}}(z) \geq \alpha, z \in \mathbb{R}, 0 \leq \alpha \leq 1\} \tag{5}$$

In above equation, the interval variable $\tilde{z}_\alpha^I$ is obtained by intersecting the membership function at $\mu_{\tilde{z}}(z) = \alpha$. This can mathematically be expressed as follows:

$$\tilde{z}_{\alpha}^{I} = \left[\underline{z}^{(\alpha)}, \bar{z}^{(\alpha)}\right] = \left\{z \in \mathbb{R} | \underline{z}^{(\alpha)} \leq z \leq \bar{z}^{(\alpha)}\right\} \quad (6)$$

where $\underline{z}$ is the lower bound and $\overline{z}$ is the upper bound of interval variable $\tilde{z}_{\alpha}^{I}$. Based on the above definition, we can perform the fuzzy finite model updating at each α-cut by using Eq. (1). Based on this view, Eq. (1) may be expressed as follows:

$$\boldsymbol{\theta}_{j+1}^{I^{(\alpha)}} = \boldsymbol{\theta}_{j}^{I^{(\alpha)}} + \boldsymbol{T}_{j}^{I^{(\alpha)}} \left(\boldsymbol{z}_{m}^{I^{(\alpha)}} - \boldsymbol{z}_{j}^{I^{(\alpha)}}\right) \quad \alpha = 0, \alpha_1, \alpha_2, \ldots, 1 \quad (7)$$

Eq. (7) may be solved using interval arithmetic, but as mentioned in the literature, the solution obtained by interval arithmetic is conservative in many cases. The authors of reference [23] provided a solution to Eq. (7) by using the kriging predictor, which is known as a meta-model in general. The solution is approximate and is case dependent. In the proposed solution, a meta model, for example, a kriging predictor, is a highly efficient surrogate for the numerical FE model. Obviously, the type of meta-model is very important and can affect the accuracy of the results. Moreover, the relationship between the input and the output is important in the selection of the meta-model and the sampling method that is used for training the meta model. As shown in [12, 23], the performance of the kriging meta-model in the prediction of FE model behaviour (particularly, with non-smooth behaviour) was excellent. Kriging also provides the residual between the model outputs and the surrogate itself that can be interpreted using probabilistic models. In the first step of fuzzy FE model updating, deterministic model updating is carried out at the highest-level membership function, i.e. at $\alpha = 1$. The updating parameters obtained by deterministic model updating can be represented by a point in the space of input parameters, as shown in Figure 3. The next step involves constructing an initial hypercube around this point and using the meta-model (the kriging model in this case) to map the initial hypercube space of updating parameters to the outputs space. Engineering expert judgment may be used to determine the dimensions of the initial hypercube. It is important to ensure that the mapping is sufficiently good to describe the relationship between the inputs and outputs accurately. This can be achieved by adding more samples when training the kriging predictor until the mean squared error at an un-sampled point falls below a threshold and the kriging model is considered sufficiently good. The inverse problem introduced by Eq. (7) is then solved using the kriging predictor. This is performed separately at each α-level. Care should be taken when determining the dimension of the initial hypercube. If the dimension is greater than the dimension of the updated hypercube at the zero level of the membership function, a new meta model need not be constructed, and this will save computational time. In the proposed method, we carry out interval FE model updating at each level of membership functions (α-level), and the upper and lower bounds of measured data at each α-level are used for interval model updating at the corresponding level. Figure 1 illustrates the proposed method of fuzzy FE model updating, and the process of interval FE model updating is shown in Figure 3. The details of interval model updating using a kriging predictor can be found in [23].

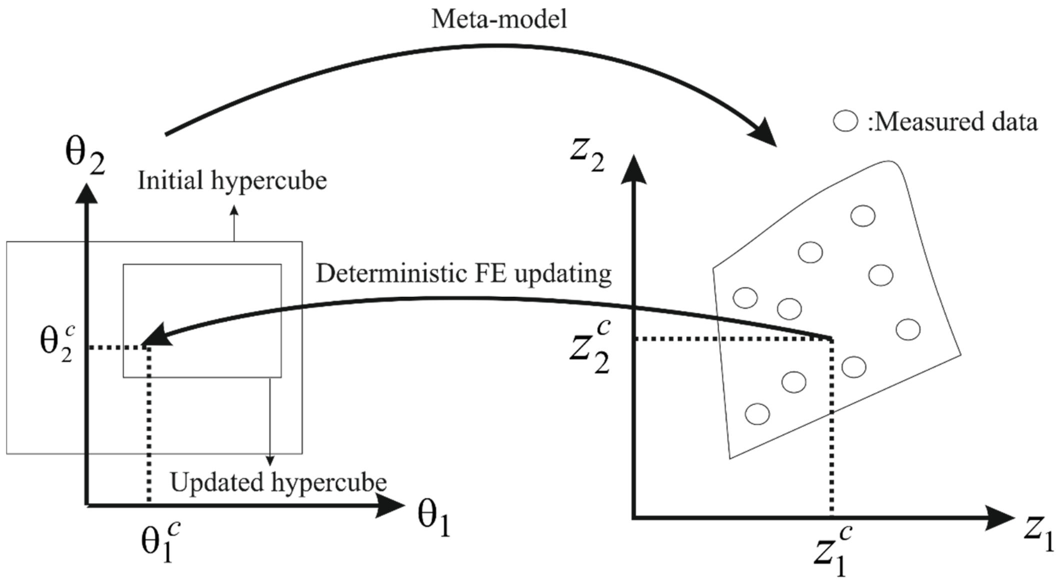


Figure 3: Procedure of interval model updating [23].

3 Case studies on the evaluation of the proposed method

3.1 Case study 1: Three degree-of-freedom mass-spring system

Figure 4 shows a three degree-of-freedom mass-spring system. The model was used to investigate the performance of the proposed fuzzy FE model updating. The deterministic parameters of the model, as shown in Figure 4, are as follows:

$$m_i = 1 \text{ kg}, i = 1,2,3 \quad k_3 = k_4 = 1 \text{ N/m and } k_6 = 3.0 \text{ N/m} \tag{8}$$

The stiffness parameters, k_1, k_2, and k_5 are considered uncertain and modelled as fuzzy variables. First, we assume that all three uncertain parameters have similar true and erroneous fuzzy membership functions. This is shown in Figure 5 (a), where $i = 1,2,5$, and therefore, k_i represents all three stiffness parameters i.e. k_1, k_2, and k_5. For this case, the measured data are simulated. For simulating the measured data, we can use the parameter vertex solution [41, 43] because a monotonic relationship exists between the outputs and inputs. This is because the output data are the eigenvalues of the dynamical system [23], and the global stiffness matrix is linearly proportional to the updating parameters. Using the simulated measured data at different level of membership functions, i.e. $\alpha = 0, 0.2, 0.4, 0.6, 0.8$ and 1, the measured fuzzy membership functions are calculated. The measured fuzzy membership functions (simulated in this case) is then utilized in the proposed FE fuzzy model updating to determine or update the fuzzy membership functions of the uncertain parameters. Figures 5 (b), (c), and (d) show the results, and an excellent agreement is seen between the measured fuzzy membership functions and the ones predicted by the numerical model.

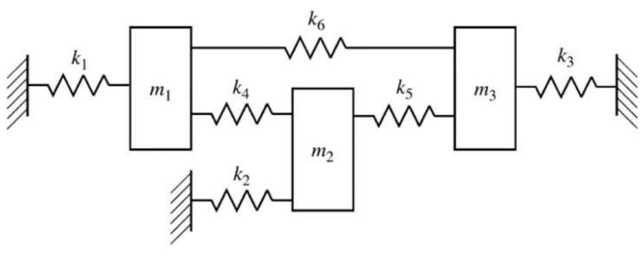


Figure 4: Mass spring system with three degrees of freedom.

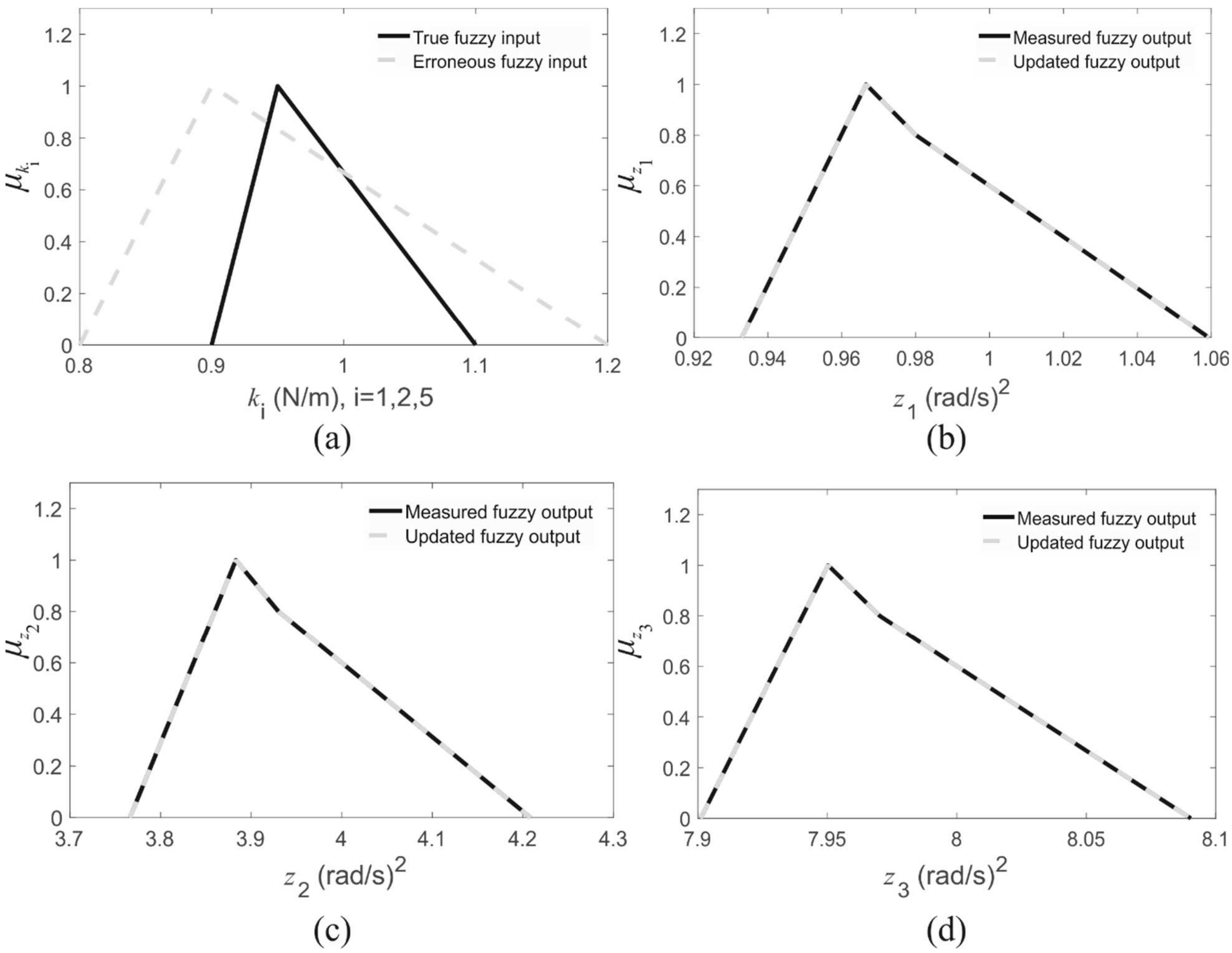


Figure 5: (a) Fuzzy membership functions of the updating parameters (solid: true; dashed: initial), (b) Fuzzy membership functions of the first eigenvalue (solid: measured; dashed: updated), (c) the second eigenvalue (solid: measured; dashed: updated), (d) and the third eigenvalue (solid: measured; dashed: updated) with no measurement noise.

Now, we assume 1% measurement noise in the measured data to show how noisy measured data can affect the accuracy of fuzzy model updating. The results are shown in Figures (6a), (6b), and (6c). As shown in the figures, the measurement noise can deteriorate the identification process. The worst case is the non-convex updated membership function of the third eigenvalue. We expect a convex fuzzy membership functions, and we proposed to fit a convex membership function to input parameters. This is illustrated in Figures (6a), (6b), and (6c). This fitting procedure will be also used in the analysis described in the next section where we deal with physical test data.

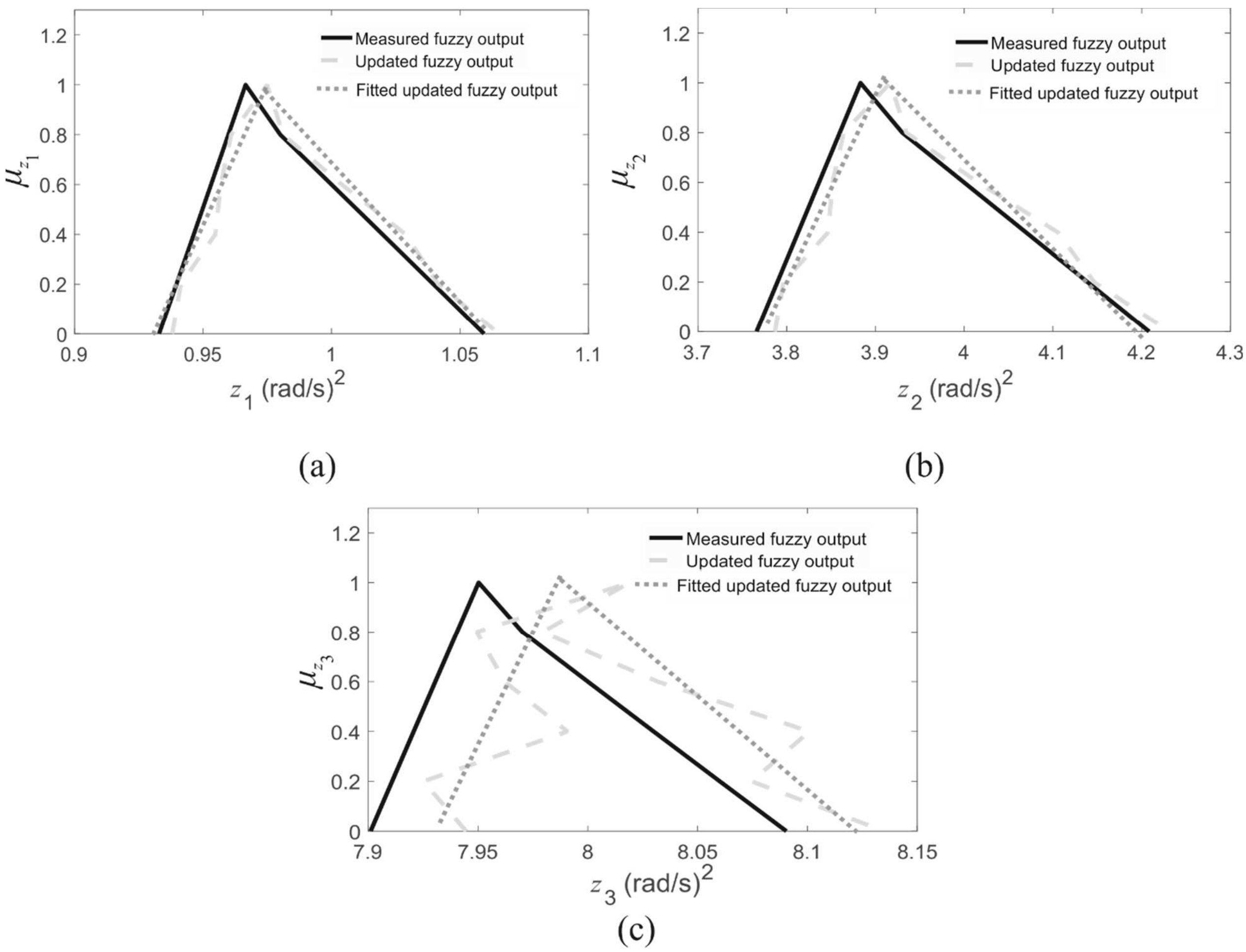


Figure 6: (a) Fuzzy membership functions of the first eigenvalue (solid: measured; dashed: updated), (b) the second eigenvalue (solid: measured; dashed: updated), (c) and the third eigenvalue (solid: measured; dashed: updated) with 1% measurement noise.

3.2 Case study 2: AIRMOD test structure using the proposed fuzzy finite element model updating

In this section, we demonstrate the application of the method to the DLR AIRMOD structure. This structure is a replica of the GARTEUR SM-AG19 benchmark, which was proposed by Balmes [19]. Figures 7 (a) and (b) shows the AIRMOD structure and the corresponding FE model. The structure consists of six aluminium beam components. The components are connected to each other using bolted joints as shown in Figure 8. The dimensions of the wing are listed in Table 1.

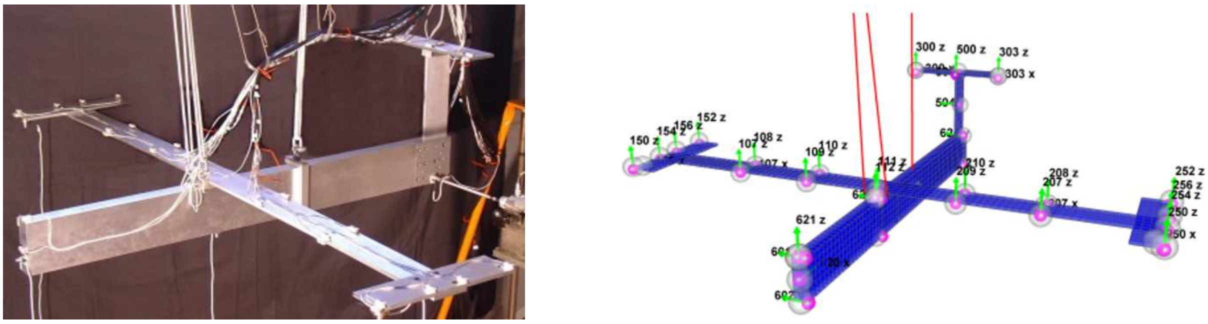
Wing span	2 m
Fuselage length	1.5 m
Tail-plane height	0.46 m

Table 1: Dimensions of the AIRMOD structure.

The mass of the structure is 44 kg. To ensure that the wing torsional modes are excited, two extra masses of 167 g were attached at the winglets tips, as shown in Figure 7 (a). MSC/NASTRAN was used to build the FE model. The details of the FE model are presented in Table 2. Rod elements were used to represent sensor cables and bungee cords.

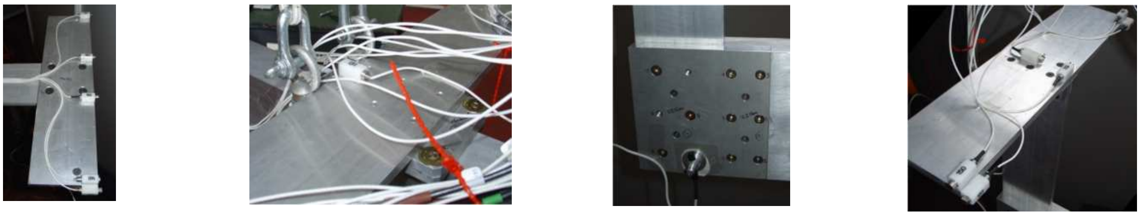
Element type	Number
CHEXA	1440
CPENTA	6
CELAS1	561
CMASS1	55
CONM2	18
CROD	3

Table 2: Details of the finite element model.



(a) (b)

Figure 7: AIRMOD (a) the physical structure and (b) the finite element model.



Winglet | Wing/Fuselage | VTP/Fuselage | VTP/HTP

Figure 8: Beam joints.

DLR designed test rigs and carried out a series of modal tests in order to assess the variability in the measured data due to different sources of uncertainty. Interested readers can see reference [3] for more details about the tests and their results. The maximum variability was shown to arise from the disassembling and reassembling of structures. Disassembling and reassembling changes the stiffness of the joints and the mass distribution. This results in significant modal variability, which is aleatory uncertainty. The first 30 natural frequencies and mode shapes of the physical structure were measured at every stage of the assembling process. For fuzzy FE model updating, we only considered 14 modes; note that this is for the interval model updating case. For deterministic model updating at a membership of one, a set of natural frequencies and mode shapes are used, and therefore, the deterministic model updating is over-determined. The initial and updated results at $\alpha = 1$, are listed in Table 3. The highlighted modes in the table, namely, modes 1–8, 10–12, 14, 19, and 20, are the ones that are used for fuzzy FE model updating because of their significant variability.

#	Mode	Experimental frequency (Hz)	Initial FE frequency (Hz)	Updated FE frequency (Hz)	Error after updating %
1	RBM Yaw	0.23	-	0.23	0.01
2	RBM Roll	0.65	0.56	0.65	-0.01
3	RBM Pitch	0.83	0.82	0.83	-0.00
4	RBM Heave	2.17	2.14	2.17	-0.02
5	2nWingBending	5.50	5.65	5.52	0.40
6	3nWingBending	14.91	15.11	14.91	-0.01
7	WingTorsionAnti	31.96	33.31	32.04	0.25
8	WingTorsionSym	32.33	33.62	32.19	-0.42
9	VtpBending	34.38	35.39	34.77	1.14
10	4nWingBending	43.89	44.66	43.85	-0.08
11	1nWingForeAft	46.71	47.21	46.72	0.02
12	2nWingForeAft	51.88	52.91	51.91	0.05
13	5nWingBending	58.59	60.59	59.35	1.29
14	VtpTorsion	65.93	67.69	65.90	-0.05
15	2nFuseLat	100.05	102.59	102.12	2.07
16	2nVtpBending	124.56	128.62	126.40	1.48
17	6nWingBending	129.38	132.08	129.20	-0.14
18	7nWingBending	141.47	145.91	142.55	0.76
19	2nHtpBending	205.59	206.73	205.51	-0.04
20	HtpForeAft	219.07	225.73	219.31	0.11
21	WingBendingRight	254.73	261.53	254.68	-0.02
22	WingBendingLeft	255.02	262.64	255.84	0.32
23	3nWingForeAft	272.08	278.71	276.00	1.44
24	WingletBendingLeft	303.96	320.15	310.77	2.24
25	WingletBendingRight	304.32	321.64	311.47	2.35
26	3nFuseLat	313.68	324.12	321.18	2.39
27	WingTorsionSym2	328.55	336.31	330.52	0.60
28	WingTorsionAnti2	331.18	341.15	333.53	0.71
29	4nWingForeAft	336.21	343.55	335.71	-0.15
30	2nFuseVert	348.68	359.54	354.85	1.77

Table 3: Deterministic model updating results; gray rows are the modes that are used for updating and the white rows are used for model validation.

The description of the 18 updating parameters is presented in Table 4. The updating parameters include stiffness and the mass parameters of the FE model of the AIRMOD structure. The stiffness parameters are categorized in two groups, namely, joint stiffnesses (θ_4 to θ_{10}) and support stiffnesses (θ_1 to θ_3 and θ_{11} to θ_{12}). Note that it is not easy to estimate the masses of the instrument cables and their distribution, and therefore, they too are considered as updating parameters. These masses may change slightly during the disassembly and reassembly of the structure. The concentrated mass parameter at the VTP joint (vertical/horizontal tail-plane joint) is represented by θ_{13}, and the distributed mass over the wings span are indicated by θ_{14} to θ_{18}. The method described in Section 2 is used to compute the measured fuzzy membership functions of the 14 natural frequencies listed in Table 3. The 14 measured fuzzy membership functions are shown in Figure 9. The figure indicates that the membership functions for the 14th and 20th natural frequencies exhibit a non-convex behaviour with two peaks in their fuzzy membership functions. This indicates clustering of the data into two distinct regions for these two modes. From an engineering point of view, one may ignore this non-convex behaviour of these two modes and only consider the upper and lower bounds at these α-levels. We also need to investigate further to understand why these two modes have non-convex properties. However, for the sake of completeness, we assume that the 14th and 20th measured fuzzy membership functions remain non-convex and propose a method to deal with this situation during fuzzy model updating. As already mentioned, fuzzy FE model updating was performed under certain levels of membership functions. Herein, α varied from 0 to 1 in steps of 0.1. Figure 9 shows how the measured data for the 14th and 20th natural frequencies are clustered for the membership functions from 0.2 to 0. 7. To deal with the non-convex measured fuzzy membership function in the fuzzy FE model updating, we propose to perform interval FE model updating for four cases when α varies between 0.2 and 0.7. At these levels of membership functions, each fuzzy variable is represented by two intervals, called left and right intervals in this paper. Considering that two modes have non-convex behaviours, there will be $2 \times 2 = 4$ possible combinations of these intervals, and therefore, four interval FE model updating exercises are required at each level. The above four possible cases for interval FE model updating can be expressed as follows:

Case 1: The left intervals of the 14th and 20th natural frequencies with intervals of the other modes

Case 2: The left interval of the 14th natural frequency and the right interval of 20th natural frequency with intervals of the other modes

Case 3: The right interval of the 14th natural frequency and the left interval of the 20th natural frequency with intervals of the other modes

Case 4: The right intervals of the 14th and 20th natural frequencies with intervals of the other modes

After applying the proposed fuzzy FE model updating along with the aforementioned classification at membership functions between 0.2 and 0.7, the updated membership functions of the uncertain parameters are determined. The results are presented in Figure 10. Note that updating parameters are normalised with respect to their initial values given in Table 4. The results show that four intervals are obtained for the updating parameters at membership functions between 0.2 and 0.7. This is attributed to the application of interval model updating for four cases at these membership functions levels. The updated fuzzy membership functions of the natural frequencies are determined by fuzzy uncertainty propagation using the trained kriging model. Fuzzy propagation is carried out at all membership functions levels using a careful selection of the updated bounds of updating parameters for each case. For uncertainty propagation at each α-cut, Monte Carlo simulation (MCS) is used. Note that the MCS is computationally efficient in this case because of the use of a kriging meta model. The kriging model is trained using 600 samples within the space of the initial updating parameters at the zero membership level. In [23], it was shown that an accurate mapping between the updating parameters and the selected natural frequencies can be achieved by using 600 samples. The membership functions obtained by the four cases are then used to compute the updated fuzzy membership functions of the natural frequencies. This should be carried out at each level of the membership function and for all of the four cases. If the updated bounds at all levels overlap with each other, the extreme points, that is the lowest and highest possible values of the updating parameters, are the updated bounds. Otherwise, at any membership functions, the level at which the updated bounds of the output quantity do not overlap with each other, we represent the fuzzy membership functions by more than one interval variable at the given level. Figure 11 shows how the method identifies two distinct interval variables for the updated 20th natural frequency fuzzy membership function. Figure 12 shows the results obtained using the proposed fuzzy FE model updating. As seen in the figure, there exists excellent agreement between the measured and updated fuzzy membership functions for the second and third natural frequencies. Further, the measured fuzzy membership functions of all the other natural frequencies are enclosed by their corresponding updated fuzzy membership functions. The updated regions at the zero level enclose all measured samples. This provides confidence that the proposed fuzzy

FE model updating not only can improve the accuracy of fuzzy membership functions of the outputs predicted by the FE model, but also can predict the non-convexity of the 20th natural frequency. The method was not successful in achieving this prediction for the 14th natural frequency. This may be explained by the fact that the measured data are contaminated by noise, and therefore, the updated intervals at all membership functions levels are greater than their corresponding experimental bounds. Note also that the errors at different levels of membership functions are reasonably low, though they appear large in Figures 12 and 13. For example, consider the 7th and 8th natural frequencies: the maximum errors of the lower and upper bounds are 1% and 0.81%, respectively, for the 7th natural frequency and 0.9% and 0.92%, respectively, for the 8th natural frequency. In addition, Figure 13 gives a two-dimensional presentation of the fuzzy membership functions without considering the effects of non-convex behaviour. Only 14 modes are shown here. The figure shows how different levels of membership functions indicate the possibility of enclosing all measured samples. Overall, the results show great improvements in the predictions of the updated fuzzy FE model considering the fact that, in this case, we are dealing with physical test data.

	Type	**Location**	**Description**	**Init. Val.**	**Unit**
θ_1	stiffness	VTP/HTP joint	sensor cable – y dir^n	1.30E+02	N/m
θ_2	stiffness	wing/fuselage joint	sensor cable – y dir^n (top)	7.00E+01	N/m
θ_3	stiffness	wing/fuselage joint	sensor cable – y dir^n (bott^m)	7.00E+01	N/m
θ_4	stiffness	VTP/HTP joint	joint stiffness – x, y dir^{ns}	1.00E+07	N/m
θ_5	stiffness	VTP/HTP joint	joint stiffness – z dir^n	1.00E+09	N/m
θ_6	stiffness	wing/fuselage joint	joint stiffness – x dir^n	2.00E+07	N/m
θ_7	stiffness	wing/fuselage joint	joint stiffness – y dir^n	2.00E+07	N/m
θ_8	stiffness	wing/fuselage joint	joint stiffness – z dir^n	7.00E+06	N/m
θ_9	stiffness	VTP/fuselage joint	joint stiffness – x dir^n	5.00E+07	N/m
θ_{10}	stiffness	VTP/fuselage joint	joint stiffness – y dir^n	1.00E+07	N/m
θ_{11}	stiffness	front bungee cord	support stiffness	1.80E+03	N/m^2
θ_{12}	stiffness	rear bungee cord	support stiffness	7.50E+03	N/m^2
θ_{13}	mass	VTP/HTP joint	sensor cables	2.00E-01	kg
θ_{14}	mass	wingtip right wing	screws and glue	1.86E-01	kg
θ_{15}	mass	wingtip left wing	screws and glue	1.86E-01	kg
θ_{16}	mass	wingtip left/right	sensor cables on wings	1.50E-02	kg
θ_{17}	mass	out^r wing left/right	sensor cables on wings	1.50E-02	kg
θ_{18}	mass	inn^r wing left/right	sensor cables on wings	1.50E-02	kg

Table 4: Initial values of the selected updating parameters.

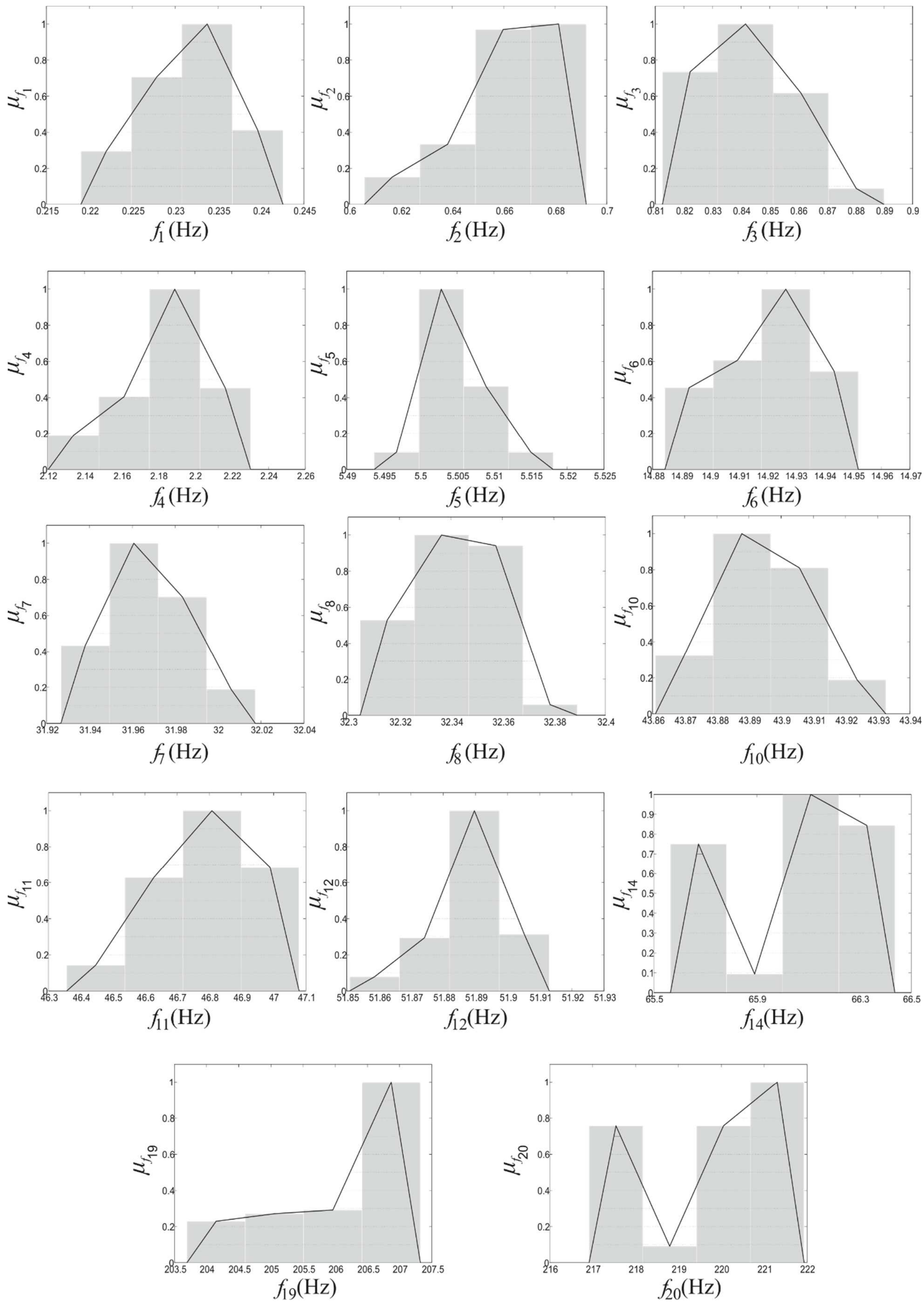


Figure 9: Measured fuzzy membership functions of the AIRMOD structures (14 natural frequencies).

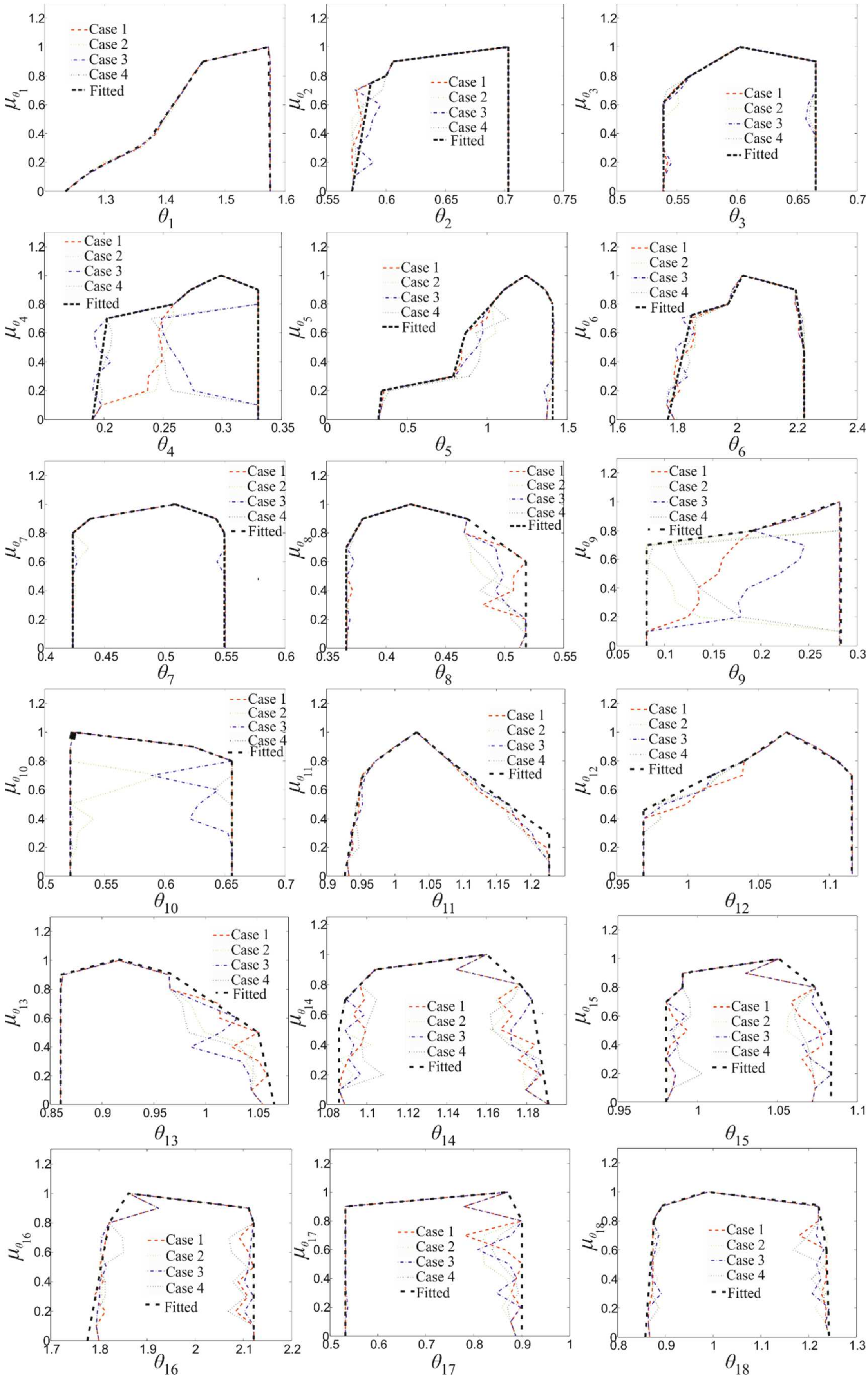


Figure 10: The updated and fitted updated fuzzy membership functions of input parameters.

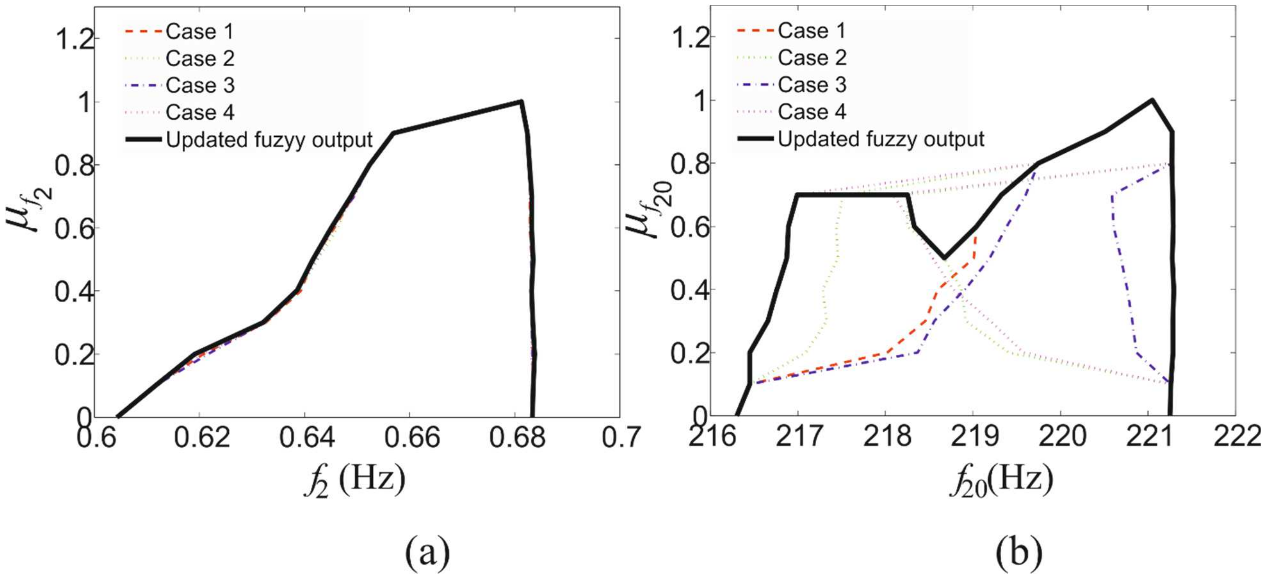


Figure 11: Calculation of the updated fuzzy membership functions in the presence of (a) convex (b) non-convex fuzzy membership functions.

5- Conclusion

The problem of fuzzy finite model updating was considered in this study. It was shown that this problem can be solved using a series of interval FE model updating problems if a fuzzy variable is represented by a number of interval variables at different levels of its membership function. A method was proposed to compute the measured fuzzy membership functions of experimental data. The histograms of the measured data were used to determine the fuzzy membership functions of the experimental data. To this end, a method is proposed to extract these membership functions from the histograms fitted to the data. The measured fuzzy membership functions are then used in fuzzy FE model updating. The solution to the fuzzy FE model updating is obtained by interval FE model updating at different levels of membership functions (α-cuts). The interval model updating is carried out using the method proposed by the authors in [23]. The proposed fuzzy FE model updating method was validated numerically by considering a simple mass-spring system with three degrees of freedom. Further, the effect of noisy measured data on the performance of the proposed method was demonstrated. The application of the proposed method was also demonstrated via a physical test. The structure considered was the DLR AIRMOD, and the variability was due to the process of disassembly and reassembly. The measured fuzzy membership functions of the natural frequencies were determined using the proposed method in this study. Two of the measured fuzzy membership functions exhibited non-convex behaviour. A method was proposed to deal with this situation. Overall, there was a good agreement between the updated fuzzy membership functions of the outputs and the measured ones with a maximum error less than 2%. The proposed method was also capable of predicting the non-convex behaviour of one of the natural frequencies.

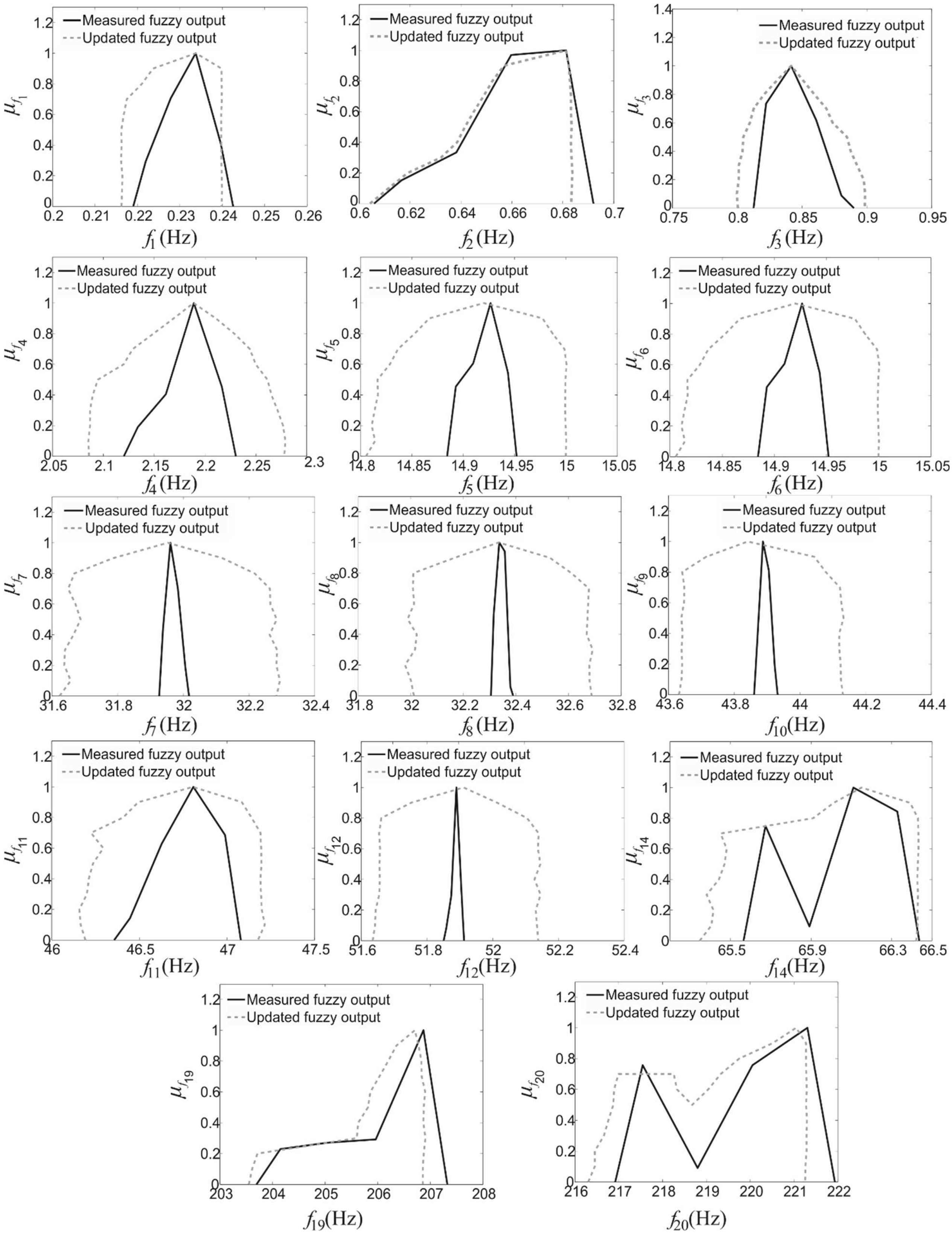


Figure 12: The fuzzy finite element model updating results using the four cases and non-convex fuzzy membership functions of input parameters, individual updated and measured fuzzy membership functions.

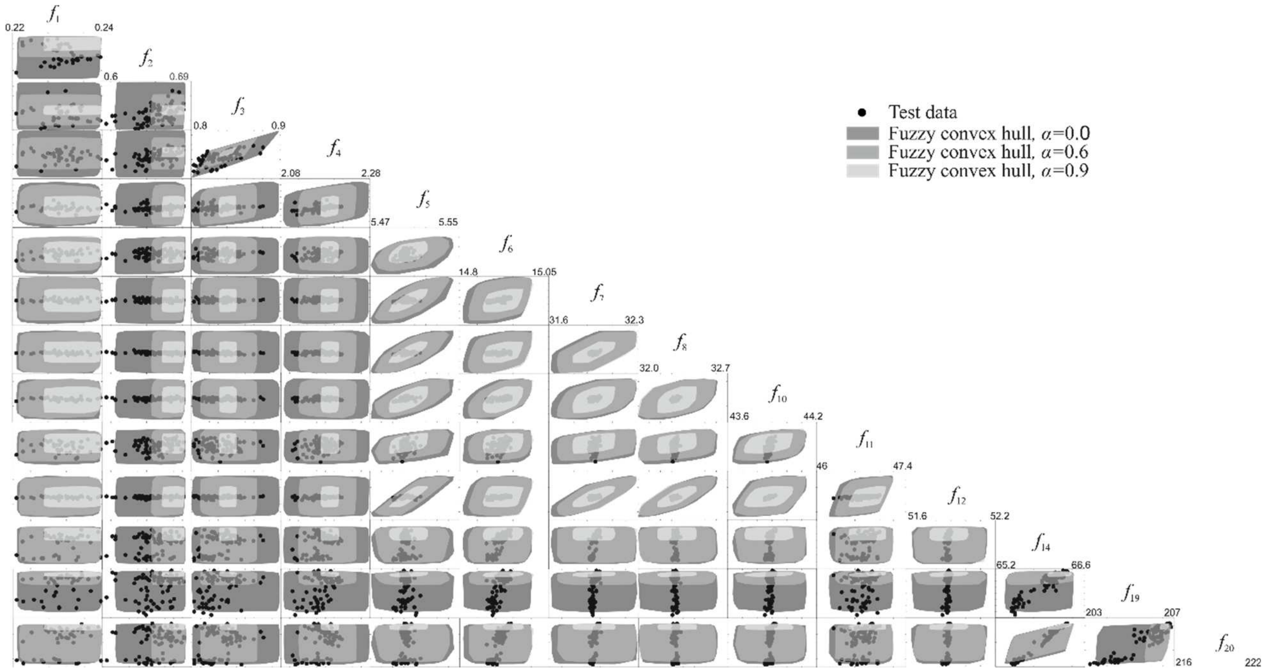


Figure 13: Two-dimensional fuzzy membership functions using convex hull (ignoring the effects of non-convex fuzzy membership functions and using the fitted fuzzy membership functions in Figure 10)

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