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WIDEBAND MEASUREMENT OF TRANSISTOR SMALL SIGNAL PARAMETERS BY TIME DOMAIN SPECTROMETRY

by

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DEPARTMENT OF ELECTRICAL AND CONTROL ENGINEERING

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SUMMARY

Time Domain Spectrometry (TDS) Methods are outlined and their application to measurement of transistor parameters is discussed. Experimental results are presented and the advantages and limitations of TDS methods are re-assessed.

Future developments are suggested.

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1. Introduction

In this report, the method of Time Domain Spectrometry (TDS) and its applications to the wideband measurement of the small signal parameters of transistors is discussed. The advantages and limitations of the method are reassessed both for present and projected systems in the light of results obtained in investigations directed particularly at establishing the accuracy and range limiting factors of present systems. Comparison is made between TDS and frequency domain (FD) measurement techniques.

The report contains two appendices, on Fourier transforms and on parameter systems. These are included not because they are original but to make the report self-contained.

1.1 History of method

Time Domain Spectrometry represents a development of so-called Time Domain Reflectometry (TDR) methods into a technique for quantitative measurement. TDR has been in use for some years and has been extensively applied to the qualitative analysis of transmission line systems.[1,2] A suitable pulse launched down the line is reflected from discontinuities associated with such features as line defects, connectors, mismatched loads, etc. By measuring the time interval between the ingoing pulse and a particular reflection, the position of the reflectance in the line can be found. By comparing the two pulses, an estimate can be made of the magnitude and nature of the reflectance. This is useful information, but is insufficient for the accurate characterisation of devices. For this, more careful analysis is required than can be realized by visual inspection. Such an extension of the technique was suggested by one of the authors several years ago, and the results obtained in early experiments[3,4] were of sufficient quality to merit the more extensive examination detailed in this report, using later and more sophisticated equipment than was available for the earlier investigation.

1.2 Time Domain and Frequency Domain

Corresponding to any realisable pulse in the time domain is a frequency distribution or spectrum, and vice versa. Pulses and spectra are related by the Fourier transform

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{i\omega t}dt \tag{1}$$

and its inverse

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{-i\omega t}d\omega \tag{2}$$

where $f(t)$ is a TD pulse and $F(\omega)$ is a FD spectrum. Appendix 1 discusses the conditions under which these transforms are valid. Further, for a linear network, the ratio of the Fourier transform of the responded and the incident pulses gives the response of the network as a function of frequency since this ratio, computed for any one frequency, represents the ratio of the spectral components, at that frequency, of the output and input signals. It follows that a single pair of pulses will yield the entire frequency response characteristic of the network, up to an upper frequency limit. This limiting frequency is related to the upper limit of component frequencies contained in the input pulse, and to bandwidth limitations in the measuring system. Thus the method is truly wideband.

The foregoing method of obtaining frequency responses has been used for the analysis of the performance of control systems. With the long response times of some such systems the real time recording of the output was simple, and the input could usually be made a good approximation to a step or impulse function, both of which have known Fourier spectra. Thus only the output function had to be Fourier transformed, either numerically or by analogue methods. Applications to transistors or any other device of very much shorter response time were not considered practicable because

(i) pulse sources and recording equipment had a limited bandwidth, and FD methods were reasonably easily available to cover this band more economically,

(ii) Fourier transforming the pulses took too long.

The first objection was overcome by the development of tunnel diode pulse generators and the wideband sampling oscilloscope which made possible the generation and accurate recording of picosecond rise time pulses. This enabled TDR systems with a bandwidth of several GHz to be constructed, using ordinary recording equipment rather than high speed photography. The second objection disappeared with the advent of easily available high speed computers.

2. T.D.S. Systems

2.1 Description

Figure 1 shows a simple T.D.S. system suitable for measuring one-port and two-port networks, which includes transistors. The input and reflected pulses are measured and recorded on the A channel, and the transmitted pulses on the B channel. The signal is carried throughout on coaxial lines of constant impedance, on which the network causes a discontinuity. The ratio of responded pulse spectrum to incident pulse spectrum is thus the reflection or transmission coefficient of the network as a function of frequency. The set of four possible ratios constitute the set of s-parameters for the network, the forward and reverse reflection and transmission coefficients (see Appendix 2).

The system shown in Fig. 1 is rather an elementary one. No biassing arrangements are shown, and no account is taken of line loss or 'A' and 'B' channel matching. Also the relative time of arrival of the responded and incident pulses are not well defined. These prevent the system from being of practical use. Fig. 2 shows a more advanced system, incorporating several hardware additions and a more refined operating technique. Such a system will be described in detail, since it has been constructed and operated, and has yielded the results which are discussed in a later section.

The method of operation is as follows. The pulse from the generator is split into two parts by the matching Tee, one part being the input signal and the other providing a time marker. The signal portion is attenuated to a suitable level, and passed through the sampler, which is of the feed through type, down the line to the network under test. The reflected and transmitted pulses are recorded at A and B respectively. The length AN_i is made large enough so that the input pulse at A has reached a constant level before the reflected pulse from the network arrives. Instead of recording the ingoing pulse as a reference input, a standard coaxial open or short circuit is placed at N_i. The pulse reflected from this to A is used as the reference for the reflected pulse. Similarly the transmitted reference pulse is obtained at B, by substituting a U-line for the network. In this way, the effects of losses in the line and in the bias injection networks are cancelled. Also, with a reference pulse for each channel, there is no need for accurate matching of the channels.

The time marker system is a very necessary addition to the simple version of the T.D.R. system. From the matched Tee, the part of the pulse generator output which will provide the marker is led down a coaxial line terminating in a matched load (50Ω). Just in front of this load is a small capacitative take-off into another coaxial line leading to an attenuator and the B channel output. The take-off consists of a modified coaxial Tee unit, in which the inner coaxial side arm does not quite touch the centre straight through conductor but is separated by a small air gap. The RC differentiating circuit produced by this capacitor and the characteristic impedance of the line provides a sharp spike from the step function output of the generator. With a further delay provided by the line length CB, the marker pulse arrives at the sampling point B. The total delay of the time marker pulse is such that it arrives at 'B' immediately before the end of the time observed on the oscilloscope, and after the pulse through the network has settled down. The peak of the time marker is taken as the time zero for the pulses observed on both A and B channels. When recording the pulses displayed on the x-y recorder, the marker pulse is automatically provided on the B output, and can be obtained on the A by superimposing the B channel after the region of interest on the A channel has been recorded. Thus a time zero is provided on both channels, dependent only on the delay produced by the line lengths, and independent of the timebase and pulse generator triggering arrangements. For the sake of convenience, the line lengths AN_i and BN_o are made equal.

The bias required by a transistor under test is provided by the Tektronix signal take-offs and probes shown in Fig. 2.* Although this is not their designed function, they perform it well. The probes contain a 500Ω ½-watt series resistor, which limits the possible bias current to about 30 mA under steady conditions. The bias voltages are prevented from reaching the sampler by the blocking capacitors shown. This is necessary since the diode sampling gates have a rather low d.c. voltage rating and are easily destroyed.

The transistors are held in G.R. transistor jigs (originally designed for use with the General Radio 1607 VHF-UHF bridge) which give a smooth transition between the coaxial line and the transistor. These provide a definite location of the measurement plane. Used with these are open and short circuit standards and U-lines, which have the same electrical length.

The system is constructed mostly with G.R. coaxial 50Ω air spaced lines and connectors, although some high quality solid dielectric cable is used in less critical parts of the marker delay lines. Care was taken in the positioning of the components along the lines to reduce reflections arriving in the signal viewing region to a minimum, or to shift them out of the region, and the choice of components of the system was made with their reflection coefficients very much in mind. Attention was also paid to the mechanical stability of the system, in particular to ensure that it was not affected by the stresses involved in changing the coaxial reflection and transmission standards and the transistor jigs.

2.2 Operation

The system was operated as described in the preceding section, and measurements were taken on transistors, passive networks and coaxial reflectance standards over a wide range of conditions and signal levels. Considerable difficulty was experienced with noise on the sampling oscilloscope, and much time and effort was wasted before the fault was realized and located. It turned out that there was low frequency noise on both channels, which showed up as high frequency noise on the x-y recorder trace. It was found that this noise was very highly correlated betweeen the channels, and was cancelled out if the oscilloscope was operated in the differential A-B mode. Unfortunately, this reduced the dual-channel oscilloscope to an effective single good channel, and although this did not prevent the making of measurements, or affect their accuracy, it made the process more inconvenient and time consuming. In spite of several appeals to the manufacturer, the fault was finally rectified only after the completion of the experiments here reported.

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*The use of these components for this purpose was suggested by M. Bradshaw and A. Jones of the Elliott-Automation Computer Research Laboratory.

The noise was traced by the manufacturers to a faulty avalanche diode. With the rectification of the fault, a number of further runs were taken on coaxial standards using the dual channel facility now available. These additional measurements served only to confirm the results given here.

With transistors, the choice of signal level was such that the change in current through the collector due to the pulse did not exceed 20% of the standing current. As a quick check that linearity was roughly maintained, the sign of the input pulse was changed and the response of the transistor monitored to see if its form was affected. 20% as a limit was thought to be a reasonable compromise between linearity and signal level for the worst case, which is a forward connected transistor in grounded emitter configuration. Here the large current gain is the difficulty, since the input must be quite small if the collector swing is not to be excessive. For 2N3572 transistors, this input was, approximately a 6 mV step. Grounded collector measurements imposed less stringent conditions since the d.c. transmission gain was small, about 2 for the above transistor, and input steps of up to 18 mV were used. In all cases the standing collector current was 10 mA.

The pulses were recorded on an x-y plotter, and the data for the computer analysis were obtained by manual amplitude sampling at suitably chosen intervals on the time axis (see Appendix 1). Care was taken to ensure that the pulse generator and oscilloscope had warmed up, and that the pulse generator was adjusted to provide a jitter-free signal. Undue delay between reference and responded pulse recording was avoided, to prevent any errors due to drift, etc. Mechanical disturbance of the system was minimised, and care was taken to see that the connectors were pushed completely home, to prevent spurious reflections and time delays.

List 1 Equipment used in T.D.S. system

Oscilloscope	140A	Main Frame	Hewlett Packard
	1425A	Sampling Timebase	Hewlett Packard
	1411A	Sampling Vertical Amplifier	Hewlett Packard
	1432A	Sampling Head	Hewlett Packard
Pulse Generator	213B		Hewlett Packard
X-Y Recorder	7004AM		Hewlett Packard
Bias networks	P6034	500Ω probes	Tektronic
	VP1	Coaxial signal take-off	Tektronix
Attenuators	874-series		General Radio
Capacitors	874-series		General Radio
Coaxial lines	874-series		General Radio
Transistor jig	874-series	P600 Grounded emitter/collector	General Radio
Coaxial open circuit	874-series	WO-10 Length matched to P600	General Radio
Coaxial short circuit	874-series	WN-10 Length matched to P600	General Radio
Coaxial U-line	874-series	U-10 Length matched to P600	General Radio
Coaxial standards	874-series	W100, W200 terminations	General Radio

3. Results

The results discussed here were taken from measurement runs on coaxial standards, passive networks, and transistors, at various signal levels and conditions, using the sampling oscilloscope in the inconvenient but noise free mode referred to above. They are thought to be representative of the results obtainable with the system previously described, and to give a fair indication of the precision attainable.

The frequency spectrum of a typical reference pulse is shown in Fig. 3. The term quasi-spectrum is used to describe the frequency-multiplied Fourier transform of a pulse. This is convenient for the discussion of step or step-like pulses, since the amplitude of the quasi-spectrum of a perfect step is a constant for all frequency. The rise time of both the oscilloscope and pulse generator was limited to 90 picoseconds, giving a bandwidth for each of 4 GHz. The combined bandwidths, and losses arising from the system imperfections, cause the amplitude of the quasi-spectrum of the input pulse to be a decreasing function of frequency. Fig. 3 shows that the amplitude decreased to half the zero frequency value at 2.3 GHz, and declines very rapidly thereafter. There is some evidence that the amplitude is beginning to level off around 4 GHz, and this is ascribed to the signal merging with the background noise level. This is borne out by the evidence of Fig. 4, where the agreement between separate pulses worsens catastrophically after 3.5 GHz.

The decline of the quasi-spectrum as a function of frequency sets the frequency limit to the usefulness of the system. Up to 2 GHz or so, the signal to noise ratio permits measurements of high accuracy to be made. Above this, the accuracy falls with increasing frequency, until at 3.5 GHz or more, the present system is useless.

Most of the results were obtained from measurement runs on coaxial standards representing nearly pure resistances in order that the accuracy of the system could be assessed. However, some measurements were taken on passive reactive networks with frequency dependent characteristics, and on bipolar transistors in both grounded emitter and collector modes.

Fig. 5 and 6 show results taken on a linear reactive passive one-port network with both the T.D.S. system, and with a F.D. system utilising 50 ohm coaxial lines, directional couplers and a vector voltmeter for the direct measurement of the complex reflection coefficient as a function of frequency. The T.D.S. system was run with a 50 mV input step. The graphs show that the T.D.S. system is capable of handling circuits which have a definite resonance and a wide variation in reflection coefficient. The results agree well with those produced by the vector voltmeter system, and also demonstrate the sensitivity of the latter to the impedance of the network being measured, one of the drawbacks of that instrument.

Figure 7 gives an indication of what is observed and recorded during the series of measurements on a transistor. The graphs are only approximately to scale, but give a good idea of the various pulse forms. The transistor was a 2N3572, which is a silicon NPN device with a nominal f_T of 1 GHz. It was connected in grounded collector configuration. Each of the six graphs spans a time of about 5 nanoseconds. The s-parameters computed from the original recordings are shown in Fig. 8.

4. Accuracy

The sources of error in the system are:-

1. Drift in the characteristics of the pulse input during recording and between recordings of responded and reference pulses. This is probably a major source of error in the present system. It can be minimised by proper warming-up of the systems and careful adjustment of the pulse generator. Also, a faster data recording system should effect a considerable reduction.

2. Non-linearity of sampler, oscilloscope amplifier, and recorder. This is probably better than the published specification for the devices, since neither the sampler nor the amplifier are required to be used at their maximum dynamic range. The input pulses to the sampler are small, and the x-y recorder can be used to provide relatively distortionless gain to reduce the amplifier swing.

3. Oscilloscope range attenuator, and choice of data sampling frequencies. These are classed together because they are easily avoidable, the first by calibration, which is simple, and the second by choosing a sufficiently small time interval between samples.

4. Sampling errors. These are quite small with manual sampling of an x-y recording, except where the slope of the graph is large. Correct choice of time scale will give a reduction in the slope, but is limited by the necessity of recording all the region of interest of the pulse. An automatic direct sampling read-out by digital voltmeter could be used to improve the present manual system. Random errors tend to cancel statistically.

5. Time jitter in pulse generation and oscilloscope timebase. The sum of the jitter from these sources as stated in the instrument specification equals 30 picoseconds, or $\pm 22°$ at 2 GHz, but normally, the jitter is much less than this, provided the pulse generator is adjusted correctly.

6. Coaxial system reflections. These can be reduced in amplitude by using good quality components, and may be reduced in number by siting potential reflectors so that their reflections do not interfere with the region of interest of the signal pulse. Since they are a permanent part of the system it is possible to make allowance for them.

7. Network and standard connection. The operational method used compares

the response of a network plus its connector(s) with the response of a standard reference component plus its connector(s). Error corrections can be made, but only to the extent of the repeatable component of the reflection from any pair of connectors. The jigs used to hold the transistor also fall into this category. These have quite appreciable reflection coefficients, due to the difficulties of matching a large diameter coaxial line with a transistor lead. Again, the repeatable part of this can be accounted for, but there remains some uncertainty as to the quality of the contact the jig makes with a transistor, particularly one with bent leads.

8. Noise on system. High frequency noise is lost by the averaging effect of the sampler and recorder as the pulse is scanned. Low frequency noise, as drift in the d.c. levels, is important. The performance of the system is dependent upon the signal to noise ratio. This, in turn, depends on the amplitudes of the reference and responded pulses, and the frequency.

Table 1 Errors in Phase Measurement on Coaxial Standards

	ERROR SOURCE	AMPLITUDE OF ERROR
1.	Drift	Not known
2.	Nonlinearity of oscilloscope	3% of F.S.D. from specification but better than 1% in use.
3.	Sampling Errors	± 0.01″ on amplitude and time marker ± 0.1″ on very steep slopes
4.	Time jitter	±30 picoseconds from specification - much less in practice
5.	Coaxial line reflections	2% to 2 GHz
6.	Network and standard connection	1% to 2 GHz for G.R. 874 connector 3% to 2 GHz for transistor jig

Table 1 gives estimates of these errors, where they are known. The figures given are really not much help in predicting the accuracy of the system. For this, the best method at present consists of measurements on coaxial standards, and comparisons between different runs on the same network.

Tables 2 and 3 give some results for various coaxial standards, for different values of the input pulse amplitude. As predicted, the errors rise with frequency. Table 2, last column, shows the results for a 50 ohm termination. This is a measure of the connector mismatch plus the drift errors. From the tables, it is a reasonable interpretation that the accuracy of the present system, without any form of correction procedure is about 3% om amplitude and 10° in phase up to 1 GHz, and 5% in amplitude and 15° in

Table 2 Percentage Errors in amplitude measurement on coaxial standards

FREQ	W100			W200			WO - WN -1.00			W50
MHz	6mV	18mV	50mV	6mV	18mV	50mV	6mV	18mV	50mV	14mV
0	-1.34	-0.82		-1.03	-0.58	-0.06	-0.24	0.82	0.00	0.082
250	-1.03	-1.66		-0.27	-1.55	-0.09	-1.34	-0.24	0.06	0.67
500	+0.23	-0.94		+1.19	-1.38	+0.21	-1.02	0.44	0.09	1.08
750	+1.29	-0.84		1.82	-1.89	+0.36	+1.21	1.86	-0.25	1.11
1000	+0.39	-2.25		1.65	-2.36	+0.45	+0.36	0.37	-0.44	0.67
1250	-0.33	-0.76		2.11	-1.34	+0.63	+0.27	0.20	0.19	0.30
1500	-1.33	-1.89		2.83	-2.29	-0.04	+3.07	0.22	0.16	0.56
1750	-2.87	-2.83		3.34	-2.77	+0.30	+2.25	3.52	-1.49	1.99
2000	+1.05	-0.79		4.45	-0.41	+1.05	+0.68	4.74	-2.05	1.62
Run number	1601/4	1611/39		1602/4	1640/39	1403/2	1603/4	1638/47	1401/2	1642/59

Table 3 Errors in phase measurement on coaxial standards

FREQ	W100			W200			WO - WN		
	6	18	50	6	18	50	6	18	50
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
250	+1.2	+1.0		-0.3	+1.5	-0.6	0.4	-2.1	0.7
500	+1.6	-2.1		-1.0	+1.7	-0.7	1.9	-3.2	1.4
750	-0.2	+2.8		-1.9	+4.8	-1.1	2.0	-5.0	2.0
1000	-2.2	+3.4		-2.0	+8.3	-0.9	1.8	-6.3	2.7
12.0	-2.9	+2.5		-2.2	+6.0	-1.1	3.3	-2.9	3.3
1500	-4.9	+2.0		-2.2	+7.4	-0.8	2.3	-9.3	4.3
1750	-4.4	+3.3		-7.2	+12.5	-0.3	2.6	-13.5	5.6
2000	-5.1	+3.1		-11.0	+12.3	-0.7	1.5	-14.5	6.6
Run number	1601/4	1647/39		1602/4	1640/39	1403/2	1603/4	1638/47	1401/2

phase up to 2 GHz, under reasonable conditions. Under optimum conditions, the results should be comparable in quality with the 50 mV results given in the tables. On the other hand, under adverse conditions (very cramped graph scale, low input step, low reflection or transmission coefficient), the errors could be greater than this.

5. Development of T.D.S. System

The present system is not ideal in use, but the addition of an automatic data sampling system would go a long way towards improving it. The convenience and speed of operation would be greatly improved, and as a bonus, accuracy would be increased by reducing the time available for drift during and between runs. Work has been started on such a system using a digital voltmeter and printer. The printer was used because of its availability, but a tape punch would be more useful. It is hoped to experiment with direct input to a small computer via a high speed analogue to digital converter.

An automatic biassing system would present few problems, but for the present experimental system, the effort is not really justified, since the bias can be set up without difficulty, and the series resistors in the injection networks provide some degree of overload protection.

A study has recently been made of the problems associated with the extension of the system bandwidth to 8 or 10 GHz. Samplers and pulse generators are now available for this range, and the main difficulty is the rise in the reflection coefficients of the components of the system. The situation is that the performance of the present coaxial system declines with frequency, and that while better lines and connectors are available, they do not offer the range of components of the G.R. system. Attenuators, reflectors, and so on are available, but the more specialised components such as transistor jigs (for frequencies greater than 2 GHz), bias injection networks, etc. are not available. For measurements on active devices, the change is probably not worth making unless there is a compelling reason for obtaining the highest possible accuracy, and a willingness to design and construct any coaxial components which are not available commercially. For precision measurements on passive networks, or on 'packaged' active networks, i.e. a network in a box with suitable connectors and independent bias supply, then the changeover is justifiable at present. A result of the study is that the careful consideration of the disposition of the components necessary to obtain any accuracy at all at 10 GHz has shown how the present system may be improved, by reducing the spurious reflections still further.

At present, the system requires that the network be physically reversed for the forward and reverse measurement, and replaced with standards for reflection or transmission reference purposes. This is time consuming, and it is possible to construct a switching arrangement to reduce this, without introducing troublesome reflections, although the cost of doing so would be several hundreds of pounds.

The data sampled from the x-y recordings are at present Fourier transformed with no further processing, no error correction other than that inherent in the operating method being used. Trials with very noisy data have shown that an improvement in accuracy is possible by smoothing the sampled data, but such techniques have not so far been applied to data of reasonable quality. It is also likely that some improvement in system accuracy can be obtained by correcting for some of the known reflections in the system.

6. Comparison with Frequency Domain Techniques*

In use, the chief difference between T.D.S. and F.D. techniques is that T.D.S. is essentially a broadband method. The same amount of effort on data collection and preparation is required for any number of frequencies of measurement, only the computer having more to do. This fact, plus the requirement for a computer, puts T.D.S. at a disadvantage where the number of frequencies is very low. However, unless completely automated, the work of data collection, calibration and error correction rises linearly with the number of frequencies, and here too computing facilities become advisable. Thus there exists a breakeven point, above which T.D.S. becomes the easier method to use.

If T.D.S. suffers from total reliance on computing, F.D. suffers from its reliance (with modern techniques) on directional couplers. These are required to separate the ingoing from the output signal, and introduce frequency dependent errors which in general must be corrected before any accuracy of measurement is obtained. Their bandwidth is limited (as is that of signal sources) and they may require to be changed in the course of a measurement run to obtain the necessary frequency range.

Both T.D.S. and current F.D. equipment rely on sampling techniques, and thus the upper frequency limit of both are related to the speed of available sampling devices. For a given speed of sampler, F.D. has a slight edge on T.D.S., except in the case of the Vector Voltmeter where the frequency limit is very much less than that of a T.D.S. system.

The ultimate accuracy obtainable from both systems would appear to depend on the repeatability of connection of the network under test, and the standards against which the network is measured. Other errors can be compensated for, or averaged out, but this non-repeatability remains, so in this also the techniques in the two domains are comparable.

It is a feature of 'state of the art' systems that they quickly become obsolescent. A new system becomes available, offering, for example, greater

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*A review of up-to-date FD instrumentation and a comparison with TDS is given in Ref. 5.

frequency range, and supercedes the previous equipment. Frequently on acquiring the new device the old is cannibalised, and therefore its breakdown value is of importance. Highly specialised components tend to be of much less use in these circumstances than general purpose ones. Here T.D.S. scores, since the main investment is in a high speed sampling oscilloscope, a pulse generator, and an x-y recorder, all generally useful.

One other advantage of the T.D.S. technique is that switching time measurements can be performed with a minimum of alteration to the system. A fast switching time measurement system requires the same type of oscillo-scope, bias sources, etc. as the T.D.S. system, and these are the major items of expenditure in both. Thus, with little addition, a T.D.S. system can be used to perform all the non-static measurements normally performed on transistors, both bipolar and field effect. F.D. measuring systems cannot perform T.D. measurements.

Table 4 gives a comparison of current T.D.S. and F.D. systems, and also includes possible extensions of T.D.S. systems.

Table 4 Range and cost comparisons for various measuring systems

SYSTEM	RANGE MHz	COST k£
H.P. Vector Voltmeter	30-1000	3.3
G.R. Admittance Bridge	25-1500	4.0
H.P. Network Analyser	110-2000	6.0
	110-4000	6.7
	110.12400	8.2
H.P. Automatic Network Analyser	110-2000	34.0
	110-12400	41.0
T.D.S. System	0-2000	3.2 or 5.2*
	0-3000	3.4 or 5.4*
	0-10000	4.6 ot 6.6*

* This includes automatic data acquisition system, additional cost, including tape punch, £2000.

7. Conclusion

The work done so far at Cranfield on T.D.S. has shown it to be a valuable measurement technique. For single spot frequency measurements, it is not as convenient as the F.D. techniques currently available, although it will perform as well as most. For wideband measurement, it is preferable to any of the other techniques, except the H.P. Automatic Network Analyser which is an order of magnitude greater in cost. It is dependent on the availability of a computer, but so, in practice, are F.D. methods if any quantity of data is to be analysed, corrected, and possibly changed from one parameter system to another.

Much work remains to be done in refining the present system, involving a careful analysis of the error sources and their reduction and correction, the automation of the data collection and sampling, and the reduction in noise. Extension of the frequency range of the system is a further worthwhile aim, in principle involving only the replacement of certain equipment items. In practice, a higher frequency capability will prove invaluable in detecting and reducing reflections in the system, with benefit to the system at all frequencies.

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APPENDIX A Fourier Analysis

The Fourier Theorem states that for a function $f(t)$ in the Time Domain there exists a corresponding function $F(\omega)$ in the frequency domain. The relationship between the two functions is given by the Fourier Transform,

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \qquad (1a)$$

and its inverse

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega \qquad (1b)$$

The Fourier theorem holds if the condition that

$\int_{-\infty}^{+\infty} |f(t)|\, dt$ is finite and unique is satisfied. This is a sufficient condition, but certain functions which do not satisfy it possess a valid Fourier transform. Such a function is the unit step function, defined by

$$H(t-\tau) = 0 \quad t < \tau$$
$$= 1 \quad t > \tau$$

which has the Fourier transform $\frac{1}{j\omega}$. For a discussion of this, see Lanzos, reference 6.

Thus, in principle, if the function $f(t)$ is given, then its Fourier transform may be calculated, either analytically or numerically. In T.D.S. work, $f(t)$ is not known analytically, and the necessary numerical integration in (1a) must be done for each frequency of interest. This is avoided by the use of the Shannon sampling theorem. Shannon[7] states that if a function $f(t)$ has a Fourier transform $F(\omega)$ which contains no components above a frequency ω_c, i.e.

$$F(\omega) = 0,\ \omega > \omega_c,$$

then it may be represented without loss of information by a series A_n, given by

$$A_n = f(n\tau),\ \tau < \frac{\pi}{\omega_c};\ \ n \text{ integer} \qquad (2)$$

Then the Fourier transform is given by

$$F(\omega) = \tau \sum_{n=-\infty}^{+\infty} A_n e^{-in\omega\tau} \qquad (3)$$

This sum is infinite or oscillatory for step or similar functions, but a modification to the Shannon theorem due to Samulon[8] overcomes this. There the sampled amplitudes are replaced by their successive differences, obtaining

$$F(\omega) = \frac{\tau e^{\frac{1}{2}i\omega\tau}}{2i\,\sin(\frac{1}{2}\omega\tau)} \sum_{n} B_n e^{-in\omega\tau} \qquad (4)$$

where $B_n = A_n - A_{n-1}$

The summation over n need only extend over the region of change of the time function.

Since in practice the requirement of an upper limit to the frequency spectrum of a pulse is usually satisfied to an adequate degree by the deficiencies of pulse generators and observing instruments if nothing else, then the equation (4) may be used to perform the required Fourier transforms. Since the summation there is much quicker to perform than the integration in (1a) on a digital computer, it is the method of choice.

The frequency limit given by the sampling interval τ is called the sampling frequency, f_s, where

$$f_s = \frac{1}{2\tau}$$

If this is not sufficiently large, and there exists an appreciable component in the analysed function of frequency greater than f_s, then the transform calculated by the sampling method will be in error, the error usually increasing with frequency. This is shown in Fig. 9.

Fourier transform programs have been written in I.C.L. 1900 FORTRAN which incorporate the above method, and these were used for the analysis of experimentally derived time functions.

Fourier Transforms and Transfer Functions

If the pulses incident to and responded from a network are f(t) and g(t) respectively, the situation may be equally well represented as the same network with frequency spectra $F(\omega)$, $G(\omega)$ as the incident and responded qualities.

The transfer function $H(\omega)$ is then defined as:

$$H(\omega) = G(\omega)/F(\omega)$$

or
$$H(\omega) = \frac{\int f(t)e^{-i\omega t}dt}{\int f(t)e^{-i\ t}dt}$$

The concept of transfer function is essentially a linear one, and the method is therefore restricted to linear networks.

APPENDIX B

Under small signal conditions, transistors may be regarded as linear devices to a reasonable approximation. As such they may be represented as two port linear networks, the theory of which is well understood. For complete characterisation of such networks, four parameters are required,[9,10] each in general a complex function of frequency.

There are a number of parameter sets in common use, the most popular to date being the hybrid (h) and the admittance (y) parameters. These fall into the class of open-closed parameter sets, meaning that an open or closed circuit is provided at one of the network points during measurement. At high frequencies, provision of open or closed circuits at the proper point on a transistor is difficult and much more so when measurements have to be made as well. It is possible to do both by reflecting impedance at the transistor port, but the line length must be adjusted for each frequency. This is impractical for T.D.S. systems. It is possible to present to the transistor an impedance which is constant and finite for all frequencies by using transmission lines at the in and out ports. Then use must be made of a finite parameter set. Of these, the scattering or s-parameters[11] are the most common, and their use is growing as transistor measurements are required at higher and higher frequencies. Insertion parameters[12] are another finite impedance set.

s-parameters are used in high frequency transmission line measurements because they are what is actually measured, the reflection and transmission coefficients of the network with both ports terminated in transmission lines of constant impedance.

parameter	coefficient
s_{11}	Forward reflected (from inport)
s_{12}	Reverse transmitted (from out to inport)
s_{21}	Forward transmitted (from in and outport)
s_{22}	Reverse reflected (from outport)

In principle, all the parameter systems are equivalent in that they contain the same information, and given any one set, any other parameter set can be calculated. In practice, experimental errors in the set which forms the starting point may become magnified in the process of transformation. Various interparameter conversion formulae are given in the references.

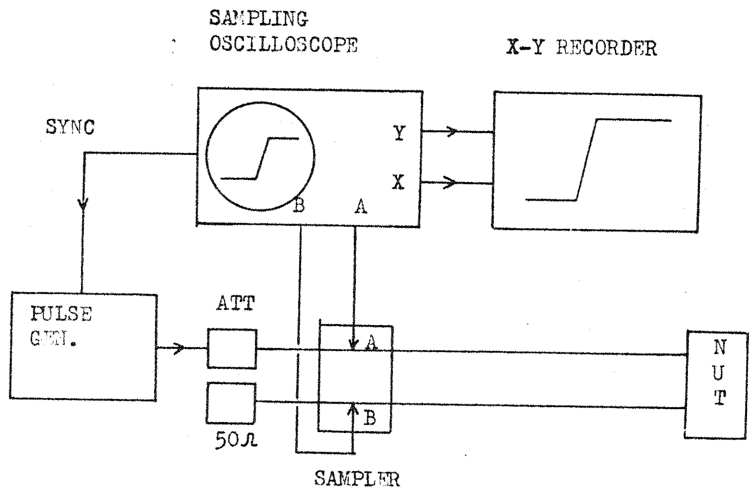


Fig. 1 Basic TDS system.

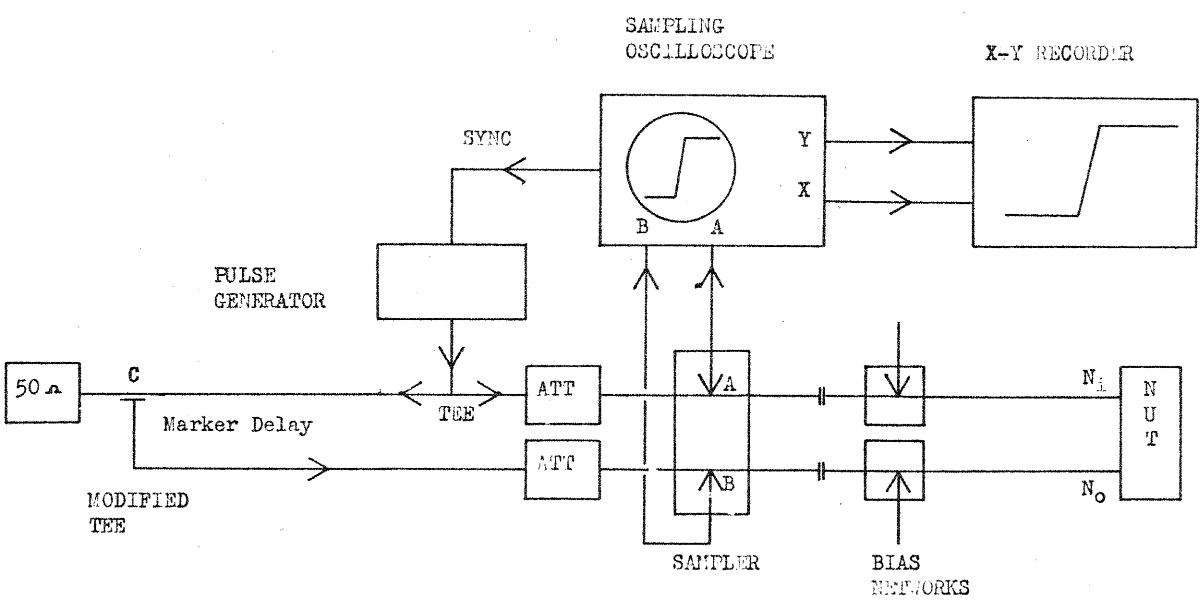


Fig. 2 Full TDS system. See List 1 for equipment listing.

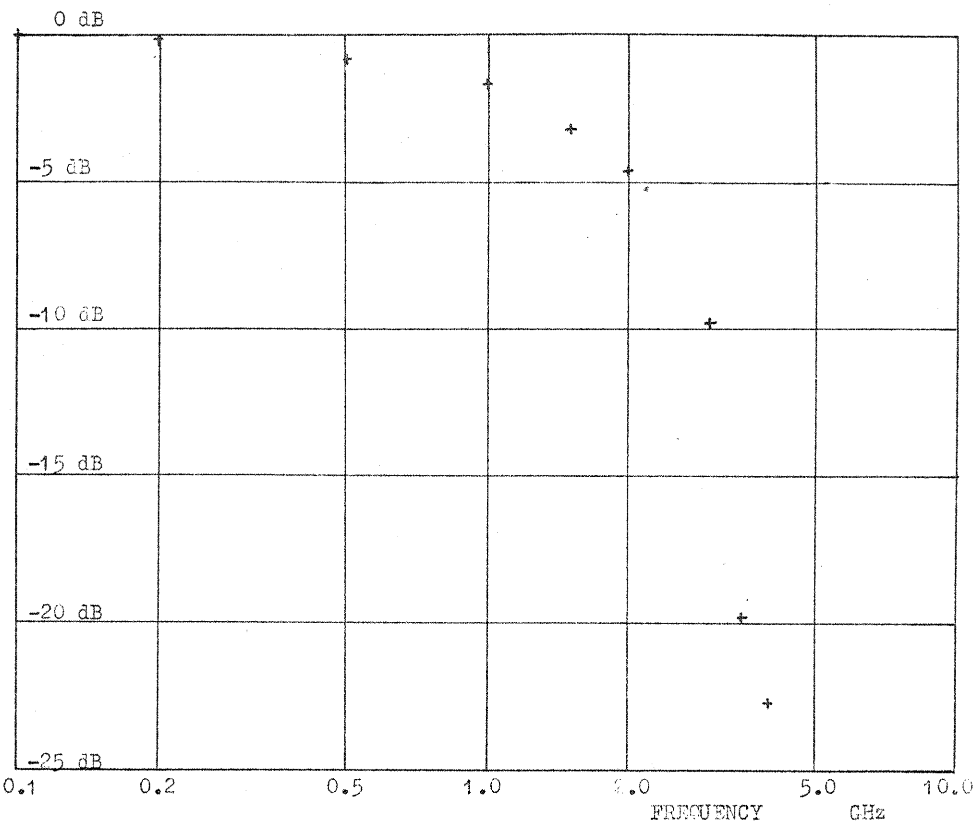


Fig. 3 Quasi-spectrum amplitude vs. frequency. Frequency on log. scale. 0 dB is zero frequency value.

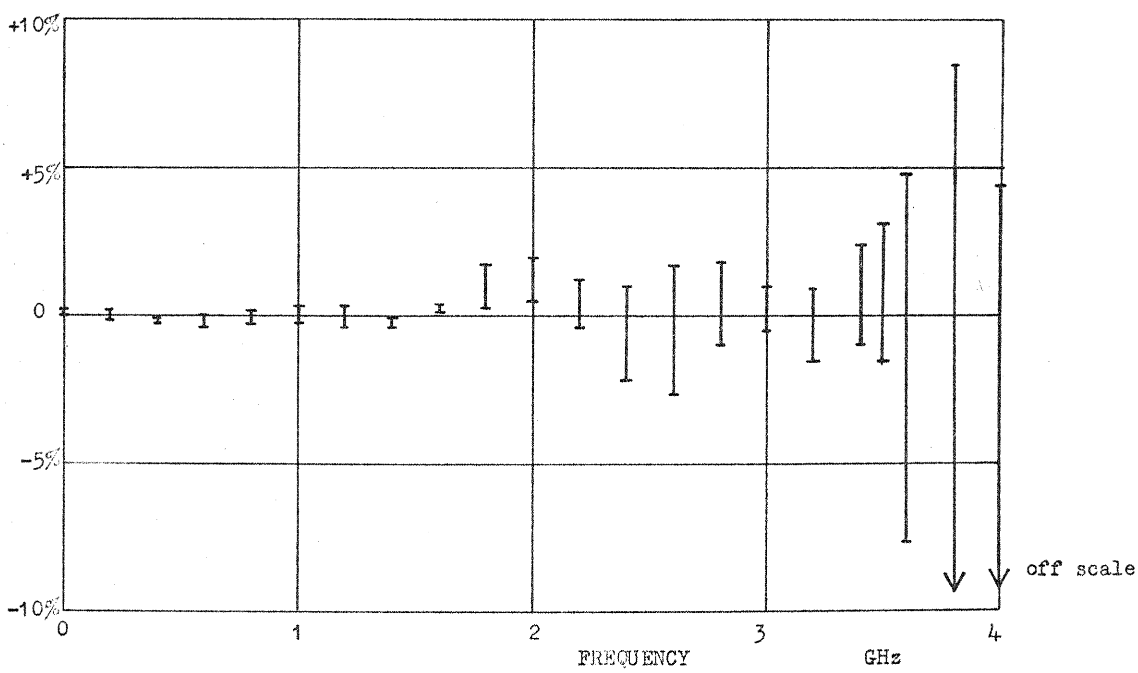


Fig. 4 Scatter between successive pulse quasi-spectra as a function of frequency. Shows deviation from unity of ratio between three pulses with a fourth, as a percentage. Note the rapid increase in scatter around 3.5 GHz.

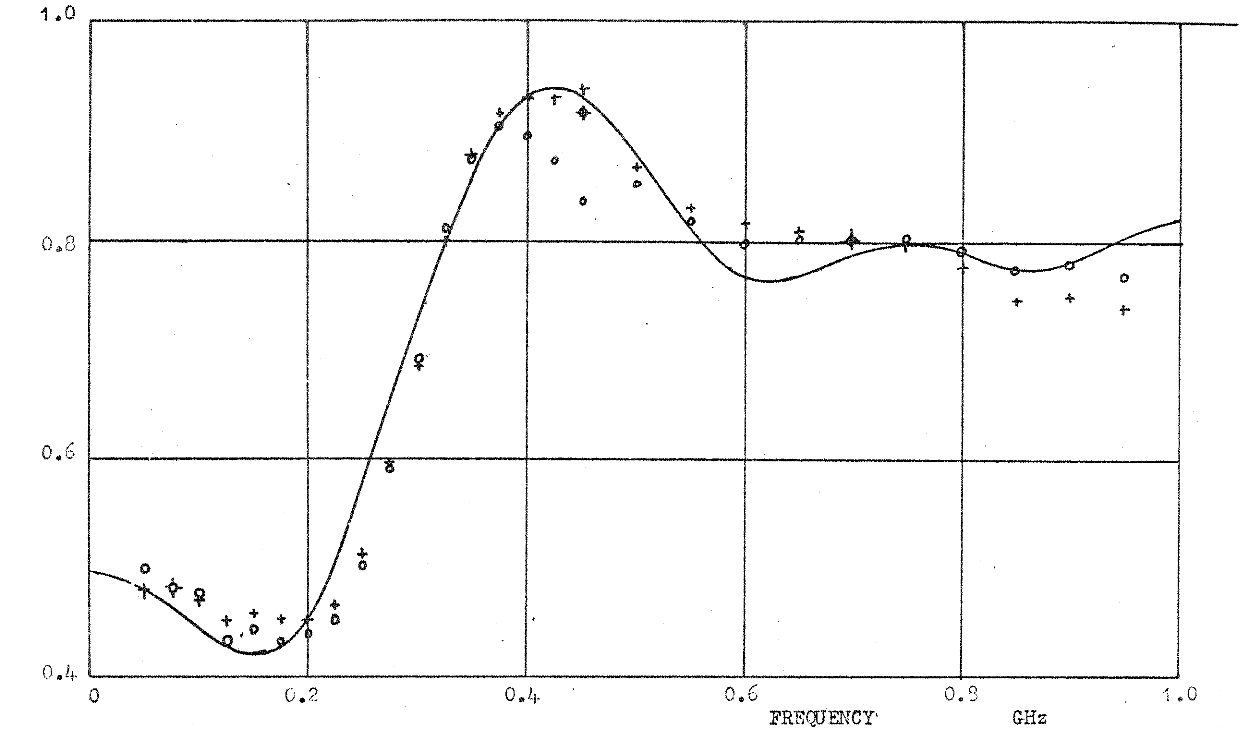


Fig. 5 Passive network reflection coefficient - amplitude.

Solid line is TDS, + is Vector Voltmeter with short circuit standard,
o is Vector Voltmeter with open circuit standard.

GMY TDS 1408/7

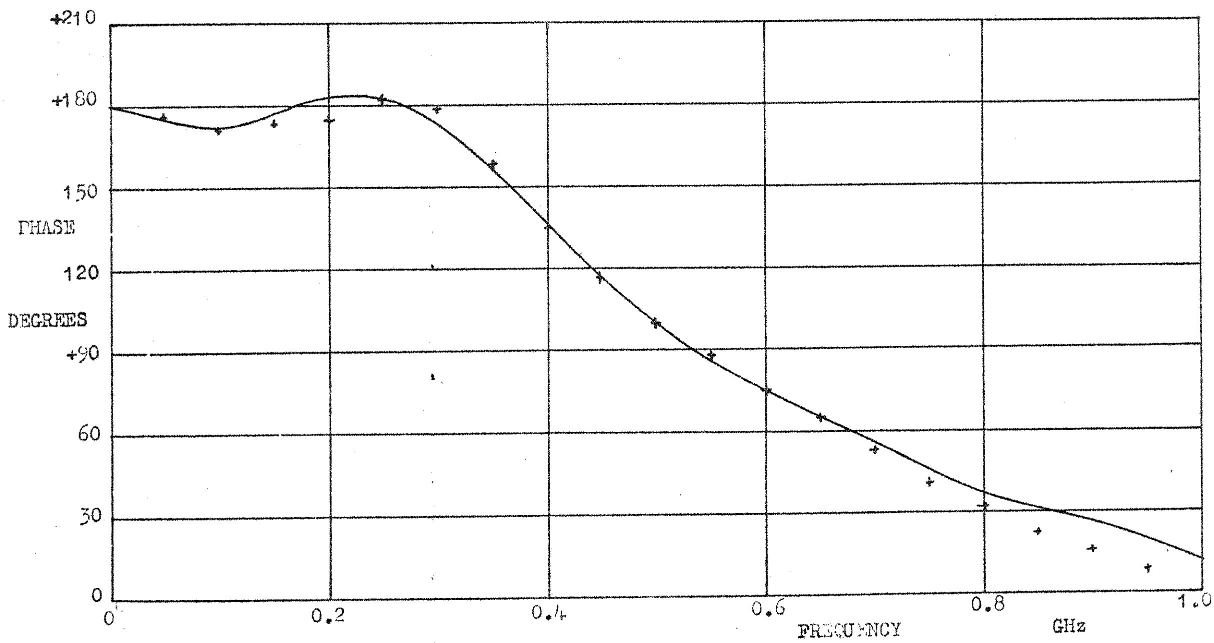


Fig. 6 Passive network reflection coefficient - phase.
Solid line is TDS, crosses are from Vector Voltmeter measurements.

GMY TDS 1408/7

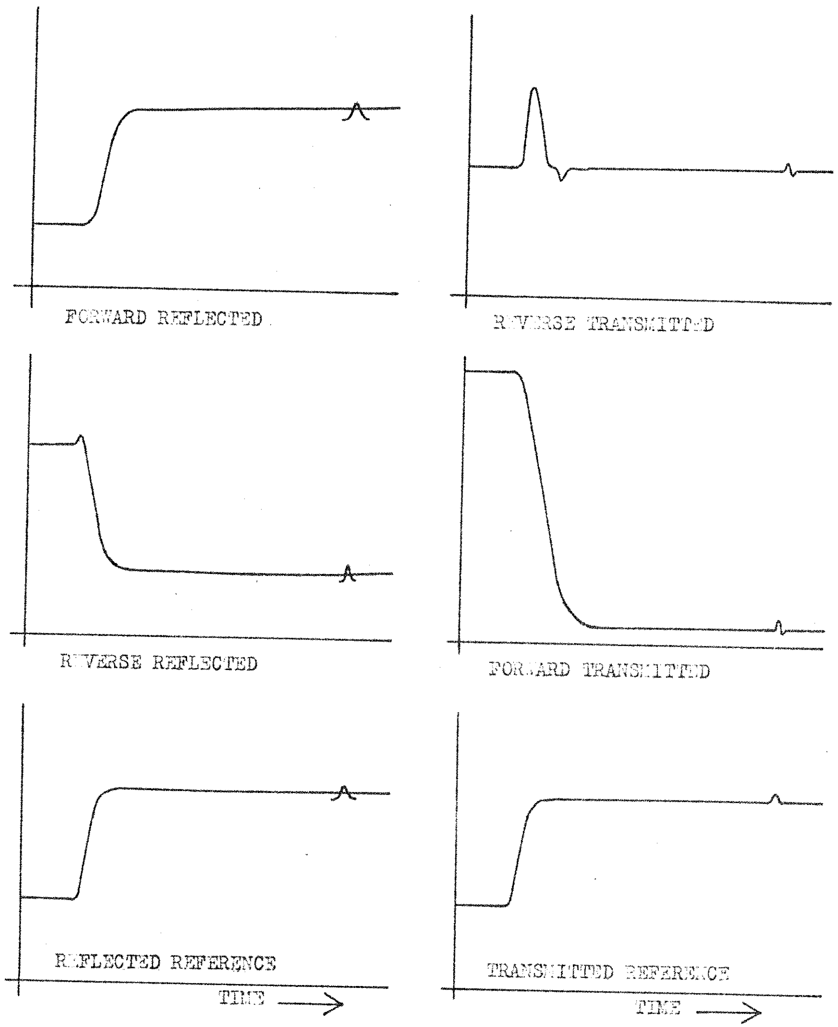


Fig. 7 Set of pulse forms observed during measurement run on 2N3572 transistor in grounded collector mode.

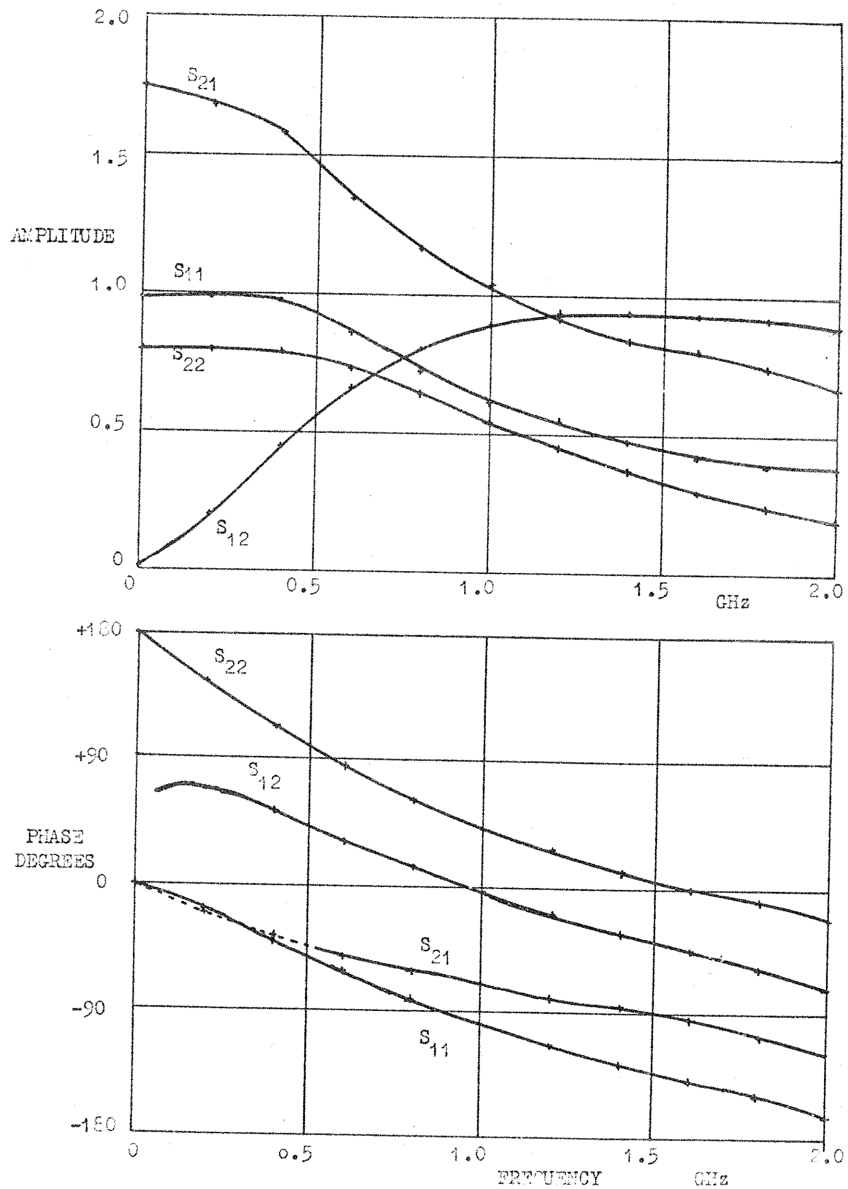
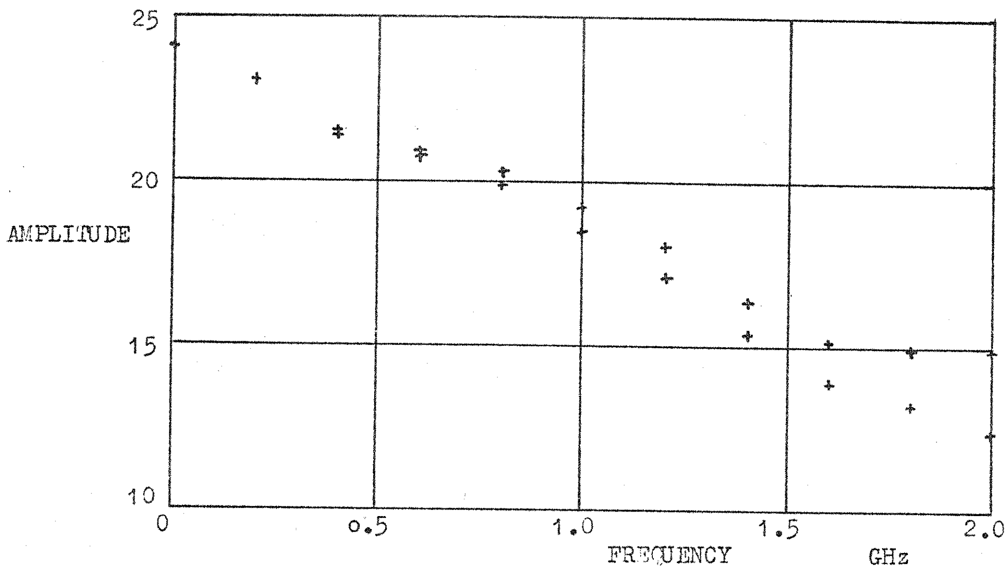
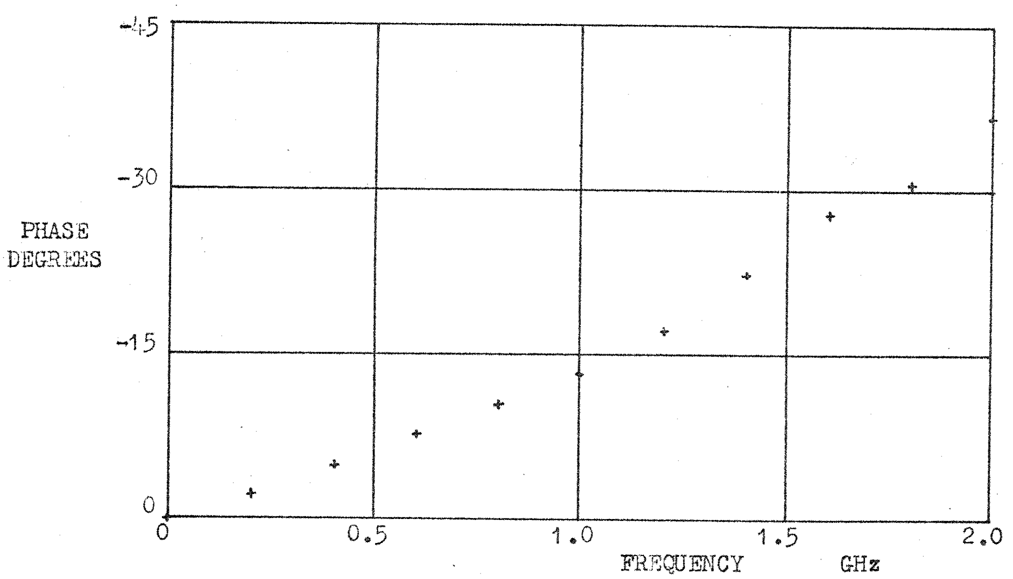


Fig. 8 s-parameters for 2N3572 by TDS. Grounded Collector.

GMY TDS 1600



Upper points are 3GHz sampling, lower are 6GHz.



Phase difference between 3 GHz and 6 GHz sampling.

Fig. 9 Effect of different sampling intervals on the quasi-spectrum of a pulse. 6GHz sampling is sufficient, 3 GHz is not.