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Aileron Reversal and Divergence of Swept Wings with Special Consideration of the Relevant Aerodynamic and Elastic Characteristics

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SUMMARY

Using oblique coordinates, the static problems of Aero-elasticity for swept wings are reduced to the solution of integral - or matrix equations, which may be solved by iteration. The present treatment also indicates the suitability of integral equations for fundamental aero-elastic investigations. It shows that the ab initio use of matrix equations may lead to more complicated calculations, and that for this reason they should be confined to computation offices, particularly since the transition from the integral equations to the matrix equations mostly used in this type of work requires only a simple formal transformation.

Application of the theory to a simple swept wing is given and its divergence speed obtained. In two Appendices methods are deduced for estimating the aerodynamic derivatives and calculating the elastic influence functions from experimental data. Both methods are applied to the case of a model wing, and the influence functions thus obtained are compared with their theoretical values. To simplify future applications of this method, the complete calculations are presented in the form of tables, suitable as computation schemes.

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NOTATION

C_{ij}	Influence coefficients for uniform wing
$C_{ij}(x)$	Influence functions for arbitrary wing
$C(x)$	Wing chord
$H(\eta,x)$	Kernel of integral equation (2.13)
$H^1(\eta,x)$, $H^2(\eta,x)$	Kernel of integral equation (2.11a)
$\bar{H}(\eta,x)$	Kernel of integral equation (3.2a)
$\bar{H}^1(\eta,x)$	Kernel of integral equation (3.3a)
$H_{ij} = H(\eta_i,x_j)$	
L	Aerodynamic Lift
L_1, M_1	Oblique components of couple about OX, OY respectively (see fig.1)
M	Aerodynamic moment
Oxyz	Oblique system of coordinate axes (Fig.1)
OX, OY	Reference axes in Oxy plane, at right angles to Oy and Ox respectively (Fig.1)
V	Forward velocity of aircraft
Z	Applied vertical force
$a_1 = \dfrac{\partial C_L}{\partial \alpha}$	
$a_2 = \dfrac{\partial C_L}{\partial \beta}$	
C_L, C_M	Aerodynamic lift and moment coefficients where the latter refers to axis Ox.
$d_1 = \dfrac{\partial C_M}{\partial \Theta}$	
$d_2 = \dfrac{\partial C_M}{\partial \beta}$	
ℓ	x coordinate of wing tip
n	Number of subdivisions of wing for transition from integral to matrix equation
$p(x)$, $q(x)$	Oblique components of rotation of wing section about Ox, Oy respectively.

Notation (contd.)

$q = \frac{1}{2}\rho V^2$ Dynamic pressure

q_R Dynamic pressure for aileron reversal

q_D Dynamic pressure for wing divergence

x, y, z Oblique coordinates (Fig.1)

x_i Wing stations for matrix equations

α Complement of angle of sweep back (Fig.1)

β Aileron angle

$\theta(x) = p \sin\alpha$ Local wing incidence

$\theta'_i = \left(\frac{d\theta}{dx}\right)_{x=x_i}$, $\theta_i = \theta(x_i)$

Δx_i Length of subdivisions for transition from integral to matrix equation

Some of the notation of the appendices is independent of that of the main part and is stated at the beginning of each appendix.

1. Introduction

Considerable attention has been given during the past years to the "static" problems of Aero-elasticity, (i.e. aileron reversal and divergence) for swept wings. The majority of reports (e.g. Ref. 1) on these subjects attempt to modify methods of analysis, used in the case of straight wings, and therefore fail to do justice to the special features of swept wings. This criticism refers in particular to the introduction of equivalent wings possessing elastic axes which are obtained by rather arbitrary assumptions referring to the root ends of the original wings (Ref.1) Other authors make use of the semi-rigid approach (Ref.2) which was already found inadequate in the case of straight wings, and which for swept wings will be just as unsatisfactory, since it involves too great a simplification of the actual deformations.

Finally, mention should be made of a recent paper (ref.3) which comes nearest to the approach to be adopted here, in that it represents the elastic structure by means of influence coefficients, to be determined by measurements on models or on the actual aircraft. Thus it is immediately seen that this method is only applicable when these measurements can be taken, and hence it is seriously restricted in its applicability. The present report offers a new approach to the problem under consideration, as far as structural representation is concerned, while the actual analytical method makes use of integral equations which are easily rewritten in the form of matrix equations if the complexity of any particular application should demand the latter. It is one of the objects of this report to illustrate the use of integral equations in problems of this kind, and to suggest that such equations present the best approach in all fundamental investigations dealing with continuous systems.

The method of solution proposed for the final equations makes use of straight forward iteration of either the integral or the matrix equation. Since the unknown function or vector in these correspond to the rate of change of twist along the wing, the initial solution assumed for the iteration may be taken identically constant along the wing, a fact which simplifies still further the rapidly converging process.

A simple problem, treated in the final section of this report, illustrates the application of the method and, in particular, the points raised above. Appendix 1 at the end of this report gives a method for estimating the aerodynamic derivatives required, while Appendix 2 deals with the experimental determination of the influence functions which are then compared with their theoretical values.

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2. Deduction of the Basic Equations

The most organic approach to any problems of aero-elasticity and general dynamics of continuous systems makes use of integral rather than matrix equations. The main reason for this fact lies with the continuous character of such physical systems. Thus it will invariably be found that integral equations allow a more lucid presentation of the problem considered, once their correct interpretation is fully understood.

On the other hand there is no point in attempting to deny the usefulness of matrix equations for the purpose of numerical work. These are most convenient whenever the physical system becomes complicated, and are of great assistance in simplifying calculations. However they bear little relation to the original system to be investigated and therefore ought to be confined to computing offices, having the purpose of providing numerical answers. There may be some justification for their use also in connection with dynamic models, whenever concentrated masses are used to represent continuous structures.

The present deduction of the basic equations for the determination of aileron reversal and wing divergence speeds makes use of the integral equation approach throughout, but the final equations will also be given in their matrix form for use in actual computations referring to more complicated structures. But it should be noted that these matrix equations are obtained from the final integral equations to which they are equivalent within the approximation introduced by the necessary transformation.

The conditions of aileron reversal and wing divergence are essentially static in character and hence the steady aerodynamic "derivatives" must be used. Thus the aerodynamic force and moment acting on the element of a swept wing will be given by

$$d\,L(x) = q\,C(x)\Big(a_1(x)\theta(x) + a_2(x)\beta\Big)\sin\alpha\,dx \qquad \text{--- (2.1)}$$

$$d\,M(x) = q\,C^2(x)\Big(d_1(x)\theta(x) + d_2(x)\beta\Big)\sin\alpha\,dx \qquad \text{--- (2.2)}$$

where the sumbols are explained in the list of notation and in Fig.1. The quantities a_1, a_2, d_1 and d_2 have been written as varying with x, the coordinate along the span. Their exact determination will often present great difficulties, since it requires reference to lifting surface theory for swept wings; and not only once, because these derivatives depend on the actual wing deformation, so that in an exact treatment they would have to be recalculated after every iteration step which involves a change in the mode of deformation of the wing. On the other hand, for most practical purposes it will be sufficient to base the values of these derivatives on results obtained by means of semi-empirical methods such as have been given by O. Schrenk (Ref.4) for straight wings, and extended to the case of swept wings by Jones (Ref.5) and other authors (Refs. 6 & 7) in America. In appendix 1 an example has been given of how one may obtain $a_1(x)$ by use of the lift distribution of the undeformed wing (Ref.5) and the so-called basic lift distribution (Ref.8) due to twisting of the wings.

/ Before ......

Before proceeding with the process of obtaining the fundamental equations of the problem, it should be pointed out that in the above equations it has been assumed that the aileron angle β is constant over the part of the wing carrying the aileron and, in addition, that the forces are transmitted directly at each station to the main structure. In other words, the existence of discreet hinges has been neglected, a procedure which is customary in this type of investigation, and which may require further examination in special cases.

The condition of aileron reversal is given by

$$\int_0^{\ell} x \, d\,L(x) = 0 \qquad \text{--- (2.3)}$$

which after substitution from (2.1) may be written as

$$\beta = - \frac{\int_0^{\ell} x \, C(x) \, a_1(x) \, \theta(x) \, dx}{\int_0^{\ell} x \, a_2(x) \, C(x) \, dx} \qquad \text{--- (2.4)}$$

where mostly $a_2(x)$ will be zero over part of the span. Substitution of (2.4) in (2.1) and (2.2) leads to expressions for the lift and moment acting on a wing element at reversal speed

$$dL(x)_R = q_R C(x) \left\{ a_1(x)\theta(x) - a_2(x) \frac{\int_0^{\ell} \xi C(\xi) a_1(\xi) \theta(\xi) d\xi}{\int_0^{\ell} \xi a_2(\xi) C(\xi) \, d\xi} \right\} \sin\alpha dx \quad (2.5)$$

$$dM(x)_R = q_R C(x)^2 \left\{ d_1(x)\theta(x) - d_2(x) \frac{\int_0^{\ell} \xi C(\xi) a_1(\xi) \theta(\xi) d\xi}{\int_0^{\ell} \xi a_2(\xi) C(\xi) \, d\xi} \right\} \sin\alpha dx \quad (2.6)$$

Next consider the representation of the elastic properties of the wing structure. Using oblique coordinates (see Fig.1) it has been shown by W.S. Hemp (Ref.9) that the rate of twist along a swept wing of constant cross section may be related to the applied Moments and forces in the following manner:

$$\frac{dp(x)}{dx} = C_{11} \, L_1(x) + C_{12} \, M_1(x) + C_{13} \, Z(x) \qquad \text{--- (2.7)}$$

where the constant influence coefficients C_{ij} may be determined theoretically or experimentally. An obvious extension, analogous to that customary in the generalisation of the Bernoulli-Euler theorem, postulates for the case of varying cross sections:

$$\frac{dp(x)}{dx} = C_{11}(x) \, L_1(x) + C_{12}(x) \, M_1(x) + C_{13}(x) \, Z(x) \qquad \text{--- (2.8)}$$

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i.e. the assumption of varying influence coefficients $C_{ij}(x)$ is introduced. It will be shown in Appendix 2 how the C_{ij} may be most easily obtained on the basis of deflection measurements taken on the wing or on a model of it. An example will be treated there and the experimentally determined influence coefficients will be compared with their theoretical values as obtained by use of formulae of reference 9.

In order to link the aerodynamic and elastic forces, one deduces from (2.5) and (2.6) the moments L_1, M_1 and force Z of (2.8) in the following manner:

$$\left.\begin{aligned} L_1(x) &= \int_x^{\ell} \big(d\, M(\eta)\big)_R \\ M_1(x) &= \int_x^{\ell} (\eta - x)\big(dL(\eta)\big)_R \\ Z\,(x) &= \int_x^{\ell} \big(d\, L(\eta)\big)_R \end{aligned}\right\} \qquad \text{--- (2.9)}$$

Further, as

$$p \sin\alpha = \theta \qquad \text{---(2.10)}$$

one obtains after substitution from (2.9) in (2.8):

$$\begin{aligned} \frac{d\theta(x)}{dx} = q_R \sin^2\alpha \Bigg[& C_{11}(x)\int_x^{\ell} C(\eta)^2 \left\{ d_1(\eta)\theta(\eta) - d_2(\eta)\, \frac{\int_0^{\ell} \xi C(\xi) a_1(\xi)\theta(\xi)\, d\xi}{\int_0^{\ell} \xi a_2(\xi) C(\xi)\, d\xi} \right\} d\eta \\ & + C_{12}(x)\int_x^{\ell} (\eta - x) C(\eta) \left\{ a_1(\eta)\theta(\eta) - a_2(\eta)\, \frac{\int_0^{\ell} \xi C(\xi) a_1(\xi)\theta(\xi)\, d\xi}{\int_0^{\ell} \xi a_2(\xi) C(\xi)\, d\xi} \right\} d\eta \\ & + C_{13}(x)\int_x^{\ell} C(\eta) \left\{ a_1(\eta)\theta(\eta) - a_2(\eta)\, \frac{\int_0^{\ell} \xi C(\xi) a_1(\xi)\theta(\xi)\, d\xi}{\int_0^{\ell} \xi a_2(\xi) C(\xi)\, d\xi} \right\} d\eta \Bigg] \end{aligned}$$

$$\text{---(2.11)}$$

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which can be written

$$\frac{d\theta(x)}{dx} = q_R \sin^2\alpha \left[\int_x^{\ell} H^1(\eta,x)\theta(\eta)d\eta - \int_0^{\ell} H^2(\eta,x)\ \theta(\eta)\ d\eta \right] \quad \text{---(2.11a)}$$

where

$$H^1(\eta,x) = C(\eta)\left[C(\eta)d_1(\eta)C_{11}(x) + a_1(\eta)\left\{(\eta-x)C_{12}(x)+C_{13}(x)\right\}\right]$$

$$H^2(\eta,x) = \frac{\eta C(\eta)a_1(\eta)}{\int_0^{\ell}\xi a_2(\xi)C(\xi)d\xi}\left[C_{11}(x)\int_x^{\ell}C(\xi)^2 d_2(\xi)d\xi \right. \quad \text{---(2.12)}$$

$$\left. + \int_x^{\ell} C(\xi)a_2(\xi)\left\{C_{12}(x)(\xi-x)+C_{13}(x)\right\}d\xi\right]$$

Finally (2.11a) may yet be simplified to give

$$\frac{d\theta(x)}{dx} = q_R \sin^2\alpha \int_0^{\ell} H(\eta,x)\theta(\eta)d\eta \quad \text{---(2.13)}$$

with

$$H(\eta,x) = \begin{cases} H^2(\eta,x) & 0 \leqslant \eta < x \\ H^1(\eta,x) - H^2(\eta,x) & x \leqslant \eta \leqslant \ell \end{cases} \quad \text{---(2.14)}$$

Equation (2.13) is the basic integral equation for the determination of aileron reversal speeds. Its matrix form which represents an approximation unless one uses an infinite number of variables, is obtained from (2.13) by selecting certain stations x_i along the wing at which (2.13) will still be assumed to hold with the one approximation that the integral is replaced by a finite sum involving the values of the variable at the selected stations. As for computational purposes the integral on the right hand side presents in practical cases great difficulty, the matrix equation to be written down now, will often be more convenient when numerical results are required.

$$\theta_i' = q_R \sin^2\alpha \sum_{j=1}^{n} H_{ji}\ \theta_j\ \Delta x_j,\ i = 1, \ldots, n, \quad \text{---(2.15a)}$$

where

$$\theta(x_i) = \theta_i,\ \frac{d\theta(x_i)}{dx} = \theta_i' \text{ etc.}$$

Δx_i = length of subdivision allotted to x_i

n = number of stations selected.

/ In many .....

In many cases it is most convenient to distribute the stations x_i uniformly along the span and to assume all the Δx_i to be equal in length. Then (2.15a) becomes

$$\theta_i' = q_R \sin^2\alpha \frac{\ell}{n} \sum_{j=1}^{n} \theta_j H_{ji} \ , \ i = 1, \ldots, n \qquad \text{---} (2.15b)$$

It should be noted at this stage that naturally the integrals involved in calculating the H_{ji} may likewise be obtained as finite sums.

Finally an equation analogous to (2.15a) will be deduced for the determination of wing divergence speeds. In this case the aileron angle β appearing in (2.1), (2.2) is identically zero, so that these equations become much simpler, and it is easily seen that this implies that $H_2(\eta,x)$ of (2.12) is zero. Thus one has for wing divergence

$$\frac{d\theta(x)}{dx} = q_D \sin^2\alpha \int_x^{\ell} H^1(\eta,x)\,\theta(\eta)\,d\eta \qquad \text{---}(2.16a)$$

and the corresponding matrix equation

$$\theta_i' = q_D \sin^2\alpha \frac{\ell}{n} \sum_{j=1}^{n} H_{ji}^1 \ \theta_j \ , \ i = 1, \ldots, n \qquad \text{---}(2.16b)$$

assuming n uniformly distributed wing stations and equal subdivisions.

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3. Method of solution of the basic equations

An iteration process is most suitable for the solution of equations of the type deduced here. However, particularly when solving the corresponding matrix equations, the presence in the equations of the variable θ as well as of its spanwise derivative $\frac{d\theta}{dx}$ is not convenient as it will require an extra stage in between each iteration step when the derivative must be obtained. For this reason the following transformation will be applied to the function $\theta(x)$ under the integrals of (2.13) and (2.16):

$$\theta(\eta) = \int_0^\eta \frac{d\theta(t)}{dt}\, dt \qquad \text{---(3.1)}$$

Assuming that $\theta(0) = 0$, i.e. that the root of the wing is at zero incidence. If this condition is not fulfilled, the transformation (3.1) involves an additive constant, the root incidence, a fact which only slightly complicates the final result. Using (3.1) and inverting the orders of integration, the basic integral equations become for aileron reversal

$$\frac{d\theta(x)}{dx} = q_R \sin^2\alpha \int_0^\ell \bar{H}(\eta,x)\, \frac{d\theta(\eta)}{d\eta}\, d\eta$$

---(3.2a)

with

$$\bar{H}(\eta,x) = \int_\eta^\ell H(t,x)\,dt$$

and for wing divergence

$$\frac{d\theta(x)}{dx} = q_D \sin^2\alpha \int_0^\ell \bar{H}^1(\eta,x)\, \frac{d\theta(\eta)}{d\eta}\, d\eta$$

with ---(3.3a)

$$\bar{H}^1(\eta,x) = \int_{\overline{x,\eta}}^\ell H^1(t,x)\,dt$$

where $\overline{x,\eta}$ = max (x,y). (This type of integration limit always occurs in the process of inverting the order of integration of double integrals of the present type, when the area covered by the integration does not cover the complete triangle formed by one of the coordinate axes and the line bisecting the right angle between them).

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The corresponding matrix equations are

$$\left\{\theta'_i\right\} = q_R \sin^2\alpha \frac{\ell}{n} \left[H_{ij}\right] \left\{\theta'_j\right\} \qquad \text{---(3.2b)}$$

and

$$\left\{\theta'_i\right\} = q_D \sin^2\alpha \frac{\ell}{n} \left[H'_{ij}\right] \left\{\theta'_j\right\} \qquad \text{---(3.3b)}$$

The transformation of the basic equations introduced above demonstrates well the suitability of the integral equation approach. At the expense of slightly more complicated coefficients H_{ij}, which normally have to be calculated once only, one has obtained equations which are suitable for a simple iteration process. Comparison of the corresponding matrix equations (2.15b) and (3.2b), (2.18b) and (3.3b) shows that such a transformation is by no means so obvious, if one only deals with equations of that type. Note also that the unknown functions or vectors in the equations (3.2) and (3.3) are different from those normally used because the transformation (3.1) was used. An alternative procedure, which however does not always lead to as simple results, would have been to integrate the equations (2.13) and (2.16) once and to invert the orders of integrations on the right hand sides.

Equations (3.2) and (3.3) may be iterated in their integral or matrix form by assuming some initial form for the rate of twist function θ' and calculating the right hand sides of the equations. The resulting functions or vectors (in the case of the matrix equations) render then the new rate of twist distribution to be used in the subsequent iteration step. After a certain number of such steps the expression for θ' will stabilize so that one obtains after cancellation of the function $\frac{d\theta}{dx}$ or the column $\left\{\theta'_i\right\}$ an equation of the type

$$K^2 = q_C \sin^2\alpha \qquad \text{---(3.4)}$$

in the case of the integral equation, and

$$K^2 = q_C \sin^2\alpha \frac{\ell}{n} \qquad \text{---(3.5)}$$

in the case of the matrix equation, where K^2 is a constant. In general there will be several values of K^2, the lowest of which will give the critical speed required, since by definition

$$q_C = \tfrac{1}{2} \rho V_C^2$$

and hence by (3.4) or (3.5)

$$V_C = \frac{K}{\sin\alpha}\sqrt{\frac{2}{\rho}} \quad \text{or} \quad V_C = \frac{K}{\sin\alpha}\sqrt{\frac{2n}{\rho\ell}} \qquad \text{---(3.6)}$$

respectively.

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Under normal circumstances, the initial function or vector for the iteration may be assumed to be constant along the wing, i.e. one may assume uniform twist. As has been noted earlier, when dealing with the aerodynamic derivatives, in an exact treatment these would have to be calculated anew after each iteration step on the basis of the newly determined mode of deformation. In most practical cases, however, such refinement will be considered too laborious and the derivatives will be based e.g. on the assumption of uniform twist.

In the next section the above theory will be applied to the very simple case of a uniform section swept wing and variations of the derivatives will be neglected for the sake of simplicity.

4. Divergence of a simple swept wing

Consider the wing shown in Fig.2 which may be taken as an approximation to the wing treated in Appendices 1 and 2 (see also Fig. 1,3) As mentioned at the end of the last section, all the characteristic quantities of the wing will be assumed constant, in fact they will be chosen within the range of values found for the model wing of Fig.3 in Appendices 1 and 2, viz:

$$C = 24\,[\text{in}] \qquad C_{12} = -17 \times 10^{-8}\left[\text{lb}^{-1}\,\text{in}^{-2}\right]$$

$$a_1 = 4.9 \qquad \ell = 76.5\,[\text{in}]$$

$$a_2 = 0.9 \qquad C_{13} = -1.55 \times 10^{-6}\left[\text{lb}^{-1}\,\text{in}^{-1}\right]$$

$$C_{11} = 30 \times 10^{-8}\left[\text{lb}^{-1}\,\text{in}^{-2}\right] \qquad \sin\alpha = 0.79$$

then by (2.12):

$$H^1(\eta,x) = 24\left[24 \times 2.5 \times 30 \times 10^{-8} + 4.9\left\{-(\eta-x)17\times10^{-8} - 1.55\times10^{-6}\right\}\right]$$

$$= \left[2.5 - 0.2\,x(\eta - x)\right] \times 10^{-4}$$

i.e. (2.17a) becomes

$$\frac{d\theta(x)}{dx} = q_D \sin^2\alpha \int_x^{\ell} 10^{-4}\left[2.5 - 0.2(\eta-x)\right]\theta(\eta)\,d\eta$$

By (3.3a)

$$\bar{H}^1(\eta,x) = \int_{x,\eta}^{\ell} H^1(t,x)\,dt$$

$$= \int_{x,\eta}^{\ell} 10^{-4}\left[2.5 - 0.2(t-x)\right]dt$$

$$= 10^{-4}\left[2.5t + 0.2\ x\ t - \frac{.2t^2}{2}\right]_{x,\eta}^{\ell}$$

$$= 10^{-4}\begin{cases} 2.5(\ell-x) + 0.2(\ell-x)\ x - 0.1(\ell^2-x^2) & \text{for } x > \eta \\ 2.5(\ell-\eta) + 0.2\ x\ (\ell-\eta) - 0.1(\ell^2-\eta^2) & \text{for } x < \eta \end{cases}$$

Hence the corresponding integral equation becomes

$$\theta'(x) = q_D\ \sin^2\alpha \int_0^{\ell} H^1(\eta, x)\ \theta'(\eta)d\eta$$

$$= q_D\ \sin^2\alpha\ x\ 10^{-4}\left[\int_0^{x} (\ell-x)\left\{2.5 + 0.2x - 0.1(\ell+x)\right\}\theta'(\eta)d\eta\right.$$

$$\left. + \int_x^{\ell} (\ell-\eta)\left\{2.5 + 0.2x - 0.1(\ell+\eta)\right\}\theta'(\eta)d\eta\right]$$

$$= 10^{-4}q_D\sin^2\alpha\left[\int_0^{x}(76.5-x)\left\{-5.15+0.1x\right\}\theta'(\eta)d\eta\right.$$

$$\left. + \int_x^{76.5}(76.5-\eta)\left\{-5.15+0.2x-0.1\eta\right\}\ \theta'(\eta)d\eta\right]$$

i.e. the following equation has to be iterated to obtain the divergence speed

$$\theta'(x) = 10^{-4}\ q_D\ \sin^2\alpha\left[(76.5-x)(0.1\ x - 5.15)\int_0^{x}\theta'(\eta)d\eta\right.$$

$$\left. + \int_x^{76.5}(76.5-\eta)(0.2\ x - 0.1\eta - 5.1)\ \theta'(\eta)d\eta\right]$$

To shorten the iteration process, let $\theta'(x) = b_o + b_1x + b_2x^2$ and deduce a system of equations for the b_i by comparing coefficients of x^o, x' and x^2. If it is then assumed that the mode has stabilised, one obtains

$$K^2\ b_o\ = -\ b_o\ 2.255\ x\ 10^5 - 6.7\ x\ 10^5 b_1 - 2.78\ x\ 10^7 b_2$$

$$K^2\ b_1\ = 5.852\ x\ b_o\ x\ 10^2 + 1.49\ x\ 10^4 b_1 + 5.685\ x\ 10^5 b_2$$

$$K^2\ b_2\ = -\ b_o\ 1.25$$

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i.e. one obtains a secular equation for K^2:

$$\begin{vmatrix} K^2 + 2.255 \times 10^5 & 6.7 \times 10^5 & 2.78 \times 10^7 \\ - 5.852 \times 10^2 & K^2 - 1.49 \times 10^4 & -5.685 \times 10^5 \\ + 1.25 & 0 & K^2 \end{vmatrix} = 0$$

i.e. $K^6 + 2.106 \times 10^5 K^4 - 3 \times 10^9 K^2 + 4.12 \times 10^{10} = 0$

and the lowest root of this equation is

$$K^2 = 13.72, \text{ and hence } K = 3.7$$

as is easily seen by inspection of the last two terms.

Therefore by (3.6)

$$V_C = \frac{3.7}{0.79} \sqrt{\frac{2 \times 10^3}{0.002378}} \text{ [in/sec] at sea level}$$

$$= \frac{3.7}{0.79} \times \sqrt{\frac{2 \times 12}{0.002378}} = 470 \text{ [ft/sec] at sea level}$$

Using the same value of K one finds from the above equations the mode of deformation of the wing to be

$$\theta'(x) = b_o\left\{1 + 3.43x - .091x^2\right\}$$

so that the above assumption of linearity of the mode of deformation, on which normally the aerodynamic derivatives are based, is not very well satisfied. Considering the low divergence speed obtained, a compressibility correction is obviously not required. It is indicated in Appendix 1, that, in case that use is made of the process given there for obtaining estimates of the distribution of the derivatives, allowance may be made for compressibility effects when deducing the underlying lift distribution.

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5. Conclusions

The method of determining aileron reversal and divergence speeds of swept wings, deduced in this report, serves well to illustrate the usefulness of oblique coordinates in work relating to such structures. Combined with the integral equation approach a compact presentation of these two problems is obtained which in some respect may be compared with the treatment of the analogous problems for straight wings given in Ref. 10 which uses only matrix equations.

The application of the method, given in section 4, demonstrates the simplicity of the required calculations. Naturally, when dealing with a more complicated type of wing, the process of computation will become more complex and the use of matrix equations may well become necessary. Nevertheless, the example treated here contains all the essential steps which will be involved when dealing with a wing with taper, crank, etc. and when variations in the influence functions and in the derivatives are to be taken into account.

In the Appendices 1 and 2 at the end of this report the initial stages to the application of the method are discussed separately and independently of the main part of the report, although some of their numerical results are used in section 4. In particular, Appendix 2 serves to compare theoretical and experimental results obtained for the influence functions. The author is indebted, as far as the former are concerned, to Mr. J.V.A. Welbourn who undertook the fairly tedious task of computing them from the formulae of Ref.9. The experimental results underlying the latter were obtained by the author as part of the aero-elastic study of the model wing under consideration.

All numerical results contained in this report, with the exception of the theoretical values of the influence functions, were obtained by use of a ten inch slide rule, so that at the most three figure accuracy can be expected. In view of the accuracy, which may be claimed for the basic physical data, it would appear doubtful whether the use of a calculating machine would help to improve the position. In general it may be said that little accuracy was lost during the computations due to subtractions of almost equal quantities.

/ References .....

LIST OF REFERENCES

No.	Author	Title, etc.
1.	Franklin W. Diederich and Bernard Budiansky	Divergence of swept wings NACA Tech Note No. 1680, 1948
2.	E.G. Broadbent and Ola Mansfield	Aileron reversal and wing divergence of swept wings RAE Report No. Structures 9
3.	E.G. Broadbent	The rolling power of an elastic swept wing. RAE Report No. Structures 85
4.	O. Schrenk	A simple approximation method for obtaining the spanwise lift distribution. Luftwissen Vol.7 1940, pp 118-120 NACA T.M. 948
5.	R. Stanton Jones	An empirical method for rapidly determining the loading distributions on swept back wings. College of Aeronautics, Cranfield Report No. 32.
6.	Franklin W. Diederich	A simple approximate theory for obtaining lift distributions over swept wings NACA R.M. L7107, 1948.
7.	Richard B. Skoog and Harvey H. Brown	A method for the determination of the spanwise load distribution of a flexible swept wing at subsonic speeds. NACA Tech. Note 2222, 1951.
8.	R. Stanton Jones	A rapid method for estimating the basic loading due to linear twist of wings of any planform. Saunders Roe A.S.R. 7
9.	W.S. Hemp	On the application of oblique coordinates to problems of plane elasticity and swept back wings. College of Aeronautics, Cranfield Report No. 31. (This work which deals with single cell boxes, has since been extended by the author to include two-cell boxes. The results of the extended theory were the subject of special lectures given in December 1951 at Cranfield.)
10.	William T. Thomson	Aileron reversal speed by influence coefficients and matrix iteration Journ. Aero. Sciences Vol.13 1946, pp 192-4.

APPENDIX 1

Determination of Lift Slope Distribution along the Span of a Swept Wing

Notation

$\bar{a}_1 = \frac{\partial C_L}{\partial \alpha}$ of whole aircraft

$\Delta a_1'$ Local variation of a_1 from its value relating to the undeformed wing.

a_1 Local lift slope of deformed wing.

C Local chord of wing

$\bar{C}$ Geometric mean chord of wing

a_1' Local lift slope of undeformed wing

C_L Local lift coefficient

$\bar{C}_L$ Mean lift coefficient for whole wing

C_{LB} Basic lift coefficient

A Aspect ratio

$K_\eta = \frac{CC_L}{\bar{C}\bar{C}_L}$ loading coefficient of undeformed wing

$K_\eta' = \frac{CC_{LB}}{\bar{C}\varepsilon}$ basic loading coefficient

ε Twist at wing tip

$\eta = \frac{x}{\lambda}$ non-dimensional coordinate along Ox

$\lambda = \frac{C_t}{C_r}$ taper ratio

Λ Angle of sweep back of $\frac{1}{4}$ chord line

/ As mentioned .....

As mentioned in the main part of this report, a method will be given here for estimating the distribution of lift slope along the span. The method is approximate and makes use of references 5 & 8 by R. Stanton Jones, which contain a semi-empirical approach to the determination of the lift distribution of swept wings. These reports are based on a large number of experimental and theoretical results published during recent years. The first person to suggest a similar method for straight wings was O. Schrenk (Ref.4), and since then various related procedures have been suggested and used by different authors (Refs.5-8).

In all these approaches the lift distribution is divided into two parts, the first of which refers to the undeformed wing (Ref.5), while the second, the so-called basic loading (Ref.7), gives the correction to be applied to the former in the case when the wing is twisted.

First consider the distribution of lift slope along the undeformed wing. From Figs. 7 & 8 of Ref.5, after some preliminary calculations which involve the following data referring to the model wing, also considered in Section 4 and Appendix 2:

$$A = 4.5 , \quad \lambda = 0.475 , \quad \Lambda = 40^o$$

one finds the following values for the load coefficient

$$K_\eta = \frac{CC_L}{\bar{C}\bar{C}_L} \text{ at the wing stations } \eta :$$

η	0	0.1	0.2	0.382	0.6	0.707	0.8	0.85	0.923	0.96
K_η	1.164	1.166	1.169	1.157	1.059	0.965	0.844	0.754	0.574	0.444

But from the expression for the load coefficient, which is only a function of η, it follows that

$$C_L = K_\eta \frac{\bar{C}}{C} C_L$$

and hence

$$a_1' = K_\eta \frac{\bar{C}}{C} \bar{a}_1$$

For the wing under consideration

$$\bar{C} = 28.7 [in] , \ C(\eta) = C_r\{1-(1-\lambda)\eta\} = 39.3(1-0.525\eta) \ [in]$$

$$\bar{a}_1 = 4.0$$

and therefore

$$a_1' = K_\eta / 0.342 \ (1 - 0.525\eta)$$

/ Using .....

Using the above values of K_η, the corresponding values of a_1' are:

η	0	0.1	0.2	0.382	0.6	0.707	0.8	0.85	0.923	0.96
a_1'	3.41	3.60	3.82	4.23	4.51	4.49	4.26	4.08	3.26	2.62

Next consider the deformed wing. The basic loading coefficient $K_\eta' = \frac{cC_{LB}}{\bar{c}\varepsilon}$ is graphed for various wing stations, angles of sweep back and aspect ratio in ref. 8. The data given in this reference refer to a wing twisted uniformly from zero incidence at the root to 1° nose downward at the tip. It is also indicated there that without great difficulty, allowance could be made for parabolic or even higher order twist, although computations would then become more complicated. In the present case the values of K_η' are:

η	0	0.1	0.2	0.3	0.4	0.45	0.5
K_η'	0.0132	0.0118	0.0093	0.006	0.0021	0	-0.0019
η	0.6	0.7	0.8	0.9			
K_η'	-0.0054	-0.0079	-0.0091	-0.0075			

But as the twist corresponding to these values of K_η' is uniform and equals one degree washout at the tip, the local twist at any station is η° and hence the local change of K_η' per radian is

$$\frac{K_\eta' \times 57.3}{\eta} = \frac{cC_{LB}}{\bar{c}\eta} \times 57.3$$

since $\varepsilon = 1$. But as C_{LB} is the basic lift coefficient which varies with η, one can interpret $C_{LB} \times 57.3/\eta$ as local change in $dC_L/d\alpha = a_1'$ per radian and hence

$$\Delta a_1' = -\frac{K_\eta' \times 41.8}{\eta(1-.525\eta)}$$

where the minus sign has to be introduced because K_η' corresponds to washout.

Obviously the values of $\Delta a_1'$ deduced in this way will be unreliable near the wing root, i.e. for small values of η. On the other hand, for the purpose of aileron reversal and wing divergence calculations, the outer wing is of greater importance and the a_1 obtained in the above manner may be expected to be satisfactory there. From the above formula one finds for $\Delta a_1'$ the values:

/η .....

η	0.1	0.2	0.3	0.4	0.45	0.5	0.6	0.7	0.8	0.9
$\Delta a_1'$	-5.20	-2.18	-0.99	-0.28	0	0.19	0.55	0.65	0.82	0.66

In Fig.4 the values of a_1', $\Delta a_1'$ and the actual lift slope distribution $a_1 = a_1' + \Delta a_1'$ are graphed against η.

In the above work no allowance has been made for the effect of compressibility. However, in ref.5 corrections for such effects are given, so that the above procedure would have to be repeated with different values of K_η.

Finally, a remark will be made with regard to the determination of the spanwise variations of the other derivatives appearing in the basic equations of Section 2. In most cases one will assume the aerodynamic centre to be at the quarter chord line in which case the values of $d_1(\eta) = \frac{dC_M}{d\alpha}$ follow from those of a_1 once the position of the axis Ox has been fixed. An approach similar to the above has been indicated for straight wings with flaps by O.Schrenk (Ref.4) so that the above procedure could be adopted, once the method of refs. 5 & 8 has been extended for such wings, with regard to the calculation of d_1 and d_2.

On the other hand, the R.Ae.S. Data sheets may be used to find the ratio a_2/a_1 and a_2 may then be calculated from the above distribution of a_1. This is the procedure adopted for the numerical example of Section 4.

/ Appendix 2 .....

APPENDIX 2

Experimental Determination of Influence Functions of a Swept Wing

Notation

C_K^{ij} Constant coefficients of polynomials for $C_{ij}(x)$

$\bar{C}_K^{ij} = \ell^K C_K^{ij}$

$W(x)$ Displacement of $(x,0)$ in Oz direction

$\bar{W}(x) = \dfrac{W(x)}{Z}$ displacement of $(x, 0)$ per unit load at tip.

$\bar{W}_i^j$ Measured values of $\bar{W}$ referring to a station x_i and corresponding to a test run j.

$\bar{p}(x) = \dfrac{p(x)}{Z}$ rotation of wing section per unit load at tip.

$\bar{p}_i^j$ Measured values of $\bar{p}$ referring to a station x_i and corresponding to a test run j.

$\xi = \dfrac{x}{\ell}$ nondimensional coordinate along Ox.

In ref. 9 W.S. Hemp has shown that the following relations hold between applied forces and moments and the resulting deformations in the case of swept wings of conventional construction (spars, ribs and stressed skin) and uniform cross section:

$$\frac{dp}{dx} = C_{11} L_1 + C_{12} M_1 + C_{13} Z$$

$$\frac{dq}{dx} = -\frac{d^2W}{dx^2} \operatorname{cosec}\alpha = C_{12} L_1 + C_{22} M_1 \qquad \text{--- (A2.1)}$$

where the C_{ij} are constants the values of which can be obtained from formulae given in that reference. In the present treatment shear deflections will be neglected, although Hemp has extended his theory to take them also into account. Only the first of the equations (A2.1) has been used in the main part of this report, but it will be shown below that both the above formulae have to be considered when it is required to determine the C_{ij} experimentally.

/ It has ......

It has been indicated in Section 2 of this report that an extension of the above formulae may be proposed analogous to that applied to the Euler-Bernoulli theorem, for the purpose of making these equations applicable to the case of wings of non-uniform cross section. This generalisation leads to the following equations:

$$\frac{dp}{dx} = C_{11}(x)L_1 + C_{12}(x)M_1 + C_{13}(x)Z$$
$$\frac{dq}{dx} = -\frac{d^2W}{dx^2}\operatorname{cosec}\alpha = C_{12}(x)L_1 + C_{22}(x)M_1 \qquad \text{--- (A2.2)}$$

In these formulae, by definition, if $Z(x,y)$ be the resulting force for a cross section x acting at a point (x,y)

$$L_1 = \int_x^{\ell} y\, Z\, d\xi\ , \qquad M_1 = \int_x^{\ell} (\xi - x) Z\, d\xi$$
$$Z = \int_x^{\ell} Z\, d\xi \qquad \text{--- (A2.3)}$$

Next an assumption will be introduced regarding the type of functions $C_{ij}(x)$. Since it is experimentally easier to measure deflections rather than slopes or curvatures, the equations (A2.2) have to be integrated. For this reason it will be most convenient to assume

$$C_{ij}(x) = \sum_{K=0}^{m} C_K^{ij}\, x^K \qquad \text{--- (A2.4)}$$

i.e. to replace the $C_{ij}(x)$ by polynomials approximating to their real values. It should be noted here that the approach of this Appendix is directly linked with the purpose of determining the influence coefficients by a finite number of deflection measurements and that it is of no theoretical importance.

It will now be shown that the constants C_K^{ij} can be determined from an appropriate number of deflection measurements taken along the span of the wing after a set of increasing loads has been applied independently at two points of the tip section, one of which should preferably correspond to the point $(\ell, 0)$.

/ Under ....

Under those circumstances, for a general point of application of the loads at the tip section, (A2.3) becomes

$$L_1(x) = y\,Z\ , \quad M_1 = (x-\ell)Z\ , \quad Z = Z \qquad \text{--- (A2.3')}$$

where Z is now independent of x. Substituting from (A2.3') and (A2.4) in (A2.2), integrating once and twice respectively with regard to x and using the following root conditions:

$$p(0) = 0\ , \quad W(0) = W'(0) = 0 \qquad \text{--- (A2.5)}$$

one obtains

$$\frac{p(x)}{Z} = y\sum_{K=0}^{m} C_K^{11}\frac{x^{K+1}}{K+1} + \sum_{K=0}^{m} C_K^{12}\left\{\frac{x}{K+2} - \frac{\ell}{K+1}\right\}x^{K+1} + \sum_{K=0}^{n} C_K^{13}\frac{x^{K+1}}{K+1}$$

$$\text{--- (A2.6)}$$

$$-\frac{W(x)}{Z\sin\alpha} = y\sum_{K=0}^{m} C_K^{12}\frac{x^{K+2}}{(K+1)(K+2)} + \sum_{K=0}^{m} C_K^{22}\left\{\frac{x}{K+3} - \frac{\ell}{K+1}\right\}\frac{x^{K+2}}{K+2}$$

It is now seen that the determination of the C_K^{ij} can be reduced to the solution of four systems of m simultaneous linear equations.

Before deducing these sets of equations, it is convenient to make the equations (A2.6) partly non-dimensional by putting

$$x = \xi\ell\ , \quad \ell^K C_K^{ij} = \bar{C}_K^{ij} \qquad \text{--- (A2.7)}$$

and to introduce the experimental measurements in the form of rotations p and deflections W per unit load, i.e. let

$$\frac{p}{Z} = \bar{p}\ , \quad \frac{W}{Z} = \bar{W} \qquad \text{--- (A2.8)}$$

As one is working in the elastic range, this last step will help to eliminate experimental errors by balancing the deformations taken for a set of loads Z. Finally let the superscripts 1 and 2 attached to $\bar{p}$, $\bar{W}$ serve to distinguish between the two test runs necessary; in particular let 1 refer to the case when the loads are applied at $(\ell,0)$ and 2 when the point of application is (ℓ,y).

/ With ....

With the above convention the following systems of simultaneous equations are obtained:

$$\left\{\frac{-\bar{W}^1_i}{\ell^3 \sin\alpha}\right\} = \left[\left(\frac{\xi_i}{K+3} - \frac{1}{K+1}\right)\frac{\xi_i^{K+2}}{K+2}\right]\left\{\bar{C}^{22}_K\right\}$$

$$\left\{\frac{\bar{W}^1_i - \bar{W}^2_i}{y\,\ell^2 \sin\alpha}\right\} = \left[\frac{\xi_i^{K+2}}{(K+1)(K+2)}\right]\left\{\bar{C}^{12}_K\right\}$$

$$\left\{\underline{\bar{p}^1_i} - \ell\sum_{j=0}^{n}\bar{C}^{12}_j\left(\frac{\xi_i}{j+2} - \frac{1}{j+1}\right)\xi_i^{j+1}\right\} = \left[\frac{\xi_i^{K+1}}{K+1}\right]\left\{\bar{C}^{13}_K\right\} \qquad \text{--- (A2.9)}$$

$$\left\{\frac{\bar{p}^2_i - \bar{p}^1_i}{\ell\,y}\right\} = \left[\frac{\xi_i^{K+1}}{K+1}\right]\left\{\bar{C}^{11}_K\right\}$$

which can be easily solved by Cramer's Rule, particularly if the degree of the polynomials (A2.4) replacing the $C_{ij}(x)$ is not larger than two. In many cases such polynomials will be quite satisfactory.

The remaining part of this Appendix will be devoted to an application of the above method to the 1 : 5 scale model of a fighter aircraft with 40° sweepback of the quarter chord line (see Fig.3). In order to simplify the procedure, the axis Ox was placed halfway between the outer spars. Deflection readings were taken for the front and rear spars at the following stations:

Rib station 2, 3, 5, 7, 9

and the corresponding displacements W of the Ox axis and rotations p about that axis calculated; (note that the change in "incidence" obtained directly from the measurements equals p $\sin\alpha$)for loads, increasing in steps of 5 lb to a total of 20 lb, applied to the points (ℓ,0) and (ℓ,10"). The resulting values $\bar{p}^1$, $\bar{p}^2$, $\bar{W}^1$ and $\bar{W}^2$ are given in Table 1 and graphed in Figs. 5 & 6.

Using values of $\bar{p}$ and $\bar{W}$ referring to the rib stations 3, 5 and 9, the elements of the systems of equations (A2.9) have been evaluated in Tables 2 - 5 which at the same time provide schedules for further applications of the method. Underneath each table appear the values of the relevant determinants for the application of Cramer's rule and the values of the constants $\bar{C}^{ij}_K$ and C^{ij}_K.

/ Using ....

Using the coefficients C_K^{ij}, determined above, the approximations (A2.4) to the $C_{ij}(x)$ have been calculated for certain stations (see Table 6) and graphed in Fig.7. For comparison the theoretical values of all but one of the C_{ij} for these stations are likewise given in the table. It is seen that on the whole agreement between the experimental and theoretical values is fair, with the exception of the values of C_{12}, and C_{22} at the two outboard stations. As this is the first time that the formulae for the two-cell box have been used, the present results can only give an indication of what may be expected in future applications. Definite sources of error in the present model may be expected to arise from the cutout (Fig.3), although the discrepancy between the experimental and theoretical results does not occur until further out along the span, and from the fact that the theoretical results were referred to an axis Ox coinciding with the centre spar while the experimental results assumed that axis along the centre line between the outer spars.

But the last condition is again sufficiently met at the outboard stations. On the other hand, C_{12} and C_{22} depend to a large degree on the moments of inertia of the wing sections, and it appeared from the computations that use of larger values for these quantities would reduce the corresponding values of the influence coefficients. In fact, the contribution of the spar-boom areas to the moments of inertia increases along the span until at the tip it almost equals that from the skins. For these reasons it will be suggested here, that, in view of the small cross sections of the model wing near the tip and the difficulty involved in constructing this part of the wing, the actual model is stiffer than predicted by calculations based on nominal sizes and dimensions. This last effect may yet be accentuated by the large number of closely spaced rivets holding the skins to the spars. Perhaps it should still be mentioned that the discrepancy for C_{22} would also have been observed if use had been made of simple beam theory as

$$C_{22} = \frac{1}{E\, I \sin\alpha}$$

Finally there arises the question as to the correctness of including constant terms in the polynomials (A2.4) representing the influence functions, for obviously $C_{ij}(x)$ should be zero at the root. However, similarly as in Appendix 1, it may be reasoned that conditions near the root will be of little importance. On the other hand, one may equally well exclude the constant terms without making the computations much more complicated, but this has not been done in the present calculations.

/ Table 1 ....

TABLE 1

Experimental Data

Rib Station	2	3	5	7	9
ξ_i	0.0825	0.198	0.454	0.716	0.905
$\bar{p}_i^1$ [rad/lb]	0.454×10^{-5}	0.745×10^{-5}	2.54×10^{-5}	4.69×10^{-5}	7.46×10^{-5}
$\bar{p}_i^2$ [rad/lb]	0.537×10^{-5}	1.005×10^{-5}	3.13×10^{-5}	6.60×10^{-5}	14.80×10^{-5}
$\bar{W}_i^1$ [in/lb]	0.079×10^{-3}	0.236×10^{-3}	1.22×10^{-3}	3.38×10^{-3}	6.00×10^{-3}
$\bar{W}_i^2$ [in/lb]	0.069×10^{-3}	0.236×10^{-3}	1.22×10^{-3}	3.46×10^{-3}	6.42×10^{-3}

TABLE 2

Determination of C^{22}_K

(1) i	(2) ξ_i	(3) $\xi_i^2/2$	(4) $\frac{\xi_i}{3}-1$	(5) $\xi_i^3/3$	(6) $\frac{\xi_i}{4}-\frac{1}{2}$	(7) $\xi_i^4/4$	(8) $\frac{\xi_i}{5}-\frac{1}{3}$
1	0.198	0.0196	-0.934	0.00259	-.451	.000384	-.294
2	0.454	0.103	-0.849	0.0312	-.387	.0106	-.243
3	0.905	0.410	-0.698	0.247	-.274	.168	-.152

i	(9) (3)x(4)	(10) (5)x(6)	(11) (7)x(8)	(12) $-\bar{W}_i^1/\ell^3 \sin\alpha$
1	-.0183	-.00116	-.000113	$-.666\times10^{-9}$
2	-.0875	-.0124	-.00257	-3.45×10^{-9}
3	-.286	-.0676	-.0255	-16.93×10^{-9}

det (9), (10), (11) = -1.127×10^{-6}

det (12), (10), (11) = -4.52×10^{-14}; $\bar{C}^{22}_0 = +4.02\times10^{-8}$; $C^{22}_0 = 4.02\times10^{-8}$

det (9), (12), (11) = $+12.82\times10^{-14}$; $\bar{C}^{22}_1 = -11.37\times10^{-8}$; $C^{22}_1 = -.149\times10^{-8}$

det (9), (10), (12) = -56.1×10^{-14}; $\bar{C}^{22}_2 = +49.8\times10^{-8}$; $C^{22}_2 = .0085\times10^{-8}$

/ Table 3 .....

TABLE 3

Determination of C_K^{12}

(1) i	(2) ξ_i	(3) $\xi_i^2/2$	(4) $\xi_i^3/6$	(5) $\xi_i^4/12$	(6) $\frac{\bar{W}_i^1 - \bar{W}_i^2}{y \ell^2 \sin\alpha}$
1	.198	.0196	.00129	.000128	0
2	.454	.103	.0156	.00353	0
3	.905	.410	.124	.056	$-9.06\text{x}10^{-9}$

det (3),(4),(5) = $3.79\text{x}10^{-6}$

det (6),(4),(5) = $-.232\text{x}10^{-13}$; $\bar{c}_o^{12} = -.0612\text{x}10^{-7}$; $c_o^{12} = -.612\text{x}10^{-8}$

det (3),(6),(5) = $+5.07\text{x}10^{-13}$; $\bar{c}_1^{12} = +1.34\ \text{x}10^{-7}$; $c_1^{12} = +.175\text{x}10^{-8}$

det (3),(4),(6) = $-15.7\text{x}10^{-13}$; $\bar{c}_2^{12} = -4.14\ \text{x}10^{-7}$; $c_2^{12} = -.00709\text{x}10^{-8}$

TABLE 4

Determination of $C_K^{\prime\prime 22}$

(1) i	(2) ξ_i	(3) $\xi_i^2/2$	(4) $\xi_i^3/3$	(5) $\frac{\bar{p}_i^2 - \bar{p}_i^1}{\ell y}$
1	.198	.0196	.00259	$.34\text{x}10^{-8}$
2	.454	.103	.0312	$.771\text{x}10^{-8}$
3	.905	.410	.247	$9.6\text{x}10^{-8}$

det (2),(3),(4) = $1.098\text{x}10^{-3}$

det (5),(3),(4) = $.468\text{x}10^{-10}$; $\bar{c}_o^{\prime\prime 22} = .426\text{x}10^{-7}$; $c_o^{\prime\prime 22} = 4.26\ \text{x}10^{-8}$

det (2),(5),(4) = $-4.06\text{x}10^{-10}$; $\bar{c}_1^{\prime\prime 22} = -3.7\text{x}10^{-7}$; $c_1^{\prime\prime 22} = -.484\text{x}10^{-8}$

det (2),(3),(5) = $9.3\text{x}10^{-10}$; $\bar{c}_2^{\prime\prime 22} = 8.47\text{x}10^{-7}$; $c_2^{\prime\prime 22} = .0145\text{x}10^{-8}$

/ Table 5 .....

TABLE 5

Determination of C_K^{13}

(1) i	(2) ξ_i	(3) $\frac{\xi_i}{2} - 1$	(4) ξ_i^2	(5) $\frac{\xi_i}{3} - \frac{1}{2}$	(6) ξ_i^3	(7) $\frac{\xi_i}{4} - \frac{1}{3}$	(8) $\bar{C}_{12}^{i-1}$
1	.198	-.901	.0392	-.434	.00776	-.284	- 4.68x10^{-7}
2	.454	-.773	.206	-.349	.0936	-.220	102.4 x10^{-7}
3	.905	-.547	.819	-.198	.741	-.107	-316.5 x10^{-7}

i	(9) $(8)^0$x(2)x(3)	(10) $(8)^1$x(4)x(5)	(11) $(8)^2$x(6)x(7)	(12) (9)+(10)+(11)
1	.835x10^{-7}	- 1.742x10^{-7}	.696x10^{-7}	-.211x10^{-7}
2	1.64 x10^{-7}	- 7.37 x10^{-7}	6.51 x10^{-7}	.78 x10^{-7}
3	2.31 x10^{-7}	-16.63 x10^{-7}	25.10 x10^{-7}	10.78 x10^{-7}

i	(13) $\bar{p}_i^1/\ell$	(14) (13)-(12)	(15) $\xi_i^2/2$	(16) $\xi_i^3/3$
1	.973x10^{-7}	1.184x10^{-7}	.0196	.00259
2	3.32 x10^{-7}	2.54 x10^{-7}	.103	.0312
3	9.75 x10^{-7}	-1.03 x10^{-7}	.410	.247

det (2),(~~16~~ 15),(~~17~~ 16) = 1.098 x 10^{-3}

det (14),(~~16~~ 15),(~~17~~ 16) = .500x10^{-9}; $\bar{C}_o^{13}$= .456x10^{-6}; C_o^{13}= .456x10^{-6}

det ((2),(14),(~~17~~ 16) = 2.43x10^{-9}; $\bar{C}_1^{13}$= 2.22x10^{-6}; C_1^{13}= .029x10^{-6}

det (2),(~~16~~ 15),(14) = -6.3x10^{-9}; $\bar{C}_2^{13}$= -5.74x10^{-6}; C_2^{13}= -.000994x10^{-6}

/ Table 6 ....

TABLE 6

Comparison of Experimental and Theoretical Values of C_{ij} (Fig.7)

Station	$C_{11} \times 10^8$		$C_{12} \times 10^8$		$C_{22} \times 10^8$		$C_{13} \times 10^6$
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.
.0825	1.8	2.2	.21	-2.28	3.42	3.92	.6
.310	.93	3.5	-.44	-3.83	5.28	6.11	.584
.596	12.32	9.4	-7.35	-9.96	14.92	15.95	-.306
.811	29.97	32.2	-17.03	-28.42	27.58	46.1	-1.547
1.0	52.27	68.5	-28.81	-53.1	42.62	86.2	-3.16

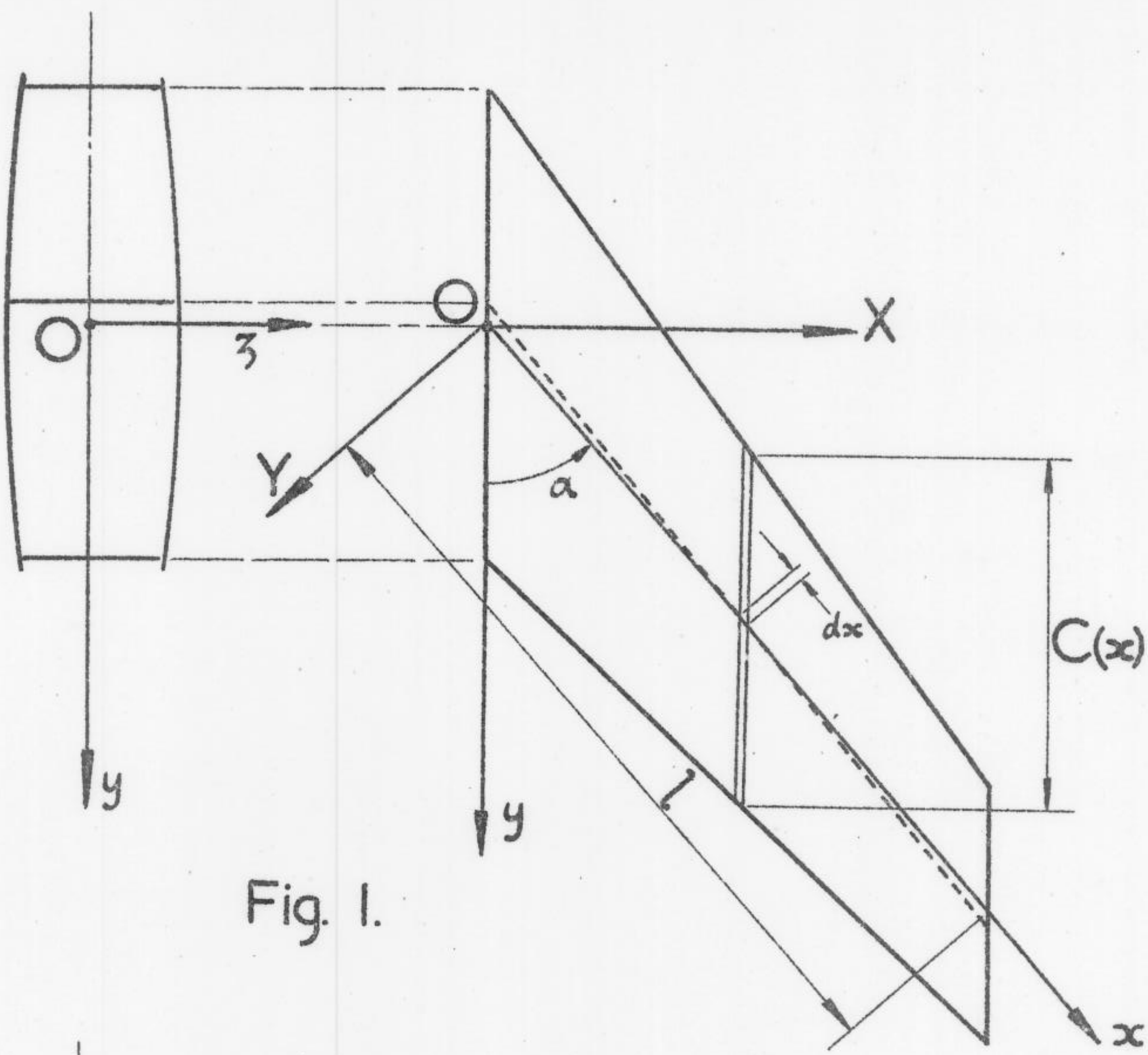


Fig. 1.

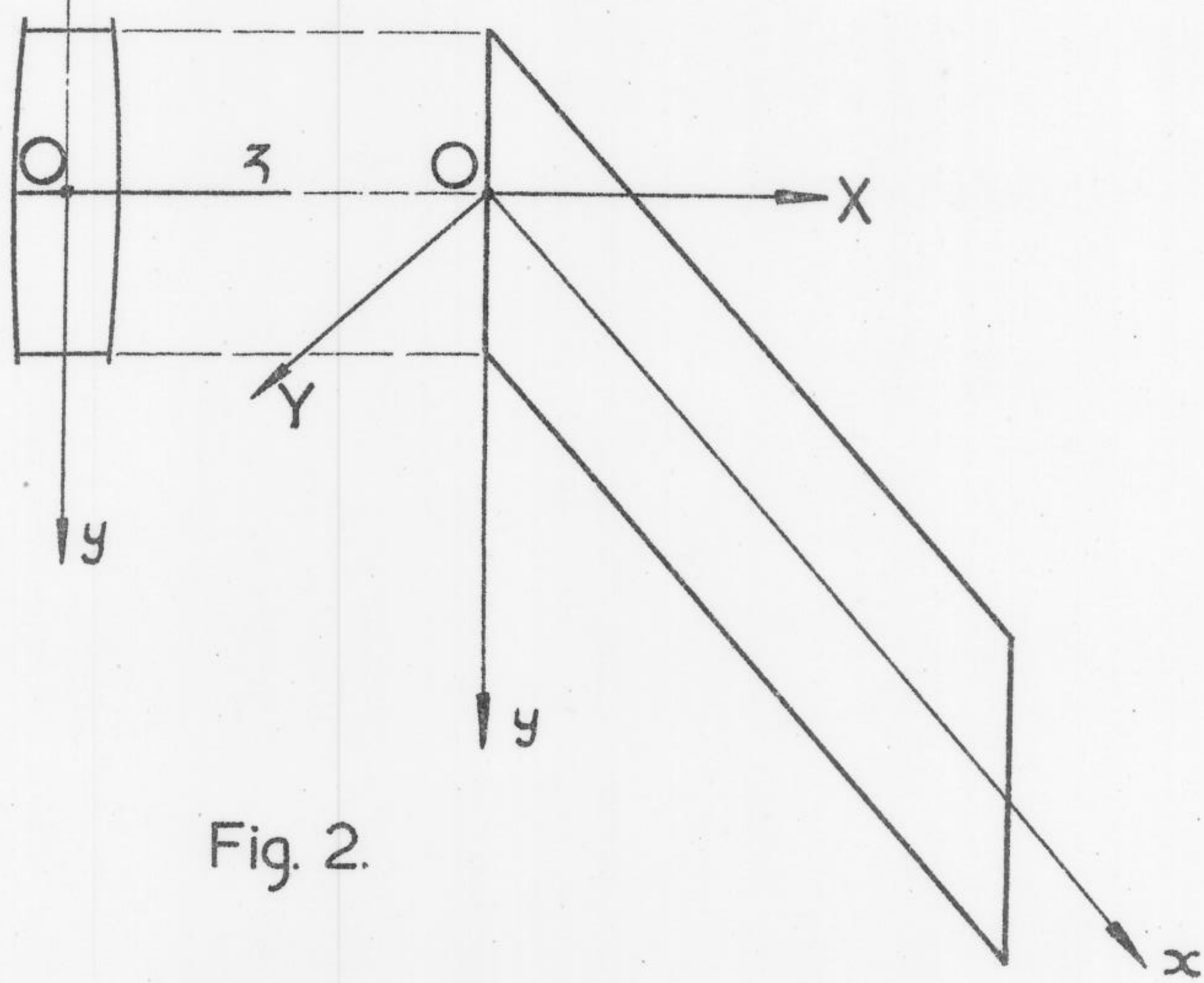


Fig. 2.

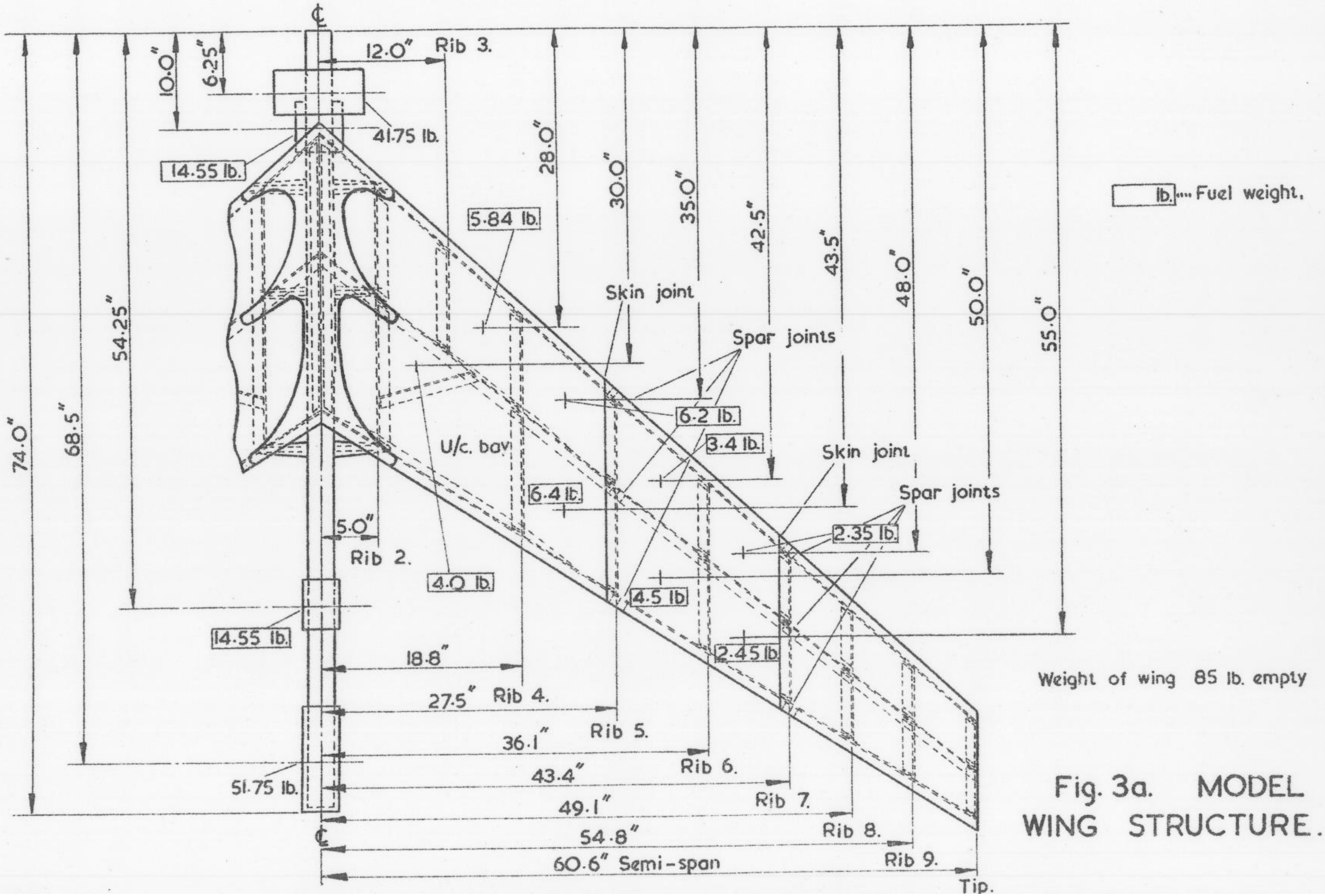
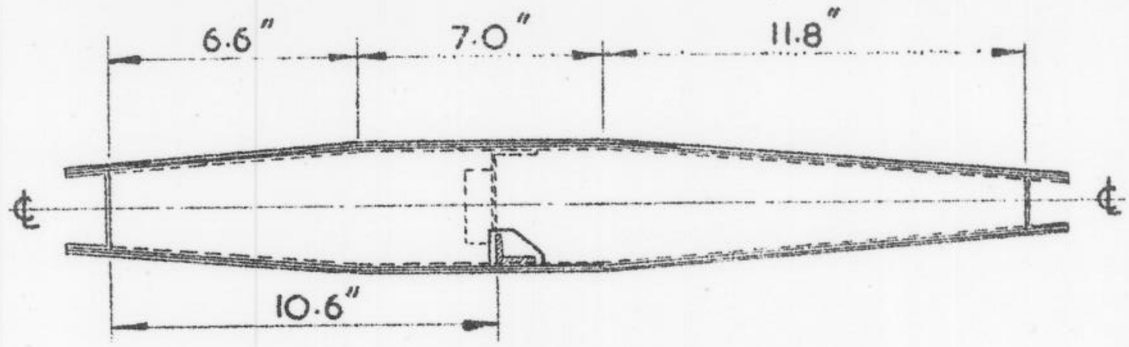
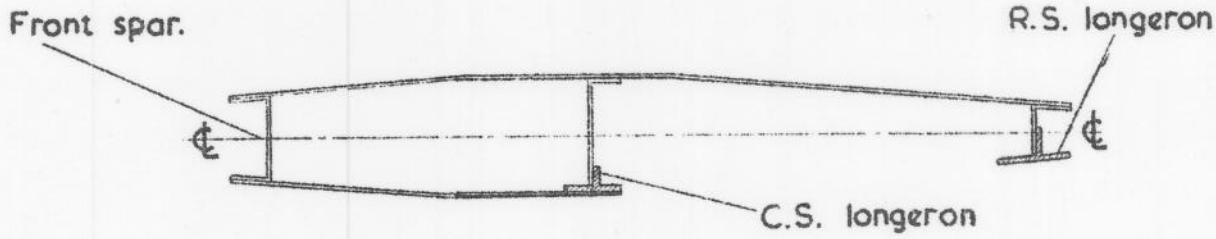


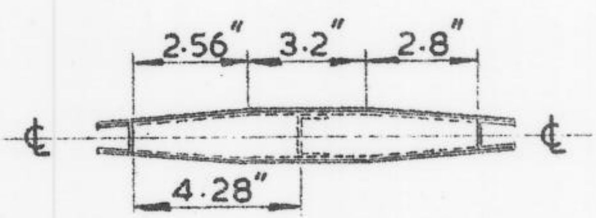
Fig. 3a. MODEL WING STRUCTURE.



RIB 1.



SECTION THROUGH U/C BAY.



RIB AT TIP.

Item	Material
Ribs 1 and 2	18g Lt. Alloy
Ribs 3 and 4	24g Lt. Alloy
Ribs 5 to tip	26g Lt. Alloy
Spars 1st section	22g Lt. Alloy
Spars 2nd section	24g Lt. Alloy
Spars 3rd section	26g Lt. Alloy
Skin 1st section	18g Lt. Alloy
Skin 2nd section	20g Lt. Alloy
Skin 3rd section	26g Lt. Alloy

Fig. 3b.

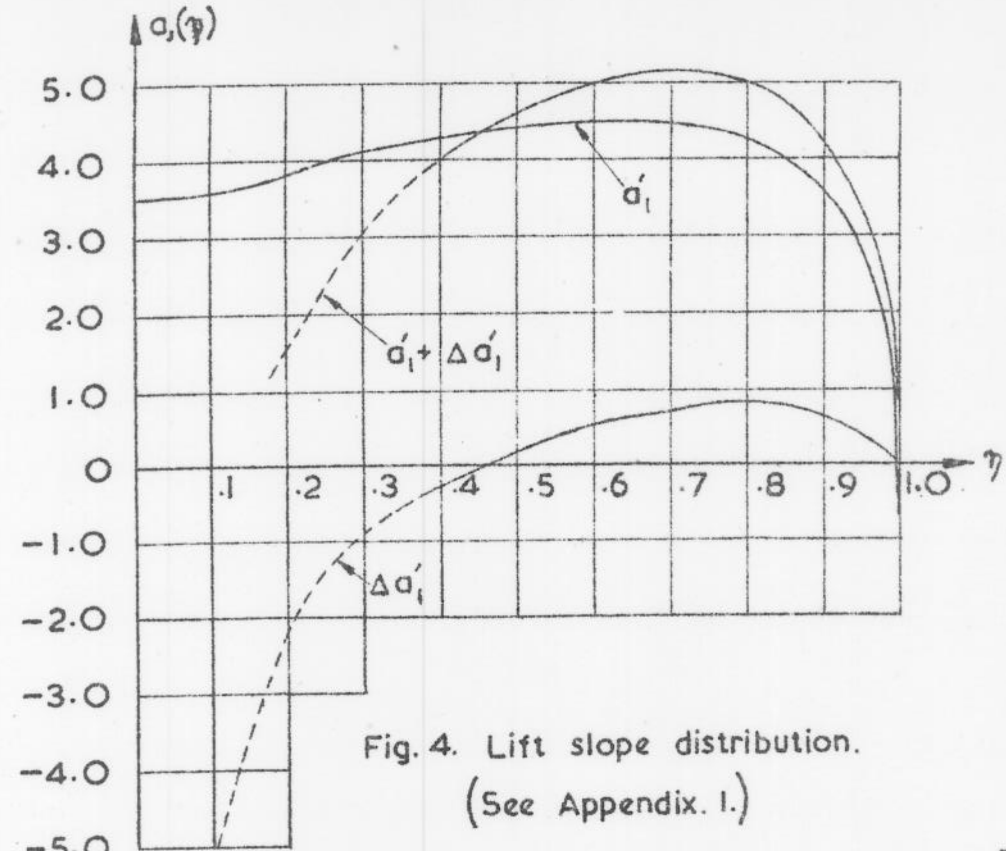


Fig. 4. Lift slope distribution. (See Appendix. 1.)

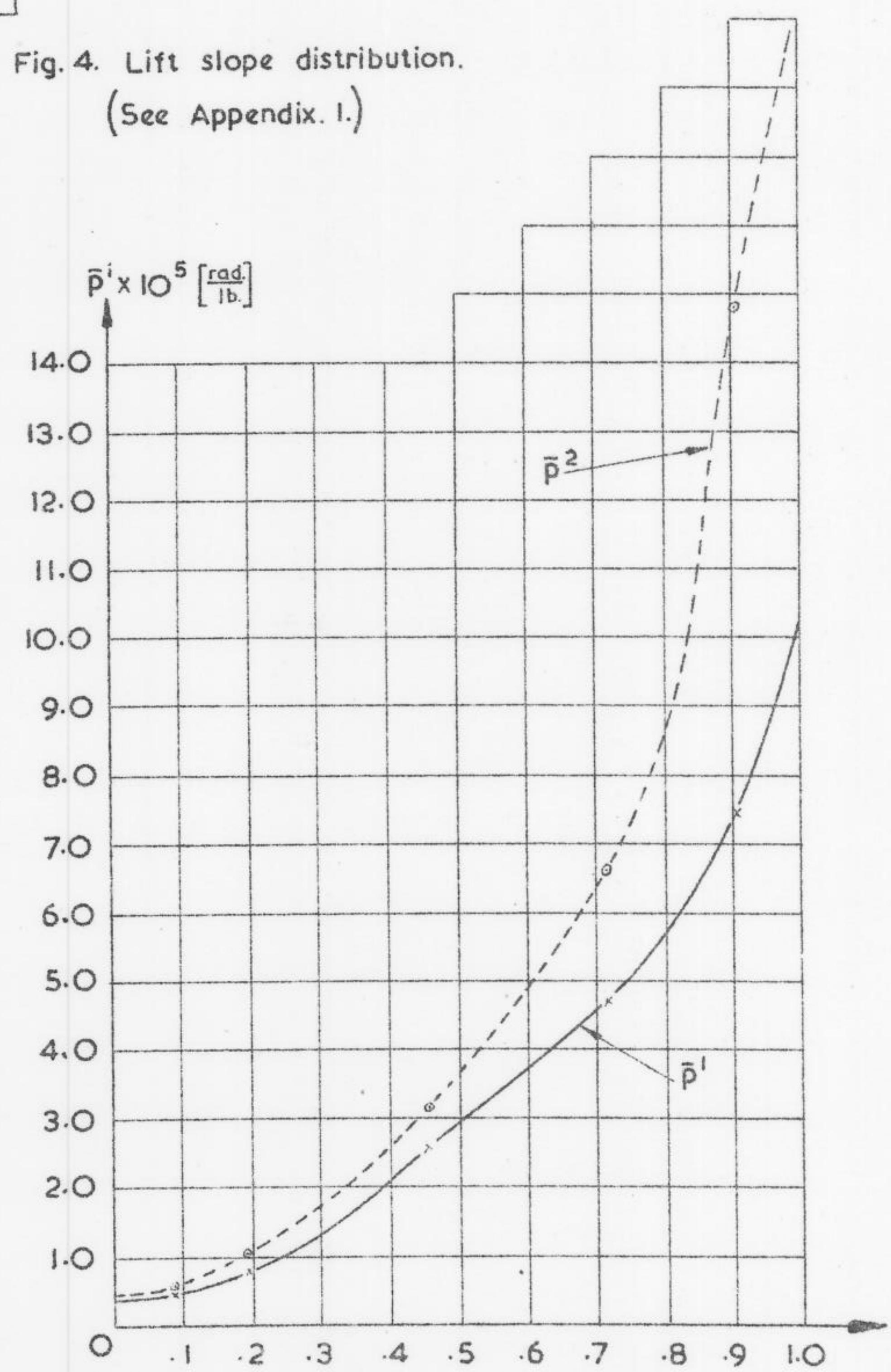


Fig. 5. Measured twists per unit load at tip. (Table. 1.)

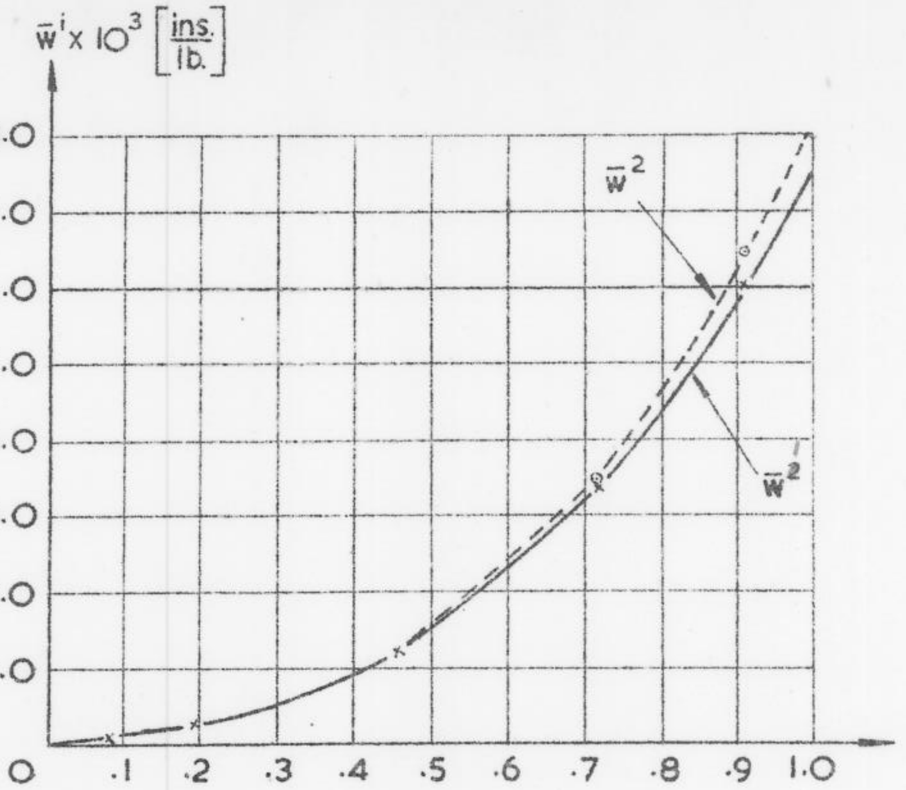


Fig. 6. Measured deflections per unit load at tip (Table. I.)

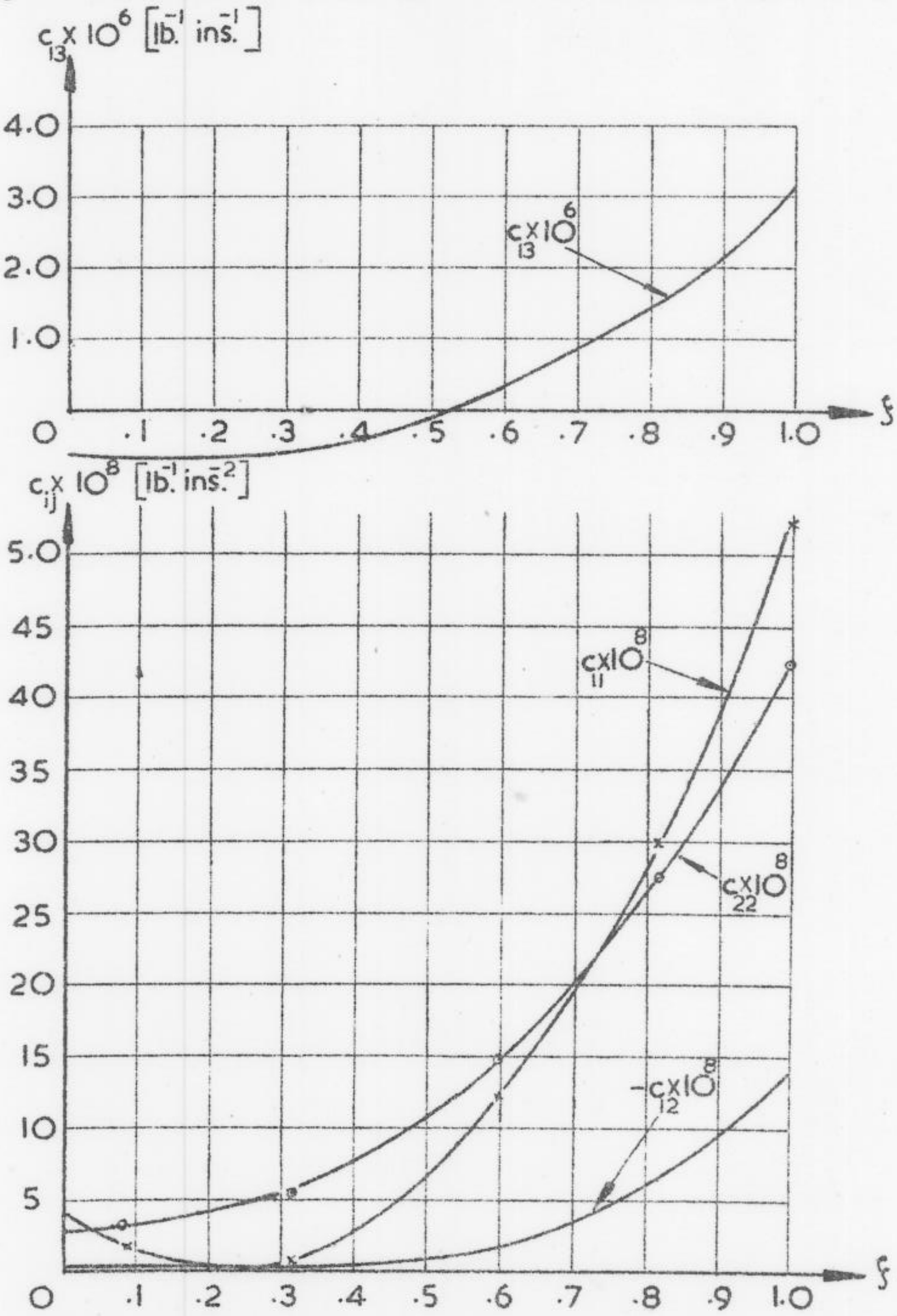


Fig. 7. Experimental and theoretical influence functions.