An Efficient Constrained Weighted Least Squares Method with Bias Reduction for TDOA-Based Localization

Liang Zhang, Tao Zhang and Hyo-Sang Shin

Abstract—**This paper addresses the source location problem by using time-difference-of-arrival (TDOA) measurements. The two-stage weighted least squares (TWLS) algorithm has been widely used in the TDOA location. However, the estimation accuracy of the source location is poor and the bias is significant when the measurement noise is large. Owing to the nonlinear nature of the system model, we reformulate the localization problem as a constrained weighted least squares problem and derive the theoretical bias of the source location estimate from the maximum-likelihood (ML) estimation. To reduce the location bias and improve location accuracy, a novel bias-reduced method is developed based on an iterative constrained weighted least squares algorithm. The new method imposes a set of linear equality constraints instead of the quadratic constraints to suppress the bias. Numerical simulations demonstrate the significant performance improvement of the proposed method over the traditional methods. The bias is reduced significantly and the Cramér–Rao lower bound accuracy can also be achieved.**

Index Terms—**TDOA, Bias reduction, weighted least squares, maximum-likelihood estimation.**

I. INTRODUCTION

Source location via time-difference-of-arrival (TDOA) measurements has drawn considerable attention thanks to its importance in the applications of sensor networks, radar, and underwater navigation[1][2]. Compared with the location method based on the time-of-arrival (TOA), the TDOA-based localization has the advantage that there is no need to synchronize sensor clocks with that of the target[3].

The TDOA-based target localization problem is essentially an optimization problem. It suffers from high nonlinearity and many linear methods have been used in the source location[4]. Among the existing methods, the two-stage weighted least squares (TWLS) algorithm is well known for its low computational complexity and proven approximate efficiency[5]. In recent years, some improved methods have been proposed.

An improved algebraic solution for TDOA localization in the presence of sensor position errors was proposed by Liu et al.[6] An efficient estimator for TDOA-based source localization was proposed[7]. It could achieve the Cramer-Rao lower bound (CRLB) accuracy with the minimum number of sensors. Besides, the hybrid systems that combine TDOA and other noisy measurements also draw considerable attention. A two-step least-square location estimator was developed for a hybrid TDOA/angle-of-arrival (AOA) location scheme and it gives a much higher location accuracy than TDOA only location[8]. For the frequency difference of arrival (FDOA)/TDOA-based localization system, a weighted least-squares minimization method was employed, which did not require initial solution guesses to obtain a location estimate[9]. Noroozi et al[10] proposed an improved algebraic solution using TDOA and FDOA and their solution was proved to be less time-consuming than the traditional method. Passive coherent locator (PCL) is usually used in the radar system. A new fusion strategy was performed at the signal processing level based on the TDOA/PCL measurements[11]. The theoretical performance of the above hybrid systems gains achievable over the localization technique using only one kind of measurements. All of these methods can lead to a closed-form solution and attain the CRLB performance at low and moderate noise levels.

However, the performance of the closed-form solution will rapidly degrade as the measurement noise increases. The maximum-likelihood (ML) estimation is optimal for the TDOA-based localization problem. The challenge of the ML estimation lies in its nonlinear and nonconvex nature. One resolution to the problem is the linearization based on Taylor expansion[12]. Some efficient iteration-based weighted least squares methods were proposed[13][14]. These iteration-based methods heavily depend on the quality of the initial estimate. The convergence of the ML estimation is not guaranteed and a local minimum solution could be attained if bad initial estimates are selected. Another resolution to the high nonlinearity issue

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in the ML problem is the use of the Lagrange multiplier technique[15][16][17]. It achieves remarkably better performance than the TWLS approach especially for the higher measurement noise level[15]. Convex relaxation methods are also applied to the TDOA-based source location problem[18][19][20]. They generally outperform the closed-form solution methods when the noise is considerable. Besides, the semidefinite programming (SDP) methods combined with reformulation linearization (RLT)[21] and the mixed SDP/ second-order cone program(SOCP)[22] were proposed to improve the location accuracy. Both of the methods show superior performance over the TWLS method. However, the SDP method is more costly in terms of computational complexity.

According to the analysis in [23], the noisy measurement will lead to a bias in the closed-form solution. It is necessary to reduce the bias to improve the estimation accuracy when the measurement noise is large. The weighted total least squares (TLS) technique was used to handle the noise correlation between the regressor and regressand[24][25]. The bias can be reduced significantly but the estimation variance is higher than the original solution. In 2012, two methods, namely, "BiasSub" and "BiasRed", were proposed[26]. Some researchers follow the idea of [26] and bias-reduced methods considering the sensor errors were proposed [27][28]. A bias-reduced nonlinear weighted least squares (WLS) method was proposed in [29]. It derived the bias of the WLS solution and subtracted it from the solution.

Generally speaking, the bias-reduced methods can be divided into two types: direct deviation refinement (DDR) and constrained deviation refinement (CDR). DDR method, such as "BiasSub" and the method in [29], requires the covariance matrix of the measurements to be known perfectly and the bias is subtracted from the WLS solution. CDR method, such as "BiasRed", only needs to know the structure of the noise. Thus, "BiasRed" is more practical in the TDOA-based location. To our knowledge, the BiasRed method still suffers from the nonlinear nature of the ML problem. The framework of the BiasRed method is the same as TWLS. It is essentially a two-stage weighted least squares method. The difference is that a quadratic constraint is imposed on the first stage in the BiasRed method. The rest of the algorithm is the same as that of the TWLS. The result from the first stage will have bad performance when the measurement noise is large and the bias-reduced method will lead to a worse solution. To mitigate the issue, we will explore a method to deal with the problem of the nonlinear nature and the bias simultaneously.

This paper proposes a new bias-reduced method. The bias of the ML has been investigated in many papers[29][30][31]. We will first analyze the theoretical bias of the ML problem. Note that the derivation of theoretical bias here is different from that of [30], in which the measurement noise is considered. Then, an iterative constrained weighted least-squares algorithm is developed to handle the nonlinear and bias-reduced problem.

Our goal is to improve the performance of the TDOA-based localization in the case of large measurement noise. The main contributions of the paper are twofold. One is that we derive the theoretical bias of the source location estimate from the ML estimation. The other is that we propose a bias-reduced method, which outperforms the traditional methods especially in the case of large measurement noise.

The structure of the paper is as follows. The first section is the introduction and includes the current research status and the contributions of the paper. The second section analyzes the TDOA positioning method and bias. The third section gives the detailed derivation of the bias-reduced method. The fourth section verifies the effectiveness of the proposed algorithm through simulation. The last section presents summary and future prospects. The main symbols and notations used are shown bellows.

TABLE I
NOMENCLATURE

Symbol	Explanation
M	Number of sensors
s_i	Position of the ith sensor
$\mathbf{u}_o$	The true source location
N	The system dimension
r_{i1}	Range difference between sensor i and the reference sensor
r_i	Range between sensor i and the source
n	TDOA noise vector
n_{i1}	Measurement noise with respect to r_{i1}
Q	Covariance matrix of n
$\hat{u}_0$	The estimated source location
CRLB	the Cramer-Rao lower bound
tr(*)	The trace of the matrix *
E(*)	the expectation of parameter *
I_{NxN}	The identity matrix with dimension of N
$\partial *$	Differential of the parameter *
$\odot$	The element-by-element product
W	the weighted matrix
$\Delta *$	The matrix * with noise term
J	Cost function of the WLS formulation
B_1	Coefficient matrix relating the noise to the residual vector
h, h_1	Regressand of the WLS formulation
G, G_1	Regressor of the WLS formulation
y, θ, V	Augmented vector
Z	Augmented matrix
λ	Lagrange multiplier

II. TDOA POSITIONING ANALYSIS

A. System model

Consider a scenario of M passive sensors with known positions to locate the source position in N-dimensional ($N = 2$ or 3) space. The position of the ith sensor is known and denoted by s_i ($i = 1,2,3, \dots M$). The source location is unknown and denoted by $\mathbf{u}_o$. In general, the first sensor (s_1) is usually selected as the reference. The actual TDOA measurement, denoted by r_{i1}, between sensor pairs i and 1 is

$$r_{i1} = r_i - r_1 + n_{i1} \qquad (1)$$

where $r_i = \|\mathbf{u}_o - s_i\|$, ($i = 2,3, \dots M$). n_{i1} is the measurement

noise. It is assumed that the TDOA noise vector $\boldsymbol{n} = [n_{21} \cdots n_{i1}]$ is zero-mean Gaussian distributed with covariance matrix $\boldsymbol{Q} = E_n(\boldsymbol{n}\boldsymbol{n}^T)$. Note that r_{i1} is actually the range difference which is the TDOA multiplied by the known signal propagation speed c. We shall use time differences and range differences interchangeably throughout the paper as they are differed by a constant scaling factor.

According to the analysis in [5], the squared distance between the source and sensor i can be simplified as

$$r_{i1}^2 + \|\boldsymbol{s}_1\|^2 - \|\boldsymbol{s}_i\|^2 + 2(\boldsymbol{s}_i - \boldsymbol{s}_1)^T\boldsymbol{u}_o + 2r_{i1}\|\boldsymbol{u}_o - \boldsymbol{s}_1\|$$
$$= 2\|\boldsymbol{u}_o - \boldsymbol{s}_i\|n_{i1} + n_{i1}^2 \quad (2)$$

Taking all the TDOA measurements into consideration, (2) can be extended as follows.

$$\boldsymbol{h} - \boldsymbol{G}\boldsymbol{y} = \boldsymbol{B}_1\boldsymbol{n} + \boldsymbol{n} \odot \boldsymbol{n} \quad (3)$$

where $\boldsymbol{h} = \begin{bmatrix} r_{21}^2 + \|\boldsymbol{s}_1\|^2 - \|\boldsymbol{s}_2\|^2 \\ \vdots \\ r_{M1}^2 + \|\boldsymbol{s}_1\|^2 - \|\boldsymbol{s}_M\|^2 \end{bmatrix}$,

$\boldsymbol{G} = -2\begin{bmatrix} (\boldsymbol{s}_2 - \boldsymbol{s}_1)^T & r_{21} \\ \vdots & \vdots \\ (\boldsymbol{s}_M - \boldsymbol{s}_1)^T & r_{M1} \end{bmatrix}$, $\boldsymbol{y} = [\boldsymbol{u}_o^T \quad r_1]^T$,

$\boldsymbol{B}_1 = 2diag([r_2 \cdots r_M])$.
and "$\odot$" denotes the element-by-element product.

Let the weighted matrix $\boldsymbol{W} = (\boldsymbol{B}_1\boldsymbol{Q}\boldsymbol{B}_1^T)^{-1}$ and define the cost function $J(\boldsymbol{y})$ as follows.

$$J(\boldsymbol{y}) \triangleq (\boldsymbol{h} - \boldsymbol{G}\boldsymbol{y})^T\boldsymbol{W}(\boldsymbol{h} - \boldsymbol{G}\boldsymbol{y}) \quad (4)$$

In (4), the unknown parameter in $\boldsymbol{B}_1$ can not be obtained in advance. We adopt an iterative approach. The matrix $\boldsymbol{B}_1$ is set as an identity matrix. Then a rough solution $\boldsymbol{y}$ can be obtained With the solution $\boldsymbol{y}$, then $\boldsymbol{B}_1$ and $\boldsymbol{W}$ can be determined. In general, obtaining an approximation of the weighted matrix $\boldsymbol{W}$ is sufficient to obtain the exact final solution. As the minimum cost function (4) is not sensitive to the noise in the weighted matrix.

Considering the relationship between $\boldsymbol{u}_o$ and r_1, (4) can be reformulated as a constrained optimization problem.

$$min(\boldsymbol{h} - \boldsymbol{G}\boldsymbol{y})^T\boldsymbol{W}(\boldsymbol{h} - \boldsymbol{G}\boldsymbol{y})$$
$$s.t.\ r_1 = \|\boldsymbol{u}_o - \boldsymbol{s}_1\| \quad (5)$$

B. Constrained weighted least squares solution

A general solution to (5) is the two-stage weighted least squares[5]. However, it suffers from high nonlinearity and large measurement noise. A constrained weighted least-squares solution will be analyzed.

To be able to combine the constraint with the cost function in (5), $(\boldsymbol{u}_o - \boldsymbol{s}_1)$ in the constraint should be treated as one vector. The first estimated parameter in $\boldsymbol{y}$ should be changed from $\boldsymbol{u}_o$ to $\boldsymbol{u}_o - \boldsymbol{s}_1$.

Thus, we rewrite (3) as

$$\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta} = \boldsymbol{B}_1\boldsymbol{n} + \boldsymbol{n} \odot \boldsymbol{n} \quad (6)$$

where $\boldsymbol{\theta} = [(\boldsymbol{u}_o - \boldsymbol{s}_1)^T \quad r_1]^T$.

$\boldsymbol{h}_1 = \begin{bmatrix} r_{21}^2 - (\boldsymbol{s}_2 - \boldsymbol{s}_1)^T(\boldsymbol{s}_2 - \boldsymbol{s}_1) \\ \vdots \\ r_{M1}^2 - (\boldsymbol{s}_M - \boldsymbol{s}_1)^T(\boldsymbol{s}_M - \boldsymbol{s}_1) \end{bmatrix}$,

$\boldsymbol{G}_1 = -2\begin{bmatrix} (\boldsymbol{s}_2 - \boldsymbol{s}_1)^T & r_{21} \\ \vdots & \vdots \\ (\boldsymbol{s}_M - \boldsymbol{s}_1)^T & r_{M1} \end{bmatrix}$

Then, the optimization problem given in (5) can be reformulated as follows.

$$J(\boldsymbol{\theta}) = min(\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta})^T\boldsymbol{W}(\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta})$$
$$s.t.\ \boldsymbol{\theta}^T\boldsymbol{\Sigma}\boldsymbol{\theta} = 0 \quad (7)$$

Where $\boldsymbol{\Sigma} = \text{diag}(1\quad 1\quad -1).\ (N = 2)$
or $\boldsymbol{\Sigma} = \text{diag}(1\quad 1\quad 1\quad -1).\ (N = 3)$

We apply the Lagrange multiplier technique to find the solution to the optimization problem given in (7). It can be reformulated as follows.

$$L(\boldsymbol{\theta}, \lambda) = (\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta})^T\boldsymbol{W}(\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta}) + \lambda\boldsymbol{\theta}^T\boldsymbol{\Sigma}\boldsymbol{\theta} \quad (8)$$

The estimate of $\boldsymbol{\theta}$ is obtained by differentiating $L(\boldsymbol{\theta}, \lambda)$ respect to $\boldsymbol{\theta}$ and then equating the results to zero:

$$\frac{\partial L(\boldsymbol{\theta}, \lambda)}{\partial\boldsymbol{\theta}} = 2(\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1 + \lambda\boldsymbol{\Sigma})\boldsymbol{\theta} - 2\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1 = \boldsymbol{0} \quad (9)$$

The solution to (9) is

$$\widehat{\boldsymbol{\theta}} = (\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1 + \lambda\boldsymbol{\Sigma})^{-1}\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1 \quad (10)$$

where $\widehat{\boldsymbol{\theta}}$ is the estimated solution of (9).

$\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1 + \lambda\boldsymbol{\Sigma}$ is a symmetric matrix. Substituting (10) into the constraint $\boldsymbol{\theta}^T\boldsymbol{\Sigma}\boldsymbol{\theta} = 0$ yields.

$$\boldsymbol{h}_1^T\boldsymbol{W}_1^T\boldsymbol{G}_1(\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1 + \lambda\boldsymbol{\Sigma})^{-1}\boldsymbol{\Sigma}(\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1 + \lambda\boldsymbol{\Sigma})^{-1}\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1$$
$$= \boldsymbol{h}_1^T\boldsymbol{W}^T\boldsymbol{G}_1\boldsymbol{\Sigma}^{-1}(\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{\Sigma}^{-1}$$
$$+ \lambda\boldsymbol{I})^{-1}(\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{\Sigma}^{-1} + \lambda\boldsymbol{I})^{-1}\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1 \quad (11)$$
$$= 0$$

where $\boldsymbol{I}$ is a $(N + 1) \times (N + 1)$ identity matrix.

λ is calculated by eigenvalue factorization method and $\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{\Sigma}^{-1}$ can be factorized as follows.

$$\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{\Sigma}^{-1} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{-1} \quad (12)$$

where $\boldsymbol{\Lambda} = \text{diag}(\eta_1 \cdots \eta_{N+1})$.

Substituting (12) into (11), the polynomial equation concerning λ can be obtained as follows.

$$f(\lambda) = \sum_{i=1}^{N+1}\frac{p_iq_i}{(\lambda + \eta_i)^2} \quad (13)$$

where $\boldsymbol{p} = [p_1 \cdots p_{N+1}] = \boldsymbol{U}^T\boldsymbol{\Sigma}^{-T}\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1$
and $\boldsymbol{q} = [q_1 \cdots q_{N+1}] = \boldsymbol{U}^{-1}\boldsymbol{G}_1^T\boldsymbol{W}\boldsymbol{h}_1.$

It can be efficiently solved by finding the roots of a polynomial equation[15]. Substituting the real λ into (10) provides the estimated value of $\boldsymbol{\theta}$, which is the optimal solution that minimize $J(\boldsymbol{\theta})$ while satisfying the constraint.

C. Bias analysis

According to the analysis in [23], the bias is mainly caused by two factors. One is the nonlinearity issue in the ML problem and the other is the noisy measurement. Both of them will be considered in the paper.

The detailed derivation process is shown in the appendix.

The bias of the constrained weighted least-squares solution is as follows.

$$E_n(\Delta\boldsymbol{u}_0) = E_n(\boldsymbol{\alpha}) + E_n(\boldsymbol{\beta}) \quad (14)$$

where $E_n(\boldsymbol{\alpha})$ is given by (44) and $E_n(\boldsymbol{\beta})$ is given by (48).

From (14), the bias of the constrained weighted least-squares solution is significant when the measurement noise is large.

To improve the positioning accuracy, the estimation bias should be reduced.

III. ITERATIVE CONSTRAINED WEIGHTED LEAST SQUARES METHOD FOR BIAS REDUCTION

The bias of the ML problem is theoretically analyzed in

section II. In the case of large measurement noise, the bias is significant. We will introduce a novel bias-reduced method, which is different from the existing methods.

The original closed-form solution minimizes

$$min(h_1 - G_1\theta)^T W(h_1 - G_1\theta)$$
$$s.t. \ \theta^T\Sigma\theta = 0 \tag{15}$$

We introduce an augmented matrix Z and V.

$$Z = [-G_1 \quad h_1]$$
$$V = [\theta^T \quad 1]^T \tag{16}$$

Z contains measurement noise and it is decomposed as.

$$Z = \Delta Z + Z^0 \tag{17}$$

where Z^0 is a matrix without any measurement noise. ΔZ is the noise term, which can be expressed as follows.

$$\Delta Z = 2[0_{(M-1)\times N} \quad n \quad \tilde{B}_1 n]$$
$$\tilde{B}_1 = diag(r_{21} \quad \cdots \quad r_{i1}) \tag{18}$$

Substituting (18) into (15) yields the cost function

$$J(\theta) = V^T Z^{0^T} WZ^0 V + V^T\Delta Z^T W\Delta Z V + 2V^T\Delta Z^T WZ^0 V \tag{19}$$

The third term $2V^T\Delta Z^T WZ^0 V$ vanishes in the expectation because ΔZ is zero-mean. Take the expectation of $J(\theta)$ to obtain the cost function on the average.

$$E[J(\theta)] = V^T Z^{0^T} WZ^0 V + V^T E[\Delta Z^T W\Delta Z]V \tag{20}$$

We regard the second term on the right-hand as a constant constraint to the cost function. Thus, we find V by

$$minV^T Z^T WZV$$
$$s.t. \ V^T\Omega V = k \tag{21}$$

where $\Omega = E[\Delta Z^T W\Delta Z]$ and constant k can be any value.

$$\Omega = E[\Delta Z^T W\Delta Z]$$
$$= \begin{bmatrix} 0_{N\times N} & 0_{N\times 2} \\ 0_{2\times N} & \tilde{\Omega} \end{bmatrix} \tag{22}$$

and

$$\tilde{\Omega} = 4\begin{bmatrix} tr(WQ) & tr(W\tilde{B}_1 Q) \\ tr(\tilde{B}_1 WQ) & tr(\tilde{B}_1 W\tilde{B}_1 Q) \end{bmatrix} \tag{23}$$

The traditional bias-reduced method is based on (21). The solution to (21) neglects the constraint relationship between u_o and r_1 in the first stage. A second step to reduce the nonlinear error is necessary. The drawback of the traditional method is that the source location in the first stage suffers from the measurement noise and an inaccurate result will lead to a local minimum solution in the second stage.

Thus, an iterative approximation method is developed to solve the bias-reduction problem. The new iterative technique follows the idea given in [32]. But a different solution to the iterative approximation problem is designed in the paper, which takes the constant constraint caused by the noisy measurements into consideration.

According to (7) and (21), it can be obtained

$$minV^T Z^T WZV$$
$$s.t. \ V^T\Omega V = k$$
$$V^T\Sigma_1 V = 0 \tag{24}$$

where $\begin{cases} \Sigma_1 = diag([1 \quad 1 \quad -1 \quad 0]) \quad N = 2 \\ \Sigma_1 = diag([1 \quad 1 \quad 1 \quad -1 \quad 0]) \quad N = 3 \end{cases}$

By using Lagrange multiplier λ, we obtain the auxiliary cost function

$$minV^T Z^T WZV + \lambda(k - V^T\Omega V)$$
$$s.t. \ V^T\Sigma_1 V = 0 \tag{25}$$

It is difficult to obtain a globally optimal solution. If we replace one of the variable V with a known estimate $\hat{V}$, the non-convex constraint becomes a linear equality constraint.

$$minV^T Z^T WZV + \lambda(k - V^T\Omega V)$$
$$s.t. \ \hat{V}^T\Sigma_1 V = 0 \tag{26}$$

(26) shows the main difference between the proposed method and the traditional method. The linear equality constraint in (26) can be reformulated as follows.

$$PV = 0 \tag{27}$$

where $P = \hat{V}^T\Sigma_1$ and $\hat{V}$ is a known vector.

Based on the generalized inverse theory of a matrix [33], the solution to (27) can be obtained as follows

$$V = (I - P^+ P)\xi \tag{28}$$

where $\xi \in R^{N+1}$ is any vector. $P^+ = P^T(PP^T)^{-1}$.

The general solution to (27) can be expressed as

$$V = U\xi \tag{29}$$

where $U = (I - P^+ P)$.

Substituting (29) into (25) yields the cost function

$$J(\xi) = min(U\xi)^T Z^T WZU\xi + \lambda(k - (U\xi)^T\Omega U\xi) \tag{30}$$

It is an ML problem and the optimal solution ξ satisfies

$$\frac{\partial J(\xi)}{\partial\xi} = 2U^T Z^T WZU\xi - 2\lambda U^T\Omega U\xi = 0 \tag{31}$$

We have the following relationship

$$U^T Z^T WZU\xi = \lambda U^T\Omega U\xi \tag{32}$$

We need to minimize λ. Through generalized singular value decomposition (GSVD) theory, the optimal solution ξ is the eigenvector that corresponds to the minimum generalized eigenvalue of the pair $(U^T Z^T WZU \quad U^T\Omega U)$.

When ξ is obtained, substitute it into (29) and we will have the optimal solution V.

From the above discussion, we finish the derivation of the bias-reduced method. However, there are two problems with the above method.

a) Since r_i in the matrix B_1 is unknown, B_1 cannot be obtained.

b) The initial value $\hat{V}$ in (26) is important and it can not be obtained.

To solve the above problems, we adopt an iterative approach. The matrix B_1 is set as an identity matrix. Then a rough solution θ can be obtained through the method in Section II. In general, obtaining an approximation of the weighted matrix W is sufficient to obtain the exact final solution. As the minimum cost function (7) is not sensitive to the noise in the weighted matrix.

When the rough value of θ is obtained, the matrix B_1 can be obtained as follows.

$$\begin{cases} B_1 = 2diag([r_2 \quad \cdots \quad r_i]) \\ r_i = \|\theta(1:N) + s_1 - s_i\| \quad i = 2, \ldots M \end{cases} \tag{32}$$

The rough value can be selected as an initial value in (26). The iterative constrained weighted least squares method is formally presented in Algorithm 1.

Sometimes, the iterative method cannot guarantee convergence to the global solution of the ML problem. A divergence factor σ is adopted in the iterative method. If the equation diverges, the estimated value will be far from the initial estimate and we will take the solution in section II-B as step 8 shows.

Algorithm 1: A bias-reduced method based on iterative constrained weighted least squares

Step 1. Set $B_1 = I_{M-1}$ and solve the constraint WLS optimization problem (7) by (9)-(13). A rough value θ^0 can be obtained.

Step 2. Initialize $k = 0$. Define a convergence threshold ε, a divergence threshold σ and a maximum number of iterations τ. Reformulate the weighting matrix $\mathbf{W}$ with θ^0. Set the initial value $\tilde{\theta}^k = \theta^0$ and $\hat{V} = [\tilde{\theta}^{k^T} \quad 1]^T$

Step 3. Set k=k+1. formulate the approximate linear constraint WLS optimization problem (26) with $\hat{V}$.

Step 4. Solve the approximate problem (30) based on GSVD to obtain the estimated value of ξ.

Step 5. Obtain the estimate $\tilde{\theta}^{k+1}$ with ξ according to (29).

Step 6. Reformulate the weighting matrix $\mathbf{W}$ with $\tilde{\theta}^{k+1}$. Set $\hat{V} = [\tilde{\theta}^{k+1^T} \quad 1]^T$.

Step 7. If $\|\tilde{\theta}^{k+1} - \tilde{\theta}^k\| < \varepsilon$ or k> τ, go to step 9, otherwise, go to step 3.

Step 8 If $\|\tilde{\theta}^{k+1} - \theta^0\| > \sigma$, set $\tilde{\theta}^{k+1} = \theta^0$.

Step 9. $u_o = \tilde{\theta}^{k+1}(1:N) + s_1$.

IV. SIMULATION TEST AND ANALYSIS

To verify the effectiveness of the proposed algorithm, simulations are performed in this section. We apply the proposed algorithm to two localization scenarios and the results are compared with that of several existing methods. 1000 Monte Carlo simulations are performed for each test.

Symbols used for the simulations are as follows:
1) 'TWLS' denotes the two-step weighted least squares algorithm described in [5].
2) 'BiasRed' denotes the bias-reduced method described in [26].
3) 'L-WLS' denotes the Lagrange- weighted least squares described in section II without reducing the bias.
4) 'Proposed method' denotes the proposed bias-reduced method described in section V.
5) 'TheoryBias' denotes the theoretical bias norm calculated by $\|\Delta u_{theory}\|$ and Δu_{theory} is obtained from (14).

Note: The CDR method has an advantage over the DDR method in that it only requires the structure of $\mathbf{Q}$[26]. The 'BiasRed' and 'The proposed' both belong to the CDR method. Thus, the method in [26] and the proposed method are mainly used for comparison.

The localization accuracy is evaluated in terms of the root mean square error (RMSE) and the bias norm of the source position, which is defined as follows.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{L}\|\hat{u}_o - u_o\|^2}{L}}$$

$$\text{BiasNorm} = \left\|\frac{\sum_{i=1}^{L}\hat{u}_o}{L} - u_o\right\| \tag{33}$$

where u_o denotes the true source position. L=1000 is the number of ensemble runs.

The TDOA noise is Gaussian and its covariance matrix is equal to the noise power times $\mathbf{Q} = (I_N + 1_N 1_N^T)/2$. At a given SNR, the noise power is obtained as follows with the signal propagation speed $sec = 3 \times 10^8$ m/s.

$$\sigma_\delta^2 = \frac{1}{8\pi^2 SNR(16 \times 10^{18})} sec^2 \tag{34}$$

A. Scenario 1- The impact of the measurement noise

We will compare the location performance under the condition of different measurement noise. We consider the sensor-source geometry, where the sensor network has an array of eight sensors and their positions are given by $s_i = \left[12\cos\left(\frac{\pi}{6}(i-2)\right) \quad 12\sin\left(\frac{\pi}{6}(i-2)\right)\right]^T$, $(i = 2,\dots 8)$. The reference sensor is located at $s_1 = [0 \quad 0]^T$. The source is located at $u_0 = \left[250\cos\left(\frac{\pi}{16}\right) \quad 250\sin\left(\frac{\pi}{16}\right)\right]^T$. The noise power is varied from -15 to 10 dB.

Comparison of the RMSE and bias norm with different measurement noise is shown in Fig. 1 and Fig. 2.
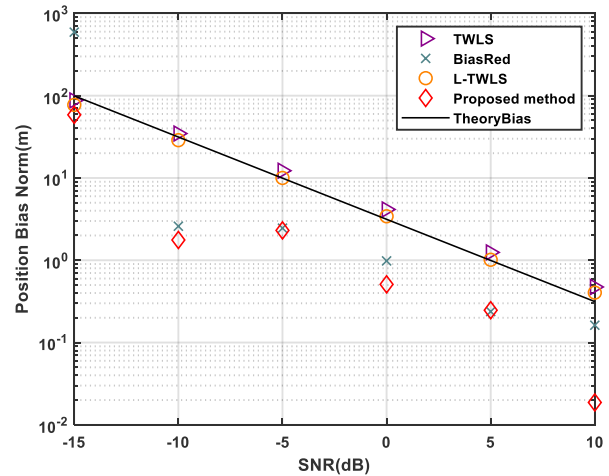

Fig. 1 Comparison of bias norm in the TDOA source location
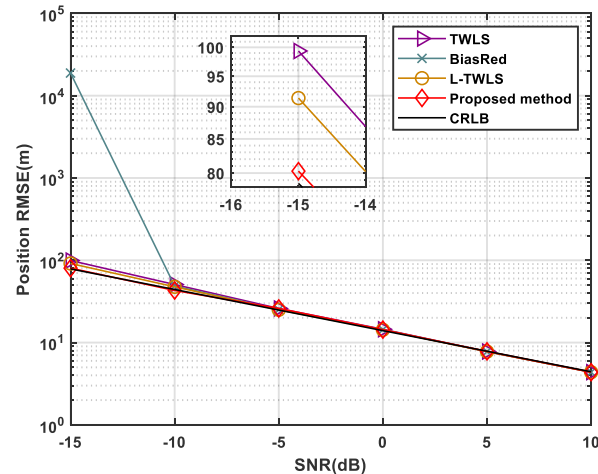

Fig. 2 Comparison of the RMSE with different measurement noise.

In Fig. 1, the black line is the theoretical bias calculated by (28). Fig. 1 indicates the bias of the location increases with the decrease of SNR. The bias of the BiasRed and the proposed method can be significantly reduced. For example, when SNR = -10 dB, the bias norm from BiasRed and the proposed method is 2.60m and 1.76m respectively. The proposed method outperforms the BiasRed method in terms of bias-reduction in the given condition. When SNR = -15 dB, the bias norm from

BiasRed is larger than the theoretical bias norm. However, the bias norm of the proposed method is still the lowest among these methods. In Fig. 2, the black line is the trace of the CRLB. It indicates the proposed method achieves the CRLB accuracy very well no matter what value is the SNR. The proposed method has the smallest RMSE among all the methods, e.g., when SNR=-15dB, the RMSE of the proposed method is 80.3m, which is smaller than the TWLS (99.4m), BiasRed (18847.7m), and L-TWLS (91.4m). It has an 11.1-m reduction in RMSE as compared with the L-TWLS. The RMSE of BiasRed rapidly increases when SNR=-15dB. It indicates the BiasRed method suffers from the large measurement noise. The result is consistent with the analysis in section V. Thus, the proposed method has better performance.

Fig. 3 compares the theoretical and simulation bias norm values. The actual bias norm is from the L-TWLS method. The bias theoretically found matches very well with the actual bias norm. It can be seen in the figure that the proposed method can reduce the bias significantly, e.g., when the SNR is -10 dB, the proposed method has a 27-m reduction in bias compared to the L-TWLS method. It demonstrates the superb performance of the proposed bias reduction procedure.
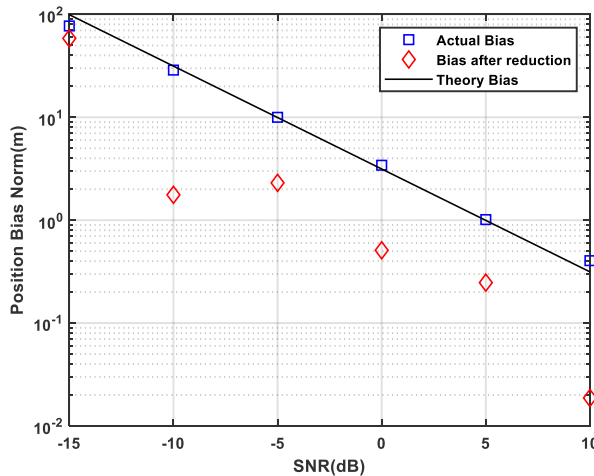

Fig. 3 Theoretical and simulation bias values of TDOA source location estimate from the proposed method

The proposed method is superior to the traditional methods especially in the condition of large measurement noise. It can reduce the bias significantly and achieve the CRLB accuracy very well.

B. Scenario 2- The impact of the sensor numbers

This section compares the location performance under the condition of different sensor numbers. The sensor network has an array of several sensors and their positions are given by $s_i = \left[12cos\left(\frac{\pi}{6}(i-2)\right) \quad 12sin\left(\frac{\pi}{6}(i-2)\right)\right]^T$. In this scenario, the sensor number (i) is varied from 8 to 12. The reference sensor is located at $s_1 = [0 \quad 0]^T$. The source is located at $u_0 = \left[350cos\left(\frac{\pi}{16}\right) \quad 350sin\left(\frac{\pi}{16}\right)\right]^T$. The noise power is set as -6 dB.

A comparison of the RMSE and bias norm with different sensor numbers are shown in Fig. 4 and Fig. 5.
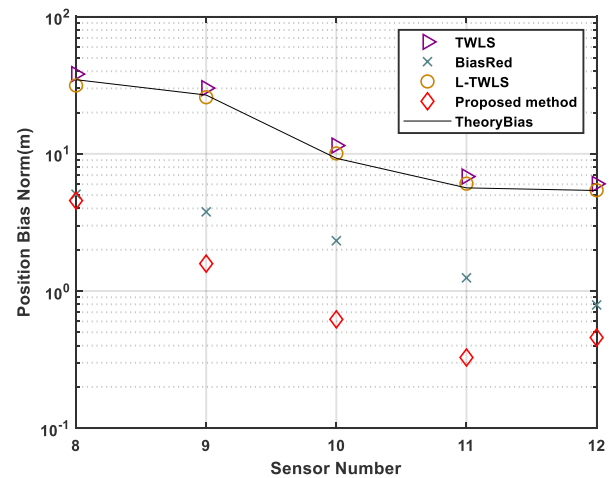

Fig. 4 Comparison of bias norm in TDOA source location estimates from different solution methods
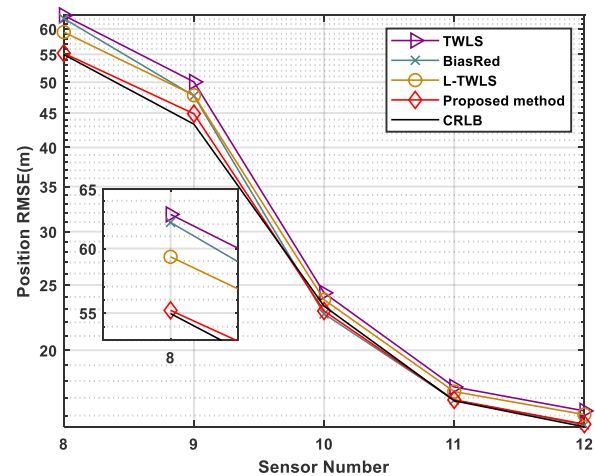

Fig. 5 Comparison of the RMSE with different sensor numbers

Fig. 4 compares the bias norm of the proposed method and other methods. The bias of the BiasRed and the proposed method can be significantly reduced. However, the bias from the BiasRed method is larger than that of the proposed method. For example, when the sensor number is 9, the bias norm from BiasRed and the proposed method is 3.78m and 1.58m respectively. Fig. 5 compares the RMSE of the proposed method and other methods. All the algorithms perform exhibit reasonable performance when the sensor number is large. The proposed method achieves the CRLB performance best no matter how the sensor number changes. However, traditional methods such as TWLS, BiasRed, and L-TWLS cannot achieve the CRLB performance when the sensor number decreases. In comparison, the proposed method provides a more stable performance both in terms of RMSE and bias.

Fig. 6 compares the theoretical and simulation bias norm values. The actual bias norm is from the L-TWLS method. The theoretical bias norm matches very well with the actual bias norm. It can be seen in the figure that the proposed method can reduce the bias significantly, e.g., when the sensor number is 9, the proposed method has a 24.29-m reduction in bias.
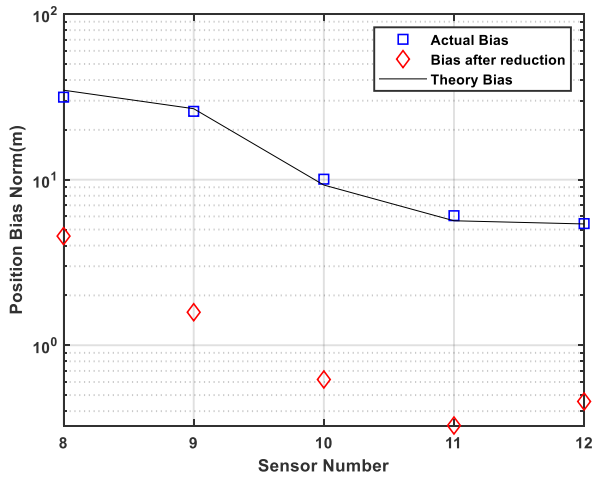
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Fig. 6 Theoretical and simulation bias values of TDOA source location estimate from the proposed method

From the above figures, the proposed method has a more stable performance both in bias-reduction and RMSE. The RMSE of the proposed method is always closer to the CRLB accuracy.

To analyze the overall computational complexity of these methods, the running times of different methods are compared. In general, the higher the computational complexity, the longer the running time.

The average running times of different methods as sensor number varies are compared. The average running times, using MATLAB in a personal computer with core (TM) i5-4460, and the main frequency is 3.2 GHz are plotted in Fig. 7.
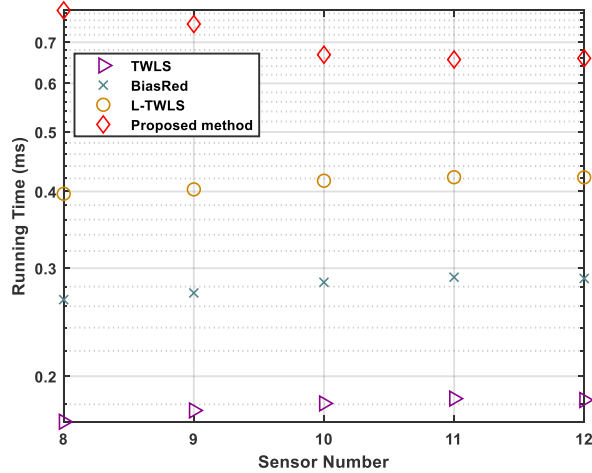

Fig. 7 Comparison of the average running times of different methods

It can be seen from Fig. 7 that the traditional TWLS method requires the least running time but its estimation accuracy is the worst. The proposed method and the traditional BiasRed method require the same order of magnitude of running time, that is, around 0.7ms and 0.3ms, respectively. Although the proposed method requires the most running time, its estimation accuracy is the highest. In the engineering aspect, if the algorithms are implemented on some chips with high configurations, which will meet the requirements of the computational efficiency, the proposed method is a suitable choice.

V. CONCLUSION

We have presented a novel bias-reduced method in the paper. The bias of the ML problem is theoretically analyzed in the paper. Traditional bias-reduced methods suffer from the high nonlinearity of the ML problem and the noisy measurement. The CDR methods are usually based on the assumption that the estimation parameters are irrelevant and a further step is carried out to reduce the nonlinear error. However, the error will rapidly increase if the measurement noise is large. The proposed method can handle the issue with the problem of the nonlinear nature and the bias simultaneously and it is more robust as it tolerates better noisy measurements. Several scenarios have been illustrated to verify the effectiveness of the proposed algorithm. Our algorithm outperforms traditional methods especially in the condition of large measurement noise. It can reduce the bias significantly and achieve the CRLB accuracy very well.

Although the study proves to be more effective than traditional methods when the measurement noise changes, there are still many challenges in its practical application. Our future work includes the robust localization method in the presence of TDOA measurement outliers and the successful application on the real scenarios, such as the underwater passive navigation[2].

APPENDIX

According to (7), the ML problem can be expressed as follows.

$$J(\boldsymbol{\theta}) = min(\boldsymbol{h_1} - \boldsymbol{G_1}\boldsymbol{\theta})^T \boldsymbol{W}(\boldsymbol{h_1} - \boldsymbol{G_1}\boldsymbol{\theta}) \tag{35}$$

The solution $\boldsymbol{\hat{u}_0}$ satisfies

$$P(\boldsymbol{\hat{u}_0}) = \left.\frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}}\right|_{\boldsymbol{u_0}=\boldsymbol{\hat{u}_0}} = 0 \tag{36}$$

Due to the nonlinear nature of the estimation problem, the Taylor series expansion of $P(\boldsymbol{\hat{u}_0})$ at $\boldsymbol{u_0}$ is

$$P(\boldsymbol{\hat{u}_0}) \triangleq \frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}} + \frac{\partial^2 J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}\partial \boldsymbol{u_0}^T}(\boldsymbol{\hat{u}_0} - \boldsymbol{u_0}) = 0 \tag{37}$$

Form (35), $\frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}}$ can be obtained as

$$\frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}} = -2\boldsymbol{F_1}^T \boldsymbol{W}(\boldsymbol{h_1} - \boldsymbol{G_1}\boldsymbol{\theta}) \tag{38}$$

where $\boldsymbol{F_1} = -2\begin{bmatrix} r_{21}\boldsymbol{\rho}^T + (\boldsymbol{s_2} - \boldsymbol{s_1})^T \\ \vdots \\ r_{M1}\boldsymbol{\rho}^T + (\boldsymbol{s_M} - \boldsymbol{s_1})^T \end{bmatrix}$, $\boldsymbol{\rho} = \frac{(\boldsymbol{u_o}-\boldsymbol{s_1})}{\|\boldsymbol{u_o}-\boldsymbol{s_1}\|}$.

According to (38), $\frac{\partial^2 J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}\partial \boldsymbol{u_0}^T}$ can be obtained as follows.

$$\boldsymbol{H} = \frac{\partial^2 J(\boldsymbol{\theta})}{\partial \boldsymbol{u_0}\partial \boldsymbol{u_0}^T} = \boldsymbol{A} - \boldsymbol{B} \tag{39}$$

where $\boldsymbol{A} = 2\boldsymbol{F_1}^T \boldsymbol{W} \boldsymbol{F_1}$.

$$\boldsymbol{B} = 2\left(\sum_{i=1}^{M-1}\sum_{j=1}^{M-1} \boldsymbol{X}_{ij} n_{j1}\boldsymbol{f}_i + \sum_{i=1}^{M-1}\sum_{j=1}^{M-1} \boldsymbol{W}_{ij}\boldsymbol{\varepsilon}_j\boldsymbol{f}_i\right)$$

$\boldsymbol{X} = \boldsymbol{W}\boldsymbol{B_1}$, $\boldsymbol{f}_i = -2r_{i1}\frac{\boldsymbol{I}_{N\times N}-\boldsymbol{\rho}\boldsymbol{\rho}^T}{\|\boldsymbol{u_o}-\boldsymbol{s_1}\|}$,

$\boldsymbol{\varepsilon}$ is a column vector formed by the diagonal elements of $\mathbf{Q}$.

$\boldsymbol{H}^{-1}$ can be approximated as follows[30].

$$\boldsymbol{H}^{-1} = (\boldsymbol{A} - \boldsymbol{B})^{-1} \approx \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}^{-1} \tag{40}$$

According to (6), the bias can be expressed as follows.

$$\Delta u_0 = \hat{u}_0 - u_0 \approx -H^{-1}\frac{\partial J(\theta)}{\partial u_0}$$
$$= 2A^{-1}F_1^T W(B_1 n + n \odot n) \quad (41)$$
$$+ 2A^{-1}BA^{-1}F_1^T W(B_1 n + n \odot n)$$
$$= \alpha + \beta$$

where $\alpha = 2A^{-1}F_1^T W(B_1 n + n \odot n)$,
$\beta = 2A^{-1}BA^{-1}F_1^T W(B_1 n + n \odot n)$.

Note that F_1 is correlated to the measurement noise n, which should be considered in evaluating the expectation. Thus, we can express

$$\begin{cases} F_1 = \Delta F_1^0 + F_1^0 \\ A = \Delta A + A^0 \end{cases} \quad (42)$$

where $\Delta A = \Delta F_1^{0^T} W F_1^0 + F_1^{0^T} W \Delta F_1^0$,

$$F_1^0 = -2\begin{bmatrix} r_{21}^0 \rho^T + (s_2 - s_1)^T \\ \vdots \\ r_{i1}^0 \rho^T + (s_i - s_1)^T \end{bmatrix}, r_{i1}^0 \ (i=2,\dots M) \text{ is the true}$$

TDOA measurement.

$$\Delta F_1^0 = -2\begin{bmatrix} n_{21}\rho^T \\ \vdots \\ n_{2i1}\rho^T \end{bmatrix} = -2n\rho^T.$$

First, consider the measurement noise and α can be expressed as follows.

$$\alpha = 2DB_1 n + 2Dn \odot n - 2A^{0^{-1}}\Delta A DB_1 n + \\ 2A^{0^{-1}}\Delta F_1^{0^T} W B_1 n \quad (43)$$

where $D = A^{0^{-1}}F_1^{0^T}W$.

Thus, the expectation of α is

$$E_n(\alpha) = 2DE_n[n \odot n] - 2A^{0^{-1}}E_n[\Delta A D B_1 n] \\ + 2A^{0^{-1}}E_n\left[\Delta F_1^{0^T} W B_1 n\right] \\ = 2DE_n[n \odot n] \quad (44) \\ - 2A^{0^{-1}}F_1^{0^T}WE_n[\Delta F_1^0 R n] \\ + 2A^{0^{-1}}E_n\left[\Delta F_1^{0^T} S n\right]$$

where $R = DB_1, S = WB_1 - WF_1^0 DB_1$.

Then $E_n(\alpha)$ can be expressed as follows.

$$E_n(\alpha) = 2D\varepsilon - 2A^{0^{-1}}F_1^{0^T}W\alpha_1 + 2A^{0^{-1}}\alpha_2 \quad (45)$$

where $\alpha_1 = -2\begin{bmatrix} \sum_{i=1}^{M-1}\rho^T R_i Q_{1,i} \\ \vdots \\ \sum_{i=1}^{M-1}\rho^T R_i Q_{N,i} \end{bmatrix}$,

$\alpha_2 = -2\rho Tr(SQ)$.

ε is a column vector formed by the diagonal elements of Q. R_i, q_i and S_i are the ith column of R, S, and Q.

Considering the measurement noise, β can be expressed as follows.

$$\beta = 2(\Delta A + A^0)^{-1}(B^0 + \Delta B)(\Delta A + A^0)^{-1}(\Delta F_1^0 + F_1^0)^T W(B_1 n + n \odot n) \quad (46)$$

where $B = B^0 + \Delta B$, ΔB is the noise term.

Ignoring the third-order and high-order noise term, β can be expressed as.

$$\beta = 2A^{0^{-1}}B_x^0 A^{0^{-1}}F_1^{0^T}WB_1 n \quad (47)$$

where $B_x^0 = 2\sum_{i=1}^{M-1}\sum_{j=1}^{M-1}X_{ij}n_{j1}f_i$.

Thus, the expectation of β can be expressed as follows.

$$E_n(\beta) = 4A^{0^{-1}}\sum_{i=1}^{M-1} C_i \quad (48)$$

where $C_i = \sum_{j=1}^{M-1}X_{ij}f_i A^{0^{-1}}F_1^{0^T}WB_1 Q(j)$,

$Q(j)$ is the ith column of Q. As the high-order noise term in (46) is neglected, (48) is not related to the noise in the matrix h_1 and matrix G_1.

From the above analysis, the bias can be expressed as follows.

$$E_n(\Delta u_0) = E_n(\alpha) + E_n(\beta) \quad (49)$$

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An efficient constrained weighted least squares method with bias reduction for TDOA-based localization

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