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INVESTIGATION INTO THE POSSIBILITY OF APPLYING LUBRICATION THEORY TO THE SELECTION OF OPTIMUM CLEARANCE FITS FOR PLAIN HYDRODYNAMICALLY LUBRICATED BEARINGS

by

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- by -

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List of symbols

h	=	Film thickness
θ	=	Angular co-ordinate measured from 'Attitude Line'
e	=	Distance between shaft centre and journal centre
c	=	Radial clearance
ϵ	=	Eccentricity ratio = $\frac{e}{c}$
ϕ	=	Attitude angle
W	=	Load per inch of bearing
U	=	Surface speed of rubbing
z	=	Co-ordinate direction in axis of shaft and journal
p	=	Pressure in lubricant film
η	=	Viscosity of lubricant
R_1	=	Radius of journal
R_2	=	Radius of shaft
R	=	$\frac{1}{2}(R_1 + R_2)$
D	=	Diameter of shaft/journal
L	=	Length of bearing
Δ	=	Sommerfeld Number = $\frac{W}{\eta U}\left(\frac{c}{R_1}\right)^2$
h_o	=	Minimum film thickness = $c(1 - \epsilon)$

1. Introduction

BS1916 limits and fits for engineering is based on ISA bulletin 25 which was first issued in 1921 and designed for use with the metric system. It provides a wide range of tolerances for holes and shafts from which it is possible by suitable selection to satisfy a wide range of engineering requirements.

The basic principle underlying the system is the allocation of letters to signify the magnitude of the displacement of that part of the tolerance zone which is nearest to the basic size and a number to indicate the magnitude of the tolerance. Capital letters are used for holes, small letters for shafts. The numbers referred to as fundamental tolerances are common to holes and shafts.

In applying the system to inch sizes care was taken to ensure that fit combinations (H6/k4) would provide the same type of fit in the inch system as was established in the metric system throughout the whole of the size range. It was also recommended that a unilateral hole basis be adopted in which the hole is the standard member and the tolerance specified as a value from nominal to plus an amount determined by the number signifying the magnitude of the tolerance. Different types of fit are obtained by selecting from the standard shafts available the one whose fundamental deviation from basic size is considered to be the most suitable and allocating to it a suitable standard tolerance.

When ISA bulletin 25 was issued great care was taken in selecting the disposition of the sizes allocated to standard shafts so that when one standard shaft was used with one standard hole it would provide a fit which would be functionally similar throughout the complete range of sizes. The decision of I.S.O. and B.S.I. to extend the size range from 20 inches to 200 inches caused reasonable doubts to exist about the reliability of this simple rule that one fit combination would be functionally similar throughout the whole of the extended size range. As this problem was examined in more detail cases were found where the above rule was not satisfactory over the extended or the smaller size range covered by the original standard (0.040″ to 20″).

In considering the possibility of applying theoretical analysis to this problem it was clear that the most difficult section was that concerned with the selection of clearance fits for hydrodynamically lubricated plain bearings. The present paper is a preliminary survey into this problem and the detailed analysis is restricted to the application of lubrication theory to the selection of what appear to be optimum clearance fits for hydrodynamically lubricated bearings and the means that may be used for comparing these results with the recommendations of I.S.A. bulletin 25 and BS1916.

Figure 1 is a simplified representation of a plain bearing operating under conditions of hydrodynamic lubrication.

When an ideal bearing is working under the conditions for which it has been designed, the shaft is supported by an oil film which ensures that no metallic contact takes place between the mating members. In a practical situation the problem is more complex than as shown in Figure 1, because errors of roundness, parallelism, alignment of axis of shaft with axis of hole, foreign matter in lubricant etc., create conditions which make the gap separating the shaft from the hole smaller at particular places in the bearing assembly than the mean gap.

To avoid metallic contact between the mating surfaces in this more complex arrangement the minimum oil film thickness must be large enough to cater for the geometric and other errors as enumerated above. To meet this condition it is necessary to ascertain the conditions which will provide the maximum value for minimum oil film thickness and from this, establish tolerances for mean size of shaft and hole and the tolerances for errors in geometric shape alignment, etc.

2. Bearing theories

The theory of hydrodynamic lubrication was first proposed by Osborne Reynolds in 1886, but the full solution of Reynold's equations for journal bearings was not completed until 1949 by Cameron and Wood (1). Following this paper an approximate theory which was very close to the exact theory was proposed by Ockvirk (2) which applied to journal bearings of short length.

The geometry of a journal in a bearing is as shown by Figure 2.

Since the lubricant film is thin compared with the journal radius, the film may be assumed to be unwrapped as shown in Figure 3, and the co-ordinates may be taken as x in the circumferential direction z in the direction of the journal axis, y in the radial direction from the centre of the bearing.

The Reynolds equation, relating the film thickness h, the pressure p, the velocity of sliding U and the fluid viscosity η for an incompressible fluid is given by:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\ \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\ \frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x}$$

Since $x = R\theta$, this equation becomes:

$$\frac{1}{R^2}\ \frac{\partial}{\partial \theta}\left(\frac{h^3}{\eta}\ \frac{\partial p}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\ \frac{\partial p}{\partial z}\right) = 6\ \frac{U}{R}\frac{\partial h}{\partial \theta} \qquad (1)$$

The film thickness, from the geometry of Figure 2 is given by:

$$h = c + e\cos\theta = R_1 - R_2 + e\cos\theta$$

$$= c(1 + \epsilon\cos\theta) \qquad (2)$$

where $\epsilon = \frac{e}{c}$ is defined as the eccenctricity ratio.

Substituting equation (2) into (1) gives:

$$\frac{\partial}{\partial\theta}\left[(1 + \epsilon\cos\theta)^3 \frac{\partial}{\partial\theta}\right] + R^2 \frac{\partial}{\partial z}\left[(1 + \epsilon\cos\theta)^3 \frac{\partial p}{\partial z}\right]$$

$$= \frac{6\eta UR}{c^2} \frac{\partial}{\partial\theta}(1 + \epsilon\cos\theta) \qquad (3)$$

This equation cannot be solved by simple methods, but Cameron and Wood (1) used the relaxation method on a computer and obtained an exact solution, as shown by Figure 4. The boundary conditions to be satisfied by equation (3) were stipulated to be:

$$p = 0 \quad \text{at} \quad \theta = 0$$

$$p = \frac{\partial p}{\partial\theta} = 0 \quad \text{at} \quad \theta = \pi + \alpha$$

These boundary conditions are known as Reynold's conditions. It is known that the pressure in the film falls to zero at some co-ordinate $\pi + \alpha$, and for angles greater than $\pi + \alpha$, the pressures remain at or very near zero.

If the boundary conditions that $p = \frac{\partial p}{\partial\theta} = 0$ at $\theta = \pi + \alpha$ were changed to $p = 0$ at $\theta = 0, 2\pi$, a dubious theory as given by Sommerfeld (3) would result. This would predict large negative pressures in the oil film whilst it is known that liquids cannot sustain such pressures, and cavitation takes place. The solution as given by Figure 4 shows the variation of eccentricity ratio ϵ with diameter/length ratio for the bearing for different values of the reciprocal of the Sommerfeld number $(\frac{1}{\Delta})$. It is noted that this theory is the only exact theory for journal bearings where the oil flow in both the axial direction and the circumferential directions have been taken into account. From this figure, it is possible to calculate minimum oil film thicknesses in a bearing at any given conditions of load, speed, dimensions etc.

3. Minimum film thickness calculations

An example was taken for a 2 in. diameter bearing 1 in. long, running on a 200 lb/in. load at 2000 R.P.M. The effect of diameter clearance on the minimum film thickness is shown in Figure 5. The calculations are summarised in the following Table No. 1.

Table 1

2" diameter bearing, 1 in. long, 2000 RPM (U = 210 in/sec.)
Viscosity of oil = 4.65×10^{-6} Reyns, Load = 200 lb/in.

Diametral clearance $2c\ 10^{-3}$ in.	1.0	2.0	3.0	6.0	12.0	17.0	24.0
Sommerfeld No. Δ	.0858	.343	.77	3.08	12.3	24.7	49.2
$\frac{1}{\Delta}$	11.65	2.92	1.3	.325	.0812	.0405	.0203
Eccentricity ratio ϵ	0.11	0.35	0.55	0.78	0.93	0.96	0.98
Minimum film thickness 10^{-4} in.	4.45	6.5	6.75	6.6	4.2	3.4	2.4

From Figure 5, it is seen that the minimum oil film thickness falls with clearance as the diametral clearance is greater or less than 0.003 in. In the case of increasing clearance, the eccentricity ratio of the bearing increases to 1 whilst in the case of decreasing clearance, the eccentricity ratio decreases to zero. Thus the minimum film thickness, as the diametral clearance tends to zero, tends to half the clearance. It is seen that although the film thickness does not fall away very rapidly as the diametral clearance increased, the optimum clearance is 0.003 in. to provide for the maximum lubricant film with the minimum of eccentric running of the shaft.

It should be noted that the theoretical assumptions do not include all the important parameters when the diametral clearances are very small. Heating due to friction should be considered. As the temperature of oil increases the viscosity will decrease until a stable operating temperature is established.

A second example for an 8 in. bearing is shown in Figure 6, where the same type of curve as in Figure 5 is obtained. The calculations for this bearing are summarised in Table No. 2.

Table 2

8″ diameter bearing, 4 inches long, 1200 RPM Viscosity of oil = 4.65×10^{-6} Reyns Load = 2000 lb/in						
Diametral clearance 2c 10^{-3} in.	2.0	4.0	6.0	12.0	24.0	48.0
Sommerfeld No. Δ	.0535	.214	.481	1.92	7.7	30.8
$\frac{1}{\Delta}$	18.7	4.67	2.08	0.52	0.13	.0324
Eccentricity ratio ϵ	.085	.24	.44	0.75	0.89	0.96
Minimum film thickness 10^{-4} in.	9.15	15.2	16.8	15	13.2	9.6

It is seen that an optimum diametral clearance at 0.008″ is obtained. Thus from the two cases considered as examples the optimum diametral clearances for the minimum eccentricity of shaft to hole with the maximum value for minimum oil film thickness is between 0.001″ to 0.0015″ per inch of bearing diameter. This is in agreement with the simple rule of 0.001″ per inch of bearing diameter used extensively in practice. Further diagrams showing the change of minimum oil thickness with change of clearance for various operating conditions are given in Figures 7, 8, 9 and 10.

4. Clearance fits for pressure lubricated bearings

The curves giving minimum oil film thickness to clearance ratio illustrated in Figures 7 to 10 show that film thickness is reduced as speed is reduced.

In cases where speed and loading are variable and combinations of low speed and high loads may persist for long periods, hydrodynamic lubrication is not satisfactory and must be replaced by a pressure fed system providing hydrostatic lubrication.

The ideal clearances for pressure fed lubrication systems is not the same for the whole of the size range as that required for hydrodynamic lubrication and further work which is outside the scope of the present paper is required to provide design data for this type of bearing.

5. Effect of clearance on stiffness of hydrodynamically lubricated bearings

The stiffness of the hydrodynamic bearing at any given clearance may be obtained by plotting the minimum film thickness against $\frac{W}{\eta U}$.

From Figure 9, we have the following results:

(a) Diametral clearance 10×10^{-3} in.

Minimum film thickness 10^{-3} in.	3.45	3.2	2.85	2.45
$\frac{W}{\eta U}$ 10^6	.513	.855	1.28	1.7

(b) Diametral clearance 20×10^{-3} in.

Minimum film thickness 10^{-3}	4.4	3.56	3.0	2.4
$\frac{W}{\eta U}$ 10^6	.513	.855	1.28	1.7

(c) Diametral clearance 30×10^{-3} in.

Minimum film thickness 10^{-3} in.	4.6	3.55	2.5	2.0
$\frac{W}{\eta U}$ 10^6	.513	.855	1.28	1.7

The above tables (a), (b) and (c) are plotted in Figure 11.

$$\text{Gradient of graph in Figure 11} = \frac{d\left(\frac{W}{\eta U}\right)}{dh}$$

$$\text{Stiffness of bearing} = \frac{dW}{dh}$$

Thus stiffness = $\eta U \times$ gradient of graph

For diametral clearance = 10×10^{-3}, $\frac{d\left(\frac{W}{\eta U}\right)}{dh} = 1.1 \times 10^9$

Assuming that $\eta = 4.65 \times 10^{-6}$ Reyns

$U = 10^3$ in/sec.

Stiffness = $1.1 \times 10^9 \times 4.65 \times 10^{-6} \times 10^3$ = 5.1×10^6 lb/in.

6. Application of clearance fits to bearings

From the above theoretical analysis it is seen that a characteristic curve of the type A.B.C. shown in Figure 12 is obtained for the relationship between clearance and minimum oil film thickness. For the ideal bearing it would seem desirable to maintain minimum oil film thickness at the largest possible value in order to ensure full fluid lubrication and no metallic contact between shaft and hole.

The diagram Figure 12 also shows:

1. Ideal clearance is that which gives the largest possible value for minimum oil film thickness.

2. At a predetermined value for minimum oil film thickness which is less than the maximum, it is possible to specify the extent to which actual clearance may depart from the ideal and by this means establish the fit tolerance for a specified set of conditions.

From the diagram it will also be seen that the ideal clearance is not in the centre of the tolerance zone but nearer the minimum clearance value. This indicates that in the production of shafts and holes for bearings the ideal size is nearer maximum metal than minimum metal conditions.

With the above method as a basis for establishing fit clearance the shaft and hole tolerances for a specified set of operating conditions for a wide range of sizes can be selected. The following is suggested as a recommended method of procedure.

1. Specify the operating conditions.

2. From the specified conditions calculate and finally draw a family of curves of the type shown in Figure 12 for a number of suitably distributed sizes in the size range to be covered.

3. From the above diagrams declare for each of the selected sizes.

   I. Ideal clearance (D. Figures 12 and 13)
   II. Minimum clearance (E. Figures 12 and 13)
   III. Maximum clearance (F. Figures 12 and 13)
   IV. Fit tolerance (G. Figures 12 and 13)
   V. Maximum value for minimum oil film thickness (H. Figure 12)
   VI. Minimum value for minimum oil film thickness (J and V. Figure 13)
   VII. Errors of geometric shape in hole (K. Figure 13)
   VIII. Errors of geometric shape in shaft (L. Figure 13)
   IX. Errors of alignment between shaft and hole (M. Figure 13)
   X. Maximum size of foreign matter in oil (M. Figure 13)
   XI. Maximum distortion due to loading (P. Figure 13)
   XII. Maximum distortion due to heating (R. Figure 13)
   XIII. Ideal mean size of hole (S. Figure 13)
   XIV. Tolerance for hole (T. Figure 13)
   XV. Ideal mean size of shaft (W. Figure 13)
   XVI. Tolerance for shaft (X. Figure 13).

When the above values have been established further curves can be drawn showing relationship between nominal size of bearing and the following features.

1. Tolerance and ideal mean size of hole in relation to basic size (T and S Figure 13).
2. Tolerance and ideal mean size of shaft in relation to basic size (X and W Figure 13).
3. Tolerance for errors of geometric shape in hole.
4. Tolerance for errors of geometric shape in shaft.

When calculating the tool errors from the individual errors of geometric shape etc., the individual errors should not be added algebraically because these vary independently and the maximum probable error due to these causes is given by the square root of the sum of the squares of the individual errors.

$$\text{Probable maximum error} = \sqrt{J^2 + K^2 + L^2 + M^2 + N^2 + P^2}$$

When the values for 1 and 2 above have been established and plotted it will be possible to superimpose the curves showing these values on charts showing the recommended values as given in I.S.O. and B.S.I. publications. It is known that some differences will be found. These can be investigated by field work designed to collect data from existing good practice and followed by some experimental work on any particular arrangement for which additional data appears to be necessary.

From a programme of the type described above it should be possible to issue a series of recommended tolerances for holes and shafts to suit a wide range of engineering requirements over a wide range of sizes. From the wide range of fundamental deviations and fundamental tolerances available in I.S.A. bulletin 25 and BS 1916 it should be possible to select standard holes and shafts to satisfy the fit conditions established by the above analysis.

7. Interference fits

The problems associated with calculating the amount of interference necessary for particular design requirements is much more simple than that required for clearance fits because a sound theoretical basis for determining the optimum amount of interference for different types of fit and different sizes is well established. In spite of this, additional work is still required to provide the designer with easily assimilated information on the selection of interferance fits for different requirements and for comparing the theoretically calculated values with good current practice.

Further work as suggested above is under consideration as phase two of the present programme.

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2. Ockvirk, F.W. — 1952 Tech. Notes NACA Washington No. 2808 'Short bearing approximation for full journal bearings'

3. Sommerfeld, A. — The Hydrodynamic Theory of lubrication friction. Zeit. Math. and Phys. Vol. 50, No. 1 and 2, 1904, p. 97-155.

4. Professor J. Loxham and J. Hemp. — 'The application of hydrostatic bearings to high precision machine tools'.

5. B.S. 1916
Part I Limits and fits for engineering
Part II Guide to the selection of fits.

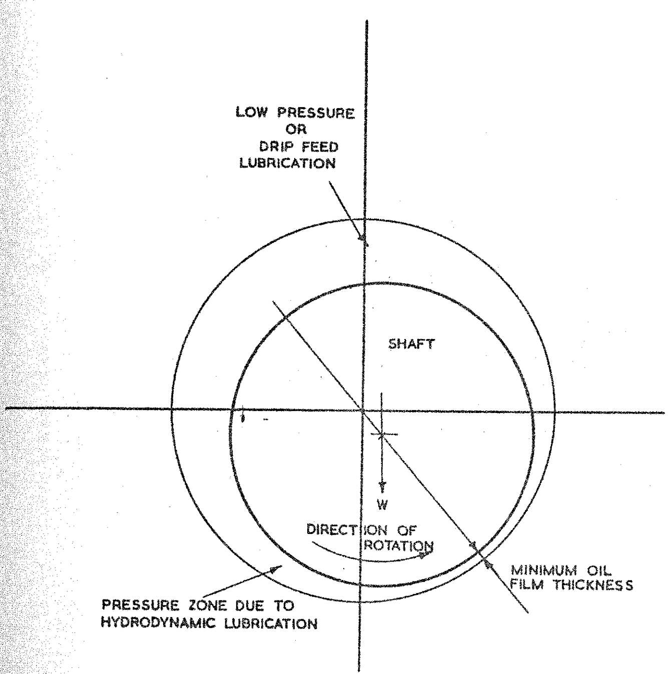

FIG. 1. SIMPLE DIAGRAMMATIC REPRESENTATION OF HYDRODYNAMIC LUBRICATION.

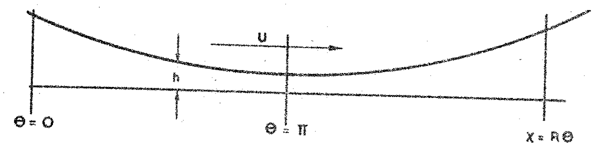

FIG. 3. OIL FILM UNWRAPPED TO MAKE CIRCUMFERENCE OF HOLE STRAIGHT.

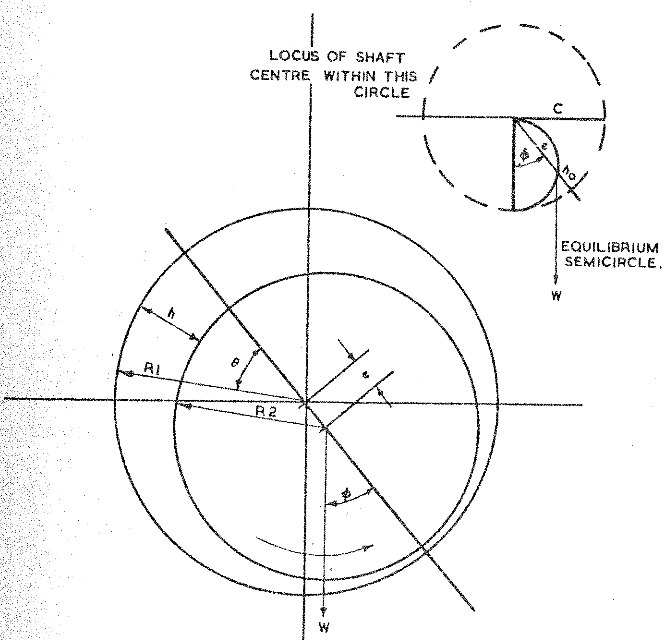

FIG.2. GEOMETRY OF HYDRODYMAMICALLY LUBRICATED BEARING.

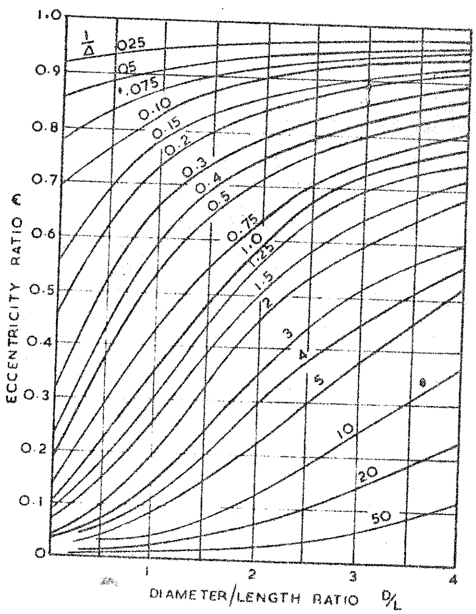

FIG. 4

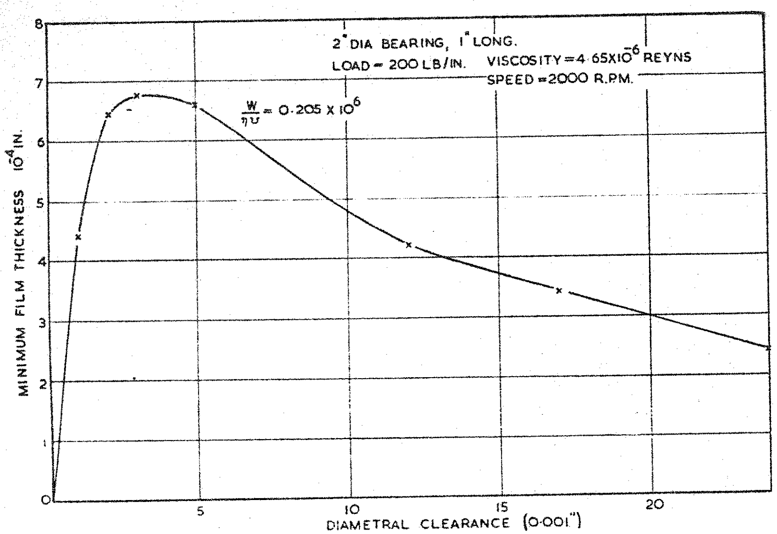

FIG. 5 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE

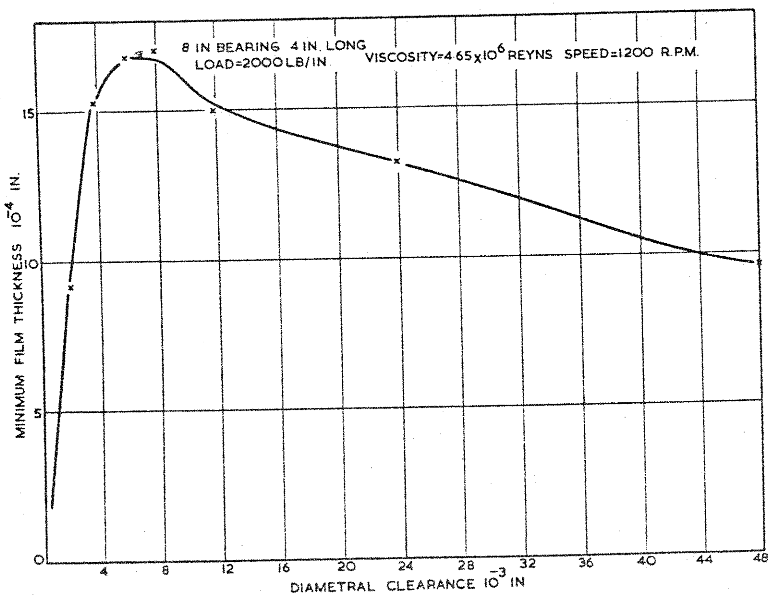

FIG 6 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE.

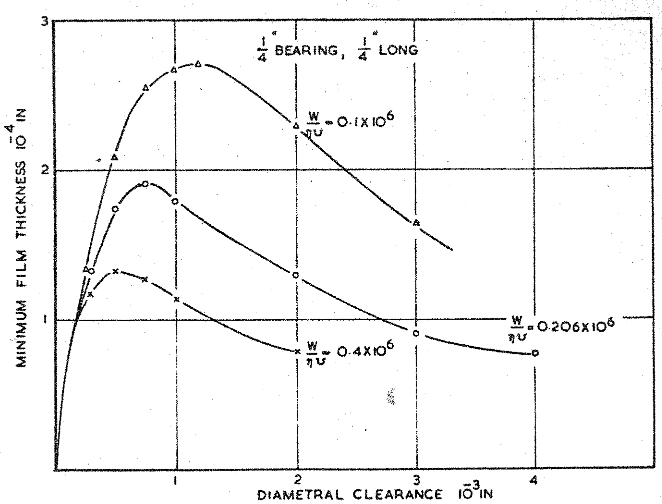

FIG. 7 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE

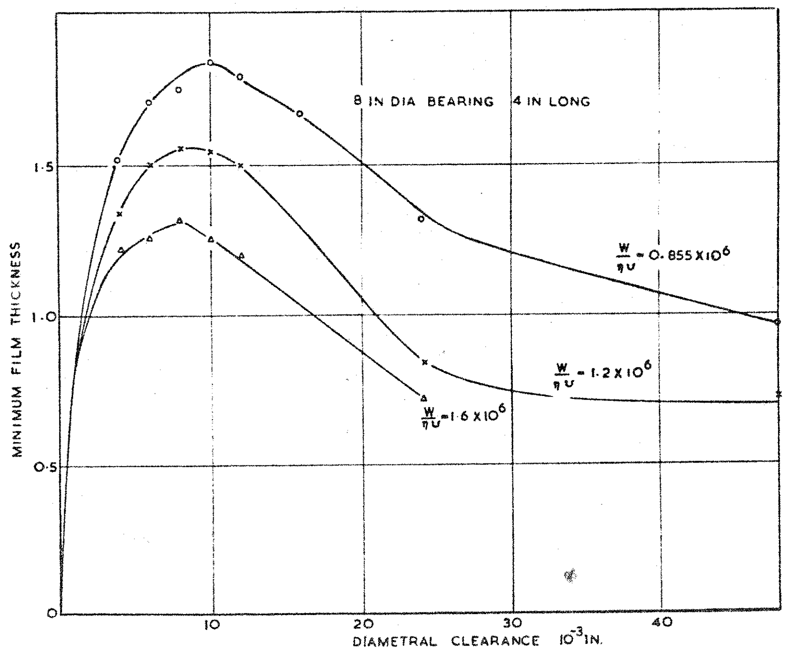

FIG. 8 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE

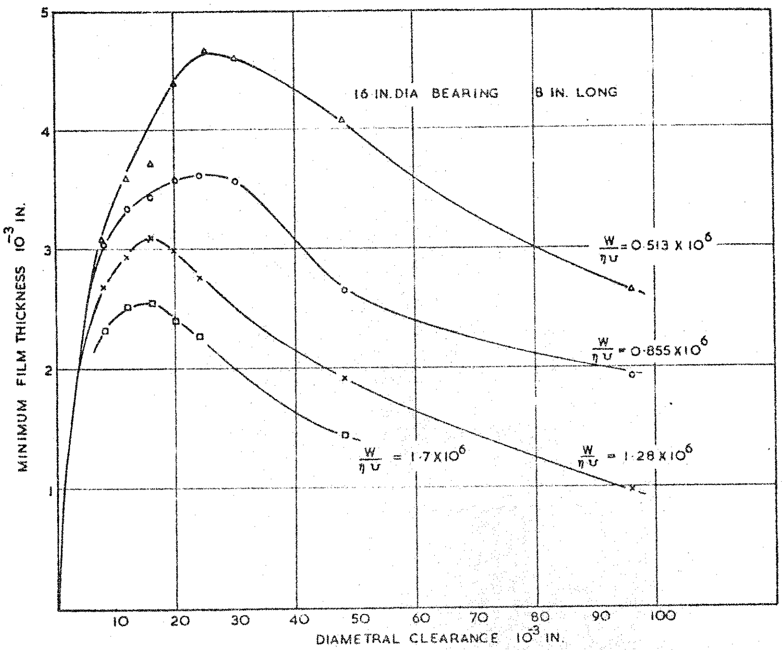

FIG. 9 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE

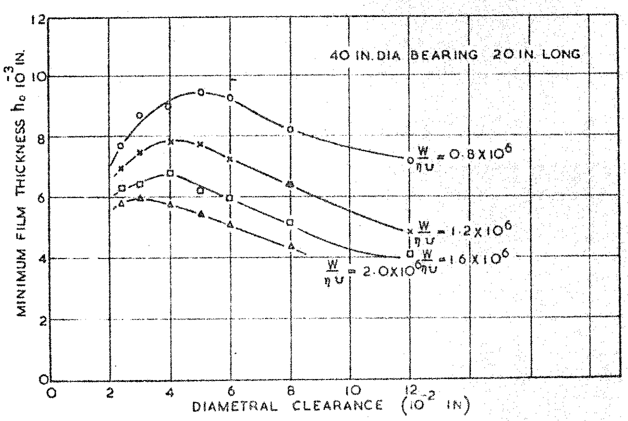

FIG. 10 VARIATION OF MINIMUM FILM THICKNESS WITH DIAMETRAL CLEARANCE

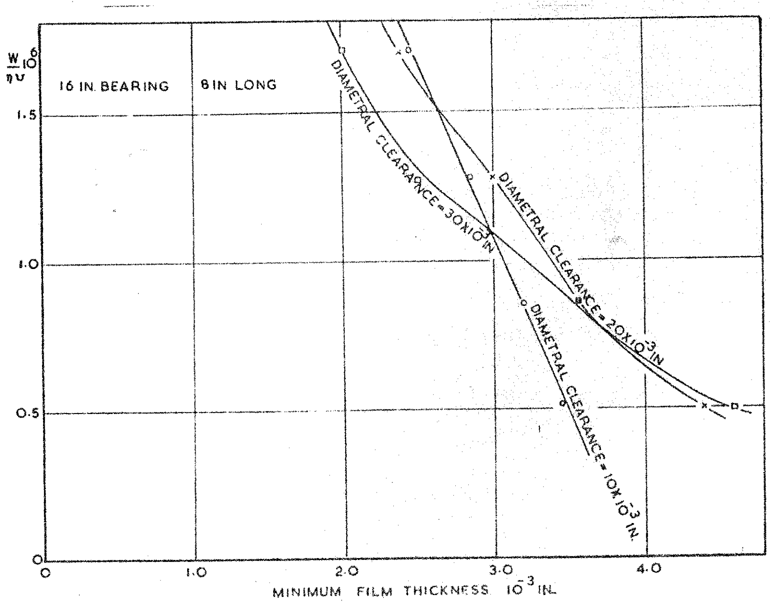

FIG. 11 VARIATION OF $\frac{W}{\eta U}$ WITH MINIMUM FILM THICKNESS

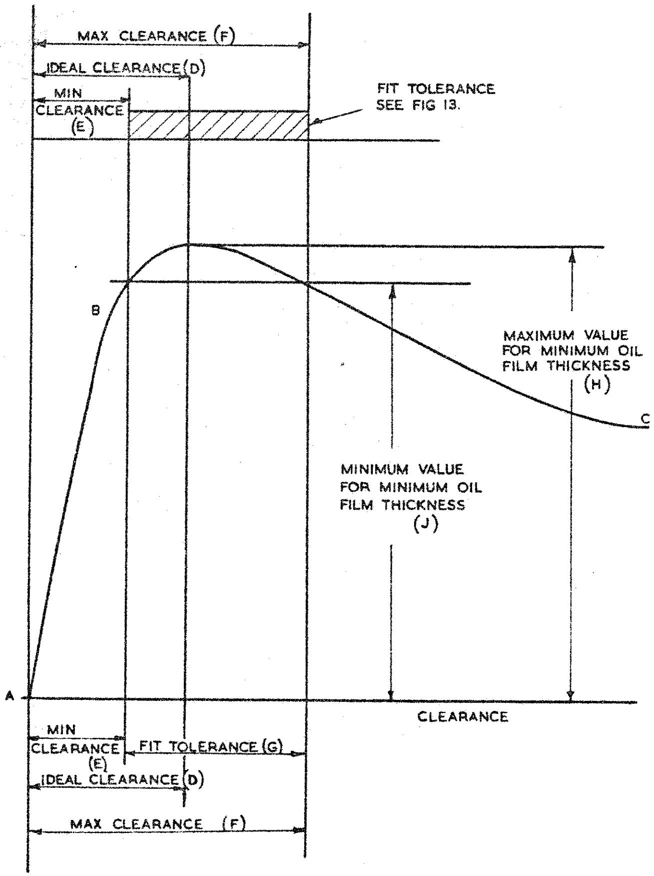

FIG. 12. RELATIONSHIP BETWEEN CLEARANCE AND MINIMUM OIL FILM THICKNESS.

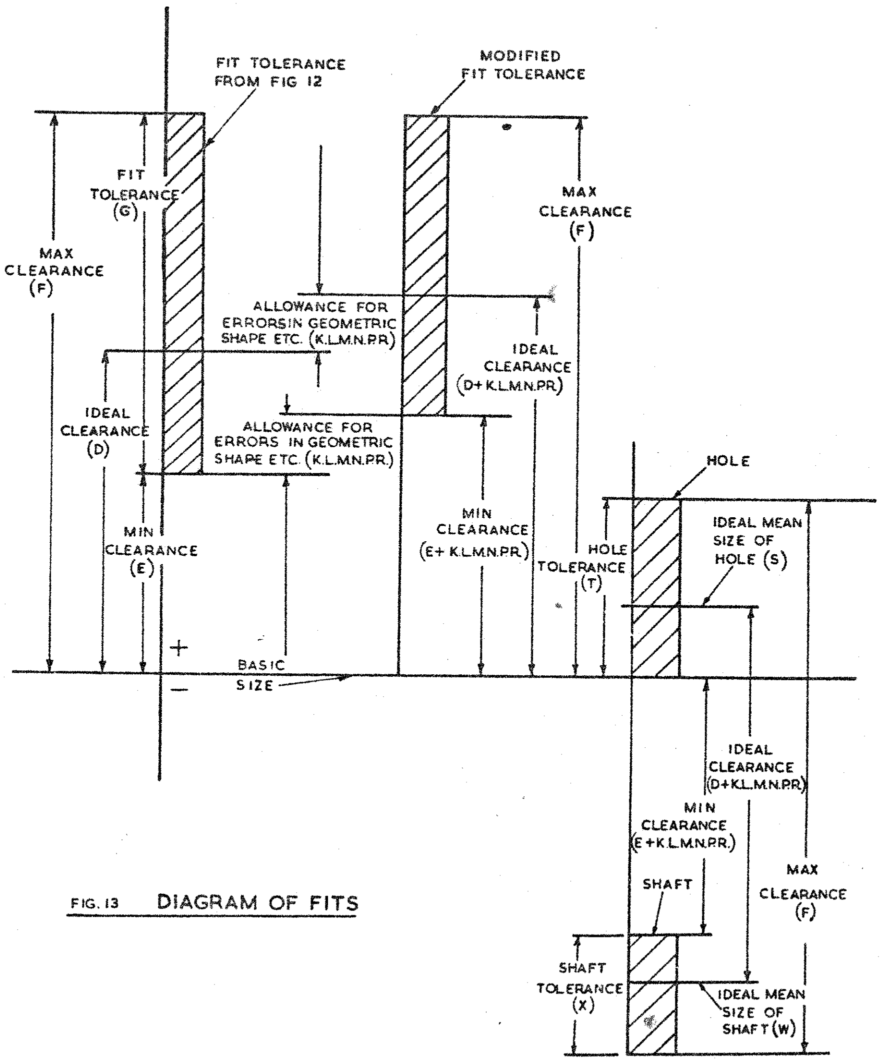

FIG. 13 DIAGRAM OF FITS