THE COLLEGE OF AERONAUTICS

CRANFIELD

MINIMUM PROPULSION FOR SOFT MOON LANDING OF INSTRUMENTS

by

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LIST OF SYMBOLS

A	area	ft^2 (in^2)
a	acceleration	ft/sec^2
c	effective exhaust velocity $\frac{F}{\dot{m}_p}$	ft/sec
C_F	nozzle thrust coefficient $\frac{v_e}{c^x}$	
c^x	propellant characteristic velocity $\frac{P_c A_t}{\dot{m}_p}$	ft/sec
d	diameter	ft (in)
F	thrust	pdl (Lb)
g_o	earth standard gravity 32.174	ft/sec^2
g_L	local gravity	ft/sec^2
K	scaling constants (Defined in Appendices 1 and 6)	
L^x	combustion chamber characteristic length	ft (in)
L	length	ft (in)
M	mass	lb
$\dot{m}$	time rate of change of mass	lb/sec
P	pressure	pdl/ft^2 (Lb/in^2)
r	propellant oxidant/fuel mixture ratio	
R_p	solid propellant utilisation ratio $\frac{\text{mass of propellant burnt}}{\text{mass of propellant carried}}$	
S	material stress	pdl/ft^2 (Lb/in^2)
t_m	thickness of material	ft (in)
t_b	engine burning or firing time	sec.
V	volume	ft^3 (in^3)
v	velocity	ft/sec
ρ	density	lb/ft^3 (lb/in^3)
η	efficiency factor	

Continued... ...

List of Symbols (Continued)

ϵ	area ratio	A_e/A_t
ϵ_p	solid propellant loading fraction	$\frac{\text{propellant volume}}{\text{case volume}}$
α	nozzle divergence half angle	

Suffixes

o	refers to state before firing
1	refers to state after firing
c	combustion chamber. t - throat. e - exit.
p	propellant ox - oxidant m - material

The equations formed in the analysis, and the values of the scaling constants are all based on the poundal, pound mass, second unitary system. In the text an engineering system of units is used based on the standard pound force (Lb) and pound mass (lb)

$$1 \text{ Lb.} = 32.174 \text{ pdl.}$$

1. Introduction

This paper examines some of the problems that are posed in landing an instrumented package on the moon. The basic requirement is to put down this payload, in one piece, using a minimum of initial vehicle mass in the process. Very approximately, a thousand pounds of earth launching vehicle mass are meeded for each pound required by the moon landing vehicle. Because of its importance in this application a considerable part of the paper is devoted to outlining an optimisation procedure that will ensure that thrust levels, and other propulsion parameters do in fact result in a minimum vehicle.

As there are no other papers specifically on the problem of soft moon landings, a certain amount of space has been devoted to earth/moon trajectories, and also various approach and landing techniques. Nevertheless, the main intention has been to look at this problem from the propulsion aspect. To this end much has had to be omitted, and many problems simplified or ignored. In particular no examination has been made of either the earth launching vehicle, or the earth-moon carrier. Very little thought has been given to the problem of guidance and control. The author has not philosophised upon the possible contents of the instrumented package.

The optimisation procedure presented has been formed to permit the assessment of both solid and liquid propellant units in order to determine their efficiency in meeting the specification. Six liquid propellant combinations have been considered. Scaling rules have been derived in order to permit assessment of the relative merits of pressurisation and turbo pumps, the selection of optimum tank, and combustion chamber pressures, expansion ratios, etc. In some instances engineering detail has been considered to clarify particular points, but this paper is not intended as a design study. At a number of points, where insufficient data has been found, or where a problem has been intransigent, "bulldozer" methods have been applied.

2. Trajectories

The minimum energy manoeuvre converting a circular orbit at 500 miles above the earth into an ellipse with an apogee just at moon radius is extremely susceptible to errors of velocity and direction. Following the work carried out at Douglas (Ref. 1) a trajectory has been selected using about 1% excess energy which is much less touchy on errors. Figure 1 gives the basic details of such a trajectory.

Such a trajectory and velocity will result in the landing vehicle possessing an energy height very close to infinity with respect to the moon.* It is now proposed to examine some possible moon approach and landing techniques. Broadly speaking, the moon may be approached in two manners - hit or miss.

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* Energy height is the sum of potential and kinetic energies expressed in terms of height units. Energy height equals real height at any instant when a body is at rest with respect to the gravitational centre.

2.1. Collision Course

With such a course, without propulsion, the landing vehicle would impact with very nearly escape velocity. Propulsion is therefore required in a vertical descent bringing the velocity to zero at the instant of contact.

2.2. Hyperbolic - elliptic course (Fig. 2)

The landing vehicle approaches the moon on a predetermined hyperbola. At closest approach a thrust impulse parallel to the moon's surface can convert the hyperbola into a grazing ellipse. Through a number of circuits fine thrust control could bring the grazing approach down to a mile or even less. Final landing could then be achieved with reverse thrust reducing the elliptic velocity down to circular and on down to zero (Fig. 3). A second rocket directed downwards takes most of the weight once the radial velocity has been reduced below circular.

2.3. Trajectory analysis

From the purely propulsion point of view, the hyperbolic-elliptic course demands a somewhat smaller total velocity increment. At the same time it requires a number of periods of thrust. Either thrust level or burning time would have to be flexible to allow for inaccuracies during manoeuvres. This approach would require accurate attitude control with respect to spherical coordinates based on the centre of the moon (because of possible three dimensional errors). Up until the moment when thrust is applied the vehicle's energy height is constant. Its value can be predetermined, and will be true regardless of all trajectory errors. Therefore, information on the height at closest approach defines the hyperbola and the negative velocity increment required to convert hyperbola to surface grazing ellipse. Further height data would be adequate for corrections and landing.

The collision course approach requires accurate delivery by the carrier vehicle. The landing vehicle requires accurate location of two dimensions normal to a moon radius and a coarse control on the other axis to prevent spinning. (A very crude self-energising sun seeker would be suitable). If the thrust level and burning time are both prefixed, extremely accurate determination of firing height is needed. Decreased demand on accurate ignition height can be achieved by use of a very small supplemental rocket fired low down when height estimation errors are of small real magnitude.

Difference in height at two preset instants towards the end of firing, would define trajectory and the firing height for the supplemental rocket. (See Fig. 4).

3. Specification and analysis

The collision course approach is selected. This is because it requires less rigorous attitude control and is less demanding on propulsion. In fact it requires less operating parts.

Table 1 lists the pertinent moon data required from this work.

3.1. Specification

For a payload of 100 lb. it is required to design a landing vehicle of minimum mass. No "in flight" acceleration limit is imposed on the payload. The landing vehicle guidance has an acceleration limit of 400 ft./sec^2. A single stage main propulsion system is required. No thrust control of magnitude or direction is required from the main propulsion system. No arrangements are required for thrust out-off, other than the normal end of burning.

3.2. Specification Analysis

With the intention of ensuring that the vehicle will, if anything, have something in hand, final performance estimation is based on meeting the full moon escape velocity of 7693 ft./sec. assuming that the surface gravitational acceleration of 5.91 ft./sec^2 is constant at all heights.

3.3. Propulsion Aspects

It is required to examine in detail the main propulsion system components, masses and performance, in order to determine the method which can meet the specification in terms of velocity increment and minimum vehicle mass. The study is to include the use of both presently available, and higher performance propellants.

4. External Ballistics

In order to ascertain the approximate mass of the vehicle, and its sensitivity both to exhaust velocity and component masses, the concept of an ideal vehicle is introduced. (See Appendix 1).

This vehicle has a constant ejection rate of propellant in terms of mass flow rate and exhaust velocity resulting in a constant thrust throughout the firing period. For such a vehicle in vertical descent (without atmosphere) the velocity relationship :-

$$\Delta v = c \log_e \frac{M_o}{M_1} - \overline{g_L} \quad t_b \qquad \text{ft./sec.} \qquad \ldots\ldots 1.5$$

can be used. In order to relate the burning time (t_b) with the initial to final mass ratio $\frac{M_o}{M_1}$, and to permit assessment of component mass effects, the ideal vehicle definition is extended. The all burnt mass of the vehicle (M_c) is taken as the sum of the payload, a mass scaling in direct proportion to propellant mass flow rate, and a mass scaling in direct proportion to the total mass of propellant carried.

$$M_1 = M_L + K_1 \dot{m}_p + K_2 \dot{m}_p t_b \qquad \ldots\ldots 1.1$$

$$M_o = M_1 + \dot{m}_p t_b \qquad \ldots\ldots 1.2$$

The K_2 value will relate to tank mass in the liquid propellant engine or to the engine case mass in a solid. K_1 relates to pipelines, valves, and combustion chamber for liquids.

In terms of the payload to initial mass ratio $\frac{M_L}{M_o}$ these equations may be brought together :-

$$\frac{M_L}{M_o} = 1 - \left(1 - \frac{1}{M_o/M_1}\right)\left(\frac{K_1}{t_b} + K_2 + 1\right) \qquad \ldots\ldots 1.6$$

where $\frac{M_o}{M_1}$ and t_b are related as in equation 1.5.

It is now possible to determine the payload to initial mass ratio for any values of the effective exhaust velocity (c), and the scaling constants K_1 and K_2. A number of these 'ideal vehicle' curves are presented. In Figure 5 $\frac{M_o}{M_L}$ has been plotted against final vehicle acceleration for the following values :-

$c = 10,000$ and $8,000$ ft./sec.

$K_1 = 0$

$K_2 = 0.08$ and 0.16

It is assumed that this is representative of a solid propellant landing vehicle both with respect to exhaust velocity and K values. The curves all show the same trend - that minimum initial mass is attained with very high final acceleration conditions obtained with comparatively high thrust and short burning time. This is due to $K_1 = 0$, there being no mass on the vehicle proportional to flow rate, hence the velocity loss due to gravity ($g_L t_b$) can be minimised. The other point to be noted is the susceptibility of the M_c/M_L ratio to change of exhaust velocity and its comparative insensibility to change of K_1 :-

1% change in c — 1.25% change in M_o/M_L

1% change in K_1 — 0.3% change in M_o/M_L

Figure 6 represents the equivalent liquid propellant curves for the following :-

c = 10,000 and 8,000 ft./sec.

K_1 = 5 and 10

K_2 = 0.02 and 0.04

An immediate difference is apparent. M_L/M_o is no longer minimum at very high thrust level and final acceleration. This is due to K_1, there being a mass on the vehicle proportional to propellant mass flow rate and thrust. Hence a minimum value of M_o/M_L is achieved at a point intermediate to a low thrust, high g loss and a high thrust, and a large mass proportional to flow rate. It is worthy of note that at the higher values of K_1 an optimisation procedure is most important, a minimum M_o/M_L is most sensitive to vehicle final acceleration and thrust level. Once again the large sensitivity to exhaust velocity should be observed plus the comparative insensitivity to changes in either K_1 or K_2 :-

1% change in c — 1.25% change in M_o/M_L

1% change in K_1 — 0.105% change in M_o/M_L

1% change in K_2 — 0.06% change in M_o/M_L

Using these graphs permits the approximate thrust sizes at which scaling rules and real constants can be determined. Values selected are:-

Solid F = 1429 lb.
= 46,000 pdl.

Liquid F = 267 lb.
= 86,000 pdl.

5. Ideal Rocket Engine Performance

The definition of the ideal vehicle is extended to include the fact that it is powered by an ideal rocket engine. Such an engine is defined as follows :- it operates with perfect gases which are in equilibrium in the combustion chamber, and whose composition does not alter through the expansion. The cycle is isentropic, and the gases leave the nozzle without divergence loss. There is no external pressure. (See Appendix 2).

The thrust of such a rocket operating in space is given by :-

$$F = \dot{m}_p \, v_e + P_e A_e \quad \text{(pdl.)} \qquad \ldots\ldots \; 2.5$$

It is now required to relate equation 2.5 to the propellant performance parameter - the characteristic velocity (c^x), the engine geometry parameters, throat area (A_t) and exit/throat area ratio.

The relevant parameters based on the ideal rocket definition are :-

$$c^x = \frac{P_c A_t}{\dot{m}_p} \quad \text{ft./sec.} \qquad \ldots\ldots \; 2.1$$

$$C_F = \frac{v_e}{c^x} \qquad \ldots\ldots \; 2.2$$

$$\frac{A_e}{A_t} = \epsilon = \left(\frac{P_c}{P_e}\right)^{\frac{1}{\gamma}} \frac{\gamma}{C_F} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \qquad \ldots\ldots \; 2.4$$

These combine and simplify to give the effective exhaust velocity :-

$$c = \frac{F}{\dot{m}_p} = c^x \left(C_F + \frac{P_e}{P_c} \epsilon\right) \quad \text{ft./sec.} \qquad \ldots\ldots \; 2.8$$

from which may be derived directly the thrust and propellant mass flow rate relationships.

Also may be derived the expression for throat area :-

$$A_t = \frac{F}{P_c \, C_F + P_e \, \epsilon} \quad \text{ft}^2 \qquad \ldots\ldots \; 2.9$$

Once a propellant has been selected these parameters may be used to determine nozzle geometry and performance for the required value of thrust.

6. Propellant Selection

6.1. Liquid propellants

It is required to examine the landing vehicle performance for a number of propellants including both presently available and possible future combinations, The fundamental need is that the combination should be storable from the moment of loading on earth, through the launching phase, and then from 2 to 5 days thereafter Nevertheless, it is felt that this should not override consideration of any attractive propellants as storage means may be found. In consequence six propellant combinations have been considered :-

99.6% hydrogen peroxide with kerosine and also hydrazine.

Liquid oxygen with kerosine, unsymmetrical dimethyl hydrazine and liquid hydrogen.

Liquid fluorine with liquid hydrogen.

Hydrogen peroxide/hydrazine has been chosen as an excellent example of a normally storable propellant combination. 99.6% peroxide has a somewhat better performance with hydrazine than the other storable oxidants - dinitrogen tetra oxide and nitric acid - and is thought to be available and in use. Peroxide/hydrazine are a self-igniting pair.

The other selections are a fairly obvious choice from the field of interest. Fluorine/hydrogen are the only other combination which are self-igniting.

Table 2 lists the relevant data for these combinations.

6.2. Solid Propellants

Insufficient unclassified data are available to extend this study into the real field of solid propellants. In consequence it has been found necessary to define a non-existent propellant having a performance thought to be within reach of the best propellants existing today. This is the only solid propellant studied in this paper. Table 3 lists the relevant data.

No suggestion is made as to its possible composition.

7. Liquid Propellant System Masses and Scaling Laws

Scaling laws have been derived for the major components of the complete liquid propellant system. These are needed so that a better appreciation can be made of a number of problems which arise. The laws then permit a more accurate determination of the vehicle constants.

7.1. Combustion chamber and nozzle (See Appendix 3)

Some consideration of a number of different combustion chamber shapes was made. Mainly, from the viewpoint of simplifying the analysis a spherical shape was decided upon. A very interesting alternative, the diverging reactor (Ref. 2) was seriously considered (Fig. 7). Unfortunately the scaling rules for such a system did not appear reliable. In consequence it is not considered further.

A volume scaling law is required for the spherical chambers. The combustion chamber volume/throat area ratio L^x (the combustion chamber characteristic length) is used in this paper. This parameter has not been selected because of its reliability, but in the absence of any alternative. An L^x of 5 ft. (60 inches) is used throughout this paper.

The basic design assumed is that the chamber is cooled by the usual double wall arrangement. It is assumed that the outer wall carries the entire pressure load. The inner wall takes the bursting load due to excess coolant pressure. On the inside wall of the chamber is an unstressed, flame sprayed heat barrier of zirconia. All chambers are made of stainless steel, and a safe working stress of 75% (150,000 Lb/in^2) of normal has been used because of possible outside wall heating. In addition, a safety factor of 1.5 has been used. This uncertainty factor has been used to make allowance for the nozzle cut out.

7.2. Nozzle (See Appendix 3)

The ideal rocket definition includes the assumption of no thrust loss due to divergence. It is therefore assumed that a "tulip" nozzle is used. As the profile of such a nozzle is complex, no attempt has been made to estimate its scaling laws. Instead a "similar nozzle" has been defined whose mass, throat, and exit areas are identical to the tulip type. The "similar nozzle" is straight sided with an expansion half angle of 15^o. The nozzle is continuous with the combustion chamber, has the same materials, total thickness and safety factors.

7.3. Valves and Pipelines (See Appendix 4)

Precise scaling laws would require a number of separate system detail designs. In order to circumvent this it has been vaguely assumed that all systems are similar and that the mass of valves and pipelines is proportional to the propellant volume flow rate.

7.4. Turbo pump and gas generator (See Appendix 4)

Again, it has been necessary to generalise. It is assumed that the masses of gas generator, turbine, case and shaft are proportional to the total propellant pumping work rate. Pump masses are assumed proportional to propellant volume flow rate.

7.5. Tanks (See Appendix 4)

A simple scaling law has been derived for tanks stressed as thin spheres. Where the same material is used for both tanks, and no problems arise from 'storability' aspects, this is directly applicable.

Consideration has been given to the special problem of storing the liquids - oxygen, fluorine, and hydrogen. Attention has been given to the problem of tank volume requirements at the end of the voyage when the propellant temperature will be different from the loading temperature.

This problem is extremely serious with the three propellants mentioned due to their low critical pressures and temperatures. Very large changes of vapour pressure and density with temperature occur at certain points. The main question to be answered is to what extent heat barriers should be used to keep propellant temperature down and hence keep down propellant density and required tank volume. The importance of this can be underlined by mentioning that for a temperature rise from 90°K to 153°K the volume requirement for liquid oxygen doubles !

Examination of this problem has not yet been taken further. In this paper, scaling rules have been developed for the three propellants, based on a crude assumption: for liquid oxygen, and fluorine, the tank mass is double the mass required for storage when the vapour pressure is equal to one atmosphere. This is perfectly reasonable for the pressurised tank requirement since vapour pressure can be used for motivation. For hydrogen, because of its very low critical temperature and pressure, a value of 4 has been taken. It is roughly assumed that this value accounts for both extra tank volume required due to temperature rise of the liquid, plus a tank insulation. In order that comparison can be made, scaling rules have also been derived for the three propellants, assuming storage at one atmosphere vapour pressure without excess volume or heat barrier.

8. Solid Propellant Engine Masses and Scaling Laws (See Appendix 5)

Scaling laws have been derived for both engine case and nozzle.

8.1. Engine case

The engine case is a cylinder, and in the scaling, allowance has been made for both cigarette burning and case bonded internal burning arrangements. Both have been stressed as thin cylinders.

With the case bonded arrangement it is assumed that very little case heating occurs, and a value of $S = 75\% = 150{,}000$ Lb./in^2 has been used. The propellant utilisation ratio $(R) = 0.9$, the loading fraction 0.8 and the safety factor at 2.

In comparison the wall heating effects in the cigarette burning arrangement are severe. No precise data has been derived for the zirconia heat barrier, but it should be appreciated that the heat ejection rate of the outer case is due entirely to radiation in this particular application. A value for S of 54% = 108,000 Lb./in^2 has been taken. Coupled with $\eta = 2$, this results in a case mass for the cigarette burning application twice that of the case bonded one.

8.2. Nozzle

An alternative model has been used to derive the scaling relation ship. As the nozzle is uncooled, allowance has been made for a zirconia heat barrier. Material strength is assumed to be down to 54% of maximum working stress. This is the same figure that was used in a similar situation for the cigarette burning arrangement of case.

9. Analysis - Scaling Constants and Complete Vehicle

The analysis of the scaling rules of the various engine components has revealed that in both liquid and solid propellant systems, nozzle mass is a function of the propellant mass flow rate to the $\frac{4}{3}$ rd power. This means that the simplified analysis of Appendix 1 has to be extended to include this factor. It is important to note that in space operation the nozzle mass is several orders of magnitude greater than the combustion chamber alone. Considerable errors can, therefore, arise from the assumption that the complete thrust chamber scales in proportion to propellant mass flow rate.

The complete vehicle analysis is treated in Appendix 6. A simple graphical method of determining vehicle characteristics for a given propellant combination is described.

10. Discussion

10,1. Scaling constants

The scaling laws discussed in sections 7 and 8 have been used to derive the scaling constants K_1, K_2, and K_3 for the solid propellants and all the liquid propellant combinations for both pressurised and turbo/pumped systems. These are all listed in Table 7. These tables are enlightening in that they give immediate information on component sizes. The K_1 values detail component mass per unit mass flow rate, the K_2 values the tank (or solid propellant case) mass per unit mass of propellant carried. The K_3 values give the nozzle mass for unit mass flow rate (but scales away from this proportional to mp $\frac{4}{3}$).

Whilst it is interesting to note these values, particularly the effects of using low density propellants, they cannot be used for direct comparison in that they do not take into account either propellant performance, or the optimisation procedure.

10.2. Results obtained - Vehicle mass and performance

At the moment of completion of this note, it has been found possible to complete the analysis on six sets of 'vehicle system/propellant combination' scaling constants. These are for the propellant combinations of hydrogen peroxide and hydrazine, and also liquid oxygen and kerosine. Each is considered for both pressurised, and turbo pumped systems. In addition, with the liquid oxygen combination, both systems have been calculated for the two storage methods used for cryogenic liquids in this paper One is based on storage at a maintained temperature of 90°K throughout the journey, without any mass allowance for preventing heating, the second, and more realistic case, assumes that the oxygen temperature rises through the journey to 153°K and has in consequence a density one half of that taken in the previous case.

The results are plotted on an initial to payload mass ratio $\frac{M_o}{M_L}$ against vehicle final acceleration (a_1) in Fig. 8. The most remarkable point about the results so far obtained is the narrow range of minimum $\frac{M_o}{M_L}$ covered by all the groups, there being less than 0.1 $\frac{M_o}{M_L}$ between them. The actual minimum values are as follows :-

Oxygen and kerosine. Pressurised (at 153°K) $\frac{M_o}{M_L}$	=	2.396
Oxygen and kerosine. Pressurised (at 90°K)	=	2.316
Hydrogen peroxide and hydrazine. Pressurised	=	2.320
Oxygen and kerosine. Pumped (at 153°K)	=	2.310
Oxygen and kerosine. Pumped (at 90°K)	=	2.305
Hydrogen peroxide and hydrazine. Pumped	=	2.302

Consider first the pressurised systems, and accept the scaling procedure as correct. The storable propellant combination shows the expected advantage over the other when allowance is made for extra oxygen tankage. When no allowance is made for the extra tank volume (again, is a most unlikely possibility) the oxygen combination shows an extremely small advantage. With the turbo pumped systems, tank mass is a very small part of the complete system mass. Because of this the differences between the three systems are insignificant.

In conclusion it is felt that considerable caution should be exercised in using these results to make comparison between pressurised and turbo pumped systems. The small apparent improvement obtained using turbo pumps is entirely dependent on the precise values given to the scaling constants, and could be upset by very small changes in their magnitude.

The solid propellant analysis has not been proceeded with at the present moment, as a spot check revealed that it was considerably outside the range of interest. The value derived is :-

$$\text{Solid propellant (Case bonded)} \qquad \frac{M_o}{M_L} = 2.70$$

10.3. Deficiencies in the present results

Each of the 22 sets of constants given in Table 7 represents a different 'vehicle system/propellant combination' group. Each group may be analysed in order to determine the engine operating values of thrust and burning time that together meet the specification, and result in a minimum vehicle mass for a given payload. Such an approach will only result in an absolute minimum mass vehicle for a given group when all the possible changes in operating parameters have been considered. The more important of these operating parameters are now discussed.

10.3.1. Combustion Pressure.

With the scaling rules it is possible to derive further sets of constants for a number of combustion pressures. It is then possible to ascertain the optimum pressure for each 'vehicle system/propellant combination' group. Because the data relating to the combustion pressure, propellant characteristic velocity available to the author at the present moment is inadequate it has not been possible to proceed further. Whilst inter-related data is required over a wide pressure range, particular interest centres on the possibility of operation at very low pressure values. In this paper liquid propellant systems are given a value of 500 Lb/in^2. and the solid propellants at 1000 Lb/in^2. There is no reason for assuming that these represent optimum for the conditions considered.

10.3.2. Expansion Ratio

Again the scaling rules permit the examination of the relative effects of changes in nozzle mass and effective exhaust velocity. Ignoring problems of nozzle rigidity there will exist an optimum expansion ratio which will be different for each set of scaling constants and propellant parameters. Data is available to do this, but it has not been found possible to include the problem within the investigations so far undertaken.

Throughout this paper an expansion ratio of 10,000 has been used. Tables 5 and 6 indicate changes of nozzle parameters and effective exhaust velocity for a constant thrust nozzle based on the propellants used in this paper.

10.3.3. Mixture ratio - liquid propellants

Examination in this paper has been limited to a single mixture ratio for each propellant combination. The ratio used is the one producing maximum effective exhaust velocity in an earth surface environment with expansion to one atmosphere. Higher effective exhaust velocities may be attainable with slightly different mixtures due to the larger expansion ratios being used. More important than this is the effect of mixture ratio on component mass. For example, with the use of liquid hydrogen it appears obvious that a mixture ratio deficient in hydrogen would result in an optimum combination of tank mass and effective exhaust velocity. To a lesser extent this will be truefor all propellant combinations.

The three effects, of combustion pressure, expansion ratio and mixture ratio, require integrated consideration with the system in order to determine their combined effects on vehicle mass.

10.3.4. Solid propellant burning rate

The question of whether to use the cigarette burning arrangement, or the less massive case bonded internal burning conduit system cannot be decided without adequate data on the surface burning rates of real propellants. As these are not available in the open literature (for the high performance propellants demanded by the moon landing application) the problem has had to be left unanalysed.

There is, however, one aspect worthy of attention, and that arises from the possible need for physical methods of controlling burning rate. The discussion here is based on the assumption that high performance propellants tend to have very fast burning rates, and the fact that in this application, with burning times of 30 seconds or so, slow burning rates are needed to utilise the case bonded layout. It is required to examine the situation in order to determine the possibility of decreasing burning rate by environmental control.

Two parameters are available which appear worthy of consideration. Since burning rate is a function of combustion pressure, low pressure operation should help towards a solution. In itself the low combustion pressure need not affect performance since a potentially infinite expansion ratio is available. It also appears possible to achieve low burning rates by use of propellant grain at low temperature at the moment of firing. It might in consequence be worth while storing the grain in a refrigerated state during the intervening journey.

11. Conclusions

The main aim of this paper has been to investigate the approximate size scale for a moon landing vehicle and its propulsion system. The work so far completed, based on a 100 lb. payload, indicates that some 230 - 240 lb. of landing vehicle are required. Such a project would need an earth launching vehicle in the 250,000 - 350,000 lb. mass range. Although these figures are based on a 100 lb. payload the proportions are scalable over quite a wide range, and can in consequence be applied to other proposals made in the light of more detailed study on actual payload requirements, for example Gatland's 'Migrant' study vehicle.

In order to meet the needs of this paper it was found necessary to develop a vehicle and propulsion system scaling procedure. A great deal of work is still required to be carried out in order to fully utilise the method derived. At present, 22 sets of scaling constants have been obtained for six liquids, and one hypothetical solid propellant. The full procedure for determining the minimum vehicle mass has only been applied to six sets of these scaling constants. In addition to the completion of the study discussed in this paper more work is required on the combined effects of combustion pressure, expansion ratio and mixture ratio. It is hoped that computer time may be made available in order to continue and widen the various aspects still requiring consideration.

The work so far completed shows that there is a considerable mass advantage obtained by using liquid propellants. In spite of this the possible reliability advantages of the solid unit cannot be ignored. From the liquid propellant aspect, therefore, studies are needed in the field of ultra simple systems, probably based on single explosive valve operation.

Because of the inadequate data available on the subject of high performance solid propellants existing today, and their possible improvements in the future, it has been found impossible to do justice to this side of the study. In consequence the inter-related problems met in trying to obtain the optimum balance between performance requirements, and component masses has had to be ignored.

12. Acknowledgements

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APPENDIX 1.

SIMPLIFIED ANALYSIS. IDEAL VEHICLE. VERTICAL DESCENT

The ideal vehicle is defined as having constant mass flow rate of propellant, effective exhaust velocity and thrust. The all burnt mass of the vehicle is expressed as the sum of the payload, a mass scaling in direct proportion to propellant mass flow rate (e.g. analogous to a liquid propellant thrust chamber, pipe lines and valves) and a mass scaling in direct proportion to the total mass of propellant carried (e.g. liquid propellant system tanks or a solid propellant combustion chamber case).

Then

$$M_1 = M_L + K_1 \dot{m}_p + K_2 \dot{m}_p t_b \quad \text{(lb.)} \qquad \text{......} \; 1.1$$

$$M_o = M_1 + \dot{m}_p t_b \quad \text{(lb.)} \qquad \text{......} \; 1.2$$

Since engine thrust $F = \dot{m}_p c$ (pdl) the final or maximum acceleration of the vehicle is

$$a_1 = \frac{F}{M_1} = \frac{\dot{m}_p c}{M_1} \quad \text{(ft./sec}^2\text{)} \qquad \text{......} \; 1.3$$

$$= \frac{\dot{m}_p c}{M_o - \dot{m}_p t_b} \quad \text{(ft./sec}^2\text{)} \qquad \text{......} \; 1.4$$

For a vertical descent the vehicle velocity increment may be written

$$\Delta v = c \log_e \frac{M_o}{M_1} - \overline{g_L} t_b \quad \text{(ft./sec.)} \qquad \text{......} \; 1.5$$

It is required to determine the variation of the intial/payload mass ratio for various values of the constants c, K_1 and K_2. This may be done at a number of a_1 values. (Alternatively values of F, or $\dot{m}_p$ could be specified). The initial/payload mass ratio may be expressed in terms of the constants and a_1 using equations 1.2 and 1.4.

$$\frac{M_L}{M_o} = 1 - \frac{a_1}{c + a_1 t_b} \left[K_1 + t_b (K_2 + 1) \right] \qquad \text{......} \; 1.6$$

The initial/final mass ratio may be treated in the same manner using equations 1.1, 1.3 and 1.6

$$\frac{M_o}{M_1} = \left[1 - \frac{a_1}{c} (K_1 + K_2 t_b) \right] \frac{M_o}{M_L} \qquad \text{......} \; 1.7$$

For each selected value of a_1 the burning time (t_b) value must satisfy equation 1.5 for the velocity increment (Δv) and mean local gravitational acceleration ($\overline{g_L}$) involved. This step may be carried out graphically or otherwise.

Plots of $\frac{M_o}{M_L}$ (and M_o for a payload of 100 lb.) vs. a_1 for the following values of the constants

Δv = 7600 ft./sec.

c = 8000 and 10,000 ft./sec.

K_1 = 0, 5 and 10 $\sec^{-1}$

K_2 = 0.02, 0.04, 0.08 and 0.16

are given in figures 6 and 7.

APPENDIX 2.

IDEAL ROCKET ENGINE

In this paper an ideal rocket engine is defined as operating with a perfect gas. The gas is in equilibrium in the combustion chamber. Its composition does not alter through the expansion process. The usual ideal cycle assumptions of zero friction and heat loss are made. Also the gas leaves the exit without divergence loss and enters an environment of zero pressure.

It is required to determine the mass flow rate of propellant, the effective exhaust velocity, throat area and exit/throat area ratio for engines of specified thrust. A number of operating conditions in terms of combustion pressure (P_c) and expansion pressure ratio P_c/P_e are to be investigated. The characteristic velocity c^x and γ have been used as basic parameters, as a number of different propellant combinations are to be considered.

The following equations are derived from nozzle flow and the ideal engine definition :-

$$c^x = \frac{P_c A_t}{\dot{m}_p} \left[\frac{\text{pdl.}}{\text{ft}^2} \quad \frac{\text{ft}^2 \text{ sec.}}{\text{lb.}} \right] \quad \text{ft./sec.} \qquad \ldots\ldots \; 2.1$$

$$C_F = \frac{v_e}{c^x} \qquad \ldots\ldots \; 2.2$$

which expands to

$$C_F = \gamma \sqrt{\frac{2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \right]} \qquad \ldots\ldots \; 2.3$$

Also

$$\frac{A_e}{A_t} = \epsilon = \left(\frac{P_c}{P_e}\right)^{\frac{1}{\gamma}} \frac{\gamma}{C_F} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \qquad \ldots\ldots \; 2.4$$

The total thrust of an ideal engine operating in space is given by :-

$$F = \dot{m}_p v_e + P_e A_e \quad \text{pdl.} \qquad \ldots\ldots \; 2.5$$

Which, expanding and substituting from 2,1, 2,2, and 2.4 gives :-

$$F = \dot{m}_p\, c^x \left(C_F + \frac{P_e}{P_c}\,\epsilon\right) \quad \text{pdl.} \qquad \ldots\ldots \quad 2.6$$

$$\therefore \quad \dot{m}_p = \frac{F}{c^x \left(C_F + \frac{P_e}{P_c}\,\epsilon\right)} \quad \text{lb./sec.} \qquad \ldots\ldots \quad 2.7$$

and

$$c = \frac{F}{\dot{m}_p} = c^x \left(C_F + \frac{P_e}{P_c}\,\epsilon\right) \quad \text{ft./sec.} \qquad \ldots\ldots \quad 2.8$$

Also from 2.1 and 2.6

$$A_t = \frac{c^x\, \dot{m}_p}{P_c} = \frac{F}{P_c\, C_F + P_e\, \epsilon} \quad \text{ft}^2 \qquad \ldots\ldots \quad 2.9$$

APPENDIX 3.

MASS OF THRUST CHAMBER - LIQUID PROPELLANT ENGINES

The term thrust chamber is taken to include combustion chamber, nozzle and injector.

1. Combustion Chamber Shape

All chambers are spherical with smoothed entry into the nozzle.

2. Combustion Chamber Volume

At the present moment adequate scaling methods are not available. Since all chambers are the same shape L^x is assumed a valid scaling concept. In the absence of adequate data a value

$$L^x = \frac{V_c}{A_t} = 60 \text{ in.} \qquad \ldots\ldots \quad 3.1$$

is used throughout the calculations.

3. Chamber Construction

All chambers are cooled. Construction is assumed to be of conventional double wall with coolant enclosed. Both walls are taken as of equal thickness. Because walls are very thin, nucleate boiling might occur. All chambers are therefore assumed to have an inner heat barrier coating of zirconia having a mass per unit area equal to the single wall onto which it is attached (by flame spraying).

4. Stressing

It is assumed that the chamber outer wall carries the entire pressure load and is everywhere below 550^{o}K. The inside wall takes the inward bursting load due to the excess pressure of the coolant. The heat barrier is unstressed.

5. Injector

No specific allowance has been made for the injector. Rather hopefully it has been assumed that the absent mass at the throat could be accounted to this. This is not serious since both injector and combustion chamber scale approximately on propellant mass flow rate. It is assumed that adequate allowance has been made.

6. Combustion Chamber Mass

It is required to relate combustion chamber mass with the performance parameters, thrust, nozzle thrust coefficient, combustion pressure, area and pressure expansion ratios and the propellant characteristic velocity. From geometry of a sphere :-

$$d_c = \left(\frac{6\,V_c}{\pi}\right)^{\frac{1}{3}} \qquad \ldots\ldots \quad 3.2$$

$$\therefore \quad \text{surface} = \pi\left(\frac{6\,V_c}{\pi}\right)^{\frac{2}{3}} \qquad \ldots\ldots \quad 3.3$$

$$\therefore \quad \text{mass} = \pi\left(\frac{6\,V_c}{\pi}\right)^{\frac{2}{3}} \rho_m\, t_m \qquad \ldots\ldots \quad 3.4$$

for a single thickness.

Stressing as thin sphere

$$S = \frac{P_c \;\; \eta\, d_c}{4\, t_m} \qquad \ldots\ldots \quad 3.5$$

Substituting 3.1, 3.2 and 3.5 in 3.4

$$\text{mass} = 1.5\, A_t\, L^x\, P_c\, \eta\, \frac{\rho_m}{S} \qquad \ldots\ldots \quad 3.6$$

for a single thickness.

Since in the combustion chamber there will be two sheets of metal and one of zirconia, each of equal mass per unit area

$$M_c = 4.5\;\; A_t\;\; L^x\;\; P_c\;\; \eta\;\; \frac{\rho_m}{S} \qquad \ldots\ldots \quad 3.7$$

which, using equation 2.7 to eliminate A_t gives

$$M_c = \frac{4.5\, L^x\; \eta\;\; F}{C_F + \frac{P_e}{P_c}\;\; \epsilon} \quad \frac{\rho_m}{S} \qquad \ldots\ldots \quad 3.8$$

It should be noted that the value of S should be an available maximum working stress for a temperature of 550°K. The value of η was chosen after some thought as 2.5. It was intended with this rather high value to make allowance for the fact that the effects of stressing the injector and nozzle had not been taken into account.

7. Nozzle Mass

In order to eliminate thrust losses due to divergence a "tulip" nozzle profile has been assumed in the performance calculations. At a given combustion pressure the profile will change in a complex manner with different gas temperatures and composition. It is not possible therefore to make a reasonable assessment of nozzle mass for the actual profile. It has therefore been assumed that the real nozzle mass is equal to a "similar nozzle" having the same throat and exit areas, but with a complete divergence angle of 30° ($2\ \alpha$).

From geometry it can be shown that the surface area of such a shape is

$$\text{surface} = \frac{A_t}{\sin \alpha} \ (\epsilon - 1) \qquad \ldots\ldots \quad 3.9$$

The mass of the nozzle has been based on the assumption that the equivalent thickness is the same as the combustion chamber, i.e. three times the calculated "cold stress" thickness. No allowance has been made for the possibility of decreasing thickness towards the exit. Nor has any allowance been made for the fact that cooling may not be required near the exit. Equation 3.9 may be rewritten using equations 2.7, 3.2 and 3.5 :-

$$M_n = \left(\frac{F}{C_F + \frac{P_e}{P_c}\,\epsilon} \right)^{\frac{4}{3}} \quad \frac{0.75\eta \ (\epsilon - 1)}{\sin \alpha} \frac{\rho_m}{S} \left(\frac{6\ L^x}{\pi \ P_c} \right)^{\frac{1}{3}} \quad \ldots \ 3.10$$

8. Thrust Chamber Mass

The mass of the complete thrust chamber is the sum of equations 3.8 and 3.10. Throughout this paper stainless steel, with a working stress of 150,000 Lb./in^2 at 550°K, has been used. Combustion chambers for pressurised and turbo-pumped systems have been taken as identical.

All chambers have been based on a dimensional design thrust of 267 lb. for P_c = 500 Lb./in^2.

APPENDIX 4.

COMPONENT MASSES - LIQUID PROPELLANT SYSTEMS

It is required to calculate the masses of the various components which may occur in a liquid propellant system. A method is required which will permit an assessment of the performance of vehicles operating with various propellants at selected combustion pressures, using either pressurised tanks or turbo pumps for propellant motivation.

1. Tank Mass

Tanks are spherical. Using an identical method to that outlined in Appendix 3, equations 3.2 to 3.5.

$$M_T = \dot{m}_p \, t_b \, \frac{1.5 \, P_T \, \eta}{\bar{\rho}_P} \, \frac{\rho_m}{S} \qquad \ldots\ldots \quad 4.1$$

Equation 4.1 can be used directly for the mass of both tanks if they are of the same material.

Equation 4.1 is used as it stands for propellants which present no thermal insulation problems, and use the same material for each propellant. (e.g. Hydrogen peroxide and hydrazine both use stainless steel).

Where different materials are required the equation is modified to

$$M_T = 1.5 \, \dot{m}_p \, t_b \, P_T \, \eta \left[\frac{r}{r-1} \left(\frac{1}{\rho} \, \frac{\rho_m}{S} \right)_{OX} + \frac{1}{r-1} \left(\frac{1}{\rho} \, \frac{\rho_m}{S} \right)_{Fuel} \right] \ldots \quad 4.2$$

Throughout this paper tank pressures have been taken as

$$P_T = (200 + P_c) \ \text{Lb./in}^2$$
$$= 700 \ \text{Lb./in}^2$$

for pressurised systems and 50 Lb./in^2 for pumped systems. η has been taken as 1.5 in all cases.

Equation 4.2 may also be used in the calculation of tank mass for the propellants, oxygen, fluorine, and hydrogen, where either considerable density change or heat barrier is required. In this paper it is assumed that for oxygen and fluorine, the density at the end of the voyage will be half the loading density, without any heat barrier. For hydrogen, a figure of 1/4 loading density has been used, of which half the extra tank mass is considered due to real density change, the remainder due to some unspecified heat barrier.

2. Valve and Pipeline Mass

It is assumed that this is directly proportional to propellant volume flow, then :-

$$M_v = 90 \frac{\dot{m}_p}{\rho_p} \quad \text{lb.} \qquad \ldots\ldots \ 4.3$$

The value of this constant and that in 4.4 have been based on figures given by Baxter (Ref. 3).

3. Turbo/pump and Gas Generator Mass

It is assumed that the gas generator, turbine, shaft and casing have a mass directly proportional to the rate at which pumping work is being done on the propellants

$$M_g = 1.83 \ \dot{m}_p \ \Delta P \times 10^{-6} \quad \text{lb.} \qquad \ldots\ldots \ 4.4$$

The pumps and their cases are assumed to have a mass proportional to propellant volume flow rate

$$M_{pu} = 10 \frac{\dot{m}_p}{\rho_p} \quad \text{lb.} \qquad \ldots\ldots \ 4.5$$

The value of ΔP in equation 4.4 has been taken as $(P_c + 200 - P_T)$ Lb/in^2 which, for the cases considered has a value of 650 Lb/in^2.

APPENDIX 5.

MASS OF THRUST CHAMBER - SOLID PROPELLANT UNIT

1. Case Mass

Neither shape, nor length/diameter ratio are defined. It is therefore assumed that the real case mass, excluding the nozzle divergent section, is equal to the mass of a cylinder containing the same volume, stressed to the same pressure, of the same material and temperature as the real case.

Then case volume may be expressed as :-

$$V_c = \frac{\dot{m}_p \ t_b}{\rho_p \ \epsilon_p \ R_p} \qquad \ldots\ldots \quad 5.1$$

where ϵ_p is the propellant loading fraction and R_p the propellant utilisation ratio and both equal one in the cigarette burning arrangement.

From geometry of cylinder :-

$$V_c = \frac{\pi}{4} D_c^2 \ L \qquad \ldots\ldots \quad 5.2$$

Then based on surface

$$\text{Mass} = \pi D_c \ L \ t_m \ \rho_m \qquad \ldots\ldots \quad 5.3$$

Substituting 5.1 and 5.2 into 5.3 :-

$$\text{Mass} = \frac{\dot{m}_p \ t_b}{\rho_p \ \epsilon_p \ R_p} \ \frac{4}{D_c} \ t_m \ \rho_m \qquad \ldots\ldots \quad 5.4$$

Stressing as thin cylinder

$$S = \frac{P_c \ \eta \ D_c}{2 \ t_m} \qquad \ldots\ldots \quad 5.5$$

$$\therefore \quad M_c = \dot{m}_p \ t_b \ \frac{2 \ \eta \ P_c}{\rho_p \ \epsilon_p \ R_p} \ \frac{\rho_m}{S} \qquad \ldots\ldots \quad 5.6$$

Equation 5.6 is directly valid for a case bonded internal burning grain. A value $\eta = 2$ has been used. Stainless steel case with S = 75% of normal maximum working stress.

For a cigarette burning charge with an internally sprayed zirconia heat barrier of mass per unit area equal to case material :-

$$M_c = \dot{m}_p \; t_b \; 4 \; \eta \; \frac{P_c}{\rho_p} \quad \frac{\rho_m}{S} \qquad \ldots\ldots \; 5.7$$

Again $\eta = 2$. A value of S = 54% has been used in this application.

2. Nozzle Mass

Nozzle in uncooled andhas a heat barrier of zirconia. As in Appendix 3, the nozzle is tulip shaped to eliminate divergence losses. The estimation of mass is based on the same "similar nozzle" with an expansion half angle of 15°.

From 3.9 :-

$$\text{Mass} = \frac{A_t}{\sin \alpha} \; (\epsilon - 1) \quad \rho_m \; t_m \qquad \ldots\ldots \; 5.8$$

In this application nozzle thickness is based on a thin cylinder

$$t_m = \frac{P_c \; \eta \; D_n}{2S} \qquad \ldots\ldots \; 5.9$$

where $$D_n = \left(\frac{A_t + A_e}{2} \right)^{\frac{1}{2}} \qquad \ldots\ldots \; 5.10$$

Introducing a factor of 2 to take into account the zirconia

$$M_n = (c^x \, m_p)^{\frac{4}{3}} \quad \frac{(\epsilon - 1)\eta}{P_c \; \sin\alpha} \quad \left(\frac{\epsilon + 1}{2} \right)^{\frac{1}{2}} \quad \frac{\rho_m}{S} \qquad \ldots\ldots \; 5.11$$

which may be written in terms of thrust and the nozzle coefficients

$$M_n = \left(\frac{F}{C_F + \frac{P_e}{P_c} \epsilon} \right)^{\frac{4}{3}} \quad \frac{(\epsilon - 1)\eta}{P_c \; \sin \alpha} \quad \left(\frac{\epsilon + 1}{2} \right)^{\frac{1}{2}} \quad \frac{\rho_m}{S} \qquad \ldots\ldots \; 5.12$$

This method of estimation of nozzle mass gives values within a few per cent of the alternative method, the result of which is derived in equation 3.10.

APPENDIX 6.

COMPLETE IDEAL VEHICLE ANALYSIS

The propulsion system scaling rules indicate that the full definition of the ideal vehicle must include a mass scaling in proportion to $\dot{m}_p^{4/3}$. From this the all burnt mass of the vehicle is expressed as

$$M_1 = M_L + K_1 \dot{m}_p + K_2 \dot{m}_p t_b + K_3 \dot{m}_p^{4/3} \qquad \ldots\ldots \quad 6.1$$

Velocity increment and burning time are given by

$$\Delta v = c \log_e \frac{M_o}{M_1} - \overline{g_L} \; t_b \qquad \ldots\ldots \quad 1.5$$

$$t_b = \frac{M_o - M_1}{\dot{m}_p} \qquad \ldots\ldots \quad 6.2$$

Rewriting 6.1 and 6.2 in terms of $\frac{M_o}{M_1}$, and eliminating M_o and M_1

$$t_b \left[\frac{1}{\frac{M_o}{M_L} - 1} - K_2 \right] = \frac{M_L}{\dot{m}_p} + K_1 + K_3 \dot{m}_p^{\frac{1}{3}} \quad \ldots \quad 6.3$$

A graphical method permitting the determination of $\frac{M_o}{M_1}$, t_b, $\frac{M_L}{M_c}$, etc : for a given 'propellant combination/vehicle system' group (for known values of Δv, c, K_1, K_2 and K_3) at various $\dot{m}_p$ (or thrust) levels is as follows:

Graph 1. Plot equation 1.5 as $\frac{M_o}{M_1}$ vs. t_b .

Graph 2. Plot left hand side of equation 6.3 vs. t_b (using the relationship between t_b and $\frac{M_o}{M_1}$ given in equation 1.5, and plotted in graph 1). Values of the right hand side of equation 6.3 are then determined for various $\dot{m}_p$ values. Then from graph 2 t_b can be obtained. Graph 2 can then be used to give $\frac{M_o}{M_1}$. M_1 and M_o are then determined separately from equations 6.1 and 6.2.

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TABLE 1

RELEVANT EARTH AND MOON DATA

	Moon	Earth
Diameter (miles)	2159	7926
Surface gravity (ft/sec^2)	5.19	32.17
Escape velocity (ft/sec)	7693	36,677

Minimum velocity (at earth surface) to reach moon	34,800 (ft/sec)
Circular velocity at 500 miles above earth surface	24,800 (ft/sec)
Minimum velocity increment (500 circular to moon)	10,000 (ft/sec)

References 1 and 5.

TABLE 2

LIQUID PROPELLANT DATA

Combination 1 99.6% Hydrogen Perozide and Kerosine ($\frac{C}{H}$ = 6)
" 2[x] 99.6% Hydrogen Peroxide and 100% Hydrazine
" 3 Liquid Oxygen and Kerosine ($\frac{C}{H}$ = 6)
" 4 Liquid Oxygen and Unsymmetrical Dimethyl Hydrazine
" 5 Liquid Oxygen and Liquid Hydrogen
" 6[x] Liquid Fluorine and Liquid Hydrogen

[x] Self-igniting combination

Performance data based on P_c = 500 Lb/in^2

Propellant Combination →	1	2	3	4	5	6	
c^x	5320	5660	5740	5975	7950	8275	ft/sec
γ	1.2	1.22	1.24	1.24	1.26	1.33	
T_c	2939	2861	3461	3394	2790	3033	^oK
m	22	19	22	20	9	8.9	$\frac{lb}{lb.mol}$
r $(\frac{Ox}{Fuel})$	6.5	1.7	2.3	1.4	3.5	4.5	
ρ_{Ox}	87.0	87.0	71.3	71.3	71.3	94.3	$\frac{lb}{ft^3}$
ρ_{Fuel}	49.8	62.4	49.8	48.9	4.41	4.41	$\frac{lb}{ft^3}$
ρ_{Prop}	79.9	77.4	61.2	59.9	16.23	19.98	$\frac{lb}{ft^3}$

References 4 and 5

TABLE 3.

SOLID PROPELLANT DATA

Values are assumed typical of best available at present time. They are not representative of any specific type of propellant.

Performance data based on $P_c = 1000$ Lb/in^2

C^x	=	4920	ft/sec
γ	=	1.26	
T_c	=	2750	^oK
ρ_p	=	100	lb/ft^3

TABLE 4.

MATERIALS DATA USED IN PAPER

1.	**Stainless steel** (Armco PH 15-7 M_o)	
	Maximum working stress at 20°C	200,000 Lb/in^2
	Maximum working stress at 300°C	160,000 Lb/in^2
	Density	0.277 lb/in^3
	Strength/Mass at 20°C	7.25×10^5 Lb.in./lb.
2.	**Aluminium Alloy**	
	Maximum working stress at 20°C	70,000 Lb/in^2
	Density	0.10 lb/in^3
	Strength/Mass at 20°C	7×10^5 Lb.in/lb
3.	**Titanium Alloy** (6A1 - 4V)	
	Maximum working stress at 20°C	160,000 Lb/in^2
	Density	0.16 lb/in^3
	Strength/Mass at 20°C	10^6 Lb.in/lb.

Reference 5 and Manufacturers data.

TABLE 5.

NOZZLE EXIT PRESSURE ~ SIZE RELATIONSHIP
FOR CONSTANT THRUST AND COMBUSTION PRESSURE

Data based on $P_c = 1000$ Lb/in^2

$F = 1429$ Lb

$C^x = 4920$ ft/sec

$\gamma = 1.26$

P_e Lb/in^2	14.7	10	2	1	0.1
$\frac{P_c}{P_e}$	68	100	500	1000	10000
C_F	1.575	1.61	1.75	1.79	1.9
v_e ft/sec	7750	7930	8620	8820	9350
$\epsilon = \frac{A_e}{A_t} =$	7.8	10.5	36	60	200
A_t in^2	0.845	0.833	0.784	0.772	0.745
A_e in^2	6.59	8.75	28.2	46.4	149.0
D_t in	1.036	1.029	0.998	0.991	0.973
D_e in	2.90	3.28	6.00	7.68	13.76
Surface area in^2	22.2	30.5	105.7	176	571
$\dot{m}_p$ lb/sec	5.53	5.44	5.11	5.05	4.86
c ft/sec	8300	8450	9000	9120	9460

TABLE 6.

PERFORMANCE AND NOZZLE PARAMETERS FOR VARIOUS LIQUID PROPELLANTS

FOR CONSTANT THRUST AND COMBUSTION PRESSURE

Data based on P_c = 500 Lb/in^2

P_e = 0.05 Lb/in^2

F = 267 Lb

Propellant combination →	1	2	2^x	3	4	5	6
C_F	1.98	1.95	1.84	1.92	1.92	1.885	1.8
v_e ft/sec	10530	11040	10420	11010	11480	14990	14900
$\varepsilon = \frac{A_e}{A_t}$	459	416	67.1	383	383	344	251
A_t in^2	0.263	0.268	0.28	0.273	0.273	0.278	0.293
A_e in^2	120.8	111.6	18.8	105	105	103	73.5
D_t in	0.578	0.584	0.597	0.589	0.593	0.595	0.61
D_e in	12.4	11.9	4.89	11.53	11.53	11.44	9.67
Surface area in^2	466	430	71.5	403	403	369	283
$\dot{m}_p$ lb/sec	0.797	0.76	0.795	0.765	0.734	0.563	0.569
c ft/sec	10780	11,300	10,800	11,220	11,710	15,260	15,100

^{x} Data based on P_e = 0.5 Lb/in^2

TABLE 7.

SCALING CONSTANTS FOR LIQUID PROPELLANT SYSTEMS

Propellant Combination →	1	2	3	4	5	6
K_1 (chamber)	0.206	0.215	0.222	0.231	0.308	0.320
K_1 (valves	1.126	1.162	1.471	1.502	5.560	4.510
K_1 (Pressurised system)	1.332	1.381	1.693	1.733	5.868	4.830
K_1 (gas generator)	5.525	5.525	5.525	5.525	5.525	5.525
K_1 (pumps)	0.125	0.129	0.163	0.167	0.616	0.501
K_1 (Turbo/pumped system)	6.982	7.035	7.381	7.425	12.009	10.856
K_2 (at 700 Lb/in^2)[1]	-	-	0.0773	0.0914	0.674	0.563
K_2 (at 700 Lb/in^2)[2]	0.0456	0.0496	0.0478	0.0618	0.185	0.157
K_2 (at 50 Lb/in^2)[1]	-	-	0.00553	0.00654	0.0481	0.0483
K_2 (at 50 Lb/in^2)[2]	0.00326	0.00354	0.00342	0.00442	0.0132	0.0112
K_3 ($\frac{P_c}{P_e}$ = 10,000)	3.425	3.350	3.15	3.32	4.23	3.36

[1]Allowance made for temperature rise, and/or heat barrier

[2]Based on density at 273 °K or boiling point at 1 atmosphere (lowest value). No allowance made for heat barrier mass.

SCALING CONSTANTS - THE SOLID PROPELLANT SYSTEM

$K_1 = 0$ $\qquad$ $K_2 = 0.36$[1], 0.18[2] $\qquad$ $K_3 = 0.48$

[1]Cigarette burning arrangement

[2]Case bonded, internal burning conduit.

TOTAL VELOCITY INCREMENT EARTH SURFACE TO MOON 35,250 ft./sec.
VELOCITY INCREMENT 500 MILE CIRCULAR ORBIT TO MOON 10,500 ft./sec.

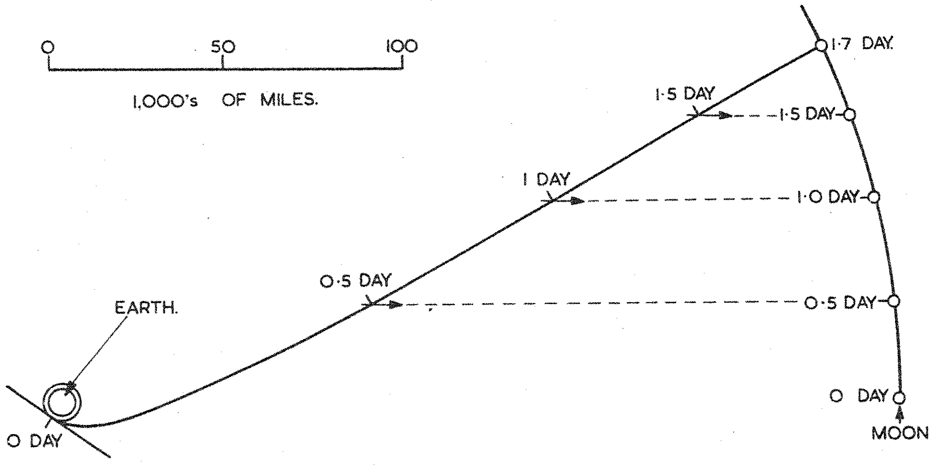

FIG. 1. TYPICAL EARTH-MOON TRAJECTORY. BASED ON REF 1.

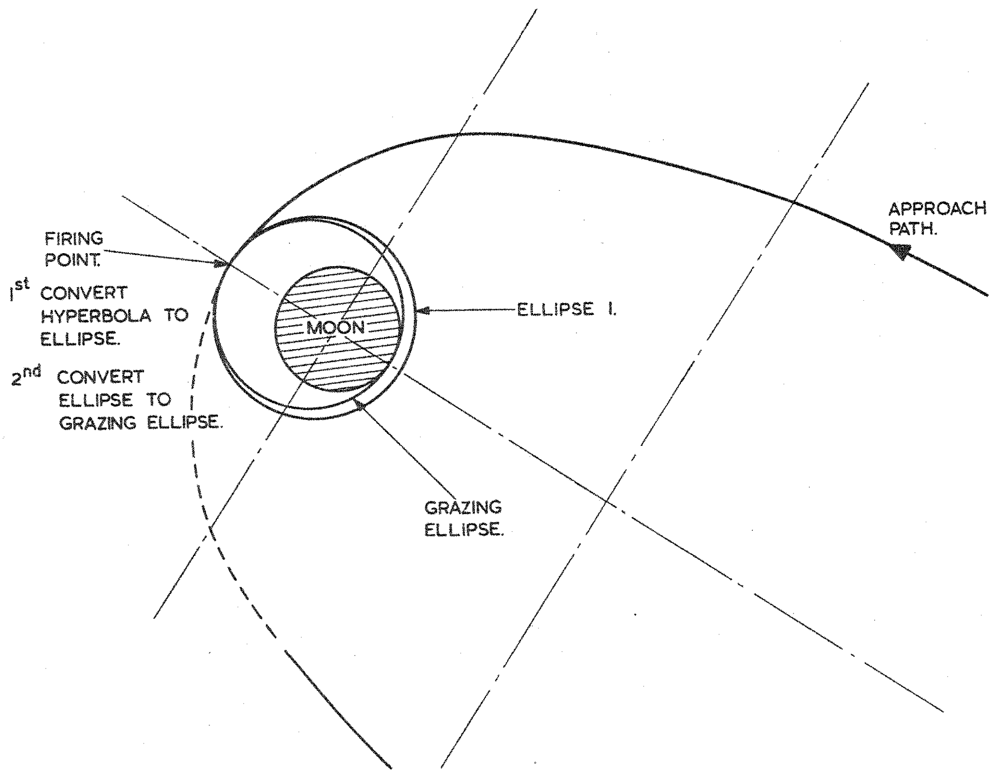

FIG. 2. MOON LANDING. HYPERBOLIC ELLIPTIC APPROACH.

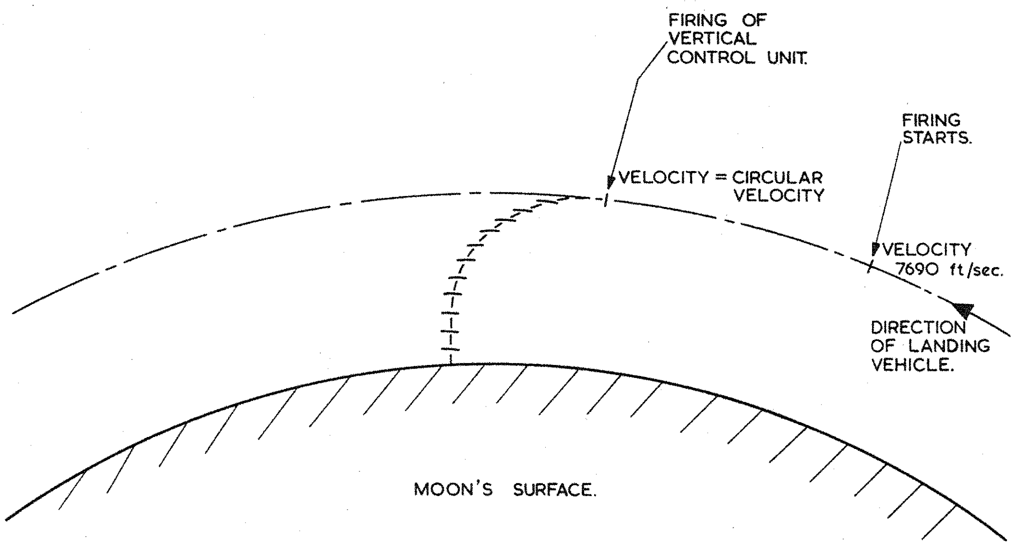

FIG. 3. GRAZING ELLIPSE LANDING TECHNIQUE.

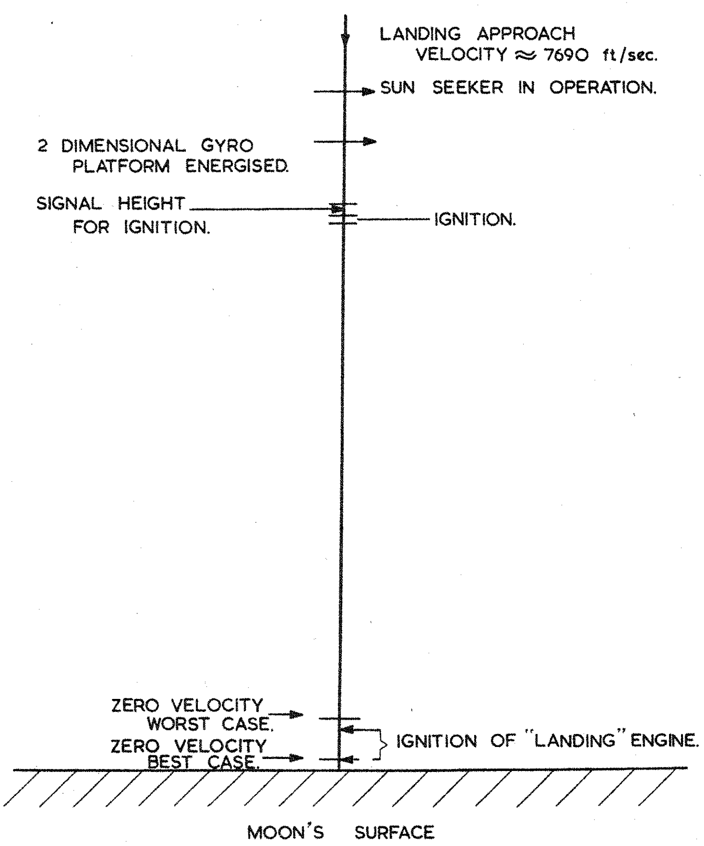

FIG. 4. VERTICAL DESCENT LANDING TECHNIQUE.

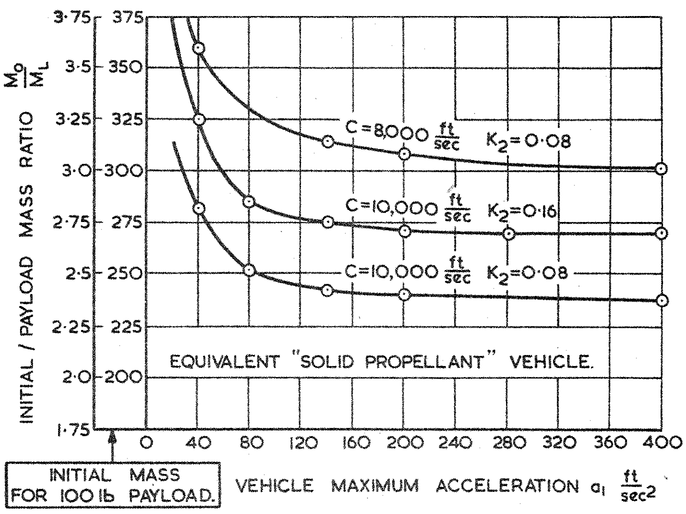

FIG. 5. IDEAL VEHICLE: MASS/ACCELERATION $K_1 = 0$

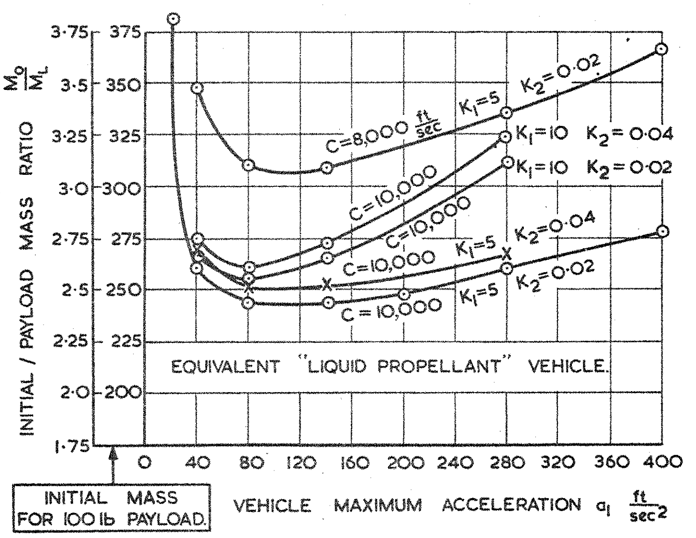

FIG. 6. IDEAL VEHICLE: MASS/ACCELERATION K_1 & $K_2 \neq 0$

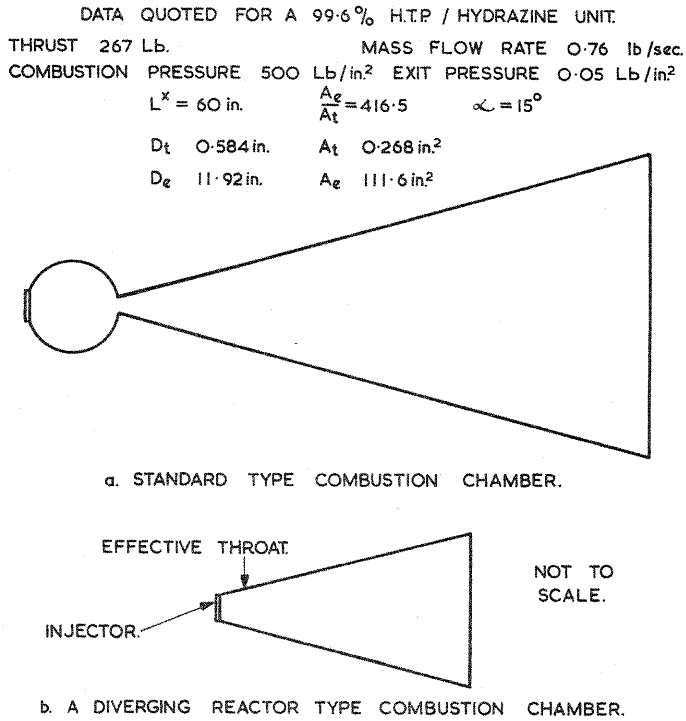

FIG. 7. LIQUID PROPELLANT COMBUSTION CHAMBERS.

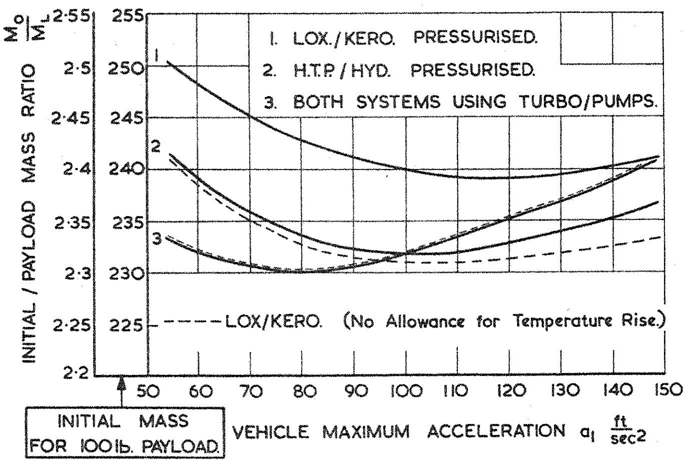

FIG. 8. REAL VEHICLE MASS/ACCELERATION.