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THE COLLEGE OF AERONAUTICS

CRANFIELD

STABILITY OF A ROTATING CYLINDRICAL SHELL PARTIALLY FILLED WITH LIQUID

by

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Experiments on the Stability of a Rotating Cylindrical Shell partially filled with liquid

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1. Introduction and summary

The theory of the stability of a gyroscope which has a right circular cylindrical cavity partially filled with liquid has been developed by Stewartson (reference 1), who has shown that instabilities occur at and near those filling-ratios * for which the frequencies of free oscillations of the liquid filling coincide with the frequencies of precession and nutation of the empty gyroscope when the mass of liquid is small compared with the mass of the empty gyroscope. When applied to the motion of a gyrostat, Stewartson's theory shows that there is an infinite number of filling-ratios at which instability is to be expected theoretically, but, in practice, it is to be expected that the effects of viscosity in the liquid filling will damp out all but the graver modes of oscillation, and one of the objects of the present experimental investigation was to determine how many of the instabilities could be detected. Stewartson's theory also predicts the range of filling-ratios for which the gyrostat should be unstable, and a second object of the experiments was to verify these predicted ranges.

The gyrostat used for the experiments is illustrated in Figures 1 and 2, and is described in more detail below. It was found that the first two instabilities predicted by Stewartson could be detected with certainty (some imagination

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* The filling-ratio is defined to be the volume of liquid in the cavity divided by the volume of the cavity.

being required for the detection of any of the others), and that the filling-ratios at which they occurred agreed accurately with the theoretical predictions. But the agreement between theory and experiment on the matter of the range of instability was not as good: possible reasons for this disagreement are discussed below, in section 4.

2. The principal features of the gyroscope

The gyroscope on which the experimental observations were made is illustrated in Figures 1 and 2. The rotor (A, Fig. 2) had a cavity $1\frac{1}{8}$ ins. in diameter and $3\frac{3}{8}$ ins. long, and was supported by two ball races mounted in a cage (B). The rotor was driven through a flexible coupling by a small 3-phase induction motor (C) which was also mounted in the cage. The motor was fed from a variable frequency alternator, the electrical connexions being through some special woven copper wire of great flexibility. The cage was connected by gymbal bearings (D) to a comparatively massive support, the inner gymbal having small lead weights (E) attached, which were adjusted to make the equivalent moments of inertia of the gymbal system the same about both axes of rotation. The cage was adjustable in the inner gymbal, and was positioned in such a way that the centre of mass of the whole system (without liquid) coincided with the centre of rotation, thus making the system into a gyrostat, and in this state the moment of inertia about the gymbal axes was 8.95 lb. ins^2. The direct measurement of the equivalent moment of inertia of the rotating parts about their axis of symmetry would have been difficult, so this moment of inertia was inferred from a dynamic experiment in which the frequencies of nutation and rotation were measured by stroposcopic means: the ratio of these frequencies was 0.112, from which it follows that the required equivalent moment of inertia was $0.112 \times 8.95 = 1.002$ lb. ins^2. The frequency of precession was very small, and was less than 1 cycle/min. at a rotor speed of 6000 r.p.m with the cavity full of liquid, which was the state of maximum unbalance.

3. Experimental method

The rotor was filled by a graduated hypodermic syringe through a hole bored in the upper axis, and it was found that 55.3 ml. of liquid were required to fill it completely, corresponding to 47.5 grms. of liquid, the liquid used being a mixture of light lubricating oil and liquid paraffin, having a kinematic viscosity of 23.9 centi-stokes at 70°F and 12.8 centi-stokes at 100°F. The experimental procedure was to weigh the rotor plus any residual liquid, to introduce the liquid 1 ml. at a time and observe the instability or stability after each addition.

For the first two sets of observations, rotor speeds of 4,000 and 5,000 r.p.m were used, the gyrostat was disturbed slightly from equilibrium, and the stability was estimated qualitatively as stable, neutral, slightly unstable, unstable, and very unstable. This estimation could be made consistently with a little practice, but it was felt that some more quantitative estimate was required. To achieve this, use was made of the fact that a light far up in the laboratory roof was reflected conveniently in the cap of the upper rotor bearing, and that this reflection traced out a small circle whose diameter was proportional to the amplitude of the nutational motion. This circle was observed through a slot, cut in metal sheet, which was $\frac{1}{8}$ in, wide over half its length, and $\frac{1}{4}$ in. wide over the other half, and by moving this slot slightly along its length, and measuring the time taken for the circle to increase in diameter from just filling the narrow portion to just filling the wider portion, the rate of build-up of the oscillation could be determined. This crude but effective method of instrumentation proved to be most convenient in use; precautions were taken to ensure that the relative positions of gyrostat, slot, and observer's eye were maintained during an observation. In this way observations of instability were made with a rotor speed of 6,000 r.p.m. It had been intended to repeat these with other rotor speeds, but the violent instabilities proved to be too much for the gymbal bearings, and further experiments will have to await reconstruction of these, and also a realignment of the rotor bearings which have been slightly upset by the comparatively large forces involved in these oscillations.

4. Experimental results and comparison with theory

The results of the tests at 4,000, 5,000 and 6,000 r.p.m are shown in Figure 3. Figures 3(a) and 3(b) show the qualitative estimates for 4,000 and 5,000 r.p.m respectively, and Figure3(c) shows the quantitative results for 6,000 r.p.m.

In Stewartson's notation, for resonance, when

$$t = \frac{\text{nutational frequency}}{\text{rotational frequency}} = 0.112$$

n = number of waves circumferentially

$p+\frac{1}{2}$ = number of waves axially

the theoretical filling-ratios are shown in Table I.

Table I

p \ n	1	2	3
0	-	-	-
1	0.66	-	-
2	0.23	-	-
3	0.14	0.78	-
4	0.10	0.60	0.92
5	0.08	0.50	0.83

It will be seen from Figure 3 that the main resonance $(n,p) = (1,1)$ and the second resonance $(1,2)$ can be detected with certainty, and that the experimental values of the filling-ratios for resonance agree well with the theoretical predictions. Other minor resonances appear to occur experimentally at 6,000 r.p.m but their positions do not agree with any of the filling-ratios in Table I, and it is possible that these resonances may be spurious. However, there is no doubt about the existence of the main resonance at filling-ratio 0.66 and the results at this resonance can be used for somparison with Stewartson's criterion for instability.

If t_o is the ratio of nutational to rotational frequencies for a given filling ratio, and $\gamma = \sqrt{\rho b^4/cL\,\sigma}$, where ρ = density of liquid, b = radius of cavity, c = length of cavity, L = moment of inertia about a transverse axis, $\sigma = 1$, then stewartson's stability number A is given by

$$A = \frac{t - t_o}{\gamma R_n}$$

where R_n is a number which depends upon $c/b(2p+1)$, t_o and the filling-ratio. According to the theory, the gyrostat is unstable if $-1 < A < 1$.

In the present case $\gamma = 0.00567$. The limits of filling-ratio for instability are a little vague in Fig. 3, but it seems that the gyrostat is unstable between filling ratios of 0.63 and 0.70. Dr. Stewartson has calculated that the corresponding values of A are 4 and -16 respectively, so the experimental range of A for instability is $-16 < A < 4$, which is considerably greater than the theoretical range. Thus it appears that the theory has failed in the detailed matter of predicting the range of filling-ratios for which instability occurs.

It is interesting to speculate on possible reasons for this failure. It seems that there are three major factors which could affect the motion, and which are neglected in Stewartson's theory, namely, the effect of gravity, the displacement of the axis of rotation from the axis of the cylinder, and the effect of non-linear terms. In the experiments, the effect of gravity is noticeable, but, the higher the rotational speed, the broader is the range of instability, so this effect is working in the wrong direction for our purposes, in as much as the effect of gravity should be less and less imprtant as the rotation speed increases. The initial displacement of the axis of rotation was kept small and the axis of rotation never made an initial angle of more than about 3° with the axis of nutation (which was approximately vertical in all tests); nevertheless the extra inertia forces might be significant, and to test for this, some large initial displacements in nutation were applied to the gyrostat. It was found that the range of instability was increased by doing this, and in fact the gyrostat could be made unstable for almost all filling-ratios tested by giving it a sufficiently voilent initial impulse. Unfortunately these tests have not been made systematically owing to the bearing failures described above. The effects of non-linear terms are inextricably bound up with these latter results, and are probably important for large initial displacements, but not for the smaller displacements used in the tests. It would be interesting to observe the liquid movement through a transparent cylinder, but a suitable modification of the present apparatus would be difficult to achieve in view of the high rotational speeds and the necessity for thin walls to reduce refraction effects.

Reference

1. K. Stewartson. The Stability of liquid-filled shell. W.R.(D.). Report No. 17/53 or A.C.12556, LSP 114 (1953)

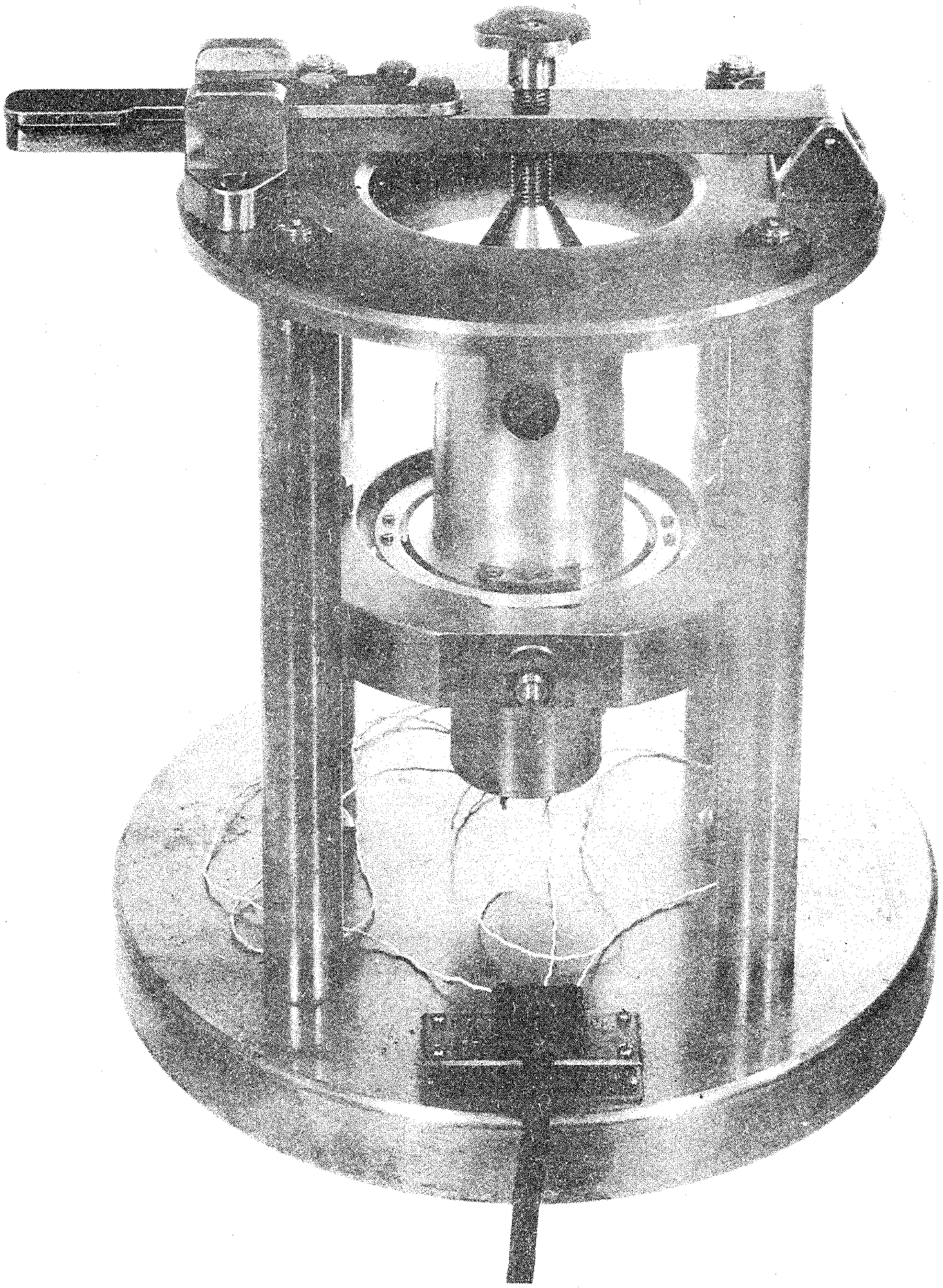


FIG. I THE GYROSCOPE AND SUPPORTS

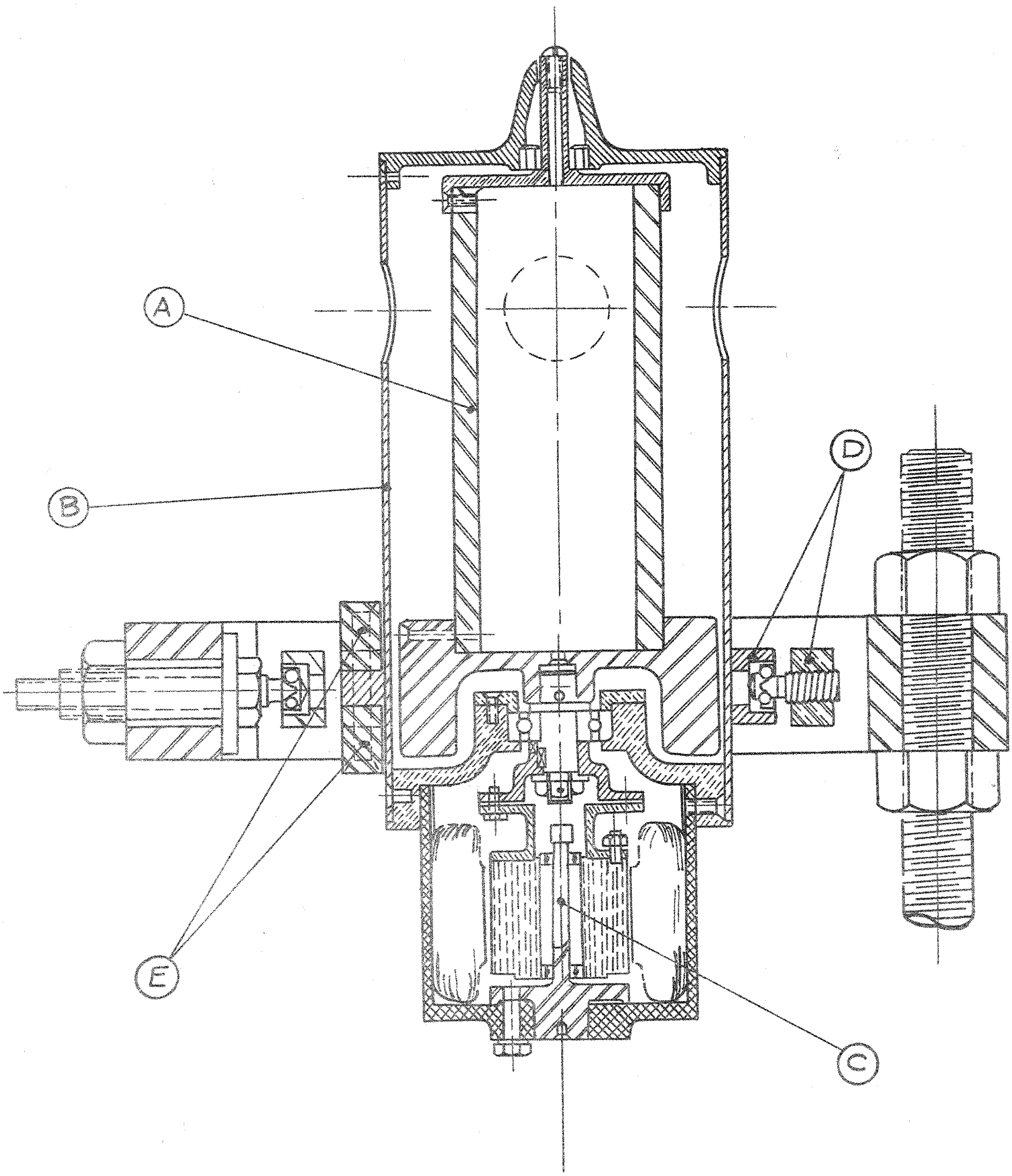
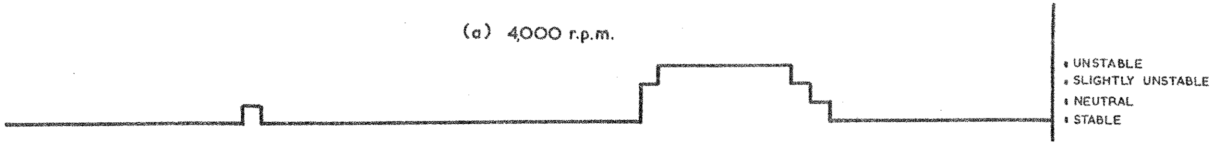
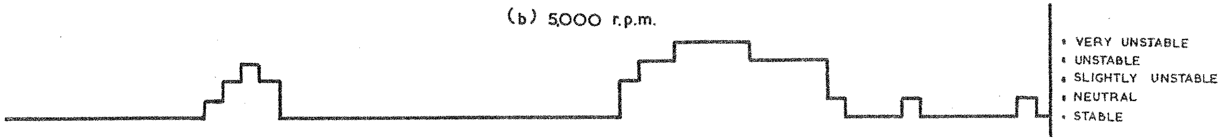


FIG. 2 DETAILS OF ROTOR AND MOTOR

(a) 4,000 r.p.m.



(b) 5,000 r.p.m.



(c) 6,000 r.p.m.

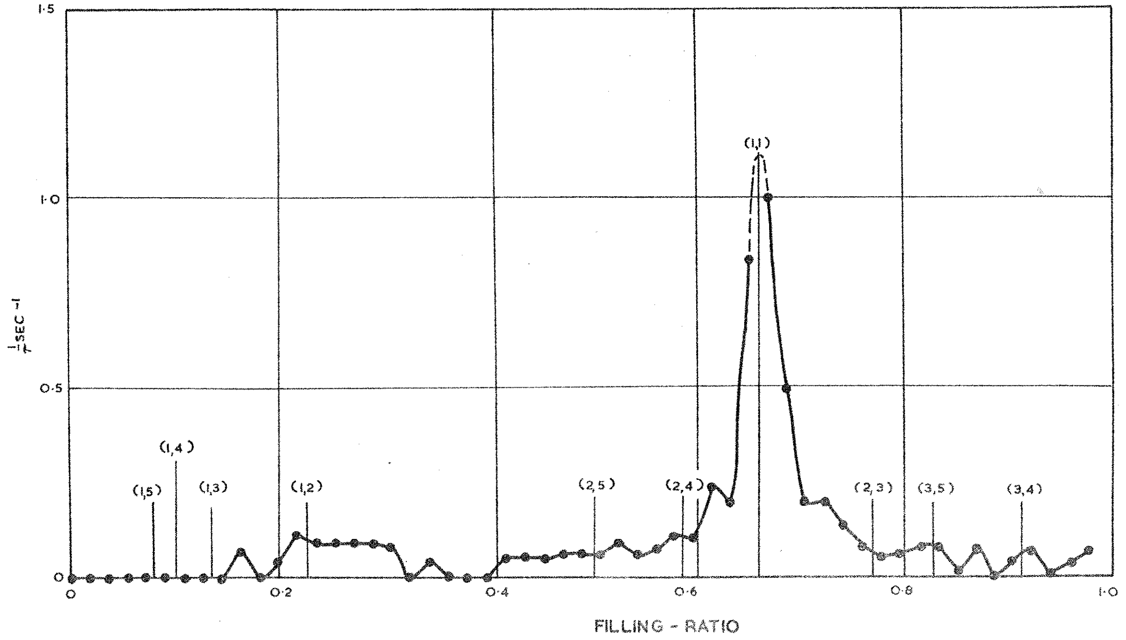


FIG. 3. EXPERIMENTAL RESULTS. (a) and (b), Qualitative estimates of instability for rotor speeds of 4,000 and 5,000 r. p. m. v. filling ratio. (c) Reciprocal of time (secs) to double amplitude for rotor speed of 6,000 r. p. m. v. filling ratio.