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An Autocode Programme to Determine the Flatness of a Surface Table Using the Least Square Mean Plane Criteria

- by -

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Introduction

The calculations involved in determining the flatness of a surface table from measurements made with instruments such as a block level or auto collimator can be lengthy, particularly where a large number of ordinates are taken. A solution to this problem is presented in this note in the form of an Autocode programme which will cater for up to 100 ordinates.

The datum from which the variations in the flatness of the surface table are determined is the least squares mean plane. This may be defined as being that plane which makes the sum of the squares of the errors relative to it a minimum.

Determination of errors relative to the least square mean plane

Let the equation of the mean true plane be

$$\eta' = ax + by + c$$

The error at a point i is

$$\eta_i - \eta'_i = \eta_i - (ax_i + by_i + c)$$

It can be shown that the mean true plane passes through the centroid $\bar{X}$, $\bar{Y}$, $\bar{Z}$ and that we may eliminate c by taking the origin at the centroid.

Thus the equation to the mean true plane becomes

$$\eta' = ax + by$$

where

$$a = \frac{\Sigma y_i^2 \; \Sigma \eta_i x_i - \Sigma x_i y_i \; \Sigma \eta_i y_i}{\Sigma x_i^2 \; \Sigma y_i^2 - \Sigma (x_i y_i)^2}$$

and

$$b = \frac{\Sigma x_i^2 \; \Sigma \eta_i y_i - \Sigma x_i y_i \; \Sigma \eta_i x_i}{\Sigma x_i^2 \; \Sigma y_i^2 - \Sigma (x_i y_i)^2}$$

Note that if the X Y axes are parallel to the axes of symmetry of the plan view then $\Sigma x_i y_i$ is always zero.

Let the observed points be

$$X_i, \quad Y_i \quad \text{and } Z_i$$

Then

$$\Sigma x_i^2 = \Sigma X_i^2 - \frac{\Sigma X_i^2}{n}$$

$$\Sigma y_i^2 = \Sigma Y_i^2 - \frac{\Sigma Y_i^2}{n}$$

$$\Sigma \eta_i^2 = \Sigma Z_i^2 - \frac{\Sigma Z_i^2}{n}$$

$$\Sigma x_i y_i = \Sigma X_i Y_i - \frac{\Sigma X_i Y_i}{n}$$

$$\Sigma x_i \eta_i = \Sigma X_i Z_i - \frac{\Sigma X_i Z_i}{n}$$

$$\Sigma y_i \eta_i = \Sigma Y_i Z_i - \frac{\Sigma Y_i Z_i}{n}$$

The equation to the mean true plane is given by

$$\eta' = ax + by$$

or

$$Z' - \bar{Z}' = a(X - \bar{X}) + b\,(Y - \bar{Y})$$

$$\therefore \quad Z' = aX + bY + \bar{Z} - a\bar{X} - b\bar{Y}$$

$$= aX + bY + \frac{(\bar{Z}Z - a\,\Sigma\,x - b\bar{Z}Y)}{n}$$

and the error of a point i on the surface relative to the least squares mean plane is given by $Z_i - Z_i^1$. The Autocode programme to obtain the error $Z_i - Z_i^1$ from a series of observed points X_i, Y_i, Z_i is included in Appendix 1.

References

1. Engineering Dimensional Metrology, L. Miller.
2. The Pegasus Autocode - Ferranti Handbook.

Appendix 1

Autocode Programme for least squares mean plane.

```
D
N
LEAST SQUARES MEAN PLANE
 X    Y    Z  ERROR
NABC
J1.0
STOP
v100=TAPE*
v200=TAPE*
v300=TAPE*
v30=n0
n1=0
v1=0
v2=0
v3=0
v5=0
v7=0
v9=0
v11=0
v13=0
2)v1=v1+v(100+n1)                ΣX
v2=v2+v(200+n1)                  ΣY
v3=v3+v(30C +n1)                 ΣZ
v4=v(100+n1)Xv(100+n1)           X²
v5=v5+v4                         ΣX²
v6=v(200+n1)Xv(200+n1)           Y²
v7=v7+v6                         ΣY²
v8=v(300+n1)Xv(300+n1)           Z²
v9=v9+v8                         ΣZ²
v10=v(100+n1)Xv(300+n1)          XZ
v11=v11+v10                      ΣXZ
v12=v(200+n1)Xv(300+n1)          YZ
v13=v13+v12                      ΣYZ
n1=n1+1
→ 2,n1=n0
v14=v1Xv1
v14=v14/v30                      Σx²
v14=v5-v14
v15=v2Xv2
v15 =v15/v30
v15=v7-v15                       Σy²
v16=v16/v30
v16=v9-v16                       Ση²
v17=v1Xv3
v17=v17/v30
v17=v11-v17                      Σxη
v18=v2Xv3
v18=v18/v30
v18=v13-v18                      Σyη
v19=v15Xv17                      Σy² Σxη
v20=v14Xv15                      Σx² Σy²
v19=v19/v20                      a
v21=v14Xv18                      Σx² Σηy
v21=v21/v20                      b
```

v22=v21Xv2	bΣY
v23=v19Xv1	aΣX
v22=v22+v23	bΣY + aΣX
v22=v3-v22	ΣZ - aΣX - bΣY
v22=v22/v30	$\frac{\Sigma Z - a\Sigma X - b\Sigma Y}{no}$
n1=0	
3)v24=v19Xv(100+n1)	aX
v25=v21Xv(200+n1)	bY
v26=v24+v25	
v26=v26+v22	Z'
v27=v(300+n1) ,v26	Z-Z'
PRINTv(100+n1),3040	PRINT X
PRINTv(200+n1),4040	PRINT Y
PRINTv(300+n1),4040	PRINT Z
PRINTv27,4041	PRINT ERROR
n1=n1+1	
→ 3,n1≠n0	
Xn1=0	
PRINT n0,3000	PRINT n0
PRINT v19,4042	PRINT a
PRINT v21,4042	PRINT b
PRINT v22,4062	PRINT CONSTANT
(→0)	

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