

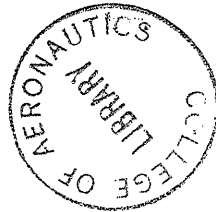
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An Autocode Programme to Determine the
Flatness of a Surface Table Using the
Least Square Mean Plane Criteria

- by -

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Introduction

The calculations involved in determining the flatness of a surface table from measurements made with instruments such as a block level or auto collimator can be lengthy, particularly where a large number of ordinates are taken. A solution to this problem is presented in this note in the form of an Autocode programme which will cater for up to 100 ordinates.

The datum from which the variations in the flatness of the surface table are determined is the least squares mean plane. This may be defined as being that plane which makes the sum of the squares of the errors relative to it a minimum.

Determination of errors relative to the least square mean plane

Let the equation of the mean true plane be

$$\eta' = ax + by + c$$

The error at a point i is

$$\eta_i - \eta'_i = \eta_i - (ax_i + by_i + c)$$

It can be shown that the mean true plane passes through the centroid \bar{X} , \bar{Y} , \bar{Z} and that we may eliminate c by taking the origin at the centroid.

Thus the equation to the mean true plane becomes

$$\eta' = ax + by$$

where

$$a = \frac{\sum y_i^2 \sum \eta_i x_i - \sum x_i y_i \sum \eta_i y_i}{\sum x_i^2 \sum y_i^2 - \sum (x_i y_i)^2}$$

and

$$b = \frac{\sum x_i^2 \sum \eta_i y_i - \sum x_i y_i \sum \eta_i x_i}{\sum x_i^2 \sum y_i^2 - \sum (x_i y_i)^2}$$

Note that if the $X Y$ axes are parallel to the axes of symmetry of the plan view then $\sum x_i y_i$ is always zero.

Let the observed points be

$$X_i, Y_i \text{ and } Z_i$$



Then

$$\Sigma x_i^2 = \Sigma X_i^2 - \frac{\Sigma X_i^2}{n}$$

$$\Sigma y_i^2 = \Sigma Y_i^2 - \frac{\Sigma Y_i^2}{n}$$

$$\Sigma \eta_i^2 = \Sigma Z_i^2 - \frac{\Sigma Z_i^2}{n}$$

$$\Sigma x_i y_i = \Sigma X_i Y_i - \frac{\Sigma X_i Y_i}{n}$$

$$\Sigma x_i \eta_i = \Sigma X_i Z_i - \frac{\Sigma X_i Z_i}{n}$$

$$\Sigma y_i \eta_i = \Sigma Y_i Z_i - \frac{\Sigma Y_i Z_i}{n}$$

The equation to the mean true plane is given by

$$\eta' = ax + by$$

or

$$Z' - \bar{Z}' = a(X - \bar{X}) + b(Y - \bar{Y})$$

$$\therefore Z' = aX + bY + \bar{Z} - a\bar{X} - b\bar{Y}$$

$$= aX + bY + \left(\frac{\bar{Z}Z}{n} - a \frac{\Sigma x}{n} - b \frac{\bar{Z}Y}{n} \right)$$

and the error of a point i on the surface relative to the least squares mean plane is given by $Z_i - Z_i^1$. The Autocode programme to obtain the error $Z_i - Z_i^1$ from a series of i observed points X_i, Y_i, Z_i is included in Appendix 1.

References

1. Engineering Dimensional Metrology, L. Miller.
2. The Pegasus Autocode - Ferranti Handbook.

Appendix 1

Autocode Programme for least squares mean plane.

```

D
N
LEAST SQUARES MEAN PLANE
  X   Y   Z  ERROR
NABC
J1.0
STOP
v100=TAPE*
v200=TAPE*
v300=TAPE*
v30=n0
n1=0
v1=0
v2=0
v3=0
v5=0
v7=0
v9=0
v11=0
v13=0
2)v1=v1+v(100+n1)
v2=v2+v(200+n1)
v3=v3+v(300+n1)
v4=v(100+n1)Xv(100+n1)
v5=v5+v4
v6=v(200+n1)Xv(200+n1)
v7=v7+v6
v8=v(300+n1)Xv(300+n1)
v9=v9+v8
v10=v(100+n1)Xv(300+n1)
v11=v11+v10
v12=v(200+n1)Xv(300+n1)
v13=v13+v12
n1=n1+1
→ 2,n1=n0
v14=v1Xv1
v14=v14/v30
v14=v5-v14
v15=v2Xv2
v15=v15/v30
v15=v7-v15
v16=v16/v30
v16=v9-v16
v17=v1Xv3
v17=v17/v30
v17=v11-v17
v18=v2Xv3
v18=v18/v30
v18=v13-v18
v19=v15Xv17
v20=v14Xv15
v19=v19/v20
v21=v14Xv18
v21=v21/v20

```

ΣX
 ΣY
 ΣZ
 X^2
 ΣX^2
 Y^2
 ΣY^2
 Z^2
 ΣZ^2
 XZ
 ΣXZ
 YZ
 ΣYZ

 Σx^2

 Σy^2

 $\Sigma \eta^2$

 $\Sigma x\eta$

 $\Sigma y\eta$
 $\Sigma y^2 \Sigma x\eta$
 $\Sigma x^2 \Sigma y^2$
a
 $\Sigma x^2 \Sigma \eta y$
b

