

# Cranfield

College of Aeronautics Report No. 8911  
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Preliminary Studies for Aircraft Parameter Estimation  
Using Modified Stepwise Regression

H A Hinds and M V Cook

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College of Aeronautics  
Cranfield Institute of Technology  
Cranfield, Bedford MK43 0AL, England



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## 1.0 INTRODUCTION.

Parameter estimation is the computational process by which the coefficients in a mathematical description of a dynamic system may be estimated from recorded input-output response data. In recent years parameter estimation methods have found extensive use in aircraft applications since it is notoriously difficult to obtain estimates for aerodynamic stability coefficients by traditional methods with any degree of confidence. However, most parameter estimation methods make considerable use of statistical techniques and therefore also have a degree of uncertainty associated with the results. Thus in order to develop confidence in the methods it is desirable to have as much visibility of the computational method as possible. Clearly this is not always easy to achieve when a complex method is applied to a complex aircraft model.

One of the more recent advances in parameter estimation is the use of the Modified Stepwise Regression (MSR) method, pioneered in the U.S.A. at the NASA Langley Research Center by Klein, Batterson and Murphy in the early 1980's. In their work the aircraft equations of motion are in general form, with the aerodynamic forces and moment coefficients expressed as polynomials in response and input variables. The modified stepwise regression is constructed to force a linear model for the aerodynamic coefficient first, then it adds significant non-linear terms and deletes non-significant terms from the model.

An advantage of the method is its relative simplicity in that explicit statistical descriptions of the noise associated with the measured data are not generally required. The method has been, and continues to be developed and has successfully been applied to a number of free flight aircraft and aircraft models. An example of the achievements may be found in reference 1. At Cranfield, MSR will be applied to a relatively simple aircraft model where some of the measured data required is not easily obtained. However, it is hoped to confirm that the MSR works equally as well as when it is applied to more complex models. It may also prove possible to provide an enhanced visibility of the analytical techniques involved in the computation of parameters.

## 2.0 LITERATURE SURVEY.

In November 1988 a literature survey was carried out at Cranfield using an on-line data base system. This survey was used to compliment literature already held on the subject of parameter identification. As a result of this, it is hoped that the list of references found to date is fairly comprehensive and provides a good basis for research into modified stepwise regression at Cranfield.

The majority of the references obtained as a result of this on-line search relate to work done by V. Klein and his colleagues in the U.S.A. They did much of the pioneering work in the use of a MSR method to identify aircraft stability and control parameters. Other references obtained from the on-line search were of more general interest as they were not concerned specifically with the MSR method of parameter identification.

More recent applications of MSR have been concerned with the identification of the stability and control derivatives of a large scale free flying fighter aircraft model. This work was carried out by the RAE and used flight test data obtained from their High Incidence Research Model aircraft, (HIRM), which was flown in the U.S.A with NASA assistance, reference 2. Within the U.K. further work is being done on HIRM aircraft by the RAE.

Past experience at Cranfield involving the use of the Hawk Model in the dynamic wind tunnel facility ( references 3 and 4 ), indicated that the aircraft model is not easily controlled in the wind tunnel without the use of stability augmentation of some kind being implemented. Thus recent surveys of literature have looked at the theoretical requirements of wind tunnel experiments for determining aircraft derivatives. Another area being looked into is that of the design and application of digital filters to facilitate the derivation of various angular attitude rates which cannot be measured directly from the experimental rig.



### 3.0 The Experimental Facility

A relatively simple dynamic wind tunnel test facility has been designed and built at the College of Aeronautics. Work commenced in 1979 and the detail design development and application has been the subject of a number of M.Sc. and Ph.D. research topics in the intervening years. Figure 1 shows the Hawk aircraft model suspended in a frame which has been positioned in the wind tunnel. Also shown in the photograph are the electronic control unit for the model and the wind tunnel controls.

### 3.1 DESCRIPTION OF THE FACILITY.

During most of the development of the experimental facility a 1/12 scale model of the B.Ae. Hawk aircraft was used. The B.Ae. Hawk was chosen as, when suitably scaled the model had sufficient internal volume for the suspension gimbal and control servos and a reasonable amount of aerodynamic and performance data was available for the aircraft. The model was constructed using standard aeromodelling techniques to provide a light-weight structure to allow for the weight of the enclosed equipment and ensure that dynamic scaling requirements were met. The model is controlled by tailplane, ailerons and rudder which are driven by small precision servo-actuators. Control signals to and from the model, together with power-supply cables, are grouped together to form a trailing umbilical connection to the control unit.

When suspended in the wind tunnel the model has four degrees of freedom since both longitudinal and lateral translation are suppressed. Figure 2 illustrates the motion freedom of the model. The suspension system consists of a vertical rod mounted in bearings at its upper and lower ends, so that it may rotate about its vertical axis. The rod is supported in a large transportable Dexion framework to which it is rigidly attached by wire bracing. The whole assembly, complete with model, can be removed from the wind tunnel as a unit.

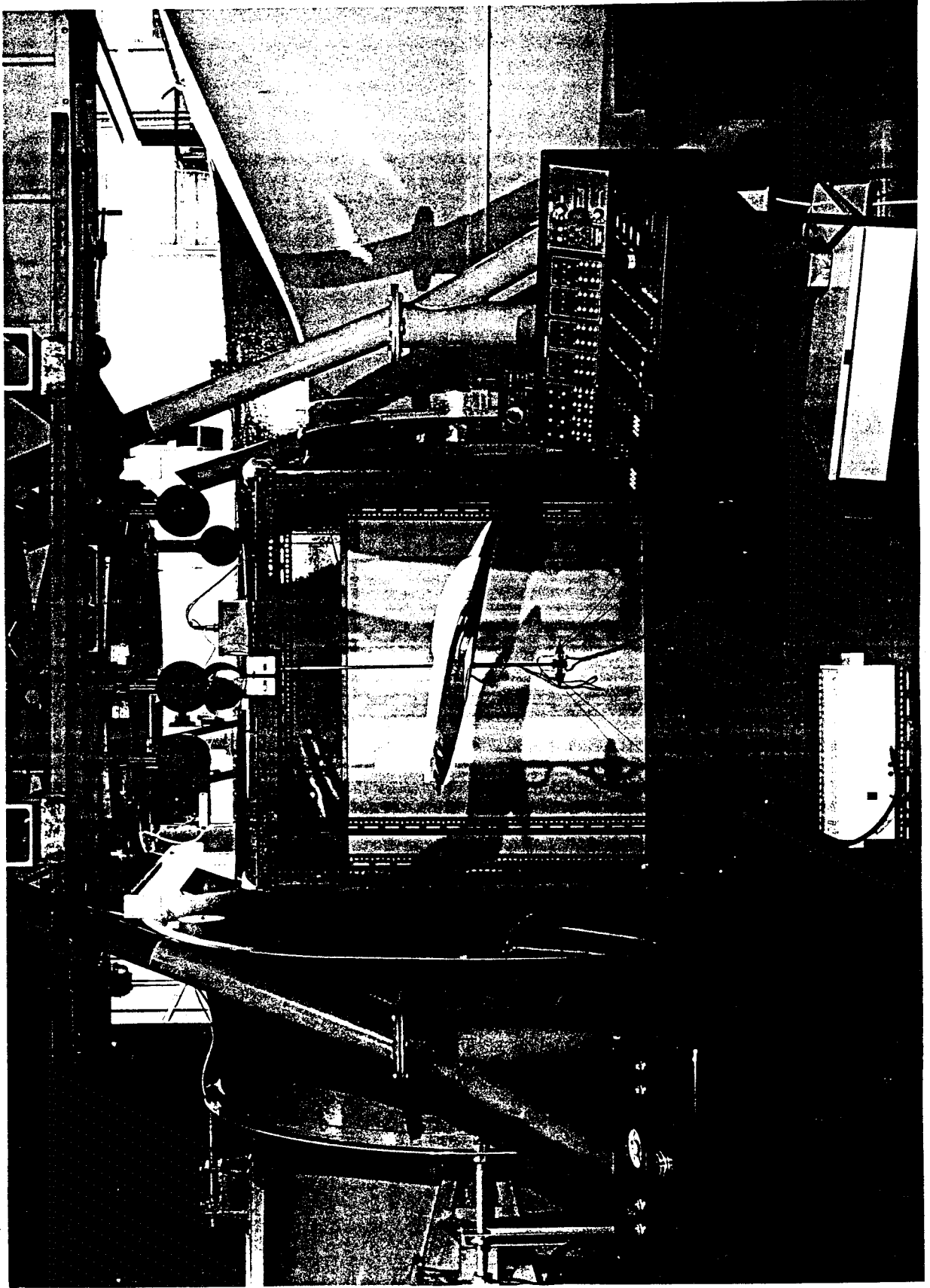
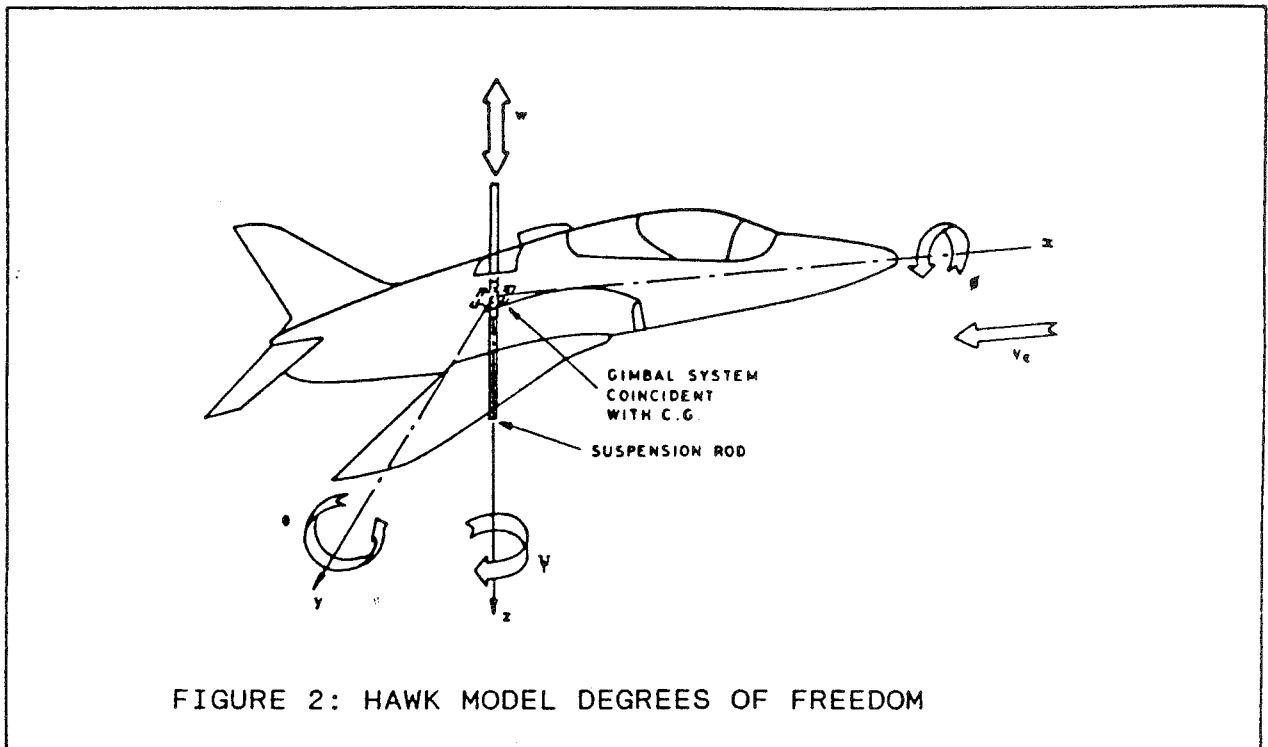


FIG 1: EXPERIMENTAL FACILITY



A sleeve is keyed to the vertical rod so that it may slide freely in a vertical sense but is constrained to rotate with the rod. The sleeve then forms part of the suspension gimbal which is mounted in the model. The model is thus free to rotate in pitch and roll about the sleeve. Rotation in yaw is about the vertical axis of the rod and vertical translation involves the sleeve sliding on the rod. Angular motion in each axis is sensed by means of potentiometers and is limited to  $\pm 30^\circ$ . Vertical motion is possible over approximately 0.75m.

The electronic control unit was designed and built as a small, self-contained, transportable console which employs analogue circuitry throughout. Construction of the control unit is highly modular, to facilitate functional changes, and it provides the following facilities;

- (i) electrical power supplies;
- (ii) input and output interfaces with the model;
- (iii) primary control of the model;
- (iv) programmable analogue computer elements for feedback purposes;
- (v) output signal interfaces for recording and display;
- (vi) input and output interfaces to an external computer.

In earlier work with the dynamic rig all data analysis was based on the use of recorded response time histories using a six-channel pen recorder, since a computer based data acquisition facility was not available. However, subsequent development of the rig necessitated the addition of a digital data-acquisition system in the form of a signal processor to digitise analogue data into the form needed for a digital computer link up.

The data acquisition system which will be used during this research programme is shown schematically in Figure 3. Analogue signals are fed from the Hawk model to the electronic control panel where the signals are sampled and digitised using a Cambridge Electronic Design (CED) 1401 processor. The recorded data is then stored on the hard disk of the host IBM PC-AT micro-computer. Using its own processors, clocks and memory, the 1401 may be programmed by the host computer to simultaneously sample various input channels to the CED at certain desired frequencies. The software to control the CED 1401 is written in TURBO-PASCAL and is run on the actual IBM.

Trials have been carried out to record data using the system described above and the system seemed to work very well. The main concern when working with this system will be to try to cut down the amount of noise associated with the analogue signal passing through various wire cables and the electronic control unit.

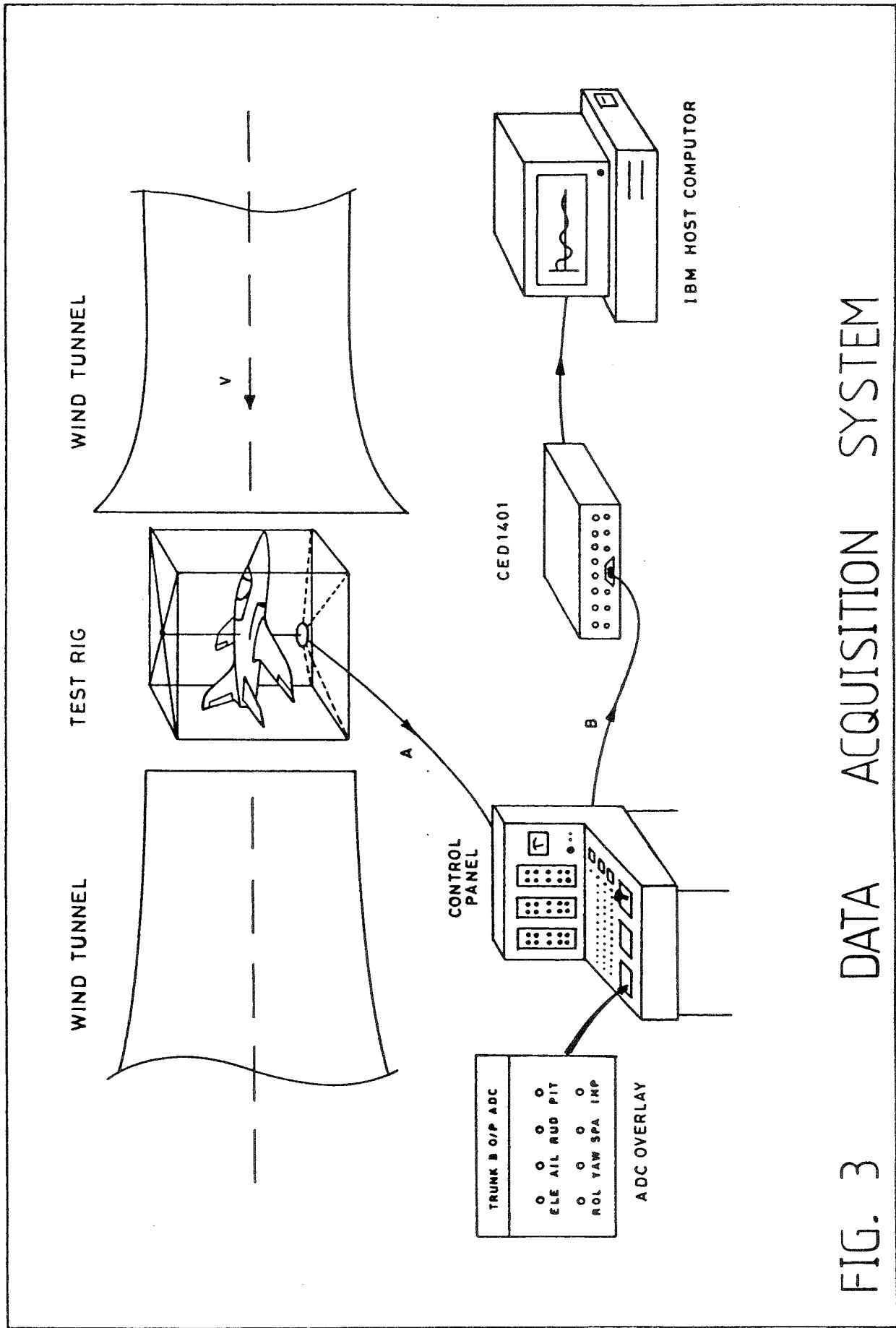


FIG. 3 DATA ACQUISITION SYSTEM

### 3.2 MODEL CALIBRATIONS AND DYNAMIC STABILITY.

Calibrations of the control surface angles have been carried out on the Hawk model, reference 5 . The calibrations are in the form of linear graphs of elevator, aileron and rudder angles against input and output voltages.

When the Hawk model is flown in the wind tunnel a number of things become apparent. The first problem which arises is to do with the mechanical friction between the model gimbal sleeve and the vertical rod. With the wind on the Hawk model is initially at rest on a restraining collar fixed onto the rod. As the tailplane angle is increased to encourage the model to rise vertically into a "flying" position nothing appears to happen at first because of friction in the system. Then, as the tailplane angle is increased further the model suddenly takes off and rises very quickly up the the vertical rod. A great deal of care is required in order to not damage the model.

Another control problem which arises when the model is flown in the wind wind tunnel is that it is difficult to get the model into a trimmed condition. Very small variations in the air flow of the tunnel can affect the model and take it out of trim. A way to improve this situation is to use the analogue circuitry of the electronic control panel to artificially augment the stability of the model. It is thought that the gimbal pivot point in the Hawk model may need to be moved slightly to improve the controllability of the model. It is planned to conduct some experiment to examine the static stability of the Hawk model and to also look at the theoretical requirements of wind tunnel experiments which involve the determination of aircraft stability and control derivatives using a dynamic rig.

#### 4.0 AIRCRAFT EQUATIONS OF MOTION.

The Modified Stepwise Regression procedure will be written as a FORTRAN 77 program in order to computerise the method. It will be necessary to test the program with data produced by aircraft simulation programs. These programs have been written in the Advanced Continuous Simulation Language ACSL and details of the programs are given in section 6.0. The actual experimental setup with the Hawk model in the wind tunnel is such that only four degrees of freedom of the aircraft motion are available. However, it is considered prudent that the MSR program should initially be written using a full set of equations of motion, as is the case with the aircraft simulation programs.

Appendix A describes the full scale dimensional equations of longitudinal and lateral motion for small perturbations when referred to body axes. The appropriate reduced order equations of motion for the model in the wind tunnel are described. Also given, for both the full scale and reduced model cases, are the equations of motion in the format required for the MSR procedure.

#### 5.0 ESTIMATION OF FULL SCALE HAWK DERIVATIVES.

To produce a simulation of the full scale Hawk it is necessary to estimate a complete set of stability and control derivatives for the aircraft. These derivatives need to be entered into the ACSL simulation programs to define the aircrafts motion. A BAe document, reference 6 giving graphical details of various performance and stability and control data was used to obtain the data required. A flight case was chosen so that it fell into the limited flight envelope range which can be produced using the dynamic rig in the wind tunnel. Details of the flight case chosen and the estimation of the various dimensional derivatives is given in Appendix B.

## 6.0 ACSL AIRCRAFT SIMULATION

Mathematical models of the aircraft have been written and a computer simulation of the aircraft has been programmed using the Advanced Continuous Simulation Language (ACSL). It is intended that the computer simulation shall be used as a tool for comparison with wind tunnel tests and for evaluating the MSR software during development of the FORTRAN 77 program.

The ACSL simulation is used to model the basic aircraft equations of motion so that any response to inputs on the real aircraft may be reproduced by the simulation model. Thus any aircraft may be used initially to test the program. To simulate a particular aircraft it is simply a case of re-setting the numerical values of the stability and control derivatives in the program and defining the appropriate flight conditions. It was decided to use Phantom (F4) data to test the simulation program since a design package on the BBC microcomputers had previously been used to produce graphs of the step and impulse responses of the F4. The response graphs from this work were used to compare the responses obtained from the ACSL simulation. The results of this work may be found in reference 5.

When a set of stability and control derivatives had been estimated for the full scale Hawk the longitudinal and lateral ACSL programs were changed as appropriate. The relevant flight conditions were also inserted into the programs. The simulation programs were then run to verify that the correct modes of motion for given control surface inputs were obtained. For example, the longitudinal phugoid oscillation characteristics were checked. The results of some of these tests are given in Appendix C.

In the initial design of the ACSL programs it was not thought necessary to consider the coupled responses of aircraft motion. So the simulation was split into two distinct programs which model the longitudinal and lateral equations of motion for small perturbations separately. Both ACSL simulation programs, LONG.CSL and LAT.CSL, take the same basic form with each one split up into the sections described overleaf in Table 1.



TABLE 1: ACSL PROGRAM STRUCTURE.

PROGRAM TITLE

INITIAL REGION

specify the flight condition

set values of stability and control derivatives

set control surface angle and duration of deflection

END OF INITIAL

DYNAMIC REGION

specify the time for the simulation to run

specify the intervals at which data is to be saved

check if control surface angle should be reset

DERIVATIVE REGION

specify integration algorithm required

specify time step

define equations of motion

integrate states

perform any angular conversions necessary

END OF DERIVATIVE

END OF DYNAMIC

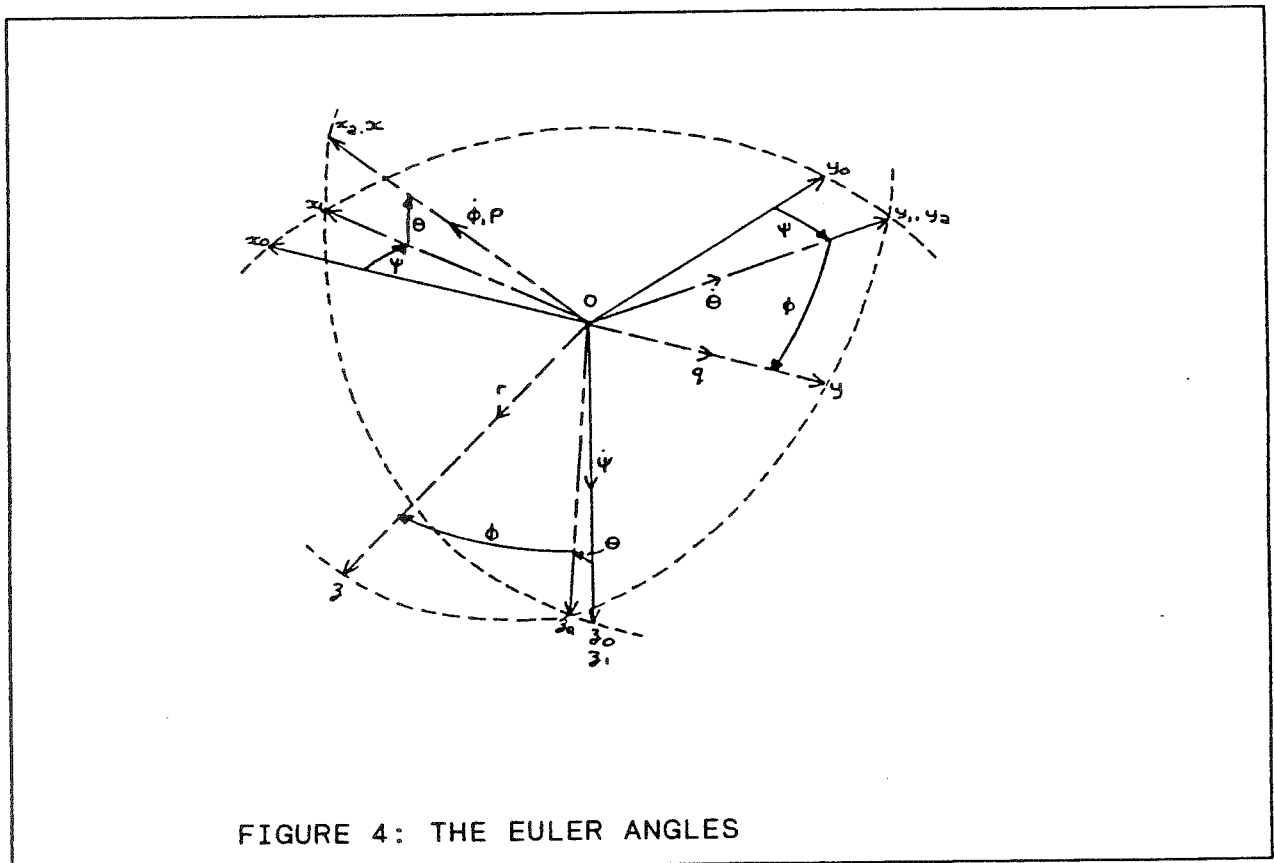
END OF PROGRAM

References 5 and 7 contain further information about the development of the simulation programs and also contain details of how the control surface inputs have been modelled.

## 7.0 HAWK MODEL KINEMATICS.

The Hawk model is suspended on a vertical rod in the wind tunnel by means of the gimbal shown in Figure 5. This means that the kinematics of the Hawk model need to be considered in order to check that the gimbal design does not require any extra angular transformations of data recorded from the dynamic rig. Appendix D defines the various axes systems and transformations which are normally required to define the aircraft motion variables.

On examination it was found that the gimbal was in fact designed so that any perturbed angles recorded by the yaw, pitch and roll potentiometers correspond directly to the Euler Angles defined in Figure 5 below. The linear and rate transformations of Equations D1 and D2 of Appendix D may be applied directly with the required angles  $\psi$ ,  $\theta$  and  $\phi$  being measured directly from the potentiometer readings.



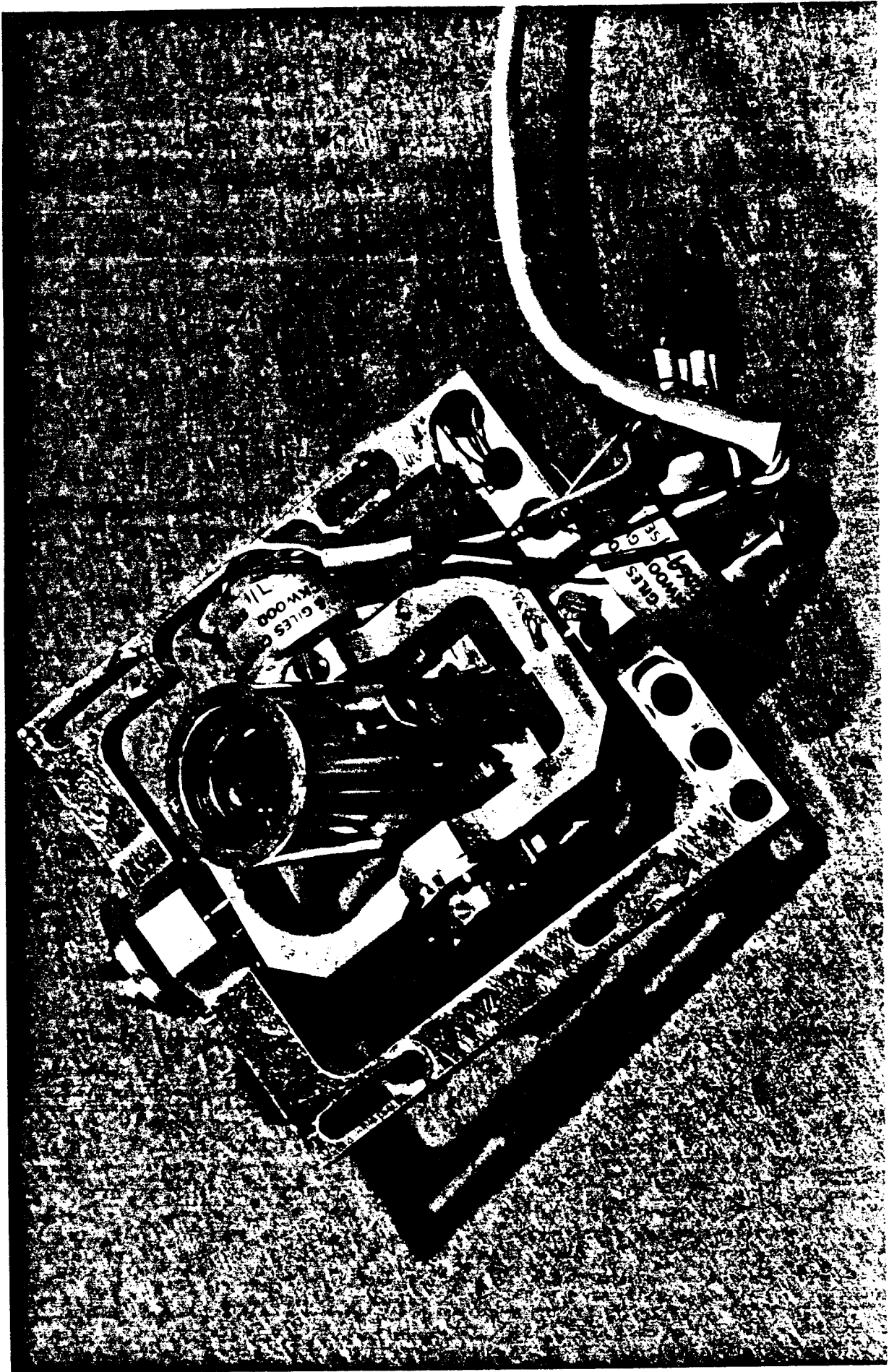


FIG 5: MODEL GIMBAL

The datum axes of the Hawk model are defined to be stability or wind axes and in the steady state these axes are coincident with the wind tunnel/earth axes. Because the resultant aircraft velocity  $V$  is produced horizontally by the wind tunnel the flight path angle  $\gamma$  is zero. This means that the angle of incidence of the aircraft is equal to its pitch angle  $\theta$ . ie.  $\alpha = \theta$ . Also, further constraints on the lateral motion of the model means that the angle of sideslip  $\beta$  is equal to minus the yaw angle, ie.  $\beta = -\psi$ .

Because of model constraints in the wind tunnel the linear transformation of Equation D1 will apply in the following form:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \tilde{A} \begin{pmatrix} 0 \\ 0 \\ z_0 \end{pmatrix} \quad \text{as there is no forward or lateral translation of the model allowed.}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \tilde{A} \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} \quad \text{where } V \text{ is equal to the tunnel speed.}$$

Another practical constraint is that the model is only free to rotate  $\pm 30^\circ$  in any direction. However, as small perturbation equations of motion are being used this should not prove to be a problem.

## 7.1 ATTITUDE RATE INFORMATION.

In an ideal parameter estimation process it is necessary to have a measure of the control variables and all of the corresponding response variables. When, for practical reasons this is not possible alternative ways have to be found for synthesizing estimates of the missing variables, the way in which this is done and the number of missing variables determines how well the estimation process will work.

Currently, in the dynamic model the control variables, aileron, elevator and rudder angles are measured from signals derived from the servo-actuator feedback circuitry. Roll, pitch and yaw attitudes are measured with precision potentiometers built into the suspension system and recently, with some additional equipment it has become possible to measure vertical displacement, velocity and acceleration. Space is also available within the model to install either a small rate gyro or an accelerometer on a temporary basis.

The reconstruction and introduction of the missing variables into the parameter estimation process can be done in a variety of ways. However, most can be expected to introduce additional problems and the objective in this context will be to find an acceptable method for reconstructing variables. In the present application the main problem is to derive rate and acceleration signals from attitude signals. This may be achieved by differentiation using analogue or digital methods, reference 7. But in either case the by-product of the process is usually unacceptable noise levels. Alternatively, state estimation methods may be used but these are complex and require some knowledge of the model which is the subject of the parameter estimation. It is intended that, as far as circumstances will allow alternative means for reconstructing the missing variables will be evaluated.

## 8.0 MODIFIED STEPWISE REGRESSION.

Linear regression is employed to estimate a functional relationship of a dependent variable to one or more or more independent variables. It is assumed that the dependent variable can be closely approximated as a linear combination of the independent variables. For the system identification of an aircraft operating at low angles of attack, the mathematical model structure for aerodynamic forces and moments is linear and may be written in the following form as described in reference 1.

$$y(t) = \theta_0 + \theta_1 x_1(t) + \theta_2 x_2(t) + \dots + \theta_{n-1} x_{n-1}(t) \quad \text{Eqn. 1}$$

Where:  $y(t)$  represents the resultant coefficient of aerodynamic force or moment ( X, Y, Z, L, M, N ) at time  $t$ . These are the dependent variables.

$\theta_1, \theta_2, \dots, \theta_{n-1}$  are the stability and control derivatives; and  $\theta_0$  is the value of any particular coefficient corresponding to the initial steady flight conditions.

$x_1, x_2, \dots, x_{n-1}$  are the independent aircraft state and control variables ( $\alpha, q, \beta, p, r, \eta, \xi, \zeta$ ) and may also include combinations of these variables at time  $t$ .

Modified stepwise regression is a procedure which starts with linear terms only in a mathematical model of the aircraft and inserts or rejects non-linear independent variables into the regression model until the regression equation is satisfactory. The complexity of such an application arises from the additional non-linear terms in the equations of motion and just how many additional terms are required to adequately describe the motion is not very clearly defined. If too many parameters are sought from an estimate made on the basis of a limited number of data measurements, a reduced accuracy in the parameter estimates can be expected or the process might fail altogether.

## 8.1 MATHEMATICAL MODEL FORMULATION.

The MSR procedure is implemented computationally by disassembling the equations of motion into a set of linear simultaneous equations. Each equation, representing one degree of freedom, is re-formatted to the form required for the regression analysis. The mathematical format required for the MSR is best shown by the following example which is taken from the longitudinal equations of motion:

$$\dot{w} = z_0 + z_u \cdot u + z_w \cdot w + z_q \cdot q + z_\eta \cdot \eta \quad \text{Eqn. 2}$$

This is of the form:

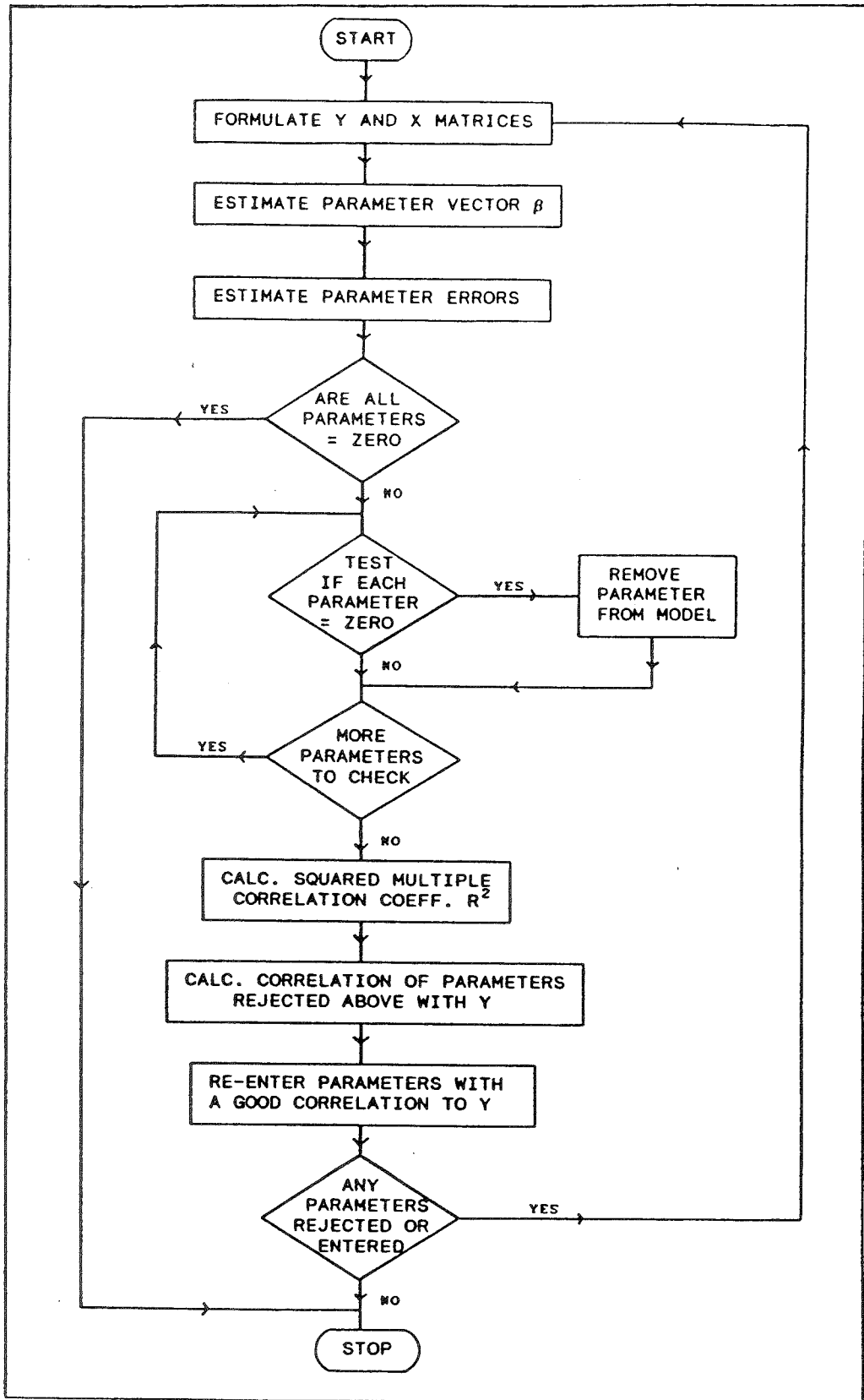
$$y(t) = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \dots + b_{n-1} \cdot x_{n-1} \quad \text{Eqn.3}$$

If a sequence of N readings of y and the x's, (ie  $\dot{w}, u, w, q, \eta$ ), are taken at times  $t_1, t_2, \dots, t_N$  and denoted by  $y(i), x_1(i), x_2(i), \dots, x_{n-1}(i)$  where  $i = 1, 2, \dots, N$  then the data acquired can be related by the following set of N linear equations:

$$y(i) = b_0 + b_1 \cdot x_1(i) + b_2 \cdot x_2(i) + \dots + b_{n-1} \cdot x_{n-1}(i) + \epsilon(i) \quad \text{Eqn.4}$$

$\epsilon(i)$  is the equation error which is introduced here as equation 4 is only an approximation of the actual aerodynamic relationship. Further, Equation 4 may be expressed in matrix form as  $Y = \underline{b} \cdot X$ . The Modified Stepwise Regression procedure is then carried out as described in Appendix E. A computer flow diagram for the MSR process is shown in Figure 6 and the output is an estimate of the coefficients in the regression equations from which the aerodynamic stability derivatives may be deduced.

FIGURE 6: MSR FLOWCHART





## 8.2 MSR SOFTWARE.

Before any software was written to computerise the MSR procedure a number of decisions were taken. The first being to write the program in FORTRAN 77 to run on the College of Aeronautics VAX computer rather than the program being PC based. It was also decided to generate all of the FORTRAN code needed from scratch and not to use any routines such as those provided by the National Algorithms Group, ie NAG routines. There follows a short list of some of the subroutines which have been written for the MSR program with a few comments being made as necessary:

1. To find the transpose of a matrix.
2. To multiply two matrices together.
3. To find the inverse of a matrix.

This routine has been written using a Gaussian Elimination Method to find the inverse. Although about 200 sets of readings will be recorded it is only the matrix ( $X^T X$ ) which needs to be inverted. The maximum order of this matrix is 6 and so relatively few numerical rounding-up errors are expected and computations involving double precision variables will be employed. However, it will still be necessary to check this stage for an ill-conditioned matrix.

Other routines have been written to carry out the calculations described in Appendix E. At the present time the MSR program is being "debugged" to sort out the difficulties which have arisen from the different numbers of independent variables and subscripts required at various stages of the MSR procedure.

It is planned to fully test the program in the near future using data generated from the ACSL simulation programs for the full scale Hawk aircraft. There is also a lot of general data available from statistical books which are not related to aircraft parameter estimation but will never the less provide another means with which to test the MSR program, eg reference 8.

## 9.0 CONCLUSIONS.

To date the experimental facility and data recovery system is fully operational although some work has yet to be done designing and implementing experiments suitable for the present application. Mathematical models of the aircraft have been written and a computer simulation of the aircraft has been programmed using the Advanced Continuous Simulation Language (ACSL). The MSR computer program is being designed, using FORTRAN and it is expected that preliminary testing will take place in the very near future.

The proposed work programme for the next year is best summarised by the following list:

1. Completion of outstanding test rig evaluations are to be carried out. This includes the measurement of aircraft model inertias and mechanical damping in the suspension system.
2. To review the data acquisition interface between the test rig and IBM PC.
3. To continue development of parameter estimation programs and evaluation with ACSL simulations of the Hawk aircraft.
4. Preliminary trials with the wind tunnel model will be carried out to demonstrate the data acquisition and analysis programs.
5. Design and implementation of autostabilisation loops around the wind tunnel model to facilitate experimental requirements to control the model in a "trimmed" state. The mathematical models will be changed to encompass this as required.

## LIST OF SYMBOLS

$C_x$	drag coefficient
$C_y$	sideforce coefficient
$C_z$	lift coefficient
$C_l$	rolling moment coefficient
$C_m$	pitching moment coefficient
$C_n$	yawing moment coefficient
$I_x, I_y, I_z, I_{xz}$	aircraft inertias
$\hat{L}_v, \hat{L}_p, \hat{L}_r, \hat{L}_\xi, \hat{L}_\zeta$	dimensional rolling moment derivative due to sideslip, roll rate, yaw rate, etc.
$\hat{M}_u, \hat{M}_w, \hat{M}_\dot{w}, \hat{M}_q, \hat{M}_\eta$	dimensional pitching moment derivative due to forward velocity, side velocity, etc.
$\hat{N}_v, \hat{N}_p, \hat{N}_r, \hat{N}_\xi, \hat{N}_\zeta$	dimensional yawing moment derivative due to sideslip, roll rate, yaw rate, etc.
$p, q, r$	rate of roll pitch and yaw respectively
$u, v, w$	components of velocity
$V$	total velocity
$\hat{X}_u, \hat{X}_w, \hat{X}_\dot{w}, \hat{X}_q, \hat{X}_\eta$	dimensional drag force derivative due to forward velocity, side velocity, etc.
$\hat{Y}_v, \hat{Y}_p, \hat{Y}_r, \hat{Y}_\xi, \hat{Y}_\zeta$	dimensional sideforce derivative due to sideslip, roll rate, yaw rate, etc.
$\hat{Z}_u, \hat{Z}_w, \hat{Z}_\dot{w}, \hat{Z}_q, \hat{Z}_\eta$	dimensional lift force derivative due to forward velocity, side velocity, etc.
$\alpha$	angle of attack, $\tan^{-1}(w/u)$
$\beta$	angle of sideslip, $\sin^{-1}(v/V)$
$\theta, \phi, \psi$	attitude in pitch, bank and azimuth
$\eta, \xi, \zeta$	control surface angle of elevator, aileron and rudder respectively

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APPENDIX A

FULL SCALE AND REDUCED EQUATIONS OF MOTION



## FULL SCALE EQUATIONS

### A.1 LONGITUDINAL EQUATIONS OF MOTION.

The general dimensional equations of longitudinal symmetric motion for small disturbances (when referred to body axes) may be written as follows (reference 9):

$$m\dot{u} - \dot{\hat{x}}_u \cdot u - \dot{\hat{x}}_w \cdot w - \dot{\hat{x}}_w \cdot \dot{w} + (mW_e - \dot{\hat{x}}_q) \cdot q + mg_1 \cdot \theta = \dot{\hat{x}}_\eta \cdot \eta \quad (A1)$$

$$-z_u \cdot u - z_w \cdot w + (m - z_w) \cdot \dot{w} - (mU_e + z_q) \cdot q + mg_2 \cdot \theta = z_\eta \cdot \eta \quad (A2)$$

$$-\dot{m}_u \cdot u - \dot{m}_w \cdot w - \dot{m}_w \cdot \dot{w} - \dot{m}_q \cdot q + I_y \cdot \dot{q} = \dot{m}_\eta \cdot \eta \quad (A3)$$

where "·" denotes a dimensional coefficient;

In the special case of wind axes and level flight,  $\theta = 0$  giving

$$g_1 = g \cos \theta_e = g ; \quad g_2 = g \sin \theta_e = 0 ;$$

$$U_e = V \cos \alpha_e = V ; \quad W_e = V \sin \alpha_e = 0 ;$$

and since small perturbations are assumed  $\dot{\theta} = q$ .

In the state space format of  $\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u}$ , equations A1-A3 may be written as shown in equation A4 below, reference 7. This is in fact the format required for the MSR procedure:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} x_u & x_w & x_q & -g \\ z_u & z_w & z_q & 0 \\ m_u & m_w & m_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{pmatrix} \begin{pmatrix} \eta \end{pmatrix} \quad (A4)$$

where:

$$\dot{\hat{x}}_u = \dot{\hat{x}}_u / m ; \quad \dot{\hat{x}}_w = \dot{\hat{x}}_w / m ; \quad \dot{\hat{x}}_w = \dot{\hat{x}}_w / m ; \quad \dot{\hat{x}}_q = \dot{\hat{x}}_q / m ; \quad \dot{\hat{x}}_\eta = \dot{\hat{x}}_\eta / m .$$

$$\dot{z}_u = \dot{z}_u / m ; \quad \dot{z}_w = \dot{z}_w / m ; \quad \dot{z}_w = \dot{z}_w / m ; \quad \dot{z}_q = \dot{z}_q / m ; \quad \dot{z}_\eta = \dot{z}_\eta / m .$$

$$\dot{m}_u = \dot{m}_u / I_y ; \quad \dot{m}_w = \dot{m}_w / I_y ; \quad \dot{m}_w = \dot{m}_w / I_y ; \quad \dot{m}_q = \dot{m}_q / I_y ; \quad \dot{m}_\eta = \dot{m}_\eta / I_y .$$

(definitions continued overleaf)



$$x_u = \left( \frac{\dot{x}_w \dot{z}_u}{(1-\dot{z}_w)} + \dot{x}_u \right); \quad x_w = \left( \frac{\dot{x}_w \dot{z}_w}{(1-\dot{z}_w)} + \dot{x}_w \right); \quad x_q = \left( \frac{(U_e + \dot{z}_q) \dot{x}_w}{(1-\dot{z}_w)} + (\dot{x}_q - W_e) \right);$$

$$z_u = \left( \frac{\dot{z}_u}{1-\dot{z}_w} \right); \quad z_w = \left( \frac{\dot{z}_w}{1-\dot{z}_w} \right); \quad z_q = \left( \frac{U_e + \dot{z}_q}{1-\dot{z}_w} \right);$$

$$m_u = \left( \frac{\dot{m}_w \dot{z}_u}{(1-\dot{z}_w)} + \dot{m}_u \right); \quad m_w = \left( \frac{\dot{m}_w \dot{z}_w}{(1-\dot{z}_w)} + \dot{m}_w \right); \quad m_q = \left( \frac{(U_e + \dot{z}_q) \dot{m}_w}{(1-\dot{z}_w)} + \dot{m}_q \right);$$

$$x_\eta = \left( \frac{\dot{x}_w \dot{z}_\eta}{(1-\dot{z}_w)} + \dot{x}_\eta \right); \quad z_\eta = \left( \frac{\dot{z}_\eta}{1-\dot{z}_w} \right); \quad m_\eta = \left( \frac{\dot{m}_w \dot{z}_\eta}{(1-\dot{z}_w)} + \dot{m}_\eta \right)$$

## A.2 LATERAL EQUATIONS OF MOTION.

The general dimensional equations of lateral asymmetric motion, referred to body axes, for small disturbances may be written as follows, reference 9:

$$m \dot{v} - \dot{Y}_v \cdot v - (mW_e + \dot{Y}_p) p + (mU_e - \dot{Y}_r) r - mg_1 \phi - m g_2 \psi = \dot{Y}_\xi \cdot \xi + \dot{Y}_\zeta \cdot \zeta \quad A5$$

$$-I_v \dot{v} + I_x \dot{p} - I_p \cdot p - I_{xz} \dot{r} - I_r \cdot r = I_\xi \cdot \xi + I_\zeta \cdot \zeta \quad A6$$

$$-N_v \dot{v} + I_{xz} \dot{p} - N_p \cdot p + I_z \dot{r} - N_r \cdot r = N_\xi \cdot \xi + N_\zeta \cdot \zeta \quad A7$$

In the special case of wind axes and level flight,  $\theta_e = 0$  giving

$$g_1 = g \cos \theta_e = g; \quad g_2 = g \sin \theta_e = 0;$$

and since small perturbations are assumed the following relationship

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{pmatrix} \cdot \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad A8$$

reduces to  $\dot{\phi} = p; \quad \dot{\theta} = q; \quad \dot{\psi} = r.$

The lateral equations of motion in the standard state variable form  $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$ , ie the form required by the MSR may be written

$$\begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} y_v & y_p & y_r & g & 0 \\ l_v & l_p & l_r & 0 & 0 \\ n_v & n_p & n_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} y_\xi & y_\zeta \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \quad A9$$

where:

$$\dot{y}_v = \dot{Y}_v/m; \quad \dot{y}_p = \dot{Y}_p/m; \quad \dot{y}_r = \dot{Y}_r/m; \quad \dot{y}_\xi = \dot{Y}_\xi/m; \quad \dot{y}_\zeta = \dot{Y}_\zeta/m$$

$$l_v = \dot{L}_v/I_x; \quad l_p = \dot{L}_p/I_x; \quad l_r = \dot{L}_r/I_x; \quad l_\xi = \dot{L}_\xi/I_x; \quad l_\zeta = \dot{L}_\zeta/I_x$$

$$\dot{n}_v = \dot{N}_v/I_z; \quad \dot{n}_p = \dot{N}_p/I_z; \quad \dot{n}_r = \dot{N}_r/I_z; \quad \dot{n}_\xi = \dot{N}_\xi/I_z; \quad \dot{n}_\zeta = \dot{N}_\zeta/I_z$$

$$e_x = I_{xz}/I_x; \quad e_z = I_{xz}/I_z; \quad E_{xz} = 1 + e_x e_z$$

$$y_v = \dot{y}_v; \quad y_p = (\dot{y}_p + W_e); \quad y_r = (\dot{y}_r - U_e);$$

$$l_v = \left\{ \frac{\dot{l}_v}{E_{xz}} + \frac{e_x \dot{n}_v}{E_{xz}} \right\}; \quad l_p = \left\{ \frac{\dot{l}_p}{E_{xz}} + \frac{e_x \dot{n}_p}{E_{xz}} \right\}; \quad l_r = \left\{ \frac{\dot{l}_r}{E_{xz}} + \frac{e_x \dot{n}_r}{E_{xz}} \right\};$$

$$n_v = \left\{ \frac{-e_z \dot{l}_v}{E_{xz}} + \frac{\dot{n}_v}{E_{xz}} \right\}; \quad n_p = \left\{ \frac{-e_z \dot{l}_p}{E_{xz}} + \frac{\dot{n}_p}{E_{xz}} \right\}; \quad n_r = \left\{ \frac{-e_z \dot{l}_r}{E_{xz}} + \frac{\dot{n}_r}{E_{xz}} \right\};$$

$$y_\xi = \dot{y}_\xi; \quad l_\xi = \left\{ \frac{\dot{l}_\xi}{E_{xz}} + \frac{e_x \dot{n}_\xi}{E_{xz}} \right\}; \quad l_\zeta = \left\{ \frac{\dot{l}_\zeta}{E_{xz}} + \frac{e_x \dot{n}_\zeta}{E_{xz}} \right\}$$

$$y_\zeta = \dot{y}_\zeta; \quad n_\xi = \left\{ \frac{-e_z \dot{l}_\xi}{E_{xz}} + \frac{\dot{n}_\xi}{E_{xz}} \right\}; \quad n_\zeta = \left\{ \frac{-e_z \dot{l}_\zeta}{E_{xz}} + \frac{\dot{n}_\zeta}{E_{xz}} \right\}$$

## REDUCED MODEL EQUATIONS OF MOTION.

### A.3 LONGITUDINAL EQUATIONS

Equations (A1, A2, A3) are for free flight aircraft. However, with the Hawk model in the wind tunnel longitudinal translation is suppressed, which means Eqn (A1) may be removed completely. Further, when considering wind axes (rather than body axes) and assuming the tunnel speed remains constant ( $u=0$ ) the following conditions apply:

$$\alpha_e = W_e = 0 \qquad u = 0$$

$$U_e = V \cos 0 = V \qquad \dot{u} = 0$$

In the case of horizontal steady flight we also have  $\theta_e = \alpha_e = W_e = 0$  giving  $g_1 = g$  and  $g_2 = 0$ .

Thus the dimensional equations for semi-free flight are

$$-\dot{z}_w \cdot w + (m - \dot{z}_w) \cdot \dot{w} - (mU_e + \dot{z}_q) \cdot q = \dot{Z}(t) \qquad \text{A10}$$

$$-\dot{M}_w \cdot w - \dot{M}_w \cdot \dot{w} - \dot{M}_q \cdot q + I_y \cdot \dot{q} = \dot{M}(t) \qquad \text{A11}$$

The format of the equations required for MSR is as follows:

$$\begin{pmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} z_w & z_q & 0 \\ m_w & m_q & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} z_\eta \\ m_\eta \\ 0 \end{pmatrix} \begin{pmatrix} \eta \end{pmatrix} \qquad \text{A12}$$

$$\text{where } z_w = \left( \frac{\dot{z}_w}{1 - \dot{z}_w} \right); \quad z_q = \left( \frac{U_e + \dot{z}_q}{1 - \dot{z}_w} \right); \quad z_\eta = \left( \frac{\dot{z}_\eta}{1 - \dot{z}_w} \right)$$

$$m_w = \left( \frac{\dot{m}_w \cdot \dot{z}_w}{(1 - \dot{z}_w)} + \dot{m}_w \right); \quad m_q = \left( \frac{(U_e + \dot{z}_q) \dot{m}_q}{(1 - \dot{z}_w)} + \dot{m}_q \right); \quad m_\eta = \left( \frac{\dot{m}_w \cdot \dot{z}_\eta}{(1 - \dot{z}_w)} + \dot{m}_\eta \right)$$

#### A.4 LATERAL EQUATIONS OF MOTION

In the case of semi-free flight in the wind tunnel Eqn A5 is removed as lateral translation of the aircraft model is suppressed. Thus, referred to wind axes we are left with only

$$-\hat{L}_v \cdot v + I_x \cdot \dot{p} - \hat{L}_p \cdot p - I_{xz} \cdot \dot{r} - \hat{L}_r \cdot r = \hat{L}(t) \quad A13$$

$$-\hat{N}_v \cdot v + I_{xz} \dot{p} - \hat{N}_p \cdot p - I_z \cdot \dot{r} - \hat{N}_r \cdot r = \hat{N}(t) \quad A14$$

$-\hat{L}_v \cdot v$  and  $-\hat{N}_v \cdot v$  have been retained as in experimental work we will take the sideslip angle and yaw angle to be equivalent.

The dynamic model aircraft is free to rotate in yaw but there is no translation in the y-direction so that the aerodynamic sideforces and gravity components are balanced by the support system. However, the fact that  $dy/dt = 0$  implies that the lateral acceleration,  $\dot{v} - pW_e + rU_e = 0$  A15

Therefore, the lateral equations of motion, (incorporating equations A13, A14 and A15), with respect to wind tunnel simulations for steady horizontal datum flight may be expressed in the standard state variable form  $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ , reference 7:

$$\begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 0 & W_e & -U_e & 0 & 0 \\ l_v & l_p & l_r & 0 & 0 \\ n_v & n_p & n_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \quad A9$$

where  $E_{xz} = 1 + e_x e_z$

$$l_v = \left\{ \frac{\overset{\circ}{l}_v}{E_{xz}} + \frac{e_x \overset{\circ}{n}_v}{E_{xz}} \right\}; \quad l_p = \left\{ \frac{\overset{\circ}{l}_p}{E_{xz}} + \frac{e_x \overset{\circ}{l}_p}{E_{xz}} \right\}; \quad l_r = \left\{ \frac{\overset{\circ}{l}_r}{E_{xz}} + \frac{e_x \overset{\circ}{l}_r}{E_{xz}} \right\};$$

(definitions continued overleaf)

$$n_v = \left\{ \frac{-e_z \dot{l}_v}{E_{xz}} + \frac{\dot{n}_v}{E_{xz}} \right\}; \quad n_p = \left\{ \frac{-e_z \dot{l}_p}{E_{xz}} + \frac{\dot{n}_p}{E_{xz}} \right\}; \quad n_r = \left\{ \frac{-e_z \dot{l}_r}{E_{xz}} + \frac{\dot{n}_r}{E_{xz}} \right\};$$

$$\text{and } l_\xi = \left\{ \frac{\dot{l}_\xi}{E_{xz}} + \frac{e_x \dot{n}_\xi}{E_{xz}} \right\}; \quad l_\zeta = \left\{ \frac{\dot{l}_\zeta}{E_{xz}} + \frac{e_x \dot{n}_\zeta}{E_{xz}} \right\}$$

$$n_z = \left\{ \frac{-e_z \dot{l}_\xi}{E_{xz}} + \frac{\dot{n}_\xi}{E_{xz}} \right\}; \quad n_\zeta = \left\{ \frac{-e_z \dot{l}_\zeta}{E_{xz}} + \frac{\dot{n}_\zeta}{E_{xz}} \right\}$$

**APPENDIX B**

**ESTIMATE OF FULL SCALE HAWK DERIVATIVES**



## ESTIMATION OF FULL SCALE HAWK DERIVATIVES.

### FLIGHT CASE DEFINITION and HAWK DESIGN DETAILS:

A/C SPEED             $M = 0.31$                              $V = 105.5 \text{ m/sec}$   
A/C MASS              $m = 9000 \text{ lb}$                              $m = 4082.4 \text{ kg}$   
A/C HEIGHT           $\text{Sea Level}$   
A/C C.G. at           $h_g = 0.275 \bar{c}$

WING AREA                             $s = 179.635 \text{ ft}^2 = 16.6887 \text{ m}^2$   
WING SPAN                             $b = 30.808 \text{ ft} = 9.3903 \text{ m}$   
HORIZONTAL TAIL ARM             $\bar{l}_T = 14.109 \text{ ft} = 4.299 \text{ m}$   
INCLINATION OF FUSELAGE  
DATUM TO AIRSTREAM             $\alpha_f = 4^\circ$

### MOMENT OF INERTIA ABOUT LONGITUDINAL, LATERAL, VERTICAL BODY AXES

$I_x = 5346.7 \text{ kg.m}^2$   
 $I_y = 19534.4 \text{ kg.m}^2$   
 $I_z = 23786.5 \text{ kg.m}^2$

PRODUCT OF INERTIA             $I_{xz} = 816.74 \text{ kg.m}^2$

### CONVERSION FACTORS:

1.  $\rho V S = 2156.81 \text{ kg/sec}$
2.  $\rho V S \bar{l}_T = 9272.11 \text{ kgm/sec}$
3.  $\rho V^2 S = 227543.02 \text{ kgm/sec}^2$
4.  $\rho S (\bar{l}_T)^2 = 377.83 \text{ kgm}$
5.  $\rho V S (\bar{l}_T)^2 = 39860.79 \text{ kgm}^2/\text{sec}$
6.  $\rho V^2 S \bar{l}_T = 978207.43 \text{ kgm}^2/\text{sec}^2$
7.  $(1/2)\rho V S b = 10126.53 \text{ kgm/sec}$
8.  $(1/2)\rho V^2 S b = 1068348.60 \text{ kgm}^2/\text{sec}^2$
9.  $(1/4)\rho V S b^2 = 47545.56 \text{ kgm}^2/\text{sec}$



### 3.1 LONGITUDINAL DERIVATIVES AND MODES OF MOTION.

$$\dot{X}_u = X_u * \rho VS = -64.71 \text{ kg/sec} \Rightarrow x_u = -0.016$$

$$\dot{X}_w = X_w * \rho S \bar{l}_T = 0.0 \text{ kg} \Rightarrow x_w = 0.0$$

$$\dot{X}_w = X_w * \rho VS = +107.841 \text{ kg/sec} \Rightarrow x_w = 0.026$$

$$\dot{X}_q = X_q * \rho VS \bar{l}_T = 0.0 \text{ kgm/sec} \Rightarrow x_q = 0.0$$

$$\dot{X}_\eta = X_\eta * \rho V^2 S = 0.0 \text{ kgm/sec}^2 \Rightarrow x_\eta = 0.0$$

$$\dot{Z}_u = Z_u * \rho VS = -884.37 \text{ kg/sec} \Rightarrow z_u = -0.217$$

$$\dot{Z}_w = Z_w * \rho S \bar{l}_T = 0.0 \text{ kg} \Rightarrow z_w = 0.0$$

$$\dot{Z}_w = Z_w * \rho VS = -5478.297 \text{ kg/sec} \Rightarrow z_w = -1.342$$

$$\dot{Z}_q = Z_q * \rho VS \bar{l}_T = -5628.104 \text{ kgm/sec} \Rightarrow z_q = -1.379$$

$$\dot{Z}_\eta = Z_\eta * \rho V^2 S = -89196.856 \text{ kgm/sec}^2 \Rightarrow z_\eta = -21.849$$

$$\dot{M}_u = M_u * \rho VS \bar{l}_T = -120.536 \text{ kgm/sec} \Rightarrow m_u = -0.005$$

$$\dot{M}_w = M_w * \rho S (\bar{l}_T)^2 = -92.19 \text{ kgm} \Rightarrow m_w = -0.0047$$

$$\dot{M}_w = M_w * \rho VS \bar{l}_T = -1066.292 \text{ kgm/sec} \Rightarrow m_w = -0.048$$

$$\dot{M}_q = M_q * \rho VS (\bar{l}_T)^2 = -23039.658 \text{ kgm}^2/\text{sec} \Rightarrow m_q = -1.669$$

$$\dot{M}_\eta = M_\eta * \rho V^2 S \bar{l}_T = -383457.144 \text{ kgm}^2/\text{sec}^2 \Rightarrow m_\eta = -19.527$$

#### SHORT PERIOD PITCHING OSCILLATION:

$$(s^2 + 2\rho_{sp} \omega_{sp} s + \omega_{sp}^2)$$

$$\omega_{sp} = 2.8 \text{ rad/sec}$$

$$\rho_{sp} = 0.54$$

$$\Rightarrow s = (-1.512 \pm 2.357i)$$

#### PHUGOID:

$$(s^2 + 2\rho_p \omega_p s + \omega_p^2)$$

$$\omega_p = 0.077 \text{ rad/sec}$$

$$\rho_p = 0.065$$

$$\Rightarrow s = (-0.005 \pm 0.077i)$$

#### CHARACTERISTIC EQUATION:

$$\Delta(s) = s^4 + 3.034s^3 + 7.876s^2 + 0.096s + 0.046 = 0$$

LATERAL DERIVATIVES AND MODES OF MOTION.

$$\dot{Y}_v = Y_v * \rho V S = -864.879 \text{ kg/sec} \Rightarrow y_v = - 0.212$$

$$\dot{Y}_p = Y_p * (1/2)\rho V S b = 0.0 \text{ kgm/se} \Rightarrow y_p = 0.0$$

$$\dot{Y}_r = Y_r * (1/2)\rho V S b = 0.0 \text{ kgm/sec} \Rightarrow y_r = 0.0$$

$$\dot{Y}_\xi = Y_\xi * \rho V^2 S = 0.0 \text{ kgm/sec}^2 \Rightarrow y_\xi = 0.0$$

$$\dot{Y}_\zeta = Y_\zeta * \rho V^2 S = -31173.394 \text{ kgm/sec}^2 \Rightarrow y_\zeta = + 7.636$$

$$\dot{L}_v = L_v * (1/2)\rho V S b = -486.073 \text{ kgm/sec} \Rightarrow l_v = - 0.085$$

$$\dot{L}_p = L_p * (1/4)\rho V S b^2 = -20206.865 \text{ kgm}^2/\text{sec} \Rightarrow l_p = - 3.780$$

$$\dot{L}_r = L_r * (1/4)\rho V S b^2 = +5943.196 \text{ kgm}^2/\text{sec} \Rightarrow l_r = + 1.038$$

$$\dot{L}_\xi = L_\xi * (1/2)\rho V^2 S b = -188136.189 \text{ kgm}^2/\text{sec}^2 \Rightarrow l_\xi = -34.842$$

$$\dot{L}_\zeta = L_\zeta * (1/2)\rho V^2 S b = +30982.109 \text{ kgm}^2/\text{sec}^2 \Rightarrow l_\zeta = + 5.075$$

$$\dot{N}_v = N_v * (1/2)\rho V S b = +875.945 \text{ kgm/sec} \Rightarrow n_v = + 0.040$$

$$\dot{N}_p = N_p * (1/4)\rho V S b^2 = -3138.007 \text{ kgm}^2/\text{sec} \Rightarrow n_p = - 0.002$$

$$\dot{N}_r = N_r * (1/4)\rho V S b^2 = -10550.361 \text{ kgm}^2/\text{sec} \Rightarrow n_r = - 0.479$$

$$\dot{N}_\xi = N_\xi * (1/2)\rho V^2 S b = +25640.366 \text{ kgm}^2/\text{sec}^2 \Rightarrow n_\xi = + 2.274$$

$$\dot{N}_\zeta = N_\zeta * (1/2)\rho V^2 S b = -107903.209 \text{ kgm}^2/\text{sec}^2 \Rightarrow n_\zeta = - 4.711$$

ROLL SUBSIDENCE MODE:  $(1 + sT_R)$ ;  $T_R = 0.33 \text{ sec}$ ;  $s = -3.0 \text{ sec}^{-1}$

SPIRAL MODE:  $(1 + sT_S)$ ;  $T_S = 91.74 \text{ sec}$ ;  $s = -0.0109 \text{ sec}^{-1}$

DUTCH ROLL:  $(s^2 + 2\rho_{dr}\omega_{dr}s + \omega_{dr}^2)$

$\omega_{dr} = 2.0 \text{ rad/sec}$        $\rho_{dr} = 0.178$        $\Rightarrow s = (-0.356 \pm 1.968i)$

CHARACTERISTIC EQUATION:

$$\Delta(s) = s( s^4 + 3.753s^3 + 6.198s^2 + 12.188s + 0.132 ) = 0$$



APPENDIX C

FULL SCALE HAWK SIMULATION RESULTS



## FULL SCALE HAWK SIMULATION RESULTS.

The longitudinal and lateral ACSL programs were run to compare the aircraft responses obtained with those predicted by the BAe report (reference 6) and those obtained using a control system design package on the BBC microcomputer. Figures C1 - C5 show the various longitudinal and lateral modes obtained using the ACSL simulation. TABLE C1 summarises the frequencies, damping ratios and time constants obtained from reference 7, the BBC package and measured from the ACSL responses, figures C1 - C5.

TABLE 1: MODES OF MOTION OF THE BAe HAWK.

### SPPO:

BAe	$s = (-1.512 \pm 2.357i);$	$\omega_{sp} = 2.8 \text{ rad/sec};$	$\rho_{sp} = 0.54$
BBC	$s = (-1.512 \pm 2.231i);$	$\omega_{sp} = 2.7 \text{ rad/sec};$	$\rho_{sp} = 0.56$
ACSL GRAPH	$s = (-0.805 \pm 2.155i)$	$\omega_{sp} = 2.3 \text{ rad/sec}$	$\rho_{sp} = 0.68$

### PHUGOID:

BAe	$s = (-0.005 \pm 0.077i);$	$\omega_p = 0.077 \text{ rad/sec};$	$\rho_p = 0.065$
BBC	$s = (-0.002 \pm 0.071i);$	$\omega_p = 0.069 \text{ rad/sec};$	$\rho_p = 0.073$
ACSL GRAPH	$s = (-0.001 \pm 0.071i);$	$\omega_p = 0.071 \text{ rad/sec};$	$\rho_p = 0.070$

### ROLL SUBSIDENCE:

BAe	$s = (-3.0 \text{ sec}^{-1});$	$T_R = 0.33 \text{ sec}$
BBC	$s = (-3.8 \text{ sec}^{-1});$	$T_R = 0.26 \text{ sec}$
ACSL GRAPH	$s = (-2.3 \text{ sec}^{-1});$	$T_R = 0.43 \text{ sec}$

### SPIRAL MODE:

BAe	$s = (-0.0109 \text{ sec}^{-1});$	$T_s = 91.73 \text{ sec}$
BBC	$s = (-0.0005 \text{ sec}^{-1});$	$T_s = 2000 \text{ sec}$
ACSL GRAPH	$s = (-0.0321 \text{ sec}^{-1});$	$T_s = 31.11 \text{ sec}$

### DUTCH ROLL:

BAe	$s = (-0.356 \pm 1.968i);$	$\omega_{dr} = 2.0 \text{ rad/sec};$	$\rho_{dr} = 0.178$
BBC	$s = (-0.320 \pm 2.090i);$	$\omega_{dr} = 2.1 \text{ rad/sec};$	$\rho_{dr} = 0.153$
ACSL RAPH	$s = (-0.152 \pm 2.044i)$	$\omega_{dr} = 2.05 \text{ rad/sec};$	$\rho_{dr} = 0.148$

FIGURE C1: LONGITUDINAL SHORT PERIOD PITCHING OSCILLATION.

INPUT: IMPULSE TO ELEVATOR.

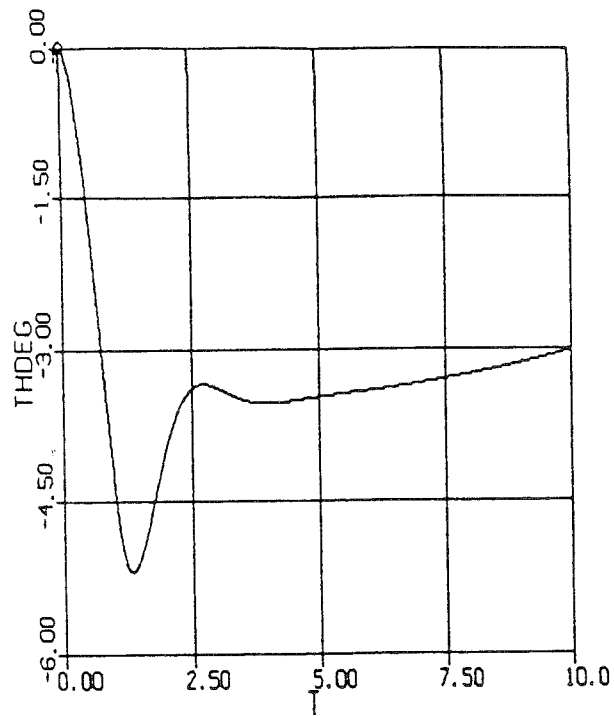


FIGURE C2: LONGITUDINAL PHUGOID OSCILLATION.

INPUT: IMPULSE TO ELEVATOR.

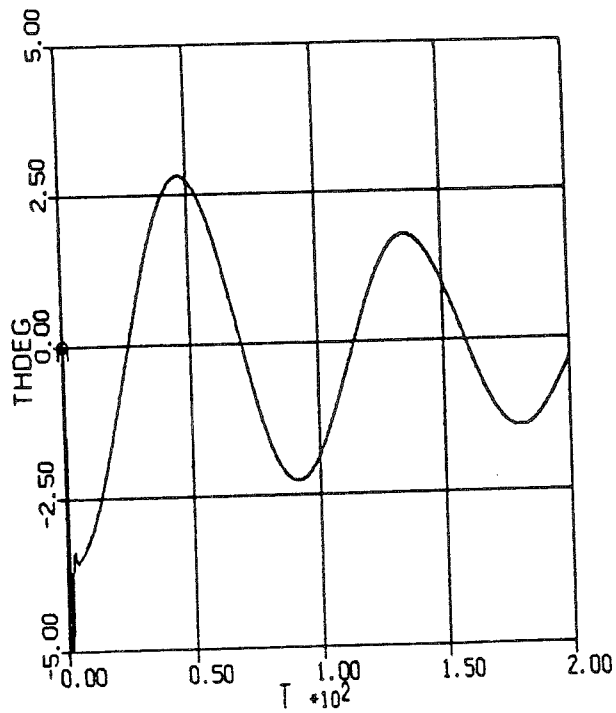


FIGURE C3: LATERAL ROLL SUBSIDENCE MODE.

INPUT: STEP TO AILERON.

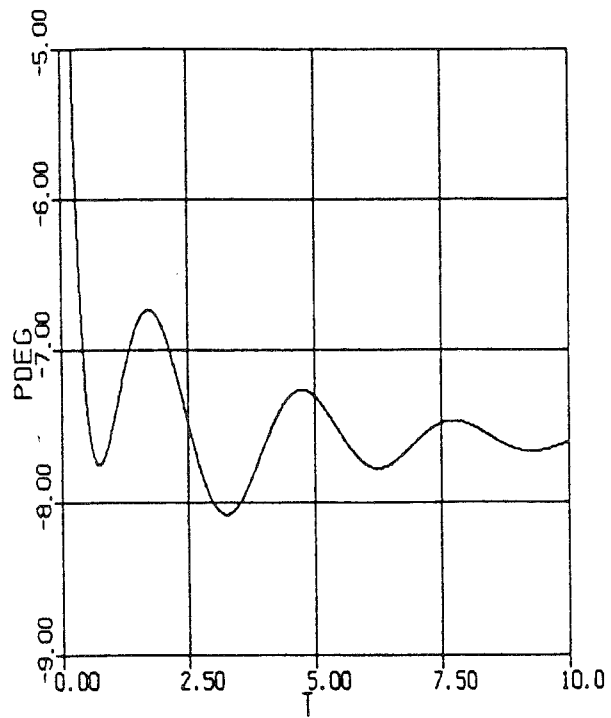


FIGURE C4: LATERAL SPIRAL MODE.

INPUT: STEP TO RUDDER.

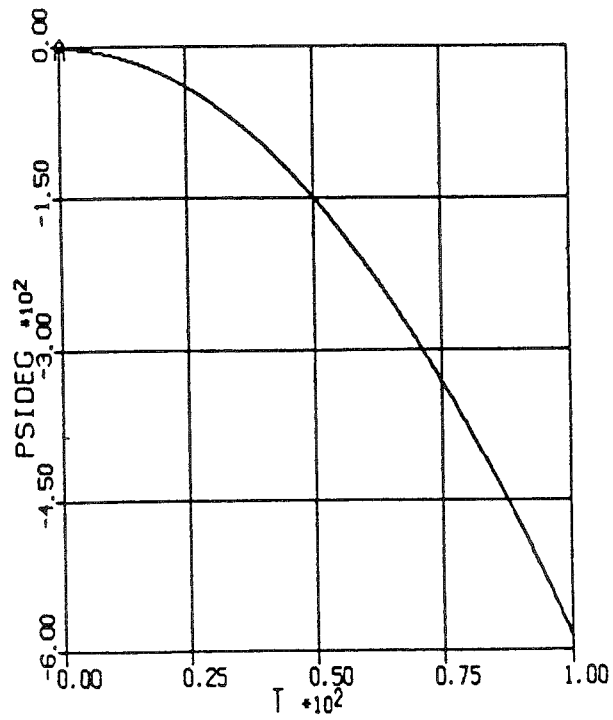
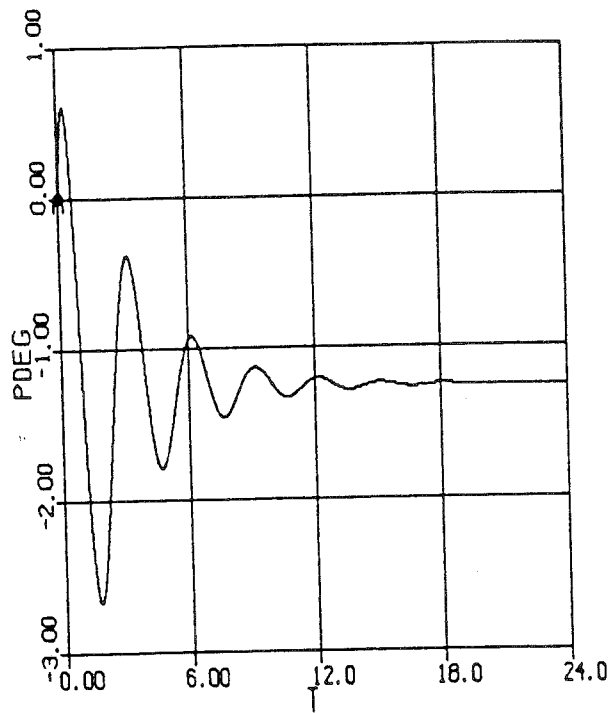




FIGURE C5: LATERAL DUTCH ROLL OSCILLATION.  
INPUT: IMPULSE TO RUDDER.



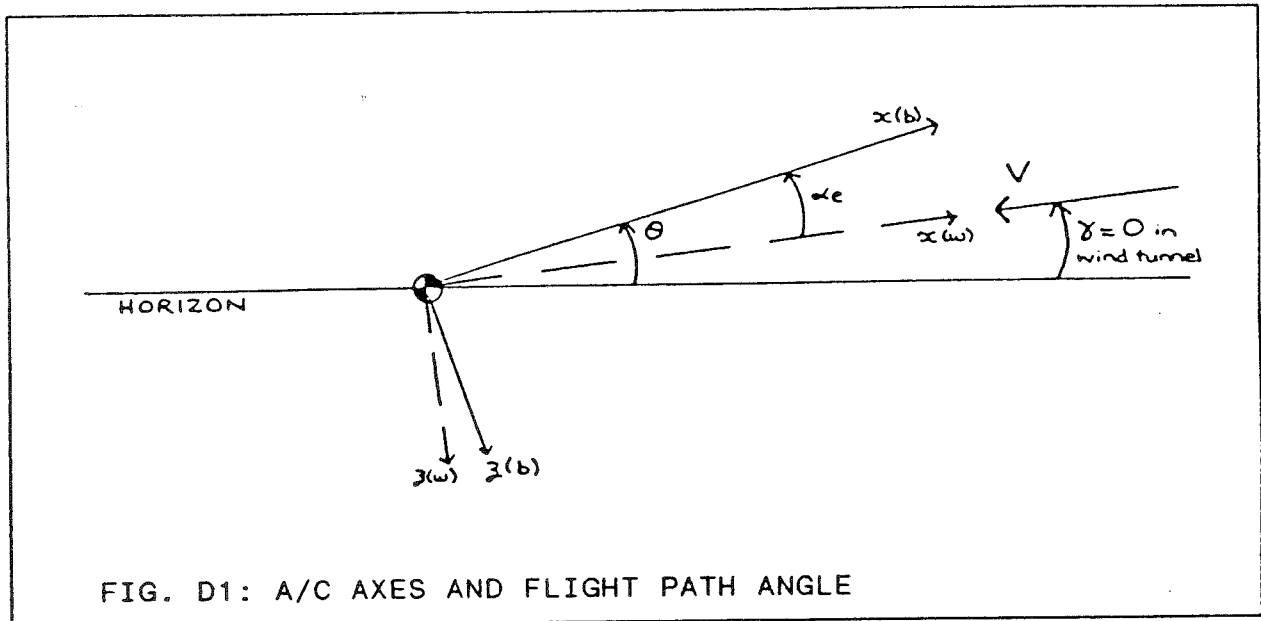
APPENDIX D

AXES SYSTEMS AND TRANSFORMATIONS



## AXES SYSTEMS AND TRANSFORMATIONS.

It is convenient to define a set of axes  $(Oxyz)_{\text{wind}}$  fixed in the aircraft such that the  $Ox$  axis is coincident with the resultant total velocity vector  $V$  in the plane of symmetry of the aircraft. This axis system is referred to as wind or stability axes and is equivalent to body axes rotated through the body incidence angle  $(\alpha_e)$  about the  $Oy$  axis. Figure D1 shows the relationship between body and wind axes.



In a disturbance the attitude of the aircraft is defined by the orientation of the disturbed body axes  $(Oxyz)$  with respect to the steady state datum body axes  $(OX_0Y_0Z_0)$ . The angular attitude of the aircraft may be established by considering the rotation, about each axis in turn, which is necessary to bring  $(OX_0Y_0Z_0)$  into coincidence with  $(Oxyz)$ . Referring to Figure 5, the angles  $\psi$ ,  $\theta$  and  $\phi$  define the aircraft attitude with respect to the datum and are called the Euler Angles.

In order to transform the linear quantities of displacement, velocity and acceleration or force it is usual to consider vector quantities  $X_0Y_0Z_0$  in the first axes set  $(OX_0Y_0Z_0)$  and use Figure 4 to define their angular relationship with the transformed vector quantities  $xyz$  in the second axes set  $(Oxyz)$ .

Transforming  $X_0Y_0Z_0$  by rotations through the yaw angle  $\psi$ , the pitch angle  $\theta$  and then the roll angle  $\phi$  leads to the following transformation relationship, reference 10:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\sim}{A} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \quad \text{Eqn. D1}$$

where

$$\underset{\sim}{A} = \begin{pmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi & \sin\psi\sin\theta\sin\phi & \cos\theta\sin\phi \\ -\sin\psi\cos\phi & +\cos\psi\cos\phi & \\ \cos\psi\sin\theta\cos\phi & \sin\psi\sin\theta\cos\phi & \cos\theta\cos\phi \\ +\sin\psi\sin\phi & -\cos\psi\sin\phi & \end{pmatrix}$$

The transformation matrix for angular perturbation quantities is that which relates attitude rates to body rates. If the angular velocities with respect to earth axes ( $OX_0Y_0Z_0$ ) are  $\dot{\phi}$ ,  $\dot{\theta}$  and  $\dot{\psi}$  and the angular velocities of the disturbed body fixed axes ( $Oxyz$ ) are  $p, q$  and  $r$ , then the following linear relationships between the angular velocities in the two axes systems may be deduced from Figure 4 :

$$\begin{aligned} \text{roll rate} \quad p &= \dot{\phi} - \dot{\psi}\sin\theta \\ \text{pitch rate} \quad q &= \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\ \text{yaw rate} \quad r &= -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi\cos\theta \end{aligned}$$

$$\text{i.e.} \quad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\phi \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \quad \text{Eqn. D2}$$

N.b. For small perturbations the first order approximations  $p = \dot{\phi}$ ,  $q = \dot{\theta}$  and  $r = \dot{\psi}$  may be made.

APPENDIX E

MSR STATISTICAL THEORY



## STATISTICAL THEORY

Consider a sequence of N readings of y and x's, which are taken at times  $t_1, t_2, \dots, t_N$  and denoted by  $y(i), x_1(i), x_2(i), \dots, x_{n-1}(i)$  where  $i = 1, 2, \dots, N$ . The data acquired can then be represented by the following set of N linear equations:

$$\hat{y}(i) = \beta_0 + \beta_1 \cdot x_1(i) + \beta_2 \cdot x_2(i) + \dots + \beta_{n-1} \cdot x_{n-1}(i) + \epsilon(i) \quad E1$$

$\epsilon(i)$  is the equation error which is introduced here as equation E1 is only an approximation of the actual aerodynamic relationship. In Eqn E1,  $\beta_1$  to  $\beta_{n-1}$  are the stability and control derivatives to be estimated;  $\beta_0$  is a constant dependent on the initial steady-state flight conditions;  $x_1$  to  $x_{n-1}$  are the measured independent aircraft state and control variables; and finally, y is the measured dependent variable, ie the output variable.

The way in which the MSR proceeds in order to estimate the stability and control parameters ( $\beta$ ) is best shown by breaking the method down into the following steps:

**STEP 1** The mathematical model is formulated using the output y and the appropriate x variables for the current iteration of the MSR procedure. This is done by noting that Eqn E1 may be expressed in matrix form as:

$$\begin{matrix} \begin{pmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{pmatrix} \\ Y \end{matrix} = \begin{matrix} \begin{pmatrix} 1 & x_{1(1)} & x_{2(1)} & \dots & x_{n-1(1)} \\ 1 & x_{1(2)} & x_{2(2)} & \dots & x_{n-1(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1(N)} & x_{2(N)} & \dots & x_{n-1(N)} \end{pmatrix} \\ X \end{matrix} \begin{matrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{n-1} \end{pmatrix} \\ \beta \end{matrix} + \begin{matrix} \begin{pmatrix} \epsilon(1) \\ \epsilon(2) \\ \vdots \\ \epsilon(n-1) \end{pmatrix} \\ \epsilon \end{matrix} \quad E2$$

**STEP 2** For  $N \gg n$ , the first estimate of the derivatives,  $\beta$ , can be made using the method of least squares as given below:

$$\beta = \left( X^T X \right)^{-1} X^T Y \quad E3$$

where  $\beta$  is the  $n \times 1$  vector of parameter estimates, Y is the  $N \times 1$  vector of measured variables of  $Y(i)$ , and X is the  $N \times n$  matrix of measured independent variables.



It should be noted that the properties of the least-squares estimates ( $\beta$ ) depend upon the following assumptions concerning the measured dependent variables and equation error [ $\epsilon(i)$ ]:

1.  $\epsilon$  is a stationary vector with zero mean value
2.  $\epsilon$  is uncorrelated with  $x$
3.  $X$  is a deterministic quantity (ie. the state and input variables are measured without errors)
4.  $\epsilon(i)$  is identically distributed and uncorrelated with zero mean and variance  $\sigma^2$

Under assumptions 1 and 2 the estimates for  $\beta$  are unbiased. In practice assumptions 3 and 4 are unobtainable and some error is introduced into the value of the parameter estimates.

**STEP 3** This step is used to calculate the standard errors involved in the parameter estimates  $\beta$ . The covariance matrix of parameter errors has the form:

$$E \left\{ (\beta - b) (\beta - b)^T \right\} = \sigma^2 \left( X^T X \right)^{-1} \quad E4$$

For an estimate of this covariance matrix,  $\sigma^2$  may be replaced by its estimate

$$s^2 = \frac{1}{N - n} \sum_{i=1}^N \hat{\epsilon}^2(i) \quad \text{where } \hat{\epsilon}(i) = y(i) - \hat{y}(i) \quad E5$$

$y(i)$  is the recorded output and  $\hat{y}(i)$  the estimated output given by

$$\hat{y}(i) = \beta_0 + \beta_1 \cdot x_1(i) + \beta_2 \cdot x_2(i) + \dots + \beta_{n-1} \cdot x_{n-1}(i) \quad \text{ie. Eqn E1}$$

Thus in order to calculate the covariances of E4 the following intermediate equations are evaluated:

(i) The residual sum of squares (RSS) =  $Y^T Y - \beta^T X^T Y$  E6

(ii) The value of residual variance  $s^2(\epsilon) = \text{RSS}/(N-n)$  E7  
follows from E6 and provides an indication of the overall error in the regression equation at this stage.

An estimate of the standard error  $s_{(\beta_j)}$ ,  $(j=0,1,\dots,n-1)$  of each individual parameter estimate  $\beta_j$  is made next. If the matrix  $(X^T X)^{-1}$  is denoted as

$$(X^T X)^{-1} = \begin{pmatrix} C_{00} & C_{01} & \dots & C_{0n-1} \\ C_{10} & C_{11} & & C_{1n-1} \\ \vdots & \vdots & & \vdots \\ C_{n-10} & C_{n-11} & \dots & C_{n-1n-1} \end{pmatrix} \quad \text{E8}$$

then the estimated standard error  $s_{\beta_j}$  for each parameter estimate  $\beta_j$  is calculated as

$$s_{\beta_0} = s \sqrt{C_{00}} ; \quad s_{\beta_1} = s \sqrt{C_{11}} ; \quad \dots ; \quad s_{\beta_{n-1}} = s \sqrt{C_{n-1}} ; \quad \text{E9}$$

where  $s = \sqrt{s^2(\epsilon)}$  and is obtained from Eqn E7.

**STEP 4** This step consists of a test which is carried out on the overall regression equation E1 to examine the possibility that all of the parameter estimates  $\beta$  are equal to zero. The null and alternative hypotheses for this test are formulated as:

$$H_0: \beta_1 = \beta_2 = \dots = \beta_{n-1} = 0$$

$$H_1: \text{not all } \beta_j = 0$$

The null hypothesis  $H_0$  is rejected if  $F > F(v_1, v_2, \alpha_p)$  where:

$$F = \frac{\beta^T X^T Y - N \bar{y}^2}{(n-1) s^2(\epsilon)} ; \quad \bar{y} = \frac{1}{N} \sum y(i) \quad \text{E10}$$

and where  $F$  is a random variable having an F-distribution with  $v_1 = n-1$  and  $v_2 = N-n$  degrees of freedom. Tabulated values of the F-distribution,  $F(v_1, v_2, \alpha_p)$ , may be found in statistical reference tables such as Ref.13. These tables give the values of the F-distribution for various significance levels  $\alpha_p$ . For example, if the value of  $F$  (calculated from E10 above) is greater than  $F(v_1, v_2, \alpha_p)$  when  $\alpha_p = 0.05$ , then it is possible to say with a confidence of 95% of being correct that not ALL of the derivatives  $\beta_j$  are zero, although one or two may be zero.

If at least 100 sets of observations  $N$  have been recorded the effect of  $n$  (the number of independent variables) on the tabulated values of  $F$  is small and a value of 12 may be chosen, ref.14.

**STEP 5** The significance of individual terms in the regression is examined next using a partial F-test. The hypotheses used are:

$$H_0: \beta_j = 0$$

$$H_1: \beta_j \text{ is not } = 0$$

For each independent variable the testing criterion calculated is

$$F_p = \frac{[\beta_j]^2}{s^2_{\beta_j}} \quad \text{E11}$$

where  $s^2_{\beta_j}$  is the variance estimate of  $\beta$  obtained from E9.

The null hypothesis is rejected if  $F_p > F(v_1, v_2, \alpha)$  where  $v_1 = 1$  and  $v_2 = N-n$ . Therefore, if  $F_p > 12$ , it may be assumed that the parameter being tested is not equal to zero and should be kept in the regression equation. If  $F_p < 12$  there is a chance that  $\beta_j = 0$ . This partial F-test is applied to all the parameter estimates in the current mathematical model. If one or more parameter is found to have a value of  $F_p < 12$  then the parameter with the lowest value of  $F_p$  is taken to be equal to zero and the corresponding x variable is rejected from the regression equation as only one variable may be rejected at a time.

**STEP 6** At this stage in the MSR procedure it is worth calculating the squared multiple correlation coefficient  $R^2$ . This coefficient is used as an indication of how "well" the independent variables  $x_1, x_2, \dots, x_{n-1}$  (which are in the current regression equation) correlate with the dependent variable  $y$ . The closer the value of  $R^2$  to 1, the better the correlation and the confidence in the mathematical regression model.  $R^2$  is given by E12:

$$R^2 = \frac{\sum_i [\hat{y}(i) - \bar{y}]^2}{\sum_i [y(i) - \bar{y}]^2} = \frac{\beta^T X^T Y - \bar{N} y^2}{Y^T Y - \bar{N} y^2} \quad \text{E12}$$

and is related to the variable F by 
$$F = \frac{N - n}{n - 1} \cdot \frac{R^2}{1 - R^2} \quad \text{E13}$$

**STEP 7** The variables which are not in the current mathematical model are examined next to see if adding any one of these variables would improve the goodness-of-fit of the regression model to the measured data. The variables are looked at in turn to see how well they correlate with the recorded values of  $y(i)$  given the variables which are already in the regression equation.

Consider for example, the case where the current regression equation contains the two variables  $x_2$  and  $x_3$ , say, and the other variables  $x_1, x_4, x_5, \dots, x_{n-1}$  are not in the equation. The mathematical model being used to fit the experimental measured data would be:

$$y = \beta_0 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3 + \epsilon \quad \text{E14}$$

A new independent variable  $z_1$  is then constructed by finding the residuals of  $x_1$  after regressing it on both  $x_2$  and  $x_3$ , ie. the residuals from fitting the model

$$x_1 = \beta_0 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3 + \epsilon \quad \text{E15}$$

The variable  $z_1$  is therefore given by

$$z_1 = x_1 - ( \beta_0 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3 ) \quad \text{E16}$$

Similarly, the variables  $z_4, z_5, \dots, z_{n-1}$  are formed by regressing the variable  $z_4$  on  $(x_2, x_3)$ ,  $z_5$  on  $(x_2, x_3)$ , etc. A new dependent variable  $y^*$  is represented by the residuals of  $y$  regressed on  $(x_2, x_3)$  using the model given by Eqn E14. This yields

$$y^* = y - \beta_0 - \beta_2 \cdot x_2 - \beta_3 \cdot x_3 \quad \text{E17}$$

A new set of correlations which involve the variables  $y^*, z_1, z_4, z_5, \dots, z_{n-1}$  is formulated. These partial correlations can be written as  $r_{jy.23}$  meaning the correlations of  $z_j$  and  $y^*$  are related to the model containing the variables  $x_2$  and  $x_3$ . The expressions for the partial correlation coefficients  $r_{jy.23}$  is given by equations E18 - E21 (shown overleaf) where  $y$  is replaced by  $y^*$  and  $x_j$  replaced by  $z_j$ .

The correlation coefficient is given by the expression;

$$r_{jy} = \frac{s_{jy}}{(s_{jj} s_{yy})^{1/2}} \quad \text{E18}$$

where

$$s_{jy} = \sum_N [x_j(i) - \bar{x}_j][y(i) - \bar{y}] \quad \text{E19}$$

$$s_{jj} = \sum_N [x_j(i) - \bar{x}_j]^2 \quad \text{E20}$$

$$s_{yy} = \sum_N [y(i) - \bar{y}]^2 \quad \text{E21}$$

$$\bar{x}_j = \frac{1}{N} \sum_N x_j(i); \quad \bar{y} = \frac{1}{N} \sum_N y(i) \quad \text{E22}$$

The next variable selected to enter the regression equation is the one whose partial correlation coefficient is the greatest. However it is necessary to perform one more test on the variable which has been chosen to enter before it is fully accepted into the regression equation. If the variable to enter is say  $x_5$  a new set of parameter estimates  $(\beta_0, \beta_2, \beta_3, \beta_5)$  is made using the method described in steps one and two. A partial F-test is performed on the variable  $x_5$  by calculating

$$F_p = \frac{[\beta_j]^2}{s^2 \beta_j} \quad \text{(E11)}$$

and if  $F_p > 12$ , the variable is allowed to enter the equation.

#### ITERATIONS OF THE MSR.

At every step of the regression, the variables incorporated into the model in previous stages and any new variable entering the model are reexamined. The partial  $F_p$  criterion, given by equation E11, is evaluated for each variable and compared with the preselected percentage point of the appropriate F-distribution. This provides information about the contribution made by each variable to the fit of the regression model to the recorded data.

Any variable which provides a nonsignificant contribution (ie small value of  $F_p$ ) is removed from the model. A variable which may have been the best single variable to enter at an early stage may, at a later stage, be superfluous because of the relationship between it and other variables now in the regression. The process of selecting and checking variables continues until no more variables can be admitted to the equation and no more need to be rejected.

During each iteration of the MSR the squared multiple correlation coefficient  $R^2$  is also calculated and noted (step 6). The value of  $R^2$  should get closer and closer to 1 as terms are rejected and reentered into the regression model until no further improvement in  $R^2$  is found. If it is found that the MSR cycles between two models the model with the value of  $R^2$  which is closer to one would be chosen as the best fit to the recorded data.