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## Output-only identification of rigid body motions of floating structures: a case study

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### Abstract

In order to identify rigid body motions of floating offshore structures, output-only techniques are very useful for developing low-cost intermediate-scale experimental activities directly into the sea, instead of wave tanks. A crucial parameter, however, is the length of the response records used as input for the identification process, since short records may result in significant loss of accuracy, while long ones may be incompatible with the assumption of stationarity of the sea state. This work presents a sensitivity study conducted on a numerical model of a spar structure, identified by means of Enhanced Frequency Domain Decomposition method. An overview on the efficiency of the method is given for various lengths of response record, along with practical indications on the minimum values acceptable.

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### 1. Introduction

Operational Modal Analysis (OMA) indicates a class of engineering methods for dynamic identification of civil structures in operational conditions [1], i.e. under the effect of ambient or operational loads such as wind, traffic, etc. These methods, also referred to as output-only identification techniques, prove to be very useful especially in the case of large structures (buildings, bridges, etc.), where the classical input-output methods of Experimental Modal

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Analysis (EMA), based on the artificial excitation of the structure at reasonably high levels, cannot be applied. Although significantly cheaper with respect to EMA techniques, OMA methods do not rely on the measurement of the input load but require some assumptions, concerning the characteristics of the system (linear, time-invariant, classically and usually lightly damped), the operational load (white noise) and the choice of the measurement points, which should be such that the contributions of all the structural modes to the overall response of the system can be observed. Under these hypotheses, it is possible to develop output-only techniques both in the time and in the frequency domain, aimed to identify modal characteristics of the structures, i.e. mode shapes, natural frequencies and damping ratios. Time-domain (parametric) techniques use discrete response time series or their correlation functions, while frequency-domain (non-parametric) methods use the power spectral density (PSD) matrices of the response. Comparative studies between time and frequency domain methods, as that of Grocel [2], show that the two categories are comparable in terms of efficiency, however the properties of Fourier transform may affect the efficiency of frequency-domain techniques at low frequencies, resulting in the need of longer response time histories to be recorded, in order to achieve the proper resolution in frequencies. Fields of application of OMA are multiple and range from dynamic identification of large structures to monitoring activities, damage detection, vibration level estimations, fatigue estimations, etc. Recently, Ruzzo et al. [3] proposed to use OMA techniques for the identification of rigid body motions of floating offshore platforms, particularly in the context of intermediate-scale open-sea experimental activities. This kind of activities has been recently proposed [4] as an alternative to small-scale experiments in controlled environment (wave tanks and ocean basins), because it would allow a significant reduction of costs, due to the absence of any artificial wave generation, as well as a reduction of the scale effect, since larger scale models would be tested in a more relevant environment and for longer durations. The uncontrolled nature of the load (incoming irregular waves) fits well with output-only identification techniques, which prove to be accurate also in the case of narrow-banded wave spectra, as long as the wave peak frequencies are sufficiently far from the modal natural frequencies to be identified.

The present paper is based on the work of Ruzzo et al. [3], who applied the Enhanced Frequency Domain Decomposition (EFDD) method to identify the modal properties of a numerical model of spar floating platform implemented in ANSYS AQWA, proving that high-precision results can be obtained using realistic input wave spectra. The same numerical model is adopted here, and a sensitivity study is carried out to evaluate the effect of the response record length on the results of the identification process. When applying EFDD method to the response time histories of a real floating structures, indeed, it must be considered that the maximum record length may be limited, due to the requirement of stationarity of the wave process (sea state) [5]. The results obtained provide an useful insight into the behavior of the EFDD method for relatively short response records, as well as a benchmark for upcoming experimental activities on intermediate-scale models of floating structures.

## 2. Description of the method and of the case study

### 2.1. EFDD method

Frequency Domain Decomposition method was firstly proposed by Brinckner et al. [6] and then enhanced in order to make possible the estimation of modal damping ratios [7]. It has been selected for the application on floating structures due to its simplicity and capacity of dealing with closely spaced and repeated modes. A detailed analytic treatment of the method is out of scope in this work, hence only a brief description is given in the following, referring in particular to the enhanced version of the method, i.e. EFDD.

The equation of motion of a linear, time-invariant and classically damped dynamic system, may be written as:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{f}(t) \quad (1)$$

being  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  the inertia, damping and stiffness matrices,  $\mathbf{f}(t)$  and  $\mathbf{y}(t)$  the external force and response vectors. In general, the system of coordinates chosen for Eq. 1 is such that the matrices are not diagonal, i.e. the degrees of

freedom are generally coupled between each other. However, provided that the system is linear, the response vector  $\mathbf{y}(t)$  and its covariance matrix  $\mathbf{C}_{\mathbf{y}\mathbf{y}}(\tau)$  can be written in terms of modal coordinates  $\mathbf{q}(t)$  and mode shape matrix  $\Phi$  as:

$$\mathbf{y}(t) = \Phi \mathbf{q}(t) \quad (2)$$

$$\mathbf{C}_{\mathbf{y}\mathbf{y}}(\tau) = E[\mathbf{y}(t+\tau)\mathbf{y}(t)^H] = \Phi \mathbf{C}_{\mathbf{q}\mathbf{q}}(\tau) \Phi^H \quad (3)$$

On assuming that modal coordinates are uncorrelated under the white noise input, the covariance matrix  $\mathbf{C}_{\mathbf{q}\mathbf{q}}$  of the modal coordinates is diagonal. By taking the Fourier transform at both sides of Eq. 3 one obtains:

$$\mathbf{G}_{\mathbf{y}\mathbf{y}}(\omega) = \Phi \mathbf{G}_{\mathbf{q}\mathbf{q}}(\omega) \Phi^H \quad (4)$$

where the PSD matrix  $\mathbf{G}_{\mathbf{q}\mathbf{q}}$  of the modal coordinates is diagonal, with positive real components. Eq. 4 represents a modal decomposition of the PSD matrix  $\mathbf{G}_{\mathbf{y}\mathbf{y}}$  of the response, which is formally equivalent to a Singular Value Decomposition (SVD), being  $\mathbf{G}_{\mathbf{q}\mathbf{q}}$  diagonal. The application of FDD method descends directly from this consideration. Indeed, given the response time series  $\mathbf{y}(t)$  under operational loads,  $\mathbf{G}_{\mathbf{y}\mathbf{y}}$  can be easily calculated by means of Fourier transform, while  $\Phi$  and  $\mathbf{G}_{\mathbf{q}\mathbf{q}}$  can be estimated at each discrete frequency value through SVD. Keeping in mind that SVD algorithm conventionally sorts singular values in descending order, the first singular value plot will usually show a peak at each modal frequency, where the corresponding mode dominates the structure dynamics. The singular value corresponding to the peak will be an estimate of the modal PSD at the associated natural frequency and the corresponding singular vector will be an estimate of the mode shape. In cases when a resonating mode does not dominate the response of the system, e.g. because of the presence of a closely-spaced or a repeated mode, the corresponding peak is shifted at the second or at the successive singular values and the same happens, correspondingly, to the singular vectors. In order to obtain also a damping estimation, the modal PSD is finally estimated over the rest of the frequency domain, using a similarity criterion on the mode shapes, namely Modal Assurance Criterion (MAC). In detail, the  $i$ -th singular value at a frequency  $\omega$  is assumed to be representative of the modal PSD ordinate of the  $j$ -th mode as long as the singular vector  $\phi_i(\omega)$  at  $\omega$  is sufficiently similar to the mode shape  $\phi(\omega_{n,j})$  at modal frequency  $\omega_{n,j}$ . according to the following criterion:

$$m_{ij}(\omega) = |\phi_i(\omega) \cdot \phi(\omega_{n,j})| > m_{\min} \quad (5)$$

The limit value  $m_{\min}$  chosen in this study is 0.82, as in [3]. The scalar product of Eq. 5 is named modal coherence. Once the modal PSD is reconstructed on the largest portion of the frequency domain possible, modal damping ratio is estimated by means of classical techniques, such as that of the logarithmic decrement.

## 2.2. Case study

The structure considered in this work is a spar floating support for offshore wind turbines, inspired to the OC3-Hywind spar buoy [8], but simplified to meet the linearity requirement of the method. The spar support is basically a slender, vertical, ballast-stabilized floating platform, anchored to the seabed through catenary mooring lines. The structure has been modeled using the numerical software ANSYS AQWA (see Fig.1) and the response in terms of the six rigid body motions (surge, sway, heave, roll, pitch and yaw) under irregular waves has been obtained. It must be said that the wind turbine (rotor+tower) has been represented in parked conditions, i.e. only in terms of mass properties. The hull of the structure has been represented through 104 Line Bodies and 11 Point Masses, while the mooring system through 3 Non-Linear Catenary lines, whose behavior is however almost linear in the range of motions of interest for the present work. Aggregate characteristics of the model are reported in Table 1 in terms of mass and stiffness matrices.

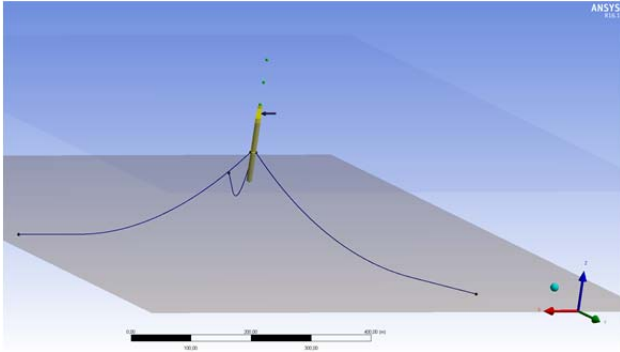


Fig. 1. Numerical model of the floating spar structure considered in this work.

Table 1. Mass and stiffness characteristics of the model.

Matrix indices	Total mass (structural + added mass)	Total stiffness (linearized)
11, 22	$1.623 \cdot 10^7$ kg	$6.478 \cdot 10^4$ N/m
33	$7.997 \cdot 10^6$ kg	$3.523 \cdot 10^5$ N/m
44, 55	$3.461 \cdot 10^8$ kgm <sup>2</sup> /rad	$1.355 \cdot 10^9$ Nm/rad
66	$1.181 \cdot 10^8$ kgm <sup>2</sup> /rad	$9.834 \cdot 10^7$ Nm/rad
15, 24, 42, 51	$1.379 \cdot 10^8$ kgm/rad	0.0 N/rad

Because the model has symmetric properties, specifically for surge-sway and roll-pitch degrees of freedom (see Table 1), two modes have the same frequency (repeated). Although damping mechanisms of spar floating structures are pretty complex, including low radiation damping, viscous drag damping, etc., only a linear damping matrix has been used in this work, since identification procedure can only return a set of damping ratios under the assumption that the model is linear. It is relevant to observe that this choice is consistent with the classical approach followed for any study of floating platform dynamics in the frequency domain, which depends on the linearization of all the terms of the equation of motion. Since Ruzzo et al. [3], working on the same structure, showed that EFDD efficiency proves pretty good for four very different damping combinations, the same linear damping matrix of the first of those four cases has been adopted also for this work. In particular, it is representative of low damping conditions of the structure and has been obtained by setting a damping ratio of 0.01 for all the six rigid modes. The characteristics of the input sea state used for the sensitivity study are: significant wave height  $H_s = 2.00$  m, peak frequency  $\omega_p = 1.05$  rad/s, JONSWAP spectrum [5] and propagation direction  $\theta = 45.0^\circ$ . The true mode shape matrix  $\Phi$  and the natural frequency vector  $\Omega$  of the numerical model, obtained by modal analysis, are:

$$\Phi = \begin{bmatrix} 0.707 & 0.707 & 0 & 0.115 & 0.115 & 0 \\ 0.707 & -0.707 & 0 & 0.115 & -0.115 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.018 & -0.018 & 0 & -0.698 & 0.698 & 0 \\ 0.018 & 0.018 & 0 & -0.698 & -0.698 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \quad \Omega = \begin{bmatrix} 0.063 \\ 0.063 \\ 0.205 \\ 0.202 \\ 0.202 \\ 0.912 \end{bmatrix} \quad (\text{rad / s}) \quad (6)$$

### 3. Numerical results and conclusions

A typical singular value plot obtained through the application of EFDD to the response of the numerical model in terms of rigid body motions is shown in Fig. 2.

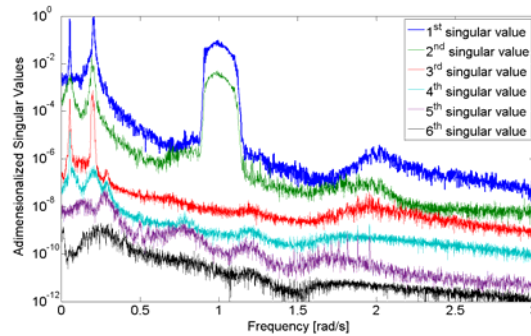


Fig. 2. Typical singular value plot obtained for the case study.

As it can be observed, all the six peaks relative to the structural modes are well-separated from the peak of the input spectrum, and fall in a frequency range where the external excitation is flat and can be assumed white. In particular, the first and the third mode present a peak in the first singular value (blue line), while the fourth mode is shifted to the second singular value (green line) and the other three modes are shifted to the third singular value (red line). With respect to the sixth mode (pure yaw) it must be said that the simplified AQWA model used in this study is not able to represent the yaw motion due to waves, hence only a small peak at a frequency close to 0,290 rad/s is present in the singular value plot. As a consequence, the corresponding estimation of modal natural frequency and modal damping ratio achieved by means of EFDD method are not reliable and will not be taken into account in the sensitivity study. The results obtained by Ruzzo et al. [3] showed that EFDD method succeeds in identifying mode shapes and modal natural frequencies of the structure, while the precision of the damping estimations depends on the position of the mode within the singular value plot. In particular, very good estimations can be achieved for the first singular value, while accuracy deteriorates as the relative importance of the mode decreases. However, these encouraging results were obtained processing relatively long response spectrum time histories. According to Grocel [2], the efficiency of the frequency domain identification methods depends on the ratio between the record length  $\Delta t$  and the longest modal natural period of the structure  $T_{\max}$ . In particular, he suggested  $\Delta t \geq 2000 T_{\max}$  which is not a feasible value for our purposes, since it would consist in almost 199,500 s of length record (more than two days), resulting in the impossibility to fulfill the assumption of stationarity of the sea state. Ruzzo et al. [3] used  $\Delta t = 20,000$  s, i.e. about  $200 T_{\max}$ , which is more realistic but still relatively difficult to realize practically.

The sensitivity study proposed in this work investigate how the accuracy of the method deteriorates for decreasing record length, keeping into account a range of lengths from  $200 T_{\max}$ , where the feasibility of the method has already been demonstrated, to  $10 T_{\max}$ . The 20,000 s long response obtained by ANSYS AQWA has been hence subdivided in smaller and smaller portions and EFDD method has been applied to each of them, in order to estimate the modal parameters. Finally, the results obtained have been used to investigate the effects of the decreasing response record length on the accuracy of the method. The accuracy in terms of mode shapes has been evaluated by considering the modal coherence between the true and the estimated mode shapes, calculated as in Eq. 5, while modal natural frequencies and damping ratios have been simply compared to the corresponding expected values. Fig. 3 shows the resulting modal coherence function versus record length, where  $m = 1$  would indicate a perfect estimation of the mode shape, Fig. 4 shows the damping estimations for the first and the third mode, which fall in the first singular value in all the cases considered, and Table 2 shows the modal natural frequencies estimation for the first five modes, being the pure yaw mode (sixth) excluded, as aforementioned.

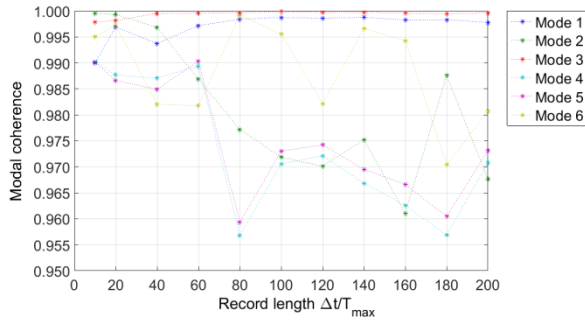


Fig. 3. Modal coherence of the estimated mode shapes with respect to real ones.

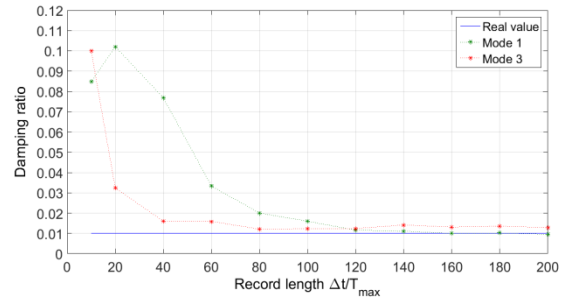


Fig. 4. Damping ratio estimations for the first and the third modes.

Table 2. Modal natural frequencies estimation [rad/s].

$\Delta t/T_{max}$	200	180	160	140	120	100	80	60	40	20	10
Mode 1	0.055	0.055	0.055	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.061
Mode 2	0.055	0.056	0.055	0.056	0.056	0.056	0.056	0.054	0.054	0.054	0.061
Mode 3	0.205	0.206	0.205	0.205	0.205	0.207	0.205	0.207	0.207	0.207	0.199
Mode 4	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.199	0.199	0.199	0.199
Mode 5	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.199	0.199	0.199	0.199

The results obtained clearly show how the estimation of mode shapes is accurate for the whole set of cases considered, being the modal coherences always superior to 0.950. In addition, the accuracy seems to be pretty independent from the length of the record. Similarly, modal natural frequency estimation is also almost invariant, except for small variations obtained for the shortest record, due to the coarser discretization of frequency domain. Quite the opposite, modal damping ratio estimation remains stable for record lengths superior to a certain value (ranging from about 80 to 120  $T_{max}$ ) and then deteriorates quickly, resulting in significant overestimations. A very similar behavior has been observed also for the other modes that, however, have not been included in Fig. 4 since the corresponding damping estimations are not accurate, due to the low position of the modes in the singular value plot.

In conclusion, the results obtained suggest that mode shape and modal natural frequency estimates are not significantly affected by the record length, while modal damping ratio estimate can lead to significant overestimations if the record is too short. With reference to the case study considered, a minimum length of 120 times the longest natural period can be suggested for practical applications of the method.

**References**

[1] E. Reynders, System identification methods for (operational) modal analysis: review and comparison. Archives of Computational Methods in Engineering, 2012, 19(1):51-124.  
 [2] J. Grocel, Comparison of deterministic and stochastic method in Operational Modal Analysis in application for civil engineering structures, Procedia Engineering, 2014, 91, 130-135.  
 [3] C. Ruzzo, G. Failla, M. Collu, V. Nava, V. Fiamma, F. Arena, Operational Modal Analysis of a spar-type floating platform using Frequency Domain Decomposition method, Energies, 2016, 9(11), 870.  
 [4] C. Ruzzo, V. Fiamma, V. Nava, M. Collu, G. Failla, F. Arena, Progress on the experimental set-up for the testing of a floating offshore wind turbine scaled model in a field site, Wind Engineering, 2016, 40 (5), 455-467.  
 [5] P. Boccotti, Wave Mechanics for Ocean Engineering, Elsevier Science, New York, 2000.  
 [6] R. Brincker, L.-M. Zhang, P. Andersen, , Modal identification from ambient response using Frequency Domain Decomposition. Proceedings of 18th IMAC, 2000.  
 [7] R. Brincker, C. Ventura, P. Andersen, , Damping estimation by Frequency Domain Decomposition. Proceedings of 19th IMAC, 2001.  
 [8] J. Jonkman, Definition of the floating system for phase IV of OC3. Technical Report NREL/TP-500-48191, 2010.

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