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Region of attraction Analysis for Adaptive Control of Wing Rock System

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Abstract: This paper introduces a numerical method for region of attraction (ROA) estimation of the adaptive control system in polynomial form. A classical adaptive controller with sigma modification on suppressing the wing rock motion is revisited. Using the nonlinear analysis technique based on the sum of squares optimization, ROA of the origin is computed with respect to the potential measurement error. The obtained result gives a solid guarantee on the allowable initial conditions where the adaptive controller is still able to suppress the wing rock motion. It provides confidence for the adaptive controller operating under some unforeseen conditions in practice.

Keywords: Adaptive control, region of attraction, SOS optimization, wing rock, nonlinear analysis

1. INTRODUCTION

Typical adaptive control architectures are inherently nonlinear, which presents a number of challenges in the robustness analysis of the closed-loop system (Heise & Holzapfel, 2014; Ioannou & Fidan, 2007). Therefore, the flight control branch would benefit a lot from the advances in the area of nonlinear analysis techniques.

For the flight control system, a significant stability metric is the region of attraction (ROA). It prescribes the allowable initial conditions under which the system will ultimately converge to the expected steady state; in other words, the valid area of the flight controller. The determination of ROA, especially the exact value, for a nonlinear system is much more difficult than its linear counterpart (Topcu et al., 2008). Significant efforts have been devoted to the computation or estimation of ROA (Bobiti & Lazar, 2018; Khodadadi et al., 2014; Parrilo, 2000; Topcu et al., 2008). The proposed techniques can be broadly classified into Lyapunov or non-Lyapunov methods (Najafi et al., 2016). The Lyapunov approaches include, for example, sum of squares (SOS) programming, approaches that apply both simulation and SOS programming, methods that use the theory of moments (Najafi et al., 2016). In this case, the ROA determination can be formulated into a problem with nonnegative inequalities following the Lyapunov theory. Computation algorithms based on the SOS technique provide a rather promising solution by relaxing the required nonnegative constraints into SOS constraints (Parrilo, 2000). The SOS optimization has brought significant development in the computing of ROA, reachability set, input-output gains, and robustness concerning uncertainty for nonlinear polynomial systems (Chakraborty et al., 2011). It is thus introduced in this paper to assess the robustness of adaptive control systems with respect to disturbances.

The objective of this paper is to demonstrate the ability of the SOS technique for computing ROA in the robustness analysis of the adaptive control system. The proposed method is applied on a delta wing model which encounters wing rock motion at a moderate to high angle of attack (Singh et al., 1995). The maneuverability can be limited when the aircraft keeps rocking between a certain level of roll angle. The mechanism behind the wing rock phenomenon, mathematical modeling, and the measures to avoid or suppress it (Calise et al., 2004; Dobrokhodov et al., 2011; Du et al., 2019; Elzebda et al., 1989; Singh et al., 1995), have been studied extensively due to its threat to the safety of the flight system. A description of the wing rock system in polynomial form is employed in this paper. A proven classical adaptive controller with sigma modification is demonstrated to suppress the wing rock motion (Pietrucha et al., 2009; Singh et al., 1995). The closed-loop system is reformulated under the presence of a potential measurement error, which alters the global stability of the designed controller.

The paper is organized as follows. Firstly, the wing rock dynamics and the classical adaptive controller with sigma modification are revisited in Sec. 2. Then a reformulation and the stability analysis of the closed-loop system with a disturbance are provided. In Sec. 3, the computation algorithm for ROA estimation is described and the estimation results are discussed in Sec. 4. Finally, the paper is concluded in Sec.5.

2. WING ROCK DYNAMICS AND ADAPTIVE CONTROL

2.1 Wing Rock Dynamics

The wing rock motion has been studied in the past few years (Elzebda et al., 1989; Ignatyev et al., 2017; Nayfeh et al.,

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1989; Pietrucha et al., 2009). In this study, a nonlinear mathematical model is taken from Nayfeh et al. (1989) which has been shown to have a good agreement in the description of the occurrence of wing rock motion. More importantly, the polynomial form is convenient for the ROA computation by SOS technique later on. The dynamic model is given as

$$\ddot{\phi} = (\rho U_{\infty}^2 Sb / 2I_{xx} C_l) + d_0 u \tag{1}$$

where ϕ is the roll angle, and the overdot denotes the time derivative. In general, a control effectiveness parameter d_0 is introduced along with the input signal u. As in Nayfeh et al., (1989), the roll-moment can be readily fit by polynomials for a slender delta wing which is found gives virtually perfect agreement:

$$C_l = a_0 + a_1\phi + a_2\dot{\phi} + a_3\phi^3 + a_4\phi^2\dot{\phi} + a_5\phi\dot{\phi}^2 \tag{2}$$

The aerodynamic parameters a_i are nonlinear with the angle of attack. Readers are referred to Nayfeh et al. (1989) for the details of functions a_i and their numerical values.

By defining the state vector $x = [x_1, x_2]^T = [\phi, \dot{\phi}]^T$, the system (1) can be cast as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x) + d_0 u \end{cases}, \tag{3}$$

where $x \in \mathbb{R}^2$, the Euclidean space of dimension of two, and

$$f(x) = b^T h(x), \tag{4}$$

where the system coefficient vector b

$$\begin{aligned} &b = [b_1, b_2, b_3, b_4, b_5]^T, \\ &b_i = (\rho U_{\infty}^2 Sb / 2I_{xx}) a_i \quad i = 1,...,5 \end{aligned}; \tag{5}$$

and the corresponding function vector $h(x)$ is

$$h(x) = [x_1, x_2, x_1^3, x_1^2 x_2, x_1^2 x_2]^T. \tag{6}$$

A reference model designating the desired response is given by a linear time-invariant differential equation

$$\dot{x}_m = A_m x_m \tag{7}$$

where $x_m = [x_{m1}, x_{m2}]^T$, $\varsigma > 0$, $\omega_n > 0$ and

$$A_m = [0, 1; -\omega_n^2, -2\varsigma\omega_n] \tag{8}$$

For the given wing rock model, the aerodynamic parameters for a 25deg angle of attack are used in this paper for demonstration. The dimensionless time $t^* = (4U_{\infty} / b)t$ as in Singh et al. (1995) is also introduced. Parameters in (3) are listed below with t^* as an independent variable rather than t

$$d_0 = 1, \quad b_1 = -0.02012844, \quad b_2 = 0.01051916$$
$$b_3 = 0.02596236, \quad b_4 = -0.1273338, \quad b_5 = 0.5197074$$

The reference signal is chosen as $x_m(t) = 0$. The parameters ς and ω_n are 0.707 and 0.5, respectively.

The open-loop system is simulated with $u = 0$ for two initial conditions: $\phi(0) = 6$ deg, $\dot{\phi}(0) = 419.4$ deg/s, and $\phi(0) = 30$ deg, $\dot{\phi}(0) = 1398$ deg/s. The phase plot is shown in Fig. 1 (red stars mark the starting point) where the small initial condition yields a limit cycle, whereas the roll angle diverges under the larger initial condition (dotted line).

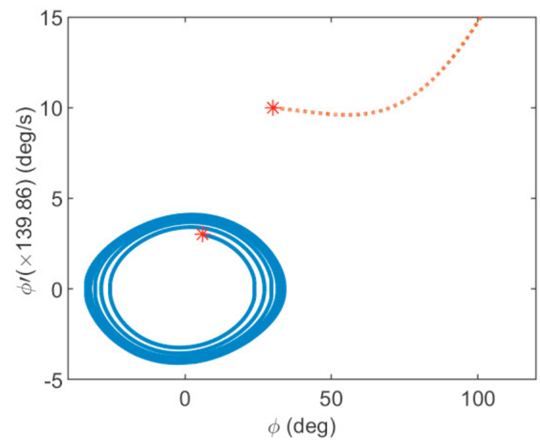


Fig. 1. Phase plot of the open-loop system

2.2 Adaptive Control

The wing rock control problem interested in this paper is taken from (Singh et al., 1995) under an assumption on the vector function $f(x)$: supposed that in (3) the parameters $b_i, i = 1,...,5$ and d_0 are unknown, but the sign of d_0 is known. Derive a control law such that $\lim[x_m(t) - x(t)] \to 0, \quad t \to \infty$. Define the trajectory tracking error

$$e = [e_1, e_2]^T = [(x_1 - x_{m1}), (x_2 - x_{m2})]^T. \tag{9}$$

Then the error dynamics are given by

$$\dot{e} = A_m e + d_m [g(x) + d_0 u], \tag{10}$$

where $d_m = [0, 1]^T$, and

$$g(x) = (b + b_m)^T h(x), \tag{11}$$

where $b_m = [\omega_n^2, 2\varsigma\omega_n, 0, 0, 0]^T$. The control law is chosen to match the uncertainty in (11) to cancel its effects

$$u = -\theta^T h(x), \tag{12}$$

where the estimated parameter $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$. Then an adaptation law for θ will be derived so that error $e(t)$ asymptotically converges to zero. θ can be defined as

$$\begin{aligned} &\theta_i^* = b_i / d_0, i = 3,4,5 \\ &\theta_1^* = (b_1 + \omega_n^2) / d_0 \quad \theta_2^* = (b_2 + 2\varsigma\omega_n) / d_0 \end{aligned}. \tag{13}$$

Thus, the parameter error vector is

$$\tilde{\theta} = [(\theta_0^* - \theta_0), \ldots, (\theta_5^* - \theta_5)] \in \mathbb{R}^6. \tag{14}$$

Then the error dynamics (10) becomes

$$\dot{e} = A_m e + d_m d_0 \tilde{\theta} h(x), \tag{15}$$

and the adaptation law guiding the variation of the parameter vector

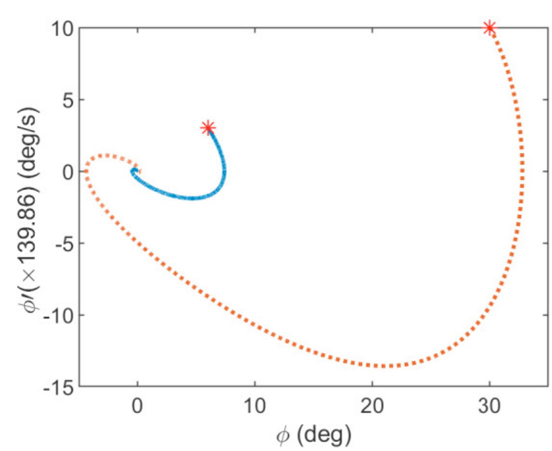
$$\dot{\theta} = \mathrm{sgn}(d_0) \Gamma (e^T P d_m) h(x) - \kappa_R \Gamma \theta, \tag{16}$$

where $\Gamma = \mathrm{diag}(\Gamma_i), i = 1,...,5$ and $\Gamma_i > 0$. P is the positive definite solution of the Lyapunov equation

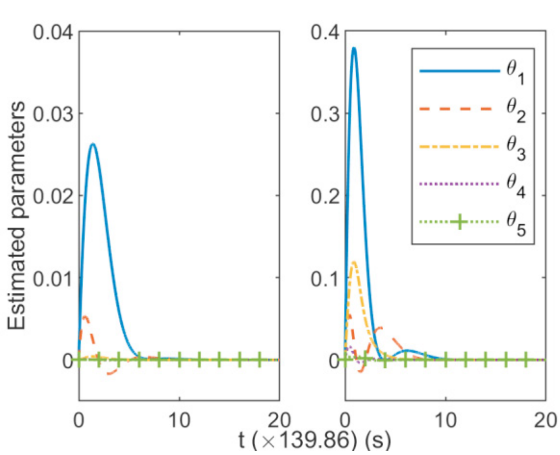
$$A^T P + PA = -Q, \ Q > 0, \tag{17}$$

where $A = A_m$. κ_R is a positive constant that forms the so-called σ- modification. The stability analysis is also given in Singh et al. (1995) The adaptation law in (16) has the effect of driving the parameter θ to zero if the equilibrium point is asymptotically stable. In this case, the error signals converge to zero, too. However, if there exist constant disturbances or commands to the control system, the σ- modification can bring unacceptably large steady-state errors (Calise et al., 2004). Thus, the robustness of the adaptive control should be re-evaluated with respect to these potential interferences. Therefore, the possible presence of disturbance in the measurement signal is modeled in the next section, intended for the estimation of the ROA thereafter.

Under two initial conditions given in Sec 2.1, the response of the closed-loop adaptive control system is shown in Fig. 2. For simplicity, the initial parameter vector θ is set to zero, the adaptation gain $\Gamma = I$, and the modification parameter $\kappa_R = 1$. As shown in Fig. 2a, Smooth convergence of roll angle and angular acceleration to zero is observed. The parameters also converge to zero as shown in Fig. 2b.



a)



b)

Fig. 2. Adaptive control for two initial conditions: a) Phase plot; b) Estimated parameters $\theta_i (i = 1,\ldots,5)$

2.3 Disturbed Adaptive Control System

Here, a common scenario in engineering applications where interference in measurement signal is considered. A quadratic form of the disturbance $d_d = \mu x_2^{\,2}$ is assumed to present during the measurement of roll angle ϕ and angular acceleration $\dot{\phi}$. The constant μ indicates the level of the disturbance. The disturbed measurement signal will be

$$x_d = [x_{1d}, x_{2d}]^T = [x_1 + d_d, x_2 + d_d]^T , \qquad (18)$$

which is used to construct control input signal

$$u_d = -\theta^T h_d(x_d) , \qquad (19)$$

where u_d is the actual signal sent to the executing agency to suppress the wing rock motion, and h_d is the disturbed five-vector function. In this case, the actual adaptation law applied to the wing rock system becomes

$$\dot{\theta} = \mathrm{sgn}(d_0)\Gamma(e^T P d_m) \cdot h_d(x_d) - \kappa_R \Gamma \theta , \qquad (20)$$

and the error dynamics

$$\dot{e} = A_m e + d_m \{d_0 \tilde{\theta} h_d(x_d) + (b - b_m)^T [h(x) - h_d(x_d)]\} , \quad (21)$$

which includes the wing rock coefficients vector b that is assumed to be unknown in the problem statement. The second term in the brace makes it difficult to determine the stability of the error system using the traditional procedure in Ioannou & Fidan (2007).

For the convenience of explanation, the disturbance level μ is specified to take the value of 0.1. Simulation of the response is similar to that of the undisturbed control system shown in Fig. 2. This corresponds to the equilibrium analysis of the closed-loop system (20) and (21), which indicates that the origin is locally asymptotically stable with eigenvalues -0.3535 + 0.3536i. However, random trajectory simulation shows that the roll angle and angular acceleration go into infinity under some initial conditions (dash line) as shown in Fig. 3, where only the variation of θ_1 in the five parameters is plotted.

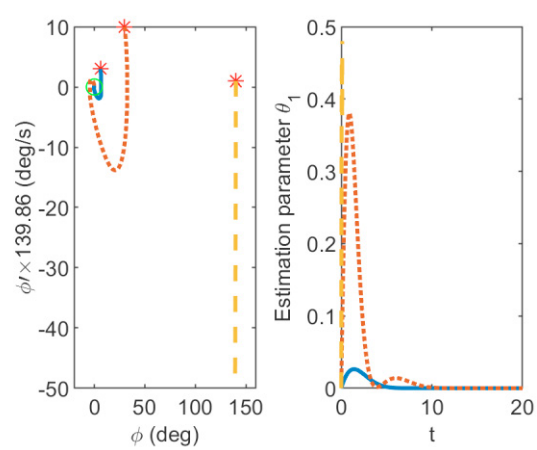


Fig. 3 Adaptive control with measurement error: small initials (solid line), large initials (dotted line), unstable initials (dash line).

The above analysis indicates that global stability cannot be achieved by the designed controller when experiencing this kind of measurement error. Thus, the information of the ROA for the origin is necessary to provide confidence for the implementation of the controller.

3. REGION OF ATTRACTION ESTIMATION

One of the challenges of adaptive control system design is to guarantee that the system is stable during the learning process. There is a number of techniques considering different metrics for the evaluation of the stability of the adaptive control systems (Falconi et al., 2016; Heise et al., 2013; Heise & Holzapfel, 2014). However, there is a lack of universal techniques applicable to different adaptive strategies. It is well known that ROA is a representative metric of a nonlinear system and can be adopted to any adaptive control. There exists a number of methods for ROA estimation, both, analytical and numerical. However, due to the complexity of the adaptive control systems, the analytical methods are hardly possible to be applied. Computational methods allow extending capabilities for ROA concept to validation and verification of adaptive control systems. Many methods make use of SOS optimization (Parrilo, 2000) to relax the original nonnegative problem into tractable SOS problems. Then SOS programming is applied to solve these problems.

It should be noted that the computational requirements for SOS optimizations grow rapidly with the number of variables and polynomial degrees. This roughly limits SOS methods to nonlinear analysis problems with at most 8–10 states and degree 3–5 polynomial models (Chakraborty et al., 2011; Parrilo, 2000). To this end, special attention should be paid to these two factors for a specific problem. For the disturbed adaptive control system considered in this paper, instead of analyzing the closed-loop system (3), (7) and (16) which yields nine variables, the system is a reduced seven-state system including only (3) and (16) is obtained by setting the desired reference signal to be zero. This has already been near the limit of the SOS method in the computation aspect especially the storage memory of an ordinary computer. Meanwhile, for time efficiency, we should also choose an appropriate algorithm for the ROA computation.

There have been various algorithms and improved procedures proposed for ROA computation (Khodadadi et al., 2014; Sidorov & Zacksenhouse, 2019). However, they are usually tested on benchmark problems with a low number of states. However, even for these problems dimensionality curse becomes a problem. In this research, we are going to consider the possibility to utilize computational methods for ROA estimation of a nonlinear system with adaptive control. In addition, the objective of this paper is to demonstrate the contribution of the SOS technique in the stability analysis of adaptive control systems. Thus, an algorithm should be selected to yield the desired solution while with a relatively low computation demand.

Let us consider an autonomous nonlinear polynomial system of the form

$$\dot{x} = F(x), \quad x(0) = x_0, \tag{22}$$

where $x \in \mathbb{R}^n$ is the state vector and $f(x)$ is the $n \times 1$ polynomial field. Without loss of generality, we assume that the origin is an asymptotically stable equilibrium point such that $f(0) = 0$. The ROA, a set of initial conditions whose trajectories will always converge back to the origin, can be defined as

$$\Omega := \left\{ x_0 \in \mathbb{R}^n : \text{If } x(0) = x_0 \text{ then } \lim_{t \to \infty} x(t) = 0 \right\}. \tag{23}$$

The approach taken in this paper is based on the proved lemma which follows the direct Lyapunov theory that specifies a sublevel set of a Lyapunov function as an estimation of the ROA for an asymptotically stable equilibrium point.

Lemma 1 (Khodadadi et al., 2014) If there exist a continuously differentiable scalar function $V(x): \mathbb{R}^n \to \mathbb{R}$ and a positive scalar $\gamma \in \mathbb{R}^+$, such that

$$V(x) > 0 \quad \forall x \neq 0 \text{ and } V(0) = 0, \tag{24}$$

$$\Omega_\gamma := \{ x : V(x) \le \gamma \} \text{ is bounded}, \tag{25}$$

$$\Omega_\gamma \subseteq \{ x : (\partial V / \partial x) F < 0 \} \cup \{0\}, \tag{26}$$

then the origin is asymptotically stable and Ω_γ is a subset of the ROA, which can be taken as an estimation. $V(x)$ can act as the so-called local Lyapunov function.

It can be seen that there are inequalities as well as set containment conditions in (24)-(26) to be satisfied. However, determining a non-negative multivariable polynomial is a challenging task. A workaround solution is to relax the nonnegativity into being SOS. This formulation transforms the original problems into SOS problems. SOS problems are treated by SOS programming. Several toolboxes such as SOSTOOLs, SOSOPTs, and solvers such as SeDuMi (Chakraborty et al., 2011) can freely available to complete the computation. For example, verifying a polynomial $s(x) \in \boldsymbol{R}(x)$ ($\boldsymbol{R}(x)$ represents the set of polynomials in $x \in \mathbb{R}^n$ with real coefficients) being an SOS polynomial is equivalent to check the existence of a positive semidefinite matrix Q, such that

$$s(x) = Z^T(x) Q Z(x) \in \Sigma_n, \tag{27}$$

where $Z(x)$ is the properly chosen vector of monomials and Σ_n denotes the set of SOS polynomial in $x \in \mathbb{R}^n$. This is the basic problem in SOS programming.

Besides, the well-known generalized S-procedure (Topcu et al., 2008) is employed to provide sufficient conditions for the set containment requirement.

Lemma 2 (Generalized S-procedure, (Topcu et al., 2008)). Given polynomials $g_0(x), \ldots, g_m(x) \in \boldsymbol{R}[x]$ and polynomials $s_1(x), \ldots, s_m(x) \in \Sigma_n$, if

$$g_0(x) - \sum_{i=1}^{m} s_i(x) g_i(x) \ge 0, \tag{28}$$

then

$$\{ x | g_1(x), \ldots, g_m(x) \ge 0 \} \subseteq \{ x | g_0(x) \ge 0 \}. \tag{29}$$

Then we can formulate the ROA estimation algorithm as:

$$\max_{V \in \Sigma_n, s_1 \in \Sigma_n} \gamma$$
$$\text{Subject to}: -[(\partial V/\partial x)F + l_1] - (\gamma - V)s_1 \in \Sigma_n \quad , \tag{30}$$

where l_1 is a small positive polynomial (typically $\varepsilon x^T x$ with some small real number ε) which is used to guarantee the derivative of V to be strictly negative. $s_1(x)$ is an SOS multiplier with a proper degree. According to Topcu et al. (2008), the degree can be chosen by

$$\begin{aligned} \max D(V \cdot s_1) &\geq \max D((\partial V/\partial x)F + l_1) \\ \min D(s_1) &\geq \min D((\partial V/\partial x)F + l_1) \end{aligned} , \tag{31}$$

where $D(s_1)$ represents the degree of the polynomial s_1. A proper choice is important to capture the feasible result of the problem (30) without introducing unnecessary computation burden, since the higher the degree of s_1, the more decision variables there are to determine.

Moreover, the problem (30) is bilinear because the decision variables in V and s_1 are coupled, and also the variable to be optimized γ is not affine. However, due to the fact that the linear solver has more advantages than the nonlinear ones such as PENMBI in YALMIP(Tibken & Youping Fan, 2006), the problem (30) has to be modified to accommodate to linear solver used in this paper. It is decoupled by choosing a feasible Lyapunov function V in advance and keep it fixed; the optimized γ is searched by bisection procedure.

A Lyapunov function can be constructed using its physical meaning for a specific problem. While a systematic way is employed here to generate an initial choice

$$V = x^T P x , \tag{32}$$

where P is the solution of Lyapunov equation (17) with $A = (\partial F/\partial x)|_{x=0}$ that is assumed to be Hurwitz. Khodadadi et al. (2014) explain why $Q = I$ is chosen but it is still a conservative choice since only the largest ball is provided. Other positive definite Q can also be customized to adapt to a specific shaped ROA when prior knowledge is available.

The optimized γ provides a lower bound for the ROA estimation.

To validate the obtained results, the exhaustive Monte Carlo search is carried out using $\theta_i = 0\,(i = 1,...,5)$ and compared with ROA approximations.

4. SIMULATION RESULTS

The MATLAB toolbox SOSTOOLs and the solver SeDuMi are used in this paper to carry out the SOS programming. For the numerical values of these parameters described in Sec. 2.1 and 2.3, the Lyapunov function obtained using (32) is

$$\begin{aligned} V &= 2.2980x_1^2 + 4.000x_1x_2 + 3.5361x_2^2 \\ &+ \theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_4^2 + \theta_5^2 \end{aligned} . \tag{33}$$

For a tolerance of 1e-3, the computed lower bound γ is 2.5017. It indicates the allowable initial conditions in the six-dimensional space. To visualize the result, a projection on $x_1 - x_2$ ($\phi - \dot{\phi}$) plane (when $\theta_i = 0$ $(i = 1,...,5)$) is demonstrated in Fig. 4 by the dotted line. The ellipse area provides confidence in the allowable initial conditions under which the wing rock motion suppression can be achieved. This result also verifies that under both the small and large initial conditions mentioned in Sec. 2.1, the designed controller is still effective even with a measurement error at $a = 0.1$. A three-dimensional demonstration of the ROA estimation is also presented in Fig. 5 by setting $\theta_i = 0\,(i = 2,...,5)$.

Monte Carlo simulation is carried out on a box region in $x_1 - x_2$ ($\phi - \dot{\phi}$) plane between $x_1 \in [-10, 4]$, $x_2 \in [-8, 10]$ and sets $\theta_i = 0\,(i = 1,...,5)$. As shown in Fig. 4, the red line bounds the region containing the initial conditions that lead to convergent trajectories. It can be seen as the actual local ROA corresponding to the origin. Initial conditions within the boxed area but lying beyond the red bound lead to the divergence of the adaptive controller.

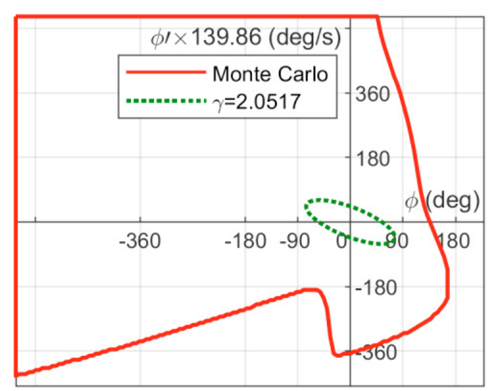


Fig. 4 Projection of ROA estimation on $\phi - \dot{\phi}$ plane

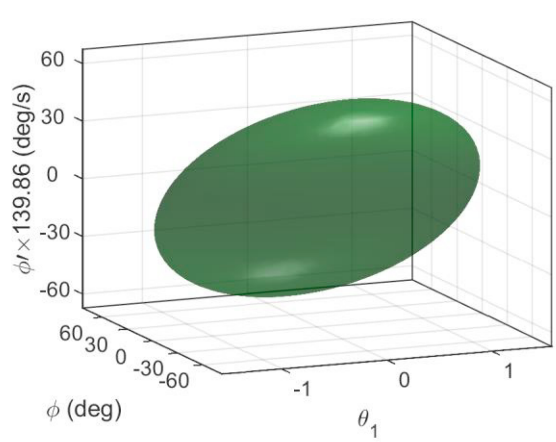


Fig. 5 ROA Estimation in the three-dimensional space

Though the estimation of ROA computed by SOS programming is less than one obtained via Monte Carlo simulation, the level set γ lies between $\phi \in [-60, 60]$deg and $\dot{\phi} \in [-6542, 6542]$deg/s . The obtained domain covers a significant part of an aircraft flight envelope, where the roll angle is from -90 to 90 deg. Besides, compared with Monte Carlo results, it gives a rather simple description of the ROA by a quadratic polynomial, which can be used easily to verify the stability at some operation points. The algorithm can also be improved for a larger inner approximation of the ROA.

5. CONCLUSIONS

The research presented in this paper assesses the robustness of a classical adaptive control system for suppressing wing rock motion under a potential disturbance in measurement signals. The lower bound of the region of attraction (ROA) of the origin is computed by a computational algorithm based on the SOS technique. Monte Carlo simulation in a two-dimensional space is carried out to obtain the upper bound. Though conservative as compared to the Monte Carlo results, the lower bound obtained by the SOS method yields a solid guarantee for the valid range of the controller and thus can be utilized for certification purposes. The obtained ROA estimation covers a significant part of the aircraft operational region. Thus, it accounts for important characteristics presented in the disturbed adaptive control system. The result provides confidence for the adaptive controller operating under some unforeseen conditions in practice. It can serve as a promising nonlinear analysis tool in adaptive control system verification and validation. Further optimization methods can also be attempted for better estimation.

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