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THE DEVELOPMENT OF HIGH FIDELITY LINEARIZED J_2 MODELS FOR SATELLITE FORMATION FLYING CONTROL

Jennifer A. Roberts[*] and Peter C. E. Roberts[†]

The inclusion of the linearized J_2 effect in the Hill equations of relative motion gives greater insight into satellite formation flying dynamics, and the opportunity to investigate alternative feedback control strategies for the station keeping task. The work of Schweighart and Sedwick is verified and extended as time varying and analytical models are developed to incorporate the J_2 perturbation and its effects on the relative motion of two or more satellites in LEO. Analysis is performed in detail to determine the best modelling strategy for satellite formation keeping in the J_2 perturbed environment. The analytical J_2 model is found to capture relative motion the most accurately, but only given specific initial conditions. The time varying model captures leader-follower motion better than the Hill equation and analytical J_2 models. LQR control laws are designed and performance evaluated for the basic Hill equations and the time varying and analytical J_2 models using Matlab/Simulink and the Satellite Tool Kit.

INTRODUCTION

The autonomous formation flying of multiple spacecraft to replace a single large satellite will be an enabling technology for a number of future missions. Potential applications include synthetic apertures for surveillance and high-resolution interferometry missions, or for taking widespread field measurements for atmospheric survey missions. Although there are potential disadvantages, the benefits of using several small, unconnected satellites could include increased reliability and redundancy, lower individual launch mass, and wider application. A number of planned missions require a formation to operate in Low Earth Orbit (LEO), for example, TechSat 21[1], XEUS[2] and the Multi-University Space Technology Nanosatellite Group (MUSTANG[3]) mission. MUSTANG is a British project, funded by the British National Space Centre (BNSC) to demonstrate distributed systems using two nano-satellites in LEO.

Many formation flying control strategies have been designed using the linearised Hill (or Clohessy-Wiltshire[4]) equations to describe the relative motion between two satellites. Hill's equations assume that both satellites must be in near-circular orbits of a similar radius, and the satellites must be separated by a distance much smaller than their radial position from the centre of the Earth. The Hill equations do not include any disturbance forces, for example J_2 geopotential due to the Earth's oblateness, and environmental forces (solar radiation pressure and aerodynamic drag). The gravitational perturbations due to the oblateness of the Earth have a significant effect in LEO on the satellite clusters' lifetime dV. The inclusion of the J_2 effect in the Hill equations model allows greater insight into satellite formation flying dynamics and provides an opportunity to investigate alternative feedback control strategies for the station keeping task.

[*] PhD Student, Space Research Centre, Cranfield University, UK
[†] Lecturer, Space Research Centre, Cranfield University, UK

Significant research has been performed into the development of LEO formation flying models which incorporate J_2 effects. Schweighart and Sedwick[5] have developed a high fidelity linearised set of differential equations for describing the relative motion of satellites in the presence of the Earth oblateness (J_2) gravity perturbation. The forces due to the J_2 effect are linearised and included in the Hill equations. The equations of motion are also modified to take into account the change in orbital time period and the nodal drift of the orbits caused by the J_2 perturbation. The cross-track motion is modelled, and analytical solutions to the relative motion equations are obtained. In their model, Schweighart and Sedwick incorporated the gradient of J_2 (∇J_2) disturbance in the relative motion equations by taking the time average. This is the only way in which a set of analytical equations of relative motion can be obtained. However, work completed at Cranfield (Izzo[6]) proposed that the gradient of J_2 terms should remain time varying, and the time averaging avoided.

Schaub and Alfriend[7] present an analytical method to establish J_2 invariant relative orbits using mean orbit elements. By establishing the conditions under which two orbits experience, on average, the same drift in the longitude of ascending node and the sum of argument of perigee and true anomaly, orbits can be identified where satellites will not separate over time due to the effects of J_2. Of course this minimises the fuel used in station keeping, however, the distance between the spacecraft may vary significantly throughout the orbits as only mean drift is considered. Schaub and Alfriend set the difference in mean orbit element drifts to zero (not the drift of each orbit) to avoid relative secular growth. Short period oscillations are ignored, as these are declared only temporary deviations. Alfriend and Schaub[8] also estimate the dV required to compensate for the relative drift due to J_2.

Alfriend, Schaub and Gim[9] and Gim and Alfriend[10] evaluate state transition matrices that include the effects of the chief satellite orbit eccentricity and the J_2 gravitational perturbations, in order to evaluate the effects of these features on the formation flying model. The state transition matrix is not obtained from solving the differential equations (of relative motion), but from the geometry of the problem whereby the deputy's relative motion is deemed a result of small changes in the chief satellite orbital elements.

Feedback control is required to minimise the error betwen the actual and desired relative satellite motion, and to maintain the formation geometry. A number of different control techniques have already been applied to the problem[11,12,13]. Some of these control strategies, designed using the Hill equations, have been performance evaluated in the perturbed environment. There are many examples of the application of linear quadratic regulator (LQR) control to spacecraft formation flying. LQR techniques, with the inherent disturbance rejection properties of the controller, provide an opportunity to reduce fuel consumption, and this tried and tested technique can be applied relatively easily.

Starin and Yedavalli[14] designed an LQR controller which allows thrust to be applied coplanar to the local horizon to achieve complete controllability of a two-satellite formation. It was shown that thrust can be avoided completely in the radial direction (the system is still controllable) and that this does not severely impact upon the performance of the controller. However, it was anticipated that problems would arise in the perturbed environment. The design was assessed using a range of initial separation conditions which were driven to zero, and the station-keeping task in the presence of orbit perturbations was not investigated.

As one of the earlier contributors to the formation-keeping problem, Ulybyshev[12] demonstrated that LQ feedback control was a good candidate for formation keeping of a large satellite constellation due to its performance and robustness. A discrete control system was applied to both the along-track relative displacement error between satellites and the orbital period displacements relative to the reference orbit (re Hill equations). The controller performance was evaluated by simulating satellite relative dynamics in the presence of aerodynamic drag where different satellites would decay at different rates.

In 1985 Vassar and Sherwood[15] designed a closed-loop formation-keeping controller for satellites in any circular orbit using digital optimal control theory. In their example they applied the controller (designed using the Hill equations) to a geosynchronous orbit and maintained the 700 metre along-track desired separation within +/- 21 metres. Their work also extended to optical sensor and thruster modelling and within the station-keeping accuracy requirement the optimal frequency of thruster firings was obtained. For their study involving a geosynchronous orbit, the effects of J_2 were not incorporated.

Sparks[16] used an LQ tracker design to minimise the error between actual and desired relative satellite motion in the presence of gravity perturbations. The elliptical formation geometry for a projected 1km circle was maintained within defined error bounds using a controller designed using the Hill equations with a modified orbital rate to account for J_2. In this case, the sampling time for the discrete controller was one hour and a tracking error of between 15 and 50 metres was achieved using a dV of 0.05m/s per day with the controller in a gravity-perturbed environment. In a second paper by Sparks[11], the formation-keeping dV using an LQR feedback law was computed through higher fidelity simulations (including higher degree and order J terms). The dV was computed as a function of the frequency of the applied control impulses. To maximise their effectiveness, impulses were applied at apogee, perigee and at points mid-way between them. In the latter case, only in-plane motion was considered.

Control techniques have also been applied using orbital elements to describe relative motion instead of the Hill equations in the formation flying problem. Schaub and Alfriend[17] and Vadali, Schaub and Alfriend[18] apply near-optimal impulsive feedback to control J_2 invariant orbits. The magnitude of the corrective impulses applied at different points in the orbit were expressed as changes in the orbital elements. The errors from desired were established at some arbitrary point in the orbit and then they were held constant during the orbit while the dV was applied.

Other authors consider the nonlinear modelling and control aspects of the problem. The design of satellite formations using the Hill equations is described by Sabol, Burns and McLaughlin[19]. Using the solutions to the Hill equations of relative satellite motion, four formations were defined: in-track, in-plane, circular, and projected circular. These results would be appropriate for application to model following control.

In this study, the model development work of Schweighart and Sedwick[5] is verified specifically against the Satellite Tool Kit (STK)[20] numerical orbit propagator. The gradient of J_2 terms are then implemented as time varying coefficients in the state matrix, and the performance of the time varying model is evaluated. The aim of the work supporting this paper was to build the analytical and numerical models describing the relative motion of two satellites subject to J_2 perturbations, and the Hill equations model in Matlab, and after verifying them against STK, to apply them to LQR design. The Matlab models are verified against the STK numerical orbit propagator at each stage of model development, and the effects of including orbital rate and nodal drift corrections are examined. The limitations of the high fidelity J_2 models are explained, and their use for LQR feedback design described and assessed.

MODEL REVIEW AND DEVELOPMENT

Equations of Motion

As stated in the introduction, Schweighart and Sedwick[5] have developed a high fidelity linearised set of differential equations for describing the relative motion of satellites in the presence of the J_2 gravity perturbation. A summary of the mathematical model development performed is given below in order to highlight the differences between their time averaged and the proposed alternative periodic time varying models. All the models derived were implemented in Matlab/Simulink.

The models were developed by initially considering the relative motion between one satellite tracing a circular reference orbit under the gravitational influence of a spherical Earth (at orbit altitude r_{ref}) and another satellite subject to the additional J_2 perturbation. Their equations of motion are given by Eqs. (1) and (2) respectively

$$\ddot{\underline{r}}_{ref} = \underline{g}(\underline{r}_{ref}) \tag{1}$$

$$\underline{\ddot{r}} = \underline{g}(\underline{r}) + \underline{J}_2(\underline{r}) \tag{2}$$

where g and J_2 are given by the well known expressions below in terms of inclination, i, and true anomaly, θ.

$$\underline{g}(\underline{r}) = -(\mu / r^3)\underline{r} \tag{3}$$

$$\underline{J}_2(\underline{r}) = -\frac{3J_2\mu R_e^{\ 2}}{2r^4}\left[\left(1 - 3\sin^2\theta\,\sin^2 i\right)\underline{\hat{x}} + \left(2\sin^2 i\,\sin\theta\,\cos\theta\right)\underline{\hat{y}} + \left(2\sin i\,\cos i\,\sin\theta\right)\underline{\hat{z}}\right] \tag{4}$$

Figure 1 illustrates the curvilinear $\underline{\hat{x}}$ (radial) $\underline{\hat{y}}$ (along-track) $\underline{\hat{z}}$ (cross-track) axis system, and $\underline{x}$ is the distance between the reference satellite (P_0) and the perturbed satellite (P_1), given by Eq. (5).

$$\underline{x} = \underline{r} - \underline{r}_{ref} \tag{5}$$

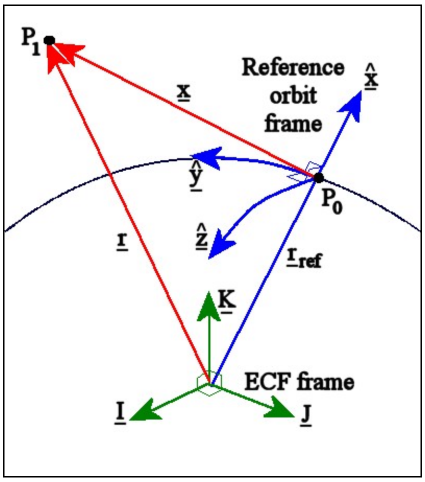


Figure 1 Axis Systems

The motion of the perturbed satellite can be linearised with respect to the reference orbit to give

$$\underline{\ddot{r}} = \underline{g}(\underline{r}_{ref}) + \nabla\underline{g}(\underline{r}_{ref})\cdot\underline{x} + \underline{J}_2(\underline{r}_{ref}) + \nabla\underline{J}_2(\underline{r}_{ref})\cdot\underline{x} \tag{6}$$

The angular velocity and velocity vector of a satellite tracing the reference orbit are defined by n and $\underline{\omega}$ where

$$n = \sqrt{\mu / r_{ref}^3} \quad \text{and} \quad \underline{\omega} = n\underline{\hat{z}} \tag{7}$$

The relative acceleration of the perturbed satellite relative to the reference, taking into account the rotation of the reference orbit frame on the reference orbit, is given by Eq. (8).

$$\underline{\ddot{x}} = \underline{\ddot{r}} - \underline{\ddot{r}}_{ref} - 2\underline{\omega}\times\underline{\dot{x}} - \underline{\dot{\omega}}\times\underline{x} - \underline{\omega}\times(\underline{\omega}\times\underline{x}) \tag{8}$$

For two satellites tracing circular orbits of similar altitude, undisturbed by additional external forces, Eq. (8) reduces to the form of the Hill or Clohessy-Wiltshire[4] equations (Eq. (9)). The notation used in this study was adopted from Schweighart and Sedwick[5] for continuity and the ^ has been omitted from the x, y and z unit vector notation from here onwards.

$$
\begin{aligned}
&\ddot{x} - 2n\dot{y} - 3n^2 x = 0 \\
&\ddot{y} + 2n\dot{x} = 0 \\
&\ddot{z} + n^2 z = 0
\end{aligned}
\tag{9}
$$

By substituting for the acceleration terms, $\ddot{\underline{r}}_{ref}$ and $\ddot{\underline{r}}$ into Eq. (8), Eq. (10) is obtained and by tensor calculus, the expressions for the gradient terms are given by Eqs. (11) and (12).

$$
\ddot{\underline{x}} + 2\underline{\omega} \times \dot{\underline{x}} + \dot{\underline{\omega}} \times \underline{x} + \underline{\omega} \times (\underline{\omega} \times \underline{x}) = \nabla \underline{g}(\underline{r}_{ref}) \cdot \underline{x} + \underline{J}_2(\underline{r}_{ref}) + \nabla \underline{J}_2(\underline{r}_{ref}) \cdot \underline{x}
\tag{10}
$$

$$
\nabla \underline{g}(r) = \begin{bmatrix} 2(\mu/r^3) & 0 & 0 \\ 0 & -(\mu/r^3) & 0 \\ 0 & 0 & -(\mu/r^3) \end{bmatrix}
\tag{11}
$$

$$
\nabla \underline{J}_2(r,\theta,i) = \frac{6J_2\mu R_e^2}{r^5} \begin{bmatrix} (1 - 3\sin^2 i \sin^2\theta) & \sin^2 i \sin 2\theta & \sin 2i \sin\theta \\ \sin^2 i \sin 2\theta & -\frac{1}{4} - \sin^2 i(\frac{1}{2} - \frac{7}{4}\sin^2\theta) & -\frac{\sin 2i \cos\theta}{4} \\ \sin 2i \sin\theta & -\frac{\sin 2i \cos\theta}{4} & -\frac{3}{4} + \sin^2 i(\frac{1}{2} + \frac{5}{4}\sin^2\theta) \end{bmatrix}
\tag{12}
$$

In Eq. (10) the values of $\underline{g}(\underline{r}_{ref})$ and $\underline{J}_2(\underline{r}_{ref})$ and the gradients of these terms are evaluated at the reference orbit. $\underline{x}$ must remain relatively small for the first order Taylor expansion to apply and therefore, the mean radii and orbital rates of the perturbed and reference satellites must be similar. An additional effect of the J_2 perturbation is to cause a drift in the longitude of ascending node and this varies with inclination. The J_2 perturbed satellite will experience this drift, but the reference orbit will not, thus the two orbits will gradually separate unless some correction to the orbital rate and nodal drift of the reference satellite is included in the model.

Schweighart and Sedwick[5] determined that the change in the orbital rate experienced by the perturbed satellite is, on average, related to the time averaged J_2 acceleration and that the drift in the longitude of ascending nodes is due to the cross-track component of J_2. The time averaged J_2 acceleration is given by Eq. (13).

$$
\frac{1}{2\pi}\int_0^{2\pi} \underline{J}_2(\underline{r})d\theta = \begin{bmatrix} -n^2 rs \\ 0 \\ 0 \end{bmatrix} \quad \text{where} \quad s = \left(\frac{3J_2R_e^2}{8r^2}\right)(1 + 3\cos 2i)
\tag{13}
$$

This results in a non-zero value for acceleration in the radial $\hat{\underline{x}}$ direction only. This can be visualised as an additional force acting to increase the Keplerian gravity term. Therefore, if a satellite is to remain in a circular orbit its orbital rate must be increased above that for Keplerian dynamics.

Originally, the reference orbit had a basic equation of motion including only the gravitational influence of a spherical Earth ($\underline{g}(\underline{r})$). The reference orbit with the modified rate has a new equation of motion (Eq. (14)) incorporating both the time averaged J_2 acceleration and cross-track component of J_2.

$$
\ddot{\underline{r}}_{ref} = \underline{g}(\underline{r}_{ref}) + \frac{1}{2\pi}\int_0^{2\pi} \underline{J}_2(\underline{r}_{ref})d\theta + [\underline{J}_2(r_{ref}) \cdot \underline{z}]\underline{z}
\tag{14}
$$

As the rate of the reference orbit has changed, so has the average angular speed of the reference satellite, and the coordinate system which is based there. The new angular speed can be found by equating the accelerations to give

$$\underline{\omega} = nc\hat{\underline{z}} \quad \text{where} \quad c = \sqrt{1+s} \tag{15}$$

The resulting linear vectorial equation of relative motion of one J_2-perturbed satellite relative to the reference, including the reference orbital rate and nodal drift correction terms is given by Eq. (16).

$$\ddot{\underline{x}} + 2\underline{\omega} \times \dot{\underline{x}} + \dot{\underline{\omega}} \times \underline{x} + \underline{\omega} \times (\underline{\omega} \times \underline{x}) = \underline{\nabla g}(\underline{r}_{ref}) \cdot \underline{x} + \underline{J}_2(\underline{r}_{ref}) + \underline{\nabla J}_2 \cdot \underline{x} - \frac{1}{2\pi}\int_0^{2\pi} \underline{J}_2(\underline{r}_{ref})d\theta - [\underline{J}_2(\underline{r}_{ref}) \cdot \underline{z}]\underline{z} \tag{16}$$

The relative motion between two perturbed satellites is the application towards which the model development was directed. Figure 2 illustrates the relative positions ($\underline{x}_1$ and $\underline{x}_2$) of two J_2-perturbed satellites (P_1 and P_2) with respect to the reference orbit (P_0).

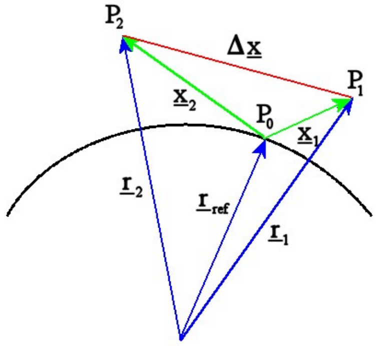


Figure 2 Relative Motion of Two Perturbed Satellites

By subtracting the relative motion equations which describe the motion of any two J_2 perturbed satellites, P_1 and P_2, with respect to the reference satellite, P_0, the equation describing the relative motion between the two perturbed satellites was obtained (Eq. (17)).

$$\Delta\ddot{\underline{x}} + 2\underline{\omega}\times\Delta\dot{\underline{x}} + \dot{\underline{\omega}}\times\Delta\underline{x} + \underline{\omega}\times(\underline{\omega}\times\Delta\underline{x}) = \underline{\nabla g}(\underline{r}_{ref})\cdot\Delta\underline{x} + \underline{\nabla J}_2(\underline{r}_{ref})\cdot\Delta\underline{x} \tag{17}$$

where

$$\Delta\underline{x} = \underline{x}_1 - \underline{x}_2 \tag{18}$$

It is important to note that the only terms affecting the relative motion between the two perturbed satellites are the gradients. The substitution of expressions into Eq. (17) results in the following equations of motion in the radial, along-track and cross-track directions.

$$\Delta\ddot{x} - 2nc\Delta\dot{y} - n^2\left[c^2 + 2 + \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)(1 - 3\sin^2 i \sin^2\theta)\right]\Delta x - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin^2 i \sin 2\theta\Delta y - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin 2i \sin\theta\Delta z = 0 \tag{19}$$

$$\Delta\ddot{y} + 2nc\Delta\dot{x} - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin^2 i \sin 2\theta\Delta x - n^2\left[c^2 - 1 - \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)\left(\frac{1}{4} + \sin^2 i\left(\frac{1}{2} - \frac{7}{4}\sin^2\theta\right)\right)\right]\Delta y + \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\left(\frac{\sin 2i\cos\theta}{4}\right)\Delta z = 0 \tag{20}$$

$$\Delta\ddot{z} - \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\sin 2i\sin\theta\Delta x + \left(\frac{6J_2n^2R_e^2}{r_{ref}^2}\right)\left(\frac{\sin 2i\cos\theta}{4}\right)\Delta y + n^2\left(1 + \left(\frac{6J_2R_e^2}{r_{ref}^2}\right)\left(\frac{3}{4} - \sin^2 i\left(\frac{1}{2} + \frac{5}{4}\sin^2\theta\right)\right)\right)\Delta z = 0 \tag{21}$$

If Eqs. 19, 20 and 21 are written together in state space form, the gradient of J_2 (∇J_2) terms create a time varying state matrix as the true anomaly varies with time. The gradient of J_2 is only constant for an equatorial orbit. In order to derive linear time invariant equations of motion, Schweighart and Sedwick[5] take the time average over one orbit of the gradient of J_2 expression to give Eq. (22).

$$\frac{1}{2\pi}\int_0^{2\pi}\nabla J_2(\underline{r})d\theta = \frac{\mu}{r^3}\begin{bmatrix} \frac{3J_2R_e^2}{2r^2}(1+3\cos 2i) & 0 & 0 \\ 0 & -\frac{3J_2R_e^2}{8r^2}(1+3\cos 2i) & 0 \\ 0 & 0 & -\frac{9J_2R_e^2}{8r^2}(1+3\cos 2i) \end{bmatrix} \tag{22}$$

The corresponding equation with time invariant coefficients describing the relative motion of two J_2 perturbed satellites is given by

$$\underline{\Delta\ddot{x}} + 2\underline{\omega}\times\underline{\Delta\dot{x}} + \underline{\dot{\omega}}\times\underline{\Delta x} + \underline{\omega}\times(\underline{\omega}\times\underline{\Delta x}) = \underline{\nabla g}(\underline{r}_{ref})\cdot\underline{\Delta x} + \frac{1}{2\pi}\int_0^{2\pi}\underline{\nabla J_2}(\underline{r}_{ref})d\theta\cdot\underline{\Delta x} \tag{23}$$

In full, the equations of relative motion for two perturbed satellites, from Eq. (23) in the radial (x), along-track (y), and cross-track (z) directions are given by Eq. (24). Clearly these equations are much simplified and comparable in form to the Hill equations.

$$\begin{aligned} &\Delta\ddot{x} - 2nc\Delta\dot{y} - (5c^2-2)n^2\Delta x = 0 \\ &\Delta\ddot{y} + 2nc\Delta\dot{x} = 0 \\ &\Delta\ddot{z} + (3c^2-2)n^2\Delta z = 0 \end{aligned} \tag{24}$$

Schweighart and Sedwick[5] developed a new cross-track model in order to rectify a problem introduced by taking the time average of gradient of J_2 where instead of rotating about the Earth's polar axis, the orbit plane was caused to rotate about a vector normal to the reference orbit. Their new equation describing cross-track motion was derived using a geometric approach based on the movement of the intersection of the orbit planes of the perturbed satellites and is given as Eq. (25). This was also implemented within the Matlab models and compared to STK.

$$\Delta\ddot{z} + q^2\Delta z = 2lq\cos(qt+\varphi) \tag{25}$$

Time Averaged and Time Varying Solutions with Initial Conditions

The linear time averaged equations describing the relative motion of one J_2 perturbed satellite relative to a reference or another J_2 perturbed satellite can be solved using standard techniques. If the equations of motion are initialised so that the satellites cross the equator at t = 0, and bounded motion is enforced for formation flying, the following relationships between the initial relative position and velocity are required. For one perturbed satellite moving relative to the orbital rate corrected circular reference, the initial conditions are given by Eqs. (26) and (27).

$$\dot{x}_0 = \frac{y_0 n(1-s)}{2c} \tag{26}$$

$$\dot{y}_0 = -2ncx_0 + \frac{3J_2R_e^2 n}{8cr_{ref}}(1-\cos 2i) \tag{27}$$

For the solutions to the nodal drift corrected equations, the initial conditions for bounded motion are given by Eqs. (28) and (29)[5]

$$\dot{x}_0 = \frac{y_0 n(1-s)}{2c} \tag{28}$$

$$\dot{y}_0 = -2ncx_0 + \frac{3J_2R_e^2n^2}{8kr_{ref}}(1-\cos 2i) \tag{29}$$

where

$$k = nc + \frac{3nJ_2R_e^2}{2r_{ref}^2}\cos^2 i \tag{30}$$

Physically, k can be considered as the frequency of equator crossing of the satellites which is modified by the shift in the line of nodes after each orbit. By integrating the perturbation equations which describe the variation with time of the orbital elements due to the normal component of the J_2 acceleration, the true anomaly can be expressed as

$$\theta = kt \tag{31}$$

The relationship in Eq. (31) was implemented within the time varying equations of motion where θ is substituted into Eqs. (19), (20), and (21).

Of greater importance for this part of the study is the relative motion of two J_2 perturbed satellites. In this case, the following initial conditions associated with the closed-form solutions were also applied during model verification. These are similar in form to the Hill equations formation flying initial conditons.

$$\Delta\dot{x}_0 = \frac{n\Delta y_0(1-s)}{2\sqrt{1+s}} \tag{32}$$

$$\Delta\dot{y}_0 = -2n\Delta x_0\sqrt{1+s} \tag{33}$$

With a time varying periodic state matrix, it is possible to derive solutions and perform a stability analysis using Floquet theory[21]. In this case the application of the initial conditions derived through the analytical solution of the time averaged equations of motion is considered, although it may be better to numerically determine the appropriate initial conditions for the time varying model, especially as the time varying model is extremely sensitive to both the orbital rate and nodal drift correction terms.

The initial conditions associated with the cross-track motion for both time varying and analytical models were user defined. For the analytical model, the inclinations of both satellites, and the initial separation and velocity are specified, and the motion is deemed uncoupled to the in-plane x-y motion. No 'formation flying' initial conditions have been derived in this case. The in-plane and cross-track time varying equations are clearly coupled, and it may be appropriate to investigate the relationship between relative positions and velocities for formation flying in all three axes.

The Use of Initial Conditions for Constellation Design

For the leader-follower case, with a desired separation in y (along-track), the initial relative position and velocity conditions for generating the relative motion of two perturbed satellites according to the time averaged analytical model, would need to be $\Delta x_0, \Delta y_0, \Delta\dot{y}_0, \Delta z_0, \Delta\dot{z}_0 = 0$. In this case it is necessary to consider the full solution to the equations of motion, including the drift and offset terms which describe unbounded motion so that when t = 0, the secular terms disappear, leaving a desired offset. The initial radial velocity condition (Eq. (34)) for an along-track separation would become

$$\Delta\dot{x}_0 = \frac{n(\Delta y_0 - \Delta y_d)(1-s)}{2\sqrt{1+s}} \tag{34}$$

As the desired separation (Δy_d) is also the initial relative position of the satellites anyway, the zero drift and offset condition becomes zero and this will apply to any case where an offset is to be maintained.

The zero drift and offset conditions in the general case will always cause the second satellite to move about the first (also perturbed), maintaining equal mean orbital rate, and have application to certain types of constellation. A constellation of three or more spacecraft could be modelled by a succession of 2-perturbed satellite motion models calculating motion of each satellite relative to a communal hub. These are also the conditions under which the model is most accurate. In the leader-follower case, the output from the relative motion model is equivalent to that obtained using the Hill equations, and unperturbed relative motion is predicted. This is however, the opportunity to demonstrate the improved modelling capability introduced by allowing the gradient of J_2 term to remain time varying.

MODEL VERIFICATION USING THE SATELLITE TOOL KIT

Model Verification Method

The mathematical models constructed within the Matlab/Simulink environment were verified against the Satellite Tool Kit (STK) Astrogator numerical orbit propagator. Astrogator is a front end for the High Precision Orbit Propagator (HPOP) in the STK software within which the user can select forces and perturbations in the simulator. In initial simulations, one satellite was created to orbit in a perfect circle at the reference orbit altitude, subject to point mass gravity, and a second satellite with the same basic properties was perturbed by the J_2 effect. Comparisons between the relative motion predicted by the Matlab models and STK were later made for scenarios where two satellites were experiencing the J_2 effect.

Any initial conditions that were applied to the Matlab models were also applied to the STK simulation. For the radial separation, the orbit eccentricity or the semi-major axis was adjusted, for the in-track separation, a non-zero value for the initial true anomaly was applied, and for the cross-track separation, the initial inclination, longitude of ascending node, and true anomaly were altered. Non-zero initial relative velocities were simulated in the STK Astrogator software by applying an impulse equivalent to the value of the relative velocity in any direction to one or both of the satellites. For each scenario, the two satellites were propagated in either 1 or 10 second time steps within STK and, and the relative position data was extracted through the Matlab Engine.

A Basic Scenario was examined in STK in order to study the effects of the gradient of J_2 term in the models where the relative motion between one J_2 perturbed and one reference satellite was calculated. The initial orbit radius of the perturbed satellite was made equal to the radius of the circular reference orbit and the initial conditions for zero drift and offset were applied to the perturbed satellite, effectively making the mean radius of the perturbed satellite equal to the reference orbit radius. The reference and perturbed satellites were then propagated and the relative positions and velocities computed. The relative position data in STK is produced in a Cartesian Local-Vertical-Local-Horizontal axis system based at the reference satellite and must be converted to curvilinear coordinates.

In the Orbital Rate Corrected scenario, the reference orbit in STK had to be artificially ‘speeded up’ to simulate the modified reference orbit in the Matlab model. Only then could relative motion data be compared. A suitable method was derived whereby the reference orbit altitude r_{ref} was temporarily lowered to provide a mean angular rate correction. An additional along-track velocity was also given to the perturbed satellite as an initial condition to generate bounded relative motion.

For the STK scenario where the relative motion of two perturbed satellites was evaluated, a reference satellite on a circular orbit was not required, despite being included in the development of the equations of relative motion. As the concept is retained within the Matlab model, it must still be accounted for through the STK initial conditions in order to allow fair comparisons between the model and STK to be made. In this case the perturbed leader satellite becomes the ‘reference’ and the relative position data is generated within STK in terms of a Cartesian set based at the leader. The reference orbit inclination and radius for a particular STK scenario are given as initial conditions to the leader, which is initially located on the reference orbit. The separations between the leader and follower satellites are implemented as 'basic properties' of the follower satellite in STK. Impulsive manoeuvres (initial relative velocities) must be applied to both perturbed satellites in order to eliminate drift and offset with respect to the 'invisible' circular reference orbit. The initial velocity conditions applied in STK were developed from Eqs. (28) and (29). The initial conditions in Matlab and STK must be similar within 10^{-6} metres of position and 10^{-9} metres per second of velocity or modelling errors will be introduced over large numbers of orbits.

Results for the Time Averaged Model

In this section, the test case used by Schweighart and Sedwick[5] is applied to illustrate the effects of the gradient of J_2 term and bounded initial conditions on relative satellite motion. In this process we verify their analytical model results. Further test cases then allow us to also assess the suitability of STK to be used as a 'real environment' within which a controller (effectively on board a satellite) can be evaluated. The methods used to implement initial conditions in STK, and to extract relative motion data were established, and some limitations of the analytical model were investigated.

During model development, the motion of one J_2 perturbed satellite relative to a satellite tracing a circular reference orbit was considered. The dashed lines in Figure 3 illustrate the effect of including the J_2 term as a disturbing function to the Hill equations of motion. The satellites start at the same point, with zero relative velocity resulting in an unbounded solution. The along-track drift illustrates the effect of J_2 in speeding up the perturbed satellite relative to the reference. In contrast, the solid line in Figure 3 illustrates the effect of including the time-averaged gradient of J_2 term, also with zero initial conditions. This shows that the gradient of J_2 multiplied by separation distance causes significant drift in the radial (x) and along-track (y) directions. In this case, the gradient of J_2 term increased the modelling error in all axes. The dotted line in Figure 3 illustrates the relative motion achieved when the orbital rate correction was made to the reference orbit. Clearly the radial drift has been eliminated, but the along-track drift remains because of the zero initial conditions. The cross-track motion is only slightly affected (by approximately 1km in 5 orbits) compared to the scale of the motion when the gradient of J_2 term and orbital rate correction are included in the equations.

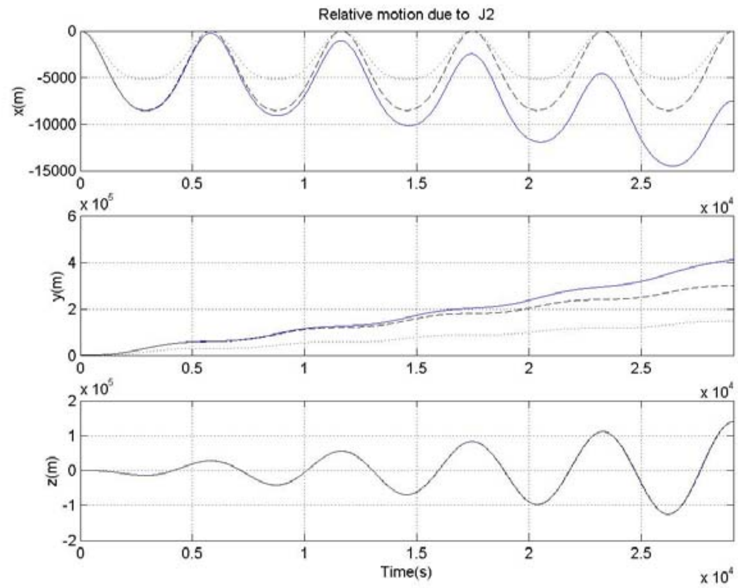


Figure 3 Motion of a J_2 Perturbed Satellite Relative to a Satellite on the Circular Reference Orbit for i=35^o, radius = 7000km, with all Initial Conditions = 0

Figure 4 illustrates the relative motion predicted by the orbital rate adjusted model with bounded in-plane motion (Eq. (35)). The cross-track motion is of course unaffected by changes in the in-plane initial conditions as the motion is decoupled in the equations.

$$\underline{\ddot{x}} + 2\underline{\omega}\times\underline{\dot{x}} + \underline{\dot{\omega}}\times\underline{x} + \underline{\omega}\times\left(\underline{\omega}\times\underline{x}\right) = \underline{\underline{\nabla g}}\left(\underline{r}_{ref}\right)\cdot\underline{x} + \underline{J}_2\left(\underline{r}_{ref}\right) + \frac{1}{2\pi}\int_0^{2\pi}\underline{\underline{\nabla J_2}}\left(\underline{r}_{ref}\right)d\theta\cdot\underline{x} - \frac{1}{2\pi}\int_0^{2\pi}\underline{J}_2\left(\underline{r}_{ref}\right)d\theta \qquad (35)$$

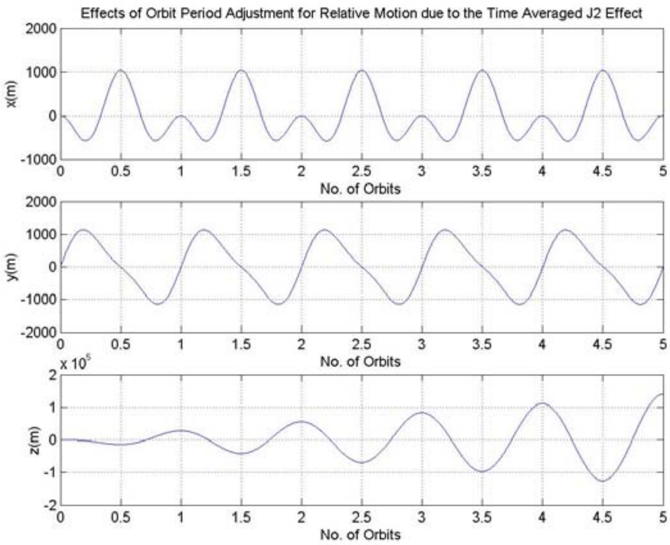


Figure 4 Bounded Motion of a J_2 Perturbed Satellite Relative to a Satellite on the Circular Reference Orbit for i=35^o, radius = 7000km according to Eq. (35)

In order to evaluate the contribution of the gradient of J_2 term, the orbital rate corrected models described by Eqs. (35) and (36) were compared to STK for a sun synchronous orbit. It should be noted that the STK scenario with which these equations were being compared also did not include nodal drift in the reference orbit motion as this was not possible to implement within STK. A sun synchronous reference orbit will naturally regress in the same manner (but not at exactly the same rate) as a J_2 perturbed satellite. This test case was chosen to reduce the additional linearisation errors introduced by the effect of J_2 on the drift in longitude of ascending node.

$$\underline{\ddot{x}} + 2\underline{\omega}\times\underline{\dot{x}} + \underline{\dot{\omega}}\times\underline{x} + \underline{\omega}\times\left(\underline{\omega}\times\underline{x}\right) = \underline{\underline{\nabla g}}\left(\underline{r}_{ref}\right)\cdot\underline{x} + \underline{J}_2\left(\underline{r}_{ref}\right) - \frac{1}{2\pi}\int_0^{2\pi}\underline{J}_2\left(\underline{r}_{ref}\right)d\theta \qquad (36)$$

Figure 5 illustrates the errors between the Matlab models and STK for bounded motion. The in-plane results show a significant modelling error reduction when the ∇J_2 term is included, but the cross-track results are worsened.

The relative motion of two J_2 perturbed satellites, separated by 100m in the along-track direction and propagated over 5 orbits in STK is shown in Figure 6.

Figure 7 illustrates the differences between both the basic Hill equations and the high fidelity linearized time averaged J_2 model when compared to the STK numerical orbit propagator for this motion, for two J_2 perturbed satellites propagated in a constellation formation. Modelling improvements are visible, with errors being reduced from a maximum of 0.5m to approximately 6.5cm over 5 orbits in the radial direction, and from over 1m to just over 30cm in 5 orbits in the along-track direction for the high fidelity model. The cross-track models (the z-Hill equation or the high fidelity model equations - Eqs. (24) and (25)) predict zero motion as the initial cross-track separation and relative velocity are zero in this case, although the STK propagator predicts a relative motion of just 2cm.

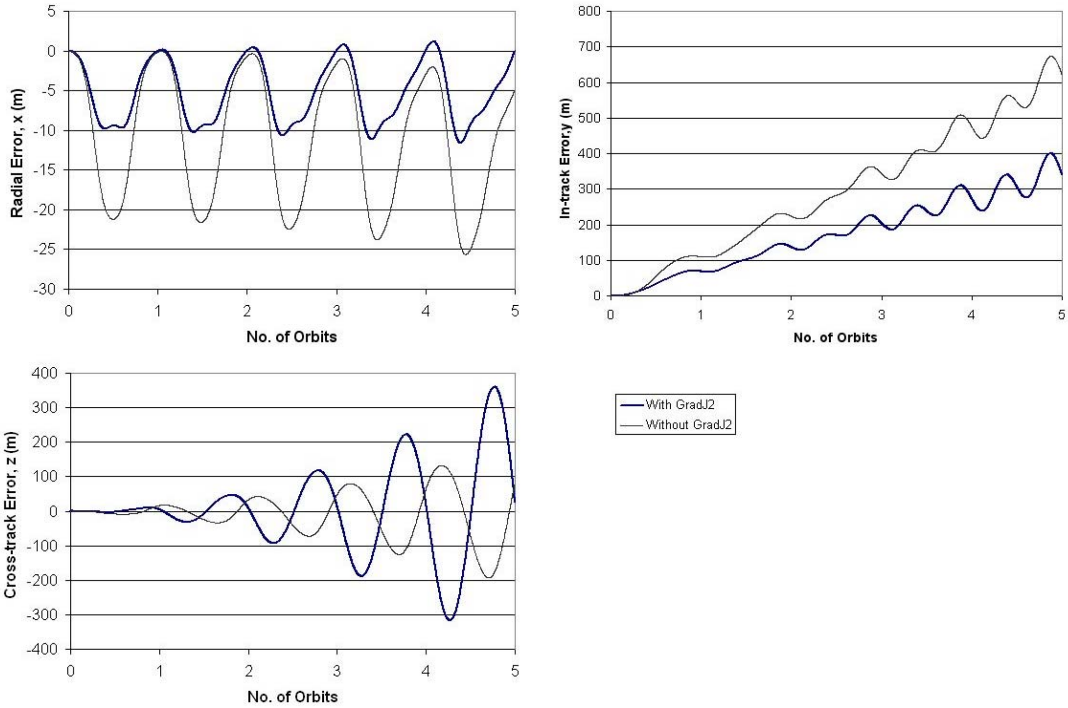


Figure 5 Effect of Gradient of J2 term on Modelling Error for a Sun Synchronous Orbit at i=97.87^{o}, radius=7000km

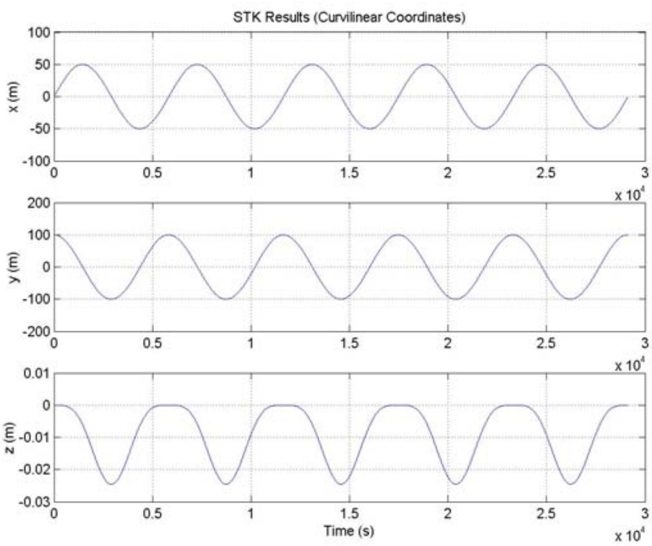
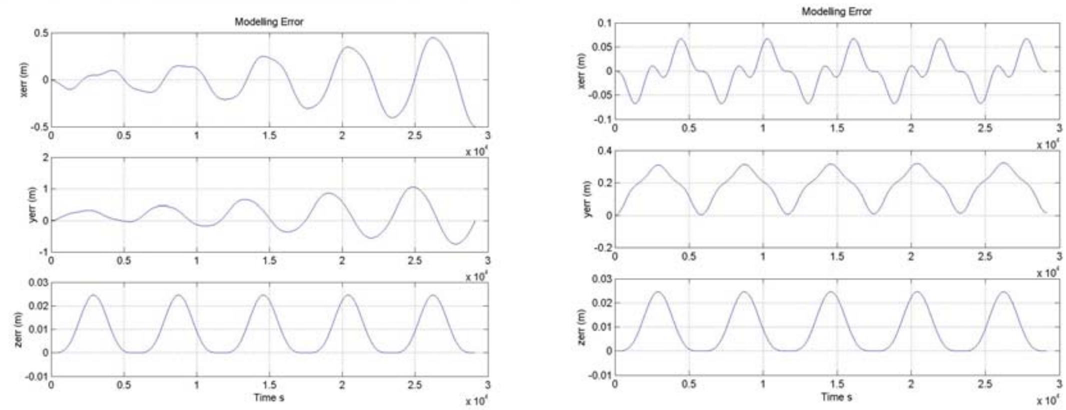


Figure 6 Relative Motion of Two J_2-Perturbed Satellites, Propagated in STK, with 100m Initial Along-Track Separation in a Sun Synchronous Orbit (i=97.87^{o}, radius=7000km) over 5 Orbits



(a) (b)

Figure 7 Error Between (a) Hill Equations and STK (b) J_2 Model and STK for Two J_2 Perturbed Satellites, Propagated in STK, with 100m Initial Along-Track Separation in a Sun Synchronous Orbit (i=97.87°, radius=7000km) over 5 Orbits

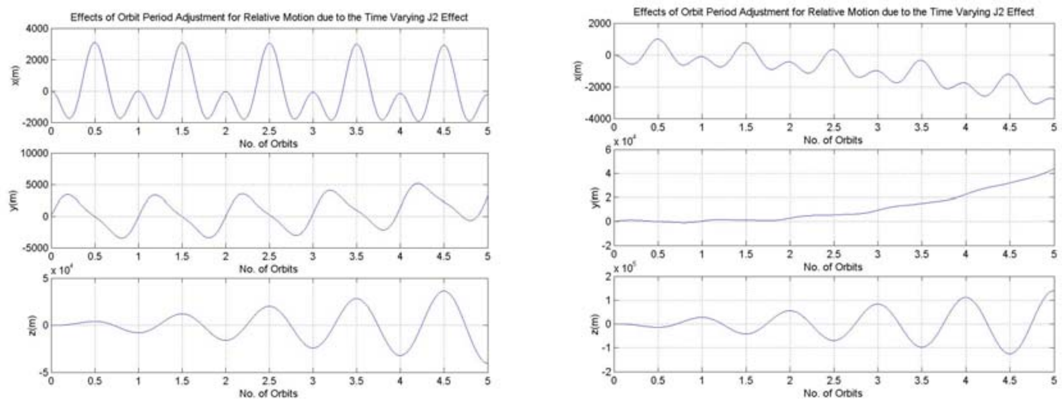
Results for the Time Varying Model

For the time varying model, the same principles of bounded motion apply, and the time varying gradient of J_2 term is equally sensitive in terms of linearisation error to the separation between the perturbed satellite and the circular reference orbit where the gradient is evaluated. In this case, the initial conditions for eliminated secular motion and offsets applied to the time invariant model were applied to the same scenarios for the time varying case.

In this section, one step in the verification process of the orbital rate corrected time varying model (Eq. (37)) is illustrated by comparing the motion of one perturbed satellite relative to the reference to the same scenario in STK.

$$\ddot{\underline{x}} + 2\underline{\omega} \times \dot{\underline{x}} + \dot{\underline{\omega}} \times \underline{x} + \underline{\omega} \times (\underline{\omega} \times \underline{x}) = \underline{\underline{\nabla g}}(\underline{r}_{ref}) \cdot \underline{x} + \underline{J}_2(\underline{r}_{ref}) + \underline{\underline{\nabla J}}_2(\underline{r}_{ref}) \cdot \underline{x} - \frac{1}{2\pi}\int_0^{2\pi} \underline{J}_2(\underline{r}_{ref}) d\theta \qquad (37)$$

The motion predicted by the time varying model for the sun synchronous case and for a lower inclination of 35° is shown in Figure 8. For the sun synchronous orbit, a gradual drift in the along-track relative motion over the 5 orbits can be seen in Figure 8a. At lower inclinations (Figure 8b), this effect is much more significant, highlighting the need to include the nodal drift effect of J_2 in the equations.



(a) (b)

Figure 8 Bounded Motion of a J_2 Perturbed Satellite Relative to a Satellite on the Circular Reference Orbit for (a) i=97.87° and (b) i=35°, radius = 7000km according to Eq. (37)

At an inclination of 90°, the period of the cross-track motion is captured, and the time averaged and time varying models behave in the same manner. If Figure 8b and Figure 4 are compared, the differences in relative motion predicted by the time varying and time averaged models can be clearly seen. When compared to STK, the time varying model introduced greater modelling error than the time averaged model in the in-plane directions, but captured the motion more accurately in the cross-track direction.

The nodal drift term 'k' was included in the time varying equations of motion, according to Eqs.(30) and (31). Figure 9 illustrates the the same motion for a sun synchronous orbit with all correction terms included. The along-track drift has been reduced, and its direction changed, but the motion is not completely bounded. Also, it was observed that the initial conditions derived from the time averaged model do not transfer directly to the time varying model.

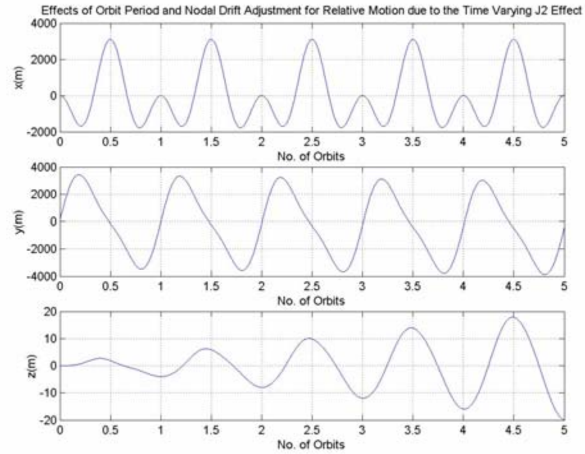


Figure 9 Bounded Motion of a J_2 Perturbed Satellite Relative to a Satellite on the Circular Reference Orbit for i=97.87°, radius = 7000km according to Eq. (16)

The difference or 'error' between the motion illustrated in Figure 6 (STK) and the time varying relative motion model is shown in Figure 10. For the same sun synchronous orbit, along-track separation, and bounded conditions, the time varying model performs worse than the Hill equations and the time averaged J_2 model over 5 orbits (compare to Figure 7). Although the time averaged J_2 force gives the correct orbital rate parameter for both the time averaged and time varying models, the remaining drift is either a function of the initial conditions, or the nodal drift term 'k' (which are related).

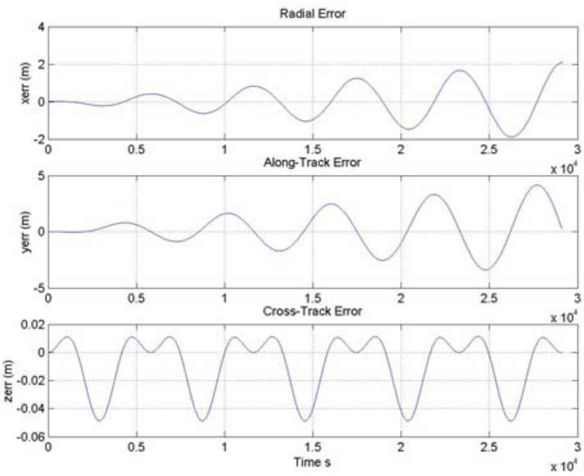
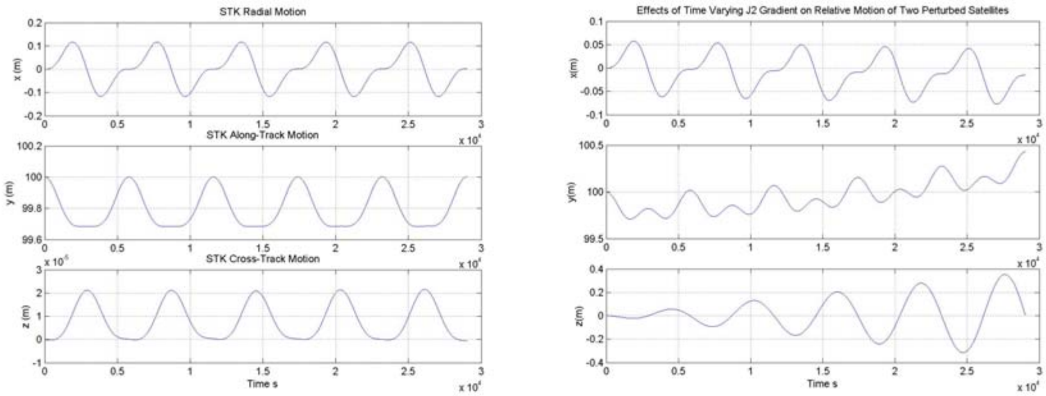


Figure 10 Error Between the Time Varying J_2 Model and STK for Two J_2-Perturbed Satellites, Propagated in STK, with 100m Initial Along-Track Separation in a Sun Synchronous Orbit (i=97.87°, radius=7000km) over 5 Orbits

MUSTANG Leader-Follower Example

For this leader-follower example, the time averaged model reduces to the Hill equations. The satellites are separated in the along-track direction, and not caused to move around each other in a 'zero drift and offset' formation. The STK orbit propagator predicts the relative motion illustrated in Figure 11a. Figure 11b shows the relative motion predicted by the time varying J_2 model. Figure 12 illustrates the modelling error between the time varying model and STK for this test case. These figures show that the time varying model captures the shape of the in-plane orbit propagator relative motion, but is subject to an along-track drift. The cross-track model does not capture the STK motion in this case, but this approaches zero for both the time varying model and STK as the inclination increases towards 90^o.



(a) **(b)**

Figure 11 Relative Motion in (a) STK (b) Matlab of Two J_2-Perturbed Satellites in Leader-Follower Formation with 100m Initial Along-Track Separation for i=85^o, radius=6978.1km over 5 Orbits

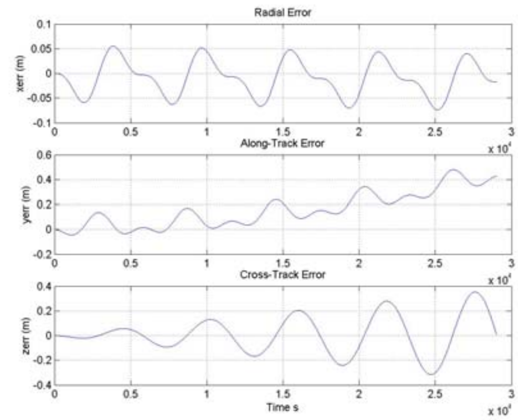


Figure 12 Error Between the Time Varying J_2 Model and STK for Two J_2-Perturbed Satellites, Propagated in STK, with 100m Initial Along-Track Separation for i=85^o, radius=6978.1km over 5 Orbits

The Effects of Different Terms and Model Limitations

Some interesting conclusions from the model developments can be drawn. The gradient of J_2 term degrades the relative motion model unless orbital rate and nodal drift corrections are made to the circular reference orbit, especially for the time varying model and regardless of the initial conditions. The analytical model is most accurate when the satellites are closer together, and therefore the zero drift and offset initial conditions give the best results. These in-plane initial conditions also reduced the errors in the cross-track direction as although they apply to the in-plane parameters, and the cross-track motion is decoupled in the time averaged model, the relative motion is coupled in the STK numerical orbit propagator. During model development, with one reference and one perturbed satellite, the inclusion of the time varying gradient of J_2 improved the cross-track model. However, this improvement did not transfer to the model with two perturbed satellites.

The time averaged relative motion model for two perturbed satellites was investigated for a range of test cases. The errors associated with the radial and along-track separations (bounded motion) were small, and the STK results were well approximated by the model, especially for an initial along-track separation. An investigation into model limitations was performed, and these were established as physical (collision with Earth if bounded motion is applied to formations with large separations), practical (location of the satellites, for example, leader-follower) and mathematical (singularities occur in the cross-track model for zero initial cross-track separations or relative velocity, and for polar and equatorial orbits and also, the lack of coupling between in-plane and cross-track motion).

The time varying model did not overcome as many of these issues as had been anticipated. Further work is required to investigate the behaviour of the time varying model in a number of different scenarios. Improvements must be made to the cross-track motion model for both the time averaged and time varying J_2 models and new initial conditions are required for the coupled time varying model.

CONTROLLER DESIGNS

Design Method

For this investigation one satellite is designated the leader, and the relative motion is always measured in a curvilinear axis system based at this satellite. The leader satellite does not regulate its own position, and follows a J_2-perturbed path relative to a circular reference orbit. The second satellite also experiences the J_2 perturbation, and maintains its position relative to the leader satellite using its own on-board sensors and thrusters. For this assessment of station keeping dV it is assumed that all the states are accurately known.

The model has not been discretised at this stage although the frequency and magnitude of thruster firings is normally an important variable in the design process. Minimising position error implies frequent corrections whereas minimising disturbances to the spacecraft and fuel use implies that a long time between formation-keeping corrections is required. The continuous application of accelerations, sampled at different time steps from the controller in this scenario, allows an initial estimate of station keeping dV to be determined.

The LQR time domain technique has been applied to the control of satellites on numerous occasions, and allows a direct multivariable design to be achieved with guaranteed closed loop stability and an ability to cope with random disturbances. By choosing the appropriate weightings for the states and control inputs, a controller can be designed and implemented in Matlab/Simulink with performance measured in terms of station keeping accuracy and cost measured in terms of fuel use. All of the formation flying models are composed of highly coupled sets of equations and LQR lends itself to the control of highly coupled systems and the optimisation problem and is therefore appropriate for the task.

In order to apply state feedback control, the equations of relative motion of two perturbed satellites were described in state space form with the state vector composed of relative positions and velocities in the three axes. The time varying state equation was also derived from Eqs. (19), (20) and (21). The feedback gain matrix was determined by minimising the linear quadratic performance index in Eq. (38).

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \tag{38}$$

Q and R were chosen, and the algebraic Riccati equation (Eq. (39)) was applied to the Hill and time averaged J_2 model equations using the state equation matrices, A and B.

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \tag{39}$$

Solving this equation enables a value for the matrix P to be obtained. The feedback gain matrix, K, is then given by Eq. (40).

$$K = R^{-1} B^T P \tag{40}$$

Evaluations were performed for the Hill and time averaged models to determine that all were controllable and observable. In order to perform the design, a suitable design point was chosen to give the system a realistic behaviour. The initial error from desired position in the radial, along-track and cross-track directions was set to 10 metres. The mass of the satellite being controlled (the follower in this case) was assumed to be 100kg, and the maximum thrust in any direction 10mN. The acceptable acceleration was therefore 10^{-4}m/s^2. The controller was tuned to allow the 10m manoeuvre (in all directions) to occur over an acceptable period of time, for example, half of an orbit. The dV was calculated by summing the magnitude of the acceleration over time. Each feedback design was implemented in Matlab/Simulink, and the response parameters were evaluated to ascertain whether the design requirements were achieved. These included closed loop eigenvalues for stability, maximum acceleration, percentage overshoot in response, time to reach within 1mm of desired offset and the dV over one orbit (including the initial manoeuvre).

This short design study demonstrated that acceptable response characteristics could be achieved using both the Hill and time averaged high-fidelity J_2 models. Only very small differences between the designs could be observed from the response graphs and data. The design for the high-fidelity J_2 controller enabled marginally greater maximum thrust to be applied within the design requirements. The most visible effect of this and the improved dynamics modelling was through the cross-track model behaviour (clear reductions in both percentage overshoot and settling time were visible) and the total dV over one orbit.

For the general application of LQR, the cost function does not have to be a quadratic function and the system does not have to be linear or time invariant. However, in this study, the time varying state matrix was applied to the standard LQR controller design at each time step in the numerical propagator and a new feedback gain matrix was found. The weighting matrices were optimised for the design point at the outset, and then the gain matrix was recalculated during the propagation in STK.

CONTROLLER EVALUATION

Controller Evaluation Method

In order to make a physical comparison between the controllers, they were implemented in the same J_2 perturbed (and realistic) space environment provided by the STK Astrogator numerical orbit propagator. The scenario implemented is illustrated in Figure 13. A desired separation was specified to the follower satellite through the 'Reference' input. The satellite 'onboard processor' used the measured relative position and velocity information to calculate the error from desired position and velocity, and multiplied the error by the feedback gains designed using the Hill and high-fidelity J_2 models. Accelerations were calculated by the controller and output to the satellite thrusters within STK. The satellites were propagated in STK in a perturbed environment (the range of disturbances applied can be user selected) while controller inputs were applied to the follower satellite. From this model, station-keeping measurements were extracted from STK and the accelerations applied were used to calculate fuel use as dV. The test cases evaluated were leader-follower formations at the i=35^o, radius=7000km design point and the MUSTANG mission (i=85^o, radius=6978.1km). In this case the MUSTANG results are considered where a 100 metre along-track separation between the leader and follower satellites was defined as the 'Reference' for the station keeping task.

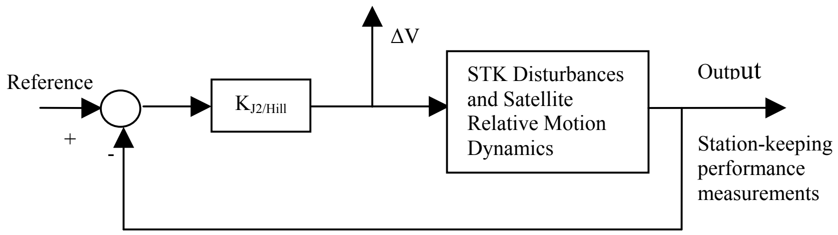


Figure 13 Block Diagram to Illustrate the Control Law Implementation with STK in the Loop

Matlab/STK Interface

The Matlab controller must be implemented in the STK environment through the STK-Matlab interface. This level of interaction is achieved through the STK Plug-In Scripts tool which allows the user to create scripts (written, for example, in Matlab) that integrate with STK's in-built software. After the declaration of inputs and outputs, the remainder of the Matlab script computes the accelerations to be applied to the controlled satellite and outputs the values back to STK under the variable name specified during the registration of variables. A set procedure allows the user to add additional forces (for example, thruster forces) at each propagation step within the numerical orbit propagator and extract certain variables made available by the STK software.

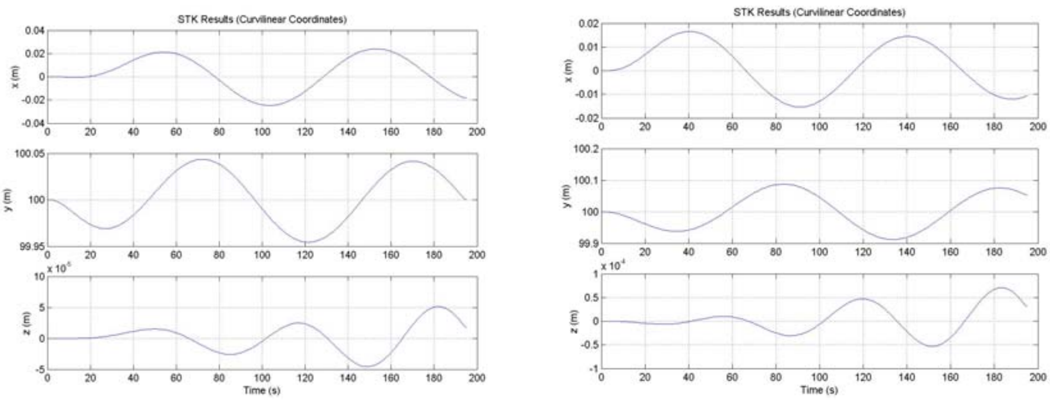
The propagation routine takes considerably longer when a script file is included. For the scenario operating without a Matlab script, the propagation of one satellite for one orbit was almost instantaneous. When the Matlab script was included to calculate motion using the simple feedback gains designed using the Hill model, the propagation time for the satellite was substantially increased. On the same processor, the propagation time for the satellite with the time varying gain recalculation increased by a further 60%.

The relative motion of the satellites can be viewed through the STK visualisation windows, and relative motion and other parameters can be obtained through the STK graph and reporting functions. Any parameters declared in the m-file and the Matlab workspace as global can also be extracted through Matlab. The calculation of dV was performed after the propagation using separate data extraction programs to obtain thruster accelerations from STK.

Controller Evaluation

Figure 14 illustrates the controlled relative motion of the satellites during the station keeping task over one orbit for MUSTANG. This can be compared to the uncontrolled motion in Figure 11a. Figure 14a illustrates the relative motion achieved using the controller gains from the Hill equations and time averaged models (there was no perceivable difference in response, and only a small improvement in dV using the high fidelity J_2 model). Figure 14b illustrates the controlled relative motion with the time varying controller embedded within the propagator software on the follower satellite.

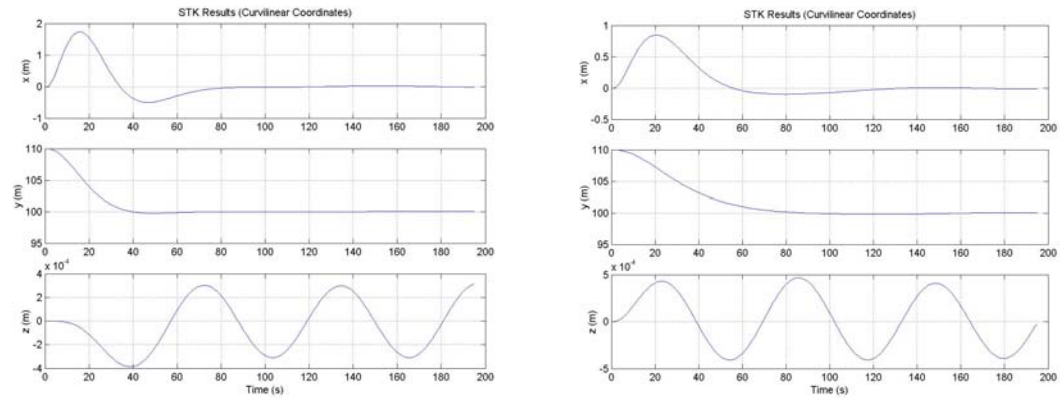
The dV required by a follower satellite to station keep in an along-track formation in the J_2 perturbed environment was found to be very small. For the Hill and time averaged J_2 models, the dV was 0.003151m/s over one orbit at $i=85^o$, and for the time varying J_2 model this reduced to 0.0026685m/s. For the lower inclination test case where $i=35^o$, the dV was just 0.0005168m/s for the time averaged J_2 model. The dV was found to reduce for lower orbit inclinations and smaller satellite separations. In the MUSTANG test case, the time varying controller used less fuel, and performed slightly differently in terms of station-keeping accuracy (Figure 14). It is not clear whether overall station keeping accuracy was improved as the along-track motion was worsened but the radial motion was slightly improved compared to the Hill and time averaged models. Through the evaluation of other test cases it was possible to show that the station-keeping performance was roughly proportional to the satellite separation for the leader-follower scenario with the set design point for the LQR controller.



(a) (b)

Figure 14 Controlled Relative Motion in STK Using the (a) Hill and Time Averaged Controller and (b) Time Varying Controller

To highlight the differences between the models, a 10 metre offset from desired separation was created in the STK scenario, and additional disturbances were selected in STK. This achieved similar results, although the dVs were all increased by a factor of approximately 10. The relative motion from the continuous controllers are illustrated for the time averaged and time varying models in Figure 15a and 15b (note the same control weighting matrices were applied). All models performed the manoeuvre within STK using a maximum initial thrust of 10mN as defined during the design process. The time varying model performed the manoeuvre over a longer time period, using less fuel and experiencing less radial relative motion.



(a) (b)

Figure 15 Controlled Relative Motion in STK given a 10 metre Offset from Desired Using the (a) Time Averaged Controller and (b) Time Varying Controller

The effects of both zero and non-zero initial velocity conditions relative to the circular reference orbit were investigated using the Hill and high fidelity J_2 model controllers. The relative velocity between the two J_2 perturbed satellites was of course still zero. Without the initial conditions (when both satellites did not keep up with the reference orbit), the dV over one orbit for the station-keeping task was found to be 0.0007044m/s. This reduced to 0.0005168m/s (a reduction of 26.6%) when an additional 2.584m/s of along-track velocity was given to each satellite. This has demonstrated that a more significant factor in reducing station keeping dV appears to be the nature of the initial conditions rather than the minor improvements in controller design. The addition of these initial velocities, particularly as the impulses are applied at the equator, effectively helps to circularise the orbits, reducing the eccentricity that the J_2 perturbation force would otherwise immediately apply.

In the test cases performed for this study, additional environmental perturbations were applied to both satellites. These were found not to affect the dV significantly for the station-keeping task. In some cases the dV was even marginally reduced. It should be noted however that the two satellites in this formation were identical in terms of their solar and aerodynamic ballistic coefficients, reducing further the relative effects of the additional disturbances applied.

CONCLUSIONS

The models have been implemented in Matlab/Simulink and the effects of different mathematical terms in the formation flying models have been investigated. These include the effects of retaining time varying parameters (gradient of J_2), and making corrections to the model to minimize linearisation errors by modifying the orbital rate and drift in the longitude of ascending node of the reference orbit. The work has enabled the 'error' magnitudes between the Matlab models and the STK numerical orbit propagator to be evaluated at each stage of model development. The use of STK for the validation process was described and model limitations established.

It has been determined that the analytical model captures relative motion well but only given specific initial conditions. However, some of the small scale effects of J_2 on relative motion are not captured due to the averaging of terms and the model reduces to the Hill equations for the leader-follower case. The time varying model is particularly sensitive to the orbital rate and nodal drift corrective terms, but captures the shape of the leader-follower motion. Further development is required for the cross-track model.

Although the implementation of Matlab controllers within STK has been successful, and a useful tool for initial station keeping dV requirements in LEO established, the benefits of using the higher fidelity models for LQR design have not been fully demonstrated. Whether or not there are real benefits in improving the models used in the design process will be more clearly demonstrated using a discretised model and impulsive control, for a specific scenario with a defined level of acceptable station keeping accuracy. The controller design process also only takes into account the dynamics illustrated in the state matrices and not the initial conditions, therefore very similar feedback controller gains were obtained. From the outset, for the leader-follower station keeping task, the results from the two LQR designs in terms of position accuracy and fuel use were expected to be very similar, especially for a continuous controller. However, the design did demonstrate that acceptable response characteristics could be achieved using both the Hill equations and high-fidelity J_2 model. Further investigations are required of the cross-track controller behaviour using constellation scenarios. The relative motion of two satellites in a 100m-separation leader-follower arrangement was very small but with the controllers, this was reduced further. The implementation of the time varying model has highlighted the additional computational effort required to run the model.

By effectively circularising their orbits, the variation in semi-major axis over the J_2 perturbed orbit is minimized. This has the effect that the orbital rate of both satellites match that of the 'invisible' reference orbit about which the high-fidelity analytical model was derived, and thus the effect of J_2 was reduced from the start of the scenario.

Further evaluation of the LQR controller design with continuous thrust for different leader-follower scenarios should be performed. Of interest would be the application of offsets in additional directions (other than along-track) and the development of the interface software to enable the formation to behave as a constellation. The current design should be modified to user-specified requirements in terms of station-keeping, perhaps using gain scheduling for progressively finer control. The development of suitable cross-track models would also be beneficial.

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