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Sliding Surface Optimization via Regional Pole Placement for a Class of Nonlinear Systems

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***Abstract*—In this paper, a new approach is introduced which combines Eigenvalue Assignment, State Dependent Riccati Equation (SDRE) and Sliding Mode Control (SMC) methods for nonlinear systems. In the classical SDRE based SMC (SDRE-SMC) approach, a nonlinear system is frozen at each time instant to obtain a linear-like structure model that is used to design a sliding surface (SS) at each time instant. This mechanism produces a state-dependent SS to hold the states on the SS. The approach proposed here is built on this mechanism and offers a new way to design a state-dependent SS for nonlinear systems so that the pointwise eigenvalues of the closed-loop system matrix of the control-free dynamics in the regular form can be kept in a specified disk. This gives a great advantage to shape the transient response characteristics. The performance of the nonlinear controller approach proposed here is investigated in simulations.**

***Keywords*—nonlinear systems; eigenvalue assignment; sliding mode control; sliding surface design.**

I. Introduction

One of the fundamental concerns in linear control theory is the locations of closed-loop system poles since their locations affect the stability and the transient characteristics of plant. Therefore, researchers have attempted to place closed-loop system poles at specific locations or in a certain region formed intentionally on the left half plane. In the literature, there are two widely used methods, known as exact pole placement and regional pole placement, to keep closed-loop system poles on the left half plane. The exact pole placement method is utilized when it is desired to place the closed-loop poles exactly at desired locations. On the other hand, the regional pole placement method provides control engineers with greater design flexibility and simplicity since it locates the closed-loop poles in a desired region [1].

Most important issue in the regional pole placement is to select an appropriate region. A region can be described in different forms such as circular, elliptical, vertical, strip, parabolic or sector regions [1]–[3]. The circular region centred on the negative real axis is often used in many studies [3]–[6]. Another issue is to place the closed-loop poles in the desired region. Furuta and Kim [2] applied Linear Quadratic Regulator (LQR) method to locate poles in a prescribed circular region, and examined the optimality and robustness of their pole assignment approach. Linear Matrix Inequality (LMI) method can be also utilized for the regional pole placement. Ling [7] used LMI approach to control a MIMO system. In another study [8], LMI approach was used to obtain controller parameters of a power system, and a state feedback controller was designed to keep the closed-loop poles in a desired region. In discrete time cases, LMI approach has also been developed to design a convex region for the regional pole placement [9].

In some approaches, pole placement methods are combined with the well-known linear control methods. Das et al. designed a PID controller by using a pole placement method, and verified the validity of their algorithm on a test setup [10]. Chang et al. developed a sliding mode fuzzy control by using the pole assignment approach [11].

One of the effective methods for some particular applications, which require a robust control, is the sliding mode control (SMC) method [12]. In the conventional SMC approach, there are two control phases. The first phase is known as a reaching phase where state trajectories are driven to a sliding surface (SS). The second phase is the sliding phase in which all states keep moving on the SS [13].

Sliding surfaces can be designed for linear and nonlinear systems. Linear SS is usually able to produce the desired control performance for linear systems. However, a linear SS may not guarantee the stability for nonlinear systems [14]. Therefore, different approaches have been developed to ensure the stability of nonlinear systems. Mobayen and Baleanu [15] introduced a new adaptive nonlinear sliding surface as a nonlinear gain function to change the damping ratio and control performance of a nonlinear system. The effectiveness of their approach was evaluated in simulations. In another approach [16], the partial feedback linearization was used for slosh control, and was tested in simulations and real-time experiments.

Optimal design of sliding surfaces has been frequently studied in recent years. In the study [17], a practical discrete-time sliding mode control method was obtained to minimize state energy during the sliding phase. In another study [18], a successive approximation method was used to design an optimal sliding mode controller that enables a missile to track a desired acceleration command. Sanjeewa and Parnichkun [19] designed LQR based SMC for stabilizing a double inverted pendulum.

In literature, another method frequently used in the design of SS is known as the State Dependent Riccati Equation (SDRE) method. Durmaz et al. [20] designed a SMC law with adaptive SS for nonlinear systems. This SDRE-SMC was applied to control a generic hypersonic aircraft. A modified SDRE-SMC method is also available in the literature to overcome the stability problem of SDRE-SMC [21] and was tested experimentally. The results showed that the modified algorithm produced smoother SS than the conventional SS. In another study [22], the performance SDRE-SMC method was experimentally verified on a 3-DOF Helicopter test bench by comparing it with Linear Time Invariant (LTI) SMC method. The comparative analysis revealed that SDRE-SMC method can relatively produce shorter settling time. SDRE-SMC method was also used to control nonlinear systems having matched or mismatched uncertainties [23]. SDRE-SMC was combined with the "unity vector approach" and applied to control a rigid satellite system. Some researchers have investigated different methods to solve SDRE for the design SMC. In an approach [24], Taylor Series was used to obtain the solution of SDRE. The number of terms in Taylor's Series was increased to obtain better approximation and performance. In [25], the integral and algebraic sliding surfaces were designed to guarantee the robustness of the controller against the parametric uncertainty.

In this study, a modified SDRE-SMC is introduced by using the regional pole placement (or eigenvalue assignment) approach to design state-dependent SS for a class of nonlinear systems. This method offers an alternative way to keep all pointwise closed-loop eigenvalues of the nonlinear null space dynamics, which is the control-free part of the nonlinear regular form, in a specified circular region at each time interval so that the desired transient response characteristics can be ensured. To the best of the author's knowledge, this study is novel in keeping the pointwise eigenvalues of the nonlinear null space dynamics in the pre-specified circular region on the left half plane at each time interval in sliding surface design of SMC. The effectiveness of the proposed algorithm is evaluated in simulations with the different values of the parameters determining a circular region, i.e. centre and radius of the circle.

The paper is structured as follows: Section II introduces briefly the traditional SDRE-SMC. Section III explains sliding surface design with state dependent eigenvalue assignment in SMC. Section IV presents a case study and simulations results. Finally, Section V gives the conclusion.

II. State Dependent SS Design

Consider the following general representation of the nonlinear dynamics

$$\dot{\varsigma} = A(\varsigma)\varsigma + B(\varsigma)u, \quad \varsigma(0) = \varsigma_0 \tag{1}$$

where $A(\varsigma)\colon \mathbb{R}^n \to \mathbb{R}^{n\times n}$ and $B(\varsigma)\colon \mathbb{R}^n \to \mathbb{R}^{n\times m}$ denote the State Dependent Coefficient (SDC) matrices, respectively. Here, $u \in \mathbb{R}^m$ is the control input and $\varsigma \in \mathbb{R}^n$ denotes the state vector. The regular form

$$\dot{z}_1 = A_{11}(z)z_1 + A_{12}(z)z_2 \tag{2}$$

$$\dot{z}_2 = A_{21}(z)z_1 + A_{22}(z)z_2 + B_2(z)u \tag{3}$$

is generated by applying the transformation $z = T(\varsigma)\varsigma$ where $z = [z_1\ z_2]^\top$ is a new coordinate system and $T(\varsigma)$ is a pointwise nonsingular matrix. Therefore, $T^{-1}(\varsigma)$ exists for all ς. Then SS can be determined the new coordinates. To design sliding surface, the switching function can be chosen as

$$\sigma(z_1, z_2) = z_2 + S(z)z_1 \tag{4}$$

where $S(z)$ is the surface slope matrix. On the sliding surface, $\sigma(z_1, z_2) = 0$, thereby $z_2 = -S(z)z_1$. Here, z_2 acts as if it is a full state feedback control input. This yields the following reduced order system defined as

$$\dot{z}_1 = (A_{11}(z) - A_{12}(z)S(z))z_1 = A_{cl}(z)z_1 \tag{5}$$

and $S(z)$ can be computed so that $A_{cl}(z)$ is a pointwise Hurwitz matrix by solving SDRE under the following assumption.

Assumption 1 (see [26])**.** *$\{A_{11}(z), A_{12}(z)\}$ is pointwise controllable for all z if $\{A(x), B(x)\}$ pair is pointwise controllable for all x.*

Finally, SDRE method produces a state dependent nonlinear SS (4) for the nonlinear systems of interest as follows:

$$S(z) = R^{-1}(z)A_{12}^\top(z)P(z)z_1 \tag{6}$$

where $P(z)$ is the solution of SDRE

$$\begin{aligned} A_{11}^\top(z)P(z) + P(z)A_{11}(z) - P(z)A_{12}(z)\times \\ R^{-1}A_{12}^\top(z)P(z) + Q(z) = 0 \end{aligned} \tag{7}$$

to minimize a quadratic cost function

$$J = \frac{1}{2}\int_0^\infty \left[z_1^\top(t)Q(z)z_1(t) + z_2^\top(t)R(z)z_2(t)\right] dt \tag{8}$$

subject to the null space dynamics (2). Here, $Q(z)$ is a semi positive definite matrix, i.e. $Q(z) \geq 0$ and $R(z)$ is a positive definite matrix, i.e. $R(z) > 0$. Now, the control input u can be derived by combining two control components in the form $u = u_{eq} + u_{sw}$ where u_{eq} is the equivalent control component and u_{sw} is the switched control component. When $\dot{\sigma}(z_1, z_2) =$

0, $\dot{z}_2 + S(z)\dot{z}_1 = 0$. Thus, the equivalent and switch control components can be defined as

$$u_{eq} = -B_2^{-1}\{A_{21}(z)z_1 + A_{22}(z)z_2 + S(z)\times [A_{11}(z)z_1 + A_{12}(z)z_2] + S(z)z_1\} \quad (9)$$

$$u_{sw} = -kB_2^{-1}\text{sgn}(\sigma(z_1, z_2)) \quad (10)$$

where $k \in \mathbb{R}^+$. Higher value k produces faster reaching time but higher chattering amplitude. The above procedure explains how to design an optimal SS in a conventional manner based on SDRE method. Now, a novel SS design incorporating an eigenvalue assignment method for SMC is introduced. This proposed approach is motivated from the theorem in [2] that was derived to put closed-loop poles of linear systems in a prescribed circular region. This study extends the previous theorem for linear systems to nonlinear systems that can be described as a state-dependent form in (1).

III. Eigenvalue Assignment for SMC

Definition 1. *A disk D, as shown in Fig. 1, is a circular region with a centre α on the real axis of the left half plane and a radius r.*

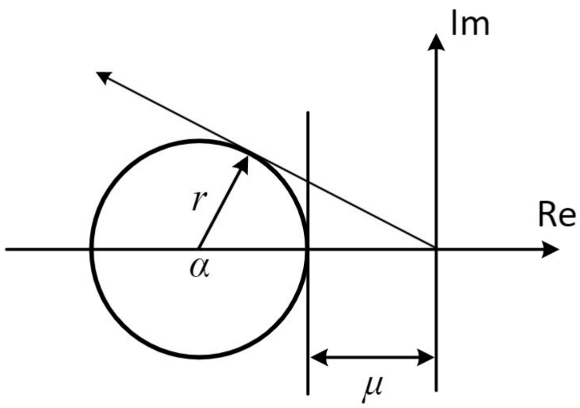

Fig. 1. Disk D on the left half plane

In nonlinear case, $A_{11}(z)$ and $A_{12}(z)$ with z satisfying the null space dynamics (2) and the range space dynamics (3) are evaluated at each instant of time t_k to obtain a linear-like structure including the constant $A_{11}(z)$ and $A_{12}(z)$ matrices given by $A_{11}(z(t_k)) = A_{11}(z_k) = A_{11_k}$ and $A_{12}(z(t_k)) = A_{12}(z_k) = A_{12_k}$ [27].

Lemma 1 (see [2])**.** *Consider the matrix equation*

$$\alpha A_{11_k}^* P_k + \alpha P_k A_{11_k} - A_{11_k}^* P_k A_{11_k} - (\alpha^2 - r^2)P_k = Q_k \quad (11)$$

*where $Q_k \geq 0$ and * denotes the conjugate of a matrix. Then, the eigenvalues of the matrix $A_{11}(z)$ at t_k, i.e. $\lambda(A_{11_k})$, are located within the disk D if there exists a positive definite solution P_k at t_k.*

Proof. Let v_k and λ_k be the eigenvector and eigenvalue of A_{11_k}, respectively. Then,

$$A_{11_k}v_k = \lambda_k v_k,\ v_k^* A_{11_k}^* = \bar{\lambda} v^* \quad (12)$$

Multiplying v and v^* with the both sides of (11) yields

$$v^*\{\alpha A_{11_k}^* P_k + \alpha P_k A_{11_k} - A_{11_k}^* P_k A_{11_k} - (\alpha^2 - r^2)P_k\}v = v^* Q_k v \quad (13)$$

Substituting (12) into (13) results in

$$(-\alpha(\bar{\lambda}_k + \lambda_k) + \bar{\lambda}_k\lambda_k + (\alpha^2 - r^2))v_k^* P_k v_k = -v_k^* Q_k v_k \quad (14)$$

With $\lambda_k = \eta_k + i\omega_k$, (14) becomes

$$(-2\alpha\eta_k + \eta_k^2 + \omega_k^2 + \alpha^2 - r^2)v_k^* P_k v_k = -v_k^* Q_k v_k \quad (15)$$

Rearranging (15) results in

$$((\eta_k - \alpha)^2 + \omega_k^2 - r^2)v_k^* P_k v_k = -v_k^* Q_k v_k \quad (16)$$

With $Q_k > 0$ and $P_k > 0$, the following inequality can be written as

$$(\eta_k - \alpha)^2 + \omega_k^2 - r^2 < 0 \quad (17)$$

This completes the proof that the eigenvalues of A_{11_k} stay inside the disk D. This also means that the eigenvalues of $A_{11}(z)$ stay inside the disk D. □

Considering Lemma 1, the following theorem gives the condition to guarantee that the closed loop matrix ($A_{cl}(z) = A_{11}(z) - A_{12}(z)S(z)$) has its eigenvalues in a desired disk when the expression $z_2 = -S(z)z_1$ defining the sliding motion is substituted into the null space dynamics of the regular form (2).

Theorem 1. *Consider the following matrix equation*

$$-\alpha A_{cl}(z)^* P(z) - \alpha P(z)(A_{cl}(z)) + A_{cl}(z)^* P(z)\times (A_{cl}(z)) + (\alpha^2 - r^2)P(z) = -Q(z) \quad (18)$$

where $Q(z) \geq 0$ and $A_{cl}(z) = A_{11}(z) - A_{12}(z)S(z)$. For a positive definite $P(z)$, $\lambda(A_{cl}(z))$ remain in the disk D.

To derive the state dependent law ($z_2 = -S(z)z_1$) required in the SS design, the following theorem is now introduced.

Theorem 2. *The following expression*

$$z_2 = -S(z)z_1 \quad (19)$$

where $S(z)$ is described by

$$S(z) = (r^2 R + A_{12}(z)^T P(z) A_{12}(z))^{-1} A_{12}^T(z)P(z)(A_{11}(z) - \alpha I) \quad (20)$$

locate all eigenvalues of $A_{cl}(z)$ inside a desired disk D. Here, $P(z)$ is a positive definite symmetric solution of the state dependent Riccati equation

$$P(z) = \frac{(A_{11}(z) - \alpha I)^\top}{r} P(z) \frac{(A_{11}(z) - \alpha I)}{r} - \frac{(A_{11}(z) - \alpha I)^\top}{r} P(z) A_{12}(z)(r^2 R + A_{12}(z)^\top \times P(z)A_{12}(z))^{-1} A_{12}(z)^\top P(z) \frac{(A_{11}(z) - \alpha I)}{r} + \bar{H}(z) \quad (21)$$

where R can be selected so that $R > 0$ and $\bar{H}(z) = H^\top(z)H(z)$ where $H(z)$ is a matrix such that the pair $(A_{11}(z), H(z))$ is observable.

Proof. Substituting (21) into (20) and rearranged equation (20) yields

$$\begin{aligned} & -\alpha(A_{11}(z)-A_{12}(z)S(z))^\top P(z)-\alpha\ P(z) \\ & (A_{11}(z)-A_{12}(z)S(z))+(A_{11}(z)-A_{12}(z)S(z))^\top \\ & P(z)(A_{11}(z)-A_{12}(z)S(z))+(\alpha^2-r^2)P(z) \\ & =-r^2(S(z)^\top RS(z)+H(z)^\top H(z)) \end{aligned} \tag{22}$$

where

$$\begin{aligned} S(z)=(r^2R+&A_{12}(z)^\top PA_{12}(z))^{-1}\times \\ & A_{12}(z)^\top P(z)(A_{11}(z)-\alpha I) \end{aligned} \tag{23}$$

and let $Q(z)$ be defined as

$$Q(z)=r^2(S(z)^\top RS(z)+H(z)^\top H(z))$$

Finally, $\lambda(A_{cl}(z))$ can be kept in the disk D by using Lemma 1 and Theorem 1. This completes the proof. □

Remark 1. *The selection of the weighting matrices R and Q in (7) has an effect of the locations of $\lambda(A_{cl}(z))$. It is possible to move $\lambda(A_{cl}(z))$ inside the disk D by changing the weighting matrices R and Q.*

The main difference between the eigenvalue assignment based SS and SDRE based SS is that $S(z)$ is determined so that the eigenvalues of the matrix $A_{cl}(z)=A_{11}(z)-A_{12}(z)S(z)$ are located within the desired disk using Theorem 2. Therefore, the slope matrix $S(z)$ of the sliding surface is needed to be changed by using (23).

IV. SIMULATION RESULTS

To investigate the performance of the modified SDRE-SMC approach based on the eigenvalue assignment method, a simulation study that compares it with the traditional SDRE based SMC is conducted. In simulations, the following fictitious system is employed as the plant

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\varsigma_1 & 1\\ -5+\varsigma_1^4 & 4+\varsigma_2^3\end{bmatrix}\begin{bmatrix}\varsigma_1\\ \varsigma_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

to be controlled. Since the fictitious plant model is already in the regular form, it does not require a coordinate transformation. This means that the mathematical model can be determined in the state z by replacing $[\varsigma_1\ \varsigma_2]$ with $[z_1\ z_2]$.

The weighting matrices R and Q in (7) for the traditional SDRE-SMC method and those in (18) for the new SMC method incorporating the eigenvalue assignment are arbitrarily selected to be $Q=I_2$ and $R=1$. The positive scalar k for the switched control component u_{sw} is selected to be 0.5 and the time interval is set to be 0.01s. All simulations are started from $\varsigma(0)=[0.5\ 0.5]^\top$. To assess the effects of the centre and radius of the disk D on the control performance, these parameters are varied within a range.

In the simulations, two different disks are considered. The first disk is located at the centre $\alpha=-2$ and has a radius of $r=1.5$. The second disk with the same radius as that of the first disk is desired to be placed where it is relatively further away from the imaginary axis on the left-half complex plane. Therefore, the centre and radius of the second disk are selected to be $\alpha=-3$ and $r=1.5$, respectively.

It is worth noting here that SDRE-SMC does not specifically aim at locating eigenvalues in a specified disk region. This means that the design parameter values of the first disk are specified so that it can contain the eigenvalue locations of the closed-loop system ($A_{cl}(z)=A_{11}(z)-A_{12}(z)S(z)$) determined by SDRE-SMC. Therefore, the centre $\alpha=-2$ and the radius of $r=1.5$ are arbitrarily selected to achieve this aim.

Fig. 2 and Fig. 3 present where the eigenvalues of $A_{cl}(z)$ are placed inside the first disk by applying the SDRE-SMC and the eigenvalue assignment based SMC (EA-SMC), respectively. Comparing the two results, it can been seen that both methods locate the closed-loop eigenvalues inside the desired disk region.

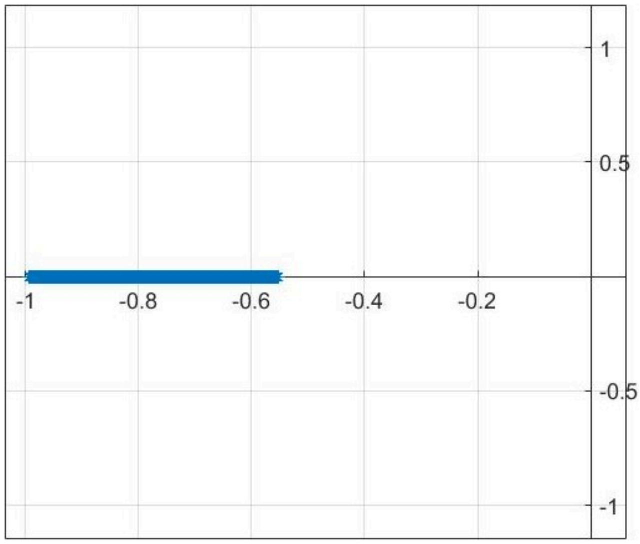

Fig. 2. The eigenvalue locations of $A_{cl}(z)$ in SDRE-SMC

Fig. 4 illustrates the state trajectories of the fictitious plant with two different controllers, i.e. the SDRE-SMC and the EA-SMC. From Fig. 4, it is apparent that the transient response characteristics of the plant model is improved by means of the EA-SMC, resulting in lower maximum overshoot and shorter settling time. This becomes possible since the pointwise closed-loop eigenvalues can be placed further away from the imaginary axis, which is clearly seen from Fig. 3. In order to examine the effects of a change in disk parameters on the system response characteristics and the eigenvalue locations, the radius of the disk is kept constant and the centre is shifted to left from the imaginary axis, as mentioned above. Therefore, the second part of the simulation study is conducted with the second disk whose centre is -3 and radius is 1.5.

The comparison of Fig. 4 and Fig. 5 reveals that the time responses obtained by using the EA-SMC is changed. In this case, the proposed SMC method enables the closed-loop eigenvalues to be kept in the second disk. As the disk is shifted to left, the transient characteristics of the system

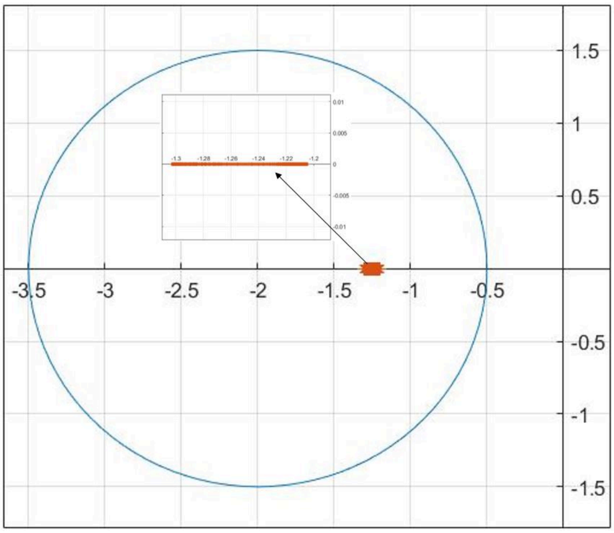

Fig. 3. The eigenvalue locations of $A_{cl}(z)$ in the EA-SMC using $\alpha = -2$ and $r = 1.5$

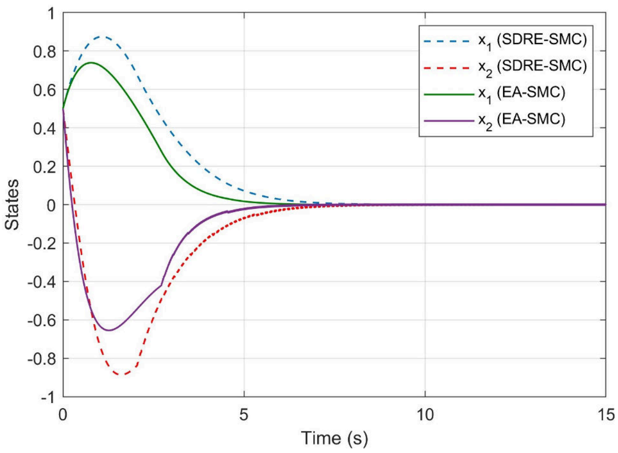

Fig. 4. Time response comparison of the EA-SMC with SDRE-SMC for the first disk

response is also changed. The settling time becomes shorter when the EA-SMC is applied to locate the eigenvalues inside the second disk. In addition, the maximum overshoot in ς_2 state is significantly decreased.

Fig. 6 shows the locations of the closed-loop eigenvalues produced by the SDRE-SMC and the EA-SMC. It is clear that the eigenvalue assignment based SDRE is capable of locating the eigenvalues inside the desired disk. However, this is not a case for the traditional SDRE-SMC.

Fig. 7 shows how the sliding surface is changed by the proposed approach and SDRE-SMC. The sliding surface is modified by the proposed approach if the eigenvalues are desired to be kept in a desired region. As a result, the considerable improvement in the settling time takes place.

V. CONCLUSIONS

This study aims to develop a new approach for designing a sliding surface enabling all eigenvalues of the closed-system

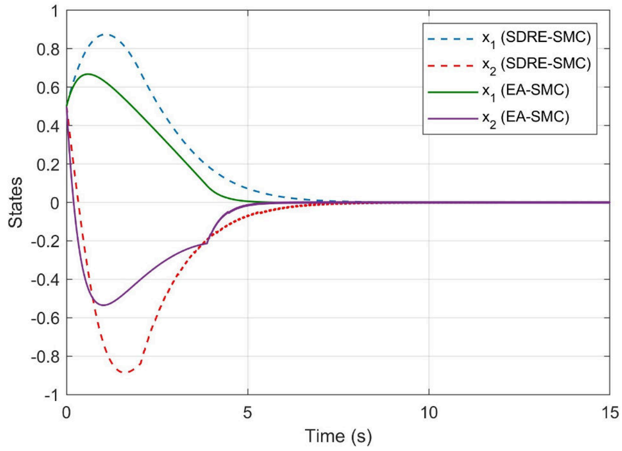

Fig. 5. Time response comparison of the EA-SMC with SDRE-SMC for the second disk

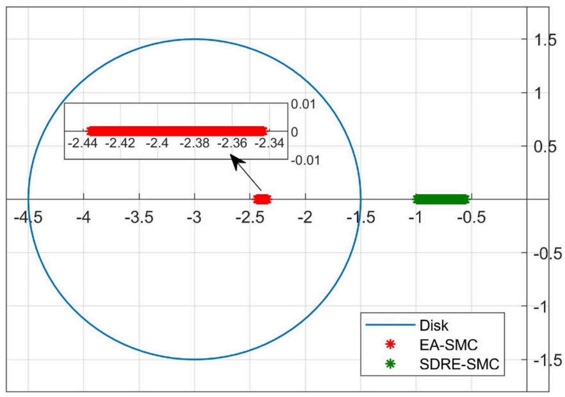

Fig. 6. The eigenvalue locations in the EA-SMC using $\alpha = -3$ and $r = 1.5$

matrix to be located in a desired region for reduced order nonlinear systems. The effectiveness of the proposed approach is investigated in simulations. To examine how effective the proposed SMC method, it is compared with the SDRE-SMC. According to the results, the proposed method is capable of locating the eigenvalues of the state-dependent closed-loop matrix in a desired region, which is not possible for the traditional based SDRE. Therefore, the proposed method offers a great advantage over the traditional method if the transient characteristics of the response is desired to be shaped. When the disk centre is moved away from origin, the eigenvalue locations can be moved away from the imaginary axis to improve the transient response characteristics.

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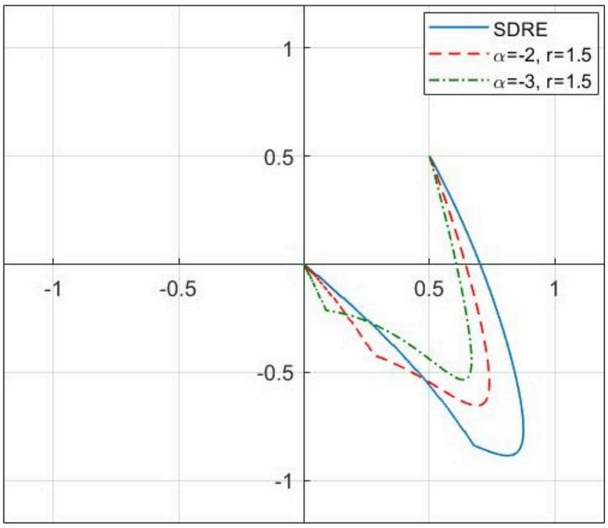

Fig. 7. Sliding surfaces in simulations

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