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GAIN SCHEDULING FOR A PASSENGER AIRCRAFT CONTROL SYSTEM TO SATISFY HANDLING QUALITIES

SCHOOL OF ENGINEERING

MSc BY RESEARCH THESIS

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Academic Year 2010-11

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Supervisor: Dr. James F. Whidborne

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Abstract

This thesis considers the problem of designing gain scheduled flight control system (FCS) for large transport aircraft that satisfy handling qualities criteria. The goal is to design a set of local Linear Time Invariant (LTI) controllers to cover the wide nonlinear aircraft operation flight envelope from the viewpoint of the handling qualities assessment. The global gain scheduler is then designed that interpolates between the gains of the local controllers in order to transfer smoothly between different equilibrium points, and more importantly to satisfy the handling qualities over the entire flight envelope. The mathematical model of the Boeing 747-100/200 aircraft is selected for the purpose of the flight controller design and handling qualities assessment.

In order to achieve an attitude hold characteristic, and also improve the dynamic tracking behavior of the aircraft, longitudinal pitch Rate Command-Attitude Hold (RCAH) controllers are designed as the local flight controllers at the specific equilibrium points in the flight envelope by means of a state space pole placement design procedure.

The handling qualities assessment of the aircraft is presented, based on which the scheduler is designed. A number of existing criteria are employed to assess the handling qualities of the aircraft, including the Control Anticipation Parameter (CAP), Neal and Smith, and C^* criteria. The gain scheduled flight controller is found to have satisfactory handling qualities.

The global gain scheduler is designed by interpolating the gains of the local flight controllers in order to transfer smoothly between different equilibrium points, and more importantly to satisfy the handling qualities over the flight envelope.

The main contribution of this research is the combination of the gain scheduling technique based on the local controller design approach and handling qualities assessment. The controllers are designed based at a number of operating points and the interpolation between them (scheduling) takes place through the scheduling scheme functions. The aircraft augmented with gain-scheduled controller performs satisfactorily and meets the requirement of handling qualities. Moreover, the performance using the gain-scheduled controller is considerably improved compared to the performance using the fixed one.

Keywords:
Gain Scheduling, Flight Control System, Handling Qualities, Rate Command-Attitude Hold, Passenger Aircraft, Flight Dynamics, Boeing 747

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Acknowledgements

Since the 3rd of January, 2010 (it was a snowy day just like today and the first day of my studies abroad), it has been one year full of adventure, challenge, enjoyment, happiness and great experiences. I have been particularly fortunate during my research to have collaborated with a broad range of School of Engineering students. I would like to take this opportunity to acknowledge the help and support of a few.

I would like to express my sincerest gratitude to my supervisor, Dr. James Whidborne for his continual advice, guidance, and support throughout the past year. His keen insights and clear guidance gave me great encouragement to carry out this research and made this thesis possible. I am also grateful to Dr. Alastair Cooke, Mr. Mike Cook and Dr. Thomas Richardson, who provided assistance and technical support, and guided me with valuable suggestions during my studies. I am indebted to Dr. Al Savvaris and Dr. Bowen Zhong, whose knowledge and experience was of great benefit to my time at Cranfield. My thanks also go to the other past and present members of the Dynamics, Simulation and Control group, especially Sunan Chumalee, Stuart Andrews, Mudassir Lone, Peter Thomas, Ken Lai, Mohmad Rouyan Nurhana, Pierre-Daniel Jameson and Neil Panchal. They were always open for discussions on my research struggles. Their friendship and support also sustained me through many challenging occasions, and helped me to move forward.

I would like to acknowledge China Scholarship Council (CSC), Commercial Aircraft Corporation of China (COMAC), and COMAC's subsidiary: Shanghai Aircraft Design and Research Institute (SADRI). My research and study would not have been possible without financial support from CSC and COMAC, and the nurturing environment of SADRI. There are a number of people, family and friends that I am grateful to and who deserve to be a part of every step forward that I have made.

Particular thanks go to the head of the Department of Flight Control System Design at SADRI, Jingzhou Zhao, who encouraged and believed in me ever since this project was first mentioned, and was continually supportive and attentive in my studies and well being in the UK. All of these stimulated me enormously to greater efforts to avoid letting him down and made me carry the fight through to the end.

I am also grateful to the encouragement and a great comfort I derived from my other colleagues of the COMAC group and Daqing Yang, who I fortunately collaborated and shared this studying abroad life with, and who were my companions endeavouring in a different field of aerospace side by side with me. Everything we have been through together in the UK will be treasured deeply.

I would like to save my deepest gratitude to my family, without whose understanding, support and encouragement I would not have achieved even a little: to my parents Chonglun Guo and Yuezhi Lee, my sister Hui Guo and all my other family members. Their love is worth far more than any degree and I am very blessed to have them there for me.

My time here at Cranfield has been memorable and valuable. I have enjoyed every aspect of this country. Once again, I would like to thank, everybody who helped me finish this research and made this Cranfield experience a most pleasant one.

Wei Guo
January, 2011

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Abbreviations

ARI	Aileron Rudder Interlink
CAD	Computer Assisted Design
CAP	Control Anticipation Parameter
CAS	Control Augmentation System
CSAS	Control and Stability Augmentation System
DASMAT	Delft University Aircraft Simulation Model and Analysis Tool
FBW	Fly By Wire
FCS	Flight Control System
FlightLab747	Flight Laboratory 747
LAF	Load Alleviation Function
LPV	Linear Parameter Varying
LTI	Linear Time-Invariant
LTV	Linear Time Varying
MAC	Mean Aerodynamic Chord
PI	Proportional-plus-Integral control
PID	Proportional, Integral and Derivative control
PIO	Pilot Induced Oscillation
RCAH	Rate Command-Attitude Hold
SAS	Stability Augmentation System
SISO	Single-Input-Single-Output
SPPO	Short-Period Pitching Oscillation

Nomenclature

A	System matrix	
A_{lin}	System matrix for linearized aircraft model	
B	Input matrix	
B_{lin}	Input matrix for linearized aircraft model	
C	Output matrix	
C_1 and C_2	Local linear controllers	
C_{lin}	Output matrix for linearized aircraft model	
D	Direct matrix	
D_{lin}	Direct matrix for linearized aircraft model	
CAP	Control Anticipation Parameter	
CAP_{open}	Control Anticipation Parameter of open-loop system	
CAP_{closed}	Control Anticipation Parameter of closed-loop system	
e	Gain scheduling factor	
EP	Equilibrium points	
p_{body}	Roll rate about body X-axis	rad/s
q_{body}	Pitch rate about body Y-axis	rad/s
r_{body}	Yaw rate about body Z-axis	rad/s
α	Angle of attack or incidence	rad
β	Angle of sideslip	rad
ϕ	Roll angle	rad
θ	Pitch angle	rad
φ	Yaw angle	rad
γ	Flight path angle	rad
ϵ_q	Integral error variable	
ζ_p	Phugoid mode damping ratio	
ζ_s	Short period mode damping ratio	
$\zeta_{s_{r1}}$, $\zeta_{s_{r2}}$	The required short period mode damping ratio	
$T_{lag_{r1}}$, $T_{lag_{r2}}$	The required integral lag time constant	
T_{θ_2}	The incidence lag variable	
ω_p	Phugoid mode natural frequency	rad/s
ω_s	Short period mode natural frequency	rad/s
$\omega_{s_{open}}$	Short period natural frequency of open-loop system	rad/s
$\omega_{s_{close}}$	Short period natural frequency of closed-loop system	rad/s
$\omega_{s_{r1}}$, $\omega_{s_{r2}}$	The required Short period mode natural frequency	rad/s
C_1, C_2	the Local controllers	

$\mathcal{F}$	The flight envelope	
h_e	Geometric altitude	m
x_e	Horizontal position along earth X-axis	m
y_e	Horizontal position along earth Y-axis	m
$\dot{p}_{body}$	Roll acceleration about body X-axis	rad/s^2
$\dot{q}_{body}$	Pitch acceleration about body Y-axis	rad/s^2
$\dot{r}_{body}$	Yaw acceleration about body Z-axis	rad/s^2
$\dot{V}_{TAS}$	Time derivative of true airspeed	m/s^2
$\dot{\alpha}$	Angle of attack or incidence rate	rad/s
$\dot{\beta}$	Angle of side-slip rate	rad/s
$\dot{\phi}$	Roll attitude rate	rad/s
$\dot{\theta}$	Pitch attitude rate	rad/s
$\dot{\varphi}$	Heading rate	rad/s
$\dot{\gamma}$	Flight path angle rate	rad/s
$\dot{h}_e$	Geometric altitude rate	m/s
$\dot{x}_e$	Horizontal ground speed along earth X-axis	m/s
$\dot{y}_e$	Horizontal ground speed along earth Y-axis	m/s
δ_e	Elevator deflection	rad
δ_{stab}	Stabilizer deflection	rad
T_n	Thrust	N
K_n	Controls fixed static stability margin	
K	Matrix of state variable feedback gains	
k_q	Pitch rate feedback gain	
k_α	Angle of attack feedback gain	
k_{ϵ_q}	Integral gain	
M	Feed forward gain matrix	
q_d	Pitch rate demand	rad/s
n_α	Normal load factor per unit angle of attack	g/rad
U_e	Axial component of steady equilibrium velocity	m/s
V_0	Steady equilibrium velocity	m/s
V_{TAS}	True airspeed	m/s
n_z	Total normal load factor	g
u	Axial velocity perturbation	m/s
v	Lateral velocity	m/s
w	Normal velocity	m/s
x	the state vector	
$\dot{x}$	the derivative of state vector	
u_{ctrl}	input vector	
y	output vector	
Δ	Characteristic polynomial: Transfer function denominator	

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Chapter 1

Introduction

1.1 Background

Gain scheduling is considered as a standard method to design Linear Time-Invariant (LTI) controllers for Linear Time Varying (LTV) or nonlinear systems in control theory. It also has widespread and successful engineering applications, an important example being its implementation in the design of the flight control system of aircraft [RS99] [LL00]. The idea behind gain scheduling is that gains of controllers are scheduled with some measured parameters of the system. These measured parameters of the system are usually referred to as the scheduling parameters. In the aerospace sector, gain scheduling technology was first used mainly on military applications. By the mid 1950s, gain scheduling technology began to overcome difficulties of implementing into the new generation of jet aircraft. A research subject of gain scheduling application in civil aircraft and other areas developed gradually since then. Recently, there has been an increasing number of research activities and a wider range of applications in the area of gain scheduling [RS99] [SG07] [RDLB03]. In the light of the significant change of the aircraft aerodynamic properties throughout the wide range of operating conditions, the design of the flight control system is a typical nonlinear control problem due directly to the responsive change in aircraft dynamics with flight condition [OSBV00]. Gain scheduling is an attractive control strategy to deal with these nonlinearities of the aircraft flight control. In the area of aircraft flight controller design, the main idea of gain scheduling methodology is to design a set of LTI controllers for a number of operating points and then interpolate the parameters or gains of the flight controllers against the current value of the scheduling parameters varying with the flight conditions over a wide flight envelope of an aircraft, instead of seeking a single robust LTI controller for the entire operating range.

For the modern civil passenger aircraft, it is quite common that some form of Flight Control System (FCS) is employed to augment the dynamic characteristics of the aircraft in order to obtain desirable flying and handling qualities. A typical FCS comprises a number of actuators, motion sensors including accelerometers and rate gyros, and air data sensors. All these signals are fed back through the controller that enacts certain flight control laws designed for a specific aircraft to control the control surface deflections and throttle [Coo07] [And10]. The control law is implemented by including one, or several control functions in the command, forward and feedback paths, each of which comprises multi-variable and separate loops for the aircraft influencing the characteristic behaviour of the augmented aircraft on the roll, pitch and yaw control axes differently.

The Control and Stability Augmentation Systems (CSAS) is an integral part of the FCS, which determines the control and stability characteristics of the augmented aircraft. The most commonly encountered CSAS control laws are equipped with rate feedback - improving artificial damping, C^* - a combination of normal acceleration and pitch rate feedback to give good dynamic handling and control sensitivity, and the longitudinal Rate Command Attitude Hold (RCAH)- that can achieve a good pitch attitude tracking characteristic. The main purpose of the FCS that includes CSAS is to improve an aircraft's flying and handling qualities, tracking ability and ride comfort [Coo10a][And10].

The handling qualities of an aircraft are the properties that describe the effectiveness and precision by which a pilot may control the aircraft in the execution of the defined flight task or mission [Coo07] [Gib99]. For more formal analytical purposes, these intangible properties must be described quantitatively rather than being expressed in terms of pilot opinion. The basic aerodynamic stability and control characteristics of the airframe, and also the effects of a installed FCS, are quantified and commonly used as indicators and measures of the handling qualities [Gib99]. Consequently, a number of criteria have been developed for the explicit purpose of ensuring good dynamic response characteristics to aid in the design of an airplane's dynamic characteristics, such as the Control Anticipation Parameter (CAP), Neal and Smith, C^*, Gibson and Bandwidth criteria.

The main contribution of this research is to combine the gain scheduling technique based on the local controller design approach and handling qualities assessment. The controllers are designed locally in a number of operating points and the interpolation between them (i.e. scheduling) takes place through the scheduling scheme functions. The aircraft augmented with gain-scheduled controller performs satisfactorily and meets the requirement of handling qualities. Moreover, the performance using the gain-scheduled controller is significantly improved compared to the performance using the fixed one.

1.2 Aim and Objectives

The aim of this project is to design a set of gain scheduling RCAH controllers for an aircraft longitudinal CSAS at different operating points to satisfy the handling qualities requirement. This focuses on the gain scheduling technique, FCS design and handling qualities assessment. With the aid of the handling qualities assessment, the schedule scheme will be designed. The study shall include the longitudinal Rate Command-Attitude Hold (RCAH) controllers design using the state space pole placement design procedure, handling qualities assessment over the whole flight envelope to identify the interpolation region, and the gain scheduling against the current value of the scheduling parameters (airspeed and altitude).

The objectives of this research are as follows:

- Design the longitudinal Rate Command-Attitude Hold (RCAH) controllers as the local flight controllers based on the specific equilibrium points.

    1. Trim and linearize the Boeing 747 series 100/200 aircraft model based on the equilibrium points along with the flight envelope.
    2. Employ the linear aircraft models on specific equilibrium points to aid in the controller design and handling qualities assessment.
    3. Apply the state space pole placement design procedure to design a longitudinal Command and Stability Augmentation System (CSAS) with Rate Command-Attitude Hold (RCAH) characteristic.

- Assess the handling qualities of the aircraft over the flight envelope.

    1. Assess the handling qualities of the aircraft with local controllers to guarantee the good dynamic response characteristics.
    2. Determine the number of local controllers to cover the entire flight envelope with satisfactory handling qualities.
    3. Identify the interpolation region by means of the handling qualities assessment of the local controllers in the whole flight envelope.

- Schedule the gains of the local controller in interpolation region of the flight envelope.

    1. Develop a gain scheduling scheme to ensure the controller gains are smoothly scheduled according to the current trimmed operating conditions achieving satisfactory performance and handling qualities.
    2. Review the influence of the flight controller with scheduled gains on the longitudinal handling qualities of the aircraft in the interpolation region.

In this research, the mathematical model of the Boeing 747-100/200 aircraft is employed for the flight controllers design and handling qualities assessment. The Boeing 747-100/200 aircraft model is an aircraft mathematical nonlinear model, and it is chosen since it is

- a well-known, popular and successful aerospace control analysis platform.

- easily available.
  The Boeing 747-100/200 aircraft model is obtained from references [EB03], [EB01] and [vdL96], which is based in Matlab/Simulink. In addition, it offers a wide range of simulation and analysis tools which make it easy to obtain linear flight dynamics models for any flight condition.

- a typical civil transport passenger aircraft.
  The Boeing 747 is an intercontinental wide-body transport with four fan jet engines. The wide array of characteristics, such as leading and trailing edge flaps, spoilers and variety of control surfaces, makes it representative of any commercial airplanes flying today.

1.3 Summary

In summary, this thesis presents a method for the design and development of gain scheduling controllers for a passenger aircraft B747 to satisfy the handling qualities requirement. With the aid of the handling qualities assessment over the whole flight envelope, the interpolation region is identified, and furthermore, the gain scheduling scheme is determined.

This thesis is organized as follows:

Chapter 2 presents a overview of the history, as well as development and state-of-the-art in flight control system, gain scheduling methodologies and aircraft handling qualities.

Chapter 3 describes the design process of a longitudinal Control and Stability Augmentation (CSAS) with Rate Command Attitude Hold (RCAH) characteristics. This comprises the application of the Boeing 747 series 100/200 aircraft mathematical model, the process of deriving the state space equations, and the design procedure of the controller.

Chapter 4 assesses the longitudinal handling qualities of the aircraft with the local controller using existing handling qualities criteria, including the Control Anticipation Parameter (CAP), Neal and Smith, and C^* criteria.

Chapter 5 discusses the handling qualities over the whole flight envelope together with a second controller design and identification of the interpolation region.

Chapter 6 derives the gain scheduling scheme to ensure the gains of the local controllers are smoothly scheduled according to the current trimmed operating conditions with satisfactory performance and handling qualities.

Chapter 7 provides a summary of findings, concluding remarks and recommendations for further work.

Chapter 2

Literature Review

This chapter presents a review of the history, the state-of-the-art of gain scheduling methodology development as applied to the aircraft flight control system, and in particular to satisfy handling qualities requirements. Thus the focus is on Flight Control System (FCS), gain scheduling technique and handling qualities assessment.

2.1 Flight Control System

The aircraft FCS enables the pilot to exercise control of the aircraft over all the flight, by controlling the aerodynamic control surface deflections and throttle to modify the aircraft dynamics, and then the aircraft is endowed with manoeuvre in pitch, roll and yaw axis [MAS08]. Normally, the FCS is designed specifically for a certain aircraft, which leads to a similar functional architecture with different details [Coo10a].

For the modern civil passenger aircraft, it is quite common that some form of FCS is employed to augment the dynamic characteristics of the aircraft in order to obtain desirable flying and handling qualities. A typical FCS comprises a number of actuators, motion sensors and air data sensors. All these signals are fed back through the controller that enacts certain flight control laws designed for a specific aircraft to control the control surface deflections and throttle [Coo07] [And10]. The control law is implemented by including one, or several control functions in the command, forward and feedback paths, each of which comprises multi-variable and separate loops for the aircraft influencing the characteristic behaviour of the augmented aircraft on the roll, pitch and yaw control axes differently.

2.1.1 Functional Description

The general objective of the flight control law design is to improve the flying and handling qualities of the basic aircraft, i.e., to enhance stability and controllability.

The pilot controls' inputs are transformed by the flight control computer into pilot control objectives which are compared with the measured aircraft states. A typical FCS is commonly comprised of measurements like air data sensors - measuring the aircraft states, such as velocity and attitude - motion sensors - measuring the aircraft attitude - as well as main functional components such as control actuators, aerodynamic control surfaces, the respective cockpit controls, connecting linkages, and the necessary operating mechanisms to control an aircraft's direction in flight [And10] [Coo10a] [MAS08]. In general, a commercial civil passenger aircraft FCS may well be divided into inner loops and outer loops with the respect of the type of control feedback loops. Inner loops are essential to determine the stability characteristics of an aircraft, e.g. the autostabiliser and the Stability Augmentation System (SAS) [Coo10a], while outer loops are in reference to optional functions, e.g. autopilot.

- Inner loops

    1. Modern high performance aircrafts rely heavily on the inner loop of the flight control system. Principal inner loop functions, such as the autostabilizer and the Control and Stability Augmentation System (CSAS), play a vital role and are continuously engaged during flight to enable the aircraft to perform manoeuvres with satisfactory flying and handling qualities. The SAS is an integral part of the FCS to augment the flying dynamic characteristics of the aircraft seeking for good flying qualities and stability, while Control Augmentation System (CAS) is additionally designed to modify the handling characteristics of the aircraft to improve the performance in manoeuvre and command tracking; together, these two systems are considered as CSAS [Coo07] [Coo10a] [Chu10] [And10].
    2. Secondary inner loop functions are concerned with automatic control of flaps, spoilers, engines, etc. without pilot intervention. In addition, it is common that many modern aircraft employ some form of active controls technology such as g-limiting, α-limiting and gust load alleviation [Coo10a] [MAS08].

- Outer loops
  The outer loops are mainly concerned with autopilot functions that are selectively engaged during the flight. The main purpose of the autopilot is either to release the pilot from continuous control of an aircraft (especially for civil passenger aircraft), or for operation in adverse conditions which are beyond the limits of human capability (particularly for military aircraft) [Coo10a].

In this research, the FCS control law function design is concerned with the CSAS of inner loop functions design. The functionality of the CSAS can be divided into three components: the feedback path, the feedforward path and the command path.

The closed-loop control and stability characteristics of the aircraft are primarily designed using the feedback path. Meanwhile, the forward path can be used both to define the closed-loop characteristics of the aircraft, and to provide control signal response shaping. Consequently; the command path may well be used to shape the command by the gain to produce the actual command signal for the closed loop without compromising the closed-loop characteristics [OB01]. The choice of the different control path can influence the aircraft dynamics differently, which relies on the requirement of flying and handling qualities. The simplified typical closed loop flight control functional structure is shown on Fig. 2.1. The closed loop transfer function thus can be obtained.

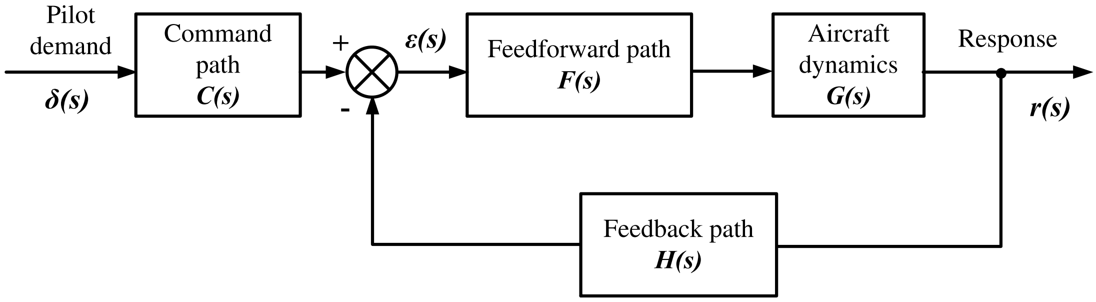


Figure 2.1: Closed Loop Control System

2.1.2 Command and Stability Augmentation System

The main purpose of the FCS, including the CSAS, is to improve an aircraft's flying and handling qualities, tracking ability and ride comfort [Coo10a] [And10]. The CSAS control law plays the key role in the entire FCS, which governs the stability and control characteristics of the aircraft seeking ideal flying and handling qualities. The decoupling of the longitudinal and lateral-directional motion of the aircraft gives rise to the possibility to design the longitudinal and lateral-directional CSAS control laws separately. The longitudinal dynamics are characterized by the Short-Period Pitching Oscillation (SPPO) mode and phugoid oscillation mode. The phugoid mode is manifested as a trimming problem, which is usually considered by the autopilot in the FCS outer loops of the aircraft. Although it is not regarded as hazardous when poorly damped, it does contribute to an increased pilot workload [Coo10b]. On the other hand, the short period mode is the main factor that needs to be augmented by the CSAS.

Before designing a CSAS, it is necessary to consider the choice of control law functions based on the design objectives for flying and handling qualities. The most commonly encountered CSAS control laws are equipped with rate feedback improving artificial damping; C^*, a combination of normal acceleration and pitch rate feedback to give good dynamic handling and control sensitivity; and the longitudinal Rate Command Attitude Hold (RCAH) achieving an excellent pitch attitude tracking characteristic. Several sub-system functional controllers of CSAS developed for different design objectives are discussed as follows:

- Rate feedback autostabilizer is used to augment the short term stability characteristics by improving the damping of the transient dynamics. As a result of the utilization of the feedback of the sensed motion rate variable, the stability of the roll mode, short period pitching mode and the dutch roll mode could be improved by the artificial damping improvement. It is considered the simplest - as well as a safe and reliable - augmentation system even though it restricts augmentation of the relaxed longitudinal static stability issue. The restrictions are due mainly to the fact it does not modify the original dynamic structure and stability and control characteristics of the aircraft [Coo10a]. Historically, the rate feedback autostabilizer has caused few problems and has generally been very successful, which generates classical pitch, roll and yaw damper. The classical yaw damper structure is shown in Fig. 2.2.

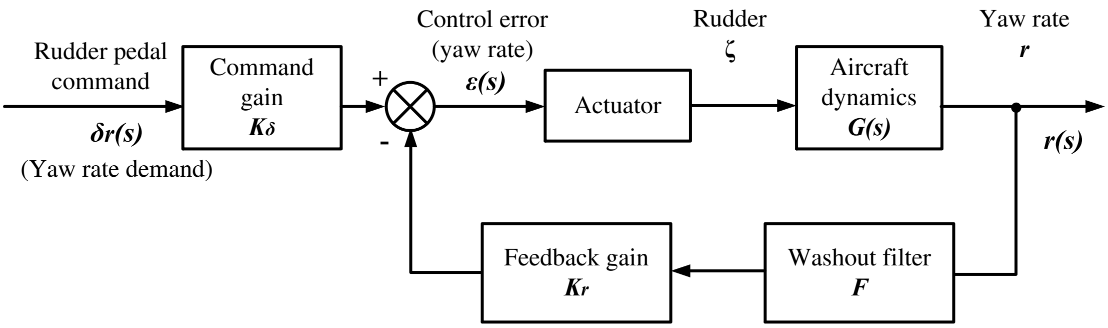


Figure 2.2: Classical yaw damper structure

- The longitudinal C^* controller refers to any longitudinal CSAS that includes a combination of normal acceleration and pitch rate feedback. Thus, a C^* controller can not only provide good short period mode damping augmentation of pitch rate, but also improve the command response characteristics of acceleration feedback [Coo10a]. The typical longitudinal C^* controller structure is presented in Fig. 2.3.

- The longitudinal RCAH controller is effective in improving the dynamic attitude tracking characteristics of the aircraft to perform precision pilot tracking tasks. Commonly a classical Proportional-plus-Integral (PI) control structure is applied in designing the RCAH controller.

  The longitudinal RCAH controller has a wide range of successful applications on a number of advanced fly-by-wire (FBW) aircraft, including the Eurofighter Typhoon, and the F-18 [Coo07] [Coo10a].

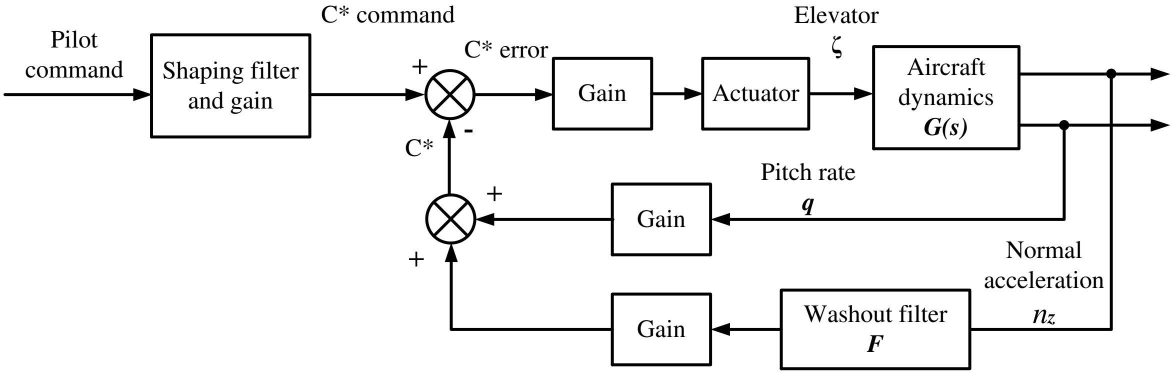


Figure 2.3: Longitudinal C^* controller structure

For the purpose of gain scheduling techniques implementation, a generic structure of the longitudinal or lateral-directional flight control law, is selected. In this research, a longitudinal FCS CSAS with RCAH characteristic is employed as a local controller to construct a global controller due mainly to the widespread application of PID and PI local control structures with scheduled gains in the FCS [LWY+08] [OSBV00].

2.2 Gain Scheduling

The idea behind gain scheduling is that the gains of controllers are scheduled with some measured parameters of the system. These measured parameters of the system are usually referred to as the scheduling parameters.

The recent considerable increasing interest in gain-scheduling design methodology is not new. Gain-scheduling is one of the most popular and conventional methods of dealing with nonlinear control systems in many engineering applications, and much of the classical gain-scheduling theory could date back to the early 1960s. However, in the early days, the robustness and performance could not be guaranteed due mainly to the absence of a sound theoretical analysis regarding the issue of the guaranteed properties for a set of control systems with scheduled gains (linear parameter-varying plants) and nonlinear plants. The focus of attentions was on the discussion of guaranteed performance; gain-scheduling methods did not start to be applied to cope with nonlinear plants until the early 1990s [SA90] [RS99]. Since then, gain-scheduling has been established as a worthwhile design methodology for nonlinear systems control and widely and successfully applied in fields ranging from aerospace to process control. Nowadays, gain scheduling is considered as a standard method to design Linear Time-Invariant (LTI) controllers for Linear Time Varying (LTV) or nonlinear systems in control theory. As such, the nonlinear system can be approximated as a Linear Parameter Varying (LPV) system. In a conventional gain-scheduling approach, the scheduling parameters are usually chosen from the variables with slow variation and could capture the plant's nonlinearities, such as altitude, velocity, the forward velocity u, and the downward velocity w [Rad04] [SA90] [RS99].

With the development of the gain-scheduling techniques, many different design notions can be interpreted as gain scheduling which is generally developed as a point of view taken in the design process. The main idea of gain scheduling method is using linear controller design techniques to address nonlinear problems by continuously varying the controller coefficients according to the current value of the scheduling parameters. Hence, the overall performance properties are determined by the local designs. Comprehensive overviews of gain-scheduling techniques can be derived from the references [LL00] and [RS99]. Recently, there has been an increasing number of research activities and a wider range of applications in the area of gain scheduling, especially in the aerospace field [RS99] [SG07].

In the area of aircraft flight controller design, gain scheduling is the most common systematic approach to cope with the nonlinearity of the aircraft dynamics over the flight envelope. In the aerospace sector, gain scheduling technology was first used mainly on military applications, e.g. missile guidance systems and autopilots for the B-52. By the end of the Second World War in the mid 1950s, gain scheduling techniques began to overcome difficulties of implementing into the new generation of jet aircraft [RDLB03]. Since then, a research subject of gain scheduling application in civil aircraft developed, and gradually, it has broadened out into more widespread engineering applications, an important example being its implementation in the design of the flight control system of an aircraft [RS99] [LL00] [Zhu06] [GZ94]. In the light of the significant change of the aircraft aerodynamic properties throughout the wide range of operating conditions, the design of the flight control system is a typical nonlinear control problem due directly to the responsive change in aircraft dynamics with flight condition [OSBV00]. Gain scheduling is an attractive control strategy to deal with these nonlinearities of the aircraft flight control. Although it does not always provide controllers that guarantee closed loop stability and performance, gain scheduling design method possesses certain engineering practical advantages in aircraft flight control system design, such as simplicity, generality, low computational complexity and ease of implementation [RLBC06].

In recent years, there have been several published studies of gain scheduling in aircraft flight control system design. Several synthesis algorithms have been exploited for systematic design of gain-scheduling controller. In order to overcome the constraints on the maximum rates of change of the scheduling parameters, the rapidly varying states such as angle of attack α are adopted as the scheduling parameters instead of the conventional scheduling parameters with slow variation [RLJD07]. It is found that the dynamic transient response in nonlinear regions has been improved. This method for scheduling state feedback gains against rapidly varying parameters is detailed in references [RDLB03], [RLJD07] and [SB92]. Gain scheduling research in aircraft flight control system design has been very active over the last decade. It could cover a wide range of subjects including the following:

- Single-input-single-output(SISO) and multivariable scheduling problem [Gar97] (with respect to input/output characteristics);
- H_∞ controller design [NRR93] and adaptive control [JAL08](due to the control techniques);
- ν-gap metric [FFN03], fuzzy clustering [OBB02] [GZ94], from interpolation techniques viewpoint;
- RCAH and coordinated turning [WSG03], from the control law functions.

The characteristic of gain scheduling methodology is to design a set of LTI controllers for a small number of operating points and interpolate the parameters(gains) of the controllers in the region between operating points over a wide flight envelope of an aircraft instead of seeking a single robust LTI controller for the entire operating range [AEJ08].

Based on the typical gain-scheduling design procedure for nonlinear plants derived from references [SA90] and [OSBV00], the design procedure of a gain scheduled flight control system design for an aircraft can be stated as follows:

1. Linearize the aircraft model around a selection of equilibrium points in the flight envelope.
2. Design a linear controller for each of the linearized models as summarized in section 2.1.2. We will refer to this as the 'local controller'.
3. Tune the coefficients (gains) of the local controller at each equilibrium point.
4. Schedule the coefficients (gains) of the local controllers resulting in a global controller- this involves the implementation of a family of linear controllers such that the controller coefficients (gains) are scheduled according to the current value of the scheduling parameters.
5. Evaluate the performance by linear analysis and non-linear simulation.

This process converges when the closed-loop aircraft dynamics are satisfactory over the entire operating range.

Most of the efforts paid on gain-scheduling flight control system design have dealt with the local controller design procedure as well as the identification of the operating points and the design of the interpolation scheme [OSBV00] [NRR93]. Three main aspects concerned about these two questions are as follows [OBB02]:

- The local controller design procedure

  Based on the identification of the operating points, the local controllers for the aircraft longitudinal motion control can be designed for the linearized aircraft model of the Boeing 747-100/200. In this research, the classical single-loop design techniques-state space pole placement is employed to develop flight control laws. There has been extensive research in using advanced control design methods to replace the classical CSAS in the flight control law design. Such methods include H_∞ multivariable design [BP02] and eigenstructure assignment [LP98]. Although replacing the classical single-loop approach with a multi-loop enhances the performance and robustness of the controller, it has not overcome difficulties of the development of an efficient method for scheduling of the multivariable controller for future industrialization [ARS01] [OBB02]. For most of the fly-by-wire (FBW) aircraft flying today, the control laws have been developed by using classical single-loop design techniques, such as frequency responses, root locus and state space pole placement.

- The identification of the operating points

For implementation of classical gain scheduling, the flight envelope is subdivided into operating regions based on information of the aircraft dynamics as a function of flight condition at different operating points. The trial-and-error method is generally employed in gain-scheduling FCS design to identify the operating points [OSBV00]. Although the operating points are selected arbitrarily and inexplicitly by this method, the iterative approach based on the past experience guarantee the satisfactory closed-loop aircraft dynamics over the entire operating range in the design process.

In this study the selection of the operating points is performed using a heuristic method based on the handling qualities assessment. Besides the local controllers' performance analysis, the aircraft handling qualities are assessed over the flight envelope. Normally, it is not necessary to design local controllers at each operating points for the purpose of covering the whole flight envelope with satisfactory handling qualities. Comparing to the conventional method of dealing with controllers at each of these equilibrium points, the whole operating range could be covered by a fewer number of controllers, which increases the efficiency of the scheduler. The advantage of applying this method to identify the operating points are that the number of local linear controllers can be kept small and more importantly the global nonlinear controller complies with the performance and handling qualities requirements.

- The design of the interpolating scheme

In this research, the conventional gain-scheduling method is considered. The parameters with slow variation- airspeed and altitude are chosen as scheduling parameters to capture the nonlinear properties of aircraft flight dynamics varying with the flight conditions. Gains of the controller can be scheduled against the current value of the airspeed and altitude following the interpolating scheme which is an important part of the gain-scheduling FCS design process. The implementation of the interpolating scheme design ensures an appropriate gain-scheduled controller, and finally determines the performance of the whole set of the controller. A number of different approaches of interpolating scheme design could be used, such as bumpless switching, interpolation techniques, and linear interpolation of the parameters [ABB02] [AEJ08]. In this thesis, the interpolating of a family of local linearized controllers with switching techniques of linear interpolation of the parameters is practiced to yield a global controller.

In recent years, Linear Parameter Varying (LPV) control has been established as an emerging advanced approach to be a reliable alternative to conventional gain-scheduling approach [EB01] [Chu10]. LPV is based on the principle of the H_∞ multi-variable control, and the excellent performance - including command tracking, disturbance attenuation, low sensitivity to measurement noise, and reasonably small control efforts - is mainly due to LPV technique theoretical property. The whole system is considered as a single dynamic system [Chu10]. Clearly, the conventional gain scheduling is a collection of dynamic systems, which means the robust performance can not be easily guaranteed for the global controller. Although this conventional approach can not guarantee the stability or robustness of the controller for each operation point of the flight envelope, this conventional approach of designing gain scheduled FCS for aircraft still has a wide range of applications in the engineering field due to its developed technique. Hence, this research contains a certain practical meaning as well.

2.3 Handling Qualities

The handling qualities of an aircraft are the properties that describe the effectiveness and precision by which a pilot may control the aircraft in the execution of the defined flight task or mission [Coo07] [Gib99]. For more formal analytical purposes, these intangible properties must be described quantitatively rather than being expressed in terms of pilot opinion. Not only the basic aerodynamic stability and control characteristics of the airframe, but also including the effects of an installed flight control system (FCS), are quantified and commonly used as indicators and measures of the handling qualities [Gib99]. Consequently; a number of criteria have been developed for the explicit purpose of ensuring good dynamic response characteristics to aid in the design of an aircraft's dynamic characteristics, such as the Control Anticipation Parameter (CAP), Neal and Smith, C^*, Gibson and Bandwidth criteria. In order to quantify the handling qualities of the B747 to aid in the identification of the operating point and accomplish the scheduler design, the existing flying and handling qualities criteria and specifications are reviewed.

Both civil and military handling qualities criteria or specifications exist which define the minimum performance requirements for a given aircraft. These aircraft handling qualities criteria and flying specifications can be specified in several ways. Military and civil flying specifications, such as MIL-STD-1797A and FAR-25 or CS-25. The US Federal Aviation Regulations (FAR-25) [Ano94] and Certification Specification for Large Aeroplanes (CS-25) [CS206] define a comprehensive suite of performance and safety requirements with which any large commercial transport aircraft must comply if it is to be granted a certificate of airworthiness. For the handling qualities assessment of a civil passenger aircraft like B747, FAR-25 and CS-25 are the most suitable criterion to comply with. Unfortunately, there are few specifications to quantify the handling qualities in FAR25 or CS-25. For more formal analytical purposes, it is necessary to describe the handling qualities quantitatively. Although these criteria are generally better developed for military combat aircraft, and are not entirely applicable to the B747 aircraft, the CAP, Neal and Smith, and C^* criteria can still be applied to assess the handling qualities of the B747 aircraft as an indicator of the degradation.

There exist many handling qualities criteria which aid the designer in the definition of the aircraft and the specification of its dynamic characteristics [Coo07]. These aircraft handling qualities criteria and flying specifications are commonly defined in terms of pole-zero specifications [Ano90]. Criteria can be in terms of minimum damping and natural frequency, or pole-position, such as the incidence lag variable T_{θ_2} [Gib99]. Criteria can also be defined in terms of frequency response, such as minimum gain and phase margins - for example the bandwidth criterion - or time response, such as the C^* Criterion [TEM66]. Criteria can also be defined based on pilot models, such as the Neal and Smith criteria which estimates aircraft flying qualities based on pilot model compensation requirements [TR71].

In general, flying specification requirements vary for different phases of a flight. Certain pilot tasks associated with different flight phases require more stringent requirements in order to achieve the mission successfully. The specification requirements are stated with respect to flight phase categories, classification of airplanes and levels of flying qualities. Those missions requiring similar flying qualities are commonly grouped together into three flight phase categories: Categories A, B, and C [Ano90] [Ano80].

Table 2.1: the Boeing 747-100/200 aircraft [Ano80]

Specification requirements	Boeing 747	Description
Classification of airplanes	Class III	Heavy transport
Flight Phase Categories	Category B	Category B: Cruise (CR)
Levels of flying qualities	Level 1	Flying qualities clearly adequate for the mission Flight Phase

The Boeing 747-100/200 aircraft is considered to be a Class III aircraft in Nonterminal Flight Phases-Category B. Level 1 flying qualities are required due to being a large, heavy aircraft, with low to medium manoeuvrability due to the necessity of passenger ride comfort. See Table 2.1.

Chapter 3

Controller Design

3.1 Aircraft Model Introduction

Before a controller of longitudinal Command and Stability Augmentation System (CSAS) can be designed, an aircraft model has to be determined. In this research, the mathematical model of the Boeing 747-100/200 aircraft is employed in aid of flight controllers design. The Boeing 747-100/200 is a wide-body commercial airliner and cargo transport with four fan jet engines. The performance characteristics are summarized in Table 3.1 below and the Boeing 747-200B during take-off is shown in Fig. 3.1.

Table 3.1: Boeing 747-100/200 Specifications [EB03] [B7407]

Cruising speed (at 35,000 ft altitude)	Mach 0.84 (893 km/h, 481 knots)
Maximum speed	Mach 0.89(955 km/h, 516 kn)
Maximum Range	6,100 statute miles (9,800 km) to 7,900 statute miles (12,700 km)
Design ceiling	13,716 m

The Boeing 747-100/200 aircraft model employed in this research is also known as FTLAB747v6.5, which is derived from the references [EB03], [EB01] and [vdL96]. FTLAB747v6.5 is developed based on Delft University Aircraft Simulation Model and Analysis Tool (DASMAT) and the Flight Laboratory 747 (FlightLab747). Both DASMAT and Flightlab747 are originally developed by Delft University of Technology. DASMAT is a general and powerful Computer Assisted Design (CAD) environment for flight dynamics and control analysis developed by C. A. A. M. van der Linden in 1996. Then, M. H. Smaili developed FlightLab747 based on DASMAT in order to study the EL AL Israel Airlines crash on October 4th 1992 near Amsterdam. Andres Marcos Esteban, who is the main developer of this model, updated an enhanced version of the Boeing 747-100/200 aircraft model - FTLAB747v6.5 in

Figure 3.1: Boeing 747-200B [B74]

2003 which is a widespread aerospace control analysis platform still using at present. FTLAB747v6.5 is an aircraft mathematical nonlinear model, which is based in Matlab/Simulink and offers a wide range of simulation and analysis tools. Meanwhile, it is also an ideal test platform to test fault tolerant control, fault detection and fault diagnostics.

In addition, FTLAB747v6.5 is well-designed with user-friendly interface. The application of this program simplified the controller design process by directly providing the procedures of trimming and linearizing the aircraft model at the defined operating points. Consequently, this nonlinear model could also be used to be an test bed to evaluate the performance and handling qualities of the aircraft with the controllers.

For the Boeing 747-100/200 aircraft model, the flight envelope considered is shown in Table 3.2. In addition, it does not include approach and take off configurations, landing gear and the ground effects [EB03].

Table 3.2: The flight envelope of the Boeing B747 aircraft model [EB03]

Parameters	**Symbols**	**Range**	**Units**
Altitude	h_e	5000 to 11000	m
True airspeed	V_{TAS}	150 to 260	m/s
Angle of attack	α	-2 to 23	deg

The aircraft is assumed to be a rigid symmetric aircraft in this model. In order to design a FCS through conventional design methods, a linearized model - that describe the aircraft's flight dynamics about one specific equilibrium point - must be developed.

The DASMAT manual [vdL96], and the FTLAB747v6.5 manual in Chapter 6 of [EB03] provide a fuller description of FTLAB747v6.5.

The trimming and linearizing process, equations of motion, and controller design will be introduced subsequently.

3.2 Trim and Linearization

It is considered that the moments acting on the aircraft are balanced when the aircraft is trimmed in steady flight. To trim an aircraft model is not as easy as to trim a real aircraft flying in the air. Trimming and linearizing are a mathematically complex procedure for an aircraft model. For linear flight controller design, special tools are provided in DASMAT to trim and linearize the aircraft at defined operating points. Therefore, it can be realized through activating the corresponding button in this B747 aircraft model. This trim and linearization routine embedded in this B747 aircraft model simplifies the process of controller design and test. However, the important thing is selection of the equilibrium points throughout the flight envelope. See Appendix C for further information of trim and linearization tools of FTLAB747v6.5.

3.2.1 Aircraft Model Trim

A trim point with specific flight configuration and condition is defined in this trim routine which is tabulated in the following Table 3.3.

Table 3.3: Aircraft Configuration of Boeing 747-100/200 in Trim Routine

Aircraft Configuration	**Value**
Initial Mass(kg)	$300,000$
x_{cg}	25% MAC
y_{cg}	0% MAC
z_{cg}	0% MAC
Flight Configuration	**Value/Status**
Altitude(h_e)	7000 m
Airspeed (V_{TAS})	241 m/s
Four engines	All working
Flight Condition	Straight-and-level trim

As shown in the Table 3.3, the specific configurations of the aircraft model are defined. The altitude ($h_e = 7000$ m) and airspeed ($V_{TAS} = 241$ m/s) represent the current operating condition illustrating the operating or equilibrium point where the aircraft is trimmed. These two flight configuration observable variables are also selected to be the scheduling parameters in this research. The trim point ($h_e = 7000$ m, $V_{TAS} = 241$ m/s) is chosen as the operating point, upon which the aircraft model is trimmed and linearized, then a flight controller is developed.

3.2.2 Aircraft Model Linearization

After flight equilibrium condition is defined in section 3.2.1, the linearization routine is performed in this section to linearize the aircraft model at the obtained operating point. Similarly, the linearization routine involves several selections and settings tabulated in Table 3.4. The linearization routine is detailed in reference [EB03].

Table 3.4: Selections in Linearize Routine

Type	**Optional Selection**	**Remark**
Control Inputs	Control Surfaces	
Mode to Linearize	Symmetric	Longitudinal motion
Observation Groups (Refer to Fig. 3.2)	`x`, `xdot`, `uctrl` (Refer to Fig. 3.3)	All the available derivatives and states are specified for each group.
Output Complexity	Compact	

The selection of mode type in linearize routine is important since it will directly affect the A_{lin}, B_{lin}, C_{lin}, D_{lin} matrices of equations of motion for linearized aircraft model. More details of the longitudinal equations of motion obtained from the trim and linearization routine will be expanded in the following section. A description of the aircraft states and selections shown in Fig. 3.2 and Fig. 3.3 with their units are given in Table C.1.

3.3 Aircraft Equations of Motion

The Boeing 747-100/200 model is trimmed and linearized on the operating point ($h_e = 7000$ m, $V_{TAS} = 241$ m/s) by the procedures performed in section 3.2to obtain the body-axes longitudinal equations of motion. In considering the current flight condition of 'Straight-and-level trim' defined in Table 3.3, the angle of attack is small, that is, $\alpha \leq 10°$, the following approximations can be made [Nel98].

$$\alpha = \tan^{-1}(\frac{w}{u}) \cong \frac{w}{u} \tag{3.1}$$

$$V_{TAS} = \left(u^2 + v^2 + w^2\right)^{1/2} \cong u \tag{3.2}$$

With reference to [Coo07], the concise form of full order equations of motion are given by,

$$\begin{bmatrix} \dot{q} \\ \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} m_q & m_u & m_\alpha & m_\theta \\ x_q & x_u & x_\alpha & x_\theta \\ z_q & z_u & z_\alpha & z_\theta \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ u \\ \alpha \\ \theta \end{bmatrix} + \begin{bmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{bmatrix} \eta \tag{3.3}$$

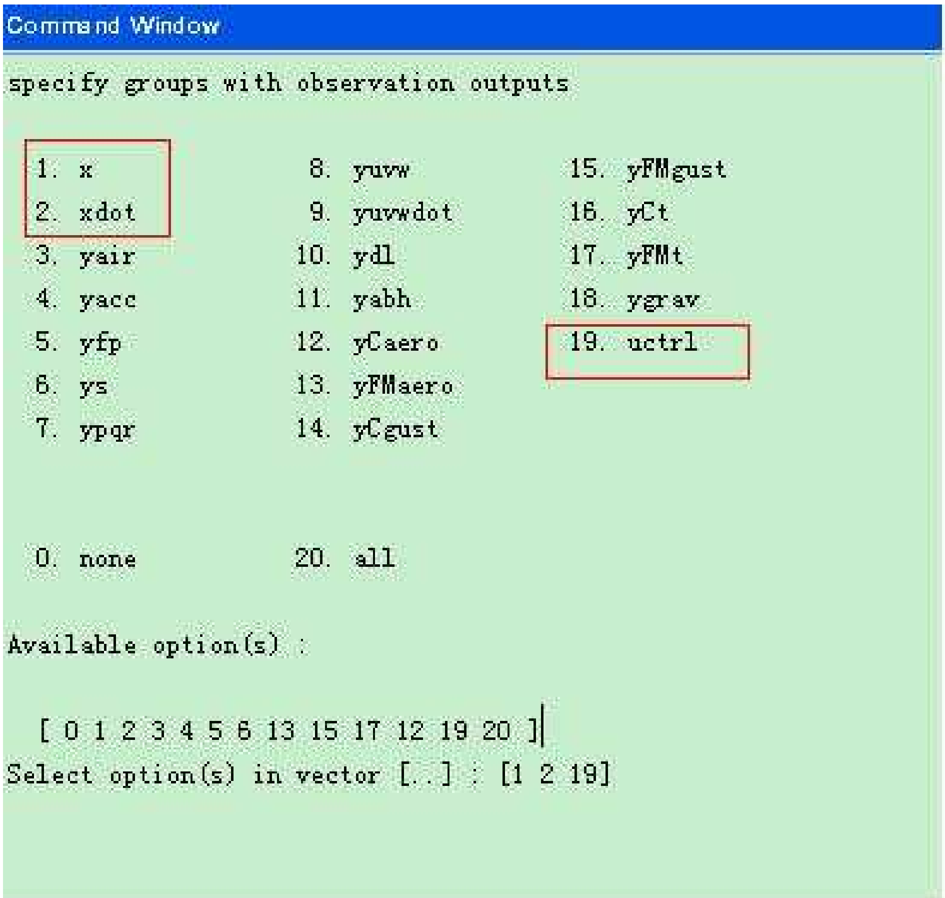


Figure 3.2: Observation outputs groups specification

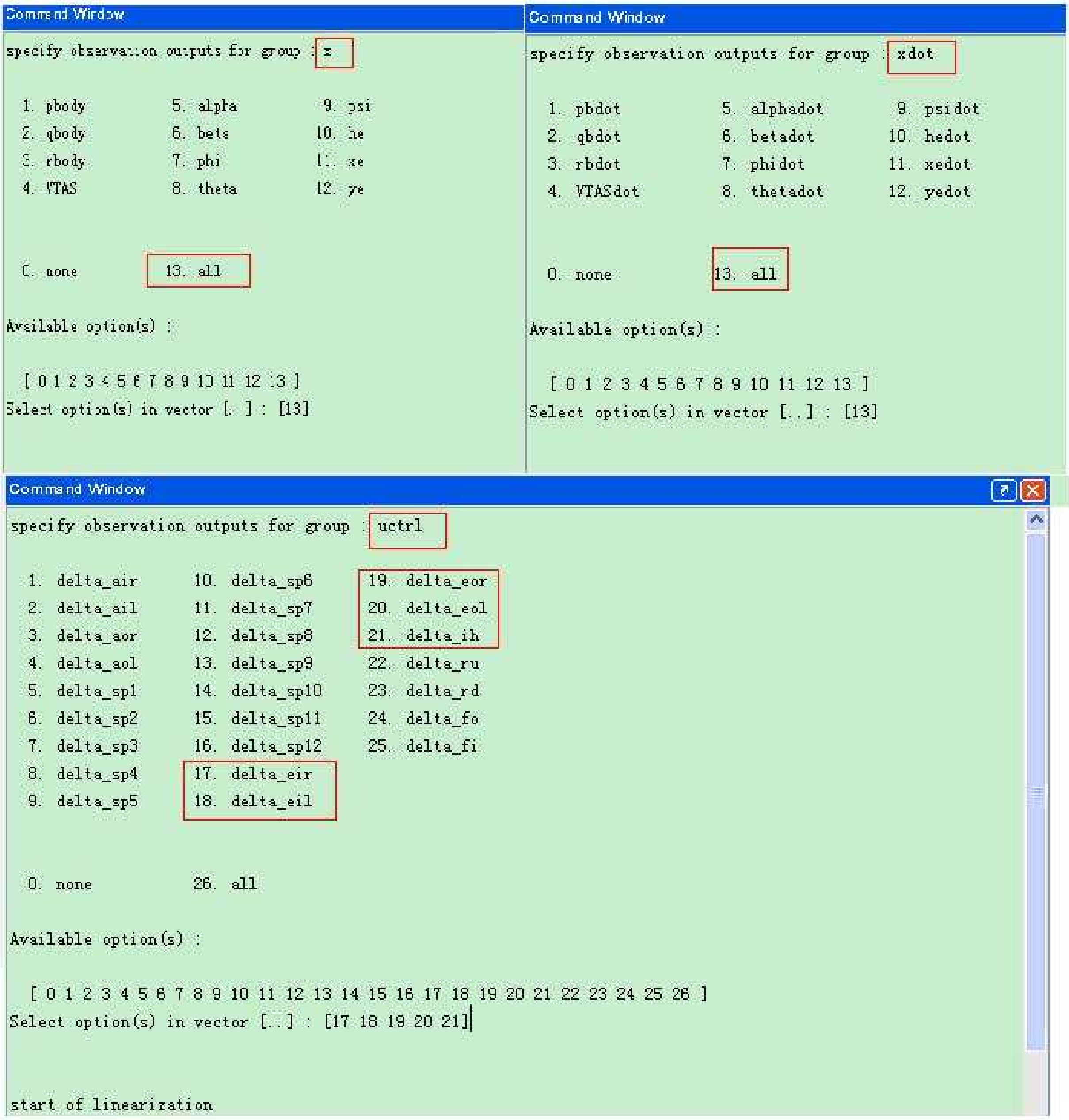


Figure 3.3: Observation outputs:`x`, `xdot`, `uctrl`

Then, the full order equations of motion are given by,

$$\dot{x}(t) = A_{lin}x(t) + B_{lin}u(t) \tag{3.4}$$

where,

$$x(t) = \begin{bmatrix} q_{body} & V_{TAS} & \alpha & \theta \end{bmatrix}^T; \tag{3.5}$$

$$u(t) = \begin{bmatrix} \delta_{stab} & \delta_e \end{bmatrix}^T \tag{3.6}$$

$$A_{lin} = \begin{bmatrix} -0.728 & -0.00048 & -1.2025 & 0 \\ -0.0839 & -0.00547 & 6.00779 & -9.78 \\ 1.0019 & -0.00036 & -0.515 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

$$B_{lin} = \begin{bmatrix} 2.3594 & 4.6099 \\ 0 & 0 \\ 0.0454 & 0.0944 \\ 0 & 0 \end{bmatrix} \tag{3.8}$$

Thus the open loop characteristic polynomial can be obtained by taking the Laplace transform of equation (3.4),

$$\Delta(s) = (s^2 + 0.002564s + 0.002525)(s^2 + 1.245s + 1.583) \tag{3.9}$$

The longitudinal stability characteristics are therefore,

$$\zeta_p = 0.0255, \qquad \omega_p = 0.0503 rad/s; \tag{3.10}$$

$$\zeta_s = 0.495, \qquad \omega_s = 1.26 rad/s; \tag{3.11}$$

3.4 Design Requirements

Before the controller can be designed, the design requirements may be determined based on the American Military Specification MIL-F-8785C [Ano80] and MIL-STD-1797A [Ano90] with respect to flight phase categories, classification of airplanes and levels of flying qualities.

According to the specification requirements stated in MIL-F-8785C and MIL-STD-1797A, the Boeing 747-100/200 aircraft is classified with Class III airplane. The aircraft flight phase required is in Non-terminal Flight Phases-Category B (cruise) and Terminal Flight Phases-Category C (takeoff and landing), and with Level 1 flying qualities. It is necessary to calculate the value of normal load factor per unit angle of attack n_α following (3.12), before determining the acceptable range of short period frequency ω_s complied with MIL-F-8785C/MIL-STD-1797A specifications.

$$n_\alpha = \frac{n_z}{\alpha} = -\frac{z_w U_e}{g} \tag{3.12}$$

Substituting equations (3.1) into equation (3.3) the following expression is obtained,

$$\begin{bmatrix} \dot{q} \\ \dot{u} \\ \frac{\dot{w}}{u} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} m_q & m_u & m_\alpha & m_\theta \\ x_q & x_u & x_\alpha & x_\theta \\ z_q & z_u & z_\alpha & z_\theta \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ u \\ \frac{w}{u} \\ \theta \end{bmatrix} + \begin{bmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{bmatrix} \eta \tag{3.13}$$

Thus, α is replaced by w as follows,

$$\begin{bmatrix} \dot{q} \\ \dot{u} \\ \dot{w} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} m_q & m_u & \frac{m_\alpha}{u} & m_\theta \\ x_q & x_u & \frac{x_\alpha}{u} & x_\theta \\ z_q \cdot u & z_u \cdot u & \frac{z_\alpha \cdot u}{u} & z_\theta \cdot u \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ u \\ w \\ \theta \end{bmatrix} + \begin{bmatrix} x_\eta \\ z_\eta \\ m_\eta \cdot u \\ 0 \end{bmatrix} \eta \tag{3.14}$$

where,

$$z_w = \frac{z_\alpha \cdot u}{u} = z_\alpha \tag{3.15}$$

z_w can be obtained from the A_{lin} matrix in equation (3.4) by $z_w = z_\alpha = A_{lin_{3,3}} = -0.515$.

Then,

$$n_\alpha = \frac{n_z}{\alpha} = -\frac{z_w U_e}{g} = -\frac{-0.515 \times 241}{9.8} = 12.67g/rad \tag{3.16}$$

Consequently, the corresponding stability modes characteristics may be determined based on MIL-F-8785C/MIL-STD-1797A specifications as shown in Table 3.5.

Table 3.5: MIL-F-8785C/MIL-STD-1797A Specifications

Symbol	**Value(rad/s)**	**Remark**
Phugoid damping ratio	$\zeta_p \geq 0.04$	Level 1 flying qualities
Short period damping ratio	$0.3 \leq \zeta_s \leq 2.0$	Level 1 flying qualities for Category B Flight Phases
Short period damping ratio	$0.35 \leq \zeta_s \leq 1.3$	Level 1 flying qualities for Category C Flight Phases
Short period frequency (rad/s)	$1.0 \leq \omega_s \leq 6.0$	Level 1 flying qualities for Category B Flight Phases
Short period frequency (rad/s)	$1.5 \leq \omega_s \leq 6.0$	Level 1 flying qualities for Category C Flight Phases

3.5 Controller Design

The first step in the CSAS design procedure is to establish a stability augmentation design objective. The MIL-F-8785C/MIL-STD-1797A specifications in Table 3.5 are compared to the longitudinal stability characteristics of the basic aircraft model obtained in cruise configuration, presented in Table 3.6. Clearly, the phugoid damping ratio should be raised to a suitable value. The phugoid or long-period mode is characterized by gradual changes in pitch angle, altitude and velocity over long periods of time. However, the phugoid mode is manifested as a trimming problem, which is usually considered by the autopilot instead of being augmented by CSAS. The short-period dynamics are characterized with Short-Period Pitching Oscillation (SPPO). The principal variables of short period mode are angle of attack α and pitch rate q [Nel98]. Pitch rate feedback to elevator could be employed to augment the aircraft, which is effective in raising both phugoid damping ratio and short period damping ratio [Coo07]. In addition, it is common that the Rate Command-Attitude Hold (RCAH) controller is included with a classical PI control structure acting on pitch rate feedback to elevator in many modern aircraft as they can provide the aircraft with better tracking performance. The typical control structure of RCAH is shown in Fig. 3.4. Proportional term can provide the desired closed loop stability and rate command performance. In addition, the integral term eliminates the rate command error resulting to the good command tracking characteristic.

Table 3.6: Design Requirements

Symbol	MIL-F-8785C/ MIL-STD-1797A	**B747 aircraft model**
Phugoid damping ratio	$\zeta_p \geq 0.04$	$\zeta_p = 0.0255$
Short period damping ratio	$0.35 \leq \zeta_s \leq 1.3$	$\zeta_s = 0.495$
Short period frequency (rad/s)	$1.0 \leq \omega_s \leq 6.0$	$\omega_s = 1.26$

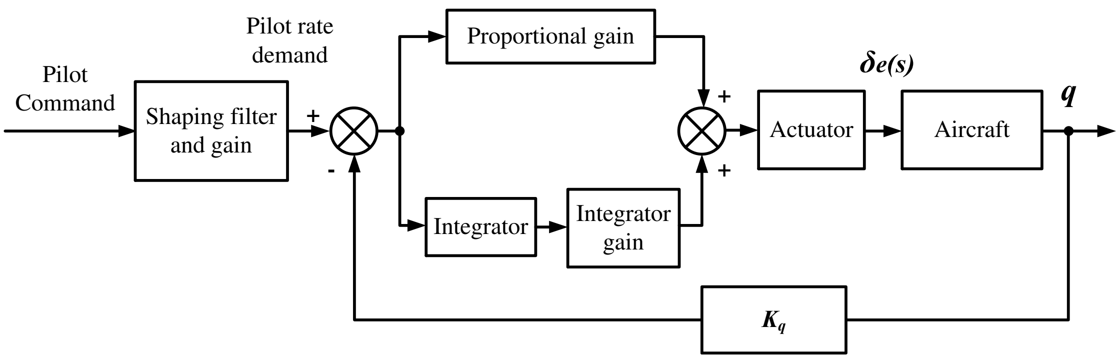


Figure 3.4: The control structure of RCAH [Coo10a]

3.5.1 Augmenting the Reduced Order Mode State Equation

The short period characteristics are more important in flying and handling qualities considerations. The reduced order state equation of aircraft short period mode can be obtained by decoupling the full order of equations of longitudinal motion (3.4), which is given by,

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -0.728 & -1.2025 \\ 1.0019 & -0.515 \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} 4.6099 \\ 0.0944 \end{bmatrix} \delta_e \tag{3.17}$$

The longitudinal equations of motion are now simplified to describe short term dynamics only. The reduced order transfer functions can be given as follows, by taking the Laplace transform of equation (3.17).

$$\frac{q(s)}{\delta_e(s)} = \frac{4.6099(s+0.4905)}{(s^2+1.243s+1.58)} \tag{3.18}$$

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{0.094396(s+49.66)}{(s^2+1.243s+1.58)} \tag{3.19}$$

The longitudinal stability characteristics of short period mode are therefore,

$$\zeta_s = 0.495, \qquad \omega_s = 1.26 rad/s; \tag{3.20}$$

The additional integrator state variable denoted $\epsilon_q(s)$ is employed to augment the state equation, which is shown in Fig. 3.5. q_d is pitch rate demand. The state equation can by written as

$$\dot{\epsilon_q}(t) = q(t) - q_d(t) \tag{3.21}$$

The open loop augmented state equation can be obtained by adding the integral

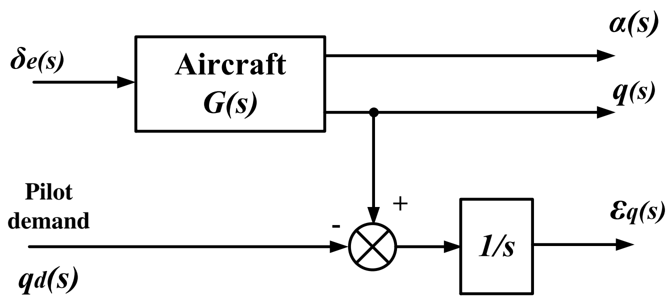


Figure 3.5: Additional Integrator

state equation (3.21) to the reduced order state equation (3.17) as follows,

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{\epsilon_q} \end{bmatrix} = \begin{bmatrix} -0.728 & -1.2025 & 0 \\ 1.0019 & -0.515 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 4.6099 \\ 0.0944 \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} q_d \tag{3.22}$$

The general pattern of open loop state equation and feedback control law are as follows,

$$\dot{x} = Ax + Bu + Nv \tag{3.23}$$

$$u = -Kx + Mv \tag{3.24}$$

where, K is the matrix of state variable feedback gains and M is the matrix of feed forward variable gains. The closed loop system state equation can be obtained in the general form by solving the equations (3.23) as,

$$\dot{x} = [A - BK]x + [BM + N]v \tag{3.25}$$

The suitable selection of gain matrices K and M is the main task in designing the RCAH controller. As can be observed from (3.25), the stability of the controller is governed by suitable selection of gain matrices K and M, which determine the system response transients of the controller. In other words, the suitable choice of gain matrices K and M is associated with the location of poles and zeros of the system in the s-plane. The design process is presented in the following sections.

3.5.2 Designing the Gain Matrix K

The pole placement approach is employed in designing the gain matrix K, where,

$$K = \begin{bmatrix} k_q & k_\alpha & k_{\epsilon_q} \end{bmatrix} \tag{3.26}$$

First of the most, to determine the location of the desirable closed-loop poles of the system base on the relevant flying qualities specification for the B747, where the American Military Specification MIL-F-8785C/MIL-STD-1797A is employed. As can be obtained from (3.22), the system is the third order, which results in the three poles in the closed loop characteristic polynomial. The three poles include a complex pair describing the short period mode characteristics and a real root representing the integral lag due to the additional integral term. The initial design decisions made through designing comprises the new stability characteristics of the augmented aircraft, which are $\zeta_{s_{r1}}$, $\omega_{s_{r1}}$ and $T_{lag_{r1}}$.

- The short period mode damping ratio of the unaugmented aircraft is $\zeta_s = 0.495$ as shown in Table 3.6. Although this value meets MIL-F-8785C/MIL-STD-1797A specification, it is decided to modify this value into $\zeta_{s_{r1}} = 0.75$ for a smaller overshoot. It is always a good initial choice as the quickest settling time after a disturbance combined with the smallest overshoot.

- The short period mode natural frequency of the unaugmented aircraft is $\omega_{s_{r1}} = 1.26$ rad/s as shown in Table 3.6. For the purpose of meeting the requirements of MIL-F-8785C/MIL-STD-1797A specification as well as obtaining the desirable response speed and settling time, the value of natural frequency ω_s is raised up from original 1.26 rad/s into $\omega_{s_{r1}} = 1.9$ rad/s.

- Not only a good knowledge of the theory of the PI controller, but also more design experiences are needed to choose the integral lag pole value $T_{lag_{r1}}$ [Coo10a]. The integral lag time constant $T_{lag_{r1}} = 1.8$ is chosen, considering it should be nearer to the short period mode frequency $\omega_{s_{r1}} = 1.9$.

The controller design procedure applied in this thesis and the decisions made above, are mainly based on the reference of 'Flight Qualities and Flight Control Lecture Notes', for further information refer to [Coo10a].

The required closed loop characteristic polynomial is thus given by,

$$\Delta(s) = (s + 1.8)(s^2 + 2.85s + 3.61) \tag{3.27}$$

The feedback gain matrix K can be calculated as follows using pole placement method 'place.m' routine in MATLAB.

$$K = \begin{bmatrix} k_q & k_\alpha & k_{\epsilon_q} \end{bmatrix} = \begin{bmatrix} 0.7331 & -1.672 & 2.8741 \end{bmatrix} \tag{3.28}$$

where, $k_q = 0.7331$ rad/rad/s, $k_\alpha = -1.672$ rad/rad/s, $k_{\epsilon_q} = 2.8741$ rad/rad/s. Thus the augmented state space can be written by,

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{\epsilon}_q \end{bmatrix} = \begin{bmatrix} -4.2927 & 6.5048 & -13.2490 \\ 0.9289 & -0.3573 & -0.2713 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 4.6099 \\ 0.0944 \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} q_d \tag{3.29}$$

3.5.3 Designing the Feedforward Gain Matrix M

Commonly, a feedforward controller is used to improve the transient performance of the closed-loop system. The feedforward gain matrix M is comprised with the only parameter-m due to the only input signal.

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{\epsilon}_q \end{bmatrix} = \begin{bmatrix} -4.2927 & 6.5048 & -13.2490 \\ 0.9289 & -0.3573 & -0.2713 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 4.6099m \\ 0.0944m \\ -1 \end{bmatrix} q_d \tag{3.30}$$

where m can be calculated according to the following equations.

$$m = \frac{k_{\epsilon_q}}{T_{lag}} = \frac{2.8741}{1.8} = 1.597 \tag{3.31}$$

3.5.4 Implementing the Controller Design

The control law (3.24) can now be given by,

$$\delta_e = -\begin{bmatrix} 0.7331 & -1.672 & 2.8741 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + 1.597 \cdot q_d \tag{3.32}$$

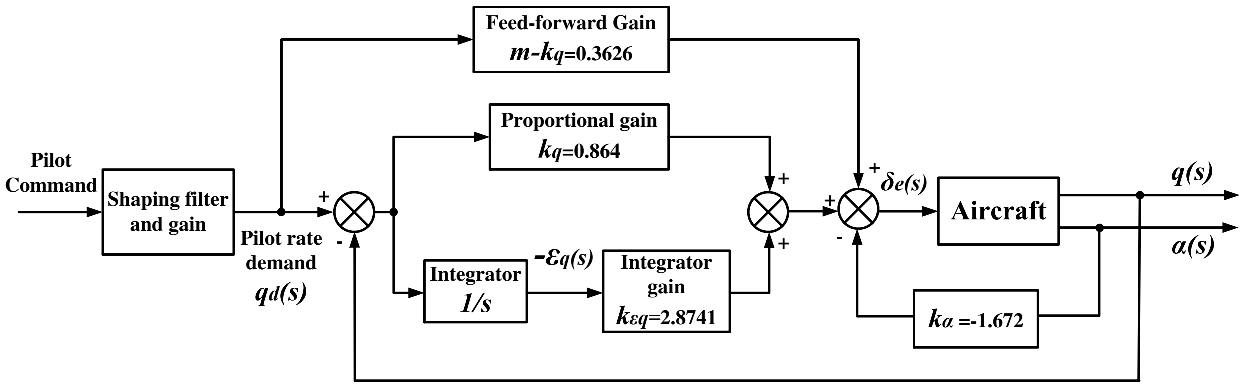


Figure 3.6: RCAH system structure

The corresponding augmented system structure is shown in Fig. 3.6. From (3.28) and (3.31), both the feedback gain matrix K and the feedforward gain matrix M are obtained. In consequence, the closed loop state equation (3.25) is given by,

$$\begin{bmatrix}\dot{q}\\ \dot{\alpha}\\ \dot{\epsilon}_q\end{bmatrix} = \begin{bmatrix}-4.2927 & 6.5048 & -13.2490\\ 0.9289 & -0.3573 & -0.2713\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}q\\ \alpha\\ \epsilon_q\end{bmatrix} + \begin{bmatrix}7.3606\\ 0.1507\\ -1\end{bmatrix} q_d \tag{3.33}$$

The closed loop transfer functions are obtained as follows.

$$\frac{q(s)}{q_d(s)} = \frac{7.3606(s+0.4905)(s+1.8)}{(s+1.8)(s^2+2.85s+3.61)} = \frac{7.3606(s+0.4905)}{(s^2+2.85s+3.61)} \tag{3.34}$$

$$\frac{\alpha(s)}{q_d(s)} = \frac{0.15072(s+49.66)(s+1.8)}{(s+1.8)(s^2+2.85s+3.61)} = \frac{0.15072(s+49.66)}{(s^2+2.85s+3.61)} \tag{3.35}$$

$$\frac{\epsilon_q(s)}{q_d(s)} = \frac{-(s+1.8)(s-4.511)}{(s+1.8)(s^2+2.85s+3.61)} = \frac{-(s-4.511)}{(s^2+2.85s+3.61)} \tag{3.36}$$

The controller is designed based on the reduced order equations of longitudinal motion. As can be observed from equations (3.34), (3.35), and (3.36), the design objective of minimizing the effect of the time lag introduced by the integral term, is achieved by near exact integral pole-zero cancellation. The response to a unit step command q_d is shown in Fig. 3.7. It is clear that a second-order-like response is achieved to pilot controls due to the cancellation of integral pole-zero.

3.5.5 Checking the Design with the Full Order Aircraft Model

In this section, the controller would applied into the full order equations of longitudinal motion to ensure that the integral lag time constant is sufficiently fast without causing intrusive issue. The unaugmented state space of full order equations of longitudinal motion obtained from (3.4) is augmented by adding the integral state

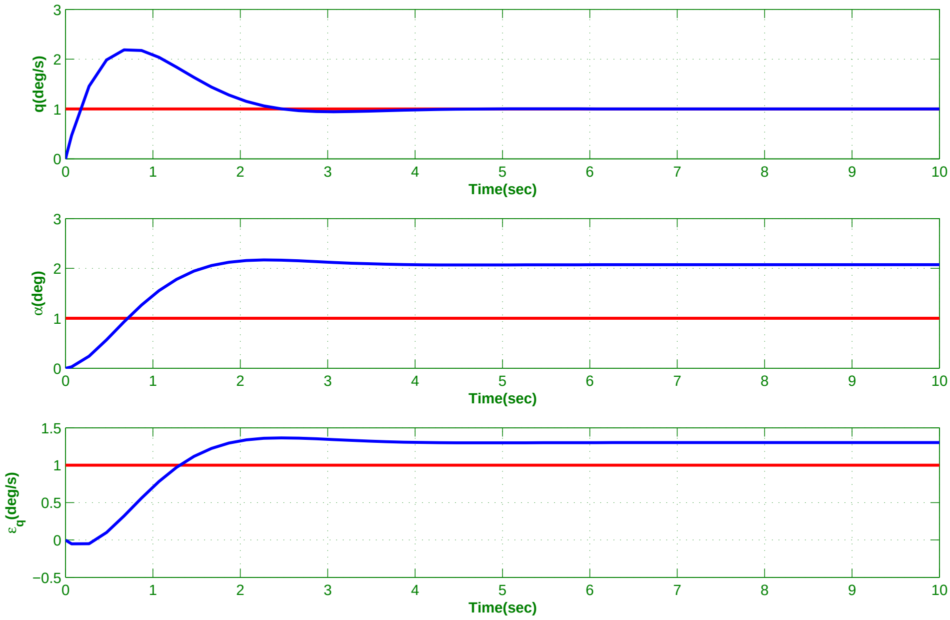


Figure 3.7: Reduced order augmented aircraft response to the unit step demand

equation (3.21) as follows,

$$\begin{bmatrix} \dot{q} \\ \dot{V}_{TAS} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\epsilon}_q \end{bmatrix} = \begin{bmatrix} -0.728 & -0.00048 & -1.2025 & 0 & 0 \\ -0.0839 & -0.00547 & 6.00779 & -9.78 & 0 \\ 1.0019 & -0.00036 & -0.515 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 4.6099 \\ 0 \\ 0.0944 \\ 0 \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} q_d \tag{3.37}$$

The control law, equation (3.24) is amended as follows.

$$\delta_e = -\begin{bmatrix} k_q & k_{V_{TAS}} & k_\alpha & k_\theta & k_{\epsilon_q} \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + m q_d \tag{3.38}$$

$$= -\begin{bmatrix} 0.7331 & 0 & -1.672 & 0 & 2.8741 \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + 1.597 \cdot q_d \tag{3.39}$$

The full order closed loop state equation can be obtained as follows by substituting equation (3.39) into equation (3.37).

$$\begin{bmatrix} \dot{q} \\ \dot{V}_{TAS} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\epsilon}_q \end{bmatrix} = \begin{bmatrix} -4.2927 & -0.0005 & 6.5048 & 0 & -13.2490 \\ -0.0839 & -0.0055 & 6.0078 & -9.7850 & 0 \\ 0.9289 & -0.0004 & -0.3573 & 0 & -0.2713 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 7.3606 \\ 0 \\ 0.1507 \\ 0 \\ -1 \end{bmatrix} q_d \tag{3.40}$$

The full order augmented aircraft transfer functions can be given as follows, by taking the Laplace transform of equation (3.40). Comparing stability characteristics of the augmented full order equations of motion to the reduced order one, the variations are tabulated in Table 3.7.

$$\frac{q(s)}{q_d(s)} = \frac{7.3606(s+0.00989)(s+0.486)(s+1.8)}{(s+1.801)(s+0.005983)(s^2+2.849s+3.61)} \tag{3.41}$$

$$\frac{V_{TAS}(s)}{q_d(s)} = \frac{0.28771(s-96.37)(s+1.8)(s+1.274)}{s(s+1.801)(s+0.005983)(s^2+2.849s+3.61)} \tag{3.42}$$

$$\frac{\alpha(s)}{q_d(s)} = \frac{0.15072(s+49.66)(s+1.8)(s^2+0.005432s+0.003401)}{s(s+1.801)(s+0.005983)(s^2+2.849s+3.61)} \tag{3.43}$$

$$\frac{\theta(s)}{q_d(s)} = \frac{7.3606(s+1.8)(s+0.486)(s+0.00989)}{s(s+1.801)(s+0.005983)(s^2+2.849s+3.61)} \tag{3.44}$$

$$\frac{\epsilon_q(s)}{q_d(s)} = \frac{-(s-4.511)(s+1.8)(s^2+0.005496s+0.003053)}{s(s+1.801)(s+0.005983)(s^2+2.849s+3.61)} \tag{3.45}$$

As can be observed from the Table 3.7, it is clear that the characteristics and

Table 3.7: Stability Characteristics Comparison

Mode	Basic aircraft	Reduced order aircraft	Full order aircraft
Short period	$\zeta_s = 0.495$, $\omega_s = 1.26$ rad/s	$\zeta_s = 0.75$, $\omega_s = 1.9$ rad/s	$\zeta_s = 0.7497$, $\omega_s = 1.9$ rad/s
Phugoid	$\zeta_p = 0.0255$, $\omega_p = 0.0503$ rad/s	N/A	$T_1 = \frac{1}{0} = \infty$s, $T_2 = \frac{1}{0.006163} = 172.41$ s
Integral lag	N/A	$T_{lag} = \frac{1}{1.8} = 0.56$ s	$T_{lag} = 0.56$ s

the integral lag time of full order aircraft are not significantly changed comparing with the short period mode. However, phugoid mode is on the other hand. The big changes occurred to the phugoid mode result from the integral feedback. The integral term of pitch rate q is equal to pitch attitude θ feedback, which is an important variable in phugoid mode, gives rise to the change in phugoid dynamics. The phugoid mode is then represented by a neutrally stable pole T_1 and a stable long time constant pole T_2 instead of the second order oscillatory which would not be obvious for pilot to handle the aircraft. Full order augmented aircraft response to the unit step demand q_d is shown in Fig. 3.8. In the first diagram, pitch rate q responds and settles to a steady value within 4 seconds after the short period transient has damped out. The characteristics are related with the integral lag time constant $T_{lag_{r1}}$ to give rise to a faster response. Since the demand q_d is a unit step for pitch rate of 1 deg/s, the pitch attitude θ ramps up at 1 deg/s which leaves the aircraft climbing.

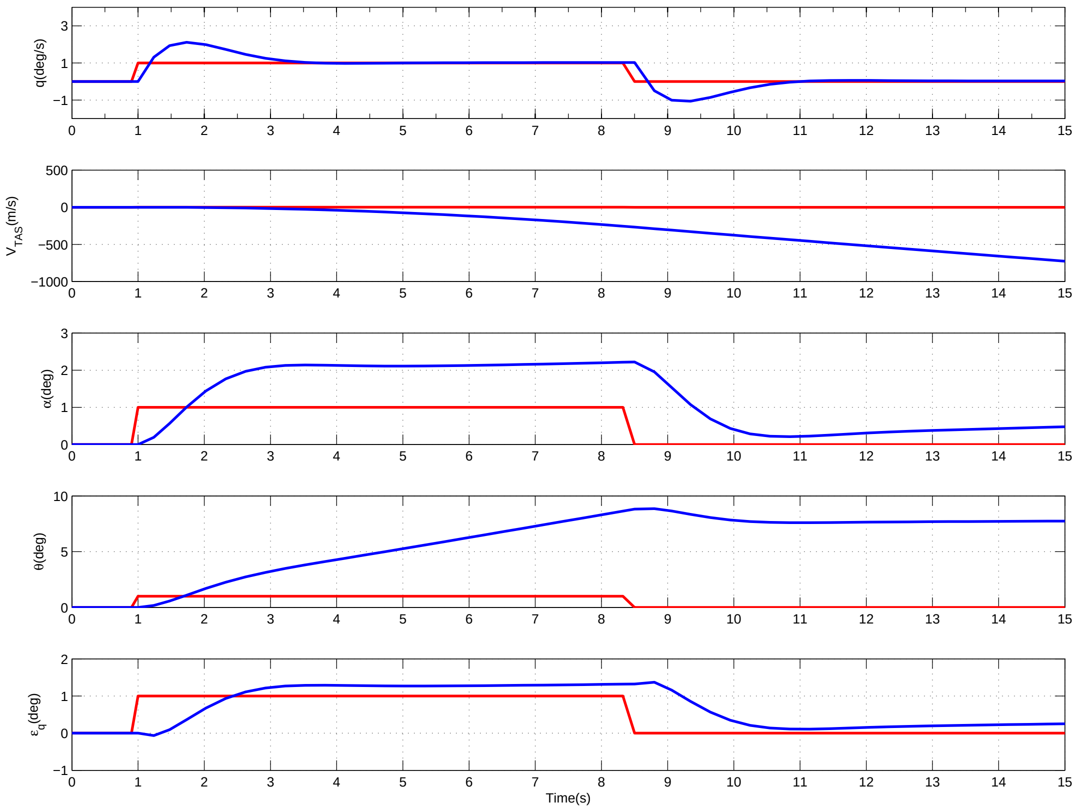


Figure 3.8: Full order augmented aircraft response to the unit step demand

Chapter 4

Handling Qualities Assessment

In this chapter, the handling qualities of the aircraft with configuration $he = 7000$m and $V_{TAS} = 241$m/s are assessed. Both basic aircraft and the aircraft augmented with a longitudinal Rate Command-Attitude Hold (RCAH) controller are discussed respectively. Many existing criteria and specifications are applied to assess the longitudinal handling qualities of aircraft. The following criteria are used in this chapter to see if they could give consistent results.

- The Control Anticipation Parameter(CAP) criterion.
- The Neal and Smith criterion.
- The C* criterion

4.1 The CAP Assessment

CAP is defined in terms of short period natural frequency ω_s and the normal load factor per unit angle of attack n_α, which is given by

$$CAP = \frac{\dot{q}(0)}{N_z(\infty)} = -\frac{g\omega_s^2}{z_w U_e} = \frac{\omega_s^2}{n_\alpha} \tag{4.1}$$

where n_α values are given by equation (3.12), and the corresponding ω_s values are obtained from Table 3.7 for basic aircraft and augmented reduced order aircraft respectively.

The value of the parameters, such as the short period frequency ω_s, and the normal load factor per unit angle of attack n_α, are given in Table 3.7 with respect to the basic and augmented reduced order aircraft models. Subsequently, the CAP parameters are calculated based on the parameters obtained, and presented in Table 4.1. The results of comparing the CAP parameters of the basic and augmented

Table 4.1: CAP parameters(he = 7000m,V_{TAS} = 241m/s)

Parameters	Open-loop	Closed-loop	Units
n_α	12.66	12.66	g/rad
ω	$\omega_{s_{open}}$=1.26	$\omega_{s_{close}}$=1.9	rad/s
CAP	CAP_{open}=0.125	CAP_{close}=0.285	-

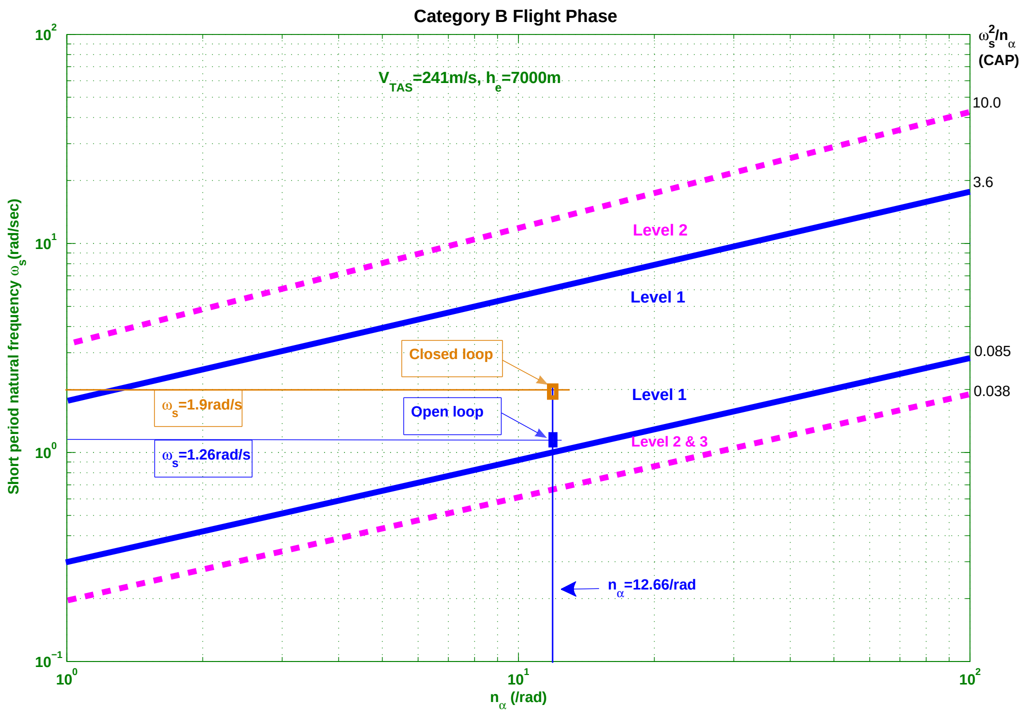


Figure 4.1: CAP assessment for Short period mode characteristics [Ano80]

aircraft are plotted against the Category B requirement defined according to MIL-F-8785C/MIL-STD-1797A specification in Fig. 4.1.

As can be observed from Fig. 4.1, both CAP parameters are located in the Level 1 region for a category B (Cruise) flight phase. However, the augmented B747 aircraft shows less marginal Level 1 flying qualities than the basic aircraft does. From Fig. 4.1, the short period natural frequency of basic aircraft $\omega_{s_{open}} = 1.26$ rad/s is shown to be located near the bottom boundary of Level 1, while the higher short period natural frequency of augmented aircraft $\omega_{s_{close}} = 1.9$ rad/s is more desirable. With this higher value, the aircraft handling qualities are now well inside the CAP Level 1 handling qualities requirements due to the aircraft being augmented by increasing the short period frequency.

4.2 The Neal and Smith Criterion Assessment

The Neal and Smith criterion has emerged as a critical measure of pilot handling opinion, in particular for the pitch tracking task. The Neal and Smith criterion assumes that the pilot adjusts the phase and gain characteristics to minimize the low-frequency droop and closed-loop resonance of the aircraft [TR71]. The criteria are developed based on the hypothesis that if satisfactory closed-loop dynamic performance can be achieved by the pilot model described by the assumed characteristics, then the satisfactory closed-loop dynamic performance will also be achievable for the human pilot with acceptable workload [Ano90].

The handling qualities of the basic aircraft as well as the augmented aircraft are assessed, and results compared by means of the Neal and Smith criterion in this section. Applying the handling qualities assessment method of the Neal and Smith criterion presented in Appendix A, the assessment results both with and without the controller engaged are shown in Fig. 4.2.

Fig. 4.2 indicates that for short period mode, an increased natural frequency from 1.26 rad/s to 1.9 rad/s effectively reduced the pilot compensation. Meanwhile, the improvement of damping ratio from 0.495 to 0.75 decreases the closed loop resonance peak substantially. Both of these endow the aircraft with the Level 1 handling qualities according to the Neal and Smith Criterion specification.

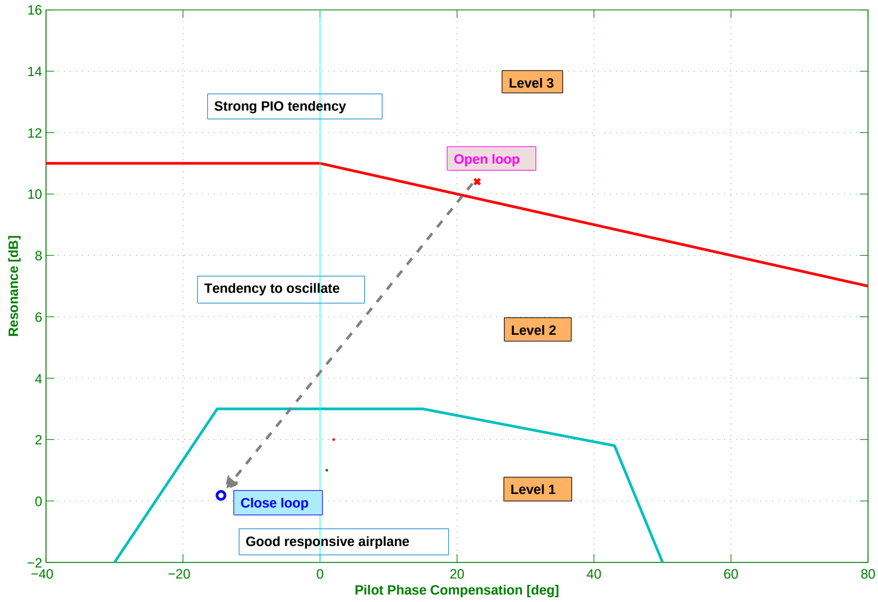


Figure 4.2: Neal and Smith criterion assessment comparison [Coo10a]

4.3 The C^* Criterion Assessment

Boeing Airplane Company developed C^* as a handling qualities criterion during the early 1960s which was later developed into the basis for flight control law design [Coo10a][Ste01]. C^* is a combination of pitch rate q and normal acceleration as shown in equation (4.2). Pitch rate dominates as a measure of aircraft handling qualities at airspeeds less than the cross over velocity V_{co}, while normal acceleration n_z dominates at airspeeds greater than V_{co}. C^* is a dimensionless response parameter comprising n_z and q as sensed at the pilot's station in the aircraft and is defined by,

$$C^* = n_z + Kq \tag{4.2}$$

The transfer function usually used in practice is given by [Coo10a],

$$\frac{C^*(s)}{\eta(s)} = \frac{1}{g\Delta(s)} \left(\left(V_{TAS} + V_{co} + ls \right) N_\eta^q(s) - sN_\eta^w(s) \right) \tag{4.3}$$

where,

- $V_{TAS} = 241\text{m/s}$.
- $V_{co} = 122\text{m/s}$; the airspeed, at which the q and n_z contributions to C^* transfer function is equal during steady manoeuvres, is referred to as the cross over velocity V_{co}. The value of the cross over velocity V_{co} based on experimental work was suggested as 122m/s in this instance. At speeds above 122m/s, the n_z contribution dominates, and at speeds below [Coo10a]the Kq contribution dominates.

- $l = 26.2\text{m}$; the distance from the pilot station to the 25% MAC as found in NASA-CR-1756 [RH71].
- The following equations obtained from (3.34) and (3.36), are the transfer functions of pitch rate and angle of attack response to elevator at configuration $V_{TAS} = 241\text{m/s}$ and $h_e = 7000\text{m}$.

$$\Delta(s) = s^2 + 2.85s + 3.61 \tag{4.4}$$

$$N_\eta^q(s) = 7.3606(s + 0.4905) \tag{4.5}$$

$$N_\eta^w(s) = V_{TAS} \cdot N_\eta^\alpha(s) = 241 \times 0.15072(s + 49.66) = 36.3235(s + 49.66) \tag{4.6}$$

where,

  - $\Delta(s)$ is the closed-loop characteristic polynomial of the aircraft short period mode.
  - $N_\eta^q(s)$ is the numerator of the reduced order closed-loop aircraft transfer function of q.
  - $N_\eta^w(s)$ is the numerator of the reduced order closed-loop aircraft transfer function of w.

Substituting the data above into equation (4.3) gives,

$$\frac{C^*(s)}{\eta(s)} = \frac{(15.82s^2 + 98.16s + 133.7)}{(s^2 + 2.85s + 3.61)} \tag{4.7}$$

The normalized C^* transfer function is given by,

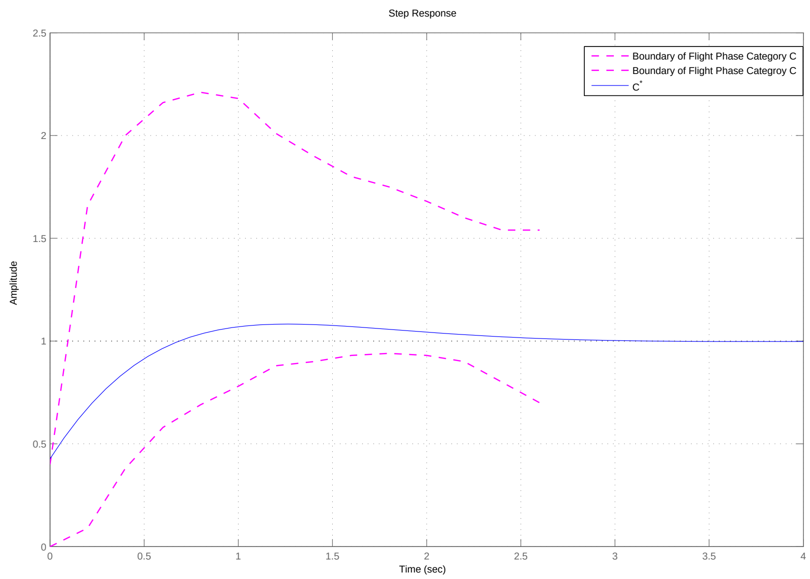
$$\lim_{s\to 0}\left(\frac{C^*(s)}{\eta(s)}\right) = 37.04 \tag{4.8}$$

$$\left(\frac{C^*(s)}{\eta(s)}\right)_{norm} = \frac{(0.4271s^2 + 2.65s + 3.61)}{(s^2 + 2.85s + 3.61)} \tag{4.9}$$

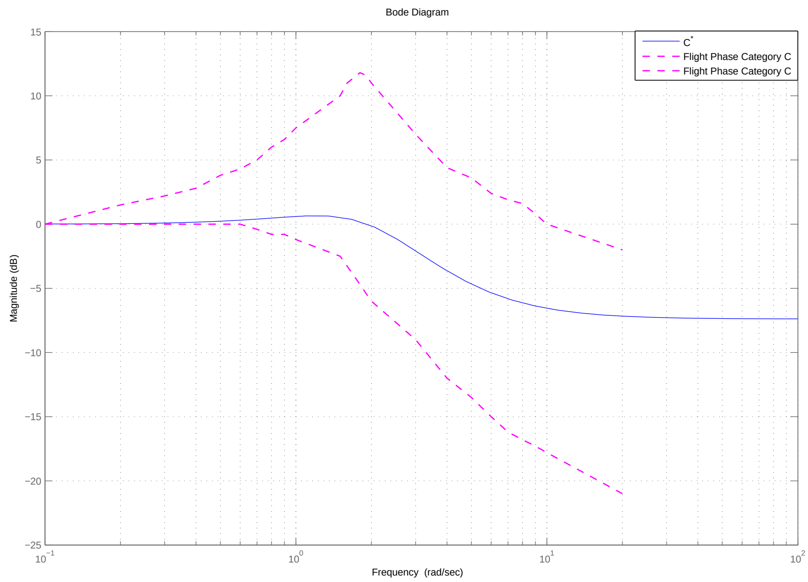
Apply the initial value theorem,

$$\lim_{s\to\infty}\left(\frac{C^*(s)}{\eta(s)}\right) = 0.4271 \tag{4.10}$$

The normalized C^* transfer function is plotted in time domain with step response and Bode plot respectively in Fig. 4.3. The envelope plotted in Fig. 4.3 represents acceptable limits of C^* defined for Flight Phase Category C. Fig. 4.3 indicates the augmented aircraft possesses acceptable handling qualities in takeoff and landing flight phase. The handling qualities assessment is limited to the flight phase category B due to the approach and take off configurations not being included in the aircraft model. However, it is considered that the C^* assessment still possesses a certain meaning to predict the handling qualities of the augmented aircraft in Flight Phase Category B, which can also be expanded into Flight Phase Category C. In other words, the C^* response of the augmented aircraft is evaluated in a envelope of a more complex and critical configuration (Flight Phase Category C), in order to systematically analyze and obtain an overview of the handling qualities of the augmented aircraft.



(a) C^* step response



(b) C^* frequency response

Figure 4.3: The C^* criterion applied to B747 [Coo10a]

Chapter 5

Assessment over the Whole Flight Envelope

This chapter presents the handling qualities assessment in the range of the entire flight envelope, together with a second controller design and identification of interpolation region.

5.1 Introduction

In order to guarantee desirable handling qualities, a set of equilibrium points is defined that covers the flight envelope. The handling qualities are assessed at each point in this chapter. For the flight conditions that handling qualities are not satisfied, a second controller is designed so that the handling qualities criteria are satisfied over the whole region therefor.

Before the different controllers and equilibrium points that are involved in the assessment are considered, it is necessary to clarify some of the terminology used in this chapter.

- The controller designed in Section 3.5 is based on the aircraft operating at a particular point in the flight envelope, namely $h_e = 7000$ m and $V_{TAS} = 241$ m/s. In the remainder of this thesis, this controller is referred to as C_1 and the corresponding operating point as $EP_{(7000,241)}$.
- The second local controller designed on operating point $h_e = 8500$m and $V_{TAS} = 180$m/s is referred to as C_2 and the operating point as $EP_{(8500,180)}$. The principle of the choice of $EP_{(8500,180)}$ for C_2 is that the whole operating range could be covered by a fewer number of controllers, which increases the efficiency of the scheduler. A heuristic method is applied to identify the operating points based on the handling qualities assessment.
- The entire flight envelope is referred to as $\mathcal{F}$.

Clearly, the performance of the controller C_1 will deteriorate as the aircraft moves away from $EP_{(7000,241)}$ unless some controller scheduling is implemented. The region $\mathcal{F}_1$ - where the aircraft with controller C_1 generally has satisfactory handling qualities - need to be identified by the Neal and Smith criterion. So that the desired global performance and handling qualities in $\mathcal{F}$ can be guaranteed, a second controller C_2 is designed with adequate handling qualities based on $EP_{(8500,180)}$. Similarly, the region $\mathcal{F}_2$ - where the aircraft with controller C_2 generally has satisfactory handling qualities - is identified as well. Consequently, in the next chapter, gains of C_1 and C_2 are smoothly scheduled with altitude and airspeed showing the flight condition transferring from $\mathcal{F}_1$ to $\mathcal{F}_2$.

The objectives in this chapter are to

1. assess the handling qualities of the aircraft with controller C_1 over a set of operating points in $\mathcal{F}$, that covers the whole of the flight envelope. The region $\mathcal{F}_1$ is then determined.

2. design a second controller based on an appropriate operating point that has adequate handling qualities, here after referred to as C_2.

3. assess the handling qualities of the aircraft with controller C_2 over $\mathcal{F}$, which results in the region $\mathcal{F}_2$.

4. divide $\mathcal{F}$ into three subsets- $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$. Based on the determination of the regions of $\mathcal{F}_1$ and $\mathcal{F}_2$; $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$ are identified, such that $\mathcal{F} = \mathcal{F}_{C_1} \cup \mathcal{F}_{C_2} \cup \mathcal{F}_{C_{12}}$ and so that the aircraft with controller C_1 generally has superior handling qualities over $\mathcal{F}_{C_1}$ and the aircraft with controller C_2 generally has superior handling qualities over $\mathcal{F}_{C_2}$. For $\mathcal{F}_{C_{12}}$, the aircraft with scheduled gains generally has superior handling qualities. Gain scheduling strategy is designed in the next chapter.

5.2 Handling Qualities Assessment of Aircraft with C_1 in $\mathcal{F}$

As pointed out earlier, the handling qualities of the aircraft with controller C_1 are assessed over a set of operating points in $\mathcal{F}$ in this section.

5.2.1 Aircraft Models and the B747 flight envelope $\mathcal{F}$

A set of the operating points along with the B747 flight envelope need to be defined, correspondingly, a set of local linear aircraft models and $\mathcal{F}$ need to be developed before the controller C_1 can be assessed.

Following the trim and linearization routine described in section 3.2, a set of local linear aircraft models can be obtained from the equations of motion based on the specific chosen equilibrium point.

Considering the aircraft model flight condition: 'Straight-and-level trim' chosen in section 3.2 and the flight envelope shown in Table 3.2, the flight envelope $\mathcal{F}$ is then determined by combining these two aspects with the B747 operating flight envelope derived from NASA-CR-2144 [HJ72]. Consequently, as shown in Fig. 5.1, the flight envelope $\mathcal{F}$ is obtained and a set of operating points is defined, based on which a corresponding set of trimmed and linearized aircraft models is developed.

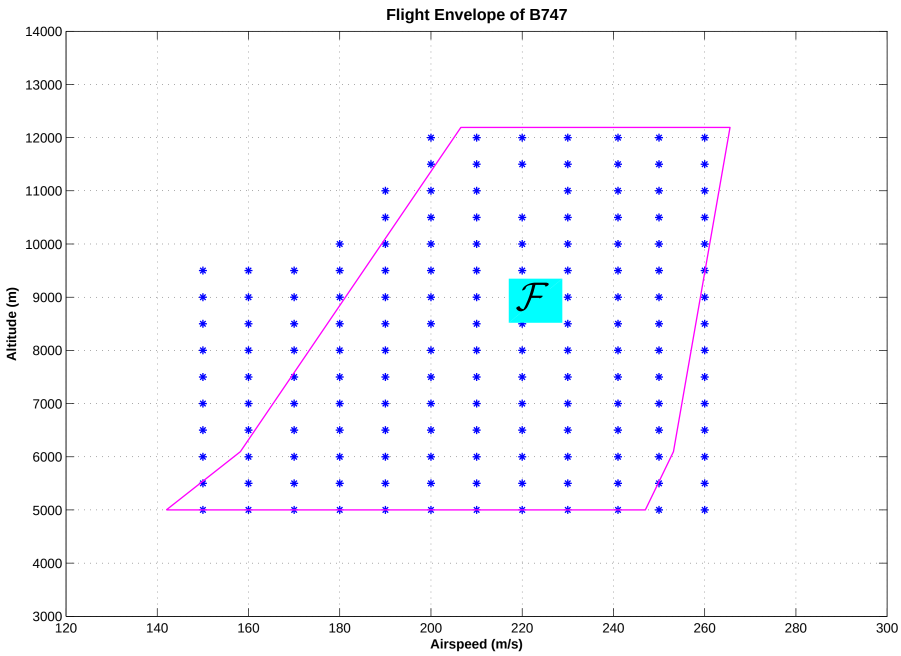


Figure 5.1: Trim and linearization equilibrium points in $\mathcal{F}$ [HJ72]

As shown in the Fig. 5.1, the trimmed and linearized aircraft models covered the entire flight envelope $\mathcal{F}$ with interval of airspeed of 10m/s and altitude of 500m. The airspeed range is from 142m/s to 260m/s, and the altitude range is from 5000m to 12190m. The sea level configuration is not considered.

5.2.2 Identification of $\mathcal{F}_1$

The handling qualities assessment of the aircraft augmented by C_1 in $\mathcal{F}$ is conducted in this section. Furthermore, $\mathcal{F}_1$, the region where the aircraft with controller C_1 generally has satisfactory handling qualities, is determined. In order to identify $\mathcal{F}_1$, controller C_1 is implemented to augment the set of aircraft models in $\mathcal{F}$ obtained in the previous section. Handling qualities are then assessed systematically.

- Implementation of controller C_1
  The control law derived from equation 3.32 is applied to augment the set of aircraft models in $\mathcal{F}$.

- Systematical handling qualities assessment
  Neal and Smith criterion are utilized to assess the handling qualities of the aircraft augmented by C_1 in $\mathcal{F}$, and these are plotted in Fig. 5.2.

Fig. 5.2 shows the variation in Neal and Smith parameters with change in altitude and airspeed, which is divided into two cases to be analyzed.

Case 1: At constant airspeed with increase in altitude For a constant airspeed, the augmented aircraft is more sluggish and tends to oscillate with strong Pilot Induced Oscillation (PIO) tendency in the case of higher altitude. While, for the basic aircraft, a strong PIO tendency tends to happen at high airspeeds and a low altitude (like $EP_{(260,5000)}$)), which gets the opposite results to the augmented aircraft in the same situation . Consequently, for the augmented aircraft, there is a general trend towards an increasing requirement of pilot compensation and resonance peak amplitude with increasing altitude, which results in a prediction of considerably degraded handling qualities.

Case 2: At constant altitude with increase in airspeed For the aircraft augmented by C_1, the resonance peak drops indicating larger damping in pitch oscillations at constant altitude with increasing airspeed from 150m/s to 260m/s, and the system is less sluggish. On the other hand the basic aircraft tends to be more sluggish and has less damping in the same situation which causes larger pilot compensation or more damping required from the FCS. Consequently, for augmented aircraft, there is a general trend towards a decreasing requirement of pilot compensation and resonance peak amplitude with increasing airspeed, which results in a prediction of desirable handling qualities.

In summary, handling qualities of the basic aircraft and augmented aircraft are quite different with variation in altitude and airspeed. The handling qualities of the basic aircraft represent the natural response of aircraft changing with airspeed and altitude, while handling qualities of augmented aircraft are more dependent on the equilibrium points which the flight controller is designed for. Overall, as predicted by the Neal and Smith criterion, handling qualities are improved enormously by flight controller C_1, especially for the equilibrium points near $EP_{(7000,241)}$ in $\mathcal{F}$. Consequently, $\mathcal{F}_1$ is now determined, which is shown in Fig. 5.3. Note that $\mathcal{F}_1$ is determined approximately - it is a 'piece-wise approximation'.

As can be indicated in Fig. 5.3, $\mathcal{F}_1$ is located in the bottom right corner of $\mathcal{F}$, and the low speed and high altitude area in $\mathcal{F}$ is not covered. In order for $\mathcal{F}$ to be covered efficiently, $EP_{(8500,180)}$ is chosen as the the second equilibrium point, based on which the second controller C_2 is designed.

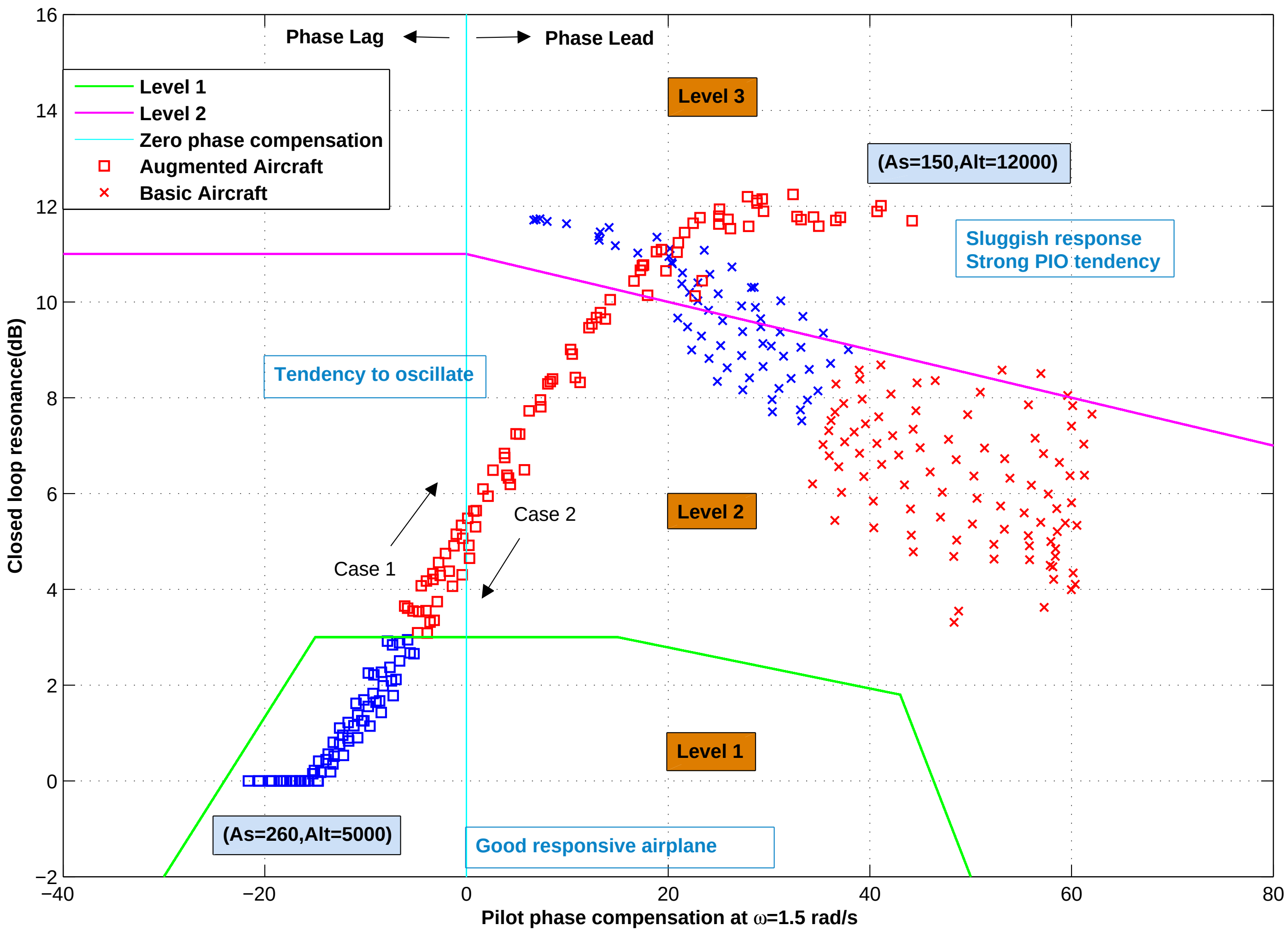


Figure 5.2: Neal and Smith parameters of C_1 with change in altitude and airspeed

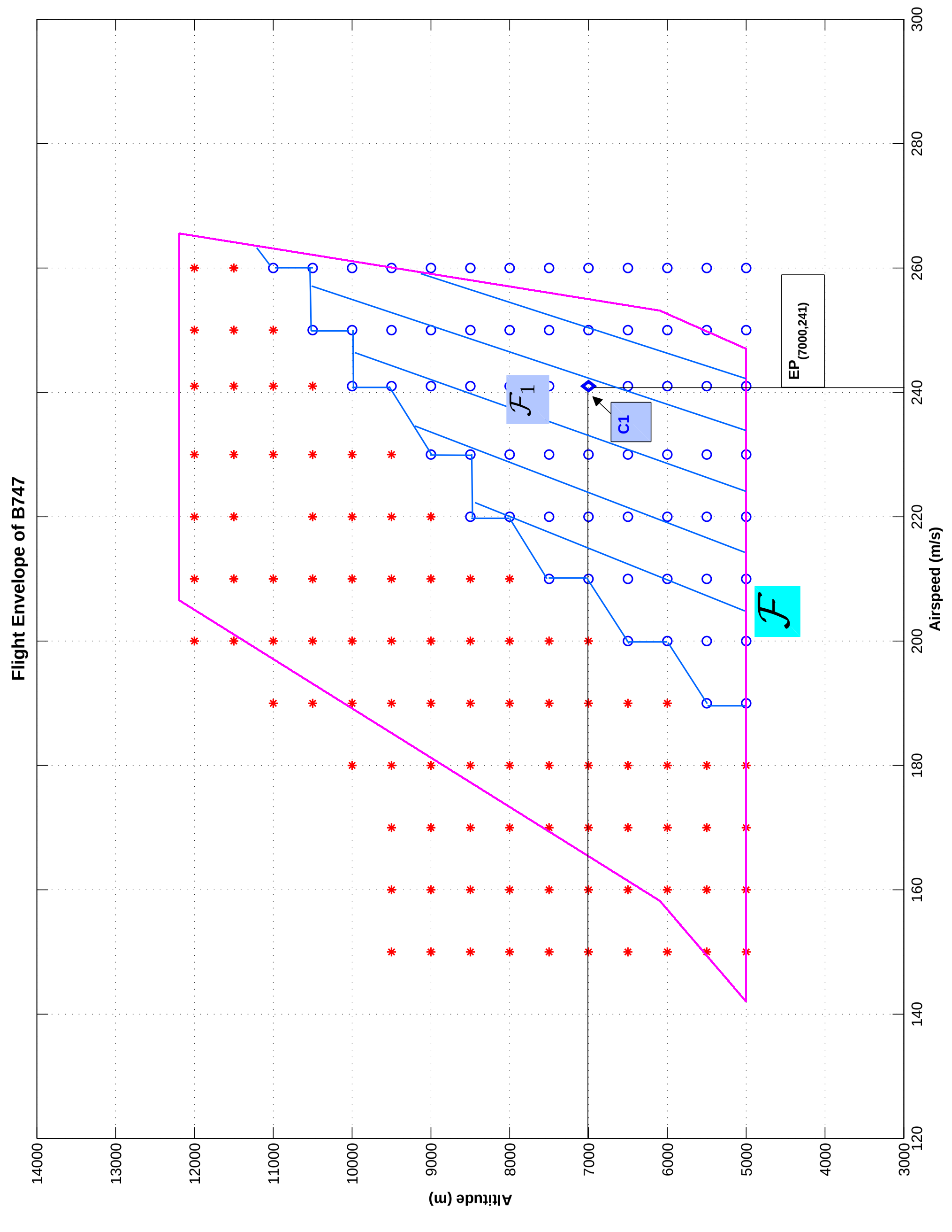


Figure 5.3: Identification of $\mathcal{F}_1$

5.3 Controller(C_2) Design for the Second Equilibrium Point $EP_{(8500,180)}$

In the previous section, equilibrium point $EP_{(8500,180)}$ has been chosen as the second equilibrium point. In this section, the controller C_2 would be designed based on $EP_{(8500,180)}$ using the similar design procedure as the controller C_1 in section 3.5, which is presented in Appendix B. Controller C_1 in chapter 3.5 and C_2 in this chapter are designed based on the equilibrium points $EP_{(7000,241)}$ and $EP_{(8500,180)}$ respectively. The main stability characteristics are tabulated in Table 5.1.

Table 5.1: Stability Characteristics Comparison-Closed Loop

Controller	C_1	C_2
Equilibrium point	$EP_{(7000,241)}$	$EP_{(8500,180)}$
Short period	$\zeta_s = 0.75$ $\omega_s = 1.9$ rad/s	$\zeta_{s_2} = 0.8$ $\omega_{s_2} = 1.7$ rad/s
Phugoid	$T_1 = \frac{1}{0} = \infty$s $T_2 = \frac{1}{0.006163} = 172.41$s	$T_1 = \frac{1}{0} = \infty$s $T_2 = \frac{1}{0.0054} = 185.19$s
Integral lag	$T_{lag} = \frac{1}{1.8} = 0.56$s	$T_{lag} = \frac{1}{1.174} = 0.85$s
n_α	$n_{\alpha_1} = 12.67$g/rad	$n_{\alpha_2} = 6.59$g/rad
Feedback gain matrix $K = \begin{bmatrix} k_q & k_\alpha & k_{\epsilon_q} \end{bmatrix}$	$\begin{bmatrix} 0.73 & -1.67 & 2.87 \end{bmatrix}$	$\begin{bmatrix} 1.68 & -3.33 & 5.76 \end{bmatrix}$
Feedforward gain M	1.60	3.84
Gain Matrix $[K, M]$	$K_1 =$ $\begin{bmatrix} 0.73 & -1.67 & 2.87 & 1.60 \end{bmatrix}$	$K_2 =$ $\begin{bmatrix} 1.68 & -3.33 & 5.76 & 3.84 \end{bmatrix}$

5.4 Handling Qualities Assessment of C_2 on $EP_{(8500,180)}$

5.4.1 CAP

As described in section 4.1, the CAP parameters on $EP_{8500,180}$ presented in Table 5.2 are calculated following the equations (4.1) and (B.9) given in previous chapter.

Table 5.2: CAP parameters ($EP_{8500,180}$)

Parameters	**Value**	**Units**
The normal load factor per unit angle of attack n_{α_2}	6.59	g/rad
Short period natural frequency of open loop $\omega_{s_{open}}$	0.743	rad/s
Short period natural frequency of closed loop $\omega_{s_{closed}}$	1.7	rad/s
Control Anticipation Parameter of open loop (CAP_{open})	0.084	-
Control Anticipation Parameter of closed loop (CAP_{closed})	0.439	-

The results of comparing the CAP parameters of the basic and augmented aircraft are shown in Fig. 5.4.

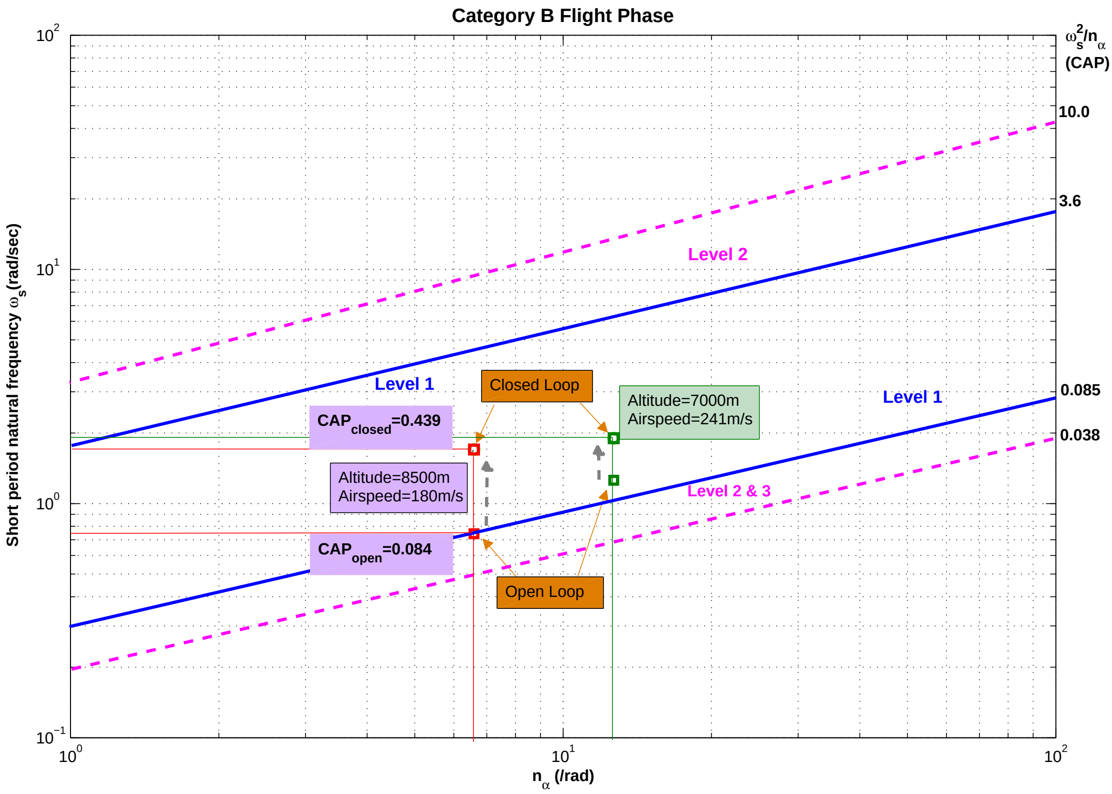


Figure 5.4: CAP assessment comparison

As can be observed from Fig. 5.4, the CAP parameter of the basic aircraft $CAP_{open} = 0.084$ is located in the region of 'Level 2 and 3'. While the CAP parameter of the closed loop is increased to be $CAP_{closed} = 0.439$ after the short period natural frequency is increased by C_2 from $\omega_{s_{open}} = 0.743$ rad/s to $\omega_{s_{closed}} = 1.7$ rad/s, which is now well inside the Level 1 handling qualities requirements of CAP criterion. Consequently, the augmented aircraft shows less marginal Level 1 flying qualities than the basic aircraft does at both $EP_{(7000,241)}$ and $EP_{(8500,180)}$.

5.4.2 The Neal and Smith Criterion

The handling qualities of the basic aircraft as well as the aircraft augmented by C_2 on $EP_{8500,180}$ are assessed, and results compared by means of the Neal and Smith criterion in this section. Details of The Neal and Smith criterion assessment can be found in Appendix A. the assessment results both with and without C_2 engaged on equilibrium points in $\mathcal{F}$ are shown in Fig. 5.5.

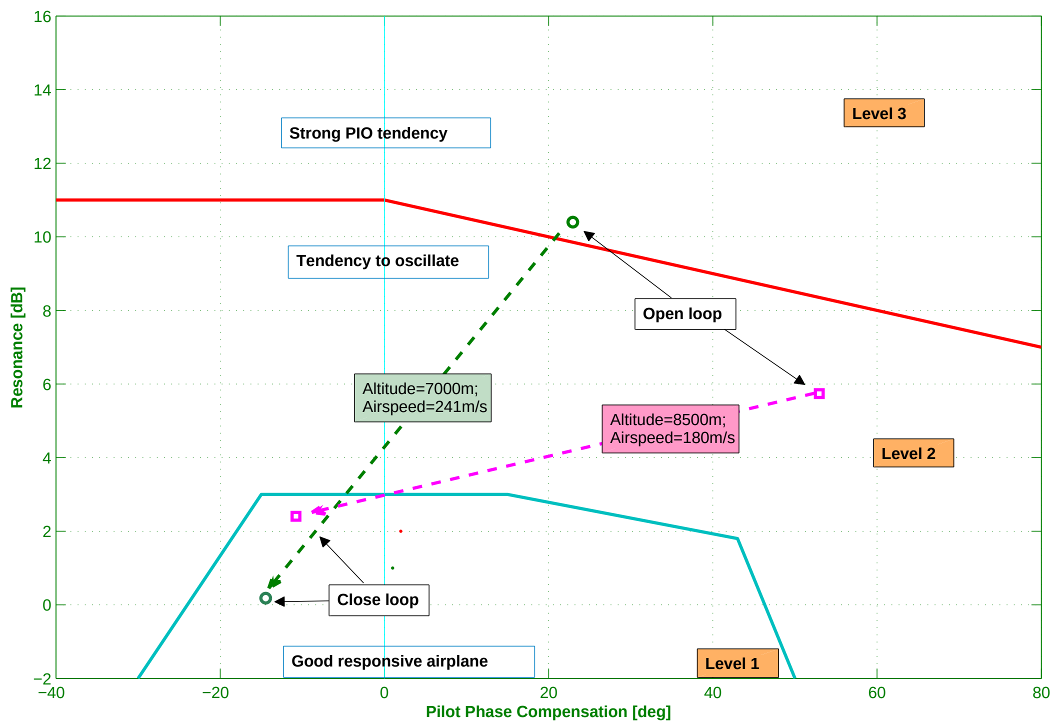


Figure 5.5: Neal and Smith assessment comparison

Fig. 5.5 indicates that for short period mode, an increasing natural frequency from 0.743 rad/s to 1.7 rad/s effectively reduced the pilot compensation. Meanwhile, the improvement of damping ratio from 0.537 to 0.8 decreases the closed loop resonance peak substantially. Both of these endow the aircraft with the desirable handling qualities according to the Neal and Smith Criterion specification. Compared with the aircraft augmented by C_1 on $EP_{(7000,241)}$, the aircraft augmented by

C_2 on $EP_{(8500,180)}$ is more sluggish and has bigger resonance peak amplitude, which leads to more marginal level 1 handling qualities in Neal and Smith criterion.

5.4.3 C^* Criterion

Following the description in section 4.3, C^* criterion is utilized here to assess the handling qualities of the aircraft augmented by C_2 in takeoff and landing flight phase.

The transfer function of C^* is given by ,

$$\frac{C^*(s)}{\eta(s)} = \frac{1}{g\Delta(s)}\big((V_{TAS} + V_{co} + ls)\, N^q_\eta(s) - sN^w_\eta(s)\big) \tag{5.1}$$

where,

- $V_{TAS} = 180$m/s.
- $V_{co} = 122$m/s;
- $l = 26.2$m, the distance from pilot station to 25% MAC, refer to NASA-CR-1756.
- The following equations are obtained from transfer function of pitch rate response to elevator in (B.17) and (B.18).

$$\Delta(s) = (s^2 + 2.72s + 2.89) \tag{5.2}$$

$$N^q_\eta(s) = 8.2995(s + 0.3482) \tag{5.3}$$

$$N^w_\eta(s) = V_{TAS} \cdot N^\alpha_\eta(s) = 180 \times 0.22633(s + 37.18) = 40.74(s + 37.18) \tag{5.4}$$

Substituting the data above into equation (5.1) gives,

$$\frac{C^*(s)}{\eta(s)} = \frac{(21.25s^2 + 110s + 89.06)}{(s^2 + 2.72s + 2.89)} \tag{5.5}$$

The normalized C^* transfer function is given by,

$$\lim_{s\to 0}\left(\frac{C^*(s)}{\eta(s)}\right) = 30.82 \tag{5.6}$$

$$\left(\frac{C^*(s)}{\eta(s)}\right)_{norm} = \frac{(0.6896s^2 + 3.571s + 2.89)}{(s^2 + 2.72s + 2.89)} \tag{5.7}$$

Apply the initial value theorem,

$$\lim_{s\to\infty}\left(\frac{C^*(s)}{\eta(s)}\right) = 0.6896 \tag{5.8}$$

The normalized C^* transfer function is plotted in time domain with step response and Bode plot respectively in Fig. 5.6. It indicates the aircraft augmented by C_2 possesses acceptable handling qualities in the takeoff and landing flight phase.

5.5 Handling Qualities Assessment of Aircraft with C_2 in $\mathcal{F}$

In this section, the handling qualities of the aircraft with controller C_2 are assessed over the set of operating points in $\mathcal{F}$ defined in section 5.2.1. A comparison of Neal and Smith parameters in closed loop and in open loop varying in altitude and airspeed is made in Fig. 5.7. Note that controller C_2, which is implemented to augment the aircraft models over $\mathcal{F}$, is obtained in section B.3.

From Fig. 5.7, the resonance peak amplitude drops and tends to converge into the Level 1 region due mainly to the increasing damping ratio induced by C_2. The assessment results of the basic aircraft located in the same position is shown in Fig. 5.2 in the Neal and Smith criterion chart. Consequently, handling qualities are improved by flight controller C_2, especially for the equilibrium points near $EP_{(8500,180)}$ in $\mathcal{F}$. Therefore $\mathcal{F}_2$ is now determined based on the handling qualities assessment above, and then is shown in Fig. 5.8.

5.6 Discussion of $\mathcal{F}_1$ and $\mathcal{F}_2$

The flight envelope $\mathcal{F}$ is now divided into several regions, and the boundaries determination is discussed in this section.

The situation of $\mathcal{F}$ covered by $\mathcal{F}_1$ and $\mathcal{F}_2$ is indicated in Fig. 5.9. Obviously, $\mathcal{F}$ is not covered quite entirely by $\mathcal{F}_1$ and $\mathcal{F}_2$. There are two aspects which have to be pointed out here.

- Firstly, the low airspeed and high altitude region $\mathcal{F}_H$ is not covered, and it follows that,

  $$\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_H \tag{5.9}$$

  According to the uncovered region $\mathcal{F}_H$ shown in Fig. 5.10, estimation and analysis are necessarily made before dealing with the issue of the division of $\mathcal{F}$ due to this unexpected region. The aircraft model augmented by C_2 on $EP_{(11500,210)}$ in the uncovered region $\mathcal{F}_H$, are assessed from the viewpoint of performance and handling qualities in the following section.

- Secondly, there is a region $\mathcal{F}_G$ where $\mathcal{F}_1$ and $\mathcal{F}_2$ overlap presented in Fig. 5.10. where,

  $$\mathcal{F}_G = \mathcal{F}_1 \cap \mathcal{F}_2 \tag{5.10}$$

  This $\mathcal{F}_G$ is a small overlap where two controllers meet to safeguard against a gap which might otherwise occur due to the coarseness of the grid of equilibrium points (shown in Fig. 5.1). A scheduled gain scheme is required in this region to perform the smooth transfer from C_1 to C_2.

5.6.1 Analysis of Aircraft Model on $EP_{(11500,210)}$ Augmented by C_2

Performance of C_2 on $EP_{(11500,210)}$ is analyzed, and the applying of controller C_2 to the full order aircraft model on $EP_{(11500,210)}$ modified the characteristics considerably. The open loop and closed loop characteristic polynomials are obtained by,

$$\Delta(s)_{openloop} = s\,(s + 1.553)\,(s + 0.001928)(s^2 + 2.513s + 2.313) \qquad (5.11)$$

$$\Delta(s)_{closedloop} = (s + 0.5719)(s + 0.1734)(s + 0.002234)(s^2 - 0.06395s + 0.0191) \qquad (5.12)$$

The short period mode of the basic aircraft is represented by two negative poles on real axises which are modified into a second order oscillatory in the closed loop. Accordingly, the short period mode is substantially modified. On the other hand, the phugoid mode is represented by a pair of unstable, oscillatory imaginary roots for open loop system. While for the closed loop, the mode is no longer oscillatory and is represented by a pair of stable, negative real poles. It means that the phugoid tendency is not specially obvious to the pilot with an intrusive handling problem. The phugoid mode of the basic aircraft is improved considerably by the controller C_2. Consequently, C_2 provides the desirable control performance to the aircraft model on $EP_{(11500,210)}$.

5.6.2 Handling Assessment for C_2 on $EP_{(11500,210)}$

The results of the handling assessment for C_2 on $EP_{(11500,210)}$ are presented in Fig. 5.11 and Fig. 5.12.

Fig. 5.11 shows the C^* transfer function plotted in the time domain with step response. It indicates the aircraft augmented by C_2 on $EP_{(11500,210)}$ possesses acceptable handling qualities.

As indicated in Fig. 5.12, the handling qualities of the aircraft with C_2 on $EP_{(11500,210)}$ degraded from Level 1 to Level 2 comparing with those equilibrium points in $\mathcal{F}_2$ shown in Fig. 5.5. The increase in the required pilot compensation, as well as the increase in the resonance peak amplitude, results in slightly degraded handling qualities predicted by Neal and Smith Criterion. However, Level 2 handling qualities are still predicted by the Neal and Smith criterion for the aircraft with C_2 on $EP_{(11500,210)}$. The resonance peak amplitude is basically less than 4 dB, and pilot phase compensation is desirably small.

5.6.3 Summary

In summary, acceptable results are obtained for the aircraft augmented by C_2 on $EP_{(11500,210)}$ based on the performance and handling qualities assessment in previous sections, though level 1 handling qualities are not obtained. Two key conclusions are as follows.

1. Level 2 handling qualities are predicted by Neal and Smith criterion for the uncovered region $\mathcal{F}_H$, and it is proposed that $\mathcal{F}_2$ is extended to embrace $\mathcal{F}_H$;

   It is conceivable that $\mathcal{F}_H$, shown in Fig. 5.10, can not be covered with level 1 handling qualities due mainly to restrictiveness of the longitudinal dynamics, limitation of aircraft models' preciseness and handling qualities assessment.

   - The restrictiveness of the longitudinal dynamics in the flight phase of low airspeed with high altitude. This is a challenge when designing controllers in this region for every aircraft. Base on the discussion above, C_2 provides the desirable control performance in the $\mathcal{F}_H$ and improves the handling qualities effectively, even though level 1 handling qualities of the aircraft are not obtained.
   - Limitation in the preciseness of the B747 aircraft model. The aircraft model is assembled on the geometry and flight dynamics in various flight conditions. The stability and control characteristics are assumed to be constant throughout the flight envelope, which are an essential factor influencing the handling qualities of the aircraft. All these parameters are not precise and continuously varying, especially for the flight dynamics in the flight phase of low airspeed with high altitude.
   - Limitation of handling qualities assessment. These handling qualities criteria are developed for a certain purpose based on the experience of a certain number of flight tests and pilot's perception [Coo07][Coo10a]. Handling qualities assessment is fairly subjective. Especially for the B747, a four-fan jet intercontinental transport aircraft, is not the intended target aircraft for applying these criteria to quantify the handling qualities.

   Base on the discussion above, it is proposed that $\mathcal{F}_2$ is extended to embrace $\mathcal{F}_H$. This means the controller C_2 is chosen to be responsible for the flight control in $\mathcal{F}_H$. However, on a certain level, it reasonably reflects the fact of the flight dynamics in practice that the handling qualities degrades in the low airspeed and high altitude flight conditions. Consequently, the relationship of regions can be expressed as follows.

   Since,

   $$\mathcal{F}_H \subset \mathcal{F}_2 \tag{5.13}$$

   Now, from expression (5.9),

   $$\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_H \tag{5.14}$$
   $$= \mathcal{F}_1 \cup \mathcal{F}_2 \tag{5.15}$$

2. Region $\mathcal{F}_G$ is approximated by the interpolation region $\mathcal{F}_{C_{12}}$ shown in Fig. 5.13, where,

   $$\mathcal{F}_{C_{12}} \cong \mathcal{F}_G = \mathcal{F}_1 \cap \mathcal{F}_2 \tag{5.16}$$

In order to provide good handling qualities and performance in $\mathcal{F}$, the overlapped $\mathcal{F}_G$ is approximated by the region $\mathcal{F}_{C_{12}}$. As shown in Fig. 5.13, region $\mathcal{F}_{C_{12}}$ is determined to be the interpolation region where gain scheduling is applied to transfer the gains from controller C_1 to C_2 in the following chapter.

5.7 Chapter-Summary

In this chapter, the local linear controllers are developed based on the handling qualities assessment over the flight envelope $\mathcal{F}$. The conclusions are summarized as follows.

- Handling qualities are assessed systematically over the flight envelope $\mathcal{F}$.

  1. Handling qualities of the aircraft with controllers C_1 are assessed over the set of operating points in $\mathcal{F}$. Level 1 handling qualities of the aircraft with C_1 are predicted by CAP as well as the Neal and Smith criterion in $\mathcal{F}_1$.
  2. A second controller C_2 is designed based on $EP_{(8500,180)}$. Adequate handling qualities of aircraft with C_2 on $EP_{(8500,180)}$ are predicted by CAP, the Neal and Smith, and C^* criterion.
  3. Handling qualities of the aircraft with controllers C_2 are assessed over the set of operating points in $\mathcal{F}$. $\mathcal{F}_2$ is identified along with the level 1 handling qualities specification of the Neal and Smith criterion.

- Region boundary of $\mathcal{F}_1$ and $\mathcal{F}_2$ is determined.

  The whole flight envelope $\mathcal{F}$ is divided into two subsets $\mathcal{F}_1$ and $\mathcal{F}_2$. From Fig. 5.13, region $\mathcal{F}_{C_{12}}$ is an approximation of region $\mathcal{F}_G$. $\mathcal{F}_{C_{12}}$ is determined as the interpolation region where gain scheduling is applied to transfer the gains from controller C_1 to C_2 in the following chapter. $\mathcal{F}_H$ is embraced by $\mathcal{F}_2$, although it means that a certain level of handling qualities have to be sacrificed. However, it reasonably reflects the fact that the handling qualities degrades in practice flight condition of low airspeed with high altitude.

  The relationship of regions can be expressed by,

$$\mathcal{F}_H \subset \mathcal{F}_2 \tag{5.17}$$

$$\mathcal{F}_{C_{12}} \cong \mathcal{F}_G = \mathcal{F}_1 \cap \mathcal{F}_2 \tag{5.18}$$

$$\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2 \tag{5.19}$$

- $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$ are defined.

As shown in Fig. 5.14, it is identified that the flight envelope $\mathcal{F}$ comprises three subsets $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$, which can be given by the expressions,

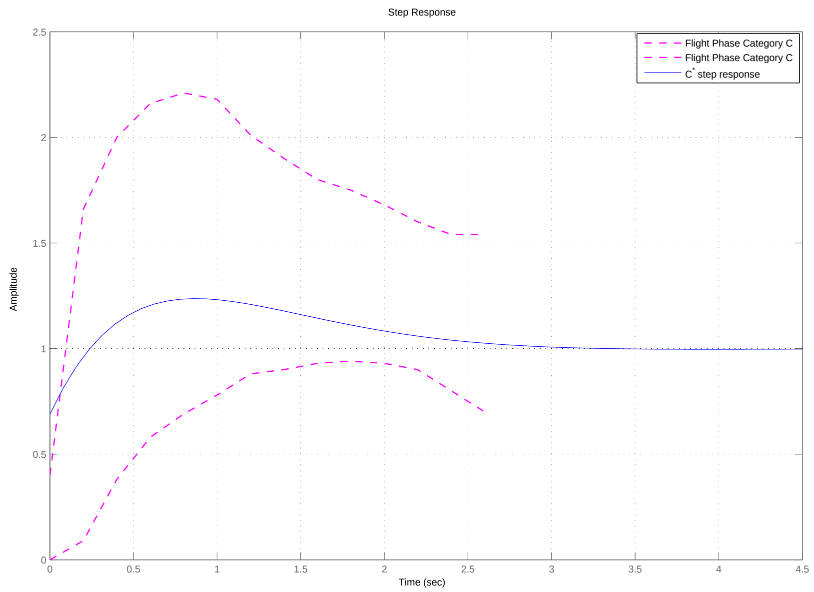
$$\mathcal{F}_{C_1} \cong \mathcal{F}_1 \cup (\mathcal{F}_2)^{'} \tag{5.20}$$

$$\mathcal{F}_{C_2} \cong \mathcal{F}_2 \cup (\mathcal{F}_1)^{'} \tag{5.21}$$

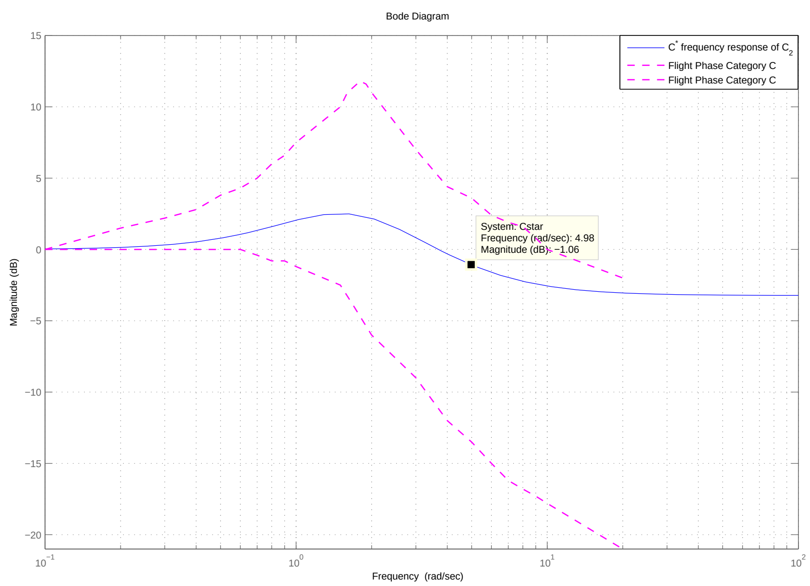
$$\mathcal{F}_{C_{12}} \cong \mathcal{F}_1 \cap \mathcal{F}_2 \tag{5.22}$$

$$\mathcal{F} = \mathcal{F}_{C_1} \cup \mathcal{F}_{C_2} \cup \mathcal{F}_{C_{12}} \tag{5.23}$$

Consequently, the aircraft with controller C_1 generally has superior handling qualities over $\mathcal{F}_{C_1}$, and the aircraft with controller C_2 generally has superior handling qualities over $\mathcal{F}_{C_2}$. For $\mathcal{F}_{C_{12}}$, gain scheduling strategy is designed in the following chapter with the aim of the aircraft augmented by the controllers with scheduled gains has superior handling qualities.



(a) C^* step response



(b) C^* frequency response

Figure 5.6: The C^* criterion applied to the B747 with C_2

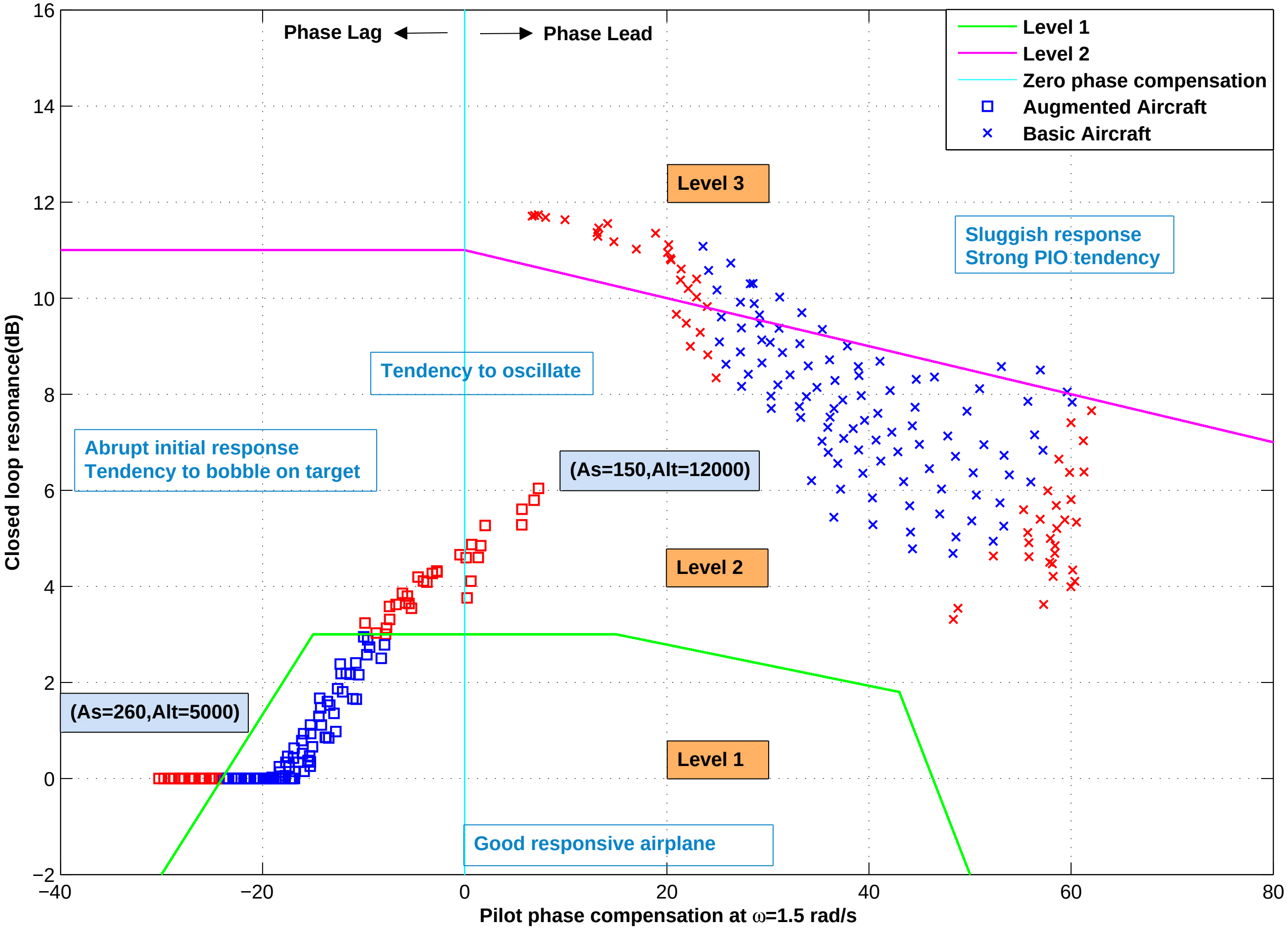


Figure 5.7: Neal and Smith criterion assessment of C_2

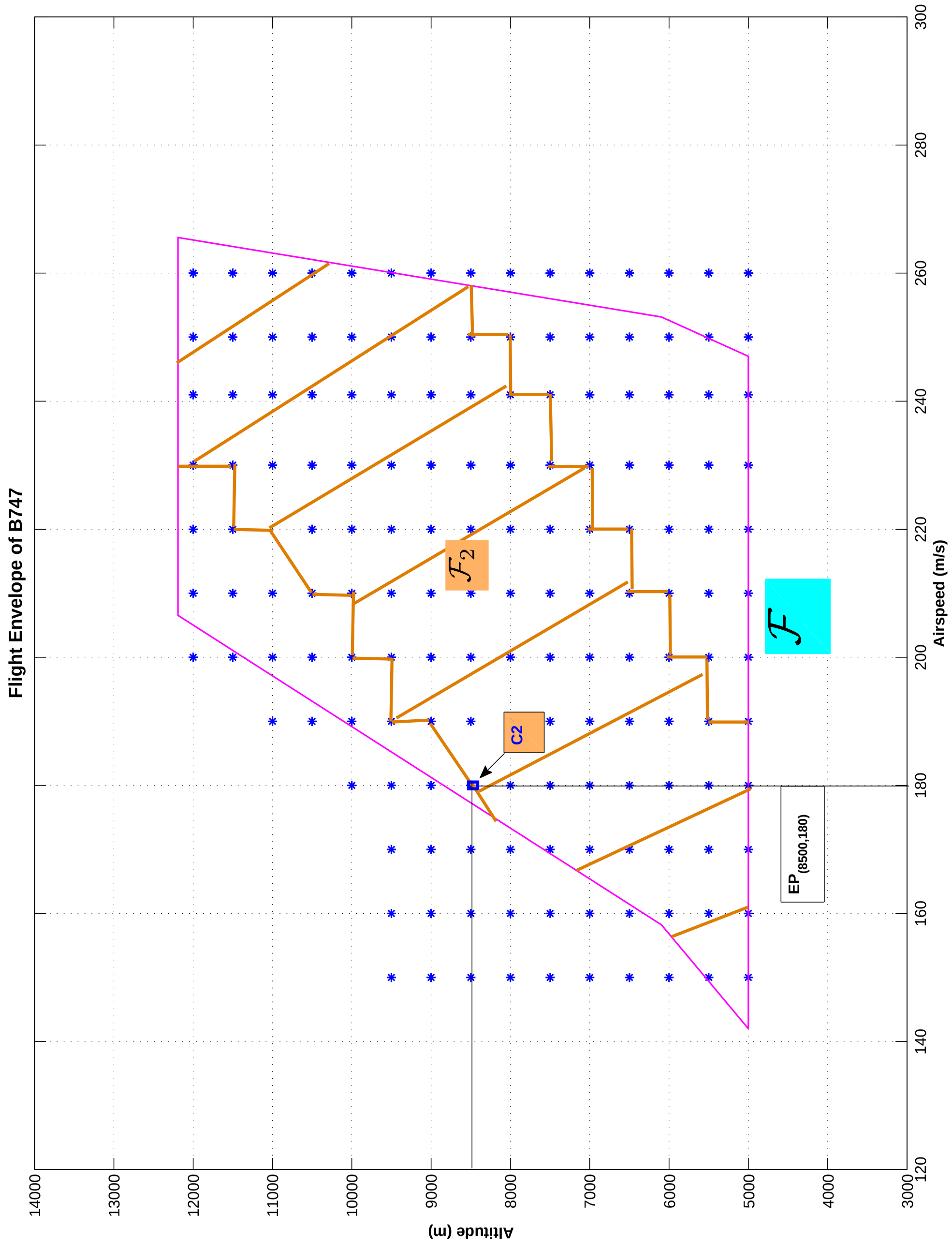
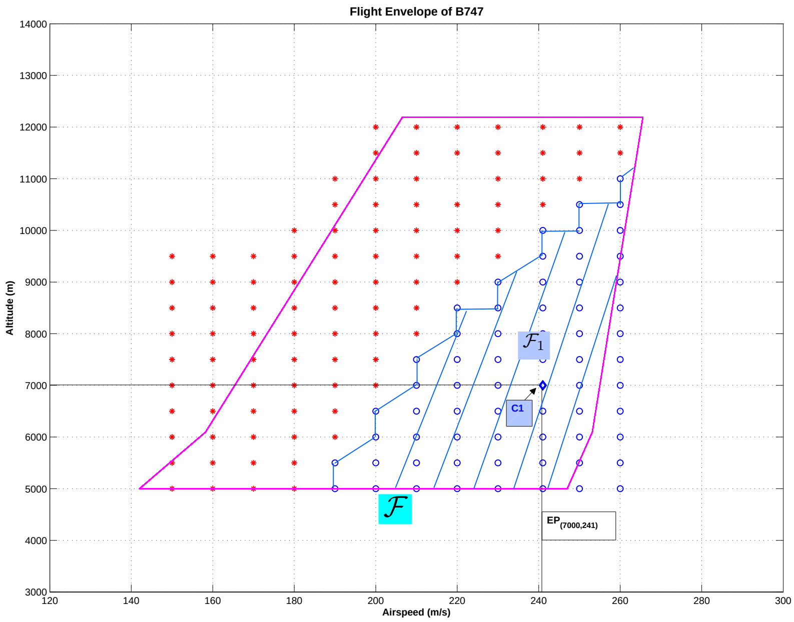
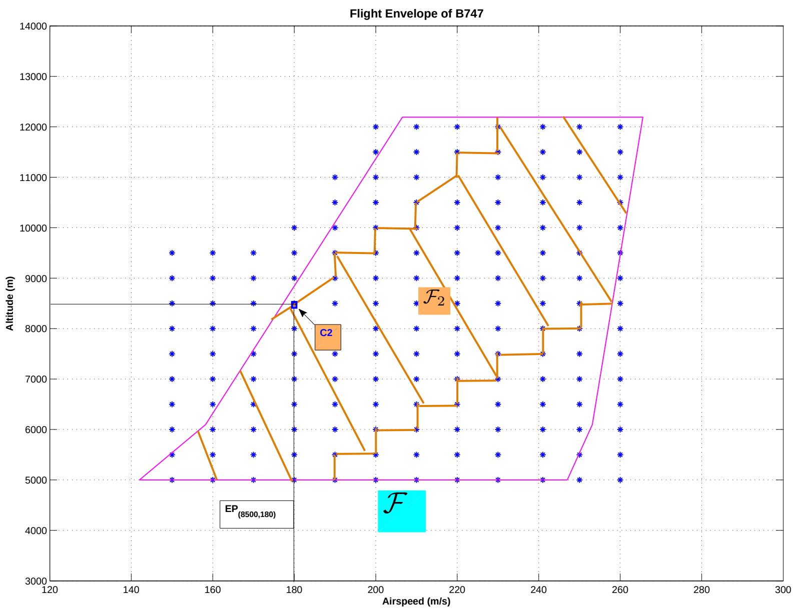


Figure 5.8: Identification of $\mathcal{F}_2$



(a) $\mathcal{F}_1$: Controller C_1 on $EP_{(7000,241)}$



(b) $\mathcal{F}_2$: Controller C_2 on $EP_{(8500,180)}$

Figure 5.9: Boundary of $\mathcal{F}_1$ and $\mathcal{F}_2$

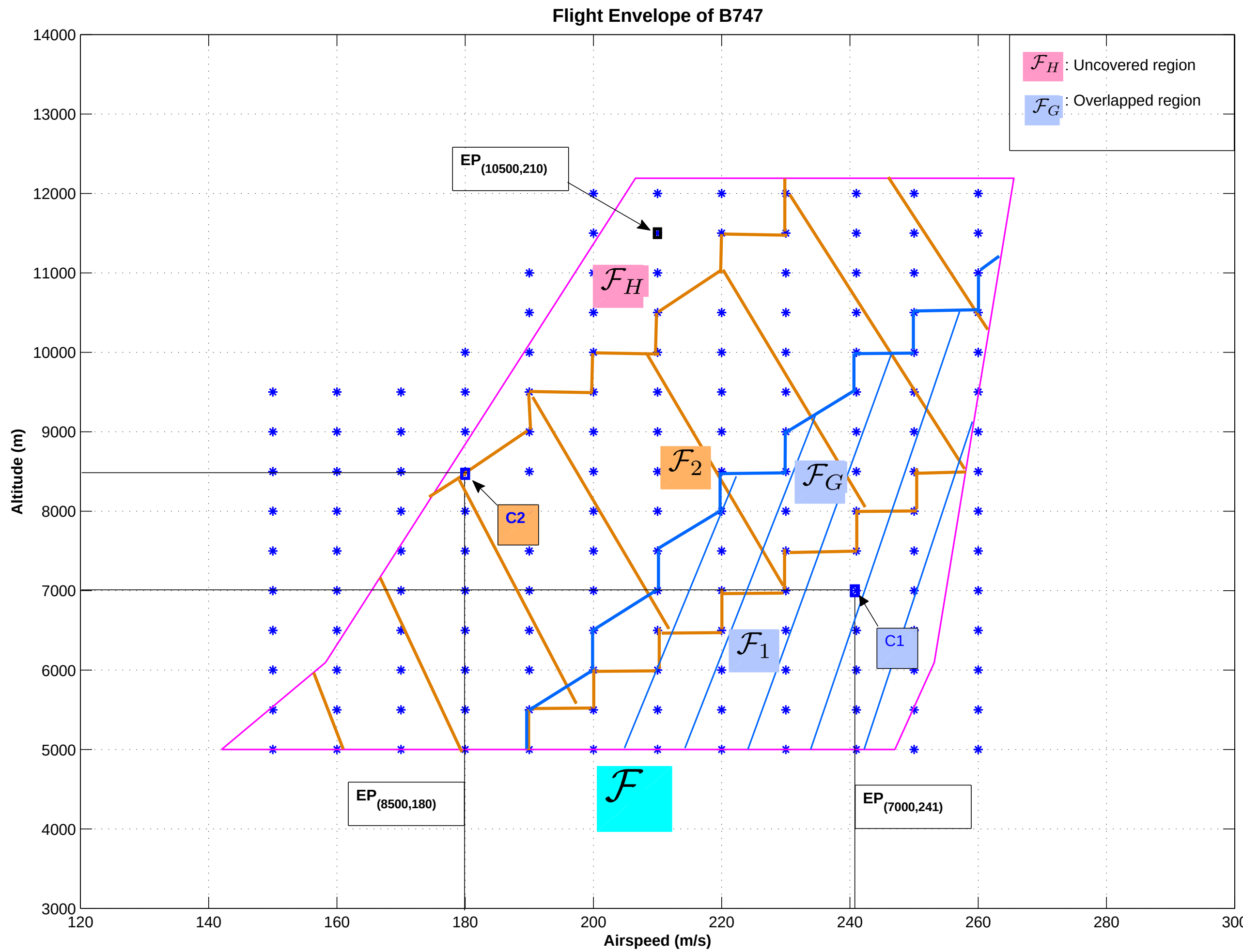


Figure 5.10: $\mathcal{F}_H$ and $\mathcal{F}_G$

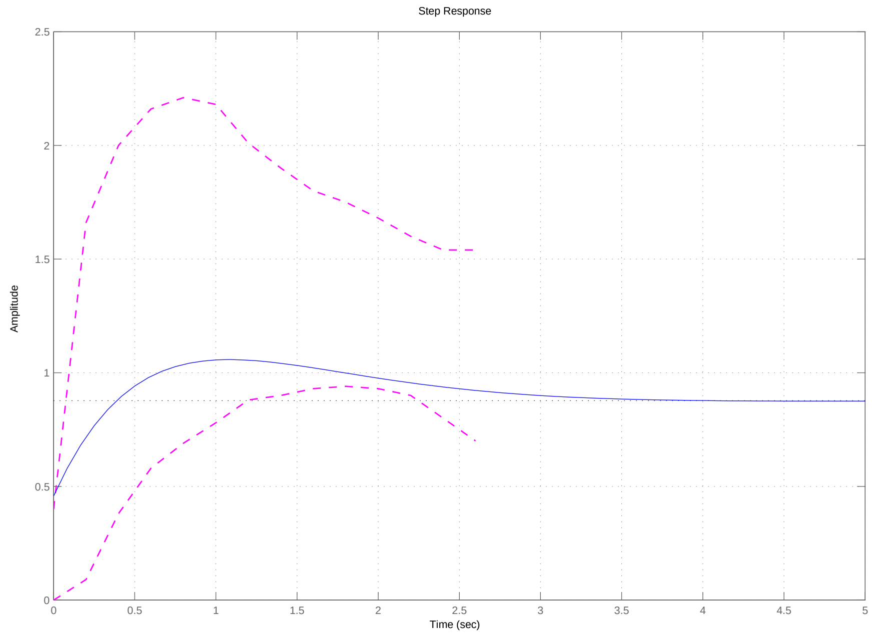


Figure 5.11: C^* criterion assessment of the aircraft with C_2 on $EP_{(11500,210)}$

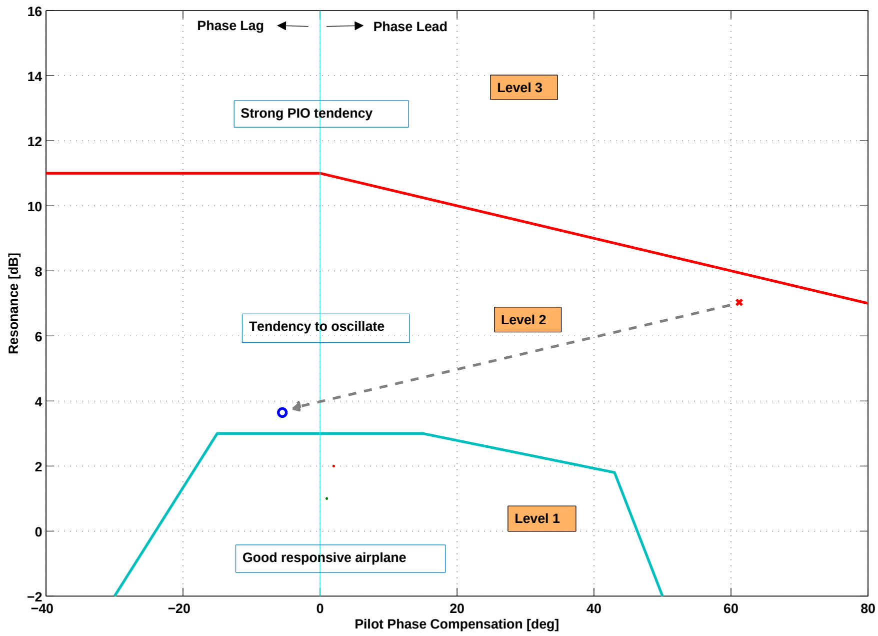


Figure 5.12: Neal and Smith criterion assessment of C_2 on $EP_{(11500,210)}$

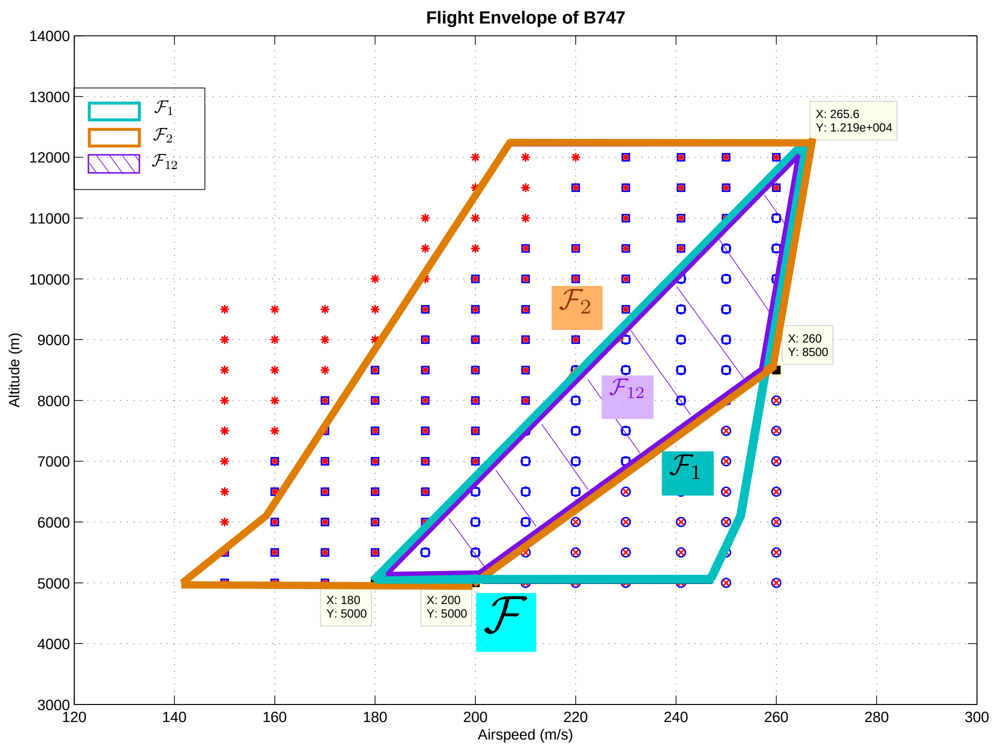


Figure 5.13: Identification of $\mathcal{F}$ with $\mathcal{F}_1$ and $\mathcal{F}_2$

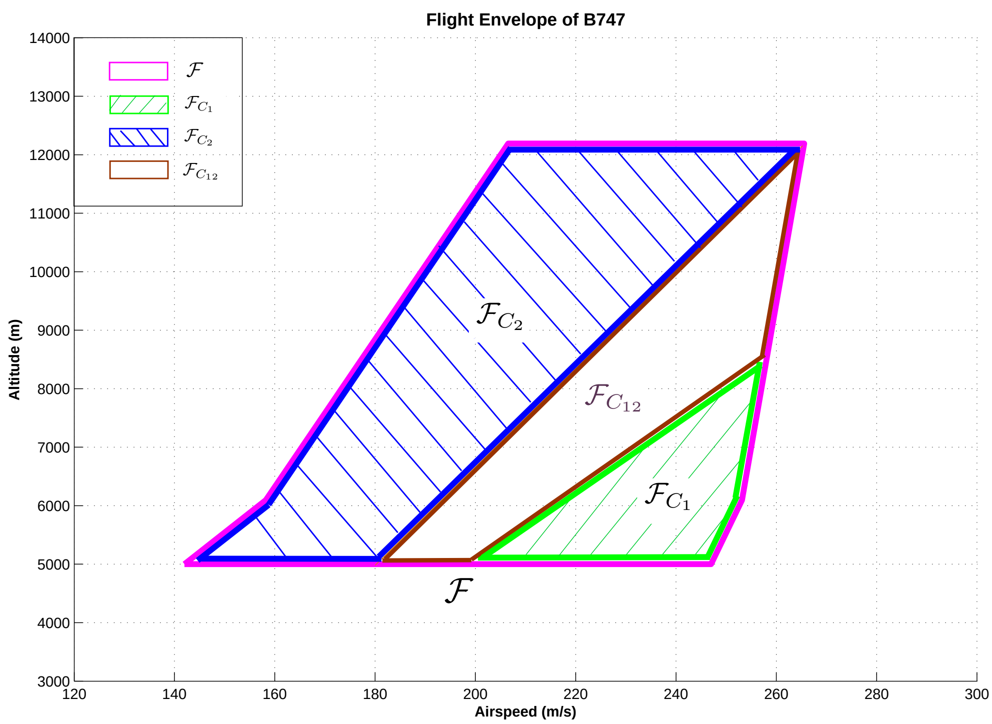


Figure 5.14: Identification of $\mathcal{F}$ with $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$

Chapter 6

Gain Scheduling

In the previous chapters, two sets of longitudinal Rate Command-Attitude Hold (RCAH) controllers C_1 and C_2 are designed for the linear aircraft models using pole placement method at the specific equilibrium points. In addition, the handling qualities of the aircraft are assessed according to the CAP, Neal and Smith, and C^* criteria over the flight envelope. In this chapter, the gain scheduling technique is conducted in interpolation region $\mathcal{F}_{C_{12}}$ with desired response about the scheduling parameters. In this thesis, the current value of the airspeed and altitude are considered as the main scheduling parameters, against which the controllers' gain matrix are scheduled. In other words, the controller gains are smoothly scheduled according to the current trimmed operating conditions.

In practice the aircraft dynamic description is nonlinear, such that the flight controllers with scheduled gains are essential if they are to be robust enough to provide good performance and endow the aircraft with desirable handling qualities throughout the flight envelope. Since the operating region $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$ are determined based on the handling qualities assessment, the nonlinear control problem is subdivided into a series of linear ones. C_1 and C_2 are designed based on two states of the scheduling parameters: $EP_{(7000,241)}$ and $EP_{(8500,180)}$. This ensures that each controller gain matrix is set for the current equilibrium point to suit each linear design condition, and therefore provides a performance that is close to optimal. For interpolation region $\mathcal{F}_{C_{12}}$, gains are calculated at specific equilibrium operating points with a certain scheduling strategy, which results in a controller that gives the desired performance about trimmed operating conditions. Consequently, a set of controllers is reconstructed by combining members of the Linear Time Invariant (LTI) local controllers.

6.1 Gain Scheduling Factor

In order to transit smoothly from one flight controller to another, a strategy of gain scheduling which is a continuous function of scheduling variables (V_{TAS} and h_e), is needed to meet a desired performance and handling qualities specification.

The gain scheduling strategy is expressed as,

$$K_i = K_1 e + K_2(1 - e) \tag{6.1}$$

where:

- K_i is scheduled gain matrix.
- K_1 and K_2 are matrices of the gains of C_1 and C_2 respectively. The values can be obtained from Table 5.1.
- Parameter e is the gain scheduling factor where $e \in [0, 1]$, and e varies according to the current value of the scheduling variables $e = f(V_{TAS}, h_e)$, where $(V_{TAS}, h_e) \in \mathcal{F}_{C_{12}}$, which is shown in Fig. 6.1 and Fig. 6.2.

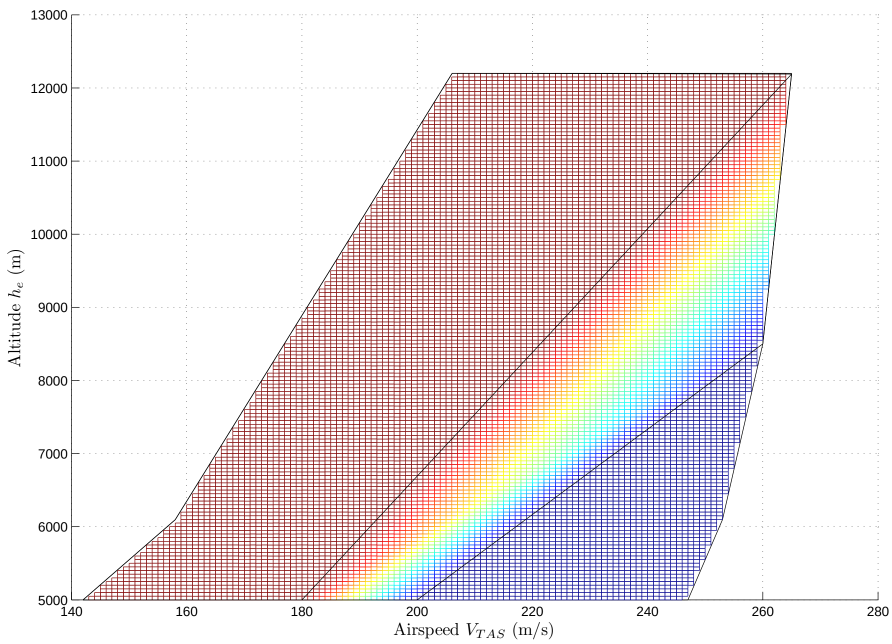


Figure 6.1: Gain scheduling factor along with the B747 flight envelope-2D

Fig. 6.1 and Fig. 6.2 show the surface where the gain scheduling factor e is interpolated linearly. The region $\mathcal{F}_{C_{12}}$ is interpolated at operating points specified by the current V_{TAS} and h_e to produce the gain scheduling factor e. The program is presented in Appendix D

6.2 Scheduled Gain Matrices

In this section, a set of typical equilibrium points are chosen to verify the gain scheduling strategy in $\mathcal{F}_{C_{12}}$.

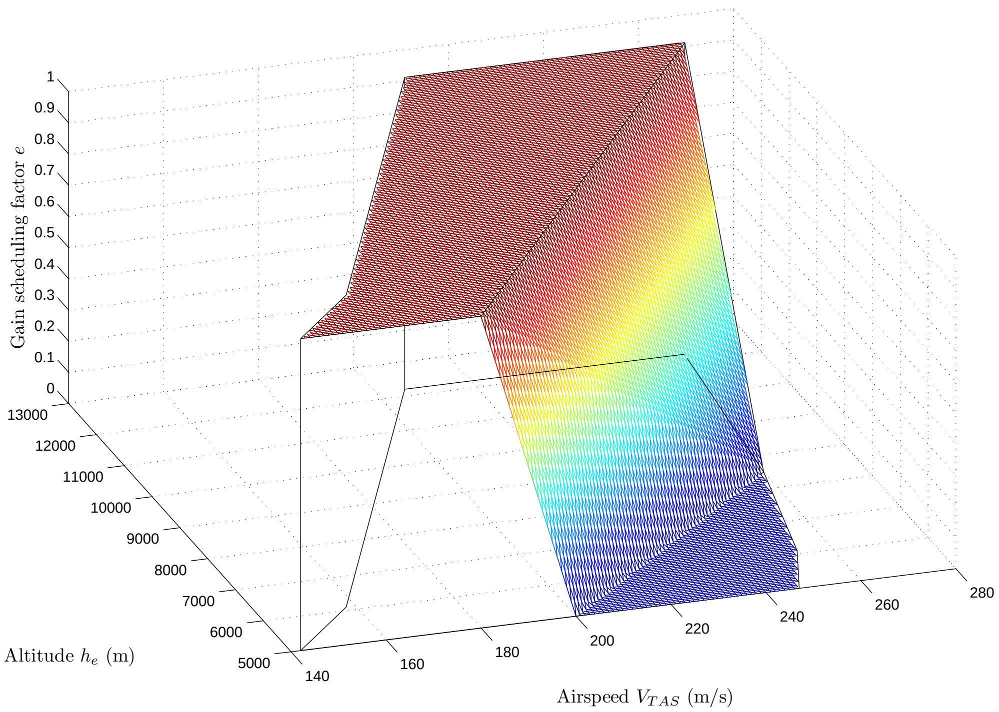


Figure 6.2: Gain scheduling factor along with the B747 flight envelope-3D

6.2.1 Gain Matrix

Equilibrium points $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ are chosen to represent the extremity and middle properties of the scheduler in $\mathcal{F}_{C_{12}}$, see Fig. 6.3.

With reference to the relationship defined in the equation (6.1), the scheduled gain matrix on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ are calculated as follows.

$$K_i = K_2 e + K_1(1 - e) \tag{6.2}$$

where,

$$K_1 = \begin{bmatrix} k_q & k_\alpha & k_{\epsilon_q} & m \end{bmatrix} \tag{6.3}$$
$$= \begin{bmatrix} 0.73 & -1.67 & 2.87 & 1.60 \end{bmatrix} \tag{6.4}$$
$$K_2 = \begin{bmatrix} 1.68 & -3.33 & 5.76 & 3.84 \end{bmatrix} \tag{6.5}$$

hence,

$$e_{(5500,200)} = 0.4286 \tag{6.6}$$
$$K_{(5500,200)} = \begin{bmatrix} 1.10 & -2.32 & 4.01 & 2.48 \end{bmatrix} \tag{6.7}$$
$$e_{(8000,230)} = 0.6237 \tag{6.8}$$
$$K_{(8000,230)} = \begin{bmatrix} 1.30 & -2.67 & 4.62 & 2.96 \end{bmatrix} \tag{6.9}$$
$$e_{(11000,260)} = 0.7652 \tag{6.10}$$
$$K_{(11000,260)} = \begin{bmatrix} 1.46 & -2.94 & 5.08 & 3.31 \end{bmatrix} \tag{6.11}$$

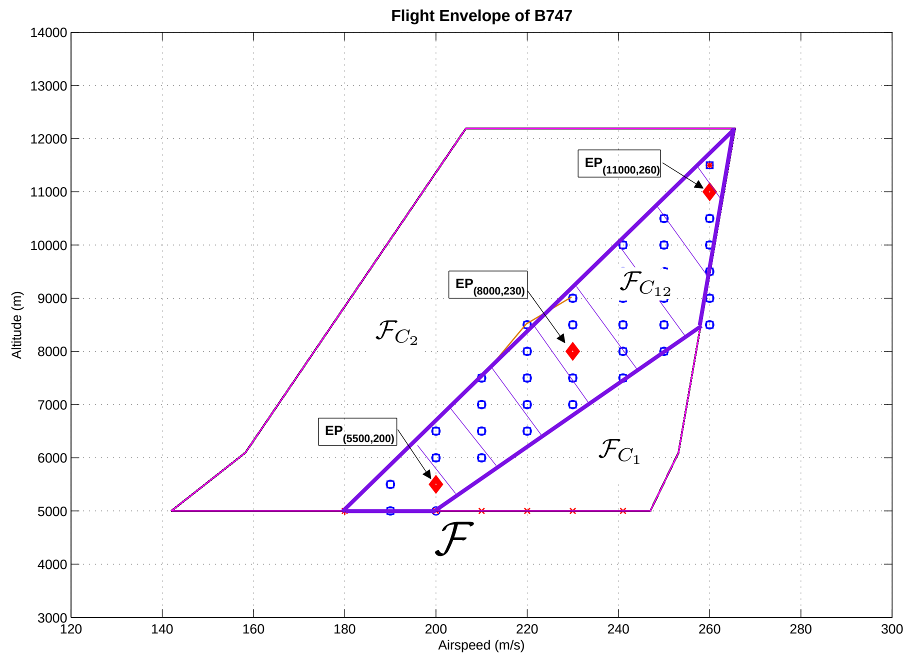


Figure 6.3: Equilibrium points $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ in $\mathcal{F}_{C_{12}}$

Fig. 6.4 shows how the gains vary as transit through equilibrium points $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$.

6.2.2 Performance

For the equilibrium points $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$, the open loop and closed loop system characteristic polynomials are obtained as follows. Note that the closed loop system characteristic polynomials are derived from the aircraft model augmented by scheduled gains from equations (6.7), (6.9) and (6.11).

$$\Delta(s)_{(5500,200)_{open}} = (s + 0.00001)(s^2 + 0.003s + 0.006)(s^2 + 1.18s + 1.47) \quad (6.12)$$
$$\Delta(s)_{(5500,200)_{close}} = s(s + 2.136)(s + 0.0058)(s^2 + 2.777s + 3.36) \quad (6.13)$$
$$\Delta(s)_{(8000,230)_{open}} = (s + 0.0004)(s^2 + 0.0026s + 0.0034)(s^2 + 1.08s + 1.38) \quad (6.14)$$
$$\Delta(s)_{(8000,230)_{close}} = s(s + 3.182)(s + 0.0053)(s^2 + 2.628s + 2.366) \quad (6.15)$$
$$\Delta(s)_{(11000,260)_{open}} = (s + 0.01)(s^2 + 0.008s + 0.002)(s^2 + 0.99s + 1.35) \quad (6.16)$$
$$\Delta(s)_{(11000,260)_{close}} = s(s + 3.565)(s + 0.0214)(s^2 + 2.569s + 1.913) \quad (6.17)$$

The characteristics of the longitudinal dynamics, which are characterized by the Short-Period Pitching Oscillation (SPPO) mode and phugoid oscillation mode, are represented by the second order oscillatory factors in the open loop at three equilibrium points $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$. The short period mode and the phugoid mode of the basic aircraft exhibit classical, second-order characteris-

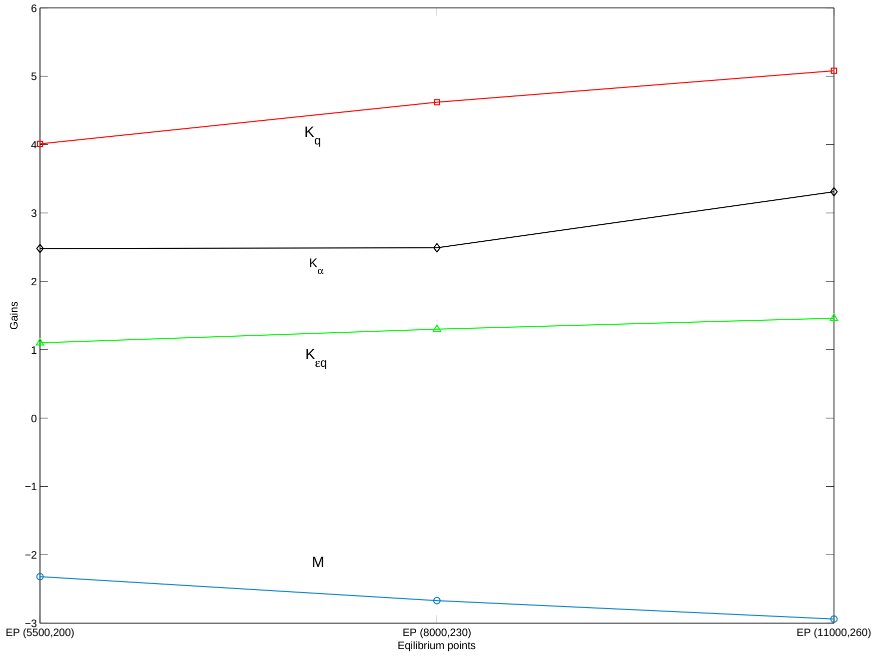


Figure 6.4: Scheduled gains of

tic which are modified considerably for the augmented aircraft. The following two aspects are discussed.

- For the short period mode, the natural frequency is decreased together with a smaller reduction in the damping ratio for the increasing altitude and airspeed in the open loop. While the short period mode is substantially modified by the scheduler in the closed loop. It is clear that the design objectives of exact integral pole-zero cancellation can not be achieved by the scheduler designed upon each equilibrium point, which is also the general case in a real application. Hence, the integral lag time constant term is inevitably induced into the system. According to equations (6.13), (6.15) and (6.17), the integral lag time constants on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ are given by,

$$T_{lag_{(5500,200)}} = \frac{1}{2.136} = 0.468s \tag{6.18}$$

$$T_{lag_{(8000,230)}} = \frac{1}{3.182} = 0.314s \tag{6.19}$$

$$T_{lag_{(11000,260)}} = \frac{1}{3.565} = 0.281s \tag{6.20}$$

  It indicates the integral lag time constants decrease with increasing airspeed and altitude, which means the aircraft response becomes more responsive to pilot control. 0.468s is the maximum value in equation (6.18), which is still adequately small, and is not especially obvious to the pilot. However, since it is still likely to induce an intrusive handling qualities problem, the handling qualities of the aircraft with scheduled gains of $K_{(5500,200)}$, $K_{(8000,230)}$ and $K_{(11000,260)}$ in region $\mathcal{F}_{C_{12}}$ are assessed in the following section.

- For the phugoid mode, it is represented by a pair of stable poles and a second order oscillation for the basic aircraft in the open loop. The natural frequency of the phugoid mode is slightly reduced with increasing altitude and airspeed. In the closed loop, the mode is no longer oscillatory and is represented by a stable and negative pair of real poles. It means that the phugoid tendency is with a stable long time constant pole, and is not especially obvious to the pilot. The phugoid mode of the basic aircraft is improved considerably by the scheduler.

6.3 Handling Qualities Assessment

A handling qualities assessment is conducted in this section, to evaluate the influence of the scheduled gains upon the handing qualities of the aircraft in $\mathcal{F}_{C_{12}}$.

Firstly, the handling qualities of the aircraft augmented by $K_{(5500,200)}$, $K_{(8000,230)}$ and $K_{(11000,260)}$ are assessed respectively based on three equilibrium points, $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ shown in 6.3. Secondly, an assessment of the aircraft augmented by scheduled gains is performed over the set of equilibrium points in $\mathcal{F}_{C_{12}}$.

6.3.1 Assessment On $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$

The CAP, Neal and Smith, and C^* criteria are employed to assess the handling qualities of the aircraft with scheduled gains on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ shown in Fig. 6.3. The results are presented in Fig. 6.5, Fig. 6.8 and Fig. 6.9.

As can be observed from Fig. 6.5, Fig. 6.8 and Fig. 6.9, a consistent assessment is obtained, which is the aircraft with scheduled gains on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$ possess satisfactory handling qualities. In particular:

- Firstly, CAP assessment shown in Fig. 6.5 indicates that there is considerable margin between the minimum CAP value of 0.085 and the closed loop CAP parameters for a Category B flight phase, say $CAP_{(5500,200)} = 0.32$,$CAP_{(8000,230)} = 0.22$ and $CAP_{(11000,260)} = 0.19$.

- Secondly, the pilot and aircraft closed loop system, as shown in Fig. A.1, are plotted on the Bode plot. As pointed out in the Neal and Smith criterion assessment on $EP_{(8000,230)}$ in the Bode plot on Fig. 6.6, the maximum low frequency droop is -2 dB which is less than the value -3 dB specified in the criterion. Comparing with the equilibrium points $EP_{(5500,200)}$ and $EP_{(11000,260)}$,

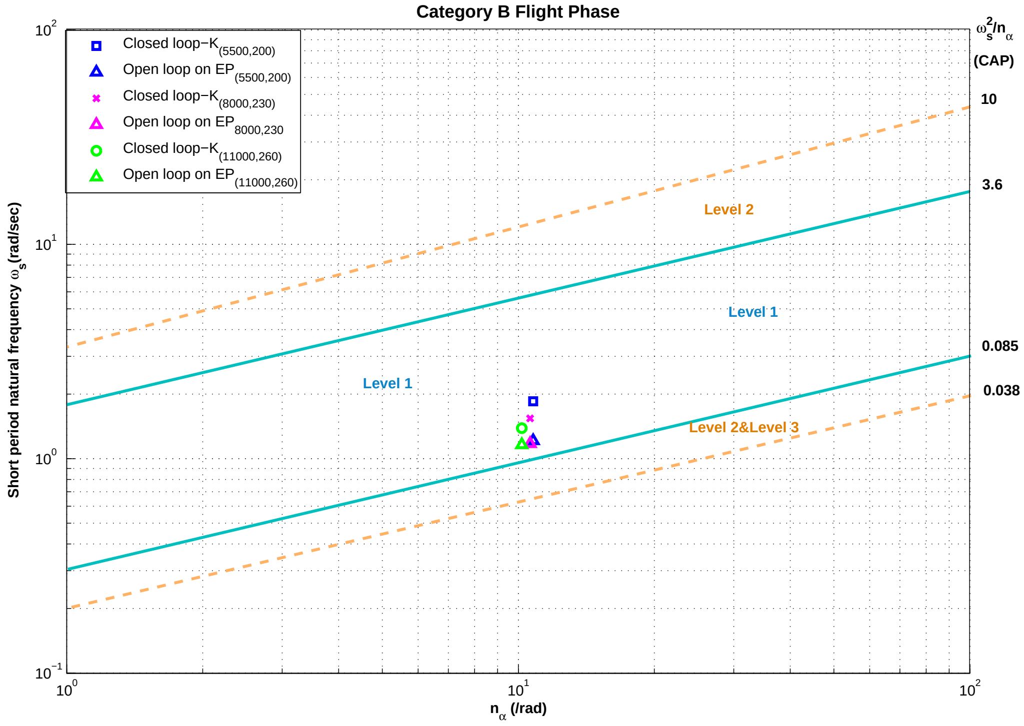


Figure 6.5: CAP criterion assessment on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$

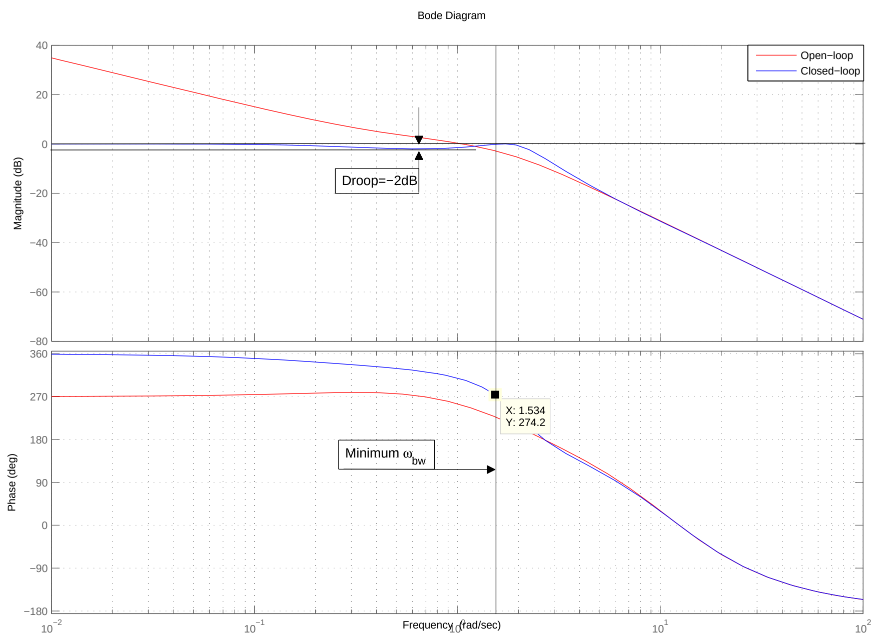


Figure 6.6: The Neal and Smith bode plot on $EP_{(8000,230)}$

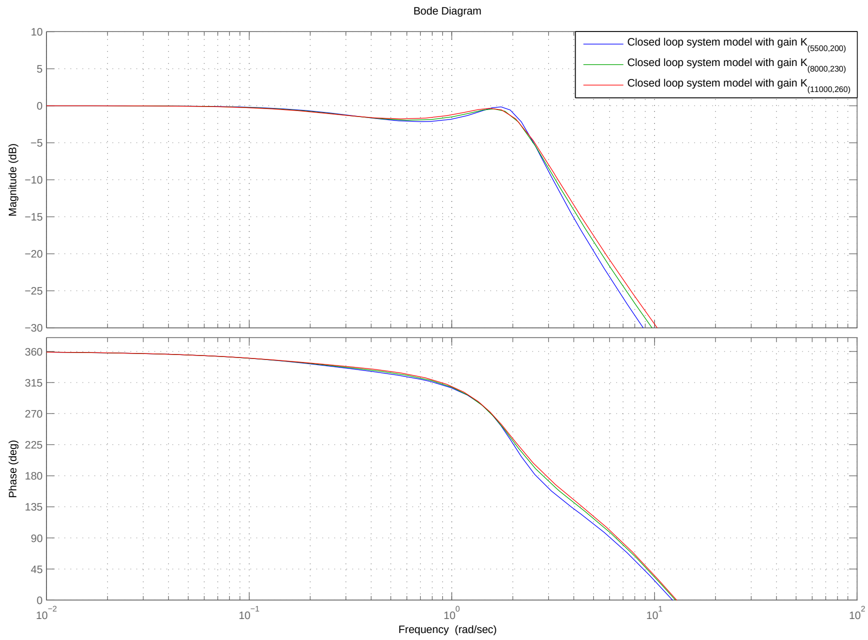


Figure 6.7: The Neal and Smith bode plot of closed loop system on 3 EPs

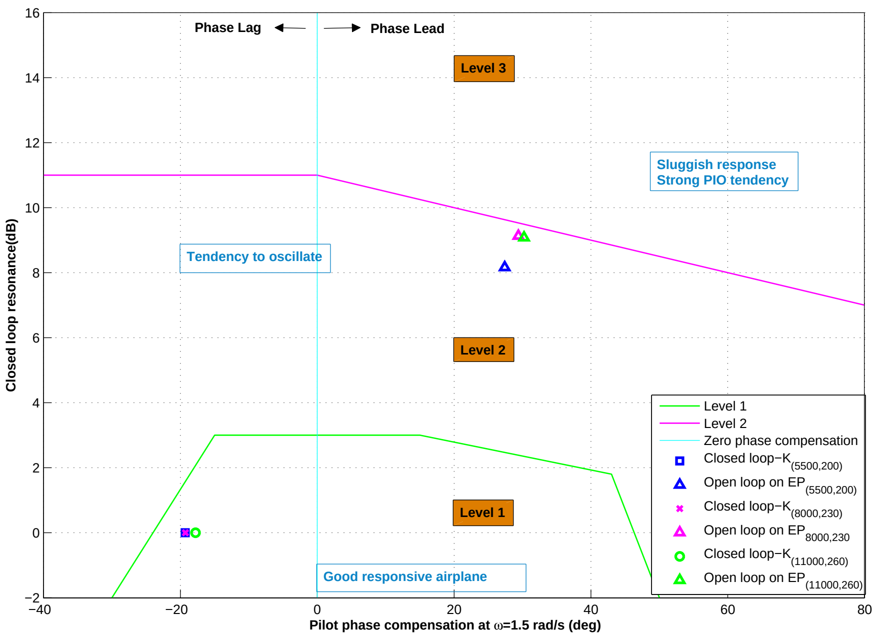


Figure 6.8: Neal and Smith criterion assessment against boundary requirement

similar assessment results are obtained in Fig. 6.7. Consequently, Level 1 handling qualities are predicted by plotting closed loop resonance and pilot phase compensation against the Neal and Smith requirements in Fig. 6.8.

- Thirdly, Fig. 6.9 plots the step response and frequency response against the C^* criterion requirements for Category C flight phase on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$. All of them indicate that the handling qualities of the aircraft with scheduled gains in the take off and landing flight phase meet the criterion requirement.

6.3.2 Assessment over Region $\mathcal{F}_{C_{12}}$

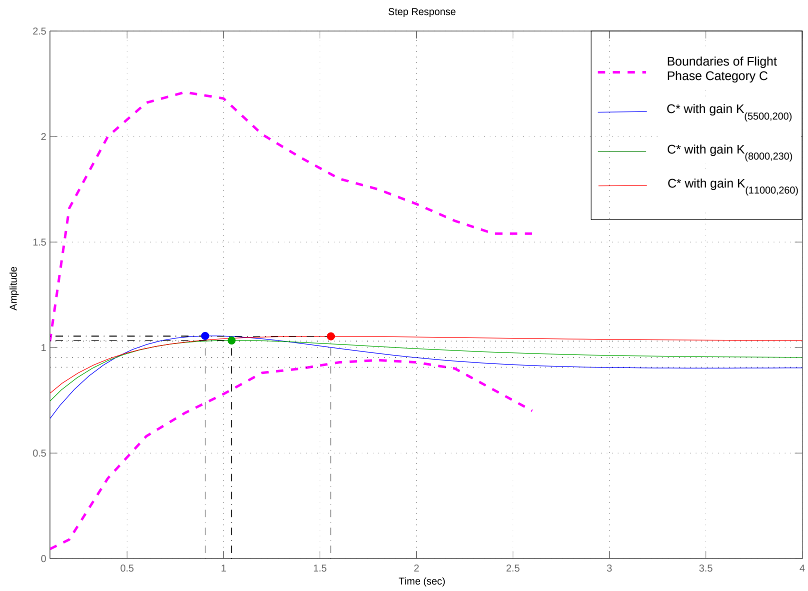
The CAP, Neal and Smith criteria are utilized in this section to assess the handling qualities of the aircraft augmented with scheduled gains over the interpolation region $\mathcal{F}_{C_{12}}$.

In order to estimate the influence of the scheduled gains upon the handing qualities of the aircraft in $\mathcal{F}_{C_{12}}$, the scheduled gains on the set of equilibrium points in $\mathcal{F}_{C_{12}}$ are calculated. Based on the gain scheduling factor in equation (6.1), the scheduled gains are calculated according to the value of h_e and V_{TAS} at each equilibrium point, which are used to augment the aircraft models in $\mathcal{F}_{C_{12}}$. The augmented aircraft models at the set of equilibrium points in $\mathcal{F}_{C_{12}}$ are now obtained. Consequently, the handling qualities assessment of these augmented aircraft models is presented in Fig. 6.10 and Fig. 6.11.

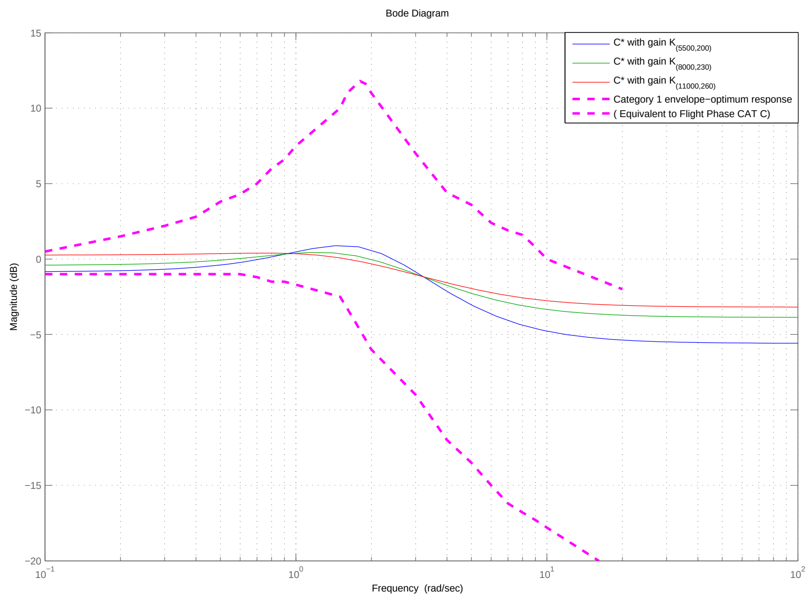
Fig. 6.10 and Fig. 6.11 indicate the CAP criterion assessment and the Neal and Smith criterion assessment over the region $\mathcal{F}_{C_{12}}$.

- The CAP assessment predicts Level 1 handling qualities in $\mathcal{F}_{C_{12}}$. For the aircraft augmented with scheduled gains, the CAP parameters are less marginally located in the Level 1 handling qualities requirements for Category B flight phase due to the short period natural frequency ω_s and damping ratio ζ_s being improved to compensate the aircraft response to the pilot control.

- The Neal and Smith criterion assessment in $\mathcal{F}_{C_{12}}$ indicates that the aircraft augmented by scheduled gains are predicted with Level 1 handling qualities, shown in Fig. 6.11.



(a) C^* criterion-step response



(b) C^* criterion-frequency response

Figure 6.9: C^* criterion assessment on $EP_{(5500,200)}$, $EP_{(8000,230)}$ and $EP_{(11000,260)}$

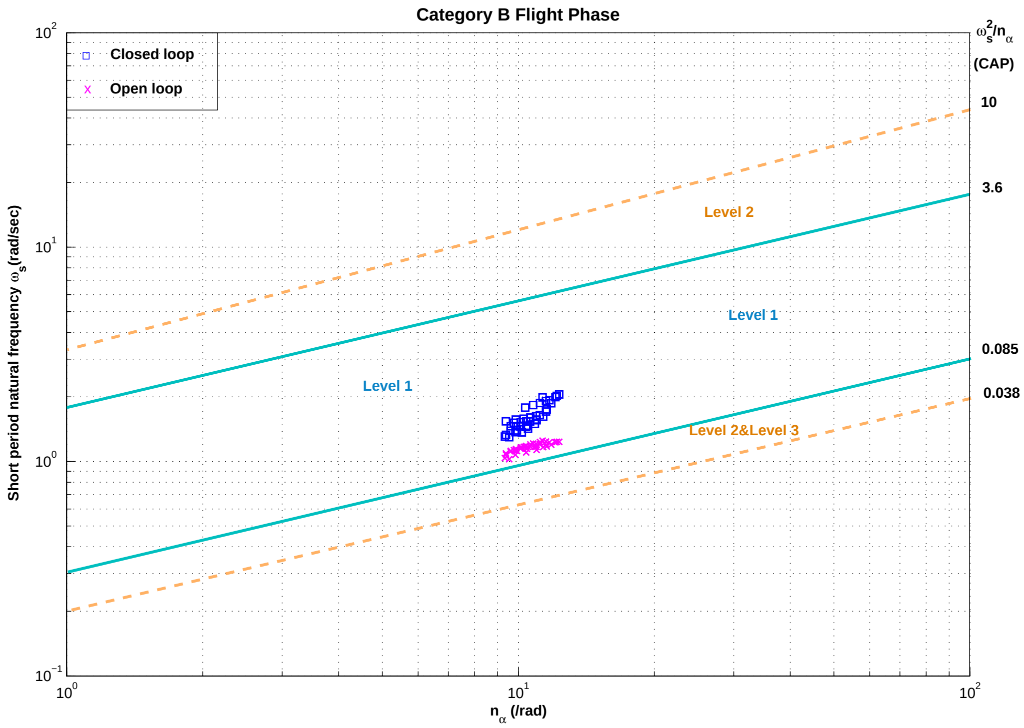


Figure 6.10: CAP criterion assessment in $\mathcal{F}_{C_{12}}$

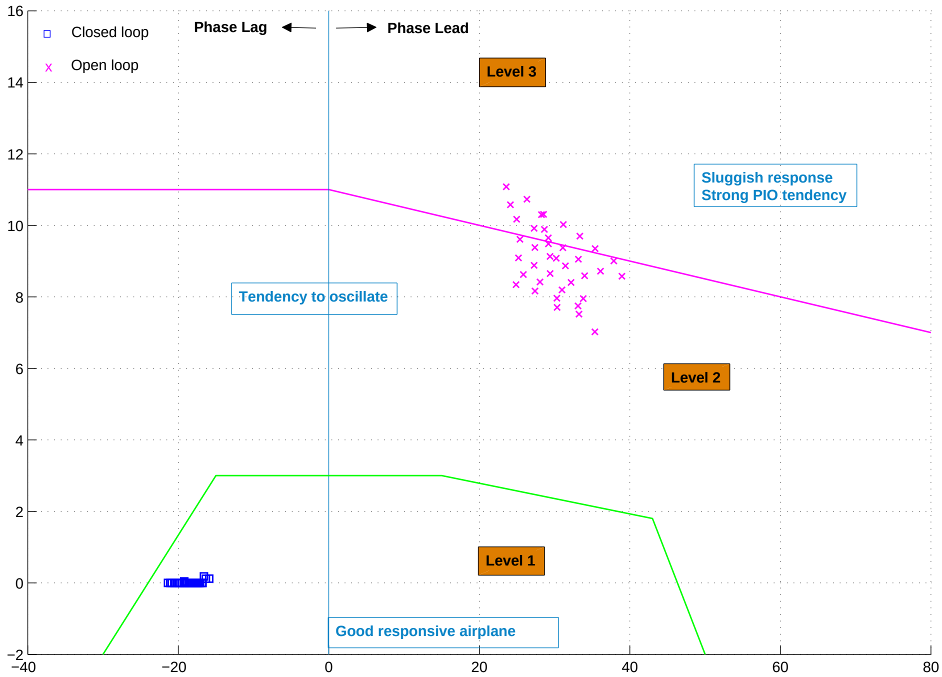


Figure 6.11: The Neal and Smith criterion assessment in $\mathcal{F}_{C_{12}}$

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Chapter 7

Conclusions

This thesis presents a method for the design and development of gain scheduling controllers for a passenger aircraft B747 from the viewpoint of handling qualities assessment. Firstly, the state space pole placement design procedure is employed to design the longitudinal Rate Command-Attitude Hold (RCAH) controllers as the local flight controllers. Secondly, handling qualities of the aircraft augmented by the local controllers are assessed over the whole flight envelope to identify the interpolation region $\mathcal{F}_{C_{12}}$. Finally, the controller gains are scheduled against the current value of the airspeed and altitude in order to obtain smooth transition from one flight controller to another in different flight conditions. A summary of the key findings and suggestions for further work are presented in this chapter.

7.1 Summary of Findings

In this thesis, the key findings comprise three major areas: flight control law design, handling qualities assessment and gain scheduling techniques.

7.1.1 Flight Control Law Design

Two sets of the longitudinal Rate Command Attitude Hold (RCAH) controllers are designed by means of pole-placement design procedure based on the existing Boeing 747-100/200 aircraft model. In order to achieve an attitude hold characteristic, and also improve the dynamic tracking behavior of the aircraft, the classical proportional-plus-integral (PI) controller structure with pitch rate q and angle of attack α feedback is implemented. In particular:

- The aircraft model is trimmed and linearized based on the application of the Boeing 747-100/200 aircraft model trim and linearization routine along with the flight envelope.

- The longitudinal flight control law is designed with RCAH characteristics. It is found that the pitch rate q and angle of attack α feedback to elevator are effective to modify the longitudinal short period mode stability characteristics of the aircraft. K_q effectively increases the short period mode damping, and K_α increases the short period mode frequency. The values of the pitch-rate feedback gain K_q, the angle of attack feedback gain K_α, the integrator gain K_{ϵ_q} and the feed-forward gain K_m are calculated by placing the short period poles at the desired locations on the s-plane.

- The longitudinal Rate Command Attitude Hold (RCAH) controllers endows the aircraft with good attitude tracking and handling characteristics due to the application of the structure including a classical PI controller; level 1 handling qualities are accessible for C_1 and C_2.

- For the local controller C_1 and C_2, the design objective of minimizing the effect of the time lag introduced by the integral term, is achieved by near exact integral pole-zero cancellation. Although the integral pole-zero cancellation can not be achieved at each equilibrium point, the assessment shows the time lag does not induce an intrusive handling qualities problem. Moreover, this is the general case in a real application, even for the perfect integral pole-zero cancellation design. The integral lag time constant term is inevitably induced into the system due to different components in the control loop, such as actuators.

7.1.2 Handling Qualities Assessment

Utilizing a number of existing flying specifications and handling qualities criteria, the longitudinal handling qualities of the B747 aircraft are assessed over the whole flight envelope. The key findings are summarized as follows:

- Handling qualities criteria are generally better developed for the military and combat aircraft.
  In this thesis, the CAP, Neal and Smith, and C^* criteria are applied in the B747 aircraft handling qualities assessment. These criteria are developed for the military and combat aircraft with the explicit purpose of ensuring good dynamic response characteristics in the execution of the defined flight task or mission. For the handling qualities assessment of a civil passenger aircraft like B747, the US Federal Aviation Regulations (FAR-25) [Ano94] or Certification Specification for Large Aeroplanes (CS-25) [CS206] might be the most suitable to comply with. Unfortunately, there are few specifications to quantify the handling qualities in FAR-25 and CS-25. For more formal analytical purposes, it is necessary for the handling qualities to be described quantitatively. Although these criteria are generally better developed for the military and combat aircraft, the CAP, Neal and Smith, and C^* criteria can still be applied to assess the handling qualities of the B747 aircraft.

- The longitudinal handling qualities of the B747 aircraft are improved by local controllers C_1 and C_2.
  Comparing the assessment of the basic aircraft with the assessment of the augmented aircraft using the CAP, Neal and Smith, and C^* criteria, consistent results can be obtained, which shows the closed loop flight controller improves handling qualities of the aircraft considerably. In particular,

  1. CAP criterion
     The CAP parameter of the augmented aircraft is well inside the boundary of CAP Level 1 handling qualities requirements. In other words, the augmented B747 aircraft shows less marginal Level 1 flying qualities than the basic aircraft does for a category B (Cruise) flight phase.
  2. The Neal and Smith criterion
     The Neal and Smith criterion, which is considered as a critical measure of pilot handling opinion for the pitch tracking task, is then used as the main method to assess the handling qualities of the Rate Command Attitude Hold (RCAH) controllers, and furthermore, to aid in the design of the scheduler.
     (a) Handling qualities assessment of the aircraft with C_1 and C_2 controllers at equilibrium design points $EP_{(7000,241)}$ and $EP_{(8500,180)}$
     It is found that for short period mode, an increased natural frequency effectively reduced the pilot compensation in order to achieve a closed loop bandwidth of $\omega_{BW} = 1.5$rad/s. Meanwhile, the improvement of the damping ratio decreases the closed loop resonance peak considerably. Both of these endow the aircraft augmented by controllers C_1 and C_2 at $EP_{(7000,241)}$ and $EP_{(8500,180)}$ with Level 1 handling qualities in the Neal Smith criterion.
     (b) Assessment of the aircraft over the flight envelope
     Firstly, handling qualities of the aircraft with C_1 and C_2 are assessed over the flight envelope to identify the operating range $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$. It is found that there is general trend towards a prediction of degrading handling qualities in the pitch tracking task in low airspeed and high altitude flight conditions. In addition, superior handling qualities are found near the equilibrium point for which the controller was designed for, this was to be expected. Secondly, the handling qualities of the aircraft with the scheduler are assessed in the interpolation region $\mathcal{F}_{C_{12}}$. It is found that the Neal and Smith criterion predicts Level 1 handling qualities of the aircraft with the scheduler in pitch tracking task.
  3. C^* criterion
     The step response and the Bode plot of the C^* transfer function are located in the acceptable limits envelope of Flight Phase Category C. It indicates the augmented aircraft possessing the acceptable handling qualities in takeoff and landing flight phase.

7.1.3 Gain Scheduling

A gain scheduling scheme that is a continuous function of the scheduling variables is designed. It is found that this scheduling scheme provides continuous and automatic gain-scheduling with respect to V_{TAS} and h_e, and a desired performance and handling qualities for the aircraft augmented by the scheduler in the flight envelope assuming frozen values of scheduling parameters - V_{TAS} and h_e.

7.2 Recommendations for Further Work

With regards to the further work on the subject, there are several directions that could be further researched and developed as grouped into the following categories: flight control, handling qualities and gain scheduling techniques.

7.2.1 Flight Control Law Design

In this thesis, the longitudinal Rate Command-Attitude Hold (RCAH) command and stability augmentation system is designed to improve longitudinal dynamic manoeuvring characteristic with the ability to undertake precision pitch attitude tracking. For the purpose of good aircraft stability in all axes, several other control laws are likely to be interesting topics for the further work.

- Longitudinal control law
  C^* includes a combination of normal acceleration and pitch rate feedback to elevator. A C^* control law design strategy embodies the desirable damping augmentation of pitch rate feedback together with the desirable command characteristics of acceleration feedback.
- Lateral control law
  The lateral roll control law provides a classical roll rate command characteristic since it utilizes roll rate feedback to improve roll damping. The basic controller structure is designed to give a rate command attitude hold control characteristic.
- Yaw control law
  Rate feedback auto-stabilizer, yaw damper with yaw rate feedback can be used to increase the damping of the dutch roll mode. A yaw damper that utilizes yaw rate feedback to improve dutch roll dynamics and incorporates an aileron-rudder-interlink (ARI) will help minimize side-slip during turn entry.

In addition to the CSAS control law mentioned above, it is increasingly common to find a Load Alleviation Function (LAF) in many large civil aircraft to improve passenger ride comfort and reduce structural dynamic deformations by managing manoeuvre and gust loads of the wing [And10]. This aspect is also very interesting for further work.

7.2.2 Handling Qualities Assessment

- The stability and control characteristics are assumed to be constant throughout the flight envelope in this thesis, and are also one of the essential factors influencing the handling qualities of the aircraft. Hence aircraft stability and control characteristics analysis is an important issue for the practical assessment, and this aspect is also very interesting for further work.

- Assessment of handling qualities by flight simulator studies [Cas03].
  Simulation of the aircraft is required to identify the issues with the parameter varying nature of the scheduled controller, especially when crossing the boundaries between $\mathcal{F}_{C_1}$, $\mathcal{F}_{C_2}$ and $\mathcal{F}_{C_{12}}$. The contents of the handling qualities assessment and performance analysis are extended to the assessment of the different flight tasks or flight conditions based on the simulation, such as takeoff, engine failure and crosswind landing configuration.

7.2.3 Gain Scheduling Technique

- Developed tools to aid the identification of the relative scheduling region
  The relative scheduling region presented here is identified based on the analysis of the handling qualities of the aircraft, and the equilibrium points are chosen by means of the heuristic method, the choice of the design points EP could be optimized in the future, so that handling qualities are optimized over the whole flight envelope.

- Advanced gain scheduling method
  In this thesis, the conventional gain scheduling approach is employed to handle the nonlinear property of the aircraft, which is successfully and popularly implemented in aircraft engineering applications. However, this approach has shortcomings: expensive, time-consuming and no guarantees on the robustness, performance and stability of the overall gain scheduled design. In recent years, Linear Parameter Varying (LPV) control has been established as an emerging advanced approach to be a reliable alternative to the conventional gain-scheduling approach [EB01] [Chu10]. LPV is based on the principle of the H_∞ multi-variable control, and guarantees excellent performance. Gain scheduling using advanced method is an interesting topic for further research.

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Appendix A

The Neal and Smith Criterion Assessment

This section discusses the Neal and Smith criterion assessment for the aircraft. Specifically it covers the routine procedures of the Neal and smith criterion [Coo10a].

A.1 Introduction

For the purpose of applying this criterion to assess the handling qualities of the aircraft, a pilot model transfer function is introduced to construct a 'pilot and aircraft closed loop system model' as shown in Fig. A.1. The handling qualities of the aircraft, which is determined by the perception of pilot to accomplish a certain task, is measured in terms of the amount of gain and phase introduced by the 'pilot and aircraft closed loop system model' in this situation. The open-loop transfer function of the pilot model is applied as follows, which includes a pure time delay, gain compensation and phase compensation term.

$$F(s) = F_p(s) \cdot \frac{\theta(s)}{\delta_e(s)} \tag{A.1}$$

$$F_p(s) = K_p e^{-\tau s} \frac{(1 + sT_{p1})}{(1 + sT_{p2})} = K_p D_p(s) P_p(s) \tag{A.2}$$

where:

- K_p is pilot gain compensation term.
- $D_p(s)$ is a pure-time delay term of τ seconds. For a typical human pilot, pilot delay is assumed to be $\tau = 0.3$s. For this analysis, the pilot's neuromuscular delay is assumed to be a second-order *Pade* approximation with 0.3s of the continuous-time delay $e^{-0.3s}$ in transfer function form. More details will be discussed in section A.3.1.

- $P_p(s)$ is phase compensation term.
- $F_p(s)$ is the pilot model.
- $F(s)$ is pilot and aircraft open loop transfer function.

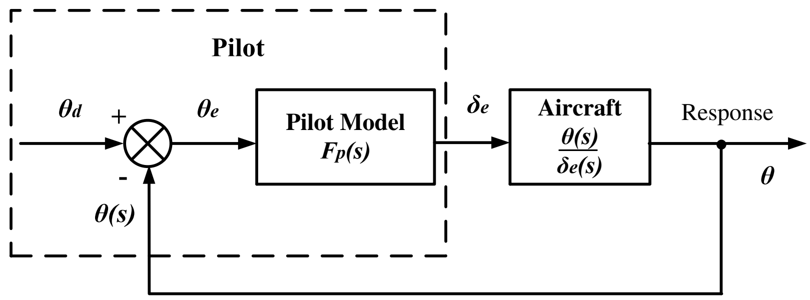

Figure A.1: Pilot and aircraft closed loop system model

The pilot and aircraft closed loop system model is shown in Fig. A.1. The handling qualities are assessed next using the gain and phase characteristics of this pilot and aircraft closed loop system model.

A.2 The criterion

The original criterion definition of the minimum bandwidth frequency is a fixed value of 3.5 rad/s [TR71]. This definition is broadened by MIL-STD-1797A [Ano90] into other flight phase categories shown in Table A.1. An minimum bandwidth of Category B $\omega_{BW} = 1.5$ rad/s is chosen here. The criterion were developed in terms

Table A.1: Bandwidth requirements of different flight phase[Ano90]

Flight Phase	**Bandwidth**
Category A	3.5 rad/s
Category B	1.5 rad/s
Landing	2.5 rad/s
Other Category C	1.5 rad/s

of limiting values for the handling parameters, the limits are defined as follows,

1. Minimum Bandwidth, $\omega_{BW} = 1.5$ rad/s at $Phase = -90$ deg.
2. Maximum low frequency, $Droop = -3$ dB.
3. Closed loop resonance; the best minimum peak value that can be achieved.

The criterion specifies the minimum closed loop bandwidth (ω_{BW}) as 1.5 rad/s when phase lag ϕ first reaches -90 deg and a maximum low-frequency droop of -3 dB. The low-frequency droop and closed-loop resonance are the main parameters which determine the handling qualities assessment of the aircraft. The boundaries of Level 1, Level 2 and Level 3 plotted in Fig. A.6 are for aircraft with the minimum bandwidth of 3.5 rad/s, and have been generated from empirical data correlation, based on a certain amount of flight tests of fighter when implementing the pitch tracking task. The empirical boundaries (for different flight phase categories) determined by flight tests, stipulate a closed-loop droop no more than -3 dB for Levels 1 and 2 and closed-loop resonance no greater than 3 dB for Level 1 in MIL-STD-1797A [Ano90]. Therefore the data do not conflict with the boundaries of Level 1, Level 2 and Level 3 for the minimum bandwidth 3.5 rad/s. Consequently, although these boundaries are not quite suitable for the aircraft with the minimum bandwidth of 1.5 rad/s in flight phase category B, they still could be used to rate the handling qualities as a reference to illustrate the improvement of the handling qualities from the basic to the augmented aircraft.

A.3 Open loop

This pilot model transfer function is required for the Neal and Smith criterion application. The pitch rate response to elevator transfer function derived from equation (3.18) is therefore,

$$\frac{q(s)}{\delta_e(s)} = \frac{k_q(s + \frac{1}{T_{\theta_2}})}{\Delta(s)} \tag{A.3}$$

$$= \frac{4.6099(s + 0.4905)}{(s^2 + 1.243s + 1.58)} \tag{A.4}$$

The pitch attitude response to elevator transfer function follows directly,

$$\frac{\theta(s)}{\delta_e(s)} = \frac{k_q(s + \frac{1}{T_{\theta_2}})}{s\Delta(s)} \tag{A.5}$$

$$= \frac{4.6099(s + 0.4905)}{s(s^2 + 1.243s + 1.58)} \tag{A.6}$$

Note that the longitudinal stability characteristics of short period mode are therefore,

$$\zeta_s = 0.495, \qquad \omega_s = 1.26 rad/s; \tag{A.7}$$

both of which meet Level 1 requirements of MIL-F-8785C specifications.

A.3.1 Frequency response of aircraft plus pilot delay

The transfer function is given by,

$$F(s) = D_p(s) \cdot \frac{\theta(s)}{\delta_e(s)} = e^{-0.3s}\frac{\theta(s)}{\delta_e(s)} \qquad = \frac{e^{-0.3s}4.6099(s+0.4905)}{s(s^2+1.243s+1.58)} \tag{A.8}$$

Now, the 2nd-order *Pade* approximation of the continuous-time delay $e^{-0.3s}$ is given by

$$Pade = \frac{s^2 - 20s + 133.3}{s^2 + 20s + 133.3} \tag{A.9}$$

The comparison of *Pade* with $e^{-0.3s}$ is shown in Fig. A.2. Then, substitute equation

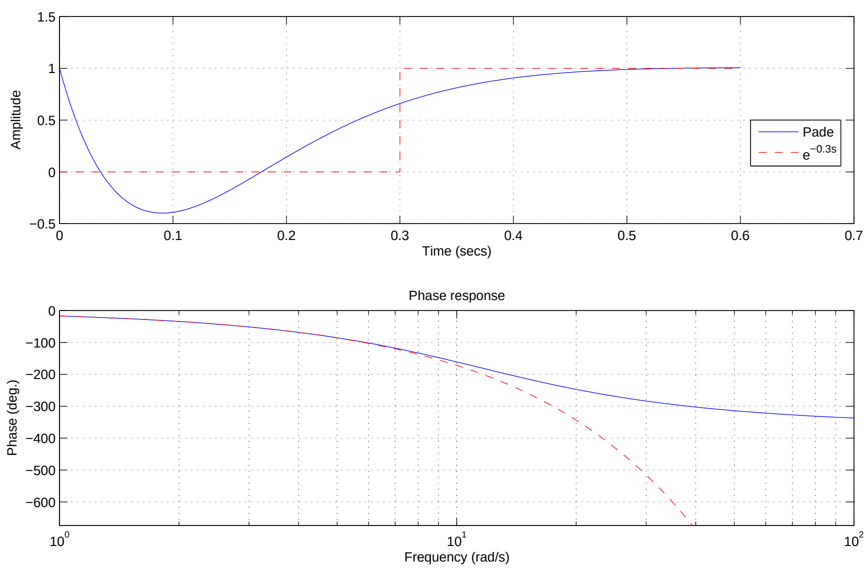

Figure A.2: Step and phase responses of the 2nd order Pade

(A.9) into equation (A.8),

$$F(s) = Pade \cdot \frac{\theta(s)}{\delta_e(s)} = \frac{s^2 - 20s + 133.3}{s^2 + 20s + 133.3} \cdot \frac{4.6099(s+0.4905)}{s(s^2+1.243s+1.58)} \tag{A.10}$$

From Fig. A.3, in order to bring the 1.5 rad/s point onto the −90 deg closed loop phase boundary and the maximum low frequency droop into coincidence with the −3 dB closed loop gain boundary, further open loop compensation of phase lead required can be given by $\phi_p = 22.95$ deg.

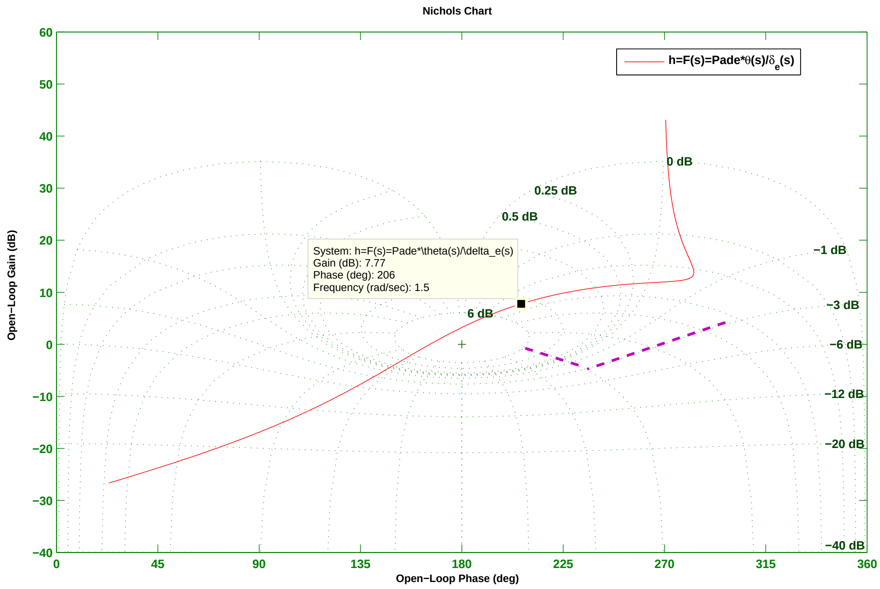

Figure A.3: Nichols chart: Aircraft with pilot delay *Pade*

A.3.2 Pilot Phase Compensation

Since phase lead $\phi_p = 22.95$ deg is required from the pilot, a lag-lead network is appropriate to the pilot model and has transfer function,

$$P_p(s) = \frac{1 + sT_{p1}}{1 + sT_{p2}} \tag{A.11}$$

where, $T_{p1} \geq T_{p2}$

$$\omega_{pk} = \left(\frac{1}{T_{p1}T_{p2}}\right)^{0.5} = 1.5 rad/s \tag{A.12}$$

$$\sin\phi_p = \frac{T_{p1} - T_{p2}}{T_{p1} + T_{p2}} = \sin(22.95°) = 0.39 \tag{A.13}$$

Solving the equations (A.12),

$$T_{p1} = 0.44 sec \tag{A.14}$$

$$T_{p2} = 1.006 sec \tag{A.15}$$

The pilot phase compensation transfer function is therefore,

$$P_p(s) = \frac{1.006s + 1}{0.4417s + 1} = \frac{2.278(s + 0.9938)}{(s + 2.264)} \tag{A.16}$$

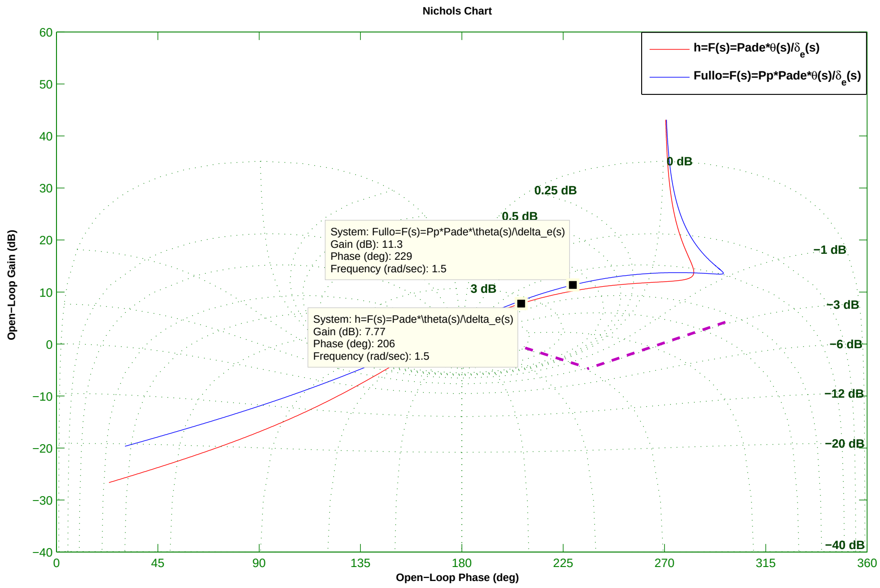

Figure A.4: Nichols chart: Aircraft with pilot delay and phase compensation

A.3.3 Fully phase compensated aircraft

The open loop gain and phase characteristics of the basic aircraft is now compensated with pilot delay and pilot phase. Fig. A.5 shows the resulting transfer function plotted on the Nichols chart, the transfer function is given by,

$$F(s) = Pade \cdot P_p \cdot \frac{\theta(s)}{\delta_e(s)} = Pade \cdot \frac{10.5013(s+0.9938)(s+0.4905)}{s(s+2.264)(s^2+1.243s+1.58)} \tag{A.17}$$

Fig. A.4 shows a gain reduction of -7.47 dB, which is the open loop gain required to bring the 1.5 rad/s point onto the -90 deg closed loop phase boundary and to lead *Droop* coincidence with -3 dB closed loop gain boundary. Therefore, open loop $K_{p_{openloop}} = 0.423$. This represents the gain K_p required from the pilot and thus completes the description of the pilot model for correct compensation according to the Neal and Smith criterion. The full pilot model (A.1) is therefore,

$$F_p(s) = K_p D_p(s) P_p(s) = K_p Pade \cdot \frac{(1+sT_{p1})}{(1+sT_{p2})} \tag{A.18}$$

$$F(s) = F_p(s) \cdot \frac{\theta(s)}{\delta_e(s)} = Pade \cdot \frac{4.4419(s+0.9938)(s+0.4905)}{s(s+2.264)(s^2+1.243s+1.58)} \tag{A.19}$$

The gain and phase characteristics of the aircraft model are compensated by the full pilot model, and is shown in Fig. A.5.

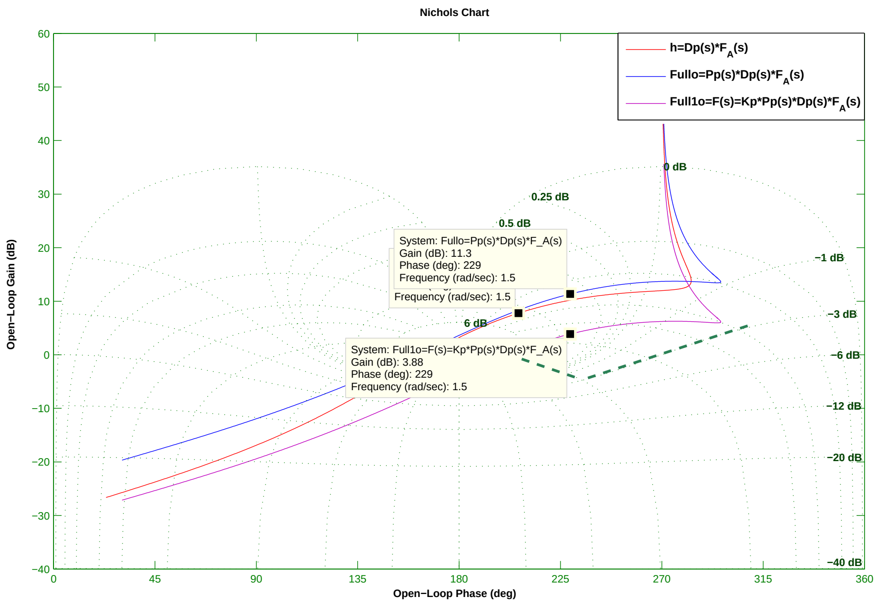

Figure A.5: Nichols chart: Fully compensated aircraft.

A.3.4 Criterion Check

Based on the full pilot compensation model established above, the handling qualities of the aircraft can be assessed against the criterion. The following parameters can be obtained from Fig. A.5:

- Resonance peak (closed loop)=10.4 dB, which is indicated by the fully compensated aircraft curve.

- Pilot phase compensation at 1.5 rad/s=22.95 deg

Fig. A.6 indicates that the basic aircraft handling qualities is Level 3 for the flight condition of $h_e = 7000$ m and $V_{TAS} = 241$ m/s. The aircraft is sluggish with Level 3 flying qualities, hence large pilot compensation is required.

A.4 Closed Loop

This pilot model transfer function is required for the Neal and Smith criterion application. The pitch rate response to elevator transfer function derived from equations

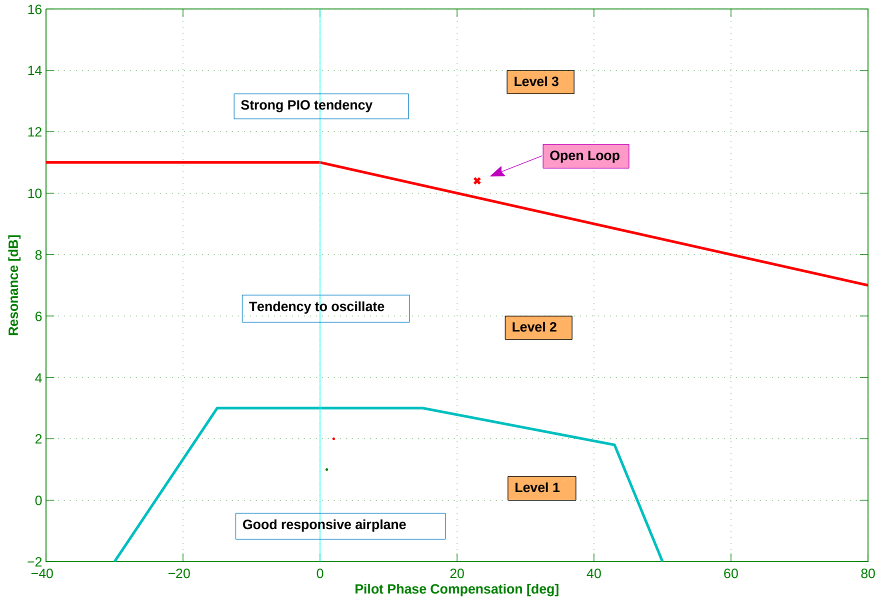

Figure A.6: Neal and Smith criterion assessment for open loop [Coo10a]

(3.34), (3.35) and (3.36) are therefore,

$$\frac{q(s)}{\delta_e(s)} = \frac{k_q(s + \frac{1}{T_{\theta_2}})}{\Delta(s)} = \frac{7.3606(s + 0.4905)}{(s^2 + 2.85s + 3.61)} \tag{A.20}$$

$$\frac{\theta(s)}{\delta_e(s)} = \frac{q(s)}{s\delta_e(s)} = \frac{7.3606(s + 0.4905)}{s(s^2 + 2.85s + 3.61)} \tag{A.21}$$

The longitudinal stability characteristics of short period mode are therefore,

$$\zeta_s = 0.75, \qquad \omega_s = 1.9rad/s; \tag{A.22}$$

both of which meet Level 1 requirements of MIL-F-8785C specifications.

A.4.1 Frequency response of aircraft plus pilot delay

The transfer function derived from equation (A.21) is given by,

$$F(s) = e^{-0.3s}\frac{\theta(s)}{\delta_e(s)} = \frac{e^{-0.3s}7.3606(s + 0.4905)}{s(s^2 + 2.85s + 3.61)} \tag{A.23}$$

Substituting *Pade* for $e^{-0.3s}$,

$$Pade = \frac{s^2 - 20s + 133.3}{s^2 + 20s + 133.3} \tag{A.24}$$

$$F(s) = Pade \cdot \frac{\theta(s)}{\delta_e(s)} = \frac{s^2 - 20s + 133.3}{s^2 + 20s + 133.3} \cdot \frac{7.3606(s + 0.4905)}{s(s^2 + 2.85s + 3.61)} \tag{A.25}$$

As mentioned in section A.4.1, reference to Fig. A.7, the following parameters are required in order to bring the 1.5 rad/s point onto the −90 deg closed loop phase boundary and the maximum low frequency droop to meet the −3 dB closed loop gain boundary, further open loop compensation of phase lag required can be given by $\phi_p = -14.46$ deg.

- Gain reduction −6.6 dB
- Phase lag −14.46 deg

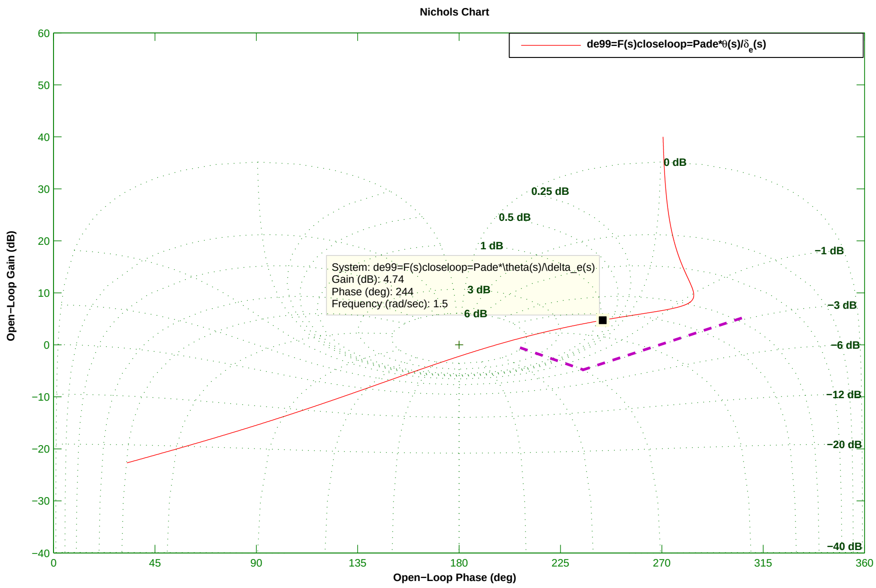

Figure A.7: Nichols chart: Augmented aircraft compensated with time delay *Pade*

A.4.2 Pilot Phase Compensation

Since phase lag $\phi_p = -14.46$deg is required from the pilot, a lag-lead network is appropriate to the pilot model and has transfer function,

$$P_p(s) = \frac{1 + sT_{p1}}{1 + sT_{p2}} \tag{A.26}$$

where,

$$\omega_{pk} = \left(\frac{1}{T_{p1}T_{p2}}\right)^{0.5} = 1.5rad/s \tag{A.27}$$

$$\sin\phi_p = \frac{T_{p1} - T_{p2}}{T_{p1} + T_{p2}} = \sin(-14.46^\circ) = -0.25 \tag{A.28}$$

Solving the equations (A.27),

$$T_{p1} = 0.52sec \tag{A.29}$$
$$T_{p2} = 0.86sec \tag{A.30}$$

The pilot phase compensation transfer function is therefore,

$$P_p(s) = \frac{1 + 0.52s}{1 + 0.86s} = \frac{0.6(s + 1.936)}{s + 1.162} \tag{A.31}$$

Augmented aircraft with time delay and phase compensation plotted in Nichols chart is shown in Fig. A.8.

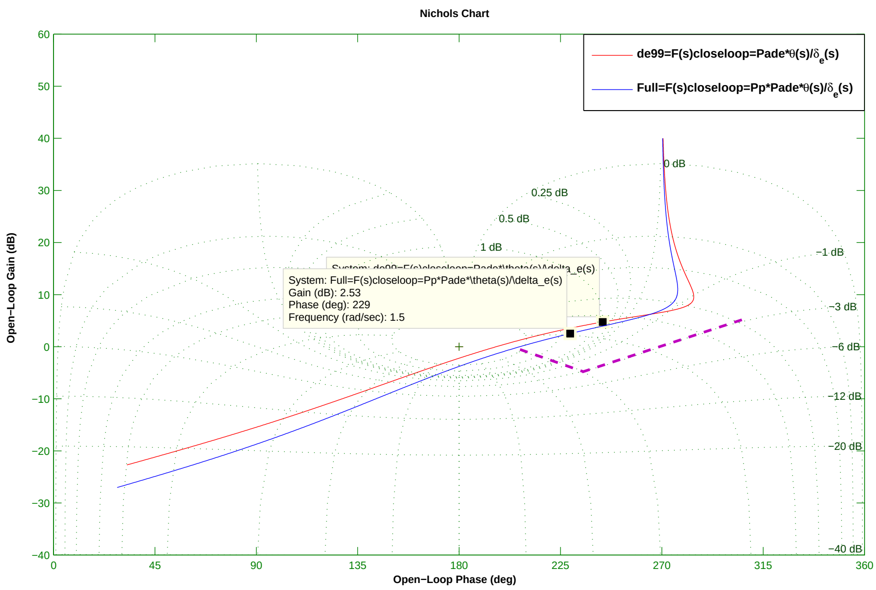

Figure A.8: Augmented aircraft with pilot time delay and phase compensation.

A.4.3 Fully phase compensated aircraft

Based on the open loop gain and phase characteristic of the augmented aircraft with pilot delay and pilot phase compensation discussed in the previous section, the following transfer function can now be obtained,

$$F(s) = D_p \cdot P_p \cdot \frac{\theta(s)}{\delta_e(s)} = Pade \cdot \frac{4.4192(s + 1.936)(s + 0.4905)}{s(s + 1.162)(s^2 + 2.85s + 3.61)} \tag{A.32}$$

Applying the method described in section A.3.3, Fig. A.8 shows that gain reduction is -5.12 dB, therefore, $K_p = 0.5548$. This parameter represents the gain K_p required from the pilot and thus completes the description of the pilot model. The final gain and phase characteristics of the augmented aircraft compensated with full pilot

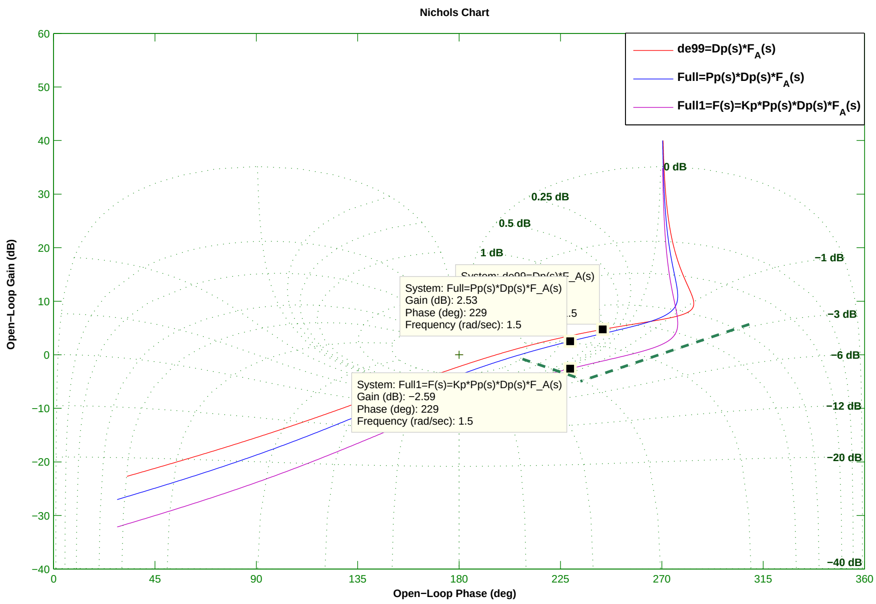

Figure A.9: Nichols chart: Fully compensated of augmented aircraft

model are plotted in Nichols chart in Fig. A.9, and Fig. A.10 shows the Neal and Smith assessment comparison against the criterion. The full pilot model open loop transfer function (A.18) is therefore,

$$F_p(s) = K_p D_p(s) P_p(s) \tag{A.33}$$

$$F(s) = F_p(s) * \frac{\theta(s)}{\delta_e(s)} = Pade \cdot \frac{2.4517(s + 1.936)(s + 0.4905)}{s(s + 1.162)(s^2 + 2.85s + 3.61)} \tag{A.34}$$

Fig. A.10 indicates that for short period mode, an increased natural frequency from 1.26 rad/s to 1.9 rad/s effectively reduced the pilot compensation in order to achieve a closed loop bandwidth of $\omega_{BW} = 1.5$ rad/s. Meanwhile, the improvement of damping ratio from 0.495 to 0.75 decreases the closed loop resonance peak immensely. Both of these endow the aircraft with the Level 1 handling qualities in the Neal and Smith Criterion.

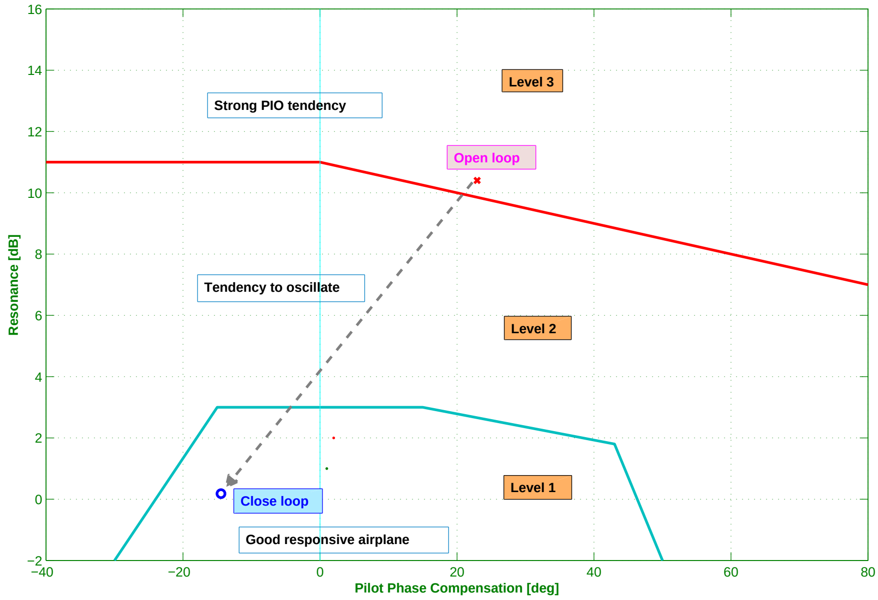

Figure A.10: Neal and Smith criterion assessment comparison [Coo10a]

Appendix B

Controller C_2 Design

B.1 Full Order of Equations of Motion

The aircraft model on operating point $EP_{(8500,180)}$ can be obtained following the trim and linearization routine procedure in section 3.2, which is given by,

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) \tag{B.1}$$

where,

$$x(t) = \begin{bmatrix} q_{body} & V_{TAS} & \alpha & \theta \end{bmatrix}^T; \tag{B.2}$$

$$u(t) = \begin{bmatrix} \delta_{stab} & \delta_e \end{bmatrix}^T \tag{B.3}$$

$$A_2 = \begin{bmatrix} -0.4390 & 0.0003 & -0.3940 & 0 \\ -0.2191 & -0.0087 & 2.0843 & -9.7803 \\ 1.0019 & -0.0006 & -0.3590 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{B.4}$$

$$B_2 = \begin{bmatrix} 1.4054 & 2.1595 \\ 0 & 0 \\ 0.0366 & 0.0589 \\ 0 & 0 \end{bmatrix} \tag{B.5}$$

Thus the open loop characteristic polynomial can be obtained by,

$$\Delta(s)_2 = (s^2 + 0.003877s + 0.007748)(s^2 + 0.8024s + 0.5505) \tag{B.6}$$

The longitudinal stability modes characteristics are therefore,

$$\zeta_p = 0.022, \qquad \omega_p = 0.088 rad/s; \tag{B.7}$$

$$\zeta_s = 0.541, \qquad \omega_s = 0.724 rad/s; \tag{B.8}$$

Normal load factor per unit angle of attack n_{α_2} on operating point $EP_{(8500,180)}$ is calculated as follows.

$$n_{\alpha_2} = \frac{n_z}{\alpha} = -\frac{z_{w_2} U_e}{g} = -\frac{-0.359 \times 180}{9.8} = 6.59 g/rad \tag{B.9}$$

where, z_{w_2} can be obtained from the A_2 matrix in equation (B.4) by $z_{w_2} = z_{\alpha_2} = A_{2_{(3,3)}} = -0.359$, and U_e in this case equal to 180m/s.

B.2 Reduced Order of Motion Equations

By decoupling the full order equations of longitudinal motion (B.1), the reduced order equations of motion of aircraft short period mode can be obtained by,

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -0.4390 & -0.3940 \\ 1.0019 & -0.3590 \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} 2.1595 \\ 0.0589 \end{bmatrix} \delta_e \tag{B.10}$$

The reduced order transfer functions can be given as follows, by taking the Laplace transform of equation (B.10):

$$\frac{q(s)}{\delta_e(s)} = \frac{2.1595(s + 0.3482)}{(s^2 + 0.7979s + 0.5523)} \tag{B.11}$$

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{0.058889(s + 37.18)}{(s^2 + 0.7979s + 0.5523)} \tag{B.12}$$

The longitudinal stability characteristics of the short period mode are therefore,

$$\zeta_s = 0.537, \qquad \omega_s = 0.743 rad/s; \tag{B.13}$$

B.3 Control Law of C_2

The pole replacement routine is performed as described in sections 3.5.1, 3.5.2 and 3.5.3. Before C_2 can be designed, the initial design decisions of the desirable locations of the short period poles on the s-plane are made. The target region $\mathcal{F}_2$ required to be covered by C_2 is with less airspeed and higher altitude than $\mathcal{F}_1$. According to the handling qualities assessment in Fig. 5.2, the aircraft in less airspeed and higher altitude operating conditions, like $\mathcal{F}_2$, requires more damping to reduce the closed loop resonance as well as less response speed and settling time. Comparing with the design decisions made for C_1 design in section 3.5.2, $\zeta_{s_{r1}} = 0.75$, $\omega_{s_{r1}} = 1.9$ and $T_{lag_{r1}} = 1.8$, the stability characteristics of the augmented aircraft by C_2 on $EP_{(8500,180)}$, are adjusted into $\zeta_{s_{r2}} = 0.8$, $\omega_{s_{r2}} = 1.7$ and $T_{lag_{r2}} = 1.5$.

The required closed loop characteristic polynomial of C_2 is thus given by,

$$\Delta(s) = (s + 1.5)(s^2 + 2.72s + 2.89) \tag{B.14}$$

Hence, the control law of C_2 is given by,

$$\delta_e = -\begin{bmatrix} 1.6755 & -3.3302 & 5.7649 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + 3.8433 \cdot q_d \tag{B.15}$$

Substituting equation (B.15) into equation (B.10) the reduced order closed loop state equation is given by,

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{\epsilon}_q \end{bmatrix} = \begin{bmatrix} -4.057 & 6.797 & -12.45 \\ 0.9033 & -0.1628 & -0.3395 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \epsilon_q \end{bmatrix} + \begin{bmatrix} 8.299 \\ 0.2263 \\ -1 \end{bmatrix} q_d \tag{B.16}$$

The reduced order augmented aircraft transfer functions are obtained as follows, by taking the Laplace transform of equation (B.16).

$$\frac{q(s)}{q_d(s)} = \frac{8.2995(s+0.3482)}{(s^2+2.72s+2.89)} \tag{B.17}$$

$$\frac{\alpha(s)}{q_d(s)} = \frac{0.22633(s+37.18)}{(s^2+2.72s+2.89)} \tag{B.18}$$

$$\frac{\epsilon_q(s)}{q_d(s)} = \frac{-(s-5.579)}{(s^2+2.72s+2.89)} \tag{B.19}$$

For the full order of equations of motion, the C_2 control law is given by,

$$\delta_e = -\begin{bmatrix} k_q & k_{V_{TAS}} & k_\alpha & k_\theta & k_{\epsilon_q} \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + m q_d \tag{B.20}$$

$$= -\begin{bmatrix} 1.6755 & 0 & -3.3302 & 0 & 5.7649 \end{bmatrix} \begin{bmatrix} q \\ V_{TAS} \\ \alpha \\ \theta \\ \epsilon_q \end{bmatrix} + 3.8433 \cdot q_d \tag{B.21}$$

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Appendix C

B747 AIRCRAFT MODEL-FTLAB747v6.5

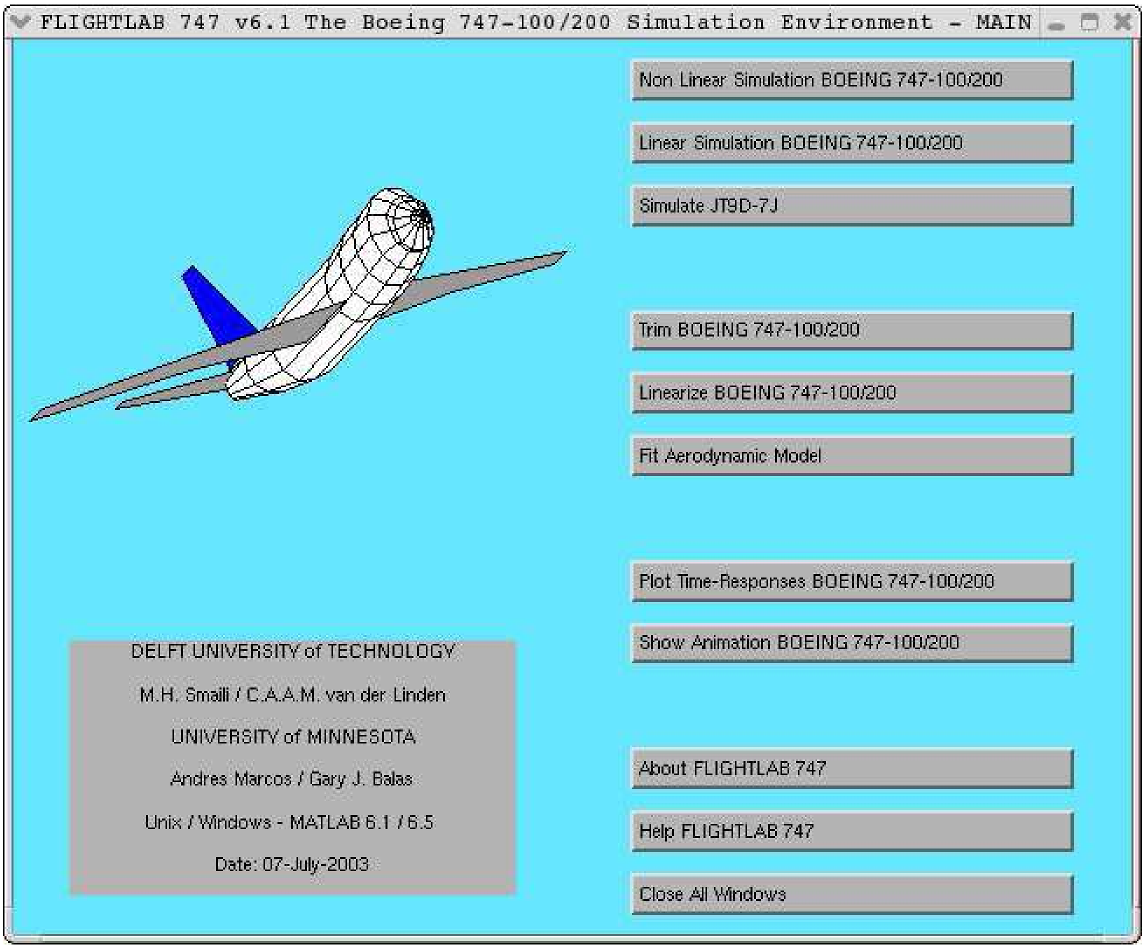

Figure C.1: FTLAB747v6.5 main menu screen

FTLAB747v6.5 is a very powerful aircraft simulation and analysis tool. Based in Matlab/Simulink, the program can be used for simulation and design respectively, and offers a wide array of analysis tools, such as trim, linearisation, flight visualization and fault detection. In order to take full advantage of this program, much more work is needed to learn the program. It is recommended to read the DASMAT manual [vdL96], and the FTLAB747v6.5 manual in Chapter 6 of [EB03], where the trim and linearization routines of FTLAB747v6.5 are presented through a step by step description of the user-supplied data and screen displays. For a specified aircraft configuration at a given operating point, a steady-state flight-condition is generated in aid of the trimming and linearization tools of this program. The original trimming and linearization tools are provided in the MATLAB M-scripts 'trim_ac.m' and'lin_ac.m' respectively. The files 'trim_ac_Modified' and 'lin_ac_Modified' is provided with this thesis, which indicates the change made to the origin software to meet the requirements of trimming and linearizing the aircraft model over the flight envelope in this project for applying, and also shows the inherent trimming and the linearizing routine procedure of the software for analysis.

Each observation group has a different number and type of output parameters. For actual broad applications, observation outputs are provided by selecting the specific parameters desired in the operating shell, a further description to the individual observations is given in reference [vdL96] and [EB03]. The symbol of the parameters chosen in this thesis are listed in Table C.1.

Table C.1: Trimming and linearisation Parameters

Symbols	Parameters	Units
delta_c	δ_c Column deflection	rad
delta_w	δ_w Wheel deflection	rad
delta_p	δ_p Pedal deflection	rad
PLA	engine power lever angle	rad
delta_stab	δ_{stab} Stablilizer knob	rad
delta_sbh	δ_{sbh} speedbrake handle	rad
delta_fh	δ_{fh} flap handle	rad
gear	the gear control	rad
delta_eil	δ_{eil} left inboard elevator	rad
delta_eir	δ_{eir} right inboard elevator	rad
delta_eol	δ_{eol} left outboard elevator	rad
delta_eor	δ_{eor} right outboard elevator	rad
delta_ail	δ_{ail} left inboard aileron	rad
delta_air	δ_{air} right inboard aileron	rad
delta_aol	δ_{aol} left outboard aileron	rad
delta_aor	δ_{aor} right outboard aileron	rad
delta_ih	δ_{ih} horizontal stabilizer	rad
delta_ru	δ_{ru} upper rudder	rad
delta_rl	δ_{rl} lower rudder	rad
T_n	engine thrust	N
x	x aircraft state variables	-
xdot	$\dot{x}$ aircraft state derivatives	-

pbdot	$\dot{p_{body}}$ roll acceleration about body X-axis	rad/s^3
qbdot	$\dot{q_{body}}$ pitch acceleration about body Y-axis	rad/s^3
rbdot	$\dot{r_{body}}$ yaw acceleration about body Z-axis	rad/s^3
VTASdot	$\dot{V_{TAS}}$ time derivative of true airspeed	m/s^2
alphadot	$\dot{\alpha}$ angle of attack rate	rad/s
betadot	$\dot{\beta}$ angle of sideslip rate	rad/s
phidot	$\dot{\phi}$ roll attitude rate	rad/s
thetadot	$\dot{\theta}$ pitch attitude rate	rad/s
psidot	$\dot{\psi}$ heading rate	rad/s
hedot	$\dot{h_e}$ geometric altitude rate	m/s
xedot	$\dot{x_e}$ horizontal ground speed along earth X-axis	m/s
yedot	$\dot{y_e}$ horizontal ground speed along earth Y-axis	m/s
yair	y_{air} airdata parameters	-
yacc	y_{acc} acceleration parameters	g
yfp	y_{fp} flight-path-related parameters	-
ys	y_s energy-related tems	-
ypqr	y_{pqr} angular axis velocities	rad/s
yuvw	y_{uvw} linear axis velocities	m/s
yuvwdot	$y_{\dot{u}\dot{v}\dot{w}}$ linear axis velocity derivatives	m/s^2
ydl	y_{dl} dimensionless axis velocities	-
yabh	$y_{\alpha\beta h}$ sensor measurements	-
yCaero	$y_{C_{aero}}$ aerodynamic force and moment coeffieients	-
yFMaero	$y_{FM_{aero}}$ aerodynamic forces and moments	-
yCgust	y_{C_g} force and moment coefficients due to turbulence	-
yFMgust	y_{FM_g} force and moments due to turbulence	-
yCt	y_{C_t} Propulsion force and moment coefficients	-
yFMt	y_{FM_t} propulsion forces and moments	m/s
ygrav	y_{gr} gravity forces	N

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Appendix D

MATLAB Program

The aircraft model itself is implemented in MATLAB, incorporating a number of user-defined functions, as well as existing blocks from the standard Simulink. Initialization and post-processing MATLAB scripts and functions are included for trim and linearisation of the model. the definition of all input parameters. is provided.

This appendix includes a description of the provided MATLAB initialization and post-processing scripts and functions (including a definition of the required inputs and outputs); a description of the required workspace variables and their correct format; a brief introduction to the provided Simulink models and libraries; and an overview of the Simulink model subsystems. Additional documentation is provided within the MATLAB program.

D.1 Gain Scheduling Factor Calculation

```
%==================================================================
% GAIN SCHEDULING FACTOR CALCULATIONS
%==================================================================
%
% Script Name :  schedule.m
% Function Name :  alpha = schedule(h,v)
%
% Purpose :  Calculate the gain scheduling factor alpha against the
% flight conditions at altitude, h, and airspeed, v.
%
% Inputs to the program :
%
% h = Altitude [m]
% v = Airspeed [m/s]
%
% Outputs of the program:
```

```
%
% alpha = gain scheduling factor [-]
%
%
% Subroutines used :
%
% griddata:  ZI = griddata(x,y,z,XI,YI,'linear') fits a surface of the
% form z = f(x,y) to the data in the (usually) nonuniformly spaced
% vectors (x,y,z).  'griddata' interpolates this surface at the points
% specified by (XI,YI) to produce ZI. The surface always passes through
% the data points.  XI and YI usually form a uniform grid (as produced
% by meshgrid).  The specified interpolation method 'linear':
% Triangle-based linear interpolation.
% %=================================================================


function alpha=schedule(h,v)

% Identify the region of the flight envelope F.
% ---------------------------------
if (((h <=12200)&&(h>=5000)&&(((68.5*v-4727-h)>0 ) ||((126.96*v-13963.76-h)>0))
&&...  ((343.43*v-80791-h)<=0)&&((182.67*v-40118.67-h)<0)) =1)
% The input flight conditions are not located in the Flight Envelope F.
alpha=NaN;


% Identify the interpolation region Fc12
% ---------------------------------

elseif ((5000<=h<=12200)&& ((58.33*v-6666.7-h)<=0)&&((84.59*v-10226-h)>=0)
)

h1=5000;V1=180; h2=12190;V2=265; h3=8500;V3=260; h4=5000;V4=200;
H0=[h1,h2,h3,h4]; V0=[V1,V2,V3,V4];
% identify the boundary of the interpolation.
alpha0=[1 1 0 0];
% max and min alpha at corners of interpolation
alpha=griddata(H0,V0,alpha0,h,v);
% Calculate the gain scheduling factor.


% Identify the region Fc2
% ---------------------------------


elseif ((5000<=h<=12200)&&(((68.5*v-4727-h)>=0 ) ||((126.96*v-13963.76-h)>=0))
&&...  ((84.59*v-10226-h)<=0))
```

```
% C2=[kq,kalpha,Keq,m]

alpha=1;


% Identify the region Fc1
% ---------------------------------

elseif((5000<=h<=8510)&& ((58.33*v-6666.7-h)>0 ) &&((343.43*v-80791-h)<0)
&&...  ((182.67*v-40118.67-h)<0))
% C1=[kq,kalpha,Keq,m]

alpha=0;

else
alpha=NaN;
% error('Inputs are out of the range.')

end

end


%======================================================================
```

D.2 Interpolation

```
%=====================================================================
% INTERPOLATION
%=====================================================================
%
% Script Name :  interpolation.m
%
% Purpose :  Schedule the gain matrices according to the scheduling
% parameters (altitude h, and airspeed, v).
%
% Inputs to the program :----------
%
% Outputs of the program:----------
%
% sub program :  schedule.m
%
%====================================================================
close all
```

```
clear
h=[5000:50:12200];
% Altitude.

v=[140:1:265];
% Airspeed.

K1=[0.73 -1.67 2.87 1.60];
% Gain matrix of Controller C1.

K2=[1.68 -3.33 5.76 3.84];
% Gain matrix of Controller C2.

for i=1:size(h,2)
for j=1:size(v,2)
ALPHA(i,j)=schedule(h(i),v(j));
% Implementing Function schedule.m .

K(:,:,i,j)=K1*ALPHA(i,j)+K2*(1-ALPHA(i,j));
% The gain scheduling strategy.

end
end


%=====================================================================
```

D.3 The Neal and Smith Criterion Assessment over the Flight Envelope

```
%========================================================================
% The Neal and Smith Criterion Assessment over the Whole Flight Envelope
%========================================================================
%
%
%==============
% INPUT DATA:
%==============
%
%
% altitude = Altitude [m]
% V0 = Airspeed [m/s]
```

```
% Vco = Cross over velocity [m/s]
% l = cg to pilot distance [m]
% lag = The integral lag time constant [-]
% k = Gain Matrix [kq ka kcq] [rad/rad/s rad/rad/s rad/rad/s]
% BW= 1.5 [rad/s]
%
%==========================
% State Space Matrices
%==========================
%
% Alin,Blin,Clin,Dlin :the full order system.
% As,Bs,Cs,Ds :  the reduced order system.
%
%
%==========================
% Transfer Function
%==========================
% Gkm is reduced order of augmented aircraft model.
% G is full order of augmented aircraft model.
% Gs is reduced order transfer function of basic aircraft model.
%
%
%
% Subroutines used :
%
% importfile.m
% createfigurens.m
%=======================================================================
%

altitude=[5000,5500,6000,6500,7000,7500,8000,8500,9000,9500,10000,10500,11000];
V0=[160,150,170,180,190,200,210,220,230,241,250,260];

for i = altitude
for j = V0
importfile(['model_',num2str(i),'_',num2str(j),'.lin']);


% short period mode
%===============================================================

g=9.8;
V=Alin(5,4); %the current airspeed.
Vco=122; % m/s
l=30; % m,the distance from the pilot station to the 25% MAC as found in
NASA-CR-1756.
```

```
% Reduced Order State Space Matrix of basic aircraft
%-------------------------------------------------------------------------
As=[Alin(1,1) Alin(1,3) 0; Alin(3,1) Alin(3,3) 0; 1 0 0 ];
Bs=[Blin(1,2); Blin(3,2); 0];
Cs=eye(3,3);
Ds=zeros(3,1);
Gs=ss(As,Bs,Cs,Ds); % Reduced order transfer function of basic aircraft
model.

% Designing the Gain Matrix K
%--------------------------------------------------------------------------
A1=As-Bs*k;
Gclos=ss(A1,Bs,Cs,Ds); % augmented with k.
zpk(Gclos);

% Designing the Feedforward Gain Matrix M
%--------------------------------------------------------------------------
N=[0;0;-1];
kcq=k(1,3);
m=kcq/lag;
B1=Bs*m+N;
Gkm=ss(A1,B1,Cs,Ds);% reduced order of augmented aircraft model.



% controller checked in full order transfer function.
%--------------------------------------------------------------------------
A=[Alin(1:4,1:4),zeros(4,1);1,0,0,0,0];
B=[Blin(1:4,2);0];
N=[0;0;0;0;-1];
K=[k(1,1) 0 k(1,2) 0 k(1,3)];
M=m;
A1=A-B*(K);
B1=B*M+N;
C=eye(5,5);
D=zeros(5,1);
G=ss(A1,B1,C,D);% augmented full order aircraft transfer function.


% THE NEAL AND SMITH CRITERION ASSESSMENT
%=========================================================================


% Frequency response of aircraft plus pilot delay
%-------------------------------------------------------------------------
```

```
s=tf('s');


[c,d] = pade(0.3,2);
u1=tf(c,d);
de99=minreal(zpk(Gkm(1,1))/s)*u1;
%closed loop Frequency response of aircraft plus pilot delay - 'pade*Gkm'.

h=zpk(Gs(1,1))/s*u1;
% open loop Frequency response of aircraft plus pilot delay- 'pade*Gs'.



[mag1,phase1]=bode(h,1.5);


[mag2,phase2]=bode(de99,1.5);
% Obtain the maginitude and Phase from the Bode Chart when frequency is
1.5 rad/s.

co90=-130.7;
% open loop phase is -130.7deg when closed loop phase is -90deg, in reference
to Flying Qualities and Flight Control Lecture Notes.

ga90=-3.78;
% open loop gain is -3.78dB when closed loop gain is -3dB, in reference
to Flying Qualities and Flight Control Lecture Notes.

Xcriopen=co90-phase1+360
% the required further open loop compensation of phase lead, in order to
bring the 1.5 rad/s point onto the -90 deg closed loop phase boundary and
the maximum low frequency droop into coincidence with the -3 dB closed
loop gain boundary.


co1=sin(Xcriopen*pi/180);
% is used to calculate the parameters of the lag-lead network for basic
aircraft system.

Xcriclos=co90-phase2+360
co=sin(Xcriclos*pi/180);
% used to calculate the parameters of the lag-lead network for augmented
aircraft system.


% Pilot Phase Compensation
%-----------------------------------------------------------------------
```

```
BW=1.5; % Bandwidth
To2=((1+co1)/(1-co1))^0.5/BW;
To1=(1-co1)/(1+co1)*To2;
Ppo=tf([To2,1],[To1,1]);
% lag-lead network of Pilot phase compensation

Fullo=zpk(Ppo*h);
% Open-loop system with Pilot Phase Compensation

T2=((1+co)/(1-co))^0.5/BW;
T1=(1-co)/(1+co)*T2;
Pp=tf([T2,1],[T1,1]);
Full=zpk(Pp*de99);% Closed-loop system with Pilot Phase Compensation


%Fully phase compensated aircraft
%------------------------------------------------------------------------


[mag3,phase3]=bode(Fullo,1.5);


[mag4,phase4]=bode(Full,1.5);
% The required gain reduction in order to bring the 1.5 rad/s point onto
the -90 deg closed loop phase boundary and the maximum low frequency droop
to meet the -3 dB closed loop.
% gain boundary
gao=ga90-mag3; % open-loop
ga=ga90-mag4; % closed-loop


Kpo= 10^(gao/20);
Kp= 10^(ga/20);
%Gain in dB=20log10(gain ratio)

Full1=zpk(Kp*Pp*de99);
% the basic aircraft system compensated by the full pilot model.

Full1o=zpk(Kpo*Ppo*h);
% the augmented aircraft system compensated by the full pilot model.

Full2=feedback(Full1,1);
% Full2 is the closed-loop of Full1.

Full2o=feedback(Full1o,1);
% Full2o is the closed-loop of Full1o.
```

```
[mag,phase]=bode(Full2o);
res_o=max(20*log10(mag))
% Resonance peak of the basic aircraft system compensated by the full pilot
model Full1.


[magc,phasec]=bode(Full2);
res_c=max(20*log10(magc))
% Resonance peak of the basic aircraft system compensated by the full pilot
model Full2.

% PLOTTING
%=========================================================================


% Boundaries
%------------------------------------------------------------------------
Level1_phase=[-30,-15,15,43,50];
Level1_res=[-2,3,3,1.8,-2];
Level3_phase=[-40,0,80];
Level3_res=[11,11,7];
naut_phase=[0,0];
naut_res=[-2,16];
createfigurens(1,1,2,2,3,3)
title(['model-',num2str(i),'-V',num2str(j)]);
plot(Level1_phase,Level1_res,'LineWidth',2)
hold on;
plot(Level3_phase,Level3_res,'r','LineWidth',2)
hold on;
plot(naut_phase, naut_res,'c')

% POINTS
%------------------------------------------------------------------------
hold on;
plot(Xcriclos,res_c,'bo',Xcriopen,res_o,'rx','LineWidth',2)
grid on;
end
end
%========================================================================
```

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References

[ABB02] N. Aouf, B. Boulet, and R. Botez. A gain scheduling approach for a flexible aircraft. In *Proceedings of the American Control Conference*, pages 4439–4442, Anchorage AK, May 2002.

[AEJ08] N. M. Ali, R. S. Ehsan, and M. R. Jahed. Aircraft controller design to enhance longitudinal maneuvers. In *Proceedings of the 27th Chinese Control Conference*, pages 258–264, Kunming,Yunnan, China, July 2008.

[And10] S. P. Andrews. *Modelling and Simulation of Flexible Aircraft: Handling Qualities with Active Load Control*. PhD thesis, Cranfield University, July 2010.

[Ano80] Anon. Military specification-flying qualities of piloted aircraft. Technical Report MIL-F-8785, Department of Defense Military Specifications and Standards, 1980.

[Ano90] Anon. Department of defense interface standard-flying qualities of piloted airplanes. Technical Report MIL-STD-1797A, Department of Defense Military Specifications and Standards, January 1990.

[Ano94] Anon. Federal aviation regulations - part 25, subpart b - flight. Technical report, Federal Aviation Administration, United States Department of Transportation,, 1994.

[ARS01] M. Attar, K. Rosenberg, and H. Smaili. Affordable digital flight control system for small commercial aircraft - adfcs. *Air & Space Europe*, 3(3-4):184–187, May-August 2001.

[B74] http://www.airliners.net/photo/USA—Air/Boeing-E-4B-(747-200B)/1885779/L/.

[B7407] 747 specifications. Technical Report Retrieved 16, The Boeing Company, December 2007.

[BP02] D. Bates and I. Postlethwaite. *Robust Multivariable Control of Aerospace Systems*. Delftse University Press, January 2002.

[Cas03] H. V. D. Castro. *Flying and Handling Qualities of a Fly-by-wire Blended-Wing-Body Civil Transport Aircraft*. Phd thesis, Cranfield University, December 2003.

[Chu10] S Chumalee. *Robust Gain-scheduled H_∞ Control for Unmanned Aerial Vehicles*. PhD thesis, Cranfield University, 2010.

[Coo07] M. V. Cook. *Flight Dynamics Principles*. Elsevier, Jordan Hill, Oxford, second edition, 2007.

[Coo10a] M. V. Cook. Flying qualities and flight control lecture notes. Cranfield University, February 2010.

[Coo10b] A. K. Cooke. Aircraft performance, stability and control lecture notes. Cranfield University, October 2010.

[CS206] Certification specification for large aeroplanes. Technical report, European Aviation Safety Agency (EASA), October 2006.

[EB01] A. M. Esteban and G. J. Balas. Linear parameter varying modeling of the Boeing 747-100/200 longitudinal motion. In *AIAA Guidance, Navigation, and Control Conference and Exhibit*, pages 1–11, Montreal, Canada, August 2001.

[EB03] A. M. Esteban and G. J. Balas. A Boeing 747-100/200 aircraft fault tolerant and fault diagenostic benchmark. Technical report, Aerospace Engineering and Mechanics Department of University of Minnesota, June 2003.

[FFN03] A. Fujimori, Terui F., and P. N. Nikiforuk. Flight control design using ν-gap metric and local multi-objective gain-scheduling. In *AIAA Guidance, Navigation, and Control Conference and Exhibit, Austin, Texas*, volume AIAA 2003-5414, pages 1–9, August 2003.

[Gar97] Sanjay Garg. A simplified scheme for scheduling multivariable controllers. *IEEE Control Systems*, (0272-1708):24–30, August 1997.

[Gib99] John Campbell Gibson. *Development of a methodology for excellence in handling qualities design for fly by wire aircraft*. Phd thesis, Delft University of Technology, February 1999.

[GZ94] P. G. Gonsalves and G. L. Zacharias. Fuzzy logic gain scheduling for flight control. In *IEEE*, number 0-7803-1896-X/94, pages 952–957, Charles River Analytics Inc. 5 Wheeler St. Cambridge, UA 02138, 1994.

[HJ72] R. K. Heffley and W. F. Jewell. Aircraft handling qualities data. Contractor Report NASA CR-2144, National Aeronautices and Space Administration, Washington, D.C. 2056, December 1972.

[JAL08] J. Jang, A. M. Annaswamy, and E. Lavretsky. Adaptive control of time-varying systems with gain-scheduling. In *American Control Conference*, pages 3416–3421, Westin Seattle Hotel, Seattle, Washington, USA, June 2008.

[LL00] D. J. Leith and W. E. Leithead. Survey of gain-scheduling analysis and design. *International Journal of Control*, 73(11):1001–1025, 2000.

[LP98] G. P. Liu and R. J. Patton. *Eigenstructure Assignment for Control System Design*. Wiley-Blackwell, Febuary 1998.

[LWY+08] C. Li, S. Wang, Z. Yue, J. Yu, and H. Wang. Anti-windup nonlinear PID controller design and its application to winged missile control system. In *Proceedings of the 27th Chinese Control Conference*, pages 377–379, Kunming, Yunnan, China, July 2008.

[MAS08] I. Moir and A. Allan Seabridge. *Aircraft Systems: Mechanical, electrical, and avionics subsystems integration*. John Wiley & Sons, third edition, 2008.

[Nel98] R. C. Nelson. *Flight Stability and Automatic Control*. McGraw-Hill, second edition, 1998.

[NRR93] R. A. Nichols, R. T. Reichert, and W. J. Rugh. Gain scheduling for h_∞ controllers: A flight control example. *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, 1(2):69–79, June 1993.

[OB01] M. Oosterom and R. Babuska. Fuzzy gain scheduling for flight control laws. In *Proceedings of the IEEE International Fuzzy Systems Conference*, pages 716–719, 2001.

[OBB02] M. Oosterom, P. Bergsten, and R. Babuska. Fuzzy gain-scheduled h_∞ flight control law design. In *AIAA Guidance, Navigation and Control Conference and Exhibit , Monterey, California*, number AIAA-2002-4748, pages 5–8, August 2002.

[OSBV00] M. Oosterom, G. Schram, R. Babuska, and H. B. Verbruggen. Automated procedure for gain scheduled flight control law design. In *AIAA Guidance, Navigation and Control Conference and Exhibit*, number AIAA-2000-4253, pages 14–17, August 2000.

[Rad04] N.G.M. Rademakers. Control of a tailless fighter using gain- scheduling. Technical report, Eindhoven University of Technology, January 2004.

[RDLB03] T. S. Richardson, P. M. Davison, M. H. Lowenberg, and M. Bernardo. Control of nonlinear aircraft models using dynamic state-feedback gain scheduling. *American Institute of Aeronautics and Astronautics*, (AIAA-2003-5503):1–9, August 2003.

[RH71] C. Rodney Hanke. The simulation of a large jet transport aircraft volume 1: Mathematical model. Contractor Report NASA CR-1756, National Aeronautics and Space Administration, Washington, D.C. 2056, March 1971.

[RLBC06] T. Richardson, M. Lowenberg, M. Bernardo, and G. Charles. Design of a gain-scheduled flight control system using bifurcation analysis. *Journal of Guidance, Control, and Dynamics*, 29(2):444–455, MarchApril 2006.

[RLJD07] T. Richardson, M. Lowenberg, C. Jones, and A. Dubs. Dynamic gain scheduled control of a hawk scale model. *Aeronautical Journal*, 111(1121):461–469, July 2007.

[RS99] W. J. Rugh and J. S. Shamma. Research on gain scheduling. *Automatica*, pages 1401–1425, December 1999.

[SA90] J. S. Shamma and M. Athans. Analysis of gain scheduled control for non-linear plants. In *IEEE Transactions on Automatic Control*, number 35 in 8, pages 898 – 907, 1990.

[SB92] S. M. Shahruz and S. Behtash. Design of controllers for linear parameter-varying systems by the gain scheduling technique. *Journal of Mathematical Analysis and Applications*, 168:195–217, July 1992.

[SG07] J. Y. Shin and I. Gregory. Robust gain-scheduled fault tolerant control for a transport aircraft. In *Proceedings of the 16th IEEE International Conference on Control Applications Part of IEEE Multi-conference on Systems and Control*, pages 1209–1214, Singapore, October 2007.

[Ste01] Anthony J Steer. *Flight Control for Advanced Supersonic Transport Aircraft Handling Quality Design.* Phd thesis, Cranfield University, February 2001.

[TEM66] H. N. Tobie, E. M. Elliot, and L. G. Malcom. A new longitudinal handling qualities criterion. In *National Aerospace Electronics Conference (NAEC)*, page 9399., Dayton, Ohio, May 1966.

[TR71] T.P.Neal and R.E.Smith. A flying qualities criterion for the design of fighter flight control systems. *Journal of Aircraft*, 8(10):803809, October 1971.

[vdL96] C. A. A. M. van der Linden. DASMAT-Delft university aircraft simulation model and analysis tool. Technical Report LR-781, Delft University of Technology, Faculty of Aerospace Engineering, Delft University of Technology, Faculty of Aerospace Engineering, P. Q. Box 5058, 2600 GB, Delt, The Netherlands., August 1996.

[WSG03] B. Walker, A. Saraf, and A. Godbole. Robust flight control for coordinated turns. In *AIAA Guidance, Navigation, and Control Conference and Exhibit, Austin, Texas*, number AIAA 2003-5553, August 2003.

[Zhu06] J. Zhu. A survey of advanced flight control theory and application. In *Proceedings of IMACS Multiconference on "Computational Engineering in Systems Applications"(CESA)*, pages 655–658, Beijing, China., October 2006.

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