

# A Likelihood-Partitioned Bayesian Framework for Lithium Sulfur Battery State Discharging of Charge Estimation

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**Abstract**—Lithium sulfur (Li-S) batteries are promising energy storage devices and alternative to lithium-ion (Li-ion) batteries in electric grid and vehicle applications. However, compared to Li-ion, the discharge voltage of Li-S is much complex and nonlinear. This results a challenging state of charge (SoC) estimation problem while Li-S is discharging. For such a problem, the traditional extended Kalman filter fails to provide accurate SoC. Therefore, this paper proposes a novel likelihood partitioned Bayesian filtering (LPBF) framework and its linearized version for SoC estimation of discharging Li-S battery cell. Though both traditional EKF and linearized LPBF use a prediction error minimization based equivalent circuit network (ECN) parameterization, the LPBF uses a partitioned ECN parameterization. The portioned models result two likelihoods, whereas the EKF uses a single state-space model throughout discharge from 100 percent SoC to zero SoC. With experiment data obtained at two different temperature conditions, numerical simulation results compare both EKF and linearized LPBF based SoC estimators. Simulation results show that the LPBF's accuracy is impressive, about 97 percent, for considered dynamic load current, operating temperature and uncertain initial SoC conditions.

**Keywords**— Battery management systems, Bayesian filters, Lithium-sulfur battery, Likelihood probability density, Nonlinear functions, State of charge

## I. INTRODUCTION

Recently, the lithium sulfur (Li-S) batteries have attracted attention of many researchers [1,2,3] as the Li-S are high energy density solid-state batteries, environmentally friendly, safe, and low cost as they use toxin-free and abundant sulfur cathode. These promising features recommend that the Li-S batteries can be alternative to lithium-ion (Li-ion) batteries for electric grid and vehicle applications. In these applications, the battery management systems (BMSs) are essential in order to ensure the safe operation, control and energy optimization. To achieve these objectives the battery state of charge (SoC) plays significant role. No sensor measures SoC directly. BMS requires appropriate SoC estimation.

The SoC estimation of discharging Li-S batteries is a difficult and complex problem than the tradition Li-ion batteries. Because the discharge open circuit voltage (Voc) of Li-S is quite different from Li-ion and has deep-dive high- and flatten low plateau [4]. For such SoC problem estimation, literature has

proposed traditional Bayesian filters [2,6], and machine learning approaches [3,5]. These estimators developed based on a first-order equivalent circuit network (ECN) model. Though they had different ECN parameterization the SoC estimation was common goal.

The SoC accuracy of all methods proposed in the literature was good. However, to improve the accuracy the adaptive neuro-fuzzy inference system (ANFIS) [3] and support vector machine (SVM) [5] hybridized with a coulomb counting (SoC) estimator. The unknown initial SoC is a major problem of CC based SoC estimator. Furthermore, the ANFIS [3] and SVM [5] did not use voltage of battery and both trains large data of estimated parameters. Contrast to [3, 5], the filters recursively estimated SoC using voltage measurement. Filters are quite power to handle sensor and parameter uncertainties. The nonlinear filter based SoC estimators [2] developed traditional Bayesian framework [7]. However, to achieve good accuracy the filters [2,6] used a partial sinusoidal function (PSF). These PSF joined the polynomial of Voc and internal resistance of Li-S battery cell. The PSF requires a parameter tuning when temperature varies. This is an additional burden of Bayesian filters. Furthermore, the extended Kalman filter (EKF) based SoC estimators [2,6] poorly performed when initial SoC is unknown. These force further research on filtering-based Li-S battery SoC estimation.

This paper proposes a novel likelihood partitioned Bayesian filtering (LPBF) framework for SoC estimation of discharging Li-S battery cells. Unlike traditional Bayesian filter framework, the proposed LPBF uses two likelihood probability density function of measured voltage. Either one of these likelihoods is recursively updated based on the ECN parameters and a priori SoC. To show the difference between traditional and proposed, the traditional EKF and linearized LPBF are compared. Having experiment test of 2.07Ah Li-S cell [8], numerical simulation presents the performance of proposed SoC estimator. The SoC by linearized LPBF outperforms than the traditional EKF. Then, the linearized LPBF is used for SoC estimation of the same Li-S cell under different temperature. Results of LPBF under two temperature conditions show that LPBF is capability and reliability to use for different condition without PSF [2,6]. Robustness of proposed linearized LPBF against initial uncertainty is investigated by considering different initial estimated SoC and actual battery SoC. The linearized LPBF is

robust against initial SoC. The estimation accuracy is very good. The error in SoC estimation of linearized LPBF is less than 3 percent of SoC.

The rest of paper is organized as follows: section II formulates an ECN model based SoC estimation problem. Section III discusses the traditional EKF that developed using Bayesian filtering framework. The new LPBF and its linearized version are studied in IV. Results and discussions are given in V. section VI concludes this paper and provides a couple of feature work.

## II. SOC ESTIMATION PROBLEM

The SoC estimation problem is formulated by modelling Li-S battery cell as a first-order ECN model, as shown in Fig 1. In this Fig 1, the SoC denotes the state of charge and the  $V_p$  represent the discharge polarization voltage of cell. Considering the two state variables, the problem for SoC estimation can be formulated by a state-space model as

$$x(k+1) = f(x(k), \theta(k), u(k)) + w(k) \quad (1)$$

$$y(k) = g(x(k), \theta(k), u(k)) + v(k) \quad (2)$$

Where  $x(k) = \begin{pmatrix} x_c(k) \\ V_p(k) \end{pmatrix}$  is the state vector and the state and measurement functions are, respectively, expressed as

$$f(x(k), \theta(k), u(k)) = \begin{pmatrix} x_{c,n}(k) - \frac{I_L(k)T_s}{C_B} \\ V_p(k) - T_s\Omega(x_c)V_{p,c}(t) + T_s\rho(x_c)R_{int}(x_c)\Omega(x_c)I_L \end{pmatrix} \quad (3)$$

and

$$g(x(k), \theta(k), u(k)) = V_{oc}(x_c) - V_p(k) - R_i(x_c)I_L. \quad (4)$$

The process noise  $w(k)$  is assumed as a Gaussian noise and expressed as

$$w(k) = \begin{pmatrix} \Delta x(k) \sim N(0, Q_x) \\ \Delta V_p \sim N(0, Q_v) \end{pmatrix}. \quad (5)$$

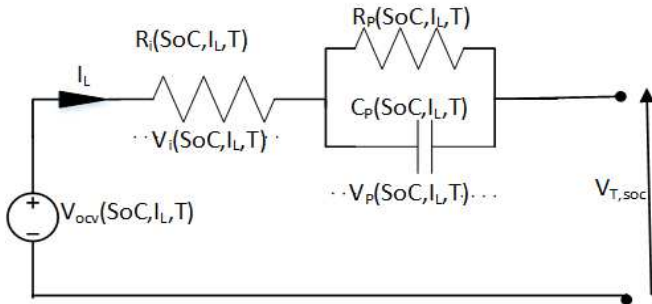


Fig.1. A first-order ECN model presents Li-S battery cell

The measurement noise  $v(k)$  is Gaussian noise with covariance,  $R_s$ . The  $w(k)$  and  $v(k)$  are the independent and identically distributed (i.i.d) Gaussian noises. The lumped parameters are obtained by prediction error method (PEM) of parameterization [4] as follow:

$$\Omega(\cdot) = \frac{1}{R_p(x_c(t), I_L, T)C_p(x_c(t), I_L, T)} \quad (4)$$

$$R_{int}(\cdot) = R_i(x_c(t), I_L, T) + R_p(x_c(t), I_L, T) \quad (5)$$

$$\rho(\cdot) = \frac{R_p(x_c(t), I_L, T)}{R_{int}(x_c(t), I_L, T)} \quad (6)$$

The  $\theta$  denotes battery parameters such as  $V_{oc}(\cdot)$ ,  $R_i(\cdot)$ ,  $R_p(\cdot)$  and  $C_p(\cdot)$ . As part of this paper contribution, these

parameters are partitioned as high and low plateau. The partitioned parameters result portioned likelihood functions, which will be studied in section IV. Contrast to likelihood partitioned filtering framework, the traditional Bayesian filtering framework is studied in the next section.

## III. TRADITIONAL BAYESIAN FRAMEWORK AND EKF

This section presents the tradition Bayesian filtering framework and its simplest nonlinear filter namely the EKF. The EKF uses PEM based ECN parameterization for SoC estimation of discharging Li-S battery cells. In this paper, the way parameterization with polynomial function for each parameter is tried with partition like Lithium-Ion cases. As the discharging Li-S behivous is quite different, the EKF fails to converge. The results will be studied in section V. Here the simple EKF algorithm without partitioned likelihood is studied follow:

### A. Traditional Bayesian filtering Framework

The traditional framework uses a priori probability distribution function (pdf) of state vector

$$p(x_s(k)|y_s(1:k-1)) = \int p(x_s(k)|x_s(k-1))p(x_s(k-1)|y_s(k-1))dx_s(k-1) \quad (7)$$

$$p(y_s(k)|y_s(1:k-1)) = \int p(y_s(k)|x_s(k), \theta_c(k-1))p(x_s(k)|y_s(1:k-1))dx_s(k) \quad (8)$$

The traditional filtering technique [7] evaluates the posteriori pdf  $\hat{x}_s(k|k)$  of state vector  $x_s(k)$  that conditioned on the available battery voltage measurement  $y_s(1:k) = \{y_s(i), i = 1, \dots, k\}$  as

$$p(x_s(k)|y_s(1:k)) = \frac{p(y_s(k)|x_s(k))p(x_s(k)|y_s(1:k-1))}{p(y_s(k)|y_s(1:k-1))} \quad (9)$$

$$\hat{\theta}(\hat{x}_c(k)) = [\hat{V}_{oc}(\hat{x}_c(k)), \hat{R}_{int}(\hat{x}_c(k)), \hat{\Omega}(\hat{x}_c(k)), \hat{\rho}(\hat{x}_c(k))] \quad (10)$$

where

$$\hat{V}_{oc}(\hat{x}_c(k)) = C_{oc,n,1}\hat{x}_c^n + C_{oc,n,2}\hat{x}_c^{n-1} + \dots + C_{oc,n}\hat{x}_c + C_{oc,n+1} \quad (11)$$

$$\hat{R}_{int}(\hat{x}_c(k)) = C_{int,1}\hat{x}_c^n + C_{int,2}\hat{x}_c^{n-1} + \dots + C_{int,n}\hat{x}_c + C_{int,n+1} \quad (12)$$

$$\hat{\Omega}(\hat{x}_c(k)) = C_{o,1}\hat{x}_c^n + C_{o,2}\hat{x}_c^{n-1} + \dots + C_{o,n}\hat{x}_c + C_{o,n+1} \quad (13)$$

$$\hat{\rho}(\hat{x}_c(k)) = C_{r,1}\hat{x}_c^n + C_{r,2}\hat{x}_c^{n-1} + \dots + C_{r,n}\hat{x}_c + C_{r,n+1} \quad (14)$$

In (11)-(14), the suffix n denotes the order of polynomial coefficient.

This framework recursively propagates priori and posteriori SoC estimation of Li-S battery when discharging from 100 to zero percent SoC . The nonlinear state and measurement functions can be approximated using various techniques such as analytical Taylor series approximation, sigma-point approximation and so on.

In this paper, the nonlinear functions are approximated using first order Taylor series approximation, which is also known as Jacobian, as it is a simple and widely used for nonlinear filtering problems. Thus, the traditional approach results a traditional extended Kalman filter (EKF). Fig 2 shows the traditional EKF, which propagates the priori and posteriori pdfs of  $x_c$ .

The traditional EKF algorithm for SoC estimation is summarized as follow:

$$p(x_s(k-1)|y_s(1:k-1)) \approx N(x_s(k-1); \hat{x}_s(k-1|k-1), P_c(k-1|k-1)) \quad (15)$$

$$p(x_s(k)|y_s(1:k-1)) \approx N(x_s(k); \hat{x}_s(k|k-1), P_c(k|k-1)) \quad (16)$$

$$p(x_s(k)|y_s(1:k)) \approx N(x_s(k); \hat{x}_s(k|k), P_c(k|k)) \quad (17)$$

This traditional SoC estimator can be compared to proposed novel approach, which will be studied in the next section.

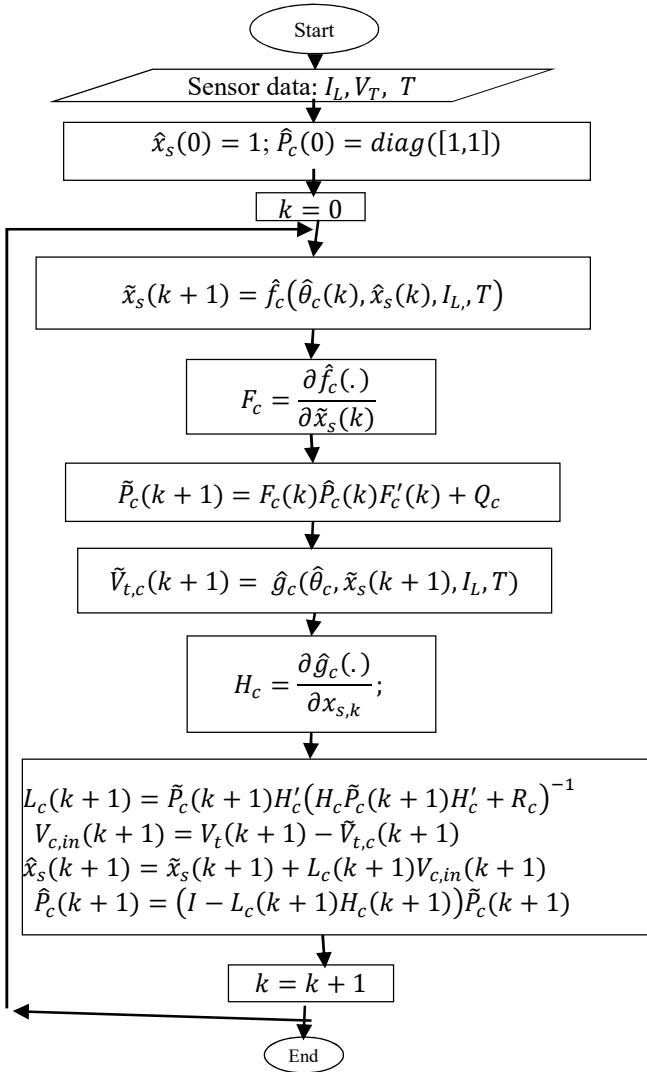


Fig. 2. The flowchart of EKF with traditional Bayesian framework.

#### IV. LIKELIHOOD-PARTITIONED BAYESIAN FRAMEWORK AND ITS LINEARIZED VERSION

Using partitioned high- and low -fitting models, two state-space models are used for formulate the LPBF. Fig 3 shows the flowchart of proposed linearized LPBF algorithm, which unlike traditional Bayesian framework [7] given in Fig.2. The LPBF computes two priori and likelihood pdfs while

estimating SoC of Li-S battery cell when fully charged or partially charged Li-S battery cell is discharging. A simple IF conditioned logical statement recursively propagates two priori and likelihoods. The theory and the concept of proposed LPBF are as follow:

$$\theta_c(k) = \begin{cases} \theta_{c,h}(k), & 1 \geq x_{s,1}(k) \geq x_j \\ \theta_{c,l}(k), & x_j \geq x_{s,1}(k) \geq 0 \end{cases} \quad (18)$$

where

$$\theta_{c,h}(k) = [V_{oc,h}(x_{s,1}) \quad R_{int,h}(x_{s,1}) \quad \Omega_h(x_{s,1}) \quad \rho_h(x_{s,1})]' \quad (19)$$

is high-plateau fitting models in the interval  $[1, x_j)$  that can be written in terms of estimated SoC,

$$\tilde{V}_{oc,h}(\hat{x}_{c,h}(k)) = C_{oc,h,1}\hat{x}_{c,h}^n + C_{oc,h,2}\hat{x}_{c,h}^{n-1} + \dots + C_{oc,h,n}\hat{x}_{c,h} + C_{oc,h,n+1} \quad (20)$$

$$\hat{R}_{int,h}(\hat{x}_{c,h}(k)) = C_{int,h,1}\hat{x}_{c,h}^n + C_{int,h,2}\hat{x}_{c,h}^{n-1} + \dots + C_{int,h,n}\hat{x}_{c,h} + C_{int,h,n+1} \quad (21)$$

$$\hat{\Omega}_h(\hat{x}_{c,h}(k)) = C_{o,h,1}\hat{x}_{c,h}^n + C_{o,h,2}\hat{x}_{c,h}^{n-1} + \dots + C_{o,h,n}\hat{x}_{c,h} + C_{o,h,n+1} \quad (22)$$

$$\hat{\rho}_h(\hat{x}_{c,h}(k)) = C_{r,h,1}\hat{x}_{c,h}^n + C_{r,h,2}\hat{x}_{c,h}^{n-1} + \dots + C_{r,h,n}\hat{x}_{c,h} + C_{r,h,n+1} \quad (23)$$

In (20)-(23), the subscript h denotes the high-plateau, and the rest of the notation represents as (24)-(28).

Like subset  $[1, x_j)$ , the fitting parameters for  $(x_j, 0]$  are obtained as follow.

$\hat{\theta}_l(\hat{x}_{c,l}(k)) = [\hat{V}_{oc,l}(\hat{x}_{c,l}(k)), \hat{R}_{int,l}(\hat{x}_{c,l}(k)), \hat{\Omega}_l(\hat{x}_{c,l}(k)), \hat{\rho}_l(\hat{x}_{c,l}(k))]'$  is low-plateau parameters that can be expressed in terms of estimated SoC as

$$\tilde{V}_{oc,l}(\hat{x}_{c,l}(k)) = C_{oc,l,1}\hat{x}_{c,l}^n + C_{oc,l,2}\hat{x}_{c,l}^{n-1} + \dots + C_{oc,l,n}\hat{x}_{c,l} + C_{oc,l,n+1} \quad (24)$$

$$\hat{R}_{int,l}(\hat{x}_{c,l}(k)) = C_{int,l,1}\hat{x}_{c,l}^n + C_{int,l,2}\hat{x}_{c,l}^{n-1} + \dots + C_{int,l,n}\hat{x}_{c,l} + C_{int,l,n+1} \quad (25)$$

$$\hat{\Omega}_l(\hat{x}_{c,l}(k)) = C_{o,l,1}\hat{x}_{c,l}^n + C_{o,l,2}\hat{x}_{c,l}^{n-1} + \dots + C_{o,l,n}\hat{x}_{c,l} + C_{o,l,n+1} \quad (26)$$

$$\hat{\rho}_l(\hat{x}_{c,l}(k)) = C_{r,l,1}\hat{x}_{c,l}^n + C_{r,l,2}\hat{x}_{c,l}^{n-1} + \dots + C_{r,l,n}\hat{x}_{c,l} + C_{r,l,n+1} \quad (27)$$

is low-plateau ECN parameterization which is fitting against  $(x_j, 0]$ . A conditional statement (18) results two priori pdfs of SoC as

$$p(x_s(k)|y_s(1:k-1)) = \begin{cases} p(x_s(k)|y_s(1:k-1), \theta_{c,h}(k-1)), & 1 \geq x_{s,1}(k) \geq x_j \\ p(x_s(k)|y_s(1:k-1), \theta_{c,l}(k-1)), & x_j \geq x_{s,1}(k) \geq 0 \end{cases} \quad (28)$$

and two likelihood pdfs as

$$p(y_s(k)|x_s(k)) = \begin{cases} p(y_s(k)|x_s(1:k-1), \theta_{c,h}(k-1)), & 1 \geq x_{s,1}(k) \geq x_j \\ p(y_s(k)|x_s(1:k-1), \theta_{c,l}(k-1)), & x_j \geq x_{s,1}(k) \geq 0 \end{cases} \quad (29)$$

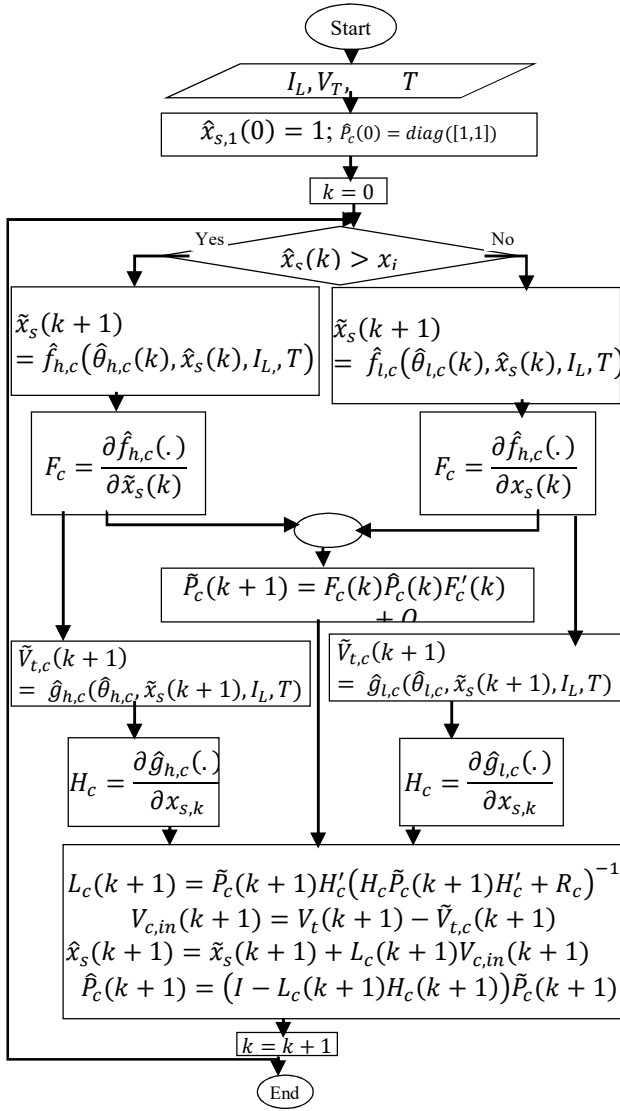


Fig. 3. Proposed linearized LPBF for SoC estimation.

In this paper, the nonlinear functions of high and low plateaus are approximated using a first order derivative of Taylor series approximation as shown in Fig 3. This proposed LPBF is simple and propagates two prior and likelihood pdfs recursively with IF logical condition. At  $k^{\text{th}}$  time instant, the IF condition on estimated SoC at  $(k-1)^{\text{th}}$  propagates any of these pdfs.

## V. SIMULATION RESULTS AND DISCUSSIONS

This section presents simulation results of LPBF SoC estimator Li-S cells. Test data of Li-S cells obtained with mixed-amplitude pulse current obtained at two different temperature at  $30^\circ\text{C}$  and at  $20^\circ\text{C}$  are considered. The results cover known initial SoC and unknown initial SoC conditions. For all simulation, the initial estimated error covariance is set to be  $P_c(0|0) = I_{2 \times 2}$

### A. EKF vs linearized LPBF for SoC estimation at $30^\circ\text{C}$ under known initial SoC

Fig 4 shows Li-S cell SoC comparison between the

traditional EKF and novel LPBF along with test data set. Subplots Fig 4(a) and Fig 4 (b) are the voltage of cell at  $30^\circ\text{C}$  discharging from full C, and mixed amplitude dynamic load current respectively. As initial SoC is known, the initial estimate state vector is set to  $\hat{x}_s(0|0) = [1, 0]^T$ . For both EKF and linearized LPBF, the process and measurement noise covariance are set to  $Q_c = \text{diag}[0.1 \times 10^{-7}, 0.1 \times 10^{-3} \text{V}^2]^T$ , and  $R_c = 0.000025 \text{V}^2$ , respectively. Under these conditions, the performances of EKF and LPBF are compared. The LPBF uses parameter given in Table III. The EKF have two different ECN parameterization; case I and case II. For case I, the coefficients of polynomials listed Table I are used, whereas Table II used for case II. Subplot 4(c) compares the EKF, LPBF and true for SoC normalization. The LPBF SoC estimator, shown by solid red-line of subplot 4(c), is an optimal as it is closely following the true SoC, black-dotted line, whereas the EKF did not. The blue dashed line and green-dashed-dotted line denote the case I and case II of EKF SoC estimator, respectively. These two lines are far away from the true SoC. Similar trend is observed in SoC Ah as shown in Fig 4(d).

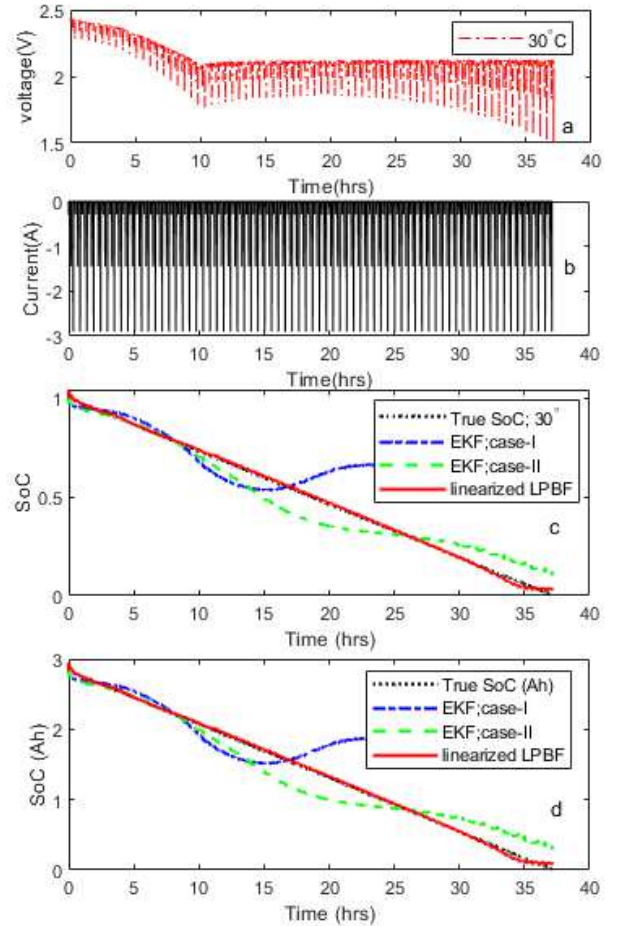


Fig. 4. SoC estimators at  $30^\circ\text{C}$  under known initial SoC condition: (a) Li-S cell voltage at  $30^\circ\text{C}$ , (b) battery discharge current (c) comparison between Linearized LPBF and EKF and true SoC (d) estimated SoC in Ah

B. Linearized LPBF at 20°C when initial estimated SoC is known and unknown.

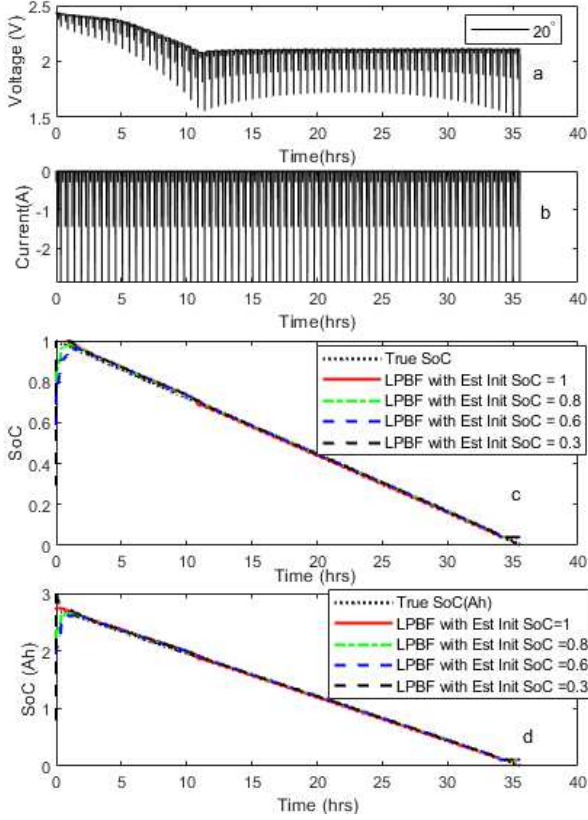


Fig. 5. Linearized LPBF SoC estimation at 20°C when estimated initial SoC is uncertain: (a) voltage; (b) current; (c) SoC estimation when initial estimated SoC is 1, 0.8, 0.6 and 0.3 (d) SoC in Ah

TABLE I

EKF CASE I : THE 4<sup>TH</sup>, 2<sup>ND</sup>, 1<sup>ST</sup> AND 1<sup>ST</sup> ORDER POLYNOMIAL COEFFICIENTS OF LI-S CELL 'S ECN PARAMETERS AT 30°C

$V_{oc}$ (V)	$R_{int}$ ( $\Omega$ )	$\rho$ (no unit)	$\omega$ (rad/s)
0.2947	-0.0012	-0.4350	0.0792
1.0784	-0.1260	0.5017	-0.0258
-1.3447	0.1816		
0.3678			
2.1006			

TABLE II

EKF CASE II : THE 6<sup>TH</sup>, 7<sup>TH</sup>, 3<sup>RD</sup> AND 3<sup>RD</sup> ORDER POLYNOMIAL COEFFICIENTS OF TYPE A CELL'S ECN PARAMETERS

$V_{oc}$ '(V)	$R_{int}$ ( $\Omega$ )	$\rho$ (no unit)	$\omega$ (rad/s)
-45.4553	-10.0168	-0.327	1.1611
124.621	45.6847	0.1058	-0.9011
-125.426	-748249	-0.0335	-0.0650
57.8445	56.6536	0.4173	0.1089
-12.3455	-21.1979		
1.0611	4.2467		
2.0968	-0.6991		
	0.2176		

TABLE III  
PARTITIONED PARAMETERS OF TYPE A CELL AT 30°C : POLYNOMIAL COEFFICIENT OF PARTITIONED ECN PARAMETERS

Plateau	$V_{oc}$ (V)	$R_{int}$ ( $\Omega$ )	$\rho$ (no unit)	$\omega$ (rad/s)
High	$[V_{oc,h}]' =$ [72.9503 -259.6135 341.4807 -195.5125 43.1183]	$[R_{int,h}]' =$ [2.001 -3.759 1.853]	$[\rho_h]' =$ [-0.1295 0.5687]	$[\omega_h]' =$ [1.0395 -0.7603]
low	$[V_{oc,l}]' =$ [-31.2310 69.3765 -60.1029 25.6103 -5.6128 0.5928 2.0974]	$[R_{int,l}]' =$ [173.7228 -515.0199 616.6895 -381.7758 129.5985 -23.1084 1.6510 0.1450]	$[\rho_l]' =$ [-2.1590 2.2718 -23.1084 1.6510]	$[\omega_l]' =$ [-1.3532 2.0927 -1.0609 0.1907]

TABLE IV

THE ECN PARAMETER FITTING MODEL OF LI-S CELL AT 20°C

Plateau	$V_{oc}$ (V)	$R_{int}$ ( $\Omega$ )	$\rho$ (no unit)	$\omega$ (rad/s)
High	$[V_{oc,h}]' =$ [108.1 -361.13 444.73 -238.18 47.03 1.88]	$[R_{int,h}]' =$ [1.07 -2.445 1.457]	$[\rho_h]' =$ [-0.0451 0.4892]	$[\omega_h]' =$ [0.41 -0.744 0.346]
low	$[V_{oc,l}]' =$ [-752.62 2085.66 -2392.87 1466.98 -517.42 105.21 -11.69 0.62 2.1]	$[R_{int,l}]' =$ [3.792 -8.212 6.428 -1.56 0.07346 0.1458]	$[\rho_l]' =$ [0.5087 -0.6572 0.6815]	$[\omega_l]' =$ [6.827 -7.422 1.685 0.140 -0.154 0.1441]

Fig 5 illustrates the LBPf's performance when the above same Li-S cell, but the temperature is 20°C . The red-line of Fig 5c and Fig5d shows the response of linearized LPBF when initial SoC is known, that the Li-S cell is discharging from 100 percent SoC and initial estimated SoC is also known. Fig 5 also illustrates the initial uncertainty by considering different initial estimated SoC, but actual battery SoC is known. The voltage and current of temperature at 20°C that given in Fig 5(a) and Fig 5(b), respectively, are used by LPBF. The ECN parameterization is given in Table IV. Like LPBF SoC estimation at 30°C, the LPBF at 20°C is accurate as shown in subplots Fig 5(c), which is SoC normalization, and Fig5(d), SoC in Ah. In both these plots, the redline denotes LPBF's SoC estimation when the initial estimated SoC is known,  $\hat{x}_c(0|0) = [1, 0]'$ , and it is closely following the true SoC, the black line. To simulate the SoC estimation performance when the initial estimated SoC is uncertain, different from the actual SoC, the LPBF runs under following four different initial estimated SoC;  $\hat{x}_s(0|0) = [1, 0]'$ ,  $\hat{x}_s(0|0) = [0.8, 0]'$ ,  $\hat{x}_s(0|0) = [0.6, 0]'$ , and  $\hat{x}_s(0|0) = [0.3, 0]'$ . The green-dashed line, blue-dashed-



dotted line and blue-dashed line of Fig 16 (c) denote the LPBF's response to  $\hat{x}_s(0|0) = [0.8, 0]'$ ,  $\hat{x}_s(0|0) = [0.6, 0]'$ , and  $\hat{x}_s(0|0) = [0.3, 0]'$ , respectively.

### C. Linearized LPBF at 20°C when both initial estimated SoC and actual battery SoC are uncertain.

Fig 6 shows the robustness of LPBF for Li-S SoC estimation against initial uncertainty in both estimated and the actual battery cell SoCs. For demonstrate this Li-S cell is discharging from the partial capacity. Fig 6(a) and Fig 6(b) show the voltage and current, respectively, of a partially discharged Li-S battery cell at 20°C As it is discharging from partial capacity, the actual battery SoC is unknown. The initial estimated state of LPBF is set to be  $\hat{x}_s(0|0) = [0.6, 0]$ , which is again uncertain. Under these uncertain initial SoC condition, the robustness of LPBF is investigated.

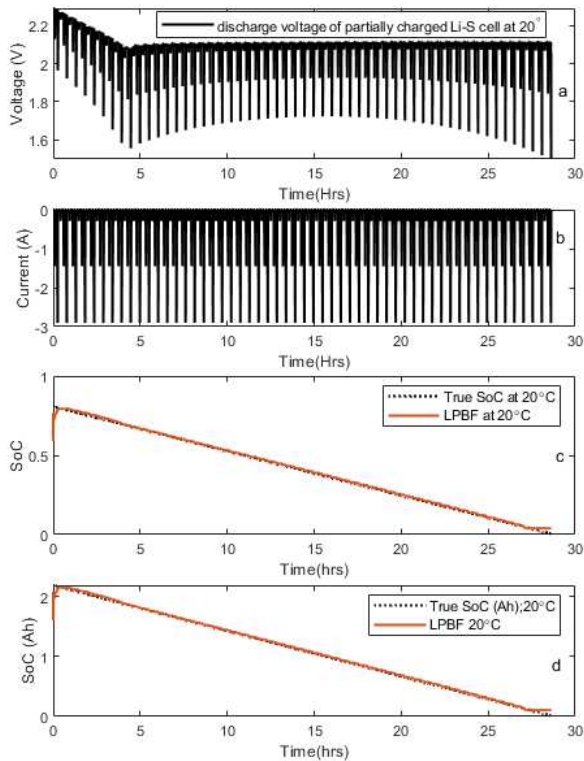


Fig. 6. Linearized LPBF when both initial SoC are uncertain at 20°C : (a) voltage; (b) current ; (c) SoC normalization; (d) SoC in Ah.

The SoC estimation by LPBF quickly converges to true SoC. The converges can be seen in the Fig 6(c) and Fig 6(d). In Fig 6(c), the estimated SoC normalization, shown by red-solid line, is compared to that of true SoC, shown by black-dotted line. Similarly, Fig6d compares SoC in Ah with its true SoC of partially discharged Li-S cell at 20°C temperature condition. For discharging from the partially-charge type battery cell, the mean error in SoC by LPBF is 0.73 percent, which is amazing and quite acceptable. The linearized LPBF is robust against the uncertainties in the both estimated and actual initial SoC of Li-S battery cells.

## VI. CONCLUSIONS AND FUTURE WORKS

A novel likelihood partitioned Bayesian filtering framework has been formulated and linearized version has been extended for SoC estimation of Li-S battery cell after modelling Li-S cell as a first-order equivalent circuit network model. The proposed linearized LPBF performed well for different temperature and known and unknown initial SoC conditions. Studing temperature variation, the reliability of algorithm for temperature sensitivity has been investigated. The robustness of novel SoC estimator is investigated by estimating SoC of cell that discharging from different initial charge capacities. Simulation results shows that the accuracy in SoC estimation by the novel approach was outstanding than the traditional approach for considered test conditions. The proposed novel SoC estimator is simple, optimal and robust against initial SoC uncertainties. Therefore, proposed novel algorithm can be used for on-board battery management systems of Li-S battery powered applications. The future research includes a further investigation on robustness of novel method against parameter and initial uncertainties, and apply the novel approach to estimate other performance metrics of battery management systems..

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