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J Pike

College of Aeronautics
Cranfield Institute of Technology
Cranfield. Bedford MK43 OAL. England

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SUMMARY

The combustion of multi-tube propellant grains is represented either by the Corner form function or an exact parallel burning layer expression. The exact expression predicts significantly different burning characteristics near the end of the burn to that of the Corner form function. When used in a typical gun firing calculation, the exact expression gives pressure histories which are closer to the experimental results. It also gives longer burn times and a reduction in the muzzle velocity of the projectile.

Introduction

For guns using solid propellant grains the burning surface area of the propellant during the firing depends by the grain shape [1,2]. It is often assumed for simplicity that the burning area of the grains varies linearly with the depth of the propellant burnt. This assumption leads to the Corner form function [2] for the for the burnt volume.

i.e.
$$z = (1-f)(1+\vartheta f) \tag{1}$$

where z is the burnt fraction of the propellant, f is the fractional depth of the grain left to burn and ϑ is a constant. The significance of the constant ϑ can be appreciated by differentiating equation (1) to give

$$dz/df = -(1 - \vartheta + 2\ \vartheta f) \tag{2}$$

where dz/df is proportional to the burning surface area of the grain and negative ϑ gives a linearly increasing burning area with decreasing f as the grain burns.

For most simple rain shapes equation (1) gives a good approximation to the burning characteristics of the grain [1]. It is also used for multi-tubular grains (shown for example in Fig.1) by redefining f to be based on 1.15 Dfrag, where Dfrag is the distance between the tubes or the outer tubes and the outer surface, as shown in Fig.1. However, multi-tubular grains break up or fragment before burnout and equation (1) is then a poor representation of the burning. It is unclear whether this error in the rate of burning of the final 15% of the mass is sufficient to cause significant changes in the internal ballistics of the gun. To investigate this, computations using the Corner form function are compared with those using an exact form function based on a parallel burning layer assumption. This assumption is itself an approximation to the actual grain burning characteristics. It assumes that the grain burns at the same rate over the whole of the grain, thus removing parallel layers of propellant as the burning proceeds. Although there are some indication that this assumption is approximately true [1], it does require that the pressure is nearly the same over the whole of the grain(including the interior of the tubes!), the propellant material is homogeneous, the external flow past the grain does not significantly affect the burning rate and that mechanical fracture of the grain does not occur before fragmentation. Whatever the influence of these effects, it is to be expected that after fragmentation the parallel layer assumption gives a more

accurate representation of the burning rate than the Corner form function. In any case, the comparison provides evidence as to the importance of the accuracy of burning models and will thus promote the introduction of more accurate models if necessary.

2. Exact form function for a multi-tubular grain shape.

Prior to fragmentation the burning area and volume of the multi-tubular grain shown in Fig.1, is given simply by

$$A = \frac{\pi}{2}(d_1^2 - 7d_2^2) + \pi\ell(d_1 + d_2) \tag{3}$$

$$V = \frac{\pi\ell}{4}(d_1^2 - 7d_2^2) \tag{4}$$

where d_1 is the outer diameter of the grain, d_2 is the tube diameter and ℓ is the grain length. These are linear functions of f (which is assumed to vary due to the burning), such that

$$d_1 = d_o - (1-f)D \tag{5}$$

$$d_2 = d_i + (1-f)D \tag{6}$$

$$\ell = \ell_o - (1-f)D \tag{7}$$

where d_o and d_i are the original grain and tube diameters and D is twice the maximum thickness to be burnt through for the grain to be fully burnt. For the 7-tube grain shown in Fig.1, D is given by

$$D = \frac{(d_o - 2W\cos(\pi/6))^2 - d_i^2 + w^2}{4W(2-\cos(\pi/6))} \tag{8}$$

where W is the distance between the tube centres or the web and is expressed in terms of the diameters by

$$W = (d_o - d_i)/4 \tag{9}$$

The volume is related to z by dividing by the origin grain volume (V_o). The burning area is related to dz/df by multiplying the area by $- D/2V_o$.

Fragmentation occurs after a depth of $(d_o - 3d_1)/8$ has been burnt through.

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That is, Dfrag. is twice this distance, and fragmentation occurs when f is Dfrag/D. The grain then divides into 12 slivers; 6 small inner slivers and 6 larger outer slivers. The calculation of their surface areas and volumes is complicated. The expressions used here are

(i) Inner Slivers

$$\alpha = \frac{\pi}{3} - 2\cos^{-1}\left(\frac{W}{d_2}\right) \tag{10}$$

$$A = \frac{3}{2}\ell d_2 \alpha + W^2 \cos(\pi/6) - \frac{3}{4}d_2^2(\sin(\pi/3 - \alpha) + d) \tag{11}$$

$$V = \ell(\frac{1}{2}W^2\cos(\pi/6) - \frac{3}{8}d_2^2(\sin(\pi/3 - \alpha) + \alpha)) \tag{12}$$

(ii) Outer slivers

$$\alpha = \pi/3 - 2\cos^{-1}((d_1^2 + 4W^2 - d_2^2)/(4d_1 W) \tag{13}$$

$$\beta = \sin^{-1}(d_1/d_2\sin(\pi/6 - \alpha/2)) \tag{14}$$

$$\alpha = \sin^{-1}(W/d_2) \tag{15}$$

$$\delta = \pi/6 + \alpha - \beta \tag{16}$$

$$A = \ell(\frac{1}{2}\alpha d_1 + \delta d_2) + d_2 W(\sin\beta - (\beta + \delta)) + \frac{1}{4}\alpha d_1^2 - \frac{1}{2}\delta d_2^2 \tag{17}$$

$$V = \ell(\frac{1}{2}d_2 W(\sin\beta - \sin(\beta + \delta)) + \frac{1}{8}\alpha d_1^2 - \frac{1}{2}\delta d_2^2) \tag{18}$$

The variation of dz/df with f from the exact form function above and from equation (2) with f equal to twice the burnt distance divided by D is shown in Fig.2. Prior to fragmentation when f = 0.6, the two models are very close. After fragmentation the Corner form function predicts a continually increasing burning area, whereas the exact shows a rapid decrease and a long tail. When equation (2) is used in practice, f is non-dimensionalised using 1.15 Dfrag. so that burning is completed when f = 0.

When these relationships are used in computational codes, it is essential to ensure that the mass of burnt and unburnt propellant is conserved. This is best achieved by expressive dz/df as a function of z, so that integration errors can be eliminated. From equations (1) and (2), we have for the Corner form function

$$\frac{dz}{df} = -\left\{(1+\vartheta)^2 - 4\vartheta z\right\}^{1/2} \qquad (19)$$

For the exact form function, it has not been found possible to express dz/df directly as a function of z and we have to use a parametric form, with dz/df and z as functions of f for example, as given by equations (3) to (18). The computationally efficient method of solving and using such relationships is to construct a "lookup" table. That is z and dz/df are tabulated for a range of f values and interpolation is used to find intermediate values. It is more convenient if the table of dz/df values is at equal z intervals and this is achieved by interpolating the lookup table values between the calculated values. The Fortran code for building a DZODF lookup table at MZ equal Z intervals between 0 and 1, is given in the Appendix. The values from this code are compared with those from equation (19) in Fig.3. We see that after fragmentation when z = 0.85, the burning rates are very different, with the exact form function falling approximately linearly to zero as z increases from 0.85 to 1. Indeed the linear assumption could be used as a quick "fix" to a computer code, to evaluate the significance of changes in the form function, although this approximation is difficult to justify theoretically as it does not give an acceptable relationship for dz/df as a function of f.

3. Effect of form function on a typical gun firing.

The WAFBC1 internal ballistics code, written by E.F. Toro [3,4] has been used to complete the conditions during the firing of a 3" Navy gun, for which experimental pressure histories are available at 3 positions in the firing chamber. The modelling of internal ballistic flows is still at a stage when the computed results are not entirely trustworthy, but the results from the code should be adequate to demonstrate the effect of varying the form function.

In Fig.4 the computed pressure distributions using the Corner form function and the exact form function are compared with the experimental pressures. The experimental pressures are shown for three positions in the firing chamber. The computed pressures are for the breech in the top left hand plot and at the three pressure gauge positions in the other three plots.

We see that in all the plots the computed pressures are nearly the same before fragmentation, which occurs at about $6\mu s$. Then the pressure predicted by using the exact form function decreases much faster than the Corner form function, until burnout occurs for the Corner form function at $6.7\mu s$. The two pressures then progressively converge to become nearly the same again when the exact form function predicts burnout at $7.8\mu s$.

When comparing these results with the experimental pressure distributions, it is difficult to justify an absolute time equivalence as the early ignition of the propellant is not well modelled in the computation. Thus the initial firing time does not form a useful reference. In Fig.4 the experimental pressures have been matched to the rising theoretical pressures, by amending each of the gauge pressure times by the same amount. When the rising pressures are matched thus, we see firstly that the peak experimental pressures are much greater than the computed pressures and secondly that the falling pressures are matched much more closely by the exact form function than the Corner form function. The use of the exact form function, then removes one defect from the pressure history, but the mismatch in the peak pressures indicates that a serious defect in the modelling (or in the experimental results!) still exists which requires further investigation. This is however beyond the scope of the present report.

The differences in the pressure between the two form functions affects the acceleration of the projectile and the predicted muzzle velocity is reduced from 843 m/s for the Corner form function to 830 m/s for the exact form function. This represents a small but significant reduction in the muzzle velocity. Another change which could be significant is that burnout occurs after 7.8 m/s instead of 6.7 m/s. This later burnout for the exact form function causes burnout to be much closer to muzzle exit of the projectile at 9.1 m/s, so increasing the risk of unburnt solid fragments of the propellant leaving the barrel. It should be noted that the long burning time of the large slivers from multi-tubular grains could be reduced by using a "slotted" rather than a circular outer profile for the propellant grains. Using such fluting the outer slivers can be made the same size as the inner slivers or even smaller, whilst the burning characteristics prior to fragmentation are virtually unchanged. Burnout then occurs more quickly

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after fragmentation, in fact in little longer time than that predicted by the Corner form function, as can be seen by referring to Fig.2. In Fig.2 the burnout of the inner slivers occurs at the kink in the exact form function at a value of f just larger than the Corner burnout value. This could then be the final burnout time, although the effect on the pressure distribution after fragmentation would need investigation.

Conclusions

The assumption of parallel burning layers gives a surface area variation (or form function) for multi-tubular grains which varies significantly from the Corner form function after grain fragmentation.

The exact form function predicts significantly longer times to grain burnout and gives pressure histories in a typical gun firing which more closely match experimental results.

The muzzle velocity is reduced by a small but significant amount.

It would be desirable to use the exact form function more generally in computational internal ballistics codes, to improve the accuracy. Fortran subroutine suitable for 7-tube grains is supplied in the Appendix.

List of Symbols

A	Surface area of burnout propellant grain
D	Twice the depth to grain burnout
Dfrag	Twice the depth to grain fragmentation
d_o	Original outer diameter of grain
d_1	Outer diameter of burning grain
d_2	Tube diameter of burning grain
d_i	Original tube diameter of grain
f	Remaining fraction of depth of grain to be burnt
ℓ	Length of burning grain
ℓ_o	Original length of grain
V	Volume of burning grain
V_o	Original grain volume
W	Web of multi-tubular grain (distance between tube centres)
Z	Fraction of grain volume burnt.
ϑ	Form function constant in eq.(1).

References

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2.   Corner. J. Theory of the interior ballistics of guns. John Wiley and Sons, New York, 1950.

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4.   Toro, E.F. Riemann problem based techniques for computing reactive two-phase flows. Numerical Combustion, Lecture notes in Physics, V.351, pp 472-481, 1987.
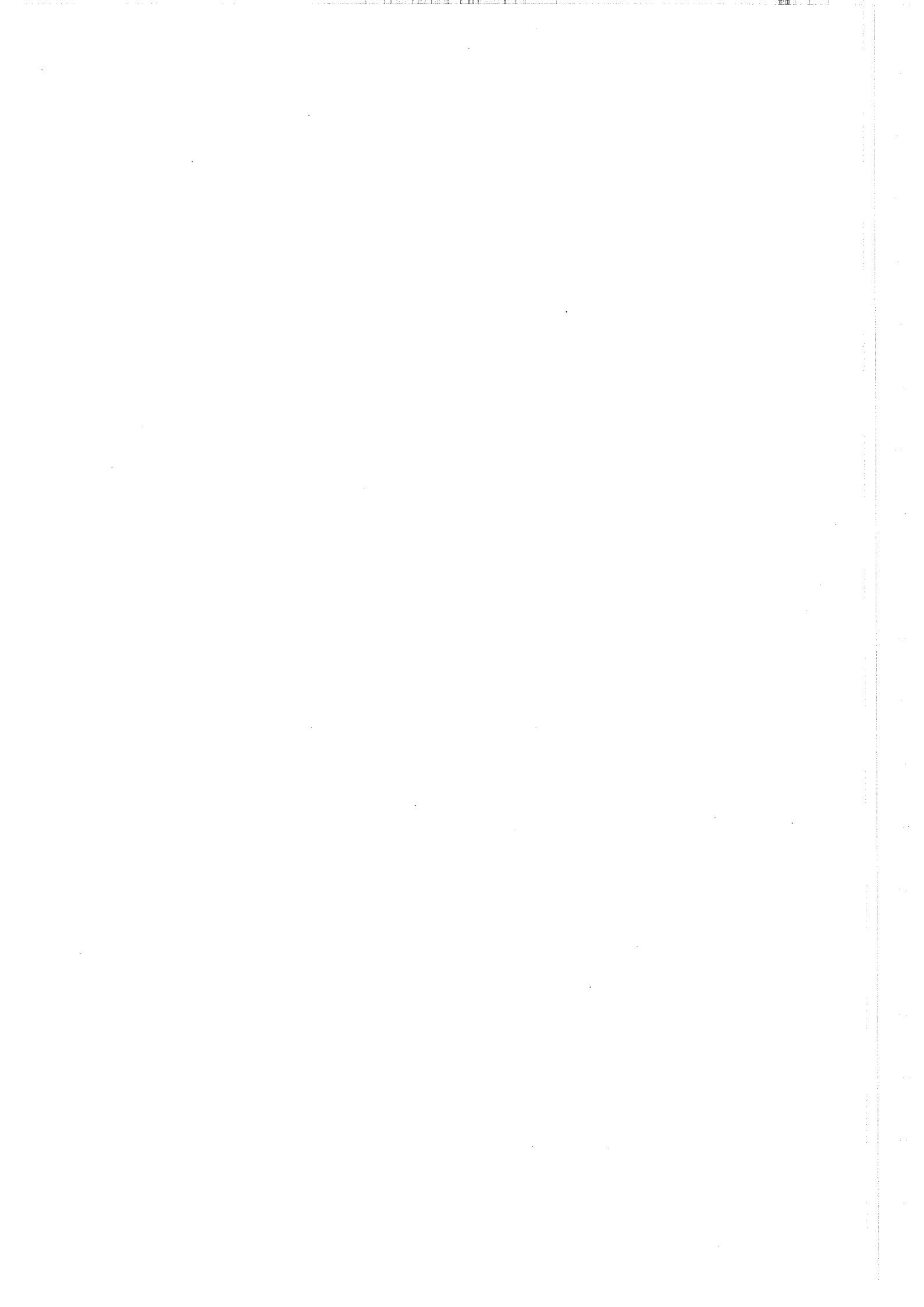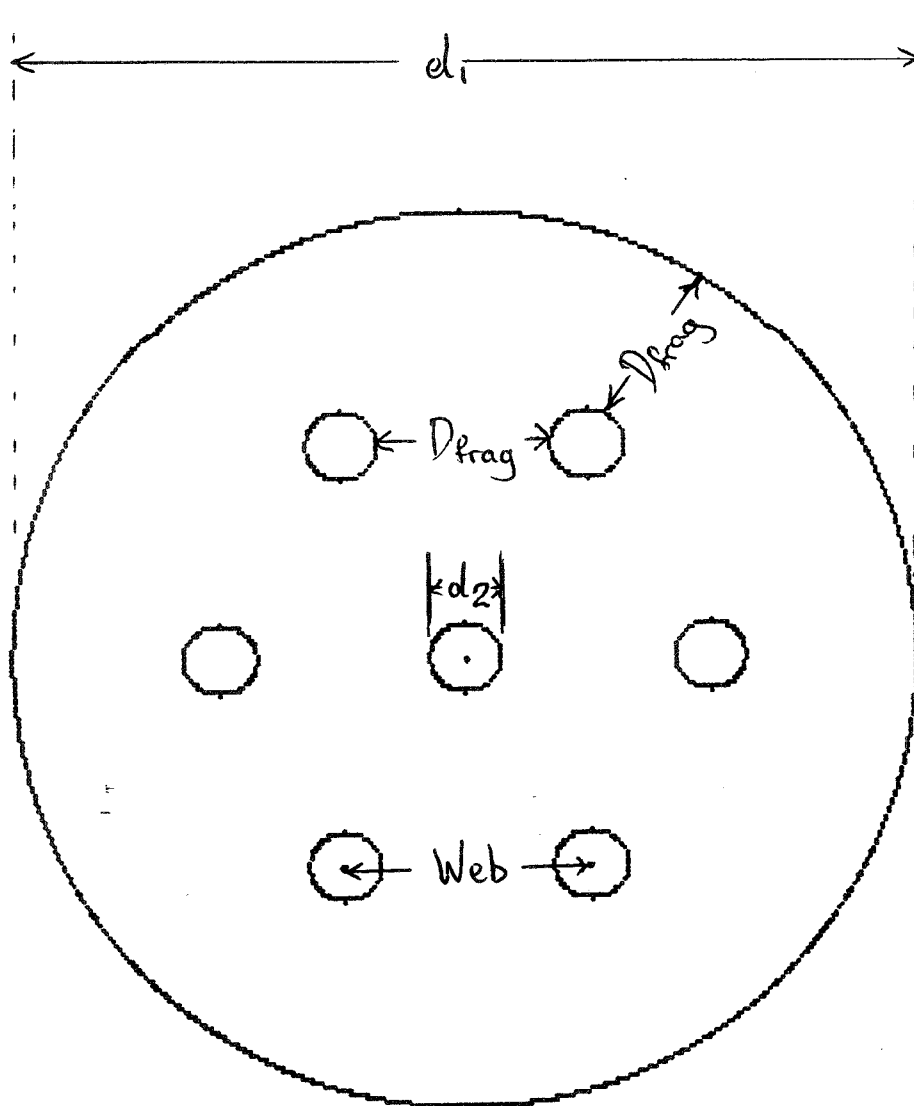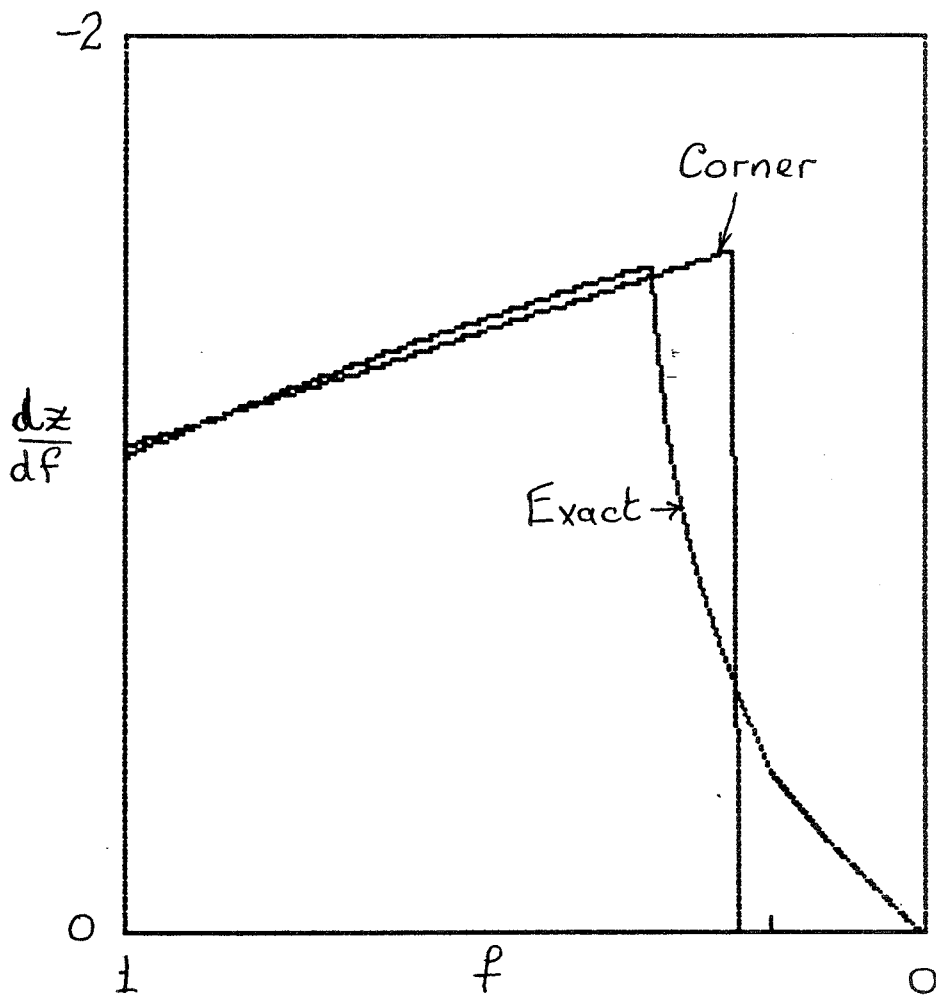
Fig.1 Cross-Section of 7-tube Grain

Fig.2 Variation of dz/df with the depth of grain left to burn for a 7-tube grain.
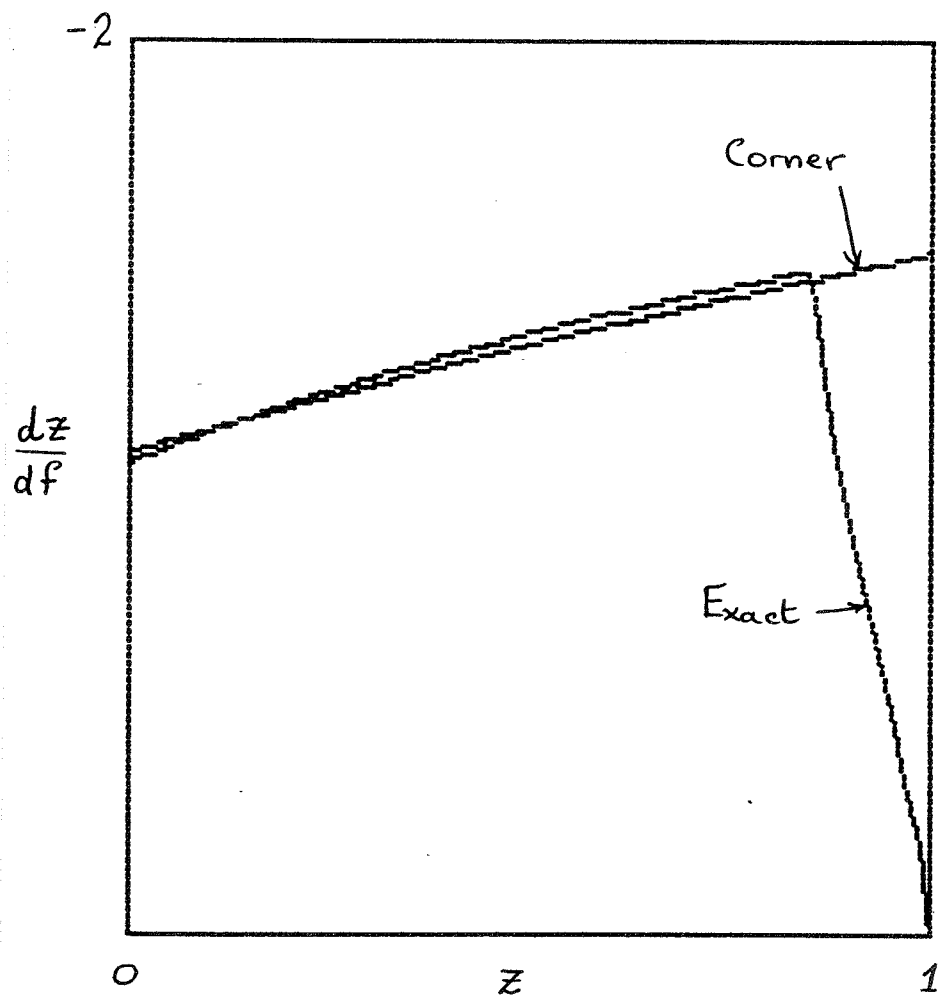
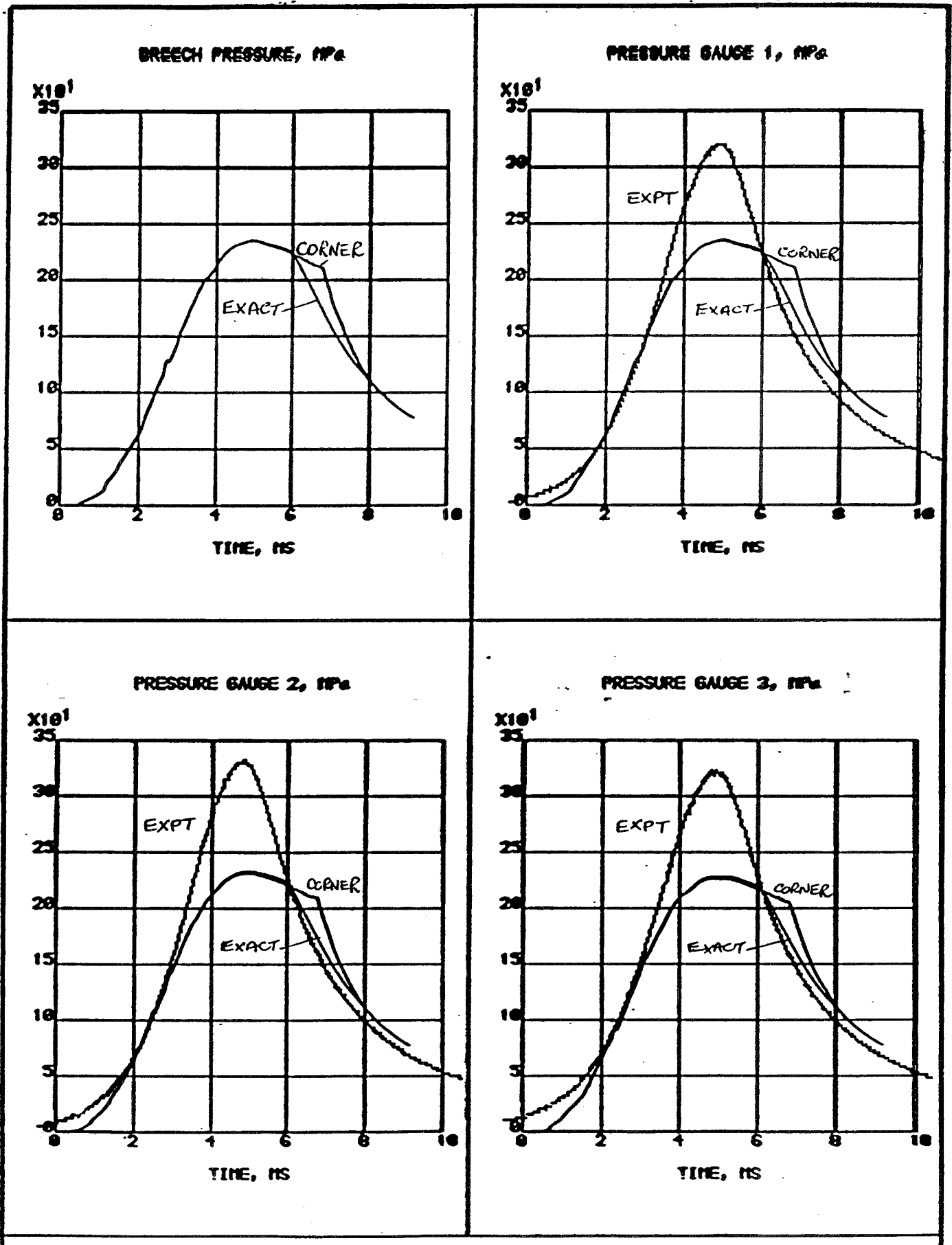Fig.3 Variation of dz/df with the burnt fraction z for a 7-tube grain.

Fig.4 Comparison of firing chamber pressures with computations using the Corner and exact form functions.

APPENDIX

```
C SURFACE AREA OF MULTI-TUBE CYLINDRICAL BURNING PROPELLANT GRAIN
C PUTS DZ/DF=C.5*D*A/V) AT EQUAL Z INTERVALS IN ARRAY DZODF
C A=SURFACE AREA BURNING,D=TWICE BURN DIST TO BURNOUT,VO=INITIAL VOLUME.
C GRLEN,GROUDI,GRINDI ARE GRAIN LENGTH,OUTER CYLINDER DIAMETER & TUBE DIAM.
C MZ IS NUMBER OF INTERVALS IN DZODF LOOKUP TABLE (MZ<=1000)
C NB. DZODF IS A DOUBLE PRECISION ARRAY DIMENSIONED 0:1000

      SUBROUTINE TUBE7(GRLEN,GROUDI,GRINDI,MZ,DZODF)
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (MD=1000)
      DIMENSION Z(0:MD),S(0:MD),DZODF(0:MD)
      IF(MZ.GT.MD)STOP 7
      PI =4.0*DATAN(1.0D0)
      C30=COS(PI/6.0)
      VO =PI/4.0*GRLEN*(GROUDI**2-7.0*GRINDI**2)
      WEB=(GROUDI+GRINDI)/4.0
      D  =((GROUDI-2.0*WEB*C30)**2-GRINDI**2+WEB**2)/4.0/WEB/(2.0-C30)
      DFRAG=(GROUDI-3.0*GRINDI)/4.0
CALC BURNT VOL FRACTION Z() & BURNING SURFACE S()
      DO 0001 LF=C,MZ
        F=REAL(MZ-LF)/REAL(MZ)
C       D1=BURNING GRAIN OUTER DIAMETER,D2=BURNING GRAIN TUBE DIAMETER
        D1=GROUDI-(1.0-F)*D
        D2=GRINDI+(1.0-F)*D
C       GL=BURNING GRAIN LENGTH, A=BURNING GRAIN SURFACE AREA
        GL=GRLEN-(1.0-F)*D
        IF(D2.LE.WEB)THEN
CALC AREA & VOLUME BEFORE FRAGMENTATION OF A SINGLE GRAIN
          A=PI/2.0*(D1*D1-7.0*D2*D2)+PI*GL*(D1+7.0*D2)
          V=PI/4.0*(D1*D1-7.0*D2*D2)*GL
        ELSE
CALC AREA & VOLUME AFTER FRAGMENTATION
C         INNER SLIVER. ALFA=ANG SUBTENDED BY SLIVER TO TUBE CENTRE
          ALFA=PI/3.0-2.0*ACOS(WEB/D2)
          IF(ALFA.GT.0.C)THEN
            PERIM=1.5*ALFA*D2
            SINNER=0.5*C30*WEB**2-0.375*D2*D2*(SIN(PI/3.0-ALFA)+ALFA)
            AINNER=GL*PERIM+2.0*SINNER
          ELSE
            SINNER=0.0
            AINNER=0.0
          ENDIF
C         OUTER SLIVER. ALFA,DELTA=ANGS SUBTEND TO GRAIN & TUBE CENTRE
          ALFA=PI/3.0-2.0*ACOS((D1*D1+4.0*WEB*WEB-D2*D2)/4.0D1/D1/WEB)
          IF(ALFA.GT.0.0)THEN
            BETA=ASIN(D1/D2*SIN(PI/6.0-ALFA/2.0))
            GAMMA=ASIN(WEB/D2)
            DELTA=PI/6.0+GAMMA-BETA
            PERIM=ALFA*D1/2.0+DELTA*D2
            SOUTER=0.5*D2*WEB*(SIN(BETA)-SIN(BETA+DELTA))
     +             +0.125*ALFA*D1*D1-0.25*DELTA*D2*D2
            AOUTER=GL*PERIM+2.0*SOUTER
          ELSE
            SOUTER=0.0
            AOUTER=0.0
          ENDIF
          A=6.0*(AINNER+AOUTER)
          V=6.0*GL*(SINNER+SOUTER)
        ENDIF
```

```
          S(LF)=A
          Z(LF)=1.0-V/VO
 C001 CONTINUE
C INTERPOLATE DZODF=0.5*D*S/VO VALUES TO EQUAL Z SPACING
       DO C003 IZ=0,MZ
          ZVAL=REAL(IZ)/REAL(MZ)
          DO 0C02 II=0,MZ-1
             IF(ZVAL.GE.Z(II).AND.ZVAL.LE.Z(II+1))I=II
 C002     CONTINUE
          DS=S(I+1)-S(I)
          DZODF(IZ)=0.5*D/VO*(S(I)+DS*(ZVAL-Z(I))/(Z(I+1)-Z(I)))
C         WRITE(6,111)20,I,ZVAL,DZODF(IZ)
 C003 CONTINUE
 0111 FORMAT(2X,2I4,6F11.5)
       RETURN
       END
```