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A Kriging approach to model updating for damage detection*

Gabriele Dessena[1] [0000−0001−7394−9303], Dmitry I. Ignatyev[1] [0000−0003−3627−3740], James F. Whidborne[1] [0000−0002−6310−8946], and Luca Zanotti Fragonara[1] [0000−0001−6269−5280]

Cranfield University, Cranfield, Bedfordshire, MK43 0AL, UK
{gabriele.dessena, d.ignatyev, j.f.whidborne, l.zanottifragonara}@cranfield.ac.uk

Abstract For complex or large structures, the model updating process can be long and tedious and numerical methods can be computationally expensive. Hence, practitioners and researchers often resort to meta-modelling techniques when large problems are met. Even so, traditional methodologies, such as the Efficient Global Optimisation, can be slow and give sub-optimal results. This work proposes a new methodology for the model updating of numerical systems based on a novel Kriging approach for the scope of damage detection and quantification. The framework proposed is based on a global-local optimisation strategy recently developed by the authors, the refined Efficient Global Optimisation, herein used to tweak finite element models' parameters to match the modal data extracted from a numerical system by using the residuals of the modified total modal assurance criterion. The main advantage to existing direct optimisation and meta-modelling frameworks is the more efficient use of computational effort for higher dimensional problems, which is verified with the use of a numerical system.

Keywords: Kriging · Finite Element Model · Finite Element Method · Model Updating · Structural Health Monitoring · Modal Analysis · MTMAC · Damage Detection.

1 Introduction

Damage is considered a change in a system, or structure, which compromises or influences its operational capability[9] and as such is a key driver, being strictly related to reliability and safety[24], in engineering design and operations. Hence, a plethora of methods for damage detection, commonly referred as structural health monitoring (SHM), have been proposed and implemented[3]. Nevertheless, the most prominent techniques are vibration-based approaches[21], which can be direct such as, [7,5], or *indirect*, such as [22,25]. The latter approach, known as model-based SHM, is the focus of this work, which ultimate goal is

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to present a new method based on an enhanced surrogate modelling technique developed by the authors, the refined Efficient Global Optimisation (rEGO). For this aim, a numerical 9-DoF mass-spring-damper system is generated with four damage scenarios. The task for the new method is to tune, or update, a finite element model (FEM) of the undamaged scenario to a damaged scenario and by matching the modal response, intended as natural frequencies and mode shapes, using the residuals of the modified total modal assurance criterion (MTMAC)[19], to localise and quantify the damage.

The remaining of this section is focused on the background related to FEM updating, surrogate modelling, and their combined implementation. On the other hand, the remainder of this work is structured as follows. Section 2 introduces rEGO, the FEM updating workflow, the numerical system, and the damage detection capabilities, while Section 3 shows the obtained results, discusses its merits and delivers the closing remarks by introducing the future opportunities for this technique.

1.1 Background

FEM updating is defined as the mathematical process by which a baseline FEM of the previously undamaged structure is tuned to achieve a good agreement with the damaged structure[15]. According to [11,18], model updating can be split in two categories: direct and indirect. Direct techniques, such as matrix update, optimal matrix, and eigenstructure assignment, use modal characteristics to update the FEM and are regarded as efficient and accurate methods; however, the requirement for precise measurements, the sensitivity to noise, the impossibility of using truncated data and the possibility of losing symmetry in the FEM matrix do not make them suitable candidates for damage detection applications via FEM updating. On the other hand, indirect, or iterative, methods can overcome the aforementioned problems[1]. Iterative methods can be divided in four main categories[1]: (i)sensitivity-based, (ii)computational intelligence-based, (iii) response surface method (RSM), and (iv)Bayesian or Monte-Carlo approaches. Only RSM methods for are discussed beyond this point as they are the focus of this work. Nevertheless, the interested reader is referred to [15] for a more profound review of the remaining approaches.

In damage detection, RSMs are used to achieve an approximation of the structural response to a given objective function [20]; generally, the objective functions target modal data or time or frequency response. RSMs are widely used in many fields of engineering and, most notably, in computational design, as their approximations can be used successfully for the research of an optimal design[23]. A general RSM procedure is defined:

1. The design of experiment (DoE) is drawn to collect a number of samples from the goal function with strategic sampling;
2. The chosen RSM is fitted to the data obtained through the DoE and a predictor is established

3. The RSM can be updated with new results strategically, such as computing the actual value of the expected minimum.
4. Iterating between points 2 and 3 until convergence is reached

It goes without saying that a minimal implementation could stop after point 2, but usually such minimal implementation does not meet the set standards. While it is not the purpose of this work to give a thorough review on meta-modelling, or RSM, the interested reader is referred to [10] for a complete scrutiny on the subject with a focus on Gaussian processes, also the core of this work. Within the realm of meta-modelling, Gaussian processes have received great attention in the two decades or so[12,10,4]; however, one of the most used techniques, Kriging, originated outside engineering, in geostatistics, in the 1950s[13]. As a stochastic-based meta-modelling technique, it is based on the relationship between the sample data (input) $\boldsymbol{X}$ and the observed response (output) $\boldsymbol{Y}$. For the sake of brevity, the full mathematical background on Kriging is not presented here, but the interested reader can consult [10,12] for a more comprehensive review.

Returning to the RSM procedure, after outlining the concept of Kriging, the second point of the list is clear. However, points 1, 3, and 4 are still to be explored. The DoE is usually carried out by selecting strategically a number of points equal to ten times the number of variables[23]. One of the most efficient strategies[10] is the Morris–Mitchell optimal latin hypercube (LH)[16,17], which ensures uniform spreading of the sample points and is used throughout this work. The reader interested in a more broad review of sampling strategies is directed to [14]. The Efficient Global Optimization (EGO)[12] is the cornerstone of the meta-modelling via Kriging. Particularly, [12] introduced a new quantity, the Expected Improvement (EI), to aid the updating of the RSM generated with Kriging. Simply speaking, the EI is the measure of how much the minimum of a function can be improved if a point at a given location is searched. In fact, the maximum absolute value of EI, along the function, is used to identify the infill point for the Kriging model. For the mathematical formulation of the EI reference is made to [12,10]. The EI is treated in this work as the informing parameter for updating the RSM model. While EGO was a pioneering strategy when released, the aforementioned classical structure was followed. The main drawback of EGO is its global only optimisation capability, which does not allow for the full convergence of the optimisation process, resulting in sub-optimal results. The authors propose to solve this constraint with a modified EGO[8], which offers a global-local search capability, allowing its application to damage detection. The modified EGO, or refined Efficient Global Optimisation (rEGO), is introduced in Section 2.

2 Methods

In this Section the rEGO workflow is introduced alongside its implementation for damage detection and, finally, the numerical system and the computational experiment setup are described.

2.1 The refined Efficient Global Optimisation

In order to enhance the capability of EGO to a global-local optimisation technique, rEGO introduces a refinement and selection technique. The refinement technique deals with an active reduction of the search domain, which is halved each time the main stopping criterion (ϵ_1), linked to EI, is achieved and the number of sample points is more that ten times the variables. The selection, inspired by multi-objective optimisation[23] and the Pareto fronts dominance[6], is chained to the refinement and promotes the selection of single points from clusters within the new refined search space. This retains only dominant points, keeps the number of sample points down and, hence, maintains the search to be computationally efficient.

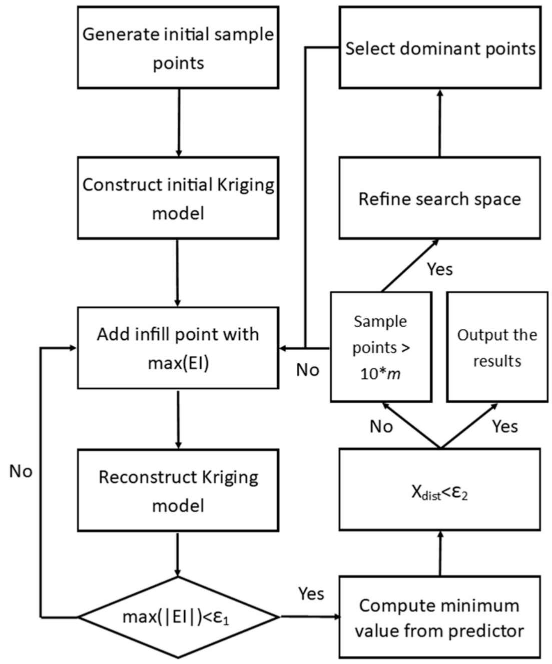

Figure 1. rEGO workflow. m stands for the number of variables.(retrieved from [8])

The workflow in Figure 1 outlines the general working principles of rEGO. The left column of the workflow represents a typical EGO algorithm, while the right column highlights the aforementioned improvements implemented by the authors. The second stopping criterion, ϵ_2, is a measure of local convergence, as it is based on the Euclidean distance between the variables of proposed minima (x_{dist}), in an hill climbing fashion. If the ϵ_2 condition is satisfied the algorithm terminates; otherwise, the number of sample points is taken under consideration and if its number is ten times the number of variables the search space is refined.

Ensuring a set number of points within the search space is pivotal to guarantee the exploration of the search space. In fact, while [12] states that obtaining at least max($|(EI)|$)$<$ 1% is a sign of good exploration, sometimes issues might arise with a low number of samples. Hence, drawing from the experience of sampling [10], the minimum size requirement for the data pool was established to be ten times the number of variables. The two requirements, EI and data pool size, ensure a good exploration of the design space and the viability of the refinement and selection technique.

2.2 Damage Detection via Model Updating

Having outlined the principles behind rEGO, the model updating and, so, the damage detection routine are described. As a model-based technique a baseline, or undamaged, FEM is generated and then the data obtained from damage scenarios are used to update the model and identify the changes in parameters which identify the damage. Modal data, natural frequencies (ω_n) and mode shapes($\boldsymbol{\phi}_n$), are taken into consideration within this work as the MTMAC residual is used. The MTMAC residuals is presented below.

$$\mathrm{MTMAC_{res}} = 1 - \prod_{i=1}^{n} \frac{\mathrm{MAC}(\boldsymbol{\phi}_i^E, \boldsymbol{\phi}_i^N)}{\left(1 + \frac{|\omega_i^N - \omega_i^E|}{|\omega_i^N + \omega_i^E|}\right)} \tag{1}$$

MAC, stands for the usual modal assurance criterion[2], n is the total number of modes under consideration, superscript E denotes for experimental results and superscript N for numerical. For the purpose of this work, N stands for the updated model and E for the damaged data. The $\mathrm{MTMAC_{res}}$ can return results only between 0 and 1, with 0 being the value that represents full agreement. The task of the rEGO is to tune the parameters of a given model to minimise the $\mathrm{MTMAC_{res}}$ response and extract the damage extend and location.

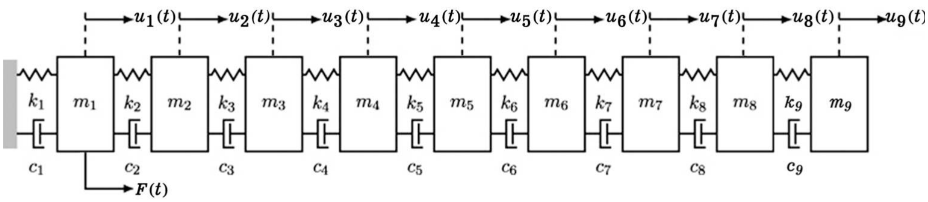

Figure 2. 9 DoF mass-spring damper system.(retrieved from [7])

The system selected for the numerical verification of rEGO as a damage detection technique is the 9 DoF mass-spring-damper system shown in Figure 2. In the undamaged system, all masses (m_{1-9}) are set to 1 kg and the springs' stiffness is $k_{1-9} = 10$ kNm^{-1}. The damping is defined as the damping ratio (ζ_n)

and it is set to 1%. Changes in damping are not considered within this study and the given value stays constant for all scenarios. The modal parameters are extracted through eigenanalysis and ζ_n is used for adjusting the values of ω_n of the damped system. The mass and stiffness matrices are built according to usual practice for such systems.

This work considers 4 damage scenarios, listed in Table 1. For all the damaged cases only the stiffness properties, as the scope of this work is solely damage detection, are tuned by the ratio of the optimisation variables ($\boldsymbol{x}$) to the baseline stiffness values, e.g. for a 10% damage at the second element k_2 is given by $x_2 \times k_2$, where x_2 should be 0.9 at convergence. The optimisation search bounds are set between $[0.7, 1]$ and 9 variables are searched to match the modal response. The

Table 1. Damage scenarios of the 9 DoF system.

Scenario #	Damage
1	Undamaged
2	10% stiffness reduction in the fourth element.
3	25% stiffness reduction in the fourth element.
4	25% stiffness reduction in the fourth element and 10% stiffness reduction in the seventh element.
5	25% stiffness reduction in the second element, 10% stiffness reduction in the fourth element and 10% stiffness reduction in the seventh element.

MTMAC takes into considerations only ω_n and $\boldsymbol{\phi}_n$, so it is beneficial to report in Table 2 the computed ω_n and in Figure 3 the $\boldsymbol{\phi}_n$. From Table 2 and Figure 3 the difference induced by damage are quite evident. These changes are detected via rEGO and, for comparison purposes, with genetic algorithm (GA) and EGO. The rEGO stopping criteria were: $\epsilon_1 = 0.1\%$ (to ensure further exploration of the design space) and $\epsilon_2 = 10^{-4}$. The same value of ϵ_1 is also selected as the EGO stopping criterion, while for the GA a function tolerance of 10^{-4} and a maximum generation number of 100 are selected. The other properties of the GA are unchanged from the standard function `ga` offered in MATLAB. GAs are not discussed in depth in this work as they are a well established method and are only used for benchmarking purposes. For all methods a Morris–Mitchell optimal LH with a number of samples ten times the number of variables is selected to define the starting population. Each optimisation run, on each scenario, for each method is run ten times for statistical significance.

3 Results and Conclusions

This section shows the results obtained for the numerical study on the 9 DoF system by comparing the results obtained for the damage detection via model updating using rEGO, EGO and GA. Particular attention is paid to the detection of stiffness changes and to the number of function evaluations needed to reach a solution by each routine.

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Table 2. Computed natural frequencies in Hz of 9 DoF mass-spring damper system.

Natural Frequencies [Hz]					
Scenario #	**1**	**2**	**3**	**4**	**5**
Mode #					
1	2.629	2.349	2.371	2.686	2.422
2	7.814	7.226	7.355	7.875	7.485
3	12.786	12.179	11.980	12.656	12.196
4	17.409	17.091	16.962	17.052	16.323
5	21.558	20.957	20.945	20.986	20.399
6	25.118	24.152	23.903	24.332	23.972
7	27.993	27.416	27.306	27.250	26.260
8	30.105	29.285	28.886	29.674	28.526
9	31.395	30.788	30.760	31.274	30.898

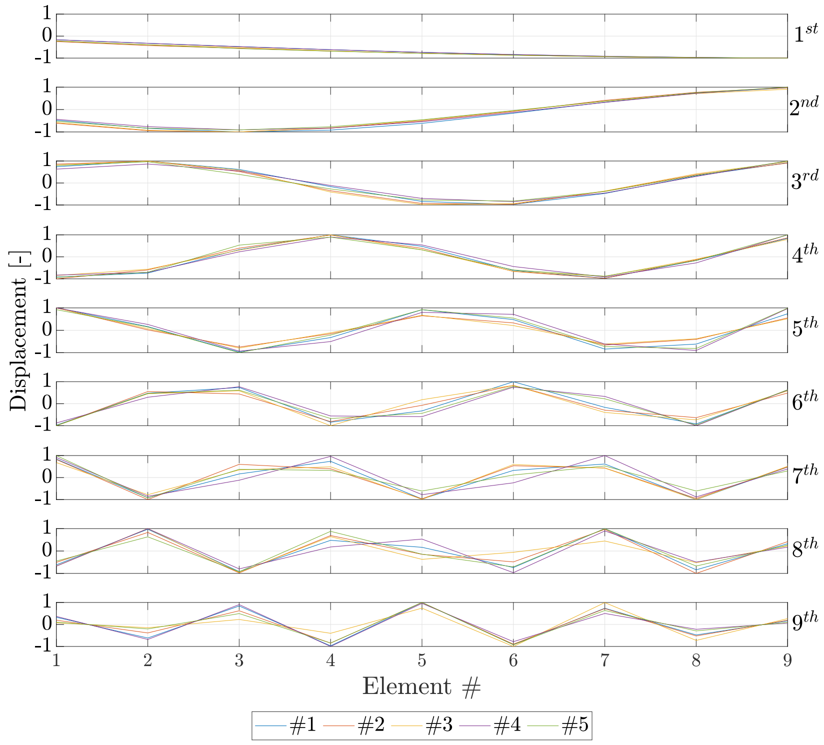

Figure 3. 9 DoF mass-spring damper system mode shapes for all scenarios. The legend refers to the scenario # and the label at the right of the plot to the mode.

The results of the damage detection routine are presented in Figure 4 as bar plots, identified as the mean (μ) parameter $\boldsymbol{x}$ change, identified as damage, over ten iterations for the rEGO, EGO , and GA implementation. Only the μ values are reported for conciseness and clarity. Table 3 presents the function

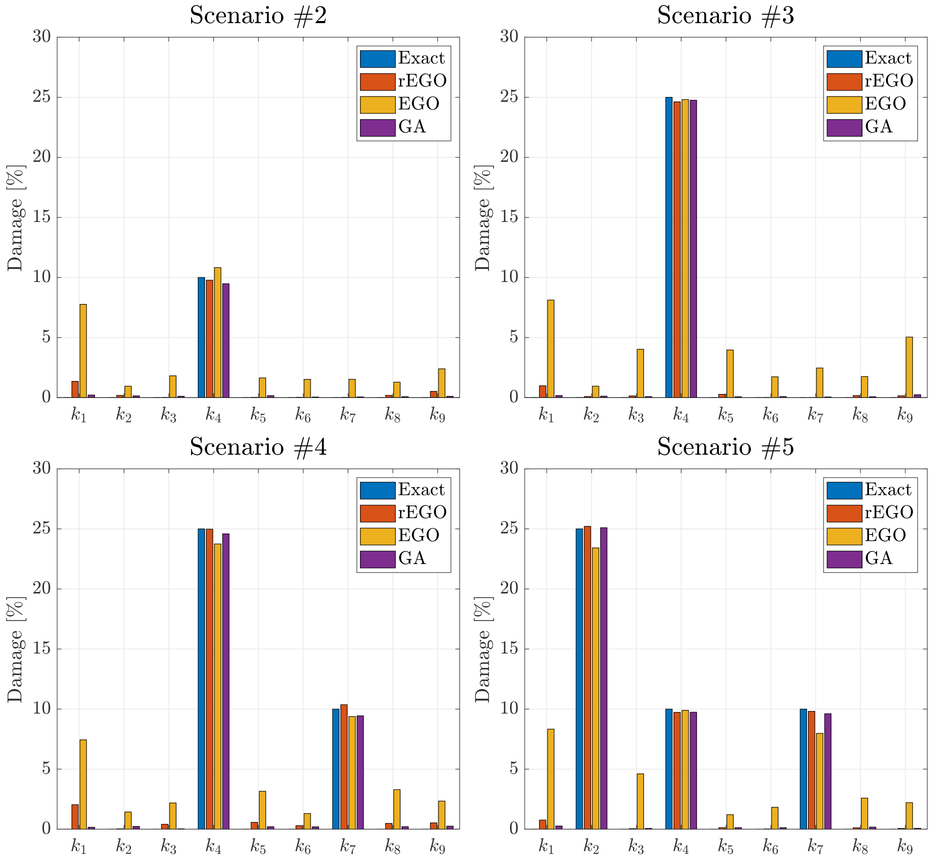

Figure 4. The mean values, over 10 iterations, of the identified damage by rEGO, EGO, and GA vs the exact value.

evaluations to convergence for each scenario. The values are presented taking into consideration the minimum and maximum evaluations needed and the μ over the ten iterations. Notably, GAs' maxima are the same for all scenarios, meaning that the hard limit for maximum generations was met in at least one realisation for the given scenarios.

Table 3. Number of function evaluations for convergence.

Scenario	**#2**			**#3**			**#4**			**#5**		
	min	μ	**max**	**min**	μ	**max**	**min**	μ	**max**	**min**	μ	**max**
rEGO	159	281	382	172	286	494	274	329	391	217	331	433
EGO	104	115	131	100	111	130	103	112	129	100	106	119
GA	15600	17899	19210	15410	17424	19210	15030	16778	19210	13700	16778	19210

From Figure 4 it is clear that rEGO has an advantage over EGO for damage detection. For all instances the damage estimated via rEGO is closer to the exact value than the one calculated via EGO. Also rEGO, detects damage as as well as GA; however, rEGO had the tendency to detect a slight, even imperceptible, damage where there was none. This is most evident in the first element of scenarios #2 and #4. Even EGO has this issue, but it is more evident and happens for all cases. Generally the GA is better at detecting undamaged locations, but at the price of a many more evaluations. As shown in Table 3, the evaluations for the GA driven search are of two order of magnitude higher than rEGO and EGO. EGO, unsurprisingly, reaches convergence with the fewest evaluations, while rEGO is able to do so with two or three times the evaluations, depending on the scenario, but, as said above, with much more success than EGO.

Concluding, rEGO proved to be a clear enhancement over EGO and an excellent compromise over the computationally expensive GA for model-based damage detection. The future plan is to implement it on existing benchmarks and experimental data to further validate its credibility for the employment in the monitoring of real structures. rEGO itself can also be used in any other domain which requires a RSM, such as computational engineering design.

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