

**CRANFIELD INSTITUTE OF TECHNOLOGY**

**SCHOOL OF MECHANICAL ENGINEERING**

**DEPARTMENT OF FLUID ENGINEERING AND INSTRUMENTATION**

**PhD Thesis**

**Academic Year 1991 - 1992**

**R. K. GRENVILLE**

**BLENDING OF VISCOUS NEWTONIAN AND  
PSEUDO-PLASTIC FLUIDS**

**Supervisor: Prof. M. Sanderson**

**April 1992**

**This thesis is submitted in partial submission for the degree of Doctor of Philosophy**

## ABSTRACT

Blend times and power consumption have been measured for Newtonian and pseudo-plastic fluids in agitated, baffled vessels with diameters of 0.30, 0.61 and 1.83 m. The viscosities of the fluids were selected to give a range of Reynolds numbers between 200 and  $10^5$ . Four impellers were tested with diameters between  $T/3$  and  $T/2$  positioned at  $T/3$  clearance above the lowest point of the vessel base. The impellers were typical of those found in the chemical industry.

The blend times were measured using a conductivity technique. A critical Reynolds number,  $Re_{crit}$  was identified for each impeller. For both the Newtonian and pseudo-plastic results the data could be divided into two groups:

$$Re > Re_{crit} \quad N\theta = \text{constant}$$

$$Re < Re_{crit} \quad N\theta \propto Re^{-1}$$

The data taken with the four impellers could be combined and expressed in terms of three dimensionless groups; power, Reynolds and Fourier numbers.

Analysis of the Newtonian blend time data shows that blending in the vessel as a whole is controlled by blending near the vessel wall. Analysis of the pseudo-plastic blend time data is based on estimation of the fluid viscosity in this region of the vessel, calculated from a torque balance between the agitator shaft and the vessel surfaces. Both Reynolds and Fourier numbers are calculated using this value of viscosity. The correlation of the pseudo-plastic blend time data, using this method, is the same as the Newtonian correlation and the two are not statistically different.

A cost-benefit analysis has been carried out to compare the running and capital costs of agitators for blending. Running costs are based on the agitator's power consumption and maintenance costs. The capital cost is based on the shaft torque since this will determine the mechanical strength of the agitator drive and shaft.

The cheapest blending of a Newtonian fluid is achieved by an agitator with a low power number impeller while the cheapest blending of a highly pseudo-plastic fluid is achieved by an agitator with a high power number impeller.

## ACKNOWLEDGEMENTS

Firstly, I would like to thank the members of the Fluid Mixing Processes consortium for funding the research reported in this thesis and for the interest they have taken in my work.

Secondly, I would like to thank the members of my Total Technology PhD Study Panel: namely Prof. Mike Sanderson of Cranfield Institute of Technology, Prof. John Smith of the University of Surrey, Mr. Mike Sweeney of Cranfield School of Management and Prof. Roger King and Mr. Colin Armstrong of the British Hydromechanics Research Group Ltd., Cranfield, for their support and advice over the last four years.

Next, I would like to thank my colleagues at BHRG Ltd. for their interest and constructive comments on my work and, in particular, I would like to thank Dr. Nick Fawcett and Mr. Joe Hannon for helping to bounce ideas around and Mr. Mike Whitton for proof reading the text of my thesis. Also, I am very grateful for the help the BHRG technicians and students gave me in gathering experimental data - especially Mr. Bob Burnapp and Mr. Kevin Lee (who also drew some of the figures in this thesis).

Finally, I would like to thank my wife, Rita, for her patience as she became a “PhD widow” for a large part of 1991 as I tried to write up the thesis and the rest of my family for their interest in my work and support over the years.

# LIST OF CONTENTS

## Page No.

ABSTRACT  
ACKNOWLEDGEMENTS  
LIST OF CONTENTS  
LIST OF FIGURES  
LIST OF TABLES  
NOMENCLATURE

|                  |                                                  |    |
|------------------|--------------------------------------------------|----|
| <u>CHAPTER 1</u> | <u>GENERAL INTRODUCTION</u>                      | 1  |
| 1.1              | INTRODUCTION                                     | 1  |
| 1.2              | NEWTONIAN AND NON-NEWTONIAN FLUIDS               | 1  |
| 1.3              | LAY OUT OF THE THESIS                            | 2  |
| <u>CHAPTER 2</u> | <u>LITERATURE REVIEW</u>                         | 4  |
| 2.1              | INTRODUCTION                                     | 4  |
| 2.2              | BLENDING IN THE TURBULENT REGIME                 | 5  |
| 2.3              | BLENDING IN THE TRANSITIONAL AND LAMINAR REGIMES | 14 |
| 2.4              | NON-NEWTONIAN FLUIDS                             | 20 |
| 2.4.1            | Determination of the Characteristic Shear Rate   | 20 |
| 2.4.2            | Blending of Non-Newtonian Fluids                 | 28 |
| 2.4.3            | Reviews                                          | 32 |
| 2.5              | DESIGN METHODS AND ECONOMIC EVALUATION           | 33 |
| 2.6              | VISCO-ELASTICITY                                 | 36 |
| 2.7              | CONCLUSIONS                                      | 38 |
| <u>CHAPTER 3</u> | <u>EQUIPMENT</u>                                 | 40 |
| 3.1              | INTRODUCTION                                     | 40 |
| 3.2              | VESSELS                                          | 40 |
| 3.3              | IMPELLERS                                        | 40 |
| 3.4              | CONDUCTIVITY PROBES                              | 41 |
| 3.5              | TEST FLUIDS                                      | 42 |
| 3.5.1            | Newtonian Test Fluid                             | 42 |
| 3.5.2            | Non-Newtonian Fluid                              | 44 |
| 3.5.3            | Tracer                                           | 45 |
| 3.6              | RHEOLOGY MEASUREMENT                             | 45 |
| 3.6.1            | Newtonian Fluids                                 | 45 |
| 3.6.2            | Non-Newtonian Fluids                             | 46 |

|                         |                                                 |           |
|-------------------------|-------------------------------------------------|-----------|
| <b><u>CHAPTER 4</u></b> | <b><u>BLENDING OF NEWTONIAN FLUIDS</u></b>      | <b>48</b> |
| 4.1                     | INTRODUCTION                                    | 48        |
| 4.2                     | METHOD                                          | 48        |
| 4.2.1                   | Instrumentation Chain                           | 48        |
| 4.2.2                   | Blend Time Determination                        | 49        |
| 4.3                     | DATA ANALYSIS                                   | 50        |
| 4.4                     | TURBULENT DATA RE-ANALYSIS                      | 52        |
| 4.5                     | ANALYSIS OF VISCOUS BLEND TIME DATA             | 55        |
| 4.5.1                   | Individual Probe Blend Times                    | 55        |
| 4.5.2                   | Root Mean Square Blend Times                    | 57        |
| 4.5.3                   | Correlation of Data                             | 58        |
| 4.6                     | DISCUSSION                                      | 60        |
| 4.6.1                   | Comparison using Power Input                    | 60        |
| 4.6.2                   | Use of the Correlation for Agitator Design      | 63        |
| 4.7                     | CONCLUSIONS                                     | 65        |
| <b><u>CHAPTER 5</u></b> | <b><u>BLENDING OF PSEUDO-PLASTIC FLUIDS</u></b> | <b>68</b> |
| 5.1                     | INTRODUCTION                                    | 68        |
| 5.2                     | PSEUDO-PLASTIC FLUIDS                           | 68        |
| 5.2.1                   | The Power Law Model                             | 68        |
| 5.2.2                   | Shear Rates                                     | 70        |
| 5.2.3                   | Wall Shear Rate Estimation                      | 70        |
| 5.2.3.1                 | The Vessel Wall                                 | 72        |
| 5.2.3.2                 | The Vessel Base                                 | 73        |
| 5.2.3.3                 | The Baffles                                     | 73        |
| 5.2.3.4                 | Combining                                       | 74        |
| 5.3                     | ANALYSIS OF PSEUDO-PLASTIC BLEND TIME DATA      | 76        |
| 5.3.1                   | Individual Probe Blend Times                    | 76        |
| 5.3.2                   | Root Mean Square Blend Times                    | 77        |
| 5.3.3                   | Correlation of Data                             | 77        |
| 5.3.3.1                 | Turbulent Data                                  | 77        |
| 5.3.3.2                 | Viscous Data                                    | 78        |
| 5.4                     | DISCUSSION                                      | 79        |
| 5.4.1                   | Controlling Viscosity                           | 79        |
| 5.4.2                   | Comparison using Power Input                    | 80        |
| 5.4.3                   | Measurement of Rheology                         | 81        |
| 5.4.4                   | Data Correlation                                | 83        |
| 5.5                     | CONCLUSIONS                                     | 84        |
| <b><u>CHAPTER 6</u></b> | <b><u>POWER MEASUREMENT</u></b>                 | <b>88</b> |
| 6.1                     | INTRODUCTION                                    | 88        |
| 6.2                     | METHOD                                          | 88        |
| 6.3                     | POWER NUMBER                                    | 90        |

|       |                                |    |
|-------|--------------------------------|----|
| 6.4   | RESULTS                        | 91 |
| 6.5   | DISCUSSION                     | 92 |
| 6.5.1 | Shear Rate and Reynolds Number | 92 |
| 6.5.2 | Comparison with Other Workers  | 92 |
| 6.6   | CONCLUSIONS                    | 94 |

## CHAPTER 7 ECONOMIC ANALYSIS 95

|       |                                                  |     |
|-------|--------------------------------------------------|-----|
| 7.1   | INTRODUCTION                                     | 95  |
| 7.2   | METHOD                                           | 95  |
| 7.2.1 | Capital Cost                                     | 96  |
| 7.2.2 | Running Cost                                     | 96  |
| 7.2.3 | Analysis                                         | 98  |
| 7.3   | RESULTS                                          | 100 |
| 7.4   | DISCUSSION                                       | 101 |
| 7.4.1 | Comparisson of Experimental and Economic Results | 101 |
| 7.4.2 | Power Number                                     | 102 |
| 7.5   | CONCLUSIONS                                      | 102 |

## CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

|       |                                       |     |
|-------|---------------------------------------|-----|
|       |                                       | 104 |
| 8.1   | INTRODUCTION                          | 104 |
| 8.2   | CONCLUSIONS                           | 104 |
| 8.2.1 | Newtonian Blend Times                 | 104 |
| 8.2.2 | Pseudo-plastic Blend Times            | 106 |
| 8.2.3 | Economic Analysis                     | 108 |
| 8.3   | RECOMMENDATIONS FOR FUTURE WORK       | 108 |
| 8.3.1 | Vessel Geometry                       | 108 |
| 8.3.2 | Fluid Physical Properties             | 109 |
|       | 8.3.2.1 Rheological Properties        | 109 |
|       | 8.3.2.2 Physical Property Differences | 110 |
| 8.3.3 | Measurement of Flow                   |     |

|            |     |
|------------|-----|
| REFERENCES | 111 |
|------------|-----|

### TABLES

### FIGURES

### APPENDIX

## LIST OF FIGURES

| <u>Figure No.</u> | <u>Title</u>                                                                                                   |
|-------------------|----------------------------------------------------------------------------------------------------------------|
| 3.1               | Vessel Arrangement                                                                                             |
| 3.2               | Pitched blade turbine                                                                                          |
| 3.3               | Flat blade turbine                                                                                             |
| 3.4               | Hydrofoil                                                                                                      |
| 3.5               | Conductivity Probe                                                                                             |
| 3.6               | Conductivity Probe Positions                                                                                   |
| 4.1               | Typical Conductivity Response                                                                                  |
| 4.2               | Typical Plot of $\sigma_c^2$ versus Time                                                                       |
| 4.3               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Newtonian data - $T/3$ Pitched blade turbine      |
| 4.4               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Newtonian data - $T/2$ Pitched blade turbine      |
| 4.5               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Newtonian data - $T/3$ Flat blade turbine         |
| 4.6               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Newtonian data - $T/2$ Lightnin A310 Hydrofoil    |
| 4.7               | Plot of Root Mean Square $N\theta$ versus Reynolds number<br>Newtonian data - $T/3$ Pitched blade turbine      |
| 4.8               | Plot of Root Mean Square $N\theta$ versus Reynolds number<br>Newtonian data - $T/2$ Pitched blade turbine      |
| 4.9               | Plot of Root Mean Square $N\theta$ versus Reynolds number<br>Newtonian data - $T/3$ Flat blade turbine         |
| 4.10              | Plot of Root Mean Square $N\theta$ versus Reynolds number<br>Newtonian data - $T/2$ Lightnin A310 Hydrofoil    |
| 4.11              | Plot of Blend Time Correlation<br>Newtonian data - $T/3$ Pitched blade turbine                                 |
| 4.12              | Plot of Blend Time Correlation<br>Newtonian data - $T/2$ Pitched blade turbine                                 |
| 4.13              | Plot of Blend Time Correlation<br>Newtonian data - $T/3$ Flat blade turbine                                    |
| 4.14              | Plot of Blend Time Correlation<br>Newtonian data - $T/2$ Lightnin A310 Hydrofoil                               |
| 4.15              | Plot of Blend Time Correlation<br>Newtonian data - All impellers                                               |
| 5.1               | Plot of Shear Stress versus Shear Rate for a Pseudo-plastic Fluid                                              |
| 5.2               | Vessel Geometry assumed for estimation of the Wall Shear Rate                                                  |
| 5.3               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Pseudo-plastic data - $T/3$ Pitched blade turbine |
| 5.4               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Pseudo-plastic data - $T/2$ Pitched blade turbine |
| 5.5               | Plot of Individual Probe $N\theta$ versus Reynolds number<br>Pseudo-plastic data - $T/3$ Flat blade turbine    |

- 5.6 Plot of Individual Probe  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/2$  Lightnin A310 & Chemineer HE3 Hydrofoils
- 5.7 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/3$  Pitched blade turbine
- 5.8 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/2$  Pitched blade turbine
- 5.9 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/3$  Flat blade turbine
- 5.10 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/2$  Lightnin A310 & Chemineer HE3 Hydrofoils
- 5.11 Plot of Blend Time Correlation  
All data -  $T/3$  Pitched blade turbine
- 5.12 Plot of Blend Time Correlation  
All data -  $T/2$  Pitched blade turbine
- 5.13 Plot of Blend Time Correlation  
All data -  $T/3$  Flat blade turbine
- 5.14 Plot of Blend Time Correlation  
All data -  $T/2$  Lightnin A310 & Chemineer HE3 Hydrofoils
- 5.15 Plot of Blend Time Correlation  
Newtonian & Pseudo-plastic data - All impellers
- 5.16 “Max-Blend” Impeller by Sumitomo Heavy Industries
- 
- 6.1 Plot of power number versus Reynolds number  
All data -  $T/3$  Pitched blade turbine
- 6.2 Plot of power number versus Reynolds number  
All data -  $T/2$  Pitched blade turbine
- 6.3 Plot of power number versus Reynolds number  
All data -  $T/3$  Flat blade turbine
- 6.4 Plot of power number versus Reynolds number  
All data -  $T/2$  Lightnin A310 & Chemineer HE3 Hydrofoils
- 
- 7.1 Plot of Costs versus power number -  $K = 2.00, n = 1.0$
- 7.2 Plot of Costs versus power number -  $K = 2.67, n = 0.9$
- 7.3 Plot of Costs versus power number -  $K = 3.55, n = 0.8$
- 7.4 Plot of Costs versus power number -  $K = 4.74, n = 0.7$
- 7.5 Plot of Costs versus power number -  $K = 6.31, n = 0.6$
- 7.6 Plot of Costs versus power number -  $K = 8.42, n = 0.5$
- 7.7 Plot of Costs versus power number -  $K = 11.22, n = 0.4$

## LIST OF TABLES

| <u>Table No.</u> | <u>Title</u>                                                                       |
|------------------|------------------------------------------------------------------------------------|
| 3.1              | Impeller Dimensions                                                                |
| 3.2              | Conductivity Probe Positions                                                       |
| 3.3              | Power Law Constant and Index of Non-Newtonian Fluids                               |
| 4.1              | Newtonian Blend Time Results - $T/3$ Pitched blade turbine                         |
| 4.2              | Newtonian Blend Time Results - $T/2$ Pitched blade turbine                         |
| 4.3              | Newtonian Blend Time Results - $T/3$ Flat blade turbine                            |
| 4.4              | Newtonian Blend Time Results - $T/2$ Lightnin A310 Hydrofoil                       |
| 4.5              | $N\theta$ versus Reynolds number - Regression Results                              |
| 4.6              | Correlation Regression Results                                                     |
| 4.7              | Comparison of Constants                                                            |
| 5.1              | Pseudo-plastic Blend Time Results - $T/3$ Pitched blade turbine                    |
| 5.2              | Pseudo-plastic Blend Time Results - $T/2$ Pitched blade turbine                    |
| 5.3              | Pseudo-plastic Blend Time Results - $T/3$ Flat blade turbine                       |
| 5.4              | Pseudo-plastic Blend Time Results - $T/2$ Lightnin A310 & Chemineer HE3 Hydrofoils |
| 5.5              | $N\theta$ versus Reynolds number - Regression Results                              |
| 5.6              | Correlation Regression Results                                                     |
| 5.7              | Comparison of Constants                                                            |
| 6.1              | Power Numbers - $T/3$ Pitched blade turbine                                        |
| 6.2              | Power Numbers - $T/2$ Pitched blade turbine                                        |
| 6.3              | Power Numbers - $T/3$ Flat blade turbine                                           |
| 6.4              | Power Numbers - $T/2$ Lightnin A310 & Chemineer HE3 Hydrofoils                     |
| 7.1              | Selling Price (Capital Cost) versus Shaft Torque<br>(from Rigden (1991))           |
| 7.2              | Power Law Constants and Indices of Fluids in Economic Analysis                     |

## NOMENCLATURE

| <u>Symbol</u>       | <u>Meaning</u>                                 | <u>Units</u>                    |
|---------------------|------------------------------------------------|---------------------------------|
| $a$                 | Constant or Exponent                           | -                               |
| $b$                 | Number of blades                               | -                               |
| $C$                 | Concentration                                  | kmol m <sup>-3</sup>            |
| $C_N$               | Natrosol concentration                         | %wt./wt.                        |
| $c$                 | Constant                                       | -                               |
| $c$                 | Clearance ( $= (T - D)/2$ )                    | m                               |
| $D$                 | Impeller diameter                              | m                               |
| $Fl$                | Pumping number ( $= Q / N D^3$ )               | -                               |
| $Fo$                | Fourier number ( $= \rho \theta / \mu T^2$ )   | -                               |
| $Fr$                | Froude number ( $= N^2 D / g$ )                | -                               |
| $g$                 | Acceleration due to gravity                    | m s <sup>-2</sup>               |
| $H$                 | Fluid depth                                    | m                               |
| $K$                 | Power law constant                             | Ns <sup>n</sup> m <sup>-2</sup> |
| $k_i$               | Constant relating to quantity $i$              | -                               |
| $K_p$               | Constant ( $= Po Re$ ) in the laminar regime   | -                               |
| $M$                 | Mixing index                                   | -                               |
| $N$                 | Impeller rotational speed                      | s <sup>-1</sup>                 |
| $n$                 | Power law index                                | -                               |
| $P$                 | Power                                          | W                               |
| $Po$                | Power number ( $= P / \rho N^3 D^5$ )          | -                               |
| $Q$                 | Flow rate                                      | m <sup>3</sup> s <sup>-1</sup>  |
| $Re$                | Reynolds number ( $= \rho N D^2 / \mu$ )       | -                               |
| $T$                 | Vessel diameter                                | m                               |
| $t$                 | Time                                           | s                               |
| $V$                 | Volume                                         | m <sup>3</sup>                  |
| $v$                 | Velocity                                       | m s <sup>-1</sup>               |
| $w$                 | Blade width                                    | m                               |
| $x$                 | Distance                                       | m                               |
| $y$                 | Exponent                                       | -                               |
| <u>Greek Symbol</u> | <u>Meaning</u>                                 | <u>Units</u>                    |
| $\alpha$            | Kinematic fluid property                       | m <sup>2</sup> s <sup>-1</sup>  |
| $\Gamma$            | Time interval                                  | s                               |
| $\dot{\gamma}$      | Shear rate                                     | s <sup>-1</sup>                 |
| $\bar{\epsilon}$    | Mean power input per unit mass                 | W kg <sup>-1</sup>              |
| $\theta$            | Blend time                                     | s                               |
| $\kappa$            | Constant relating shear rate to impeller speed | -                               |
| $\Lambda$           | Torque                                         | Nm                              |
| $\mu$               | Dynamic viscosity                              | Ns m <sup>-2</sup>              |
| $\rho$              | Density                                        | kg m <sup>-3</sup>              |
| $\sigma_i$          | Standard deviation of quantity $i$             | Units of $i$                    |
| $\tau$              | Shear stress                                   | N m <sup>-2</sup>               |

| <u>Subscript</u> | <u>Meaning</u>     |
|------------------|--------------------|
| <i>a</i>         | Apparent           |
| <i>c</i>         | Cavern             |
| <i>crit</i>      | Critical           |
| <i>FBT</i>       | Flat blade turbine |
| <i>Hyd</i>       | Hydrofoil          |
| <i>large</i>     | Large scale        |
| <i>RMS</i>       | Root mean square   |
| <i>small</i>     | Small scale        |
| <i>y</i>         | Yield              |
| 1, 2, 3          | Probe 1, 2, 3      |

Other symbols used in this thesis are defined in the text

## CHAPTER 1

### GENERAL INTRODUCTION

#### **1.1 INTRODUCTION**

Blending is a widely used operation in the chemical and process industries. Its objective is to take two or more miscible fluids and blend them to a pre-determined degree of homogeneity. The time taken to reach this degree of homogeneity is called the “blend time”.

The most common processes where blending is used as the design basis are simple mixing duties where the product is simply the mixture, reactions where the chemicals are blended in order to promote the reaction and storage vessels where “minimum motion” is required to prevent stratification or a skin forming on the surface of the fluid.

There is a great deal of useful information on the blending of low viscosity fluids and this is rarely considered a difficult duty since blend times are usually short relative to other times associated with the process, such as the time taken to fill or empty the vessel. As the fluid viscosity increases the process becomes increasingly difficult and blend times longer until they limit the rate at which the process can proceed. In these cases knowledge of the blend time is much more important.

Most of the work done in the past on the blending of viscous fluids has concentrated on those exhibiting a Newtonian rheology yet the vast majority of viscous fluids processed in industry are non-Newtonian. Thus, there is very little relevant information available to engineers who have the responsibility to select agitation equipment for non-Newtonian blending duties.

#### **1.2 NEWTONIAN AND NON-NEWTONIAN FLUIDS**

The rheology of a fluid is determined by measuring the shear stress at different shear rates using a rheometer. A Newtonian fluid is characterised by a linear relationship between shear stress and shear rate and, by definition, a constant viscosity. The analysis of blend time data taken in Newtonian fluids is fairly simple because the viscosity is the same in all regions of the vessel, ie. those experiencing high shear, near the impeller, and those experiencing low shear, near the fluid surface and vessel wall.

An agitator rotating in a moderately viscous fluid is usually operating in the transitional regime ( $100 < Re < 10^4$ ). Here, the product of the impeller rotational speed and blend time ( $N\theta$ ) is inversely proportional to Reynolds number which is a property of the fluid viscosity.

Non-Newtonian fluids are characterised by a non-linear shear stress - shear rate relationship and their viscosity is dependent on the shear rate which they experience. So the viscosity of the fluid will vary according to its position in the vessel and the type of rheology it exhibits.

This research project is concerned with the blending of shear-thinning, or pseudo-plastic, fluids. These have a low viscosity at high shear rates and high viscosity at low shear rates. Therefore, in an agitated vessel, the viscosity will be lowest near the impeller and highest near the walls and fluid surface. An important question which must be answered is can the fluid viscosity be characterised in such a way that a Newtonian type relationship between the blend time and Reynolds number be obtained? If not, can the effect of non-Newtonian behaviour on blending performance be quantified in another way?

Other questions which need to be answered for engineers selecting and designing agitators are: is a certain size and type of impeller best suited for the blending of pseudo-plastic fluids and if so, over what range of conditions can it be applied? Also, the penalties for using non-optimum impeller design need to be investigated.

### 1.3 LAY OUT OF THE THESIS

The thesis contains seven main chapters and they are briefly introduced below:

Chapter 2 is a literature review, the conclusions of which have identified the “gaps” in the current knowledge of non-Newtonian blending. This has been used to determine the experimental objectives of this research.

Chapter 3 describes the equipment, instrumentation and test fluids used in the project.

Chapter 4 covers the experimental technique, data analysis method and the results of the Newtonian blend time measurements. A new blend time correlation is presented which contains three dimensionless groups: power number,  $Po$ , Reynolds number,  $Re$ , and Fourier number,  $Fo$ .

Chapter 5 covers the measurements made in pseudo-plastic fluids. A new method of correlating the pseudo-plastic blending data is proposed and comparison between Newtonian and pseudo-plastic data is made.

Chapter 6 covers power measurements and presents plots of power number versus Reynolds number for each of the impellers tested. Comparison with literature data is made.

Chapter 7 describes a cost-benefit analysis of the research presented in this thesis.

Chapter 8 is a final general discussion of the results and presentation of the conclusions of the research. Finally, suggestions for future work are presented.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 INTRODUCTION

The performance of various impeller types is discussed in the literature review and detailed information on their geometries can be found in text books such as “Mixing in the Process Industries” edited by Harnby, Nienow & Edwards (Butterworths, 1985) or “Fluid Mixing Technology” by Oldshue (M<sup>c</sup>Graw-Hill, 1983).

The objectives of this chapter are to compare the various experimental techniques used to carry out blend time measurements in agitated vessels and the results reported in the open literature. The advantages and disadvantages of the various measurement techniques are discussed and the results compared. The literature has been divided into three sections: the first reviews the measurement of blend time in turbulent systems, the second discusses blend time in viscous, Newtonian systems and the third, viscous, non-Newtonian systems.

The subject of this thesis is blending of viscous fluids but it is important to consider turbulent blending because this gives a base-line against which the performance of agitators in viscous blending can be compared.

Methods of estimating shear rates are compared because the viscosity of a non-Newtonian fluid is determined by the shear rate it is experiencing. The viscosity of the fluid will vary according to its position in an agitated vessel and it is important to be able to identify the viscosity which controls the performance of an agitator in any non-Newtonian mixing duty.

A brief catalogue of non-Newtonian mixing reviews is followed by a review of agitator design methods available and a brief review of literature on the subject of visco-elasticity.

Finally, the work covered in this thesis is put into context with the conclusions drawn from the information available in the open literature and the identification of “gaps” in current knowledge.

## 2.2 BLENDING IN THE TURBULENT REGIME

The most widely used blend time measurement technique involves the addition of a small volume of tracer to an agitated vessel and recording the change which takes place in the vessel as the tracer is blended into the bulk fluid. The tracer will have a property which is different from the bulk fluid but does not affect the way in which the two fluids mix. The property must be identifiable and measurable as it is diluted in the bulk fluid. This property can be electrolyte concentration, temperature or acidity.

- i) Concentration differences are measured using conductivity probes. The concentration is measured at one or more points in the vessel and the blend time is defined as the time taken for the tracer to be dispersed to a specified degree of homogeneity.
- ii) Temperature differences are measured using a thermocouple and the same principle applies to this technique as to the conductivity technique, ie. a slug of hot fluid is added to the vessel as tracer and the blend time is the time taken for the temperature differences to be homogenised.
- iii) An acid, or alkaline, tracer is added to the bulk fluid in order to cause a colour change in a chemical indicator. Blend time is determined from the time taken for the last wisp of colour to disappear as mixing progresses. This technique also allows poorly mixed regions of the vessel to be identified, since this is where the last wisp of colour will disappear. One drawback with this technique is the effect of stoichiometry on the apparent blend time and this has been discussed by Norwood & Metzner (1960).

The first two methods have the advantage of being objective measurement techniques. The fluctuations recorded by the conductivity probe or thermocouple can be recorded using a chart recorder or digitally, on a computer disk, and analyzed after the experiment has been completed. The colour change technique is subjective and relies on the experimenter's judgement on when blending is complete.

Care must be taken when using conductivity or temperature differences to ensure that the density of the tracer is not significantly different from the bulk fluid in the vessel because it is possible that the density difference between the tracer and the bulk fluid will affect the way in which the blending process takes place.

Landau & Procházka (1961) compared these three blend time measurement techniques and concluded that the results obtained were mutually consistent. So it is possible to compare the blend time measurements reported in the literature, provided one of these three techniques was used and the results are compared with consideration given to the drawbacks associated with each technique.

Blend time experimental data can generally be divided into two groups: in the first, the dimensionless group,  $N\theta$ , is a constant, dependent on impeller diameter and type, and independent of Reynolds number and, in the second, the value of  $N\theta$  is dependent on impeller diameter, type and Reynolds number. This is the main contradiction in the correlation of the data and it has important implications for scale-up.

Kramers *et al.* (1953), Procházka & Landau (1961), Biggs (1963), Khang (1975), Rielly & Britter (1985), Sano & Usui (1985, 1987), Mackinnon (1987), Komori & Murakami (1988) and Raghav Rao & Joshi (1988) all used the conductivity technique to measure blend times. The majority of the work reported in these references was carried out in vessels of up to 0.61 m diameter. Only Mackinnon (1987) worked at large scale with vessels of 0.61, 1.83 and 2.67 m diameter. The methods employed and results are described below:

Kramers *et al.* (1953) measured blend times at two scales, 0.31 and 0.64 m with  $H = T$ , using two conductivity probes positioned  $T/8$  above the vessel base and below the surface of the liquid. A marine propeller, of  $T/4$  diameter, was studied at both scales and flat and curved blade Rushton turbines, of the same diameter, were studied at the large scale only. A variety of baffle types were examined. They concluded that  $N\theta$  is constant, within  $\pm 10\%$  and for scale-up, with geometrical similarity and constant impeller tip speed, constant power per unit mass is required for equal blend time. They also concluded that vortex formation increases blend time and that Froude number should be included in a mixing time correlation to take this into account.

Procházka & Landau (1961) applied the conductivity technique to measuring blend times with a propeller, pitched blade and Rushton turbines. The ratio  $D/T$  was varied between 0.10 and 0.33, which is smaller than industrial standards (where  $D/T$  is typically between 0.2 and 0.7). They correlated their results by:

$$N\theta = k \left( \frac{T}{D} \right)^{(2+a)} \quad \dots \text{Eqn(2.1)}$$

ie,  $N\theta$  is a constant, and its value is dependent on the type and geometry of the impeller.

The values of  $k$  were 3.48, 2.02 and 0.91 for the propeller, pitched blade turbine and Rushton turbine respectively. Similarly, the values of  $a$  were 0.05, 0.20 and 0.57. They also concluded that the correlation of the data was unaffected by the size of the tracer charge or the position of its addition.

Biggs (1963) used a conductivity technique to measure the time taken for homogeneity to be obtained in an agitated vessel with continuous flow in and out (known as a continuous stirred tank reactor or CSTR). The vessel was 0.24 m diameter and propellers, pitched blade, flat blade and Rushton turbines with  $D/T$  varying between 0.26 and 0.42, were studied. Three times were identified: the first was the time for the conductivity trace to reach 10% of the fully mixed value; the second was the time for the trace to reach its maximum value and the third, the “terminal mixing time” or blend time. The Rushton turbine blend times were correlated by:

$$\theta = \frac{1.55}{N^{0.84} D^{1.93}} \quad \dots \text{Eqn(2.2)}$$

Biggs compared the terminal mixing time in the CSTR to batch mixing times measured by other workers and justified this comparison because the ratio of residence time to blend time was of the order of 100:1. The conductivity fluctuations were unaffected by the flow of the fluid through the vessel.

Khang (1975) and Khang & Levenspiel (1976a, 1976b) used residence time distribution theory and probability to predict the mixing rate of a slug of tracer in an agitated vessel. A conductivity technique was used in 0.56 and 1.22 m diameter vessels using two sizes of Rushton turbine and marine propeller. The data were correlated by:

$$\left(\frac{N}{A}\right) \left(\frac{D}{T}\right)^a = k \quad \dots \text{Eqn(2.3)}$$

The value of  $a$  was 2.3 for the Rushton turbine and 2.0 for the marine propeller.  $A$  is an amplitude decay rate constant, found from analysis of the peaks and troughs in the conductivity trace for each experiment. A design method is proposed, showing how the

data correlation can be used to size agitators for blending duties.

Rielly & Britter (1985) measured mixing times in a vessel 0.29 m diameter, with  $H = T$ , using a Rushton turbine. They investigated the effect of the probe and tracer volume on the measured blend time. They concluded that a large probe volume measures a shorter blend time than a probe of small volume but the volume of tracer does not affect the measurements. They also concluded that  $N\theta$  is not constant in the turbulent regime and this may be an effect of the probe volume and its resolution. A plot of  $N\theta$  versus Reynolds number was presented but the mean value of  $N\theta$  and its standard deviation were not, so it is not possible to determine how their results compare with other workers.

Sano & Usui (1985, 1987) measured blend times, based on the conductivity technique, and impeller discharge flow, using a flow follower technique, in vessels of 0.2 and 0.4 m diameter. The impellers studied were a Rushton turbine and two-bladed paddle. They concluded that  $N\theta$  and  $Fl$  are constants, independent of Reynolds number and correlated them with the impeller dimensions. They found that the blend time was proportional to the circulation time, with the number of circulations required to achieve homogeneity given by:

$$m = 3.8 \left( \frac{D}{T} \right)^{0.5} \left( \frac{b w}{T} \right)^{0.1} \quad \dots \text{Eqn(2.4)}$$

where  $m$  is the number of circulations.

Mackinnon (1987) reviewed the research which had been carried out by the FMP consortium. FMP measured blend time using the same technique at three scales; 0.61, 1.83 and 2.87 m. Eleven radial, mixed and axial flow impellers were studied and the data taken for all impellers at all scales were correlated by:

$$Po^{1/3} N\theta \left( \frac{D}{T} \right)^{13/6} = k \quad \dots \text{Eqn(2.5)}$$

ie.  $N\theta$  is a constant for a given impeller.

Blend times were measured in three regions of differing agitation intensity; close to the

impeller, behind a baffle and halfway between the agitator shaft and vessel wall. The blend time was not found to vary significantly around the vessel.

Komori & Murakami (1988) measured power consumption and blend and circulation times in a baffled vessel 0.29 m diameter with  $H = T$  and  $H = 2T$ . The test fluid was water and the measurements were carried out at Reynolds numbers of  $2.5 \times 10^4$ ,  $4.8 \times 10^4$  and  $6.2 \times 10^4$ . The blend times were measured using a conductivity technique and the flow patterns using a laser-Doppler anemometer. The impellers were single and dual  $T/2$  flat blade turbines and the spacing between the dual impellers was varied. They concluded that turbulent blending is strongly dominated by large recirculating flows in the axial and radial directions. Greater energy efficiency is achieved by creating larger scale flows and reducing the number of small scale recirculating flows in the vessel.

Raghav Rao & Joshi (1988) have measured blend times at four scales: 0.3, 0.57, 1.0 and 1.5 m diameter and compared their results with a theoretical approach to the blending process. Rushton turbines and upward and downward pumping pitched blade turbines were compared and the effect of impeller clearance examined. They concluded that  $N\theta$  is constant and dependent on the vessel and impeller diameters, fluid depth and impeller off-bottom clearance. Of all the impellers tested, the most efficient was the downward pumping pitched blade turbine of  $T/3$  diameter, a conclusion which differs from every other worker in this area.

The thermal method was used by Havas *et al.* (1978b) and Shiue & Wong (1984).

Havas *et al.* (1978b) measured blend times at two vessel scales, 0.40 and 0.53 m., using standard propellers, proprietary S. E. M. and VEGYTERV propellers and a pitched blade turbine. They correlated their data by:

$$k = P_o (N\theta)^3 \left( \frac{D}{T} \right)^5 \quad \dots \text{Eqn(2.6)}$$

$k$  was plotted against  $D/T$  for the VEGYTERV propeller and it was found that the lowest value of  $k$  occurs when  $D/T \approx 0.55$ . This shows that this diameter will achieve a desired blend time for the lowest power input and is, therefore, the most energy efficient.

Shiue & Wong (1984) measured blend times in a 0.4 m diameter vessel using a wide variety of impeller types and sizes, including the VEGYTERV propeller and using draught tubes for the two and four blade pitched turbines studied. They compared the efficiency of the various impellers by plotting the relationship proposed by Hoogendoorn & Den Hartog (1967) and concluded that the pitched blade turbines in draught tubes were the most efficient impellers tested and Rushton turbines at  $H/2$  clearance, the least efficient.

The colour change method was used by Fox & Gex (1956), Norwood & Metzner (1960) and Brennan & Lehrer (1976)

Fox & Gex (1956) measured blend times at three scales, 0.3, 1.5 and 4.2 m diameter, using propellers and jet mixers in unbaffled vessels. The technique used at the small scale was an acid-base colour change with blend time measured with the agitator running. At the large scale, measurement of iodine value (a measure of the degree of saturation) of batches of vegetable oil was used, with the agitator stationary at the start of an experiment. They correlated their low viscosity propeller data by:

$$N\theta \propto \left( \frac{Fr}{Re} \right)^{1/6} \frac{H^{1/2} T}{D^{3/2}} \quad \dots \text{Eqn(2.7)}$$

and jet mixer data by an identical relationship, based on the Reynolds and Froude numbers at the jet nozzle.

Fox & Gex concluded that the difference in measured blend times, due to the different techniques used at small and large scales, was the order of 25%. The fact that Froude number appears in the correlation of the data may be due to the fact that blend times were measured in unbaffled vessels, as suggested by Kramers *et al.* (1953), causing a vortex to form on the liquid surface.

Norwood & Metzner (1960) measured blend times, with an acid-base colour change and fluid velocities, using an electric velocity probe, the resistance of which was a function of the velocity of the fluid flowing past it. Three scales were studied ranging from 0.14 to 0.39 m. The only impeller type studied was a Rushton turbine with  $D/T$  ranging between 0.18 and 0.74 with various blade widths. Blend times were correlated by:

$$N\theta \propto \frac{Fr^{1/6} H^{1/2} T^{3/2}}{D^2} \quad \dots \text{Eqn(2.8)}$$

Norwood & Metzner concluded that, although their correlation was of a similar form to Fox and Gex's, turbine impellers were more efficient for blending than propellers or jet mixers.

Brennan & Lehrer (1976) measured blend times in a vessel of 0.42 m diameter for Rushton and flat blade turbines of  $D/T$  between 0.15 and 0.24. A colour change technique was used to determine blend times and the main conclusion of the work was that  $N\theta \neq \text{constant}$ . They did not give a correlation of the data.

Finally, Van de Vusse (1955a, 1955b) used the Schlieren method to measure blend times in vessels ranging between 2 and 45 litres volume. The Schlieren method involves creating two layers of fluid having significant differences in refractive index. In practise this was achieved by superimposing two layers of fluid with different densities. Differences in refractive indices of the two fluids were detected by shining a beam of light through the fluid, forming Schlieren lines on a screen on the opposite side of the vessel. As mixing proceeds the intensity of the Schlieren lines decreases as the density difference and refractive indices are homogenised. The results obtained using this experimental method can be compared with those obtained using other techniques but there are two main differences which must be noted: the first is that large density differences are required to achieve clear Schlieren lines and second, the agitator must be stationary at the start of an experiment, with blend time measured from the moment the agitator starts to rotate. Four types of impeller were studied giving predominantly radial, tangential or axial flow patterns. The data were correlated by:

$$N\theta \propto \left( \frac{T}{D} \right)^3 \left( \frac{\rho N^2 D^2}{\Delta\rho g H} \right)^{-0.3} \quad \dots \text{Eqn(2.9)}$$

This correlation holds for  $Re > 10^3 - 10^4$  and shows that for a given set of experimental conditions,  $N\theta$  is a constant.

Van de Vusse (1955b) includes a section on optimum agitator design and the key rules for low viscosity mixing given are to use small vessels if possible, or several small agitators in a large vessel; the optimum  $D/T \approx 0.4$  and  $H/T \approx 0.8$  and the impeller blade width,  $w$ , should be as large as possible. The blade angle was found to have little effect

on blending efficiency.

Van de Vusse found that blend time is proportional to circulation time, calculated from the pumping capacity of the impeller.

Hiraoka & Ito (1977) re-analyzed the data of Kamiwano *et al.* (1962) and produced the following relationship:

$$N\theta = 7.2 \left( \frac{b w}{D} \right)^{-0.35} \frac{T^{1.4} H^{0.5}}{D^{1.9}} \quad \dots \text{Eqn(2.10)}$$

This shows that, for a given impeller and vessel geometry,  $N\theta$  is constant.

Several of the blend time studies have included measurements of circulation time (eg. Komori & Murakami (1988)) and blend time has been found to be proportional to circulation time. Bryant & Sadeghzadeh (1982) and Middleton (1982) both measured circulation time using a “radio pill” technique.

Bryant & Sadeghzadeh (1982) measured circulation times in a vessel of 1 m<sup>3</sup> volume using a neutrally buoyant flow-follower with an internal radio transmitter. An aerial was positioned around the impeller and each time the flow-follower passed through the impeller its passage was recorded. The mean circulation time was correlated by:

$$t = k_t \frac{H}{N D^3} \quad \dots \text{Eqn(2.11)}$$

and the standard deviation of  $t$  by:

$$\sigma = k_\sigma \frac{H^{7/3}}{N D^3} \quad \dots \text{Eqn(2.12)}$$

Equations (2.11) and (2.12) were combined to give a relationship for blend time:

$$\theta \approx k_\theta \frac{H^{11/3}}{N D^3} \quad \dots \text{Eqn(2.13)}$$

Middleton (1982) has measured circulation times in three scales of vessel between 0.18 and 4.4 m<sup>3</sup> volume using the technique proposed by Bryant & Sadeghzadeh (1982). Rushton turbines of diameters between  $T/2$  and  $T/3$  were studied and the effect of gassing was also investigated. The mean circulation time was correlated by:

$$t = 0.5 V^{0.3} \frac{1}{N} \left( \frac{T}{D} \right)^3 \quad \dots \text{Eqn(2.14)}$$

and the standard deviation of the mean by:

$$\sigma = \frac{1.23 V^{1.45}}{D^{2.5} N} \quad \dots \text{Eqn(2.15)}$$

Mean circulation times in plant scale vessels were found to be greater than would be predicted by the ratio of the vessel volume to impeller pumping capacity.

The majority of references in the literature report that  $N\theta$  is constant for a given impeller geometry. This list of references includes the one collection of data taken at large scale which used the same experimental technique at all scales (Mackinnon (1987)). Those that report the data correlated with  $N\theta$  a weak function of Reynolds number were:

- i) taken at one, small scale (Rielly & Britter (1985)).
- ii) used the subjective colour change technique (Brennan & Lehrer (1976)).
- iii) used the colour change technique at small scale and a different technique at large scale (Fox & Gex (1956)).
- iv) drew conclusions regarding the performance of batch mixing agitators from measurements made in a CSTR (Biggs (1963)).

When an agitator is to be designed for turbulent blending duty, and blend time is to be used as a design criterion, the value of  $N\theta$  should be taken as a constant, scaling-up from literature or pilot-plant data. This has the advantage of a great deal of supporting evidence and being more conservative than the alternative correlation where  $N\theta$  is

proportional to Reynolds number raised to the  $-1/6$  power.

### 2.3 BLENDING IN THE TRANSITIONAL AND LAMINAR REGIMES

The type of impeller used for the blending of viscous fluids usually depends on the viscosity of the fluid. Low and medium viscosity fluids are blended using “open” impellers, such as turbines and propellers with  $D \leq T/2$ , while high viscosity fluids are blended with “close-clearance”, or positive displacement, impellers, such as helical or anchor impellers.

One of the problems encountered in the measurement of blend times using a conductivity probe or thermocouple in viscous fluids, is the way in which the boundary layer around the probe affects the measurement being made (Metzner & Astarita (1967)). This has not been found to affect blend time measurements in medium viscosity fluids but measurement of blend time in high viscosity fluids has almost exclusively been made using acid/alkali colour change or dye addition.

Several of the references discussed in the previous section include measurements of blend times with viscous fluids. In these cases one of the objectives of the work was to examine the way in which fluid viscosity affected blending performance and to identify the Reynolds number at which the turbulent blending regime ends. The results of these are discussed below:

Van de Vusse (1955a, 1955b) correlated data taken at  $Re < 250$  by:

$$N\theta \propto \left(\frac{T}{D}\right)^3 \left(\frac{\rho N^2 D^2}{\Delta\rho g H}\right)^{-0.3} Re^{-3} \quad \dots\text{Eqn(2.16)}$$

There is a gap in Van de Vusse’s correlation where no data appear to have been taken, for  $250 < Re < 10^4$ . Other workers have found that  $N\theta \propto Re^{-1}$  in this Reynolds number range. The design rules suggested for  $Re < 250$  are the same as those for the turbulent regime, except that the maximum impeller diameter should be selected.

Fox & Gex (1956) correlated data taken for  $100 < Re < 10^4$  by:

$$N\theta \propto \frac{Fr^{1/6}}{Re} \frac{H^{1/2} T}{D^{3/2}} \quad \dots\text{Eqn(2.17)}$$

They also suggested that a different relationship between  $N\theta$  and Reynolds number may exist for more viscous fluids.

Norwood & Metzner (1960) measured blend times at Reynolds numbers down to 1, but only presented a correlation for the turbulent data, equation (2.8). Instead they plotted the turbulent correlation versus Reynolds number and identified five regimes: turbulent; late, middle and early transitional; and laminar. The boundaries of these regimes coincide with the changes in gradient of the power number versus Reynolds number curve for a Rushton turbine. The fluid viscosity started to affect blend time at a Reynolds number of about 2000 and the laminar regime started at a Reynolds number of about 5.

Landau *et al.* (1963) proposed that Reynolds number and Archimedes number should be considered in the blending of viscous fluids. Archimedes number was proposed to take account of density differences between the bulk fluid and the added tracer. They examined the rate of blending at various positions within the vessel but did not present a correlation for the data. They found that the fluid viscosity starts to affect blend time at Reynolds numbers of about  $10^4$ , depending upon the impeller type. They also found that the blend time was dependent upon the measuring position within the vessel. Figures are presented showing the agitation intensity in different regions of the vessel for different impeller types. The conclusion drawn from the work is that impellers should be fitted in draught tubes to improve flow around the vessel in viscous blending processes.

Khang (1975) measured blend times at a minimum Reynolds number of 500, for Rushton turbines and marine propellers. Like Norwood & Metzner (1960), Khang plotted the turbulent correlation, equation (2.3), versus Reynolds number for both impeller types and the boundary between the transitional and turbulent regimes was found to be at  $2 \times 10^3$  for the Rushton turbines and  $2 \times 10^4$  for the marine propellers. No method of taking the fluid viscosity into account in agitator design is given so the correlations presented can only be applied to blending in the turbulent regime.

Havas *et al.* (1978a) measured blend times for screw, helical ribbon, gate, anchor and multi-paddle agitators using the same technique as employed for their studies of low viscosity blending (Havas *et al.* (1978b)). The range of Reynolds numbers studied were 2 - 500. Plots of  $N\theta$  versus Reynolds number showed that, in the laminar region,  $N\theta$  is a constant for positive displacement impellers, such as helical ribbons and screws. The blending efficiencies of the impellers were compared by plotting the dimensionless

groups proposed by Hoogendoorn & Den Hartog (1967). The most efficient impeller was found to be the screw agitator in a draught tube while the least efficient were the multi-paddle and gate agitators.

Shiue & Wong (1984) measured blend times in viscous fluids for the same range of impellers as studied at low viscosity. Plots of  $N\theta$  versus Reynolds number were given for all impellers tested and the fluid viscosity was found to start to influence the blend time at Reynolds numbers in the region  $10^3 - 10^4$  depending upon the impeller type. Pitched blade turbines in draught tubes were found to be least affected by viscosity and were also found to be the most energy efficient agitators when compared with the others tested using the Hoogendoorn & Den Hartog (1967) technique.

Other workers who have studied the blending performance of open impellers in viscous fluids are discussed below:

Hoogendoorn & Den Hartog (1967) measured blend times in vessels of 0.24 and 1.8 m diameter using Rushton turbines, multiple pitched blade turbines, helical screws and ribbons, marine propellers and anchors. The effect of baffling and draught tubes on blending efficiency was studied. Several important results were found:

For Rushton turbines:

- |      |                            |                                       |
|------|----------------------------|---------------------------------------|
| i)   | $Re > 4 \times 10^3$       | $N\theta = \text{constant}$           |
| ii)  | $170 < Re < 4 \times 10^3$ | $N\theta \propto Re^{-1}$             |
| iii) | $Re < 170$                 | $N\theta \propto Re^{-a} \quad a > 1$ |

The increase in blend time at low Reynolds numbers was found to be caused by “dead zones” behind the baffles so blend times were measured in unbaffled vessels. Blend times were generally shorter but at  $Re < 150$  stable toroidal vortices were formed above and below the impeller. Similar results were found for the other open impeller types studied. Installing the marine propeller and multiple pitched blade turbine in a draught tube was found to improve blending at low Reynolds numbers.

The helical screw was found to behave in a similar way to the Rushton turbines, with  $N\theta$  increasing with decreasing Reynolds number. Installing the impeller in a draught tube caused a change in the performance such that, for  $0.1 < Re < 100$ ,  $N\theta$  was

constant. Similarly, over the same Reynolds number range,  $N\theta$  was constant for the helical ribbon mixer.

The anchor agitator was studied at  $100 < Re < 10^5$  and was found to be inefficient because the fluid rotated at the same speed as the impeller.

The blending performance of the impellers was compared by plotting two dimensionless groups: the first is based on the power required to achieve a certain blend time and is:

$$\frac{P \theta^2}{\mu T^3} \quad \dots \text{Eqn(2.18)}$$

This number is a constant for positive displacement impellers, such as helical ribbons and those in draught tubes, and varies with Reynolds number for open impellers in the laminar regime.

The second dimensionless group was defined as a Reynolds number from which the impeller rotational speed has been eliminated and is in fact a reciprocal Fourier number,  $1/Fo$ :

$$\frac{\rho (T/\theta) T}{\mu} \quad \dots \text{Eqn(2.19)}$$

Equation (2.18) was plotted versus (2.19) and the most efficient impellers were found to be those in draught tubes followed by the helical ribbon in the regime where equation (2.19) has a value less than one. The general conclusion of the work was that agitation of viscous fluids requires good top-to-bottom motion with all streamlines passing through the impeller and sufficient motion to disturb the laminar wall layers.

Zlokarnik (1967) measured blend times with a variety of impellers and produced a set of design rules regarding which impeller type to use under different conditions of viscosity and scale. The determining factor was the Fourier number given in equation (2.19) and the rules are summarised below:

|      |                               |                                        |
|------|-------------------------------|----------------------------------------|
| i)   | $1/Fo > 10^4$                 | Marine propeller in a baffled vessel   |
| ii)  | $2 \times 10^3 < 1/Fo < 10^4$ | Gate impeller in a baffled vessel      |
| iii) | $12.5 < 1/Fo < 2 \times 10^3$ | Paddle impeller in a baffled vessel    |
| iv)  | $0.25 < 1/Fo < 12.5$          | Paddle impeller in an unbaffled vessel |
| v)   | $1/Fo < 0.25$                 | Close clearance helical ribbon         |

Zlokarnik's list of impellers shows that the laminar regime exists at  $1/Fo < 0.25$  since this is the value below which use of a helical ribbon is recommended. This is in good agreement with the conclusions of Hoogendoorn & Den Hartog (1967) who found the laminar regime exist when  $1/Fo < 1$  for all impellers they tested.

Johnson (1967) measured blend times at two scales, 0.10 and 0.15 m, with  $H = T$ , using two sizes of Rushton turbine, a marine propeller and a helical ribbon. Blend times were determined by adding dye to the fluid in the range  $10 < Re < 100$ . The blend time data taken for the three open impellers were correlated by  $N\theta \propto Re^{-13}$ .  $N\theta$  was found to be almost constant for the helical ribbon over the range of Reynolds numbers studied, with a slight decrease with decreasing Reynolds number. This decrease was attributed to the increased drag causing an increase in bulk movement of the fluid.

Other workers have studied the blending performance of close clearance impellers in high viscosity blending duties and these are described below:

Gray (1963) measured blend times in a vessel 0.23 m diameter by 0.30 m deep with a helical ribbon and screw, four-bladed flat and curved turbines and a rod and paddle agitator in the laminar regime. A dye dispersion technique was used, placing a small quantity of coloured test fluid on the surface of the vessel contents. Blend time was determined from the time taken to fully disperse the dye after starting the agitator. The helical ribbon and off-centre helical screw were found to be the most efficient impellers for blending in this regime. Blending was found to be hindered by the formation of a "doughnut shaped" stagnant region which formed below some of the impellers and blending was found to be improved by incorporating methods of reducing its size, such as placing the screw agitator off-centre.

Novák & Rieger (1969, 1975, 1977 see also Rieger & Novák (1973, 1974)) carried out an extensive study of the power consumption and blending efficiency for several geometries of helical screw agitator, anchors and turbines using a conductivity technique in vessels ranging between 0.1 and 1.0 m diameter. For the screw agitators, they found that  $N\theta$  was constant in the laminar regime ( $Re < 100$ ), decreased in the transitional regime and had a lower constant value in the turbulent regime. The most energy efficient positions for installing screws were centrally in a draught tube or off-centre with a small gap between the screw and the vessel wall. The optimum diameter was  $T/2$ .

Coyle *et al.* (1970) measured blend and circulation times for various geometries of helical ribbon impeller in a 0.46 m diameter vessel, using a dye addition method.  $N\theta$  was found to be constant and the blend time about three times longer than the circulation time.

Hall and Godfrey (1971) have also studied blending with helical ribbon impellers and found that  $N\theta$  is independent of Reynolds number in the laminar regime and that a twin flight helix is more efficient than a single flight.

Vonken *et al.* (1964) measured blend and circulation times for an axial flow impeller in a draught tube and found that  $N\theta$  was constant in the laminar regime and decreased with increasing Reynolds number. The blend time was proportional to the circulation time and showed the same dependence on Reynolds number. A stable toroidal vortex formed at the lower end of the draught tube at  $100 < Re < 300$  which could be prevented by increasing or decreasing the impeller speed.

The effect of viscosity on the pumping capacity,  $Fl$ , of impellers has been investigated by Wong & Huang (1988). They measured pumping capacities of radial flow turbines at three scales using hot-wire anemometers in Newtonian fluids up to 50 cP viscosity. They concluded that the pumping capacity is constant for  $Re > 2 \times 10^4$  and decreases as Reynolds number decreases below this value. Tables of  $Fl$  and  $Re$  were included in this paper and these data have been correlated to show that, when  $Re < 2 \times 10^4$ , the pumping capacity is proportional to  $Re^{0.2}$ .

The conclusions that can be drawn from the work reported in the literature on blending of viscous fluids are that there are two blending regimes at Reynolds numbers outside the turbulent regime. Once Reynolds number decreases below a value of around  $10^3$  to  $10^4$ , the exact value being dependent on impeller type,  $N\theta$  is inversely proportional

to Reynolds number. At Reynolds numbers below about 200 the exponent on Reynolds number decreases from -1 to around -10 for open impellers. In practise, if high viscosity blending is required, a close-clearance impeller, such as a helix, should be used. Alternatively, a pitched blade turbine or propeller can be installed in a draught tube. The use of a helix or draught tube produces a constant value for  $N\theta$  in the laminar regime.

## 2.4 NON-NEWTONIAN FLUIDS

The papers reviewed in the previous two sections have studied the blending of low, medium and high viscosity Newtonian fluids. The effect of non-Newtonian behaviour in blending performance has been studied by several workers and these references are reviewed below.

### 2.4.1 Determination of the Characteristic Shear Rate

When studying the blending of Newtonian fluids the viscosity of the fluid does not vary with position in the vessel as it is independent of the shear rate which it is experiencing. The apparent viscosity of non-Newtonian fluids is dependent on the shear rate and, therefore, the shear rate must be taken into account in determination of the viscosity of the fluid.

The work of Metzner & Otto (1957) is the most well-known in this area, although they were using a procedure first reported by Magnusson (1952), to determine the shear rate near the impeller in order to predict its power consumption. Many workers have used this method to determine shear rates produced by other impellers but it has been found that it only works well for prediction of shear rates near the impeller. It is poor for predicting the characteristic shear rate in processes which are limited by flow in regions of the vessel away from the impeller, eg. heat transfer from coils or the vessel wall into the fluid. The various methods are described in chronological order below.

Metzner & Otto's (1957) method involves comparison of the power number versus Reynolds number curve, measured in the laminar regime where power number is inversely proportional to Reynolds number, for Newtonian and non-Newtonian, pseudo-plastic fluids. The value of the constant of proportionality,  $K_p$ , is measured in the Newtonian fluid and the relationship between power number and Reynolds number found. Power numbers are calculated from the measured power consumption in the non-Newtonian fluids. The apparent Reynolds number can then be calculated from:

$$Re_a = \frac{K_p}{Po} \quad \dots \text{Eqn(2.20)}$$

The apparent viscosity of the non-Newtonian fluid can be found by re-arranging the definition of Reynolds number:

$$\mu_a = \frac{\rho N D^2}{Re_a} \quad \dots \text{Eqn(2.21)}$$

The shear rate can then be calculated by assuming that the power law, or Ostwald model describes the rheology of the fluid:

$$\dot{\gamma} = \left( \frac{\mu_a}{K} \right)^{\frac{1}{n-1}} \quad \dots \text{Eqn(2.22)}$$

The shear rates calculated in this way are proportional to the impeller speed. This relationship could, it was claimed, be applied to any flow conditions. When the power numbers were plotted against apparent Reynolds number, the Newtonian and non-Newtonian curves coincided at  $Re < 10$  but the non-Newtonian power numbers were less than the Newtonian ones for  $10 < Re < 200$ . The two curves coincided at Reynolds numbers above 200. It was suggested that the laminar regime may extend to higher Reynolds numbers in pseudo-plastic fluids than in Newtonian fluids.

Metzner & Otto (1957) worked at vessel diameters between 0.15 and 0.56 m with  $D/T$  varying between 0.27 and 0.77 in the laminar regime and 0.18 and 0.5 in the transitional regime. Reynolds number was varied between 2 and 270.

Metzner *et al.* (1961) extended the work reported by Metzner & Otto (1957) to include other impeller types and found that the proportionality between shear rate and impeller speed held for the other impellers tested. The shear rate was related to impeller speed by:

$$\dot{\gamma} = \kappa N \quad \dots \text{Eqn(2.23)}$$

The values of  $\kappa$  were 11.5 ( $\pm 12\%$ ) for the Rushton turbine, 13 ( $\pm 15\%$ ) for the fan turbine (a pitched blade turbine) and 10 ( $\pm 9\%$ ) for single and double pitch marine

propellers. The experimental conditions were essentially the same as those used by Metzner & Otto (1957), but the Reynolds number range was extended to a minimum of 0.1 and maximum of 620.

Good agreement between the Newtonian and non-Newtonian power number - Reynolds number curves was obtained for all the impellers tested. All showed the same trend as the Rushton turbine, reported previously by Metzner & Otto (1957), that the laminar regime exists at a higher Reynolds number in non-Newtonian fluids than in Newtonian. Preliminary work with dilatant fluids indicated that a much more complex relationship between shear rate and impeller speed exists than the direct proportionality found with pseudo-plastic fluids.

Calderbank & Moo-Young (1959, 1961) proposed that the apparent viscosity of the fluid at the impeller in non-Newtonian fluids can be calculated from a similar relationship for the apparent viscosity of a fluid in pipe flow, from which the apparent viscosity of a fluid is:

$$\mu_a = \frac{K}{(8N)^{(1-n)}} \left( \frac{3n + 1}{4n} \right)^n \quad \dots \text{Eqn(2.24)}$$

This relationship assumes that the value of  $\kappa$  is 8, which is lower than the value found by Metzner *et al.* (1961) for three different types of impeller.

Rushton turbines, with  $0.33 < D/T < 0.67$ , two-bladed paddles, with  $0.33 < D/T < 0.5$ , and three and four-bladed marine propellers, with  $D/T = 0.6$  for the three-bladed propeller and 0.47 for the four-bladed propeller, were studied in a baffled vessel of 0.25 m diameter. Reynolds number was varied between 0.2 and 1000 and fluids with pseudo-plastic, Bingham plastic and dilatant rheologies were examined. Equation (2.23) was found to hold for fluids with pseudo- and Bingham plastic rheologies with  $\kappa = 10$ , while the shear rate in dilatant fluids was correlated by:

$$\dot{\gamma} = 38 N \left( \frac{D}{T} \right)^{0.5} \quad \dots \text{Eqn(2.25)}$$

Pseudo- and Bingham plastic fluids showed the same trend as observed by Metzner *et al.* (1961), for  $10 < Re < 250$ , that the power number in a non-Newtonian fluid is less

than the power number in a Newtonian fluid.

The “pipe flow” Reynolds number, calculated using the viscosity given in equation (2.24), was plotted versus apparent Reynolds number and the resulting relationship, for pseudo- and Bingham plastic fluids, can be re-arranged to give:

$$\mu_a = 0.8 \frac{K}{(8N)^{(1-n)}} \left( \frac{3n + 1}{4n} \right)^n \quad \dots \text{Eqn(2.26)}$$

No relationship could be found for the dilatant fluid data using this method of analysis. It appears that the Reynolds number determined in this way, calculated using a viscosity based on a shear rate equal to  $8N$ , over-estimates the measured apparent viscosity by 20%, leading to a constant of proportionality of 0.8.

Beckner & Smith (1966) measured power consumption of various anchor impellers in a 0.23 m diameter vessel with Reynolds number varied between 0.2 and 100. The effects of fluid depth, clearance (distance between impeller blade and vessel wall) and breaker bars, connecting the anchor blade to the shaft, were examined and the data were correlated by:

$$Po = 82 \left( \frac{T}{c} \right)^{1/4} \left( \frac{\rho N^{2-n} D^2}{K [a(1 - n)]^{n-1}} \right)^{-0.93} \quad \dots \text{Eqn(2.27)}$$

where  $c$  is the gap between the impeller blade and vessel wall and  $a$  is a constant dependent on the geometry of the impeller. It follows that the shear rate is proportional to the impeller speed, dependent on the impeller geometry and given by:

$$\dot{\gamma} = a (1 - n) N \quad \dots \text{Eqn(2.28)}$$

The constant,  $\kappa$ , is dependent on the impeller geometry, which was not the case for turbines and propellers as reported by Metzner & Otto (1957) and Calderbank & Moo-Young (1959, 1961) and the degree of non-Newtonian behaviour exhibited by the fluid, since the power law flow index,  $n$ , is included in the equation. The shear rate is directly proportional to the impeller speed, for a given impeller geometry and fluid.

Equation (2.27) shows that  $Po \propto Re^{-0.93}$ , yet it is generally accepted that, in the laminar

regime,  $Po \propto Re^{-1}$ . It is not clear why the lower exponent was obtained.

Pollard & Kantyka (1969) measured jacket and coil heat transfer coefficients and power consumption in anchor agitated vessels of 0.30, 0.60 and 1.14 m diameter for Newtonian and non-Newtonian fluids. Reynolds number was varied between 5 and  $10^6$  for power measurements and 500 and  $10^6$  for the heat transfer measurements. They concluded that, for  $Re < 50$ , the average shear rate is proportional to the impeller speed but at higher Reynolds numbers:

$$\dot{\gamma}_{av} \propto N^{3.75} \quad \dots \text{Eqn(2.29)}$$

Nagata *et al.* (1971) measured power consumption for a helical ribbon, anchor, flat blade and Rushton turbine in pseudo- and Bingham plastic fluids over a Reynolds number range between 0.2 and 1000. They concluded that, in the laminar regime, the shear rate was proportional to the impeller speed with the constant,  $\kappa$ , dependent on the impeller type for all the impellers tested. The values of  $\kappa$  were in good agreement with those reported by other workers. Good correlation between power number and Reynolds number, for Newtonian and non-Newtonian fluids, was obtained for ribbon and anchor impellers in the transitional regime using this method of shear rate estimation but it did not work for the turbine impellers tested. No alternative was proposed.

Bourne *et al.* (1981) measured heat transfer coefficients and confirmed the findings reported previously by Pollard & Kantyka (1969), that the apparent shear rate is proportional to the impeller speed only in the laminar regime and derived an equation for the shear rate as a function of the impeller speed and diameter, the flow regime and the flow index of the fluid:

$$\dot{\gamma}_{av} \propto N^{\left(\frac{3-2m+mn}{1+n}\right)} D^{\left(\frac{2(1-m)}{1+n}\right)} \quad \dots \text{Eqn(2.30)}$$

where  $-m$  is the gradient of the log power number versus log Reynolds number plot for the impeller.

Höcker *et al.* (1981) examined the “drag reducing” effects found when dilute solutions of polymers are pumped or agitated. Power consumption was measured in flat-based, baffled vessels of 0.40 m diameter using Newtonian and non-Newtonian fluids. Single

and dual Rushton turbines and dual Ekato Mig impellers were studied with  $D/T$  ranging between 0.33 and 0.7 at Reynolds numbers between 0.1 and  $10^6$ .

In order to calculate the apparent Reynolds numbers of the impellers in non-Newtonian fluids a shear rate was derived from:

$$\dot{\gamma}_{derived} = \left( \frac{P}{N^2 D^3 K K_p} \right)^{\left( \frac{1}{n-1} \right)} \quad \dots \text{Eqn(2.31)}$$

The derived shear rate was plotted against impeller speed and a linear relationship found with a gradient of about 11 for all the impellers tested. This is in good agreement with the value found by Metzner & Otto (1957).

Wichterle *et al.* (1984) measured shear rates on the blades of a Rushton turbine using an electrochemical technique in a baffled vessel 0.30 m diameter with one impeller of  $D/T = 0.33$ . Newtonian and non-Newtonian fluids were studied with Reynolds number varied between 1 and  $10^4$ . The data were correlated by:

$$\dot{\gamma}_m = (1 + 5.3n)^{\left( \frac{1}{n} \right)} Re_m^{\left( \frac{1}{n+1} \right)} N \quad \dots \text{Eqn(2.32)}$$

where:

$$Re_m = \frac{\rho N^{2-n} D^2}{K} \quad \dots \text{Eqn(2.33)}$$

It was proposed that this relationship could be used to estimate the shear rate on the blades of other impeller types.

Wichterle *et al.* (1985) used the same technique to study the distribution of shear stresses on the walls of agitated vessels. A vessel of 0.30 m diameter was studied with Rushton turbines of  $0.15 < D/T < 0.5$  with further measurements made in 0.14 and 1.0 m diameter vessels. The measured shear stress was converted to a dimensionless friction factor,  $f$ , and the maximum occurring value was correlated by:

$$f_{\max} = (1 + 15n) Re_m^{-\left(\frac{1}{n+1}\right)} \left(\frac{T}{D}\right)^{\left(\frac{2(1-n)}{(1+n)}\right)} \quad \dots \text{Eqn(2.34)}$$

The shear rate in the fluid at the wall of the vessel was found to be approximately 30% of the shear rate, measured previously, at the impeller blades and was five times higher than the shear rate estimated from the method reported by Bourne *et al.* (1981).

Tebel & Zehner (1985) studied the mixing of pseudo-plastic fluids in loop reactors and defined a shear rate representative of the mixing process, as opposed to a Metzner & Otto (1957) shear rate which is used in power correlations. They applied this analysis to blend time data from the literature and obtained a very good correlation.

Shamlou & Edwards (1985) measured power consumption of helical impellers in Newtonian and non-Newtonian fluids at 0.15 and 0.40 m scale over a Reynolds number range between 0.1 and  $10^5$ . They found that the shear rate constant,  $\kappa$ , could be correlated by:

$$\kappa = 34 - 144 \left(\frac{c}{D}\right) \quad \dots \text{Eqn(2.35)}$$

This is similar to the relationship reported by Beckner & Smith (1966). They also concluded that as the clearance between the impeller and the vessel wall increases the value of the constant,  $\kappa$ , would decrease until it reaches a limiting value of 12, close to that reported by Metzner & Otto (1957) and others.

Wang & Yu (1989) measured heat transfer coefficients achieved with a variety of impellers over a Reynolds number range between 7 and  $1.84 \times 10^4$ . They showed, from a theoretical approach, that the shear rate can be estimated from:

$$\dot{\gamma} = k \left(\frac{1}{n}\right) N^{\left(\frac{2-f(2-n)}{n}\right)} \quad \dots \text{Eqn(2.36)}$$

where:

$$f = \exp\left(\frac{-m D^2 N^{(2-n)} \rho}{K}\right) \quad \dots \text{Eqn(2.37)}$$

The values of  $k$  and  $m$  were found to be 0.4 and 0.00705 respectively.

Elson (1988, 1990) studied the growth of “caverns” in fluids possessing a yield stress as they are agitated and found that the cavern diameter could be correlated by:

$$\frac{D_c}{D} = \left( \frac{Po Re_y}{\left( \frac{H_c}{D_c} + \frac{1}{3} \right) \pi^2} \right)^{1/3} \quad \dots \text{Eqn(2.38)}$$

where  $D_c$  is the cavern diameter,  $H_c$  is the cavern height and  $Re_y$  is the yield stress Reynolds number given by:

$$Re_y = \frac{\rho N^2 D^2}{\tau_y} \quad \dots \text{Eqn(2.39)}$$

where  $\tau_y$  is the yield stress of the fluid.

Elson suggests that, in pseudo-plastic fluids, a “pseudo-cavern” exists with the impeller effectively surrounded by slow moving fluid. Substituting the shear stress for the yield stress in equations (2.38) and (2.39) and re-arranging, the shear rate on the boundary of the well-mixed and slow moving regions can be found.

It is clear from the information available in the literature that the method of shear rate determination proposed by Metzner & Otto (1957) works well for predicting the shear rate and apparent viscosity of a non-Newtonian fluid in the region of an agitated vessel near the impeller, provided it is operating in the laminar regime. Nagata *et al.* (1971) have found that it does not work for open impellers in the transitional regime and Pollard & Kantyka (1969), Bourne *et al.* (1981) and Wang & Yu (1989) have found that it does not work for determining a characteristic shear rate in an agitated vessel in which heat transfer rates are to be predicted.

Wang & Yu have shown that it is important to be able to estimate the fluid viscosity in the region of the vessel which controls the rate at which the process under investigation takes place. In their work the region in question is the wall of the vessel.

The same approach can be applied to blending provided the region in which the blending rate is controlled can be identified and the viscosity of the fluid estimated.

#### 2.4.2 Blending of Non-Newtonian Fluids

Several references report the results of blending experiments with non-Newtonian fluids. Some do not produce a correlation for prediction of blend time but a method for determining the minimum Reynolds number at which an impeller will operate efficiently. The references are described below, in chronological order.

Metzner *et al.* (1961) compared the blending of viscous Newtonian and non-Newtonian fluids using single and dual Rushton turbines, fan turbines (pitched blade turbines) and marine propellers. The vessel diameter was varied between 0.14 and 0.36 m with  $D/T$  varied between 0.28 and 0.98. Reynolds number was varied between 0.1 and 620.

The minimum Reynolds number, required for movement on the fluid surface, was identified for each impeller type at the different  $D/T$  ratios studied. Larger impellers could operate satisfactorily at lower Reynolds numbers than small ones. Of the single impellers, the fan turbine was found to be most effective but the dual Rushton turbines were most efficient of all when compared in this way. When the performance of the impellers was compared in terms of their power input there was little advantage to be gained by using multiple impellers of  $D/T > 0.5$  but there was a distinct advantage in using dual impellers when  $D/T < 0.5$ . The use of dual impellers would position one closer to the fluid surface than a single impeller, so it is not surprising that the dual impeller agitator was found to be the most efficient when judged in this way.

A design method was proposed allowing the impeller size and speed to be determined in order to achieve a desired “mixing rate”.

Godleski & Smith (1962) measured blend times, using a colour change technique, in baffled vessels of 0.14, 0.29 and 0.44 m diameter with  $T/3$  Rushton turbines at each scale. Blend times were found to be 10 to 50 times longer than predicted by the correlation of Norwood & Metzner (1957). Calculation of the apparent viscosity revealed that the data points had been taken under laminar conditions,  $Re < 10$ , although observation of the flow indicated that the flow appeared to be middle transitional, as defined by Norwood & Metzner. The blend time data appear to have been taken at Reynolds numbers below the minimum values required for motion on the fluid surface, as defined by Metzner *et al.* (1961). This may explain why Godleski & Smith found

that blend times were an order of magnitude greater than predicted by Norwood & Metzner's correlation. Also, the Norwood & Metzner approach to blend time data analysis does not take account of any pseudo-plastic behaviour a fluid may exhibit. Other workers (eg. Moo-Young *et al.* (1972) discussed below) have found that pseudo-plastic behaviour increases blend time when compared with blend times measured in Newtonian fluids.

Moo-Young *et al.* (1972) measured blend times and power consumption in a vessel of 0.29 m diameter using a  $T/3$  Rushton turbine,  $2T/3$  helical ribbon and several novel tubular agitators. Reynolds number was varied between 5 and  $10^5$ , calculated from the definition of Calderbank & Moo-Young (1959, 1961).

The Newtonian and non-Newtonian power data were found to correlate well with this Reynolds number, as reported previously, but the non-Newtonian blend times were longer than those measured in Newtonian fluids at the same Reynolds number. This was said to be due to eddies decaying more rapidly in pseudo-plastic fluids causing blend times to increase above the value expected in a Newtonian fluid, at the same Reynolds number. The fact that the Reynolds number used to correlate the data was originally developed for correlation of power data, and is not suitable for correlation of blend time data, is a more likely explanation.

Demange *et al.* (1977) measured blend times in unbaffled vessels of 0.15, 0.20 and 0.28 m diameter with Rushton turbines of 0.064, 0.076 and 0.178 m diameter. Seven pseudo-plastic fluids were tested, selected for their rheological properties and industrial relevance.

The data were correlated by:

$$N\theta = k \frac{(T/D)^a (H/D)^c}{Fr^{-f} Re^{-h}} \quad \dots \text{Eqn(2.40)}$$

The values of the constant and exponents in equation (2.40) were found to be functions of the power law index,  $n$ , of each fluid. The exact form of this function was a fourth order polynomial and a table of the constants was given. In the case when  $n = 1$  the values were found to be identical to those in the correlation of Fox & Gex (1956) for viscous Newtonian fluids. The correlation predicted mixing times to within  $\pm 11 \%$ .

Wichterle & Wein (1981) examined the effect of non-Newtonian behaviour on the minimum degree of agitation required to create motion throughout the whole vessel by studying the size of the “nucleus” of fluid in motion around the impeller in relation to the size of the vessel. This is a similar approach to that taken by Elson (1988) in the study of the motion of fluids exhibiting a yield stress.

The dimensions of the nucleus created by a Rushton and pitched blade turbine were determined by moving the impeller towards the fluid surface or the base of the vessel and recording the conditions under which the fluid moved.

Two values of Reynolds number were defined: the first is the Reynolds number at which distinctive fluid motion appears and the second, the Reynolds number at which all dead spaces disappear in the fluid,  $Re_{ww}$ . The best design criterion was found to be the second definition and this was given by:

$$Re_{ww} = \left( \frac{1.8 T}{a D} \right)^2 \quad \dots \text{Eqn(2.41)}$$

where  $a = 0.6$  for a propeller and  $= 0.3$  for a Rushton turbine.

Having calculated  $Re_{ww}$  the impeller speed is found by equating this value with the definition of Reynolds number as defined by equation (2.32). It was suggested that this design method could be applied to other impellers with the value of the constant,  $a$ , calculated from:

$$a \approx 0.375 \sqrt[3]{Po} \quad \dots \text{Eqn(2.42)}$$

where  $Po$  is the impeller power number under turbulent conditions.

Hemrajani *et al.* (1982) carried out pilot-plant studies to develop scale-up rules for blending and heat transfer in remelters in which a solid material is added to a high viscosity, non-Newtonian fluid. The solids are melted in these vessels prior to further processing. The agitator design was based on good surface motion, to incorporate the solids into the liquid, and adequate heat transfer for remelting.

Studies were carried out in a 0.3 m diameter vessel with single and dual  $T/3$  pitched blade turbines at a variety of positions in the vessel with four baffles with widths of  $0.083T$  and  $0.02T$ . A model fluid was used for the tests. The best impeller

combination, on the basis of power input, was equally spaced dual impellers with the smaller baffles.

Nouri & Whitelaw (1990) studied the flow characteristics, using laser-Doppler anemometry, of a Rushton and 60° pitched blade turbine, both of  $T/3$  diameter, in a vessel of 0.14 m diameter, in water, a viscous Newtonian and a viscous non-Newtonian fluid. The discharge from the Rushton turbine was predominantly radial at all Reynolds numbers studied while the discharge from the pitched blade turbine had an axial and radial component, with the radial component increasing with decreasing Reynolds number. This trend was observed for the Newtonian and non-Newtonian fluids. The axial components, of the discharge from the pitched blade turbines, were found to decrease dramatically at a Reynolds number of 650 in the Newtonian fluid and  $4.8 \times 10^3$  in the non-Newtonian fluid. The difference in Reynolds numbers was attributed to the drag-reducing effect of the non-Newtonian fluid and the definition of shear rate used to calculate the apparent viscosity of the fluid and, hence, Reynolds number.

Koutsakos *et al.* (1990) also used laser-Doppler anemometry to study the effect of non-Newtonian properties on the discharge from a  $T/3$  Rushton turbine in a vessel of 0.15 m diameter. Three regions were identified:

- |      |                  |                       |
|------|------------------|-----------------------|
| i)   | $Re > 10^4$      | $Fl = 0.78$           |
| ii)  | $60 < Re < 10^4$ | $Fl \propto Re^{0.2}$ |
| iii) | $Re < 60$        | $Fl \propto Re$       |

The discharge mean velocity profiles were weakly dependent on Reynolds number when  $Re > 60$ . The fluctuating velocities increase with increasing Reynolds number until the flow becomes fully turbulent, leading to the conclusion that viscosity and pseudo-plasticity reduce turbulence. Overall motion in the impeller region of the vessel was found to be proportional to the mean centre-line velocity of the discharge flow.

The exponent on Reynolds number in the region  $60 < Re < 10^4$  is in good agreement with that reported by Wong & Huang (1988) who used hot-wire anemometry to measure the pumping capacity of several radial flow turbines in viscous Newtonian fluids.

The only reference which allows blend times to be predicted for Newtonian and non-

Newtonian data is that developed by Demange *et al.* (1977). Moo-Young *et al.* (1972) measured blend times but could not correlate their data because they used a definition of Reynolds number which was originally developed for correlation of power data.

Metzner *et al.* (1961) and Wichterle & Wein (1981) have found the minimum Reynolds number at which open impellers can operate efficiently in viscous fluids and, generally, this minimum occurs at a Reynolds number of about 200. This agrees well with the findings of Hoogendoorn & Den Hartog (1967) who studied Newtonian fluids. It also agrees, indirectly, with the findings of Godleski & Smith (1962) who found that the blend times they had measured were between 10 and 50 times longer than would be predicted by the correlation of Norwood & Metzner (1960). It appears that Godleski & Smith (1962) measured blend times at Reynolds numbers below 200.

### 2.4.3 Reviews

Many reviews have been written on the problems of mixing non-Newtonian fluids and a few of these are briefly mentioned below.

Bourne (1964) wrote a survey of industrial mixing problems, with the co-operation of about fifty companies producing a wide range of chemicals. Several recommendations were made regarding the knowledge required to improve agitator design.

Su & Holland (1968a, 1968b) wrote two papers: the first was concerned with defining the types of non-Newtonian fluids which could be encountered in industry and a description of the factors which should be taken into account in agitator selection and design. The second gave a comprehensive review of the data that had been published in the literature on non-Newtonian mixing.

Ford *et al.* (1972) wrote a review of the experimental techniques used in blend time measurements and summarised many correlations from the literature.

Blasiński & Rzycki (1976, 1978) wrote two papers: the first reviewed power measurements made with open impellers in non-Newtonian fluids and the second, power measurements with close-clearance impellers.

Short & Etchells (1982) reviewed unsolved problems they had experienced as consultant engineers within a large chemical company. They highlighted the fact that, often blend

time is not an important design criterion. Maintaining a minimum degree of agitation to avoid stagnation is often more desirable. Rheological properties of interest were pseudo-plasticity, fluids exhibiting a yield stress and visco-elasticity. Although blend time is not an important design criterion, it can be used, as it has in this research project, to compare the blending efficiency of various impeller types under different operating conditions.

Silvester (1984a, 1984b) reviewed mixing of non-Newtonian fluids giving industrial examples. Topics covered included power measurement, blend times, heat transfer, flow patterns and aerated systems.

Nienow & Elson (1988) reviewed their work, with other co-workers, in the mixing of non-Newtonian fluids and covered many topics including power consumption, flow patterns, scale-up and blend times.

## **2.5 DESIGN METHODS AND ECONOMIC EVALUATION**

Reavell (1951) wrote a review of the types of mixing problems experienced in industry and discussed the advantages of using various types of impellers as solutions to these problems.

The most well-known agitator design method, available in the literature, are the “Liquid Agitation” (Chemineer (1976)) series of twelve articles, originally published in “Chemical Engineering” between December 1975 and December 1976, and later as a collection in a booklet, also known as the “Chemineer Articles”. Design methods were presented for blending, solids suspension and gas dispersion with advice on mechanical design, shaft sealing, cost estimation and scale-up.

The design method for fluid blending is based on a “bulk fluid velocity” which determines the agitation intensity. The agitation intensity is given a “ChemScale” number between 1 and 10, with one point on the scale equivalent to a bulk fluid velocity of  $0.03 \text{ m s}^{-1}$  ( $6 \text{ ft min}^{-1}$ ). Having decided on the scale of agitation required and knowing the cross-sectional area of the vessel, the pumping capacity of the impeller required to achieve that velocity, can be calculated and the impeller size and speed calculated. If the flow is not turbulent an iterative process is required to do the design. Dickey (1984) published a program, written for a Texas Instruments TI-59 programmable calculator, that performed this calculation. No way of taking non-Newtonian behaviour into account in the agitator design was included in the procedures.

Mersmann *et al.* (1976) reviewed the data on blending, solids-suspension, gas-dispersion and heat transfer available in the literature to produce design guidelines for each process. They concluded that, in the turbulent regime, the product  $N\theta$  was constant and could be estimated from:

$$N\theta = \frac{6.7}{Po^{1/3}} \left( \frac{T}{D} \right)^{5/3} \quad \dots \text{Eqn(2.43)}$$

In the laminar regime, use of a helical ribbon is recommended with  $N\theta$  having a constant value of 55. No advice is given on blending in the transitional regime or on the effect of non-Newtonian behaviour on blending.

Flagg *et al.* (1977) compared the blending and heat transfer characteristics of open and close-clearance impellers including an examination of the economics of agitator selection and design. They concluded that, at the extremes of Reynolds number encountered, ie. turbulent or laminar flow, the selection of impeller type is straightforward. In transitional flow the choice of open or close-clearance impeller is not so easy and other factors such as initial capital cost, available power supply or the need for a “self-cleaning” impeller need to be taken into account.

Cheng & Tookey (1977) described a series of pilot-plant experiments carried out to study the effect of agitator geometry and speed on the yield of a herbicide. The yield obtained at plant scale was 5 to 10% less than the yield achieved in the laboratory and the difference was attributed to lack of agitation at the plant scale. The fluid rheology changed during the reaction from low viscosity Newtonian at the start, to highly viscous non-Newtonian with a “porridge-like” consistency at the end. It was found that this property effected the flow of the fluid when agitated by an anchor impeller, which had been used in the plant. A replacement impeller was tested and installed at the plant with subsequent improvements in product yield. They concluded that detailed knowledge of the process chemistry and thermodynamics is not required to determine optimum agitator design. This is true in some cases, but there are chemical reactions where knowledge of these parameters combined with knowledge of the mixing is required to achieve an efficient reactor design.

Corpstein *et al.* (1979) reviewed the factors which must be considered in pilot-plant work. Advice is given on the type of impeller to use at given scale and fluid viscosity combination and on the interpretation of results. The relationship between the small

and large scale impeller speeds, with geometrical similarity, is:

$$N_{large} = N_{small} \left( \frac{D_{small}}{D_{large}} \right)^a \quad \dots \text{Eqn(2.44)}$$

The value of the exponent,  $a$ , establishes the resulting scale-up rule as follows:

|            |                                         |
|------------|-----------------------------------------|
| $a = 1$    | equal fluid velocity                    |
| $a = 0.75$ | equal solids suspension uniformity      |
| $a = 0.67$ | equal overall mass transfer coefficient |
| $a = 0$    | equal blend time                        |

Since scaling-up can sometimes lead to impractically large agitator specification Corpstein *et al.* recommended the use of ChemScale for interpretation of the pilot-plant data. Curves of energy per unit mass versus Reynolds number are given for various ChemScale levels of agitation. The energy per unit mass term can be rearranged to show that it is equivalent to torque per unit volume divided by the fluid density.

Advice is given on the selection and control of pilot-plant equipment, measurement and analysis of data and the economic considerations which must be given to the scale of operation.

Garrison (1983) states that the design of agitators for processing Newtonian fluids where the scale-up rules are known can be done using the Chemineer Articles (Chemineer (1976)). A review is given of the factors which must be considered when pilot-plant work is needed. The rules given are applicable to Newtonian fluids but their application to non-Newtonian fluids is discussed.

Bowen (1985) produced a design method for the scaling-up of systems where the fluid viscosity is the controlling variable. This method is based heavily on the Chemineer Articles (Chemineer (1976)) but includes refinements, such as taking the fluid depth into account in the calculations. It is shown that scale-up, on the basis of equal bulk fluid velocity, can be performed by maintaining constant torque per unit volume indicating that any size of impeller can be used to achieve a desired bulk fluid velocity.

In a second paper Bowen (1986) produced a design method for shear-sensitive systems. The shear rate is calculated from the mean velocity profile of a Rushton turbine:

$$\dot{\gamma} = k N \left( \frac{D}{T} \right)^{0.3} \left( \frac{D}{w} \right) \quad \dots \text{Eqn(2.45)}$$

with  $k = 4.2$  for calculation of the average shear rate and 9.7 for the maximum.

Equation (2.45) is based on the velocity profile measurements of other workers. Fawcett (1990) has shown that the Reynolds, or turbulent, stresses in water are several orders of magnitude greater than the viscous stresses, which are created by the mean flow from the impeller. Using Bowen's technique to estimate the shear rates in a low viscosity fluid will give a value several orders of magnitude lower than the true value because the turbulent stresses are not considered.

## 2.6 VISCO-ELASTICITY

Visco-elasticity is an important rheological effect and is recognised by phenomena such as "shaft climbing", fluid recoil or relaxation and flow reversal. It is found in polymerisation processes where small molecules react to form a long chain.

This thesis is concerned with the effect of pseudo-plasticity on the blending performance of impellers but a brief review of some of the literature available on visco-elasticity is given below.

Ulbrecht (1974) has reviewed the literature on mixing of visco-elastic fluids and reviewed (1975) more practical aspects of mixing these fluids and the influence of rheology on product quality.

Chavan *et al.* (1975) extended the work of Coyle *et al.* (1970), studying the influence of helical ribbon geometry on blending performance, to visco-elastic fluids. This was done by following the path of a small, neutrally buoyant particle through the vessel. They found that the axial component of velocity was suppressed in favour of the angular velocity with recirculating vortices into which the particle was drawn. They concluded that the consequence of this flow pattern could have a detrimental effect on the molecular weight distribution in a polymerisation reaction.

Kelkar *et al.* (1972) measured the power required to rotate spheres, discs and impellers in fluids exhibiting pure visco-elasticity, pure pseudo-plasticity and a combination of the two. They concluded that, when the two rheological properties were present, the viscous effects were much greater than the elastic effects and, as a result, the power consumption of the impellers could be calculated by considering the viscous drag effects alone.

Ide & White (1974) measured the rheological properties of styrene-polystyrene solutions at concentrations between 15 and 57% and observed the flow patterns in a vessel agitated by a sphere, disc, turbine and propeller. They found that at low concentrations the flow patterns were as expected, flow reversal taking place at high concentrations and segregated flow taking place at intermediate concentrations. They suggested that the presence of segregated regions within the reactor vessel could lead to distorted molecular weight distributions of the polymer product.

Ford & Ulbrecht (1976) measured the influence of rheological properties of polymer solutions on blend and circulation times in a vessel agitated by a helical screw impeller. They found that blend time was strongly dependent on the visco-elasticity of the fluid while circulation times were dependent on both the visco-elasticity and pseudo-plasticity. They concluded that, when scaling-up, velocity gradients do not increase by the scale factor and in certain circumstances may reduce. This reduces the normal stresses in the fluid which, in turn, reduces the secondary flow effects caused by visco-elasticity.

Carreau *et al.* (1976) measured blend and circulation times in vessels agitated by helical ribbons. They found that blend times in Newtonian fluids were 3 to 7 times shorter than those measured in visco-elastic fluids and concluded that blend time is highly dependent on the fluid's degree of visco-elasticity. They found that the axial velocity component was suppressed by visco-elasticity and the angular velocity increased. Blend time data were correlated by plotting dimensionless blend time versus a dimensionless pumping factor, which is a function of the geometrical properties of the impeller.

Boger (1980) has reviewed the development of non pseudo-plastic, elastic fluids and their use in isolating elastic effects in rheological experiments.

Greene *et al.* (1982) used streak photography to investigate the change in flow characteristics of a pitched blade turbine due to increasing visco-elasticity. They found that, as the degree of visco-elasticity increased, the axial component of flow decreased

and the radial component increased, until the flow pattern resembled that of a radial flow impeller.

Pörtner *et al.* (1988) used an optical technique to measure blend times in pseudo-plastic and visco-elastic fluids. Data were correlated by:

$$N\theta \propto \frac{Re}{1 + k Wi} \quad \dots \text{Eqn(2.46)}$$

where  $Wi$  is the Weissenberg number, which is a dimensionless group relating the inertial and elastic forces in the fluid. Visco-elasticity was found to increase blend time.

Brito-De la Fuente *et al.* (1990) measured power consumption, blend and circulation times in Newtonian and visco-elastic fluids. They found that visco-elasticity affected both power and blend time and fitted their data to a correlation which included Weissenberg number.

## 2.7 CONCLUSIONS

The objective of this section of the literature review is to summarise the work reported in the preceding sections and to put the work reported in this thesis in context.

A great deal of work has been done on the blending of low viscosity fluids in the turbulent regime ( $Re > 10^4$ ). The majority of workers agree that  $N\theta$  is constant and its value is dependent on the impeller type and diameter.

As Reynolds number decreases a value is reached where the impeller enters the transitional regime. This occurs at  $10^3 < Re < 10^4$  with the exact value dependent on impeller type. Only Hoogendoorn & Den Hartog (1967) have measured blend times at large scale using consistent experimental techniques and they found that, in the transitional regime,  $N\theta$  is inversely proportional to Reynolds number. Fox & Gex (1956) also measured blend times at large scale but used a different technique to their work at small scale.

As Reynolds number decreases further the impeller enters the laminar regime where, for open impellers,  $N\theta$  is inversely proportional to Reynolds number raised to the power

-10. All workers who have measured blend times in this regime agree that close clearance or axial flow impellers in draught tubes should be used to achieve efficient blending.

No large scale work has been carried out to measure non-Newtonian blend times and only one correlation exists which can be used to predict blend times in non-Newtonian fluids (Demange *et al.* (1977)). Also, no consistent method has been proposed for determination of shear rate and viscosities in agitated vessels in which the blend time must be predicted. For these reasons the objectives of the work reported in this thesis are:

- i) to measure blend times in Newtonian fluids at three scales using open impellers which are common in industry, to correlate the data and to give rules for their design. The range of Reynolds numbers will be between 200 and  $5 \times 10^4$ , since there is a great deal of evidence that such impellers cannot operate efficiently at lower Reynolds numbers.
- ii) to measure blend times in non-Newtonian fluids at three scales using the same impellers as tested in Newtonian fluids, to correlate the data and to give rules for their design. The same range of Reynolds numbers will be used.
- iii) to determine the characteristic shear rate in an agitated vessel in which blending takes place in order to estimate a viscosity which can be used to correlate the blend time data.
- iv) to use the correlation to explain the effect of non-Newtonian behaviour on the blending process in agitated vessels.
- v) to determine which impeller types are best suited to the blending of fluids in the transitional regime. The criteria used will be capital, running and total cost of the agitation equipment.

## CHAPTER 3

### EQUIPMENT

#### **3.1 INTRODUCTION**

The equipment used for the experimental programme reported in this thesis was developed prior to the start of the project and had been used for the measurement of power consumption and blend times in water (Mackinnon (1987)). New equipment was acquired and tested for the measurement of rheology but the basic experimental equipment and technique was the same. The equipment and test fluids are described in this chapter.

#### **3.2 VESSELS**

Three geometrically similar vessels were used for the tests with diameters of 0.305, 0.609 and 1.830 m. The vessels had a dished base with a radius of curvature equal to the vessel diameter and were fitted with four baffles, equally spaced around the circumference of the vessel. The baffles were  $T/12$  wide and mounted vertically at a spacing of  $T/60$  off the vessel wall and extended from above the fluid surface to the tan-line of the vessel.

The 0.305 and 0.609 m vessels were constructed of perspex and allowed observation of the blending process to be made. The 1.830 m vessel was constructed of steel and was fitted with four viewing windows. In all cases the vessels were filled so that the fluid depth was equal to the vessel diameter.

The vessel arrangement is shown in figure 3.1.

#### **3.3 IMPELLERS**

Much of the research carried out into blending of miscible fluids has concentrated on the performance of Rushton turbines and marine propellers. These two impellers are not widely used in industry today for blending duties. Current industrial practices were discussed with members of the FMP consortium, prior to the start of the experimental programme and every effort was made to ensure that “industrially relevant” equipment was tested.

Pitched and flat blade turbines are used because they are relatively cheap to manufacture, consisting of rectangular blades bolted or welded onto a hub. The other impeller type which is used is the hydrofoil. These have been designed to produce the same axial flow patterns as marine propellers but are claimed to have advantages in terms of manufacture and installation over the propeller (Oldshue (1983)). These tend to be more expensive to manufacture because they have profiled blades and more care needs to be taken in balancing them because they have three blades, compared with the four blades normally fitted to pitched and flat blade turbines.

The three impeller types used in the tests were:

- i) pitched blade turbines of  $T/3$  and  $T/2$  diameter.
- ii) a flat blade turbine (ie. vertical blades) of  $T/3$  diameter.
- iii) Lightnin A310 and Chemineer HE3 hydrofoils of  $T/2$  diameter.

The geometrical ratios were kept constant in scale-up and full dimensions of each impeller used are given in table 3.1 and they are shown in figures 3.2 to 3.4.

All impellers were positioned such that their centre-lines were  $T/3$  above the lowest point on the base of the vessel. They rotated clock-wise when viewed from above and the pitched blade turbines and hydrofoils pumped downwards.

### 3.4 CONDUCTIVITY PROBES

The method used for measurement of blend times relied on the detection of changes in conductivity in different parts of the vessel after an electrolytic tracer had been added on to the surface of the fluid. The conductivity changes were detected using conductivity probes placed in regions of the vessel with differing agitation intensities.

The conductivity probes used were made following the design originally described by Khang & Fitzgerald (1975). They were constructed by embedding pieces of platinum wire, soldered onto a screened signal lead, in a “bullet” of epoxy resin 6 mm in diameter and 30 mm long (Figure 3.5).

The outer electrode was coiled around the probe tip in the shape of a cone and this acted as an earth for the measuring electrode and as a screen to prevent interference

from other electrodes or earthed objects in the vessel. The probes were coated in platinum black in order to make their surfaces inert.

The probes' measuring volume was made as small as practically possible to minimize averaging effects when measuring conductivity. Their presence in the vessel would affect local mixing but this was also minimised by making the probes as small as possible and by arranging the probe supports in such a way that they did not interfere with the flow of fluid around the vessel. This was achieved by attaching the probe supports to the baffles.

As mentioned above, the three conductivity probes were positioned in regions of differing agitation intensity:

- i) Probe 1 was positioned beneath the impeller and near the blade tips in the region of highest agitation intensity.
- ii) Probe 2 was positioned halfway between the agitator shaft and vessel wall.
- iii) Probe 3 was positioned behind a baffle in the region of lowest agitation intensity.

The probes were placed at different heights and on different sides of the vessel and, on scale-up, were placed in the same positions. These positions are given in table 3.2 and shown in figure 3.6.

The instrumentation chain used for blend time measurements is described in chapter 5.

## 3.5 TEST FLUIDS

Two types of test fluid were used: the first exhibited a Newtonian rheology and the second, a non-Newtonian, pseudo-plastic rheology.

### 3.5.1 Newtonian Test Fluid

Several factors were considered in the choice of the Newtonian fluid used for the power and blend time measurements. Muskett (1984) described criteria which were set down for choice of fluid. These were:

- i) Cost: Measurements were made in the 1.830 m diameter vessel, which has an operating volume of 4.455 m<sup>3</sup>, so it was important to find a relatively cheap material that would not put too great a strain on the project budget.
- ii) Transparency: Flow visualisation was an important facet of the experimental programme so it was important to find a clear fluid which allowed all regions of the vessel to be observed during experiments.
- iii) Foaming: If the fluid was a foam stabiliser air, drawn into the fluid from the surface, would form small, stable bubbles reducing the mean density of the fluid and effecting the power consumption of the agitator. The presence of bubbles would also affect the performance of the conductivity probes, causing large fluctuations in the signal as bubbles passed between the probe electrodes.
- iv) Handling/  
Storage: The laboratory in which the experiments were performed was originally built for Civil Engineering research and did not have the facilities to handle hazardous chemicals. For this reason the choice of fluid was limited to those which did not require any special handling procedures. The fluid also had to be stable over a period of days, once it had been exposed to air.
- v) Salt Solubility: Blend times were measured using a conductivity technique and involved the addition of a small volume of tracer with a high conductivity, to the vessel and measurement of the conductivity fluctuations. For this reason the fluid selected had to be capable of dissolving an electrolyte without altering its rheology.
- vi) Disposal: As mentioned above, the laboratory did not have facilities to handle hazardous chemicals and disposal of the used materials had to be considered carefully.

Several fluids were considered: Glycerol was rejected on the grounds of cost (over £1000 per tonne) and its affinity for atmospheric moisture; Glucose syrup was rejected because its high dissolved sugar content reduced its ability to dissolve electrolytes; and Breox 75 W 30000, supplied by BP Chemicals Ltd., was found to be a good foam stabiliser.

The fluid selected was a grade of Versicol. These are polyacrylic acids, polyacrylamides and polymethacrylic acids and are supplied by Allied Colloids of Bradford, England. Several grades of Versicol were examined and Versicol E-11 was finally selected. It dissolved electrolytes readily and, at a cost of around £400 per tonne, was considered to be financially acceptable.

The visco-elasticity of Versicol E-11 solutions was checked by sending samples to the Warren Springs Laboratory in Stevenage, England. Solutions of desired viscosity were made up and sent to the laboratory and were not found to exhibit visco-elasticity as supplied but were found to evaporate readily and high concentration solutions were visco-elastic.

### **3.5.2 Non-Newtonian Test Fluid**

Many literature sources report the use of Carboxy-methyl Cellulose (CMC) solutions in the study of the mixing of non-Newtonian, pseudo-plastic fluids. The initial stage of this part of the project was to approach suppliers of CMC to discuss the requirements. Aqualon (UK) Ltd. supply a wide range of Cellulose-based polymers and recommended the use of a grade of Hydroxy-ethyl Cellulose, Natrosol 250MR.

This met all the criteria set out for the Newtonian test fluid and cost around £500 per tonne. The fact that test solutions were, at most, 3/4% wt./wt. meant that the cost of materials used for this part of the project was very low.

The visco-elasticity of the Natrosol 250MR solutions was checked by sending samples to the Department of Chemical Engineering at the University of Birmingham, England. The solutions made-up did have a normal stress component and, strictly speaking were slightly visco-elastic, but under the operating conditions the ratio between the shear and normal stresses indicated that the effect of the fluid's visco-elasticity on the flow in the vessel would be negligible. This was confirmed by the fact that none of the classic signs of visco-elastic behaviour, such as shaft-climbing or flow-reversal were observed.

Later tests were made with solutions of Natrosol 250HHR, which has a heavier molecular weight and exhibits more severe non-Newtonian behaviour. 3/4% wt./wt. solutions made up with this material did shaft-climb so the most concentrated solution of this grade used was 1/2% wt./wt.

### 3.5.3 Tracers

The tracer used for Newtonian blend time measurements was made by adding concentrated nitric acid to a Versicol E-11 solution ensuring that the viscosity of the tracer was the same as the viscosity of the fluid in the vessel being used for the experiment. The concentration of nitric acid in the tracer was 4.8 mols per litre.

The viscosity was always within  $\pm 5$  cP of the viscosity of the fluid in the vessel. The viscosity was adjusted by adding more Versicol or water to the batch of tracer being made and was checked during the experiment to ensure that it stayed within the limits required.

The tracer used for the non-Newtonian blend time measurements was made by taking a small volume of fluid from the vessel and adding sufficient salt (sodium chloride) to it to make a saturated solution. After mixing, the undissolved salt settled at the bottom of the container and clear fluid was taken from the surface for addition to the vessel.

## 3.6 RHEOLOGY MEASUREMENT

### 3.6.1 Newtonian Fluids

The viscosity of the Newtonian test fluids was measured using a Contraves Rheomat 108 portable viscometer. Its operation is based on the coaxial cylinder principle (see Solomon (1980), Hickman (1985)). The measuring system conforms to the DIN 53019 standard. The rotational speed of the bob could be raised in eight steps from 50 to 1000 rpm giving a range of shear rates for the measuring system used of 64 to 1290  $\text{s}^{-1}$ . A direct read-out of the fluid's viscosity was given at each shear rate, in Newton seconds per square metre ( $\text{Ns m}^{-2}$ ).

The viscosity was measured at the start and end of each experiment and it did not change significantly. Also, the viscosity was measured at three of four shear rates to ensure that the sample of Versicol supplied was not "out-of-spec" and exhibiting a non-Newtonian rheology. None of the samples ever failed this test.

### 3.6.2 Non-Newtonian Fluids

Solomon (1980) and Hickman (1985) have covered a great deal of the background on non-Newtonian fluid rheology and measurement and will not be repeated in this thesis. The technique used in this project to determine the fluid rheology is described below:

The rheology of the pseudo-plastic fluids was measured using a Contraves Rheomat 115 fully automated viscometer controlled by software run on a Tulip personal computer. This also works on the coaxial cylinder principle and complies with the relevant DIN standard.

The interface between the Rheomat 115 viscometer and the computer was a "Rheoanalyzer" which converted the viscometer's output into digital form.

The computer was programed with the measurement routine and the following information was input:

- i) low shear rate.
- ii) holding time at low shear rate.
- iii) high shear rate.
- iv) holding time at high shear rate
- v) time taken for shear rate to rise from low to high value (the ramp time).
- vi) time taken for shear rate to fall from high to low value.

Once the data had been taken and stored, on the computer hard disk, it could be fitted to different rheological models by the program. The power law model was selected (see chapter 6) and the constant,  $K$ , and index,  $n$ , were evaluated by performing a linear regression of:

$$\ln \tau = n \ln \dot{\gamma} + \ln K \quad \dots \text{Eqn(3.1)}$$

The low and high shear rate values were taken as 10 and 500 s<sup>-1</sup> respectively because the viscometer system could not measure accurately at lower shear rates. The holding times were 30 seconds and the ramp time was 250 s. Once the data had been recorded it could be fitted to the power law over different shear rate ranges in order to investigate the effect of the range on the values of  $K$  and  $n$ . Very little effect was observed when the data were fitted over any range within the overall sample range provided that the lower shear rate was not altered. Increasing the lower shear rate reduced the degree of pseudo-plasticity the fluid exhibited, increasing  $n$ .

The power law constants and indices of each of the Natrosol solutions used in the measurements of blend times are given in table 3.3.

## CHAPTER 4

### BLENDING OF NEWTONIAN FLUIDS

#### 4.1 INTRODUCTION

This chapter of the thesis is concerned with the measurement and analysis of blend times in viscous, Newtonian fluids. The blend time measurement method and instrumentation chain are described in section 4.2 and the data reduction method is described in section 4.3.

The data taken by FMP, reported by Mackinnon (1987), have been re-analyzed to produce a correlation of three dimensionless groups: impeller power number,  $Po$ ; Reynolds number,  $Re$  and Fourier number,  $Fo$ . The reason for re-analyzing the data and the analysis are described in section 4.4.

The analysis of the viscous Newtonian blend time data is described in section 4.5 and discussion of the results and comparison with literature is given at relevant points in each section and a general discussion is given in section 4.6. The conclusions of this section of the project are given in section 4.7.

#### 4.2 METHOD

##### 4.2.1 Instrumentation Chain

The instrumentation used for the measurement of blend times in viscous Newtonian fluids was identical to that used to measure low viscosity blend times, reported by Mackinnon (1987) and described by Ruszkowski (1985a). A brief description of the equipment is given below.

The conductivity probes, described in section 3.5, were energised by a conductivity meter manufactured by the Instrumentation Section of BHR Group Ltd. for FMP and was designed especially for blend time measurement.

The output from the meter was a 0 - 200 mV analogue signal, directly proportional to the conductivity measured. The signal was amplified to  $\pm 5$  V, passed through an anti-aliasing filter and then input to the analogue-digital converter (ADC) board of a PDP 11/23 computer. The low-pass cut-off frequency to which the filters were set was

dependent on the sampling frequency of the ADC.

The computer collected 4096 data points for each probe signal, irrespective of the sample time. The sampling frequency was adjusted so that the sampling time was always between three and four times longer than the blend time. For example, if a sampling frequency of 50 Hz was selected the sample time was 81.92 seconds and the expected blend time would be between 20 and 30 seconds.

The data were stored in the computer memory during an experiment and written to the hard disk once the sampling routine was completed.

#### 4.2.2 Blend Time Determination

The blend time measurement method used in this work involved the addition of a small volume electrolyte to the bulk fluid in the vessel and recording the conductivity at three points in the vessel, shown in figure 3.6. The tracer was added from a solenoid valve positioned above the fluid surface and its exact position is also shown in figure 3.6. A typical conductivity trace is shown in figure 4.1. The solenoid valve was activated by a Schmidt trigger which also signalled the computer to start recording the output from the conductivity meter.

The blend time was measured 8 times for each set of conditions and the data are presented in tables 4.1 to 4.4. The blend time reported is the mean of the eight repetitions and the relative standard deviation associated with each mean value is given where the relative standard deviation is given by:

$$RSD = \frac{\sigma_{\theta}}{\bar{\theta}} \times 100 \% \quad \dots \text{Eqn(4.1)}$$

### 4.3 DATA ANALYSIS

The technique used for the analysis of the viscous blend time data taken for this project was identical to the method used by the workers who measured turbulent blend times for FMP. The method is described by Ruszkowski (1985a) and is briefly described below:

Root mean square concentration has been chosen as the quantity used to characterise the blending in agitated vessels. The time varying concentration in a vessel at any point may be represented by the mean concentration in the vessel and a time-varying component:

$$C(t) = \bar{C} + C'(t) \quad \dots \text{Eqn(4.2)}$$

where:

$C(t)$  is the local concentration at time  $t$ .

$\bar{C}$  is the mean concentration.

$C'(t)$  is the local fluctuating component of concentration at time  $t$ .

The root mean square concentration fluctuation over a time  $\Gamma$  is given by:

$$\sigma_c^2 = \frac{1}{\Gamma} \int_0^\Gamma (C(t) - \bar{C})^2 dt = \frac{1}{\Gamma} \int_0^\Gamma (C'(t))^2 dt \quad \dots \text{Eqn(4.3)}$$

This approach is commonly used for dealing with turbulent fluctuating quantities such as fluid velocities measured by laser-Doppler anemometry. The situation in an agitated vessel is very complicated since the concentration fluctuations are transient, whereas the magnitude of the root mean square velocity is constant with time.  $C$  and  $\sigma_c^2$  are functions of  $\Gamma$ , the time over which the integration takes place. This problem has been overcome by calculating the root mean square concentration for small sections along the conductivity-time history. A typical plot of  $\sigma_c^2$  versus time is shown in figure 4.2.

The concentration within the vessel was measured at three points, 4096 times during an experiment, and the analysis program normalised the responses so that the magnitude of the total change in concentration was represented by 2048 computer digits, irrespective of the actual change in conductivity in the vessel.

The root mean square concentration fluctuation is then calculated for a small section of the trace. The position of the “calculation section” is then moved along the root mean square concentration fluctuation and calculated again. There were 32 points in each section and the section was moved forward by 4 points to carry out the next calculation. A series of estimates of the root mean square concentration fluctuation are obtained:

$$\sigma_c^2(i) = \frac{1}{32} \sum_{4i}^{4i+31} (C_j - C(\infty))^2 \quad i = 0, 1, 2, \dots, 1016 \quad \dots \text{Eqn(4.4)}$$

where:

$\sigma_c^2 (i)$  is the  $i^{\text{th}}$  estimate of the root mean square concentration fluctuation.

$C_j$  is the concentration of the  $j^{\text{th}}$  data point.

$C(\infty)$  is the concentration in the vessel when blending is complete.

The number of points over which the root mean square is calculated is 32 since this covers a significant portion of the circulation time in the vessel.

Having converted the data from a raw conductivity trace to root mean square concentration fluctuation versus time, the blend time was defined as the time taken for the degree of homogeneity to reach a pre-set value. The degree of homogeneity was calculated by defining a mixing index,  $M$ , where:

$$M = \frac{\Delta C - (\sigma_c^2)^{1/2}}{\Delta C} \quad \dots \text{Eqn(4.5)}$$

where:  $\Delta C = C(\infty) - C(0)$

$M$  expresses the reduction in the root mean square concentration fluctuation as a fraction of the total change in concentration in the vessel due to the injection of tracer. The value of  $M$  selected was 0.95, ie. the blend time is defined as the time taken for the blending process to be 95% complete.

The individual probe blend times,  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ , were all calculated in this way. A fourth blend time, the root mean square blend time  $\theta_{RMS}$ , was calculated to give an estimate of the blend time for the vessel as a whole. This is calculated from the data taken at each individual probe by defining a root mean square concentration in the vessel:

$$C_{j, RMS} = \left( \frac{1}{3} (C_{j, 1}^2 + C_{j, 2}^2 + C_{j, 3}^2) \right)^{1/2} \quad \dots \text{Eqn(4.6)}$$

where  $C_{j, 1}$  is the concentration of the  $j^{\text{th}}$  data point measured at probe 1, etc.

$C_{j, RMS}$  is substituted for  $C_j$  in equation (4.4) to calculate  $\sigma_{c, RMS}^2$  which in turn is substituted into equation (4.5) to calculate  $M_{RMS}$ .

All data correlation and comparison with other workers' results has been made using  $\theta_{RMS}$  since it is defined as the blend time for the whole vessel and results in the literature are presented on the same basis.

#### 4.4 TURBULENT DATA RE-ANALYSIS

Mackinnon (1987) reviewed the results of the blend time measurements made in water by the FMP consortium. The first data were fitted to:

$$Po^{1/3} N\theta \left( \frac{D}{T} \right)^{13/6} = k \quad \dots \text{Eqn(2.5)}$$

This was developed by taking the dimensionless power per unit mass, as reported by Mersmann *et al.* (1976) and Havas *et al.* (1978b), ie:

$$Po^{1/3} N\theta \left( \frac{D}{T} \right)^{5/3} = \left( \frac{\bar{\epsilon}}{T^2} \right)^{1/3} \theta = k' \quad \dots \text{Eqn(4.7)}$$

and introducing a scale factor:

$$Po^{1/3} N\theta \left( \frac{D}{T} \right)^{5/3} \left( \frac{D}{T} \right)^a = k \quad \dots \text{Eqn(4.8)}$$

The exponent  $a$  was determined to be 1/2 by analyzing the data of Khang (1975) because, when the correlation was first presented by Ruszkowski (1985b), of the six impellers tested, five were of  $T/3$  diameter and one  $T/2$ . As more data were taken they were fitted to equation (2.5) without checking to see if a better fit could be obtained with a different value of  $a$ .

Before analysis of the viscous blend time data was started the data taken in water were re-analyzed by carrying out a multiplicative regression of  $N\theta$  on  $Po$  and  $D/T$  using the Statgraphics computer package, ie:

$$N\theta \propto P_o^b \left( \frac{D}{T} \right)^c \quad \dots \text{Eqn(4.9)}$$

The values of  $b$  and  $c$  were -0.319 and -2.062 respectively with a correlation coefficient,  $r^2$ , of 91.71% and F-ratio of 139, for 26 data points.

After rounding-up these values, this showed that the value of  $a$  is 1/3, not 1/2, ie:

$$P_o^{1/3} N\theta \left( \frac{D}{T} \right)^2 = k'' \quad \dots \text{Eqn(4.10)}$$

Having carried out this regression, equation (4.10) was re-arranged and both sides multiplied by  $\rho/\mu$  to give:

$$P_o^{1/3} \frac{\rho N D^2}{\mu} = k'' \frac{\rho T^2}{\mu \theta} \quad \dots \text{Eqn(4.11)}$$

where:  $k'' = 5.20 \pm 10.0\%$

Equation (4.11) can be re-written:

$$P_o^{1/3} Re = \frac{5.20}{Fo} \quad \dots \text{Eqn(4.12)}$$

The terms on the left hand side of equation (4.12) are the impeller power number and Reynolds number which are commonly used and well-known dimensionless groups. The power number can be regarded as a drag coefficient and relates the power input by an impeller to its speed and diameter. Reynolds number is a ratio between the inertial and viscous forces acting on the fluid.

The term on the right hand side of equation (4.12) is Fourier number and has been used by other workers in this area to correlate blend time data (eg. Hoogendoorn & Den Hartog (1967), Shiue & Wong (1984)).

Fourier number is a dimensionless group used in unsteady-state mass, heat or momentum transfer problems (Parkin (1984)). It can be written in a general form as:

$$Fo = \frac{\alpha \theta}{T^2} \quad \dots \text{Eqn(4.13)}$$

$T$  is the characteristic length and  $\theta$  is the characteristic time of the process in question.  $\alpha$  is a kinematic property of the fluid and is:

- i) the diffusion coefficient in mass transfer processes.
- ii) the thermal conductivity in heat transfer processes.
- iii) the kinematic viscosity in momentum transfer processes.

The blend time in an agitated vessel is controlled by the viscosity of the fluid so the third version of Fourier number is the appropriate one to use in analysis of the data.

An important point to note regarding this analysis is that it implies that different impellers of equal diameter will achieve the same blend time at the same power input. The correlation can be re-arranged to show:

$$N\theta = k' \left( \frac{T}{D} \right)^2 \quad \dots \text{Eqn(4.14)}$$

Other workers have found that the exponent on  $T/D$  is dependent on impeller type. For example, Procházka & Landau (1961) found the exponent was 2.05, 2.20 and 2.57 for marine propellers, pitched blade turbines and Rushton turbines respectively. Khang (1975) found the exponent was 2.0 for marine propellers and 2.3 for Rushton turbines. The associated constants of proportionality were different for each impeller type whereas the analysis presented in this thesis shows that one constant can be applied to all the impeller types tested if the exponent is the same. The relative standard deviation of the constant is  $\pm 10\%$  which is the same as the deviation reported by Kramers *et al.* (1953) for their  $N\theta$  values. Since power number and  $D/T$  are constant in the turbulent regime for a particular impeller the standard deviation of the correlation constant is in fact the standard deviation of the  $N\theta$  values.

Although previous workers have reported a difference in the exponents on  $D/T$  the fact that the scatter of the data about the correlation is as good as the scatter of the raw data, and that the data correlated were taken at three scales (0.61, 1.83 and 2.67 m diameter vessels) gives confidence in the use of this correlation for agitator design and impeller comparison.

## 4.5 ANALYSIS OF VISCOUS BLEND TIME DATA

The blend time data have been analyzed and are reported in three ways:

- i) individual probe blend times which indicate how the blending process proceeds in regions of different agitation intensity.
- ii) root mean square blend times which indicate how blending proceeds for the vessel as a whole.
- iii) fitting the data to a correlation of the same form as the one developed for the turbulent data, equation (4.12)

### 4.5.1 Individual Probe Blend Times

Individual probe blend times were measured in order to investigate the way in which the blending process proceeds in regions of high and low agitation intensity. Three probes were used: the first was positioned beneath the impeller, the second half way between the shaft and vessel wall and the third, behind a baffle. Mackinnon (1987) found that the individual probe blend times measured in water were almost equal at a particular impeller speed.

The results of the viscous blending experiments are given in tables 4.1 to 4.4. The tables show vessel scale, nominal fluid viscosity, impeller speed, three individual probe blend times and root mean square blend time for each set of experimental conditions. The blend time reported is the mean of eight repetitions of the measurement and the number given in brackets beneath is the relative standard deviation of the mean, given as a percentage. The results of the individual probe measurements are plotted as  $N\theta_i$  versus Reynolds number in figures 4.3 to 4.6. The results reported by Mackinnon (1987), taken in water, are included in the plots for comparison.

The plots show that a critical Reynolds number,  $Re_{crit}$ , exists for each impeller. When

the viscosity of the fluid is low, and  $Re > Re_{crit}$ , the blend times are approximately equal. As viscosity is increased, and  $Re < Re_{crit}$ , the blend times diverge such that:

$$N\theta_1 < N\theta_2 < N\theta_3 \quad \dots \text{Eqn(4.15)}$$

The degree of divergence increased with decreasing Reynolds number for the two pitched blade and flat blade turbines while the degree of divergence is fairly constant for the hydrofoil. This appears to indicate that the way in which the tracer is blended into the bulk fluid differs for the various impellers.

Landau *et al.* (1963) carried out a similar exercise with five axial and three radial combinations of probe position (ie. fifteen possible positions) and found that the value of critical Reynolds number could not be strictly identified since it was determined by a number of factors which they did not correlate. They presented plots showing the distribution of  $N\theta$  with position for a number of impellers in baffled and unbaffled vessels at Reynolds numbers of  $2.7 \times 10^3$  and  $5.0 \times 10^4$ . The results are generally in good agreement with the results reported here. Once  $Re < Re_{crit}$ , the blend times throughout the vessel will be longer than those measured under turbulent conditions but the blend times measured near the impeller will be shorter than those measured in regions of low agitation intensity, near the vessel wall, near the fluid surface or behind a baffle.

Several workers have modelled agitated vessels as a well-mixed region near the impeller in which the agitation intensity is high and regions in which the agitation intensity is lower with transfer of fluid into and out of these regions as the blending process proceeds (eg. Khang (1975), M<sup>c</sup>Manamey (1980)). In the turbulent regime, the pumping number of the impeller,  $Fl$ , is constant and flow between the well-mixed region and other parts of the vessel will be determined by the impeller speed since:

$$Q = Fl N D^3 \quad \dots \text{Eqn(4.16)}$$

As Reynolds number is decreased a value will be reached where  $Fl$  is a function of Reynolds number and decreases as Reynolds number decreases. This has been reported by Wong & Huang (1988) who found that the boundary between these two regimes occurred at  $Re \approx 10^4$ . The most likely explanation for the increase in blend times measured near the wall and surface of the vessel is the reduction in pumping capacity and hence, reduction in transfer of well-mixed fluid to these regions and unmixed fluid into the impeller. The critical Reynolds number determined from the  $N\theta$  versus  $Re$

plots (figures 4.3 to 4.6) occurs at about the same value supporting this explanation of the process.

A point to note from these figures is that the degree of divergence between  $N\theta_1$  and  $N\theta_3$  varies with impeller type. The most marked difference is shown by the *T/2* Lightnin A310 hydrofoil where the blend times do not diverge but have a constant difference as Reynolds number decreases with  $N\theta_3$  being 2 - 3 times greater than  $N\theta_1$ . This indicates that the proportion of the energy input to the vessel devoted to flow and mixing may vary according to impeller type but, the correlation of the data (see section 4.6.1) shows that there is very little difference between the root mean square blend times when impellers of the same diameter are compared at equal power input.

Most workers have reported their data in the form of a blend time representative of the blending in the vessel as a whole. The root mean square blend times have been calculated in order to obtain a blend time for the whole vessel and are more suitable for comparison with the bulk of data in the literature.

#### 4.5.2 Root Mean Square Blend Times

The root mean square blend times are given in tables 4.1 to 4.4 and are plotted in figures 4.7 to 4.10. Again the data of Mackinnon (1987) are included for comparison.

The critical Reynolds number can be identified clearly on these plots and it can be seen that the Lightnin A310 hydrofoil is affected by viscosity at a higher Reynolds number than the other three impellers. This result has been reported by Zlokarnik (1967) and Khang (1975). Both measured blend times for marine propellers and Rushton turbines found that, for the marine propeller  $Re_{crit} \approx 2.0 \times 10^4$  and for the Rushton turbine  $Re_{crit} \approx 2.0 \times 10^3$ . The results reported here show the same trend with the highest  $Re_{crit} \approx 1.0 \times 10^4$  for the Lightnin hydrofoil and the lowest  $Re_{crit} \approx 3.0 \times 10^3$  for the flat blade turbine. The value of  $Re_{crit}$  for the two pitched blade turbines occurs at about  $5.0 \times 10^3$ . The impeller with the highest power number (the flat blade turbine) has the lowest  $Re_{crit}$  and vice versa. This is because the power input to the fluid by the four impellers at a particular Reynolds number will be different. For example, if a *T/3* flat blade turbine with  $Po = 3.0$  is compared with a *T/2* hydrofoil with  $Po = 0.3$  at equal Reynolds number the flat blade turbine will input approximately 15 times the power of the hydrofoil.

Another comparison which can be made between the data reported here and in the

literature is the relationship between  $N\theta$  and Reynolds number. It is generally accepted that, in the viscous mixing regime:

$$N\theta \propto Re^{-1} \quad \dots \text{Eqn(4.17)}$$

Most workers agree that  $x = 1$  in the transitional regime.

The Statgraphics computer package was used to perform a regression of  $N\theta$  on Reynolds number for the data taken in the viscous regime (ie.  $Re < Re_{crit}$ ) and the results are shown in table 4.5. Both pitched blade turbines fit this relationship given by equation (4.17) but the flat blade turbine and hydrofoil do not.

Zlokarnik (1967) found that the exponent,  $x$ , on Reynolds number for a marine propeller was -0.7 which agrees quite well with the value of -0.84. The value of  $x$  for the flat blade turbine, of -1.26, is probably due to the number of points in the data set.

### 4.5.3 Correlation of Data

Initially the data were plotted as  $Po^{1/3} Re$  versus  $1/Fo$ , ie. the turbulent data correlation, for each impeller, in figures 4.11 to 4.14, and all the data sets together. This is presented in figure 4.15 and shows that two mixing regimes exist. The boundary between the two regimes occurs at:

$$1/Fo \approx 10^3 \quad \dots \text{Eqn(4.18)}$$

So the data groups were split at this value using the facility in the Statgraphics package.

The data in the groups defined by equation (4.18) have been correlated by performing a regression of:

$$Po^{1/3} Re \propto Fo^y \quad \dots \text{Eqn(4.19)}$$

and the results are given in the table 4.6.

A very good fit of the data to the relationship was obtained but the exponent,  $y$ , is different for each impeller. The correlation would be easier to use, for agitator design and impeller comparison, if the exponents are equal so the value of  $y$  was rounded-up to -0.5 for each impeller and the constants and their standard deviations compared, ie.

the constants in the following two equations were compared:

$$\frac{Po^{1/3} Re}{Fo^y} = k_y \quad \dots \text{Eqn(4.20)}$$

$$\frac{Po^{1/3} Re}{Fo^{-0.5}} = k_{0.5} \quad \dots \text{Eqn(4.21)}$$

The comparison of the constants and their deviations is given in table 4.7.

The table shows that rounding-up the exponent has very little effect on the scatter of the data. Thus all the data taken at  $Re < Re_{crit}$  can be fitted to:

$$Po^{1/3} Re = \frac{183}{\sqrt{Fo}} \quad \dots \text{Eqn(4.22)}$$

The critical values of the two correlating variables were identified by solving equations (4.12) and (4.22). This shows that:

$$Po^{1/3} Re_{crit} = 6370 \quad \dots \text{Eqn(4.23)}$$

$$1/Fo_{crit} = 1225 \quad \dots \text{Eqn(4.24)}$$

$1/Fo_{crit}$  is close to the value taken to split the data ( $1/Fo = 1000$ ) as expected.

The critical value of Reynolds number for a particular impeller can be found by re-arranging equation (4.23) to show:

$$Re_{crit} = \frac{6370}{Po^{1/3}} \quad \dots \text{Eqn(4.25)}$$

This shows that an impeller with a high power number will have a lower critical Reynolds number than an impeller with a low power number which can be seen in plots 4.7 to 4.10 and has been reported by Zlokarnik (1967) and Khang (1975).

## 4.6 DISCUSSION

### 4.6.1 Comparison using Power Input

The information that will be of most interest to agitator designers is the type of impeller which gives the best performance in blending fluids for the lowest power input. The data taken in the turbulent regime, equation (4.12), can be re-arranged to obtain a relationship between blend time, impeller diameter and vessel scale:

$$\theta = \frac{5.20 T^2}{P_o^{1/3} N D^2} \quad \dots \text{Eqn(4.26)}$$

which can be re-arranged to show:

$$\theta = 5.20 \left( \frac{T^3}{P_o N^3 D^5} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.27)}$$

which can in turn be re-arranged to:

$$\theta \propto \left( \frac{1}{\epsilon} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.28)}$$

Equation (4.28) shows that if impellers are compared at equal power input, a large impeller (smaller  $T/D$ ) will give a shorter blend time. This rule has been confirmed by many other workers ( eg. Procházka & Landau (1961), Khang (1975)) but Havas *et al.* (1978b) found that the optimum diameter of a VEGYTERV propeller was approximately 55% of the vessel diameter and Van de Vusse (1955b) found that the optimum impeller diameter was approximately 40% of the vessel diameter. The impeller sizes studied by Mackinnon (1987), whose data have been re-analyzed, Procházka & Landau (1961) and Khang (1975) were less than or equal to 50% of the vessel diameter so it is likely that insufficient data were taken to identify a similar minimum in power requirement.

A similar re-analysis can be performed on the data taken in the viscous regime, equation (4.22), which shows:

$$\theta \propto \left( \frac{1}{\bar{\epsilon}} \right)^{2/3} \left( \frac{T}{D} \right)^{2/3} T^{-2/3} \quad \dots \text{Eqn(4.29)}$$

Equation (4.29) also shows that large impellers give a shorter blend time when compared at the same power input and scale. Van de Vusse (1955b) found that impellers should be as large as possible when  $Re < 250$ , but no information was presented on impellers greater than 50% of the vessel diameter in the regime studied here, ie.  $200 < Re < 10^4$ .

There are no great surprises in the results of the effect of impeller diameter on blend time in the turbulent and viscous regimes, but the effect of scale at first may appear unusual. In the turbulent regime, at constant power input per unit mass and impeller/vessel geometry, the blend time increases with the ratio between the blend times at large and small scale given by:

$$\frac{\theta_{large}}{\theta_{small}} = \left( \frac{T_{large}}{T_{small}} \right)^{2/3} \quad \dots \text{Eqn(4.30)}$$

This relationship also shows that if a process is to be scaled-up at constant blend time and geometry, the ratio in power inputs is given by:

$$\frac{P_{large}}{P_{small}} = \left( \frac{T_{large}}{T_{small}} \right)^5 \quad \dots \text{Eqn(4.31)}$$

Equation (4.29) can be re-arranged into the same form as equation (4.30) to show that, at constant power input per unit mass and impeller/vessel geometry:

$$\frac{\theta_{large}}{\theta_{small}} = \left( \frac{T_{small}}{T_{large}} \right)^{2/3} \quad \dots \text{Eqn(4.32)}$$

ie. blend time reduces as the process is scaled-up!

If a process is scaled-up at constant power input per unit mass and geometry the ratio

between the impeller speeds at large and small scale will be:

$$\frac{N_{large}}{N_{small}} = \left( \frac{D_{small}}{D_{large}} \right)^{2/3} \quad \dots \text{Eqn(4.33)}$$

The ratio between Reynolds numbers will be:

$$\frac{Re_{large}}{Re_{small}} = \frac{N_{large} D_{large}^2}{N_{small} D_{small}^2} \quad \dots \text{Eqn(4.34)}$$

Substituting equation (4.33) into (4.34):

$$\frac{Re_{large}}{Re_{small}} = \left( \frac{D_{large}}{D_{small}} \right)^{4/3} \quad \dots \text{Eqn(4.35)}$$

It has been shown that, in the viscous regime:

$$N\theta \propto Re^{-1} \quad \dots \text{Eqn(4.17)}$$

Combining equations (4.35) and (4.17) shows the relationship between  $N\theta$  at different scales:

$$\frac{(N\theta)_{large}}{(N\theta)_{small}} = \left( \frac{D_{small}}{D_{large}} \right)^{4/3} \quad \dots \text{Eqn(4.36)}$$

Cross-multiplying the impeller speeds at the two scales and substituting equation (4.33):

$$\frac{\theta_{large}}{\theta_{small}} = \left( \frac{D_{small}}{D_{large}} \right)^{2/3} \quad \dots \text{Eqn(4.37)}$$

and since blend time at constant geometry is being considered the ratio between impeller and vessel diameter is constant so equation (4.37) can be re-written:

$$\frac{\theta_{large}}{\theta_{small}} = \left( \frac{T_{small}}{T_{large}} \right)^{2/3} \quad \dots \text{Eqn(4.38)}$$

In practise, it is more likely that an agitator will be designed at constant blend time, in which case the power per unit mass required to achieve that blend time will be reduced. The ratio between the power requirements at the two scales will be given by:

$$\frac{P_{large}}{P_{small}} = \left( \frac{T_{large}}{T_{small}} \right)^2 \quad \dots \text{Eqn(4.39)}$$

#### 4.6.2 Use of the Correlation for Agitator Design

Blend times have been analyzed for  $1 < 1/Fo < 10^6$  and two regimes have been identified with the boundary occurring at:

$$1/Fo \approx 10^3 \quad \dots \text{Eqn(4.18)}$$

If  $1/Fo < 10^3$ , blend time reduces with increasing scale and, if  $1/Fo > 10^3$ , blend time increases with increasing scale provided power input per unit mass and impeller/vessel geometry are constant. This result indicates that the optimum scale for operation of an agitator, required to achieve a desired blend time, is the one which operates at the boundary between the two regimes. This is the point at which the blend time will be achieved for minimum power input per unit mass. The vessel diameter will be calculated from:

$$T = \sqrt{\frac{10^3 \mu \theta}{\rho}} \quad \dots \text{Eqn(4.40)}$$

The form of data correlation presented here is particularly useful because it allows various impeller types to be compared with one equation and it splits the variables determined by conditions at the impeller from those determined by the scale of operation and the desired process result (blend time).

If blend time is an appropriate design criterion for a particular agitation duty, the conditions determined by the scale of operation and process result will be known, ie. the vessel scale ( $T$ ), fluid physical properties ( $\rho$ ,  $\mu$ ) and desired blend time ( $\theta$ ) are

specified at the start of the design and Fourier number ( $Fo$ ) can be calculated. The mixing regime can immediately be identified by comparing the calculated value with  $Fo_{crit}$  and the value of  $Po^{1/3} Re$  can be found. This will allow the impeller type, speed and diameter to be chosen in order to satisfy the blend time requirements.

Similarly, if an existing agitator is to be rated or vendors' quotations compared, for a blending duty, the impeller type ( $Po$ ), speed ( $N$ ), diameter ( $D$ ), vessel scale ( $T$ ) and fluid properties ( $\rho, \mu$ ) will be known.  $Po^{1/3} Re$  can be calculated, the mixing regime identified, Fourier number calculated and the blend time found by re-arranging the Fourier number.

The correlation shows that different impeller types will achieve the same blend time at a given scale, provided the impellers are the same diameter and operating at the same power input per unit mass. This is true in both the turbulent and viscous mixing regimes. Thus there is no saving to be made in agitator running costs by choosing one impeller type over another.

The capital cost of an agitator can be related to the shaft torque since the a high torque, requires a greater mechanical strength of gearbox, shaft and impeller. So, savings in capital cost can be achieved by selecting an impeller which operates at a low torque. The torque is related to the impeller power by:

$$P = 2\pi \Lambda N \quad \dots \text{Eqn(4.41)}$$

When two impellers are compared at equal power input the impeller with the higher rotational speed will have the lower torque.

If a flat blade turbine, with  $Po = 3.0$ , and hydrofoil, with  $Po = 0.3$  are compared at equal impeller diameter and power input, the difference in rotational speeds will be:

$$\frac{N_{Hyd}}{N_{FBT}} = \sqrt[3]{\frac{Po_{FBT}}{Po_{Hyd}}} = 2.154 \quad \dots \text{Eqn(4.42)}$$

Therefore, a large diameter hydrofoil will give the most cost effective operation. Further analysis of the economic implications of the results of the research are given in chapter 7 of this thesis.

## 4.7 CONCLUSIONS

The conclusions which can be drawn from the re-analysis of blend time measurements made in water, reported by Mackinnon (1987) are:

- i) blend time data can be correlated by a relationship containing three dimensionless groups; power number, Reynolds number and Fourier number:

$$Po^{1/3} Re = \frac{5.20}{Fo} \quad \dots \text{Eqn(4.12)}$$

The relative standard deviation of the constant of proportionality is  $\pm 10.0\%$ .

- ii) This correlation is based on data taken with two pitched blade turbines, of  $T/3$  and  $T/2$  diameter, a flat blade turbine of  $T/3$  diameter and a Lightnin A310 hydrofoil of  $T/2$  diameter in standard, baffled vessels of 0.61, 1.83 and 2.67 m diameter. In all experiments the impeller was positioned  $T/3$  above the base of the vessel.
- iii) This relationship can be re-arranged to show that the dimensionless group,  $N\theta$ , is constant for each impeller type.
- iv) Individual probe blend times; measured beneath the impeller, halfway between the agitator shaft and behind a baffle are approximately equal.

The conclusions which can be drawn from the data taken in viscous Newtonian fluids for this research project are:

- i) two mixing regimes have been identified with the boundary between them occurring at  $1/Fo \approx 10^3$ .
- ii) for  $1/Fo > 10^3$   $N\theta = \text{constant}$   
for  $1/Fo < 10^3$   $N\theta \propto Re^{-1}$
- iii) when  $1/Fo < 10^3$  blend time can be correlated by:

$$Po^{1/3} Re = \frac{183}{\sqrt{Fo}} \quad \dots \text{Eqn(4.22)}$$

The relative standard deviation of the constant of proportionality is  $\pm 17.4\%$ .

- iv) The same four impellers were used as those used for the water tests, reported by Mackinnon (1987) in 0.30, 0.61 and 1.83 m diameter vessels of the same geometry.
- v) This relationship can be re-arranged to show that  $N\theta$  is inversely proportional to Reynolds number, as mentioned previously.
- vi) When  $Re < Re_{crit}$  individual probe blend times diverge with the longest measured behind the baffle and the shortest beneath the impeller. The degree of divergence increases with decreasing Reynolds number.

The two correlations can be re-arranged to show the dependence of blend time on power per unit mass, impeller/vessel geometry and scale:

$$\text{i) for } 1/Fo > 10^3 \quad \theta \propto \left(\frac{1}{\bar{\epsilon}}\right)^{1/3} \left(\frac{T}{D}\right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.28)}$$

$$\text{ii) for } 1/Fo < 10^3 \quad \theta \propto \left(\frac{1}{\bar{\epsilon}}\right)^{2/3} \left(\frac{T}{D}\right)^{2/3} T^{-2/3} \quad \dots \text{Eqn(4.29)}$$

- iii) in both the turbulent and viscous mixing regimes a large diameter impeller will achieve a shorter blend time than a small one at equal power input per unit mass.
- iv) on scale-up at constant  $D/T$  in the turbulent regime:

$$\text{at equal power input per unit mass:} \quad \frac{\theta_{large}}{\theta_{small}} = \left(\frac{T_{large}}{T_{small}}\right)^{2/3} \quad \dots \text{Eqn(4.30)}$$

ie. blend time increases with increasing scale of operation.

for equal blend time:

$$\frac{P_{large}}{P_{small}} = \left( \frac{T_{large}}{T_{small}} \right)^5 \quad \dots \text{Eqn(4.31)}$$

ie. the power required to achieve equal blend time on scale-up increases dramatically.

v) on scale-up at constant  $D/T$  in the viscous regime:

at equal power input per unit mass:

$$\frac{\theta_{large}}{\theta_{small}} = \left( \frac{T_{small}}{T_{large}} \right)^{2/3} \quad \dots \text{Eqn(4.38)}$$

ie. blend time decreases with increasing scale of operation.

for equal blend time:

$$\frac{P_{large}}{P_{small}} = \left( \frac{T_{large}}{T_{small}} \right)^2 \quad \dots \text{Eqn(4.39)}$$

ie. the power required to achieve equal blend time increases with increasing scale but not as quickly as in the turbulent regime.

There are no savings to be made in agitator running costs because the blend time is dependent on the power input to the fluid in both mixing regimes. Choosing a large impeller diameter will reduce the running cost required to achieve a desired blend time.

There are potential savings to be made in capital cost if an impeller with a low power number is chosen. This will run at a high speed to deliver the required power input and will therefore operate at a lower torque. Less mechanical strength will be required in the gearbox, shaft and impeller giving potential savings in the agitator's capital cost.

## **CHAPTER 5**

### **BLENDING OF PSEUDO-PLASTIC FLUIDS**

#### **5.1 INTRODUCTION**

The vast majority of viscous fluids processed in the chemical industry are non-Newtonian and a large proportion of these are pseudo-plastic. Short & Etchells (1982) have highlighted the importance of researching the effects of non-Newtonian fluid properties on agitation.

The previous chapter dealt with the results of blend time measurements in viscous, Newtonian fluids, the correlation of data and the implications for agitator selection. This chapter covers similar ground, but deals with the measurements made in viscous, pseudo-plastic fluids.

The experimental and data reduction methods used were identical to those used for the Newtonian measurements, described in sections 4.2 and 4.3.

The viscosity of the non-Newtonian fluids varies with position in an agitated vessel because the shear rate differs. The viscosity of a pseudo-plastic fluid will be low near the impeller where the shear rates are high and high near the fluid surface or the wall of the vessel where the shear rates are low. Section 5.2 discusses pseudo-plasticity, its effect on blending and proposes a method of account for it in the correlation of blend time data.

The application of this correlation method to the data is discussed in section 5.3 and a general discussion is given in section 5.4. The conclusions are given in section 5.5.

#### **5.2 PSEUDO-PLASTIC FLUIDS**

##### **5.2.1 The Power Law Model**

The rheology of a fluid is determined by measuring the shear stress over a range of shear rates. When the data are plotted, a Newtonian fluid is characterised by a straight line relationship between the two parameters. The gradient of the line is the dynamic viscosity of the fluid, ie:

$$\tau = \mu \dot{\gamma} \quad \dots \text{Eqn(5.1)}$$

A pseudo-plastic fluid is characterised by a progressively decreasing gradient on a plot of shear stress versus shear rate. The apparent viscosity of the fluid at a particular shear rate is the ratio between the corresponding shear stress and the shear rate, ie:

$$\mu_a = \frac{\tau}{\dot{\gamma}} \quad \dots \text{Eqn(5.2)}$$

At very high and low shear rates the apparent viscosity becomes constant (ie. the fluid is Newtonian) and equal to  $\mu_\infty$  and  $\mu_0$ . These relationships are shown in figure 5.1.

There are several empirical correlations which describe the relationship between shear stress and shear rate in a pseudo-plastic fluid and of these, the simplest is the power law model. This is written as:

$$\tau = K \dot{\gamma}^n \quad \text{where } n < 1 \quad \dots \text{Eqn(5.3)}$$

$K$  is a measure of the fluid's viscosity, or "thickness", and  $n$  is a measure of the fluid's deviation from Newtonian behaviour. For a Newtonian fluid,  $n = 1$  and  $K = \mu$ .

If equations (5.2) and (5.3) are combined:

$$\mu_a = K \dot{\gamma}^{n-1} \quad \dots \text{Eqn(5.4)}$$

As the shear rate increases the apparent viscosity of the fluid decreases, hence shear-thinning behaviour.

There are several objections that have been raised against the use of the power law model. Among these are:

- i) when the shear rate is zero, the predicted viscosity of the fluid is infinite.
- ii) for real fluids,  $n$  is not constant over a wide range of shear rates.
- iii) the dimensions of  $K$  are dependent on  $n$ .

The second objection to the use of the power law can be overcome by specifying the shear rate range over which the rheological measurements have been made and the values of  $K$  and  $n$  can be applied.

Other, more complex models have been proposed that overcome these objections but the power law is widely used because of its simplicity and because it has been found to adequately describe pseudo-plastic fluid rheology in many applications.

The apparent viscosity at a particular point in a non-Newtonian fluid can be estimated if the shear rate in the fluid at that point is known. One of the most important aspects of this project is to identify a method of characterising the shear rates in agitated, pseudo-plastic fluids so that the blend time data can be correlated.

### 5.2.2 Shear Rates

Examination of the Newtonian data shows that the blending in the vessel as a whole is controlled by the rate of mixing at probe 3, which was positioned behind a baffle (see figure 3.6). Thus, the viscosity in this region of the vessel controls the overall blending process. A method for estimating the characteristic shear rate is therefore required in order to calculate this viscosity. Ideally, the viscosity at probe 3 should be estimated but this is very difficult. The viscosity at the wall of the vessel can be found more easily and should be significantly different from the baffle viscosity. This viscosity has been used to correlate the blend time data and its estimation is discussed in the following section.

### 5.2.3 Wall Shear Rate Estimation

The estimation of the shear rate is based on a torque balance which equates the shear stress at the wall of the vessel with the torque per unit volume. This is a simplification of the derivation of this equation which should actually be written:

$$\begin{aligned} \text{Torque} = & \text{Shear stress at surface} \\ & \times \text{Area of surface} \\ & \times \text{Radial distance from vessel axis to surface.} \end{aligned}$$

Bird *et al.* (1960) give an equation relating the shaft torque to the fluid viscosity, the velocity gradient (or shear rate) in the fluid at the wall and base of the vessel and the pressure drop due to the fluid velocity changing as it impinges on the baffles:

$$\Lambda = \mu \iint_S R \left( \frac{\partial v_\theta}{\partial r} \right)_w dS + \iint_A R p_{baff} dA \quad \dots \text{Eqn(5.5)}$$

where:

- $v_\theta$  is the tangential component of the fluid velocity.
- $S$  is the total surface area of the vessel in contact with fluid.
- $A$  is the total surface area of the baffles.
- $R$  is the radial distance to any surface area element  $dS$  or  $dA$  from the impeller axis of rotation.
- $r$  is the distance measured into the fluid from any element of tank surface,  $dS$  or  $dA$ .
- $p$  is the pressure exerted on the baffles by the fluid.

In order to solve equation (5.5), it is necessary to have an estimate of the tangential velocity gradient at the vessel surfaces. If an average shear rate at the surfaces of the vessel is defined, this term can be re-written:

$$\mu \iint_S R \left( \frac{\partial v_\theta}{\partial r} \right)_w dS = \mu \dot{\gamma}_w \iint_S R dS = \tau_w \iint_S R dS \quad \dots \text{Eqn(5.6)}$$

No assumptions need to be made about the velocity gradient since this has been combined with the viscosity term to give the shear stress in the fluid at the vessel surfaces. In order to define an average shear rate, in equation (5.6), it has been assumed that the shear stress is constant at all points on the vessel surface. Wichterle *et al.* (1985) have measured the shear stress on the wall of an agitated vessel using an electrochemical technique and shown that it does vary with position. It is also likely to be a function of impeller type with an axial flow impeller giving higher shear stresses on the base of the vessel than a radial flow impeller which will give high shear stresses on the wall. The assumption of constant wall shear stress has been made in the following analysis even though this over simplifies the true relationship between the shear stress and position on the vessel wall because there are no relevant data which can be applied to the four impellers tested and it has been used previously by Elson (1988, 1990) in the analysis of cavern formation in agitated yield stress fluids.

Equation (5.5) can be written in terms of three components which are balanced with the shaft torque. These are:

- i) the contribution of the cylindrical vessel wall.
- ii) the contribution of the vessel base.
- iii) the contribution of the pressure drop on the baffles.

Therefore the vessel will be considered as three separate elements, each balancing a proportion of the shaft torque. If a term  $G$  is defined as the product of the area of a small element of the vessel surface and its radial distance from the vessel axis, equation (5.5) and (5.6) can be re-arranged to:

$$\Lambda = \tau_w \left[ \iint_{S_{cyl}} dG_{cyl} + \iint_{S_{base}} dG_{base} \right] + \iint_A p_{baff} dG_{baff} \quad \dots \text{Eqn(5.7)}$$

In order to carry out this integration, the vessel geometry has been simplified and this is shown in figure 5.2. The limits of each integral are also shown.

### 5.2.3.1 The Vessel Wall

The area of a small element of the vessel wall is  $dS_{cyl}$ . Therefore, the product of the surface area and radial distance for each element of the vessel wall can be written:

$$R dS_{cyl} = dG_{cyl} = R ((R d\phi) dz) \quad \dots \text{Eqn(5.8)}$$

The limits of the integration are the arc swept by the vessel wall,  $\phi$ , which ranges from 0 to  $2\pi$  radians and the axial height of the vessel wall,  $z$ , which ranges from 0 to  $\sqrt{3} T/2$ . The radial distance between the vessel axis and the surface under consideration is constant for all elements and is equal to vessel radius,  $T/2$ . The integral equation can be written:

$$G_{cyl} = \left( \frac{T}{2} \right)^2 \int_0^{2\pi} d\phi \int_0^{\sqrt{3} T/2} dz \quad \dots \text{Eqn(5.9)}$$

Integrating:

$$G_{cyl} = \frac{T^2}{4} 2\pi \frac{\sqrt{3}T}{2} = 1.360 T^3 \quad \dots \text{Eqn(5.10)}$$

### 5.2.3.2 The Vessel Base

The vessel in which the blend time measurements were made has a dished base (see figure 3.1) but, in order to simplify the integration, it has been assumed that the vessel has a flat base. The area of a small element of the vessel base is  $dS_{base}$  and the radial distance between the vessel axis and each element varies and is equal to  $r$  (see figure 5.2). Therefore, the product of the area and the radial distance for each element of the vessel base is given by:

$$r dS_{base} = dG_{base} = r ((r d\phi) dr) \quad \dots \text{Eqn(5.11)}$$

Again the arc swept by the vessel base must be considered but the axial height of the base is constant, since it has been assumed that it is flat. The radial position,  $r$ , varies from 0 to  $T/2$ . The integral can be written:

$$G_{base} = \int_0^{2\pi} d\phi \int_0^{T/2} r^2 dr \quad \dots \text{Eqn(5.12)}$$

Integrating:

$$G_{base} = \frac{1}{3} 2\pi \frac{T^3}{8} = 0.262 T^3 \quad \dots \text{Eqn(5.13)}$$

### 5.2.3.3 The Baffles

Two quantities contribute to the effect of the baffles on the torque balance: the first is the area of the baffles and the second, the change in velocity of the fluid as it impinges on the baffles.

The pressure drop due to the baffles is:

$$p_{baff} = \frac{1}{2} \rho (\Delta v)^2 \quad \dots \text{Eqn(5.14)}$$

The area of a small element of the baffle is  $dA$  and the radial position of the baffle varies and this is defined as  $r$ . The product of the area and the radial distance can be written as:

$$r \, dA = dG_{baff} = r \, (dz \, dr) \quad \dots \text{Eqn(5.15)}$$

The height of the baffles,  $z$ , ranges from 0 to  $\sqrt{3} \, T/2$  and they are  $T/12$  wide and mounted  $T/60$  off the vessel wall, giving a total distance between the inner edge of the baffle and the vessel wall of  $T/10$ . Considering one baffle, the integral can be written:

$$G_{baff} = \int_0^{\sqrt{3} \, T/2} dz \int_{(T/2 - T/60)}^{(T/2 - T/10)} r \, dr \quad \dots \text{Eqn(5.16)}$$

Integrating:

$$G_{baff} = \frac{1}{2} \frac{\sqrt{3} \, T}{2} \left( \left( \frac{29 \, T}{60} \right)^2 - \left( \frac{2 \, T}{5} \right)^2 \right) = 0.03187 \, T^3 \quad \dots \text{Eqn(5.17)}$$

#### 5.2.3.4 Combining

The three components of the equation can be combined to give:

$$\Lambda = \tau_w (G_{cyl} + G_{base}) + 4 \times p_{baff} \, G_{baff} \quad \dots \text{Eqn(5.18)}$$

$$\Lambda = \tau_w (1.360 + 0.262) \, T^3 + \left( 4 \times \left( \frac{\rho (\Delta v)^2}{2} \right) 0.03187 \, T^3 \right) \quad \dots \text{Eqn(5.19)}$$

Re-arranging equation (5.19):

$$\tau_w = \frac{1}{1.622} \left( \frac{\Lambda}{T^3} - 0.0638 \rho (\Delta v)^2 \right) \quad \dots \text{Eqn(5.20)}$$

The only quantity in equation (5.20) which is not known is the velocity of the fluid impinging on the baffles. This has been estimated from the data of Schwartzberg & Treybal (1968) who used streak-photography to measure the fluid velocities in vessels agitated by Rushton turbines. They correlated their data by:

$$v_{ST} = k_{ST} \frac{N D^2}{(T^2 H)^{1/3}} \quad \dots \text{Eqn(5.21)}$$

where  $k_{ST}$  is 1.586 for the average total velocity and 1.387 for the average vertical-plane fluid velocity. If it is assumed that the velocity of the fluid after impact on the baffles is zero, equation (5.20) can be written as:

$$\tau_w = \frac{1}{1.622} \left( \frac{\Lambda}{T^3} - 0.0638 \rho v_{ST}^2 \right) \quad \dots \text{Eqn(5.22)}$$

|                                            |                        |                 |
|--------------------------------------------|------------------------|-----------------|
| The values of $k_{ST}$ have been taken as: | Flat blade turbines    | $k_{ST} = 1.3$  |
|                                            | Pitched blade turbines | $k_{ST} = 0.75$ |
|                                            | Hydrofoils             | $k_{ST} = 0.35$ |

The reason for varying the constant with impeller type is that Fawcett (1989) found that the tangential component of velocity is largest for impellers which produce a predominantly radial flow pattern and smallest for impellers which produce a predominantly axial flow pattern. It must be stressed that these constants are assumed and further work on the measurement of fluid velocities in agitated vessels will be required to provide better confidence in this analysis.

The shear stress at the wall of the vessel has been calculated for each data point using equation (5.22). The shear rates and viscosities of the fluids have then been calculated using the power law, ie. from equation (5.3):

$$\mu_w = K \dot{\gamma}_w^{(n-1)} \quad \dots \text{Eqn(5.23)}$$

The wall viscosity is then used to calculate Reynolds and Fourier numbers.

### 5.3 ANALYSIS OF PSEUDO-PLASTIC BLEND TIME DATA

The pseudo-plastic blend time data have been analyzed in the same way as the Newtonian data:

- i) individual probe  $N\theta$  values plotted versus Reynolds number, based on the wall viscosity derived in the previous section, to show how the blending process proceeds in regions of differing agitation intensity.
- ii) root mean square  $N\theta$  values plotted versus the same Reynolds number to show how blending proceeds in the vessel as a whole.
- iii) fitting the data to the correlation developed for Newtonian data.

#### 5.3.1 Individual Probe Blend Times

As mentioned in the last chapter, individual probe blend times were measured to investigate the way in which blending proceeds in regions of differing agitation intensity.

The results of the pseudo-plastic blending experiments are given in tables 5.1 - 5.4. The tables show vessel scale, the concentration and grade of Natrosol used, the individual probe blend times and the root mean square blend time for each set experimental conditions. The blend time reported is the mean of eight repetitions and the standard deviation of the mean is given as a percentage in brackets below the blend time result. This is identical to the way in which the Newtonian data are reported. The individual probe results are plotted as  $N\theta_i$  versus Reynolds number in figures 5.3 - 5.6. The data reported by Mackinnon (1987), taken in water are included in the plots for comparison.

The plots are almost identical to figures 4.3 - 4.6 which show  $N\theta_i$  versus Reynolds number for the Newtonian fluids, ie:

- i)  $Re > Re_{crit}$   $N\theta_1 \approx N\theta_2 \approx N\theta_3$
- ii)  $Re < Re_{crit}$   $N\theta_1 < N\theta_2 < N\theta_3$

- iii) the degree of divergence of the mixing times increases with decreasing Reynolds number.

For this reason, the explanation given in section 4.5.1, for the behaviour of Newtonian fluids can be applied to the behaviour on pseudo-plastic fluids.

### 5.3.2 Root Mean Square Blend Times

The root mean square blend times are given in tables 5.1 - 5.4 and plotted in figures 5.7 - 5.10 with the data taken by Mackinnon (1987) in water included for comparison.

The trends observed in the non-Newtonian root mean square blend times are identical to those observed in Newtonian fluids, ie:

- i)  $Re > Re_{crit}$   $N\theta \approx \text{constant}$
- ii)  $Re < Re_{crit}$   $N\theta \propto Re^{-x}$

The value of  $x$  for each impeller is given in table 5.5 with the relevant statistical information. The results are similar to those found for Newtonian fluids and, as with the individual probe mixing times, the explanation presented for the blending of Newtonian fluids can be applied to pseudo-plastic fluids. The fit of data is worse than that obtained for the Newtonian data and this is probably due to errors in the estimation of the fluid viscosity.

### 5.3.3 Correlation of Data

The values of the two dimensionless groups,  $Po^{1/3} Re$  and  $1/Fo$  were calculated for all the Newtonian and pseudo-plastic data points and plotted. The boundary between the turbulent and viscous mixing regimes occurred at:

$$1/Fo \approx 10^3$$

as for the Newtonian fluids alone, so the data sets were split at this value.

#### 5.3.3.1 Turbulent Data

The turbulent data have been analyzed by performing a multiplicative regression of  $N\theta$

on  $Po$  and  $D/T$  in the same way as the Newtonian data. The result of the regression is:

$$N\theta \propto Po^{-0.357} \left( \frac{D}{T} \right)^{-1.933} \quad \dots \text{Eqn(5.24)}$$

The correlation coefficient,  $r^2$ , was 82.74% and F-ratio is 121 for 51 data points. Again, these statistics are worse than the corresponding values obtained for the Newtonian data.

After rounding-up the data can be described by equation (4.12) but with the constant of proportionality changed from 5.20 to 5.47, ie:

$$Po^{1/3} Re = \frac{5.47}{Fo} \quad \dots \text{Eqn(5.25)}$$

The relative standard deviation of the constant is  $\pm 13.6\%$ .

### 5.3.3.2 Viscous Data

The data have been correlated in the same way as the Newtonian data by performing a regression of:

$$Po^{1/3} Re \propto Fo^y \quad \dots \text{Eqn(4.19)}$$

and the results are given in table 5.6.

A very good fit of the data was obtained and, again, the exponents were rounded-up to -0.5. A comparison of the of the exact and rounded-up correlations are given in table 5.7. Rounding-up does not significantly worsen the standard deviation of the correlation so the data are presented in this form. All the viscous data (Newtonian and pseudo-plastic) are correlated by:

$$Po^{1/3} Re = \frac{193}{\sqrt{Fo}} \quad \dots \text{Eqn(5.26)}$$

The relative standard deviation of the constant is  $\pm 32.5\%$ .

Also the Newtonian and pseudo-plastic data sets for each impeller have been compared using the “Student’s t-test”. They are not statistically different at the 95% confidence level.

The Newtonian and pseudo-plastic data, substituted into the correlation, are plotted for each impeller in figures 5.11 - 5.14 and for all impellers and fluids in figure 5.15.

## 5.4 DISCUSSION

### 5.4.1 Controlling Viscosity

Moo-Young *et al.* (1972) measured blend times in Newtonian and pseudo-plastic fluids and plotted  $N\theta$  versus Reynolds number, with the fluid viscosity defined by equation (2.26). They found that the Newtonian data behaved as expected with:

$$N\theta \propto Re^{-1} \quad \dots \text{Eqn(4.17)}$$

but the pseudo-plastic data did not fit the same relationship. The exponent on Reynolds number was more negative than -1 indicating that, when compared on the basis of equal Reynolds number, blending of pseudo-plastic fluids is more difficult than Newtonian blending.

Analysis of the Newtonian data, discussed in the previous chapter, showed that the blending of the whole vessel was controlled by the longest blend time, measured behind the baffle. Blend times measured in the bulk of the vessel and near the impeller were always much shorter than the limiting blend time. For this reason, the analysis of the pseudo-plastic blend time data started with finding a method of estimating the fluid viscosity in this region of the vessel.

The method used to estimate the wall shear rate and viscosity is based on a torque balance between the agitator shaft and the baffles and vessel surfaces. This has been used in the past by Elson (1990) to correlate mixing data taken in yield stress fluids. The inclusion of the baffles in the analysis has not been done before. If the baffles are ignored, a worse correlation between the Newtonian and pseudo-plastic data is obtained. If the data are analyzed ignoring the baffles, the two sets fail the t-test and are statistically different.

The definition of Fourier number, on the basis of wall viscosity, is sensible because it

contains the controlling blend time and the viscosity in the region of the vessel where this blend time occurs.

Reynolds number is determined by conditions at the impeller and should, therefore, include the viscosity of the fluid in the impeller region. Defining Reynolds number with the wall viscosity gives a ratio of the inertial forces produced by the impeller and the viscous forces in the fluid in this region of the vessel. This is sensible because, again, the region of the vessel where the controlling blend time occurs is being considered.

The fact that a much better correlation between the Newtonian and pseudo-plastic blend time data has been produced than has appeared in the literature previously indicates that this is a good model of the process but, as mentioned previously, further work will be required for its validation.

#### 5.4.2 Comparison using Power Input

As mentioned in the last chapter, the information of most interest to agitator designers is the type of impeller which gives the best performance in blending fluids for the lowest power input. The basic conclusions drawn from the Newtonian blend time data were:

- i) for a given process operated at constant geometry ( $D/T$ ) and scale, there is no saving to be made in running cost because blend time is dependent on power input. A large diameter impeller will achieve a desired blend time for less power than a small impeller.
- ii) for a given process, there are potential savings to be made if an impeller with a low power number is chosen because it must run at a higher speed to input the same power as an impeller with a high power number.

These conclusions are true in both the turbulent and viscous regimes.

Equation (5.26) can be re-arranged to show:

$$\theta \propto \left( \frac{1}{\varepsilon} \right)^{2/3} \left( \frac{T}{D} \right)^{2/3} \frac{\mu_w}{\rho} T^{-2/3} \quad \dots \text{Eqn(5.27)}$$

This equation is identical to equation (4.29) for Newtonian fluids. The fluid physical properties do not appear in equation (4.29) because they are constant for a Newtonian fluid. They must be included when a pseudo-plastic fluid is examined because they will vary depending on the impeller type used to blend the fluid. The derivation of the wall viscosity shows that its value is a function of the torque acting on the agitator shaft - the higher the torque, the higher the wall shear rate and the lower the wall viscosity.

If a blending duty is considered at constant power input per unit mass and geometry (ie.  $D/T$  is constant) and at one scale the blend time will be determined by the wall viscosity of the fluid. An impeller which minimizes the wall viscosity at a given power input will achieve the shortest blend time. When the four impellers tested are compared at equal power input per unit mass, the highest torque is produced by the  $T/2$  pitched blade turbine and the results of the blend time measurements show that it is the best impeller to use for pseudo-plastic mixing duties.

In fact, the results imply that an impeller with a higher power number would be even more efficient for blending these fluids. Zlokarnik (1967) concluded that, for  $0.25 < 1/Fo < 10^4$ , the most energy efficient impellers to use are gates and large diameter paddles. This approach to blending viscous fluids has been pursued by two Japanese agitator manufacturers who offer wide blade paddle impellers for these duties. Sumitomo Heavy Industries have the “Max-Blend” impeller and Shinko-Pantec have the “Full-Zone” impeller which operate on the same principles as the gates and paddles tested by Zlokarnik (see figure 5.16).

### 5.4.3 Measurement of Rheology

The results of the blend time experiments show how important the measurement of fluid rheology is to agitator selection and design.

Equation (5.26) shows how the blend time varies with the viscosity of the fluid at constant power input per unit mass and impeller/vessel geometry:

$$\theta \propto \mu_w \propto K \dot{\gamma}_w^{n-1} \quad \dots \text{Eqn(5.28)}$$

Equation (5.22) and (5.23) can be combined to show the relationship between the fluid's power law constant and index and the agitator shaft torque at constant scale:

$$\dot{\gamma}_w \propto \left( \frac{\Lambda}{K} \right)^{\frac{1}{n}} \quad \dots \text{Eqn(5.29)}$$

If equations (5.28) and (5.29) are combined the effect that errors in the measurement of the power law constant and index will have on blend time can be examined:

$$\theta \propto K \left( \frac{\Lambda}{K} \right)^{\frac{n-1}{n}} \propto K^{\frac{1}{n}} \Lambda^{\frac{n-1}{n}} \quad \dots \text{Eqn(5.30)}$$

Equation (5.30) shows that, if the fluid is Newtonian, ie.  $n = 1$ , the mixing time is proportional to the fluid viscosity, as expected.

The mixing time in two fluids, 1 and 2, can be compared at constant torque. The ratio of the blend times will be:

$$\frac{\theta_1}{\theta_2} = \frac{K_1^{1/n_1}}{K_2^{1/n_2}} \Lambda^{\left[ \left( \frac{n_1-1}{n_1} \right) - \left( \frac{n_2-1}{n_2} \right) \right]} \quad \dots \text{Eqn(5.31)}$$

If subscript 1 refers to the true values of  $\theta$ ,  $K$  and  $n$  for a particular fluid and subscript 2 refers to the measured values of  $K$  and  $n$  and the predicted value of  $\theta$ , calculated from those values, then:

$$K_1 = e_K K_2 \quad \dots \text{Eqn(5.32)}$$

and:

$$n_1 = e_n n_2 \quad \dots \text{Eqn(5.33)}$$

If the measured value of  $n$  is correct but there is an error in  $K$ , the relationship between the true and predicted blend times will be:

$$\theta_1 = e_K^{1/n} \theta_2 \quad \dots \text{Eqn(5.34)}$$

The greater the degree of pseudo-plastic behaviour exhibited by the fluid, the greater the value of the exponent,  $1/n$ , and the greater the error in the predicted blend time. For example:

$$\text{if } e_K = 1.1 \text{ and } n = 0.9, \text{ then: } \theta_1 = 1.11 \theta_2$$

$$\text{if } e_K = 1.1 \text{ and } n = 0.5, \text{ then: } \theta_1 = 1.21 \theta_2$$

Similarly, if the measured value of  $K$  is correct but there is an error in  $n$ , the relationship between the true and predicted blend times will be:

$$\theta_1 = K^{\left(\frac{1-e_n}{e_n n_2}\right)} \Lambda^{\left(\frac{1+e_n}{e_n n_2}\right)} \theta_2 \quad \dots \text{Eqn(5.35)}$$

$$\text{if } e_n = 1.1 \text{ and } n = 0.9, \text{ then: } \theta_1 = K^{-0.10} \Lambda^{2.02} \theta_2$$

$$\text{if } e_n = 1.1 \text{ and } n = 0.5, \text{ then: } \theta_1 = K^{-0.18} \Lambda^{3.64} \theta_2$$

This analysis shows that errors in the measurement of the power law constant and index are magnified when the fluid is highly pseudo-plastic and accurate measurement of these parameters is more important than it is in Newtonian blending.

#### 5.4.4 Data Correlation

The scatter of the data about the correlation of the pseudo-plastic blend time data is twice as wide as the scatter of Newtonian data about their correlation.

The reason for this is most probably due to errors in the estimation of the fluid viscosity at the vessel wall,  $\mu_w$ . These errors are caused because the method used for the estimation of the fluid velocity impinging on the baffles is based on measurements made by Schwartzberg & Treybal (1968) of the fluid velocities produced by a Rushton turbine in the turbulent regime. No account of the effect of the fluid viscosity on the velocity has been included because there does not appear to be a rigorous way of doing this.

Wong & Huang (1988) measured the pumping capacity of Rushton turbines in Newtonian fluids and found that it is constant when  $Re > 2 \times 10^4$  and reduces significantly as the flow regime becomes transitional. Koutsakos *et al.* (1990) measured the pumping capacity of Rushton turbines in pseudo-plastic fluids and found that it is constant when  $Re > 10^4$  and it reduces in the range  $60 < Re < 10^4$  with  $Fl \propto Re^{-0.2}$ .

Although there is this disagreement about the way pumping capacity and flow near the impeller change with changing fluid viscosity, there is no information on the way the viscosity affects flow in the regions of the vessel away from the impeller.

Also, assumptions have been made about the velocity of the fluid after it has been in contact with the baffle and the effect of impeller type on the fluid velocity near the baffles.

In order to overcome the need to make assumptions about the fluid velocities impinging on the baffles and the effect of impeller type, measurements of the flow in this region of the vessel must be made. This could be done using laser-Doppler anemometry and would prove, or disprove, the validity of the model proposed in this thesis for the blending of viscous, pseudo-plastic fluids.

## 5.5 CONCLUSIONS

The conclusions which can be drawn from the analysis of the pseudo-plastic blend time data are:

- i) the pseudo-plastic data have been analyzed by estimating the viscosity at the wall of the vessel. This has been done because this is the region of the vessel where the overall blending in the vessel is controlled.
- ii) the wall viscosity has been estimated by carrying out a torque balance between the impeller and the wall of the vessel also taking into account the pressure drop due to the fluid flow impinging on the baffles. This calculation allows the shear stress in the fluid at the wall to be estimated from:

$$\tau_w = \frac{1}{1.622} \left( \frac{\Lambda}{T^3} - 0.0638 \rho v_{ST}^2 \right) \quad \dots \text{Eqn(5.22)}$$

- iii) with knowledge of the fluid's power law constant,  $K$ , and index,  $n$ , the viscosity is estimated from:

$$\mu_w = K \dot{\gamma}_w^{(n-1)} \quad \dots \text{Eqn(5.23)}$$

Equation (5.3) was used to find the relationship between the wall shear rate and wall shear rate.

Assumptions have been made regarding the fluid velocity impinging on the baffles, required to estimate the pressure exerted on the baffles. The work of Schwartzberg & Treybal (1968), who measured fluid velocities in the turbulent regime, has been used to estimate the velocities in the viscous regime. This is an area requiring further work.

When the data taken in the turbulent regime were analyzed in this way, the following conclusions were drawn:

- i) the pseudo-plastic rheology does not affect the blending of the fluid. All the Newtonian and pseudo-plastic, turbulent blend time data are correlated by:

$$Po^{1/3} Re = \frac{5.47}{Fo} \quad \dots \text{Eqn(5.22)}$$

The standard deviation of the constant of proportionality is  $\pm 13.6\%$ .

- ii) This correlation is based on data taken with two pitched blade turbines of  $T/3$  and  $T/2$  diameter, a flat blade turbine of  $T/3$  diameter and Lightnin A310 and Chemineer HE3 hydrofoils of  $T/2$  diameter. In all experiments the impeller was positioned  $T/3$  above the base of the vessel.
- ii) This relationship can be re-arranged to show that  $N\theta$  is constant and its value is dependent on the impeller type.
- iv) Individual probe blend times; measured beneath the impeller, halfway between the shaft and vessel wall and behind the baffle are approximately equal.

The data taken in the viscous regime were also analyzed by substitution into equation (5.22) and the conclusions drawn were:

- i) two mixing regimes exist with the boundary between them occurring at  $1/Fo \approx 10^3$ .
- ii) for  $1/Fo > 10^3$   $N\theta = \text{constant}$

for  $1/Fo < 10^3$

$$N\theta \propto Re^{-1}$$

- iii) when  $1/Fo < 10^3$  the pseudo-plastic blend time data can be correlated by:

$$Po^{1/3} Re = \frac{200}{\sqrt{Fo}} \quad \dots \text{Eqn(5.36)}$$

The standard deviation of the constant of proportionality is  $\pm 38.4\%$ .

The viscous Newtonian and pseudo-plastic correlation constants are not statistically different when compared with the “Student’s t-test” at the 95% significance level.

When  $1/Fo < 10^3$ , all the blend time data taken in the viscous regime can be correlated by:

$$Po^{1/3} Re = \frac{193}{\sqrt{Fo}} \quad \dots \text{Eqn(5.26)}$$

The standard deviation of the constant of proportionality is  $\pm 32.5\%$ .

- ii) This correlation is also based on data taken with two pitched blade turbines of  $T/3$  and  $T/2$  diameter, a flat blade turbine of  $T/3$  diameter and Lightnin A310 and Chemineer HE3 hydrofoils of  $T/2$  diameter. In all experiments the impeller was positioned  $T/3$  above the base of the vessel.
- iii) This relationship can be re-arranged to show that  $N\theta$  is inversely proportional to Reynolds number.
- iv) Individual probe blend times diverge with the longest measured behind the baffle and the shortest beneath the impeller. The degree of divergence increases with decreasing Reynolds number.

The two correlations can be re-arranged to show the dependence of blend time on power input per unit mass, impeller geometry and scale.

The conclusion drawn from the turbulent, pseudo-plastic data is identical to that drawn from the Newtonian data, and is given by:

$$\text{for } 1/Fo > 10^3 \quad \theta \propto \left( \frac{1}{\bar{\epsilon}} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.28)}$$

The conclusion drawn from the viscous, pseudo-plastic data is based on the a similar relationship between blend time and power input deduced from the Newtonian data:

$$\text{for } 1/Fo < 10^3 \quad \theta \propto \left( \frac{1}{\bar{\epsilon}} \right)^{2/3} \left( \frac{T}{D} \right)^{2/3} \frac{\mu_w}{\rho} T^{-2/3} \quad \dots \text{Eqn(5.27)}$$

This relationship shows that, when compared at constant power input per unit mass, geometry and scale, the blend time is proportional to the wall viscosity of the fluid. Since the wall viscosity is a function of the agitator shaft torque, an impeller with a high power number will achieve the shortest mixing time when compared at constant geometry ( $D/T$ ).

This conclusion is different to that drawn for Newtonian fluids where impellers of the same diameter operated at the same power input, geometry and scale achieve the same blend time. The economic implications of this conclusion are examined in chapter 7 of this thesis.

## CHAPTER 6

### POWER MEASUREMENTS

#### 6.1 INTRODUCTION

It is important to measure power consumed by an agitator during blend time measurements since this is the way in which their efficiency as blending devices is compared. The impeller that achieves a desired blend time for the minimum power input is the most energy efficient and will have the lowest running costs. It is also important to consider the torque delivered by an agitator since this determines the mechanical strength of the agitator gearbox, shaft and impellers and therefore the capital cost of the equipment.

Also, it has been shown in the previous chapter that the power input and torque can be used to determine the viscosity of the fluid in the baffle/wall region of the vessel and it is the viscosity of the fluid here that controls the blending duty in the vessel as a whole.

The method used to measure the impeller power at the 0.609 and 1.830 m scale was identical to that used by Muskett (1987) and used the system first described by Pollard (1983). No work has been done as part of this project to improve or modify the power measurement technique.

The method is briefly described in section 6.2 and the definition of power number is discussed in section 6.3. The results are presented in section 6.4 and discussed in section 6.5, including comparison with the results reported by other workers. Finally, the conclusions are given in section 6.6.

#### 6.2 METHOD

The method used to determine the power consumption of the impellers tested involves the simultaneous measurement of the shaft rotational speed and torque. Having measured the torque and speed, the power is calculated from:

$$P = 2\pi \Lambda N$$

...Eqn(4.41)

The rotational speed of the 0.609 and 1.830 m vessel shafts was measured using optical tachometers and a toothed wheel fixed on the impeller shaft. The measurements were accurate to  $\pm 1$  rpm at the 0.609 m scale and  $\pm 0.5$  rpm at the 1.830 m scale.

The shaft torque at the 0.609 and 1.830 m scale in both the Newtonian and pseudo-plastic fluids was measured using semi-conductor strain gauges mounted directly on the shaft and located above the surface of the fluid under test. Semi-conductor strain gauges were used because they are more sensitive than foil gauges. Low noise slip rings were used to transmit the strain gauge signal from the rotating shaft to the stationary pick-ups on the agitator frame. The signal was filtered, amplified and then sampled by a PDP 11/23 computer.

The strain gauges were calibrated by applying known loads to a lever arm attached to the shaft. The shaft of the 1.830 m vessel was calibrated in place with the force applied to the lever arm measured with a load cell. The shaft of the 0.609 m vessel was calibrated by removing it from the vessel, mounting it horizontally and adding weights to the lever arm. Once the calibration procedure had been completed, tests were carried out with an impeller which would give a known result. The relationship between the voltage output and applied torque was linear.

The data were analyzed by subtracting the PDP 11 computer's digital output with the shaft stationary from the output with the shaft rotating and averaging the readings. Muskett (1987) found that six windows of 2048 data points sampled at 100 Hertz were sufficient to produce an accurate average value. The filter unit was set at one quarter of the sampling frequency (on low pass) to remove high frequency noise. The arrangement was set from the experience reported by Pollard (1983). The impeller power number was calculated by a sub-routine of the sampling program.

Muskett (1987) reported that the accuracy of the torque measurements was within  $\pm 2\%$  and, allowing for error in speed measurements, the power consumption of the agitator recorded was accurate to within  $\pm 3\%$ .

At the 0.305 m scale the power consumption was estimated for the measurements made in viscous Newtonian fluids. The estimation was made using a power number design guide (Palmer (1987)). The impeller shaft rotational speed was measured using a hand-held optical tachometer.

The power consumption, at the 0.305 m scale, was measured in pseudo-plastic fluids

using a “Coesfeld Rheosyst” viscometer which gave a direct reading of the impeller rotational speed and the torque. The output from the viscometer was a voltage which was sampled by a Tulip personal computer which was also programed to calculate the impeller power number from the data. It was purchased after the Newtonian tests had been completed which is why the power numbers were estimated using the power number design guide.

### 6.3. POWER NUMBER

Having measured the torque and rotational speed, the power consumption of the impeller is calculated from equation (4.41). The power number of the impeller,  $P_o$ , can then be calculated from:

$$P_o = \frac{P}{\rho N^3 D^5} \quad \dots \text{Eqn(6.1)}$$

The power number can be considered as a drag coefficient (King (1985)), the higher its value, the higher the power input for a particular combination of impeller diameter and rotational speed. The power number is determined by several factors:

- i) the impeller geometry, ie. the type of impeller and the angle, width, thickness, length and number of the blades. These variables have been comprehensively discussed by Muskett (1987) and for this reason they are not discussed here.
- ii) the vessel geometry, ie. the type and number of baffles, the shape of the vessel base, the position of the impeller relative to the base and surface of the fluid. None of these factors were varied for the power or blend time measurements.
- iii) the fluid viscosity. The flow regimes (turbulent, transitional and laminar) are defined in terms of the relationship between power number and Reynolds number.

The relationship between power number, Reynolds number and the flow regimes can be summarised by:

|                      |                  |                        |
|----------------------|------------------|------------------------|
| Turbulent regime:    | $Re > 10^4$      | $Po = \text{constant}$ |
| Transitional regime: | $10 < Re < 10^4$ | $Po \propto Re^a$      |
| Laminar regime:      | $Re < 10$        | $Po \propto Re^{-1}$   |

The power number - Reynolds number relationship in the turbulent and laminar regimes is well defined and is the same for all impellers. The relationship in the transitional regime depends on impeller type: for some it remains at the turbulent value as Reynolds number decreases and the laminar regime is reached, ie  $a = 0$ . For others power number changes as Reynolds number decreases and it may increase or decrease.

The measurements of power made in this study have concentrated on the transitional regime where the blend time measurements have been made. No power measurements have been made in the laminar regime.

## 6.4 RESULTS

The results are given in tables 6.1 - 6.4 with the power number measured in water, reported by Mackinnon (1987) also included for comparison. The mean power number in each fluid tested is given with the relative standard deviation. The power numbers are approximately equal in all the fluids and rises slightly at the high viscosity end of the range.

Power numbers are plotted versus Reynolds number in figures 6.1 - 6.4. The definition of shear rate used to calculate the viscosity of the pseudo-plastic fluids is that given by Metzner & Otto (1957), ie:

$$\dot{\gamma} = \kappa N \quad \dots \text{Eqn(2.23)}$$

The value of  $\kappa$  used was 11 as reported by Metzner & Otto and others. The definition of Reynolds number is therefore:

$$Re_{MO} = \frac{\rho N D^2}{K (11 N)^{n-1}} = \frac{\rho N^{2-n} D^2}{K 11^{n-1}} \quad \dots \text{Eqn(6.2)}$$

Power number was also plotted versus Reynolds number derived from the wall shear rate, as discussed in the previous chapter, but the best correlation between the Newtonian and pseudo-plastic data were obtained with the Metzner & Otto Reynolds number.

## **6.5 DISCUSSION**

### **6.5.1 Shear Rate and Reynolds Number**

As discussed above, the best agreement between the Newtonian and pseudo-plastic data was obtained using Metzner & Otto's definition of shear rate. This underestimates the shear rate in the turbulent regime since it does not take account of the turbulent stresses in the fluid (Fawcett (1990)). But, at the high end of the Reynolds number range the power number is constant in Newtonian and pseudo-plastic fluids so it is impossible to determine which is the correct method of defining shear rates by simply comparing plots of power number in the two types of fluid.

At the lower end of the Reynolds number range studied, the turbulent stresses would be reduced and, although power number may rise or fall slightly before reaching the laminar regime, it does not change significantly from its turbulent value. Power number must vary sufficiently for a comparison of different shear rate estimation methods to be made.

The best agreement between the Newtonian and pseudo-plastic power numbers was obtained using the Metzner & Otto (1957) definition of shear rate, as discussed above.

The lowest Reynolds numbers at which power numbers and blend times have been measured is approximately 200. This is the Reynolds number at which the power number starts to rise significantly and there are very few data points to compare at this value. Many other workers have used the Metzner & Otto definition of shear rate to correlate their power data and this is discussed in the following section.

### **6.5.2 Comparison with Other Workers**

Many workers have measured power numbers for the same types of impeller as studied in this research.

Axial flow impellers have been studied by Landau & Procházka (1964), Havas *et al.* (1978b) and Shiue & Wong (1984) among others.

Landau & Procházka (1964) measured the power number of a marine propeller in a baffled vessel and found that its value in the turbulent regime was 0.30. As Reynolds number was decreased by increasing the fluid viscosity, the power number started to rise at a Reynolds number of around  $2 \times 10^3$ .

Havas *et al.* (1978b) measured power numbers for standard, S. E. M. and VEGYTERV propellers and found that power number is constant for  $Re > 300$  and starts to rise slightly at Reynolds numbers below this value. The turbulent power numbers of the various propellers varied between 0.098 and 0.80 depending on the diameter, clearance and propeller type. The power number was constant at Reynolds numbers greater than 300 and increased as Reynolds number decreased below this value.

Shiue & Wong (1984) also studied the VEGYTERV propeller and found that, for  $D/T = 0.325$ , the turbulent power number was 0.67. This agreed very well with the value reported by Havas *et al.* (1978b). The value at which power number started to rise in the transitional regime was around  $3 \times 10^3$ , a factor of 10 higher than reported by Havas *et al.*

The turbulent power numbers reported agree well with the power number measured for the axial flow, hydrofoil impellers studied in this project which were 0.30 in the turbulent regime. Landau & Procházka (1964) and Shiue & Wong (1984) both found the boundary between the turbulent and transitional regimes occurred at a Reynolds number of between  $2 \times 10^3$  and  $3 \times 10^3$  which is a factor of 10 higher than the value reported by Havas *et al.* (1978b) and in this thesis.

The three workers referred to above also studied pitched blade turbines and generally the same results were reported, ie. the Reynolds number at the boundary between the turbulent and transitional regime was similar to that found for the propellers studied. Landau & Procházka (1964) found that the power number was 1.45 in the turbulent regime; Havas *et al.* (1978b) found that it was around 1.45, depending on the clearance of the impeller above the vessel base and Shiue & Wong (1984) found that it was 1.74 for a four-bladed turbine and 1.20 for a two bladed-turbine. These values agree well with those measured in this project with the power number of the  $T/3$  turbine ( $w/D = 0.2$ ) measured at 1.60 at the 0.609 m scale and increasing to 1.80 at the 1.830 m scale. A similar effect was observed for the  $T/2$  turbine ( $w/D = 0.13$ ) with the power number

measured at 1.10 and 1.28 at the 0.609 and 1.830 m scales respectively.

There is less information in the literature on flat blade turbines of the type studied in this project, even though they are widely used. The bulk of work carried out with radial flow impellers has been done with Rushton turbines on which the blades are attached to a disc. These are more commonly used in gas dispersion duties and are not commonly found in blending applications. Shiue & Wong (1984) studied a turbine with curved flat blades which had a power number of 1.56 in the turbulent regime, with  $Re > 5 \times 10^3$ . The major difference between the measurements reported in this thesis and those in the literature is the behaviour of the power number in the transitional regime. Figure 6.3 shows that the power number of the flat blade turbine was constant over the range of Reynolds numbers studied. Shiue & Wong found that the power number of the curved blade turbine falls in the transitional regime reaching a minimum value of about 0.8 at  $Re = 10^3$  and rising as Reynolds number reduces further. In fact the power number curve of the flat blade turbine is very similar to those of the pitched blade turbines studied here and reported in the literature.

## 6.6 CONCLUSIONS

The conclusions which can be drawn from the results of the power measurements are:

- i) power number is constant over most of the range of speeds and fluids tested. It rises slightly at the low end of the range of Reynolds numbers studied.
- ii) the power number of the pitched blade turbines was observed to rise on scale-up. This may be due to slight geometrical differences between the two scales.
- iii) when plotted versus Reynolds number, the best agreement between the Newtonian and pseudo-plastic data was obtained using the Metzner & Otto (1957) definition of shear rate. Reynolds number is given by:

$$Re_{MO} = \frac{\rho N^{2-n} D^2}{K 11^{n-1}} \quad \dots \text{Eqn(6.2)}$$

- iv) the power numbers and the trends observed in their behaviour agree very well with those reported in the literature.

## CHAPTER 7

### ECONOMIC ANALYSIS

#### **7.1 INTRODUCTION**

The results reported and discussed in chapters 4 and 5 relate the blend time in an agitated vessel to the power input, torque, impeller-vessel geometry ( $D/T$  ratio), fluid physical properties and scale of operation. The objective of the work reported in this chapter is to take that technical data and extract economic information on which types of impeller will give the most cost-effective operation over the working life of an agitator.

The capital cost of an agitator can be related to the torque it must deliver since this determines the mechanical strength of the gearbox, shaft and impellers. The greater the torque, the greater the mechanical strength and, hence, the greater the capital cost.

Similarly, the running cost of an agitator can be related to the power required to achieve the desired blend time and the annual cost of maintenance. Maintenance costs can be related to the capital cost and are added to the power cost to give a total running cost for a years operation.

The method used to carry out the economic analysis is described in detail in section 7.2 and the results of a specific example are given in section 7.3. The results are discussed in section 7.4 and conclusions are given in section 7.5.

#### **7.2 METHOD**

The operating cost of an agitator can be divided into two parts:

- i) the capital cost which is the financial outlay required to purchase the equipment from a manufacturer.
- ii) the running cost which is the combination of the cost of buying power and the cost of maintaining the equipment.

In the economic analysis, the capital cost has been considered as a one-off payment at the start of the agitator's operating life. The running cost has been calculated for each

year, as the sum of the cost of the electricity required to drive the motor and the cost of maintenance, as a fraction of the capital cost, discounted to give a Net Present Value at the start of the operating life.

### 7.2.1 Capital Cost

The capital cost of an agitator is related to the torque which the agitator will deliver. Rigden (1991) has provided data relating the approximate selling price of agitators to the torque. The data is based on an agitator fitted with a single impeller having all parts in contact with the fluid constructed of 316 Stainless Steel. The prices also include a cartridge mechanical seal fitted to the drive. A multiplicative regression has been carried out on the data, which is given in table 7.1, to give the following relationship between capital cost (in pounds sterling),  $Co_{cap}$ , and torque (in Nm),  $\Lambda$ :

$$Co_{cap} = 488 \Lambda^{0.484} \quad \dots \text{Eqn(7.1)}$$

The correlation co-efficient,  $r^2$ , of the regression is 97.74%.

The torque is related to the power consumption and rotational speed of the impeller by re-arranging equation (4.41):

$$\Lambda = \frac{P}{2\pi N} = \frac{Po \rho N^2 D^5}{2\pi} \quad \dots \text{Eqn(7.2)}$$

### 7.2.2 Running Cost

The running cost of the agitator is made up of two components:

- i) the maintenance cost.
- ii) the power cost.

The maintenance cost of equipment used in the chemical industry can be estimated as a fraction of the capital cost (Weaver & Bauman (1974)). The fraction can vary between 4 and 10% depending on the environment in which the equipment is situated and the amount of attention it requires to keep it working. Agitators are often situated in hostile environments and it has been assumed that the maintenance cost is at the

upper end of the scale, ie:

$$Co_{main} = 0.10 Co_{cap} \quad \dots \text{Eqn(7.3)}$$

The power required to drive an agitator is given by re-arranging equation (6.1):

$$P = Po \rho N^3 D^5 \quad \dots \text{Eqn(7.4)}$$

Having calculated the power consumption of the agitator, this can be related to the annual power cost if the number of hours the agitator runs each year,  $Hpy$ , and the cost of a unit of electricity,  $Co_{kWh}$ , are known, ie:

$$Co_{pow} = P Hrs Co_{kWh} \times 10^{-5} \quad \dots \text{Eqn(7.5)}$$

The factor  $10^{-5}$  must be introduced because the power,  $P$ , is calculated in Watts and must be converted to kiloWatts and the cost of a unit of electricity,  $Co_{kWh}$ , is given in pence per kiloWatt-hour and must be converted to pounds per kiloWatt-hour.

The total running cost for one years operation can be calculated by combining equations (7.3) and (7.5):

$$Co_{run} = Co_{main} + Co_{pow} \quad \dots \text{Eqn(7.6)}$$

The total cash flow,  $Cf$ , for the agitator during its operating life,  $Yrs$ , will be:

$$Cf = Co_{run} Yrs \quad \dots \text{Eqn(7.7)}$$

One of the methods of assessing the value of an investment is to discount the cash flow back to the start of the project to obtain a Net Present Value,  $Npv$ . This is done by multiplying the cash flow by a discounting factor. The discounting factor,  $Df$ , used assumes that the cost of capital (the interest rate,  $Ir$ ) is constant over the life of the equipment and is described by Peters & Timmerhaus (1968):

$$Df = \frac{1}{Ir Yrs} (1 - \exp (-Ir Yrs)) \quad \dots \text{Eqn(7.8)}$$

The net present value of the agitator's running cost,  $Npv_{run}$ , can now be calculated from:

$$Npv_{run} = Cf Df = Co_{run} Yrs Df \quad \dots Eqn(7.9)$$

Having calculated the capital and discounted running costs of the agitator at the start of its operating life the total net present value,  $Npv_{tot}$ , of the agitator can be calculated by combining equations (7.1) and (7.9):

$$Npv_{tot} = Co_{cap} + Npv_{run} \quad \dots Eqn(7.10)$$

### 7.2.3 Analysis

The equations given in the preceding sections relate the cost of agitation to the power consumption and the shaft torque. These two parameters have been calculated for a hypothetical blending duty in fluids exhibiting different degrees of pseudo-plasticity using the “tk-Solver” computer program.

“tk-Solver” is a declarative, rule-based programming language which allows the user to work directly with a particular set of equations without having to program them in a particular sequence. Complex relationships, such as the one between the blend time in an agitated vessel, the agitator power and torque and the running and capital costs of the equipment, can be investigated quickly without resorting to time consuming computer programming and mathematics.

The analysis of the blend time data in chapters 4 and 5 has shown that a large impeller will achieve a certain blend time for less power input than a small one so the impeller diameter has been fixed at  $D = T/2$ .

The parameter which is a function of impeller type is power number,  $Po$ , so this has been varied between 0.3 and 3.0 in ten steps of 0.3. This corresponds to the range of power numbers studied in the blend time experiments with the power number of the hydrofoils equal to 0.3, the flat blade turbines equal to 3.0 and the pitched blade turbines equal to 1.0 and 1.70.

The blend time was set at 100 seconds and the diameter of the vessel and fluid depth at 5.0 m, giving an operating volume of 91.3 m<sup>3</sup>.

The viscosity of the fluid was estimated by calculating the shear stress in the fluid at the wall of the vessel, as described in section 5.2. The effect of the pressure drop due to the fluid impinging on the baffles has been ignored in this analysis because its effect is small relative to the shear stress term. So the shear stress is given by:

$$\tau_w = \frac{\Lambda}{1.622 T^3} \quad \dots \text{Eqn(7.11)}$$

The shear rate is then calculated from equation (5.3) and the viscosity from equation (5.23).

The power law index,  $n$ , has been varied between 0.4 and 1.0 in steps of 0.1 and the power law constant,  $K$ , selected so that the viscosity at the wall,  $\mu_w$ , is  $2.00 \text{ Ns m}^{-2}$  when agitated by an impeller with power number equal to 1.0. The combinations of constant and index are given in table 7.2.

Having set the impeller diameter, required blend time, fluid physical properties and scale of operation, the program calculated the impeller speed required to achieve this blend time for each value of power number by solving:

$$Po^{1/3} Re = \frac{193}{\sqrt{Fo}} \quad \dots \text{Eqn(5.26)}$$

where:  $Re = \frac{\rho N D^2}{\mu_w} \quad \dots \text{Eqn(7.12)}$

and:  $Fo = \frac{\mu_w \theta}{\rho T^2} \quad \dots \text{Eqn(7.13)}$

The only other information required relates to the economic analysis. It has been assumed that the agitator will operate for 8000 hours a year (since this assumption is commonly used in the assessment of equipment in the chemical industry), its life will be 11 years, the interest rate will be 11% ( $Ir = 0.11$ ) and the cost of a unit of electricity will be 6 pence a kilowatt-hour (Eastern Electricity (1991)).

A full list of the equations used in the “tk-Solver” program and the values assumed for each variable are given in appendix 1.

### 7.3 RESULTS

The equations listed in the previous section have been solved by the “tk-Solver” program giving a list of capital cost, discounted running cost and total cost versus power number for the seven fluids. The results are presented as plots of capital cost and the net present value of the running and total costs versus power number in figures 7.1 - 7.7.

The capital cost of the agitator for all the fluids is around k£ 30 - 40 and contributes between 20 and 25% of the total cost. The capital cost is lower for the low power number impellers and rises as power number increases. The maintenance cost also increases since this is calculated as 10% of the capital cost.

In the Newtonian fluid ( $K = 2.00$ ,  $n = 1.0$ ) the total net present value rises from k£ 142 to k£ 164 as power number rises from 0.3 to 3.0, shown in figure 7.1. The power cost is constant and the increase in running costs is caused by the increased torque increasing the maintenance costs. So, in the Newtonian fluid, there is a financial advantage in using the lowest power number impeller because it minimizes the capital cost of the agitator and it is k£ 13 cheaper than the agitator with the highest power number impeller.

The total net present value in the first pseudo-plastic fluid ( $K = 2.67$ ,  $n = 0.9$ ) also rises from k£ 151 to k£ 160 but the net present value of the running costs,  $Npv_{run}$ , falls from k£ 121 to k£ 118 as the effect of increasing the impeller’s power number reduces the viscosity of the fluid and makes the blending duty easier. This is plotted in figure 7.2. Again, the lowest power number impeller minimizes the total net present value but the difference between this and the agitator with the highest power number is reduced because the running costs are reduced.

Figure 7.3 shows the results for the next fluid ( $K = 3.55$ ,  $n = 0.8$ ). The total net present value changes from k£ 156 to k£ 152, a difference of k£ 4, as the power number increases and the increase in capital cost due to the increasing torque is more or less balanced by the decreasing power costs due to the reduced apparent viscosity of the fluid.

Figures 7.4, 7.5, 7.6 and 7.7 show the effect of further increases in the fluids' degree of pseudo-plasticity. The greatest difference between the total net present value for an agitator with a low and high power numbers is found in the most pseudo-plastic fluid ( $K = 11.22$ ,  $n = 0.4$ ) where the total net present value falls from k£ 189 for a power number of 0.3 to k£ 131 for a power number of 3.0, a difference of k£ 58, shown in figure 7.7.

## 7.4 DISCUSSION

### 7.4.1 Comparison of Experimental and Economic Results

The results of the blend time measurements in Newtonian fluids show that, when compared at equal power input, geometry ( $D/T$ ) and scale, all the impellers tested give the same performance, ie. achieve the same blend time. An impeller with a low power number will have to run at a higher speed in order to input the required power and would, therefore, operate at lower torque than an impeller with a high power number and it was suggested that, as a consequence, savings could be made in the capital cost of the agitator. A similar examination of the blend time measurements in pseudo-plastic fluids show that there is an advantage in using an impeller with a high power number because, for a given power input, it would produce a lower viscosity making the fluid easier to blend. The analysis of the costs of a hypothetical blending duty carried out in the previous section show that the experimental results can be translated into financial benefits.

The capital cost of the agitator varies between about k£ 30 and k£ 40 as the power number of the impeller increases in all the fluids examined. In a real situation it is more likely that the capital cost would be more or less constant as the range of torques may be accommodated by one size of gearbox. If the torque exceeded the limit of one gearbox it would be necessary to use the next size up in which case there would be a step in the capital cost curve.

The net present value of the running costs varies much more and ranges between k£ 92 and k£ 156. The largest difference is found in the most pseudo-plastic fluid and the lowest, in the Newtonian fluid.

The difference in the total net present value is also greatest in the most pseudo-plastic fluid but it is smallest in the fluid with  $n = 0.8$  where the increase in capital cost is more or less cancelled out by the decrease in running costs.

So, in general the experimental and cost analysis agree that there are advantages in using a high power number impeller in pseudo-plastic fluids provided that  $n < 0.8$ . If the fluid exhibits slight pseudo-plasticity, using an impeller with a low power number may produce a cheaper overall agitator than an impeller with a high power number.

#### 7.4.2 Power Number

The results predict that using an impeller with an even higher power number would reduce the cost of agitation even further. To test this, the results have been extrapolated outside the range of power numbers tested.

In the most pseudo-plastic fluid ( $K = 11.22$ ,  $n = 0.4$ ) when the power number is 3.0, the capital cost of the agitator is k£ 38.6, the net present value of the running costs is k£ 92.2 and the total net present value is k£ 130.8. If the power number is increased to 5.0, the capital cost rises slightly to k£ 39.2, the running costs fall to k£ 65.1 and the total costs fall to k£ 104.3. Similarly, if the power number is further increased to 7.0, the capital cost rises to k£ 40.1, the running costs fall to k£ 60.0 and the total costs fall to k£ 100.1.

So, increasing the power number from 3.0 to 7.0 will reduce the total cost of agitation by k£ 30.7 over the life of the plant. This can be achieved by using multiple impellers or one wide blade impeller such as a gate or paddle. As mentioned in the previous chapter, two Japanese agitator manufacturers have started marketing wide blade paddle impellers for viscous blending duties and the cost advantages predicted by this analysis should be obtained if they are used in highly pseudo-plastic fluids.

### 7.5 CONCLUSIONS

The conclusions which can be drawn from the cost-benefit analysis are:

- i) the results of the economic analysis support the findings of the experimental programme of work, ie. blending of Newtonian fluids is achieved most cost-effectively using a low power number impeller while blending of pseudo-plastic fluids should be carried out using a high power number impeller.
- ii) for the hypothetical example discussed, increasing the power number from 0.3 to 3.0 in a Newtonian fluid increased the total net present value of the agitator by k£ 22 from k£ 142 to k£ 164.

- iii) for the hypothetical example discussed, increasing the power number from 0.3 to 3.0 in a fluid with power law index,  $n$ , of 0.4 reduced the total net present value of the agitator by k£ 58 from k£ 189 to k£ 131.
- iv) the results predict that pseudo-plastic fluids should be agitated with impellers of high power number. The results have been extrapolated for the most pseudo-plastic fluid and show that if the power number is increased from 3.0 to 5.0, the total net present value of the agitator is reduced by k£ 26 from k£ 141 to k£ 104. If the power number is increased from 3.0 to 7.0 greater savings are possible with the total net present value of the agitator is reduced by k£ 31 from k£ 141 to k£ 100.
- v) the results predict that multiple impellers or wide blade gates or paddles should be used to blend pseudo-plastic fluids because they have a high shaft torque and significantly reduce the running costs of the agitator.

## CHAPTER 8

### CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

#### 8.1 INTRODUCTION

The purpose of this chapter is to summarize the conclusions of the work reported in the previous chapters of this thesis. The conclusions of the blend time measurements and their analysis are discussed in section 8.2. The recommendations for future work are given in section 8.3.

#### 8.2 CONCLUSIONS

##### 8.2.1 Newtonian Blend Times

Blend times were measured in standard geometry vessels (see figure 3.1) at three scales, 0.305, 0.609 and 1.830 m, using three impeller types commonly found in the chemical industry. These were pitched and flat blade turbines and hydrofoils. The fluids were Newtonian and ranged in viscosity between 10 and 500 cP. Also, the data reported by Mackinnon (1987), taken in water, were re-analyzed.

The blend time measurement technique used three conductivity probes placed in regions of differing agitation intensity and the individual probe blend times were recorded and a combined Root Mean Square blend time calculated.

It was found that each impeller has a critical Reynolds number, above which  $N\theta$  is constant and below which  $N\theta$  increases with decreasing Reynolds number. The lower the impellers power number, the higher the critical Reynolds number. Also, when  $Re > Re_{crit}$ , the individual probe blend times were approximately equal. When  $Re < Re_{crit}$  the individual probe blend times diverge with the shortest measured beneath the impeller and the longest behind the baffle.

The data for all the impellers were combined and correlated by three dimensionless groups. These were power number,  $Po$ , Reynolds number,  $Re$  and Fourier number,  $Fo$ .

The data could be divided into two groups. The turbulent data were taken at  $1/Fo > 10^3$  and the viscous data taken at  $1/Fo < 10^3$ .

The turbulent data were correlated by:

$$Po^{1/3} Re = \frac{5.20}{Fo} \quad \dots \text{Eqn(4.12)}$$

Similarly, the viscous data were correlated by:

$$Po^{1/3} Re = \frac{183}{\sqrt{Fo}} \quad \dots \text{Eqn(4.22)}$$

Both equations were re-arranged to show that the blend time is determined by the power input per unit mass, impeller to vessel diameter ratio and the scale of the vessel.

In turbulent blending:

$$\theta \propto \left( \frac{1}{\bar{\epsilon}} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.28)}$$

This shows that, for a given power input and impeller geometry, blend time increases with increasing scale.

In viscous mixing:

$$\theta \propto \left( \frac{1}{\bar{\epsilon}} \right)^{2/3} \left( \frac{T}{D} \right)^{2/3} T^{-2/3} \quad \dots \text{Eqn(4.29)}$$

This shows that, for a given power input and impeller geometry, blend time decrease with increasing scale which is a surprising result.

The analysis of the effect of power input on blend time shows that there are potential savings to be made in the capital cost of an agitator if an impeller with a low power number is chosen. The reason for this is that it will run at a higher speed than an impeller with a high power number to deliver the same power to the fluid and will therefore have a lower torque on its shaft. Less mechanical strength is required to deliver the torque so potential savings may be made in the capital cost of the agitator gearbox and shaft.

There are no savings to be made in running cost because a certain power will be required to achieve a given mixing time, whatever impeller type is used.

### 8.2.2 Pseudo-plastic Blend Times

The same impellers and vessels were used to study blending in pseudo-plastic fluids. The fluids used had a minimum power law index,  $n$ , of 0.55 and the power law constant ranged from 0.014 to 1.18.

The Newtonian blend time measurements had shown that the overall blending in the vessel was controlled by the time taken to achieve homogeneity behind the baffle so it was proposed that the viscosity in this region controlled the blending process. The apparent viscosity of the fluids was determined by estimating the average shear stress in the fluid at the surface of the vessel wall and base and this was done by taking an equation given by Bird *et al.* (1960) relating the torque to the tangential velocity gradient of the fluid at the vessel wall and base and the pressure drop due to the fluid impinging on the baffles to the shaft torque. Assumptions were made in order to estimate the velocity of the fluid at the baffle using an equation proposed by Schwartzberg & Treybal (1968). The apparent viscosity of the fluid was calculated for each data point.

The shear stress at the wall of the vessel is given by:

$$\tau_w = \frac{1}{1.622} \left( \frac{\Lambda}{T^3} - 0.0638 \rho v_{ST}^2 \right) \quad \dots \text{Eqn(5.22)}$$

where:

$$v_{ST} = k_{ST} \frac{N D^2}{(T^2 H)^{1/3}} \quad \dots \text{Eqn(5.21)}$$

When the data were analyzed in this way identical trends to those exhibited by the Newtonian data were observed, ie. at  $Re > Re_{crit}$ ,  $N\theta$  is constant and the individual probe blend times are approximately equal. When  $Re < Re_{crit}$ ,  $N\theta$  increases with decreasing Reynolds number and the individual probe blend times diverge.

The turbulent data were correlated by:

$$Po^{1/3} Re = \frac{5.47}{Fo} \quad \dots \text{Eqn(5.22)}$$

Similarly, the viscous data were correlated by:

$$Po^{1/3} Re = \frac{200}{\sqrt{Fo}} \quad \dots \text{Eqn(4.22)}$$

In the same way as the Newtonian data, both equations were re-arranged to show that the blend time is determined by the power input per unit mass, impeller to vessel diameter ratio and the scale of the vessel.

In turbulent blending:

$$\theta \propto \left( \frac{1}{\epsilon} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3} T^{2/3} \quad \dots \text{Eqn(4.28)}$$

This shows that, for a given power input and impeller geometry, blend time increases with increasing scale.

In viscous mixing:

$$\theta \propto \left( \frac{1}{\varepsilon} \right)^{2/3} \left( \frac{T}{D} \right)^{2/3} \frac{\mu_w}{\rho} T^{-2/3} \quad \dots \text{Eqn(5.27)}$$

In turbulent mixing the same rules regarding the relationship between power input and blend time apply but in viscous mixing there is an effect of the fluid's viscosity at the wall of the vessel. An impeller with a high power number, and shaft torque, will produce a lower viscosity and therefore, a shorter blend time for a given power input.

Thus there are savings in running cost to be made if a high power number impeller is used to blend a pseudo-plastic fluid.

### 8.2.3 Economic Analysis

The blend time correlations were programmed into the “tk-Solver” package with rules relating the capital cost of the agitator to its torque and the running cost to its power and maintenance requirement.

This shows that, for pseudo-plastic fluids, significant savings in the total operating cost of an agitator are possible if a large power number impeller is used. The increased capital cost due to the higher torque is offset by the reduced power and running cost required to achieve the desired blend time.

## 8.3 RECOMMENDATIONS FOR FUTURE WORK

### 8.3.1 Vessel Geometry

All the blend time and power measurements reported in this thesis have been carried out in dished base vessels of 0.305, 0.609 and 1.830 m diameter fitted with four plate baffles  $T/12$  wide, mounted  $T/60$  off the vessel wall. In all cases the fluid depth was equal to the diameter. This is shown in figure 3.1.

The experiments show that the greatest increase in blend time, as the fluid viscosity increases, occurs behind the baffles. This indicates that one way of improving the overall blending in the vessel would be to reduce the baffling by, for example, reducing the baffle width or removing them altogether.

The use of multiple impellers for blending viscous fluids has been tested by Metzner *et al.* (1961) and was found to be more effective than single impellers, especially for small diameter impellers.

Thus, a new area for blend time measurement would be to compare the standard geometry vessel's blending performance with vessels of other geometries including multiple impellers and those with reduced baffling.

### **8.3.2 Fluid Physical Properties**

There are two areas which would benefit from study of the effect of the fluids' physical properties on blend time. These are other rheological properties and fluids with different viscosities and densities.

#### **8.3.2.1 Rheological Properties**

Only two rheological properties were studied in this research programme. They were Newtonian and pseudo-plastic. Very few viscous fluids exhibit a Newtonian rheology and pseudo-plasticity is the commonest non-Newtonian rheology found in the chemical industry. It was important to measure blend times in Newtonian fluids since this gave a base-line against which the non-Newtonian properties of the pseudo-plastic fluids could be compared.

Among the more common rheologies are visco-elasticity which is encountered in polymer processing and Bingham plastics, or yield stress fluids, which are often high concentration slurries such as titanium dioxide and coal. In both cases, knowledge about the best type of impeller to use is important as additives are often blended into the slurries for processing reasons.

The blend time measurement technique used in this study could be applied to many other types of fluid provided care was taken to ensure that a large stagnant zone did not form around the conductivity probe.

### 8.3.2.2 Physical Property Differences

One of the commonest mixing problems in the chemical industry is the introduction of a small quantity of fluid into a larger quantity with significantly different viscosities and/or densities. The aim of the study should be to identify the best type of impeller to use for this application and to find the best place to introduce the additive.

### 8.3.3 Measurement of Flow

In the pseudo-plastic data analysis it was assumed that the volumetric flow and flow patterns were unaffected by the viscosity of the fluid. The fact that the pseudo-plastic data give a worse fit to the correlation than the Newtonian data shows that this assumption is probably wrong. Very little work has been done to measure the change in pumping capacity with viscosity, especially in pseudo-plastic fluids and no work has been done to measure the velocities near the vessel wall, base and the baffles.

It would be possible to use laser-Doppler anemometry to measure the flow and turbulence in regions of the vessel away from the impeller and this would provide useful data to use in the re-analysis of the blend time data reported here and would provide insights into the mechanisms of other mixing phenomena such as the pick-up of solids from the base of a vessel in solid-liquid mixing applications.

## REFERENCES

- Beckner, J. L. & Smith, J. M. Anchor agitated systems: power input with Newtonian and pseudo-plastic fluids, Trans.I.Chem.E., vol. 44, no. 6, pp. T224 - T263, 1966.
- Biggs, R. D. Mixing rates in stirred tanks, A.I.Ch.E.J., vol. 9, no. 5, pp. 636 - 640, 1963.
- Bird, R. B., Stewart, W. E. & Lightfoot, E. N. Transport Phenomena Ch. 6, p. 205, Wiley International Edition, John Wiley & Sons Inc., New York, 1960.
- Blasiński, H. & Rzyski, E. Mixing of non-Newtonian fluids with turbine, propeller and paddle agitators, Int.Chem.Eng., vol. 16, no. 4, pp. 751 - 754, 1976.
- Blasiński, H. & Rzyski, E. Mixing of non-Newtonian fluids with anchor, screw and ribbon mixers, Int.Chem.Eng., vol. 18, no. 4, pp. 708 - 711, 1978.
- Boger, D. V. Separation of shear thinning and elastic effects in experimental rheology, Rheology (vol.1) pp. 195 - 218, Astarita, Marrucci and Nicolais (eds.), Plenum Publishing Corp., New York, 1980.
- Bourne, J. R. The mixing of powders, pastes and non-Newtonian fluids, Chem.Engr., pp. CE202 - CE216, September 1964.
- Bourne, J. R. An analysis of the flow produced by helical ribbon impellers, Trans.I.Chem.E., vol. 47, pp. T11 - , 1969.
- Bourne, J. R., Buerli, M. & Regenass, W. Power and heat transfer to agitated suspensions: use of heat flow calorimetry, Chem.Eng.Sci., vol.36, pp. 782 - 784, 1981.
- Bowen, R. L. Agitation intensity: key to scaling up flow-sensitive liquid systems, Chem.Engng., pp. 159 - 168, 8 March 1985.
- Bowen, R. L. Unravelling the mysteries of shear sensitive mixing systems, Chem.Engng., pp. 55 - 63, 9 June 1986.
- Brennan, D. J. & Lehrer, I. H. Impeller mixing in vessels: experimental studies on the influence of some parameters and the formulation of a general mixing time equation, Trans.I.Chem.E., vol. 54, pp. 139 - 152, 1976.
- Brito-de la Fuente, E., Leuliet, J. C., Choplin, L. & Tanguy, P. A. Mixing and circulation times in rheologically complex fluids, Fluid Mixing IV, I.Chem.E. Symposium Series no. 121, pp. 75 - 96, Bradford, England, 11 - 13 September 1990.
- Bryant, J. & Sadeghzadeh, S. Terminal mixing times in stirred tanks, Proc. 4<sup>th</sup> Euro. Conf. on Mixing, paper B4, Noordwijkerhout, Holland, BHRA, 27 - 29 April 1982.

- Calderbank, P. H. & Moo-Young, M. B. The prediction of power consumption in the agitation of non-Newtonian fluids, Trans.I.Chem.E., vol. 37, pp. 26 - 33, 1959.
- Calderbank, P. H. & Moo-Young, M. B. The power characteristics of agitators for the mixing of Newtonian and non-Newtonian fluids, Trans.I.Chem.E., vol. 39, no. 5, pp. 337 - 347, 1961.
- Carreau, P.J., Patterson, I. & Yap, C. Y. Mixing of visco-elastic fluids with helical ribbon agitators, Can.J.Chem.Eng., vol. 54, pp. 135 - 142, 1976.
- Chavan, V. V., Arumugam, M. & Ulbrecht, J. On the influence of liquid elasticity on mixing in a vessel agitated by a combined ribbon-screw impeller, A.I.Ch.E.J., vol. 21, no.3, pp. 613 - 615, 1975.
- Chemineer Liquid Agitation, Reprinted from Chem.Engng., McGraw Hill Inc., New York, 1976.
- Cheng, D. C. -H. & Tookey, D. J. The effect of agitator geometry and speed on yield in batch reactors, Proc. 2<sup>nd</sup> Euro. Conf. on Mixing, paper A1, Cambridge, England, BHRA, 30 March - 1 April 1977.
- Corpstein, R. R., Dove, R. A. & Dickey, D. S. Stirred tank reactor design, Chem.Eng.Prog., pp. 66 - 74, February 1979.
- Coyle, C. K., Hirschland, H. E., Michel, B. J. & Oldshue, J. Y. Mixing in viscous liquids, A.I.Ch.E.J., vol. 16, no. 6, pp. 903 - 906, 1970.
- Demange, A. L., Forsyth, T. H. & Focht, L. G. Batch mixing times of viscous pseudo-plastic liquids, Elastomerics, pp. 33 - 37, April 1977.
- Dickey, D. S. Program chooses agitator, Chem.Engng., pp. 73 - 81, 9 January 1984.
- Eastern Electricity Tariffs for the supply of electricity, Eastn Electricity plc., 1<sup>st</sup> April 1991.
- Elson, T. P. Mixing of fluids possessing a yield stress, Proc. 6<sup>th</sup> Euro. Conf. on Mixing, pp. 485 - 492, Pavia, Italy, BHRA, 24 - 26 May 1988.
- Elson, T. P. The growth of caverns formed around rotating impellers during the mixing of a yield stress fluid, Chem.Eng.Commun., vol.96, pp. 303 - 319, 1990.
- Fawcett, N. J. S. The mean flow in the discharge of pitched blade turbines, FMP Interim Report 1046, October 1989.
- Fawcett, N. J. S. The turbulence generated at the discharge of pitched blade turbines, FMP Interim Report 1059, January 1990.
- Flagg, P. L., Hill, D. J., Molineux, K. J. & Sampson, N. A technical and economic evaluation of impellers for use on the viscous regime, Proc. 2<sup>nd</sup> Euro. Conf. on Mixing, paper A3, Cambridge, England, BHRA, 30 March - 1 April 1977.

- Ford, D. E., Mashelkar, R. A. & Ulbrecht, J. Mixing times in Newtonian and non-Newtonian fluids, *Process Tech.Int.*, vol. 17, no. 10, pp. 803 - 807, 1972.
- Ford, D. E. & Ulbrecht, J. Influence of rheological properties of polymer solutions upon mixing and circulation times, *Ind.Eng.Chem., Process Des.Dev.*, vol. 15, no. 2, pp. 321 - 326, 1976.
- Fox, E. A. & Gex, V. E. Single phase blending of liquids, *A.I.Ch.E.J.*, vol. 2, no. 4, pp. 539 - 544, 1956.
- Garrison, C. M. How to design and scale mixing pilot-plants, *Chem.Engng.*, pp. 63 - 70, 7 February 1983.
- Godleski, E. S. & Smith, J. C. Power requirements and blend times in the agitation of pseudo-plastic fluids, *A.I.Ch.E.J.*, vol. 8, no. 5, pp. 617 - 620, 1962.
- Gray, J. B. Batch mixing of viscous liquids, *Chem.Eng.Prog.*, vol. 59, no. 3, pp. 55 - 59, 1963.
- Greene, H. L., Carpenter, C. & Casto. L. Mixing characteristics of an axial impeller with Newtonian and non-Newtonian fluids, *Proc. 4<sup>th</sup> Euro. Conf. on Mixing*, paper D1, Noordwijkerhout, Holland, BHRA, 27 - 29 April 1982.
- Hall, K. R. & Godfrey, J. C. The effect of fluid properties and impeller design on the performance of helical ribbon mixers, *Chemeca '70*, pp. 111 - 127, Butterworths, 1971.
- Havas, G., Sawinsky, J. & Deák. A. Investigation of the homogenisation efficiency of the screw agitator, helical ribbon agitator, and the multi-paddle agitator in the mixing of high viscosity Newtonian liquids, *Period.Polytech.*, vol. 22, pp. 317 - 330, 1978 (a).
- Havas, G., Sawinsky, J., Deák, A. & Fekete, A. Investigation of the homogenisation efficiency of various propeller agitators types, *Period.Polytech.*, vol. 22, pp. 331 - 343, 1978 (b).
- Hemrajani, R. R., Chang, M., Florez, M. R. & Patel, R. D. Mixing and heat transfer in non-Newtonian coal liquids, *Proc. 4<sup>th</sup> Euro. Conf. on Mixing*, paper D2, Noordwijkerhout, Holland, BHRA, 27 - 29 April 1982.
- Hickman, A. D. Agitation, mixing and mass transfer in simulated high viscosity fermentation broths, *PhD Thesis*, University of Birmingham, 1985.
- Hiraoka, S. & Ito, R. Simple relationship between power input and mixing time in turbulent agitated vessel, *J.Chem.Eng.Japan*, vol. 10, no. 1, pp. 75 - 77, 1977.
- Höcker, H., Langer, G. & Werner, U. Power consumption of stirrers in non-Newtonian liquids, *Ger.Chem.Eng.*, vol. 4 pp. 113 - 123, 1981.
- Hoogendoorn, C. J. & Den Hartog, A. P. Model studies on mixers in the viscous flow region, *Chem.Eng.Sci.*, vol.22, pp. 1689 - 1699, 1967.

- Ide, Y. & White, J. L. Rheological phenomena in polymerisation reactors: rheological properties around agitators in polystyrene-styrene solutions, *J.App.Polymer Sci.*, vol 18, pp. 2997 - 3018, 1974.
- Johnson, R. T. Batch mixing of viscous liquids, *Ind.Eng.Chem., Process Des.Dev.*, vol. 6, no. 3, pp. 340 - 345, 1967.
- Kamiwano, M., Yamamoto, K. & Nagata, S. Mixing performance of various agitators, *Kagaku Kogaku*, vol. 31, pp. 365 - 372, 1962.
- Kelkar, J. V., Mashelkar, R. A. & Ulbrecht, J. On the rotational visco-elastic flows around simple bodies and agitators, *Trans.I.Chem.E.*, vol. 50, pp. 343 - 352, 1972.
- Khang. S. J. New scale-up and design criteria of stirrer agitated batch and flow mixing vessels, PhD Thesis, Oregon State University, 1975.
- Khang, S. J. & Fitzgerald, T. J. A new probe and circuit for measuring electrolyte conductivity, *Ind.Eng.Chem.Fund.*, vol. 14, pp. 208 - 213, 1975.
- Khang, S. J. & Levenspiel, O. New scale-up and design method for stirrer agitated batch mixing vessels, *Chem.Eng.Sci.*, vol. 31, pp. 569 - 577, 1976.
- Khang, S. J. & Levenspiel, O. The mixing rate number for agitator stirred tanks, *Chem.Engng.*, pp. 141 - 143, 11 October 1976.
- King, R. Mixing in the process industries, Harnby, Edwards & Nienow (eds.), ch. 14, p. 253, Butterworths, London, 1985.
- Komori, S. & Murakami, Y. Turbulent mixing in baffled stirred tanks with vertical blade impellers, *A.I.Ch.E.J.*, vol. 34, no. 6, pp. 932 - 937, 1988.
- Koutsakos, E., Nienow, A. W. & Dyster, K. N. Laser anemometry study of shear thinning fluids agitated by a Rushton turbine, *Fluid Mixing IV*, *I.Chem.E. Symposium Series* no. 121, pp. 51 - 73, Bradford, England, 11 - 13 September 1990.
- Kramers, H., Baars, G. M. & Knoll, W. H. A comparative study of the rate of mixing in stirred tanks, *Chem.Eng.Sci.*, vol. 2, pp. 35 - 42, 1953.
- Landau, J. & Procházka, J. Studies on mixing XI: experimental methods for following the homogenisation of miscible liquids in rotary mixers, *Coll.Czech.Chem.Comm.*, vol. 26, pp. 1976 - 1990, 1961.
- Landau, J., Procházka, J., Václavek, V. & Fořt, I. Studies on mixing XIV: homogenisation of miscible liquid in the viscous region, *Coll.Czech.Chem.Comm.*, vol. 28, pp. 279 - 292, 1963.
- Landau. J., & Procházka, J. Studies on mixing XVII: relation between the energy consumption and rate of homogenisation, *Coll.Czech.Chem.Comm.*, vol. 29, pp. 1866 - 1876, 1964.

- Mackinnon, C. P. Research review: comparative mixing times, FMP Report 029, April 1987.
- Magnusson, K. Effektbehovets beroende av den effektiva viskositeten vid omröring av strukturviskösa vätskor, IVA, vol. 23, no. 2, pp. 86 - 91, 1952.
- M<sup>c</sup>Manamey, W. J. A circulation model for batch mixing in agitated vessels, Trans.I.Chem.E., vol. 58, pp. 271 - 276, 1980.
- Mersmann, A., Einkenkel, W. D. & Käppel, M. Design and scale-up of mixing equipment, Int.Chem.Eng., vol. 16, no. 4, pp. 590 - 603, 1976.
- Metzner, A. B. & Otto, R. E. Agitation of non-Newtonian fluids, A.I.Ch.E.J., vol. 3, no. 1, pp. 3 - 10, 1957.
- Metzner, A. B., Feehs, R. H., Ramos, H. L., Otto, R. E. & Tuthill, J. D. Agitation of viscous Newtonian and non-Newtonian fluids, A.I.Ch.E.J., vol. 7, no. 1, pp. 3 - 9, 1961.
- Metzner, A. B. & Astarita, G. External flows of visco-elastic materials: fluid property restrictions on the use of velocity sensitive probes, A.I.Ch.E.J., vol. 13, no. 3, pp. 550 - 555, 1967.
- Middleton, J. C. Measurement of circulation within large mixing vessels, Proc. 3<sup>rd</sup> Euro. Conf. on Mixing, paper A2, York, England, BHRA, 4 - 6 April 1979.
- Moo-Young, M., Tichar, K. & Dullien, F. A. L. The blending efficiencies of some impellers in batch mixing, A.I.Ch.E.J., vol. 18, no. 1, pp. 178 - 182, 1972.
- Muskett, M. J. Gas-liquid dispersion in stirred tanks: scale-up and agitator comparison, PhD Thesis, Cranfield Institute of Technology, 1987.
- Nagata, S., Nishikawa, M., Tada, H. & Gotoh, S. Power consumption of mixing impellers in pseudo-plastic liquids, J.Chem.Eng.Japan, vol. 4, no. 1, pp. 72 - 76, 1971.
- Nienow, A. W. & Elson, T. P. Aspects of mixing in rheologically complex fluids, Ch.E.R.D., vol. 66, pp. 5 - 15, 1988.
- Norwood, K. W. & Metzner, A. B. Flow patterns and mixing rates in agitated vessels, A.I.Ch.E.J., vol. 6, no. 3, pp. 432 - 437, 1960.
- Nouri, J. M. & Whitelaw, J. H. Flow characteristics of stirred reactors with Newtonian and non-Newtonian fluids, A.I.Ch.E.J., vol.36, no. 4, pp. 627 - 629, 1990.
- Novák, V. & Rieger, F. Homogenisation with helical screw agitators, Trans.I.Chem.E., vol. 47, pp. T335 - T340, 1969.
- Novák, V. & Rieger, F. Homogenisation efficiency of helical ribbon and anchor agitators, Chem.Eng.J., vol. 9, pp. 63 - 70, 1975.

- Novák, V. & Rieger, F. Influence of vessel to screw diameter ratio on the efficiency of screw agitators,  
Trans.I.Chem.E., vol. 55, pp. 202 - 206, 1977.
- Oldshue, J. Y. Fluid Mixing Technology,  
Ch. 3, pp. 62 -62, McGraw Hill Publishing Company, New York, 1983.
- Palmer, M. H. Design guide for impeller power numbers,  
FMP Report 028, April 1987.
- Parkin, S. P. (ed.) McGraw-Hill Dictionary of Scientific and Technical Terms, 3<sup>rd</sup> edition,  
McGraw-Hill Book Company, New York, 1984.
- Peters, M. S. & Timmerhaus, K. D. Process design and economics for chemical engineers, 2<sup>nd</sup> edition,  
Ch. 5, p. 179, McGraw-Hill Book Company, New York, 1968.
- Pollard, J. & Kantyka, T. A. Heat transfer to agitated non-Newtonian fluids,  
Trans.I.Chem.E., vol. 47, pp. T21 - T27, 1969.
- Pollard, G. J. Flooding and aeration efficiency in standard stirred vessels,  
Int. Symposium on Mixing, paper C4, Mons, Belgium, Faculté Polytechnique de Mons, 21 - 24 February 1978.
- Pörtner, R., Langer, G. & Werner, U. Homogenisation of non-Newtonian fluids in stirred tanks,  
Proc. 6<sup>th</sup> Euro. Conf. on Mixing, pp. 521 - 526, Pavia, Italy, BHRA, 24 - 26 May 1988.
- Procházka, J. & Landau, J. Studies on mixing XII: homogenisation of liquids in the turbulent region,  
Coll.Czech.Chem.Comm., vol. 26, pp. 2961 - 2973, 1961.
- Raghav Rao, K. S. M. S. & Joshi, J. B. Liquid phase mixing in mechanically agitated vessels,  
Chem.Eng.Comm., vol. 74, pp. 1 - 25, 1988.
- Reavell, B. N. Practical aspects of liquid mixing and agitation,  
Trans.I.Chem.E., vol. 29, pp. 301 - 309, 1951.
- Rieger, F. & Novák, V. Power consumption of agitators in highly viscous non-Newtonian liquids,  
Trans.I.Chem.E., vol. 51, pp. 105 - 111, 1973.
- Rieger, F. & Novák, V. Power consumption for agitating visco-elastic liquids in the viscous regime,  
Trans.I.Chem.E., vol. 52, pp. 285 - 286, 1974.
- Rielly, C. D. & Britter, R. E. Mixing times for passive tracers in stirred tanks,  
Proc. 5<sup>th</sup> Euro. Conf. on Mixing, paper 36, Wurzburg, W. Germany, BHRA, 10 - 12 June 1985.
- Rigden, M. P. Private Communication, 1991.
- Ruszkowski, S. The FMP method for measuring mixing times,  
FMP Report 008, February 1985 (a).

- Ruszkowski, S. A comparison of mixing time and mixing efficiencies for a range of impellers in a 0.61 m diameter vessel, FMP Report 009, October 1985 (b).
- Sano, Y. & Usui, H. Interrelations among mixing time, power number and discharge flowrate number in baffled mixing vessels, J.Chem.Eng.Japan, vol. 18, no. 1, pp. 47 - 52, 1985.
- Sano, Y. & Usui, H. Effects of paddle dimensions and baffle conditions on the interrelations among discharge flowrate, mixing power and mixing time in mixing vessels, J.Chem.Eng.Japan, vol. 20, no. 4, pp. 399 - 404, 1987.
- Schwartzberg, H. G. & Treybal, R. E. Fluid and particle motion in turbulent stirred tanks: fluid motion, Ind.Eng.Chem.Fund., vol. 7, no.1, pp. 1 - 6, 1968.
- Shamlou, P. A. & Edwards, M. F. Power consumption of helical ribbon mixers in viscous Newtonian and non-Newtonian fluids, Chem.Eng.Sci., vol. 40, no. 9, pp. 1773 - 1781, 1985.
- Shiue, S. J. & Wong, C. W. Studies in homogenisation efficiency of various agitators in liquid blending, Can.J.Chem.Eng., vol. 62, pp. 602 - 609, 1984.
- Short, D. G. R. & Etchells, A. W. Unsolved problems of industrial mixing, Proc. 4<sup>th</sup> Euro. Conf. on Mixing, paper A1, Noordwijkerhout, Holland, BHRA, 27 - 29 April 1982.
- Silvester, R. S. Mixing time and power requirements for non-Newtonian fluids, Fluid Mixing II, I.Chem.E. Symposium Series no. 89, Bradford England, 3 -5 April 1984 (a).
- Silvester, R. S. Mixing of non-Newtonian media: a technical review, BHRA Report G0128, 1984 (b).
- Solomon, J. Mixing, aeration and rheology of highly viscous fluids, PhD Thesis, University College London, 1980.
- Solomon, J., Nienow, A. W. & Pace, G. W. Flow patterns in agitated plastic and pseudo-plastic, visco-elastic fluids, Fluid Mixing, I.Chem.E. Symposium Series no. 64, pp. A1 - A13, Bradford, England, 24 - 26 March 1981.
- Su, Y. S. & Holland, F. A. Agitation and mixing of non-Newtonian fluids - part 1, Chem.Proc.Eng., pp. 77 - 84, August 1968 (a).
- Su, Y. S. & Holland, F. A. Agitation and mixing of non-Newtonian fluids - part 2, Chem.Proc.Eng., pp. 110 - 120, September 1968 (b).
- Tebel, K. H. & Zehner, P. Mixing of power law fluids in loop reactors, Proc. 5<sup>th</sup> Euro. Conf. on Mixing, paper 32, Wurzburg, W. Germany, BHRA, 10 - 12 June 1985.
- Ulbrecht, J. Mixing of visco-elastic fluids by mechanical agitation, Chem.Engr., pp. 347 - 367, 1974.

- Ulbrecht, J. Influence of rheological properties upon the process of mixing, *Verfahrenstechnik*, vol. 9, no. 9. pp. 457 - 463, 1975.
- Van de Vusse, J. G. Mixing by agitation of miscible liquids part I, *Chem.Eng.Sci.*, vol. 4, pp. 178 - 200, 1955 (a).
- Van de Vusse, J. G. Mixing by agitation of miscible liquids part II, *Chem.Eng.Sci.*, vol. 4, pp. 209 - 220, 1955 (b).
- Vonken, R. M.,  
Goetzee, W. J. J. & Beek, W. J. Circulation flow and mixing at low Reynolds numbers, *Int. Symposium on Mixing*, paper A5, Mons, Belgium, Faculté Polytechnique de Mons, 21 - 24 February 1978.
- Wang, K. & Yu, S. Heat transfer and power consumption of non-Newtonian fluids in agitated vessels, *Chem.Eng.Sci.*, vol. 44. no. 1, pp. 33 - 40, 1989.
- Weaver, J. B. &  
Bauman, H. C The chemical engineers' handbook, 5<sup>th</sup> edition, Perry & Chilton (eds.), sect. 25, p. 25-30, McGraw-Hill Kogakusha, New York, 1974.
- Wichterle, K. & Wein, O. Threshold of mixing non-Newtonian liquids, *Int.Chem.Engng.*, vol. 21, no. 1, pp. 116 - 120, 1981.
- Wichterle, K., Kadlec, M.,  
Žák, L. & Mitchka, P. Shear rates on turbine impeller blades, *Chem.Eng.Commun.*, vol. 26, pp. 25 - 32, 1984.
- Wichterle, K., Žák, L. &  
Mitschka, P. Shear stresses on the walls of agitated vessels, *Chem.Eng.Commun.*, vol. 32, pp. 289 - 305, 1985.
- Wong, C. W. & Huang, C. T. Flow characteristics and mechanical efficiency in baffled stirred tanks with turbine impellers, *Proc. 6<sup>th</sup> Euro. Conf. on Mixing*, pp. 29 - 34, Pavia, Italy, BHRA, 24 - 26 May 1988.
- Zlokarnik, M. Eignung von ruhren zum homogenisieren von flussigkeitsgemischen, *Chem.Eng.Tech.*, vol. 39, no. 9/10, pp. 539 - 548, 1967.

Table 3.1

Impeller Dimensions

| Nominal Diameter | Impeller Type | Scale $T$<br>(m) | Diameter $D$<br>(m) | Blade Angle to horizontal<br>(deg) | Blade width $w$<br>(m) | Fluids Tested <sup>A</sup> |
|------------------|---------------|------------------|---------------------|------------------------------------|------------------------|----------------------------|
| $T/3$            | Pitched blade | 0.305            | 0.102               | 45                                 | 0.020                  | New <sup>n</sup> + Ps. pl. |
| "                | "             | 0.609            | 0.203               | "                                  | 0.041                  | "                          |
| "                | "             | 1.830            | 0.610               | "                                  | 0.122                  | "                          |
| "                | "             | 0.609            | 0.203               | 41                                 | 0.038                  | Ps. pl.                    |
| $T/2$            | "             | 0.305            | 0.151               | 45                                 | 0.020                  | New <sup>n</sup> + Ps. pl. |
| "                | "             | 0.609            | 0.300               | "                                  | 0.041                  | "                          |
| "                | "             | 1.830            | 0.915               | "                                  | 0.122                  | Ps. pl.                    |
| $T/3$            | Flat blade    | 0.305            | 0.100               | 90                                 | 0.020                  | Ps. pl.                    |
| "                | "             | 0.609            | 0.202               | "                                  | 0.041                  | New <sup>n</sup> + Ps. pl. |
| $T/2$            | Chemineer HE3 | 0.305            | 0.150               | -                                  | -                      | Ps. pl.                    |
| "                | Lightnin A310 | 0.609            | 0.298               | -                                  | -                      | New <sup>n</sup> + Ps. pl. |

A: New<sup>n</sup> = Newtonian fluids

Ps. pl. = Pseudo-plastic fluids

Table 3.2

Probe and Injector Positions

|          | Axial position              | Radial position         |
|----------|-----------------------------|-------------------------|
| Injector | $T/25$ above fluid surface  | $T/10$ from shaft axis  |
| Probe 1  | $T/50$ below impeller       | $T/8$ from shaft axis   |
| Probe 2  | $T/4.5$ below fluid surface | $T/4.7$ from shaft axis |
| Probe 3  | $T/3$ below fluid surface   | $T/2.2$ from shaft axis |

Table 3.3

Power Law Constants and Indices of Pseudo-plastic Fluids

| Natrosol Grade | Concentration<br>(wt./wt. water) | Constant $K$<br>(Ns <sup><math>n</math></sup> m <sup>-2</sup> ) | Index $n$<br>(-) |
|----------------|----------------------------------|-----------------------------------------------------------------|------------------|
| 250 MR         | 0.25                             | 0.014                                                           | 0.88             |
| "              | 0.50                             | 0.068                                                           | 0.86             |
| "              | 0.75                             | 0.258                                                           | 0.79             |
| 250 HHR        | 0.25                             | 0.092                                                           | 0.80             |
| "              | 0.50                             | 1.180                                                           | 0.55             |

Table 4.1

## Newtonian Blend Time Results

## T/3 Pitched Blade Turbine

| $T$<br>(m) | $\mu$<br>(cP) | $N$<br>(s <sup>-1</sup> ) | $\theta_1$ (s)<br>(RSD (%)) | $\theta_2$ (s)<br>(RSD (%)) | $\theta_3$ (s)<br>(RSD (%)) | $\theta_{RMS}$ (s)<br>(RSD (%)) |
|------------|---------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------------------|
| 0.305      | 500           | 12.7                      | 9.6<br>(25)                 | 9.5<br>(17)                 | 77.8<br>(47)                | 59.4<br>(48)                    |
| "          | "             | 14.0                      | 8.5<br>(12)                 | 10.8<br>(22)                | 51.0<br>(48)                | 43.7<br>(48)                    |
| "          | "             | 15.5                      | 7.3<br>(19)                 | 8.7<br>(15)                 | 41.6<br>(48)                | 34.3<br>(46)                    |
| "          | 100           | 5.87                      | 10.0<br>(11)                | 14.8<br>(17)                | 77.5<br>(10)                | 68.6<br>(11)                    |
| "          | "             | 6.98                      | 6.7<br>(10)                 | 10.1<br>(17)                | 69.0<br>(11)                | 61.1<br>(14)                    |
| "          | "             | 8.23                      | 6.7<br>(19)                 | 9.3<br>(17)                 | 45.5<br>(11)                | 39.3<br>(14)                    |
| 0.609      | "             | 3.67                      | 14.8<br>( 6)                | 14.4<br>(13)                | 29.7<br>(28)                | 28.0<br>(27)                    |
| "          | "             | 4.38                      | 12.4<br>( 7)                | 13.6<br>(35)                | 25.8<br>(38)                | 24.0<br>(43)                    |
| "          | "             | 5.05                      | 9.7<br>(12)                 | 10.7<br>(24)                | 26.0<br>(64)                | 18.0<br>(27)                    |
| "          | 40            | 1.51                      | 37.9<br>(20)                | 38.6<br>(27)                | 69.7<br>(24)                | 64.9<br>(25)                    |
| "          | "             | 3.00                      | 12.8<br>(16)                | 15.5<br>(22)                | 23.3<br>(19)                | 23.5<br>(10)                    |
| "          | "             | 4.82                      | 6.7<br>(14)                 | 9.4<br>(17)                 | 11.6<br>(15)                | 11.0<br>(17)                    |
| "          | 10            | 1.82                      | 20.5<br>(10)                | 22.2<br>( 5)                | 23.0<br>( 9)                | 23.6<br>( 8)                    |
| "          | "             | 3.65                      | 9.8<br>(16)                 | 10.4<br>(18)                | 10.8<br>(18)                | 10.8<br>(17)                    |
| "          | "             | 4.78                      | 7.4<br>(16)                 | 8.1<br>(17)                 | 8.0<br>(13)                 | 7.9<br>(14)                     |
| 1.830      | 300           | 1.38                      | 33.2<br>(23)                | 60.8<br>(21)                | 90.6<br>( 2)                | 86.9<br>( 8)                    |
| "          | 40            | 1.37                      | 27.8<br>(12)                | 29.4<br>(12)                | 29.4<br>(17)                | 30.0<br>(11)                    |
| "          | "             | 1.98                      | 20.2<br>(11)                | 20.0<br>(13)                | 21.4<br>( 4)                | 20.6<br>(15)                    |

Table 4.2

Newtonian Blend Time Results

*T*/2 Pitched blade turbine

| <i>T</i><br>(m) | μ<br>(cP) | <i>N</i><br>(s <sup>-1</sup> ) | θ <sub>1</sub> (s)<br>(RSD (%)) | θ <sub>2</sub> (s)<br>(RSD (%)) | θ <sub>3</sub> (s)<br>(RSD (%)) | θ <sub>RMS</sub> (s)<br>(RSD (%)) |
|-----------------|-----------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|-----------------------------------|
| 0.305           | 500       | 5.03                           | 11.1<br>( 4)                    | 9.4<br>(12)                     | 165<br>( 7)                     | 133<br>(18)                       |
| "               | "         | 6.33                           | 8.1<br>( 7)                     | 7.8<br>(23)                     | 116<br>(22)                     | 87.0<br>(16)                      |
| "               | "         | 7.25                           | 8.9<br>(17)                     | 8.0<br>(16)                     | 78.8<br>( 8)                    | 67.7<br>(15)                      |
| "               | 100       | 2.98                           | 10.2<br>(16)                    | 11.6<br>(23)                    | 44.4<br>( 8)                    | 41.5<br>(23)                      |
| "               | "         | 3.53                           | 7.5<br>(14)                     | 8.8<br>(18)                     | 22.9<br>(64)                    | 22.3<br>(60)                      |
| "               | "         | 4.17                           | 6.8<br>(10)                     | 7.7<br>(11)                     | 25.1<br>(10)                    | 21.6<br>(19)                      |
| 0.609           | "         | 2.12                           | 14.3<br>( 8)                    | 13.9<br>(13)                    | 16.0<br>(20)                    | 15.2<br>(18)                      |
| "               | "         | 2.52                           | 12.4<br>(10)                    | 11.4<br>(17)                    | 13.0<br>(17)                    | 12.5<br>(10)                      |
| "               | "         | 2.92                           | 10.1<br>(11)                    | 10.3<br>(27)                    | 12.3<br>(28)                    |                                   |
| "               | 40        | 1.50                           | 16.7<br>(11)                    | 18.4<br>(17)                    | 23.4<br>(24)                    | 21.7<br>(19)                      |
| "               | "         | 2.67                           | 6.5<br>(29)                     | 7.1<br>(20)                     | 10.9<br>(42)                    | 9.5<br>(29)                       |
| "               | "         | 2.92                           | 6.5<br>(40)                     | 8.2<br>(20)                     | 12.9<br>(20)                    | 11.5<br>(13)                      |
| "               | "         | 3.17                           | 6.6<br>(30)                     | 6.8<br>(22)                     | 11.4<br>(36)                    | 9.1<br>(17)                       |
| "               | 10        | 0.83                           | 21.3<br>(14)                    | 22.2<br>(12)                    | 28.7<br>(26)                    | 26.9<br>(10)                      |
| "               | "         | 1.50                           | 10.7<br>(10)                    | 12.1<br>(16)                    | 14.4<br>(12)                    | 14.0<br>(13)                      |
| "               | "         | 2.17                           | 8.6<br>(18)                     | 8.6<br>(21)                     | 9.9<br>(19)                     | 10.3<br>(15)                      |
| "               | "         | 3.00                           | 6.6<br>(16)                     | 6.6<br>(20)                     | 7.8<br>(16)                     | 8.0<br>(15)                       |

Table 4.3

Newtonian Blend Time Results

*T*/3 Flat blade turbine

| <i>T</i><br>(m) | μ<br>(cP) | <i>N</i><br>(s <sup>-1</sup> ) | θ <sub>1</sub> (s)<br>(RSD (%)) | θ <sub>2</sub> (s)<br>(RSD (%)) | θ <sub>3</sub> (s)<br>(RSD (%)) | θ <sub><i>RMS</i></sub> (s)<br>(RSD (%)) |
|-----------------|-----------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|------------------------------------------|
| 0.609           | 100       | 1.70                           | 28.4<br>(16)                    | 32.2<br>(22)                    | 202<br>(33)                     | 168<br>(33)                              |
| "               | "         | 2.55                           | 21.6<br>(51)                    | 20.5<br>(18)                    | 88.7<br>(23)                    | 71.1<br>(29)                             |
| "               | "         | 3.33                           | 11.0<br>(18)                    | 11.8<br>(14)                    | 39.7<br>(40)                    | 30.5<br>(34)                             |
| "               | 40        | 1.82                           | 20.9<br>(10)                    | 21.7<br>( 7)                    | 22.9<br>( 8)                    | 21.8<br>( 5)                             |
| "               | "         | 3.83                           | 8.8<br>(17)                     | 10.3<br>(24)                    | 8.7<br>(14)                     | 10.4<br>(16)                             |
| "               | 10        | 1.67                           | 16.3<br>( 8)                    | 21.0<br>(16)                    | 25.5<br>(28)                    | 23.6<br>(25)                             |
| "               | "         | 3.33                           | 8.1<br>( 9)                     | 8.8<br>(19)                     | 8.3<br>(11)                     | 8.7<br>(11)                              |

Table 4.4

Newtonian Blend Time Results

*T*/2 Lightnin A310 hydrofoil

| <i>T</i><br>(m) | $\theta$<br>(cP) | <i>N</i><br>(s <sup>-1</sup> ) | $\theta_1$ (s)<br>(RSD (%)) | $\theta_2$ (s)<br>(RSD (%)) | $\theta_3$ (s)<br>(RSD (%)) | $\theta_{RMS}$ (s)<br>(RSD (%)) |
|-----------------|------------------|--------------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------------------|
| 0.609           | 200              | 1.63                           | 98.8<br>(58)                | 127<br>(47)                 | 240<br>(20)                 | 218<br>(24)                     |
| "               | "                | 3.30                           | 13.7<br>(47)                | 18.9<br>(60)                | 99.3<br>(15)                | 87.4<br>(14)                    |
| "               | 175              | 2.50                           | 31.8<br>(30)                | 36.6<br>(28)                | 66.7<br>(30)                | 58.8<br>(22)                    |
| "               | 100              | 1.67                           | 29.3<br>(37)                | 29.5<br>(36)                | 97.4<br>(14)                | 91.1<br>(15)                    |
| "               | "                | 3.37                           | 23.2<br>(40)                | 16.9<br>(35)                | 47.9<br>(34)                | 43.4<br>(43)                    |
| "               | 75               | 3.17                           | 11.9<br>(21)                | 14.7<br>(11)                | 26.8<br>(12)                | 23.3<br>(17)                    |
| "               | 40               | 1.68                           | 21.8<br>(36)                | 19.6<br>(26)                | 61.9<br>(69)                | 60.8<br>(58)                    |
| "               | "                | 3.30                           | 7.3<br>(41)                 | 12.0<br>(11)                | 12.6<br>(21)                | 13.0<br>(15)                    |
| "               | "                | 4.62                           | 5.8<br>(29)                 | 7.6<br>(24)                 | 9.8<br>(48)                 | 9.4<br>(29)                     |

Table 4.5

*N*θ versus *Re* Regression Results

| Impeller         | <i>x</i> | r <sup>2</sup> (%) | F-ratio | Data Points |
|------------------|----------|--------------------|---------|-------------|
| <i>T</i> /3 4PBT | -0.989   | 93.09              | 163     | 13          |
| <i>T</i> /2 4PBT | -1.068   | 86.51              | 65      | 11          |
| <i>T</i> /3 4FBT | -1.259   | 84.71              | 23      | 5           |
| <i>T</i> /2 A310 | -0.836   | 78.95              | 27      | 8           |

Table 4.6

Correlation Regression Results

| Impeller         | <i>y</i> | r <sup>2</sup> (%) | F-ratio | Data Points |
|------------------|----------|--------------------|---------|-------------|
| <i>T</i> /3 4PBT | -0.497   | 98.46              | 769     | 13          |
| <i>T</i> /2 4PBT | -0.434   | 95.78              | 228     | 11          |
| <i>T</i> /3 4FBT | -0.425   | 94.74              | 73      | 5           |
| <i>T</i> /2 A310 | -0.488   | 94.22              | 115     | 8           |
| All              | -0.464   | 96.49              | 990     | 37          |

Table 4.7

Comparison of Constants

| Impeller         | <i>k<sub>x</sub></i> | RSD (%) | <i>k<sub>0.5</sub></i> | RSD (%) |
|------------------|----------------------|---------|------------------------|---------|
| <i>T</i> /3 4PBT | 179.5                | 10.98   | 177.7                  | 10.99   |
| <i>T</i> /2 4PBT | 238.8                | 21.80   | 186.1                  | 25.22   |
| <i>T</i> /3 4FBT | 263.2                | 12.65   | 182.0                  | 15.85   |
| <i>T</i> /2 A310 | 199.9                | 14.98   | 190.2                  | 15.18   |
| All              | 222.5                | 16.26   | 183.5                  | 17.40   |

Table 5.1

Pseudo-plastic Blend Time Results

T/3 Pitched Blade Turbine

| $T$<br>(m) | $C_N$ (%)<br>Grade | $N$<br>(s <sup>-1</sup> ) | $\theta_1$ (s)<br>(RSD (%)) | $\theta_2$ (s)<br>(RSD (%)) | $\theta_3$ (s)<br>(RSD (%)) | $\theta_{RMS}$ (s)<br>(RSD (%)) |
|------------|--------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------------------|
| 0.305      | 0.50<br>250HHR     | 8.33                      | 81.4<br>(20)                | 89.2<br>(13)                | 768<br>(10)                 | 740<br>(10)                     |
| "          | "                  | 10.0                      | 17.5<br>(12)                | 18.9<br>(33)                | 670<br>( 7)                 | 601<br>( 7)                     |
| "          | 0.25<br>250HHR     | 3.33                      | 18.0<br>(12)                | 21.1<br>(14)                | 265<br>(22)                 | 240<br>(20)                     |
| "          | "                  | 5.00                      | 10.2<br>(15)                | 18.2<br>(29)                | 91.6<br>(33)                | 83.8<br>(36)                    |
| 0.609      | 0.50<br>250HHR     | 4.17                      | 38.8<br>(14)                | 37.9<br>(31)                | 193<br>( 3)                 | 170<br>( 4)                     |
| "          | "                  | 5.50                      | 28.5<br>(16)                | 28.7<br>(25)                | 66.0<br>(11)                | 58.5<br>( 8)                    |
| "          | 0.25<br>250HHR     | 3.00                      | 17.9<br>(15)                | 18.3<br>(18)                | 39.1<br>(22)                | 29.7<br>(13)                    |
| "          | 0.75<br>250MR      | 3.33                      | 34.7<br>(14)                | 30.3<br>(14)                | 135<br>(10)                 | 115<br>(24)                     |
| "          | "                  | 5.33                      | 20.9<br>(26)                | 20.3<br>(24)                | 49.5<br>(18)                | 43.1<br>(24)                    |
| "          | 0.50<br>250MR      | 2.08                      | 23.5<br>(10)                | 29.0<br>(21)                | 103<br>(29)                 | 90.6<br>(20)                    |
| "          | "                  | 2.92                      | 18.7<br>(13)                | 20.3<br>(15)                | 44.0<br>(17)                | 40.2<br>(20)                    |
| "          | "                  | 3.75                      | 13.4<br>(17)                | 13.7<br>(17)                | 30.3<br>(12)                | 29.6<br>(19)                    |
| "          | 0.25<br>250MR      | 2.00                      | 17.4<br>(16)                | 22.8<br>(11)                | 20.6<br>(21)                | 21.9<br>(10)                    |
| "          | "                  | 2.67                      | 12.3<br>( 8)                | 14.6<br>(16)                | 17.3<br>(26)                | 17.9<br>(19)                    |
| 1.830      | 0.50<br>250HHR     | 1.67                      | -                           | 42.0<br>(26)                | 74.4<br>(13)                | 73.2<br>(12)                    |
| "          | "                  | 2.50                      | -                           | 25.4<br>(15)                | 28.4<br>( 6)                | 28.1<br>(12)                    |

Table 5.2

## Pseudo-plastic Blend Time Results

*T/2* Pitched blade turbine

| $T$<br>(m) | $C_N$ (%)<br>Grade | $N$<br>(s <sup>-1</sup> ) | $\theta_1$ (s)<br>(RSD (%)) | $\theta_2$ (s)<br>(RSD (%)) | $\theta_3$ (s)<br>(RSD (%)) | $\theta_{RMS}$ (s)<br>(RSD (%)) |
|------------|--------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------------------|
| 0.305      | 0.50<br>250HHR     | 4.17                      | 26.4<br>(13)                | 40.5<br>(22)                | 216<br>(19)                 | 190<br>(18)                     |
| "          | "                  | 5.83                      | 5.9<br>(43)                 | 9.1<br>(25)                 | 32.6<br>( 8)                | 27.2<br>( 8)                    |
| "          | 0.25<br>250HHR     | 2.00                      | 12.4<br>(21)                | 13.0<br>(16)                | 95.2<br>(14)                | 82.0<br>(15)                    |
| "          | "                  | 3.67                      | 4.3<br>(20)                 | 7.7<br>(13)                 | 24.2<br>(25)                | 19.9<br>(23)                    |
| 0.609      | 0.50<br>250HHR     | 2.50                      | 18.2<br>( 8)                | 22.0<br>( 7)                | 104<br>(17)                 | 90.8<br>(19)                    |
| "          | "                  | 4.00                      | 5.9<br>( 4)                 | 7.9<br>(30)                 | 13.4<br>(22)                | 10.9<br>(29)                    |
| "          | 0.25<br>250HHR     | 0.92                      | 15.4<br>(14)                | 18.2<br>(13)                | 88.5<br>(16)                | 79.1<br>(19)                    |
| "          | "                  | 2.00                      | 4.9<br>( 4)                 | 8.0<br>(16)                 | 16.8<br>(17)                | 14.9<br>(19)                    |
| "          | 0.75<br>250MR      | 2.00                      | 28.3<br>(20)                | 34.3<br>(15)                | 56.0<br>(22)                | 63.3<br>(10)                    |
| "          | "                  | 2.50                      | 19.3<br>(23)                | 24.4<br>(30)                | 25.5<br>(19)                | 25.0<br>(26)                    |
| "          | "                  | 3.00                      | 18.5<br>(18)                | 18.6<br>(19)                | 21.2<br>(21)                | 21.6<br>(17)                    |
| "          | 0.50<br>250MR      | 1.25                      | 21.7<br>(16)                | 24.0<br>(17)                | 29.2<br>(17)                | 28.4<br>(17)                    |
| "          | "                  | 2.08                      | 14.8<br>(12)                | 16.6<br>(13)                | 21.2<br>(13)                | 17.5<br>(10)                    |
| "          | "                  | 2.92                      | 10.7<br>(20)                | 12.9<br>(20)                | 11.9<br>(18)                | 13.4<br>(19)                    |
| "          | 0.25<br>250MR      | 1.00                      | 22.1<br>(10)                | 22.6<br>(15)                | 25.4<br>(12)                | 24.4<br>(11)                    |
| "          | "                  | 1.33                      | 14.8<br>( 9)                | 17.3<br>( 9)                | 19.1<br>(19)                | 18.5<br>(17)                    |
| "          | "                  | 1.67                      | 12.8<br>(13)                | 15.2<br>(18)                | 14.4<br>(14)                | 15.5<br>(12)                    |

Table 5.2 continued

Pseudo-plastic Blend Time Results

*T*/2 Pitched blade turbine

| <i>T</i><br>(m) | <i>C<sub>N</sub></i> (%)<br>Grade | <i>N</i><br>(s <sup>-1</sup> ) | θ <sub>1</sub> (s)<br>(RSD (%)) | θ <sub>2</sub> (s)<br>(RSD (%)) | θ <sub>3</sub> (s)<br>(RSD (%)) | θ <sub><i>RMS</i></sub> (s)<br>(RSD (%)) |
|-----------------|-----------------------------------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|------------------------------------------|
| 1.830           | 0.50<br>250HHR                    | 1.00                           | -                               | 30.6<br>(12)                    | 45.2<br>( 9)                    | 44.3<br>(10)                             |
| "               | "                                 | 1.25                           | -                               | 25.1<br>(34)                    | 27.3<br>(11)                    | 26.2<br>( 9)                             |

Table 5.3

Pseudo-plastic Blend Time Results

T/3 Flat blade turbine

| $T$<br>(m) | $C_N$ (%)<br>Grade | $N$<br>(s <sup>-1</sup> ) | $\theta_1$ (s)<br>(RSD (%)) | $\theta_2$ (s)<br>(RSD (%)) | $\theta_3$ (s)<br>(RSD (%)) | $\theta_{RMS}$ (s)<br>(RSD (%)) |
|------------|--------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------------------|
| 0.305      | 0.50<br>250HHR     | 8.67                      | 6.3<br>(24)                 | 7.3<br>(15)                 | 111<br>(25)                 | 89.6<br>(27)                    |
| "          | "                  | 10.33                     | 5.3<br>(13)                 | 8.3<br>(39)                 | 163<br>(19)                 | 127<br>(18)                     |
| "          | 0.25<br>250HHR     | 3.33                      | 9.9<br>(27)                 | 11.7<br>(14)                | 140<br>(12)                 | 121<br>(13)                     |
| "          | "                  | 5.00                      | 7.1<br>(11)                 | 12.5<br>(22)                | 91.1<br>(16)                | 73.7<br>(21)                    |
| 0.609      | 0.50<br>250HHR     | 4.00                      | 9.9<br>( 5)                 | 13.5<br>(25)                | 39.3<br>(18)                | 28.0<br>(16)                    |
| "          | 0.25<br>250HHR     | 2.00                      | 10.9<br>(19)                | 16.5<br>(19)                | 82.4<br>(20)                | 77.4<br>(18)                    |
| "          | "                  | 3.50                      | 3.6<br>(43)                 | 6.4<br>(27)                 | 15.3<br>(23)                | 13.1<br>(21)                    |
| "          | 0.75<br>250MR      | 3.00                      | 21.4<br>(24)                | 21.1<br>(15)                | 43.2<br>(18)                | 37.0<br>(17)                    |
| "          | "                  | 4.00                      | 11.5<br>(16)                | 13.2<br>(20)                | 15.6<br>(29)                | 15.6<br>(23)                    |
| "          | "                  | 5.00                      | 11.1<br>(34)                | 12.9<br>(10)                | 14.9<br>(19)                | 14.3<br>(10)                    |
| "          | 0.50<br>250MR      | 2.08                      | 16.2<br>(14)                | 16.5<br>(12)                | 23.0<br>(14)                | 18.9<br>(19)                    |
| "          | "                  | 2.92                      | 12.2<br>(12)                | 13.4<br>(18)                | 16.7<br>(18)                | 16.7<br>(17)                    |
| "          | "                  | 3.75                      | 9.5<br>(12)                 | 9.8<br>(10)                 | 11.8<br>(11)                | 11.8<br>(12)                    |
| "          | 0.25<br>250MR      | 1.33                      | 19.3<br>(14)                | 22.6<br>(17)                | 27.4<br>(19)                | 26.8<br>(16)                    |
| "          | "                  | 2.00                      | 14.6<br>(15)                | 19.5<br>(18)                | 22.2<br>(23)                | 22.0<br>(22)                    |
| "          | "                  | 2.67                      | 10.9<br>( 8)                | 11.4<br>(13)                | 11.5<br>(19)                | 11.8<br>(11)                    |

Table 5.4

## Pseudo-plastic Blend Time Results

*T*/2 Hydrofoils

| <i>T</i><br>(m) | <i>C<sub>N</sub></i> (%)<br>Grade | <i>N</i><br>(s <sup>-1</sup> ) | θ <sub>1</sub> (s)<br>(RSD (%)) | θ <sub>2</sub> (s)<br>(RSD (%)) | θ <sub>3</sub> (s)<br>(RSD (%)) | θ <sub>RMS</sub> (s)<br>(RSD (%)) |
|-----------------|-----------------------------------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|-----------------------------------|
| 0.305<br>HE3    | 0.50<br>250HHR                    | 7.50                           | 53.1<br>( 8)                    | 51.5<br>(10)                    | 292<br>(15)                     | 283<br>(14)                       |
| "               | "                                 | 9.17                           | 16.5<br>(19)                    | 14.3<br>(40)                    | 205<br>(21)                     | 160<br>(16)                       |
| "               | 0.27<br>250HHR                    | 3.67                           | 18.1<br>(18)                    | 28.8<br>(24)                    | 117<br>( 7)                     | 111<br>( 6)                       |
| "               | "                                 | 5.33                           | 11.1<br>(28)                    | 14.7<br>(28)                    | 93.5<br>(22)                    | 84.1<br>(29)                      |
| 0.609<br>A310   | 0.50<br>250HHR                    | 3.33                           | 23.8<br>(17)                    | 25.8<br>(17)                    | 71.7<br>(17)                    | 61.0<br>(17)                      |
| "               | "                                 | 5.00                           | 39.9<br>(14)                    | 28.8<br>(15)                    | 47.9<br>(14)                    | 44.3<br>(19)                      |
| "               | 0.25<br>250HHR                    | 2.00                           | 22.6<br>(28)                    | 21.5<br>(10)                    | 82.0<br>( 5)                    | 61.0<br>(18)                      |
| "               | "                                 | 3.00                           | 14.5<br>(13)                    | 13.5<br>(13)                    | 17.7<br>(13)                    | 16.8<br>(16)                      |
| "               | 0.75<br>250MR                     | 3.00                           | 25.9<br>(16)                    | 28.8<br>(22)                    | 38.9<br>( 8)                    | 36.5<br>(12)                      |
| "               | "                                 | 4.00                           | 24.0<br>(26)                    | 26.5<br>(24)                    | 31.6<br>( 7)                    | 29.3<br>(15)                      |
| "               | "                                 | 5.00                           | 15.7<br>(16)                    | 19.4<br>(15)                    | 23.2<br>(11)                    | 21.2<br>(11)                      |
| "               | 0.50<br>250MR                     | 2.08                           | 16.6<br>(20)                    | 21.8<br>( 8)                    | 25.0<br>(16)                    | 24.7<br>(16)                      |
| "               | "                                 | 2.92                           | 20.3<br>(12)                    | 23.3<br>(12)                    | 27.3<br>(12)                    | 23.4<br>( 6)                      |
| "               | "                                 | 3.75                           | 11.2<br>(18)                    | 14.6<br>( 9)                    | 15.0<br>(21)                    | 14.7<br>( 9)                      |
| "               | 0.25<br>250MR                     | 1.33                           | 24.3<br>(24)                    | 29.8<br>(24)                    | 39.4<br>(27)                    | 38.4<br>(27)                      |
| "               | "                                 | 1.67                           | 16.8<br>(23)                    | 20.3<br>(15)                    | 19.9<br>(15)                    | 18.4<br>(12)                      |
| "               | "                                 | 2.00                           | 15.8<br>(19)                    | 16.7<br>(24)                    | 16.8<br>(16)                    | 17.0<br>(17)                      |

Table 5.5

*N*θ versus *Re* Regression Results

| Impeller         | <i>x</i> | r <sup>2</sup> (%) | F-ratio | Data Points |
|------------------|----------|--------------------|---------|-------------|
| <i>T</i> /3 4PBT | -0.636   | 73.41              | 29      | 11          |
| <i>T</i> /2 4PBT | -0.842   | 84.94              | 74      | 14          |
| <i>T</i> /3 4FBT | -0.894   | 67.23              | 26      | 13          |
| <i>T</i> /2 A310 | -0.960   | 64.21              | 24      | 14          |

Table 5.6

Correlation Regression Results

| Impeller         | <i>y</i> | r <sup>2</sup> (%) | F-ratio | Data Points |
|------------------|----------|--------------------|---------|-------------|
| <i>T</i> /3 4PBT | -0.583   | 94.76              | 182     | 11          |
| <i>T</i> /2 4PBT | -0.515   | 96.29              | 338     | 14          |
| <i>T</i> /3 4FBT | -0.483   | 90.80              | 119     | 13          |
| <i>T</i> /2 A310 | -0.431   | 84.89              | 74      | 14          |
| All              | -0.453   | 89.71              | 463     | 52          |

Table 5.7

Comparison of Constants

| Impeller         | <i>k<sub>x</sub></i> | RSD (%) | <i>k<sub>0.5</sub></i> | RSD (%) |
|------------------|----------------------|---------|------------------------|---------|
| <i>T</i> /3 4PBT | 141.4                | 21.09   | 193.7                  | 24.68   |
| <i>T</i> /2 4PBT | 171.9                | 18.89   | 183.5                  | 19.43   |
| <i>T</i> /3 4FBT | 188.6                | 30.40   | 175.8                  | 31.17   |
| <i>T</i> /2 A310 | 265.7                | 30.16   | 206.6                  | 38.68   |
| All              | 237.2                | 32.19   | 200.0                  | 38.36   |

Table 6.1

Power Numbers

T/3 Pitched Blade Turbine

| <i>T</i><br>(m)    | $\mu$ (cP)/<br><i>C<sub>N</sub></i> (%) | <i>P<sub>O</sub></i><br>(-) | RSD<br>(%) |
|--------------------|-----------------------------------------|-----------------------------|------------|
| 0.305 <sup>A</sup> | 500                                     | 1.54                        | 1.47       |
| "A                 | 100                                     | 1.41                        | 2.04       |
| "                  | 0.50 HHR                                | 2.53                        | -          |
| "                  | 0.25 HHR                                | 1.57                        | 6.08       |
| 0.609              | 100                                     | 1.51                        | 0.14       |
| "                  | 40                                      | 1.62                        | 2.62       |
| "                  | 10                                      | 1.60                        | 0.58       |
| "B                 | 1                                       | 1.56                        | -          |
| "C                 | 0.50 HHR                                | 1.25                        | 2.38       |
| "C                 | 0.25 HHR                                | 1.15                        | 3.80       |
| "C                 | 0.75 MR                                 | 1.21                        | 0.94       |
| "C                 | 0.50 MR                                 | 1.23                        | 1.06       |
| "C                 | 0.25 MR                                 | 1.20                        | 3.14       |
| 1.830              | 300                                     | 1.65                        | -          |
| "                  | 40                                      | 1.77                        | -          |
| "B                 | 1                                       | 1.80                        | -          |
| 2.670 <sup>B</sup> | 1                                       | 1.80                        | -          |

A: Power number estimated from Palmer (1987).

B: Power number data from Mackinnon (1987).

C: Blade angle = 41°.

Table 6.2

Power Numbers

T/2 Pitched Blade Turbine

| <i>T</i><br>(m)    | $\mu$ (cP)/<br><i>C<sub>N</sub></i> (%) | <i>P<sub>O</sub></i><br>(-) | RSD<br>(%) |
|--------------------|-----------------------------------------|-----------------------------|------------|
| 0.305 <sup>A</sup> | 500                                     | 1.72                        | 2.27       |
| "A                 | 100                                     | 1.45                        | 0.74       |
| "                  | 0.50 HHR                                | 1.22                        | -          |
| "                  | 0.25 HHR                                | 1.08                        | -          |
| 0.609              | 100                                     | 1.10                        | 0.23       |
| "                  | 40                                      | 1.00                        | 0.45       |
| "                  | 10                                      | 1.10                        | 4.72       |
| "B                 | 1                                       | 1.12                        | -          |
| "                  | 0.50 HHR                                | 1.03                        | 3.99       |
| "                  | 0.25 HHR                                | 1.00                        | 0.21       |
| "                  | 0.75 MR                                 | 1.03                        | 0.45       |
| "                  | 0.50 MR                                 | 0.99                        | 1.22       |
| "                  | 0.25 MR                                 | 1.05                        | 0.41       |
| 1.830 <sup>B</sup> | 1                                       | 1.28                        | -          |
| 2.670 <sup>B</sup> | 1                                       | 1.28                        | -          |

A:      Power number estimated from Palmer (1987).

B:      Power number data from Mackinnon (1987).

Table 6.3

Power Numbers

T/3 Flat Blade Turbine

| $T$<br>(m)         | $\mu$ (cP)/<br>$C_N$ (%) | $P_o$<br>(-) | RSD<br>(%) |
|--------------------|--------------------------|--------------|------------|
| 0.305              | 0.50 HHR                 | 2.77         | -          |
| "                  | 0.25 HHR                 | 2.97         | 1.76       |
| 0.609              | 100                      | 2.71         | 0.48       |
| "                  | 40                       | 2.70         | 0.58       |
| "                  | 10                       | 2.72         | 0.75       |
| "A                 | 1                        | 2.85         | -          |
| "                  | 0.50 HHR                 | 3.23         | -          |
| "                  | 0.25 HHR                 | 3.02         | 0.87       |
| "                  | 0.75 MR                  | 3.03         | 1.90       |
| "                  | 0.50 MR                  | 2.83         | 1.11       |
| "                  | 0.25 MR                  | 2.96         | 2.13       |
| 1.830 <sup>A</sup> | 1                        | 2.98         | -          |

A: Power number data from Mackinnon (1987).

Table 6.4

Power Numbers

T/2 Hydrofoils

| <i>T</i><br>(m) | $\mu$ (cP)/<br><i>C<sub>N</sub></i> (%) | <i>P<sub>o</sub></i><br>(-) | RSD<br>(%) |
|-----------------|-----------------------------------------|-----------------------------|------------|
| 0.305           | 0.50 HHR                                | 0.55                        | 11.6       |
| "               | 0.25 HHR                                | 0.33                        | 0.86       |
| 0.609           | 200                                     | 0.41                        | 3.13       |
| "               | 175                                     | 0.33                        | -          |
| "               | 100                                     | 0.30                        | 0.71       |
| "               | 75                                      | 0.29                        | -          |
| "               | 40                                      | 0.29                        | 1.39       |
| "A              | 1                                       | 0.29                        | -          |
| "               | 0.50 HHR                                | 0.26                        | 2.67       |
| "               | 0.25 HHR                                | 0.25                        | 2.83       |
| "               | 0.75 MR                                 | 0.28                        | 2.15       |
| "               | 0.50 MR                                 | 0.25                        | 0.46       |
| "               | 0.25 MR                                 | 0.30                        | 0.51       |

A:        Power number data from Mackinnon (1987).

Table 7.1

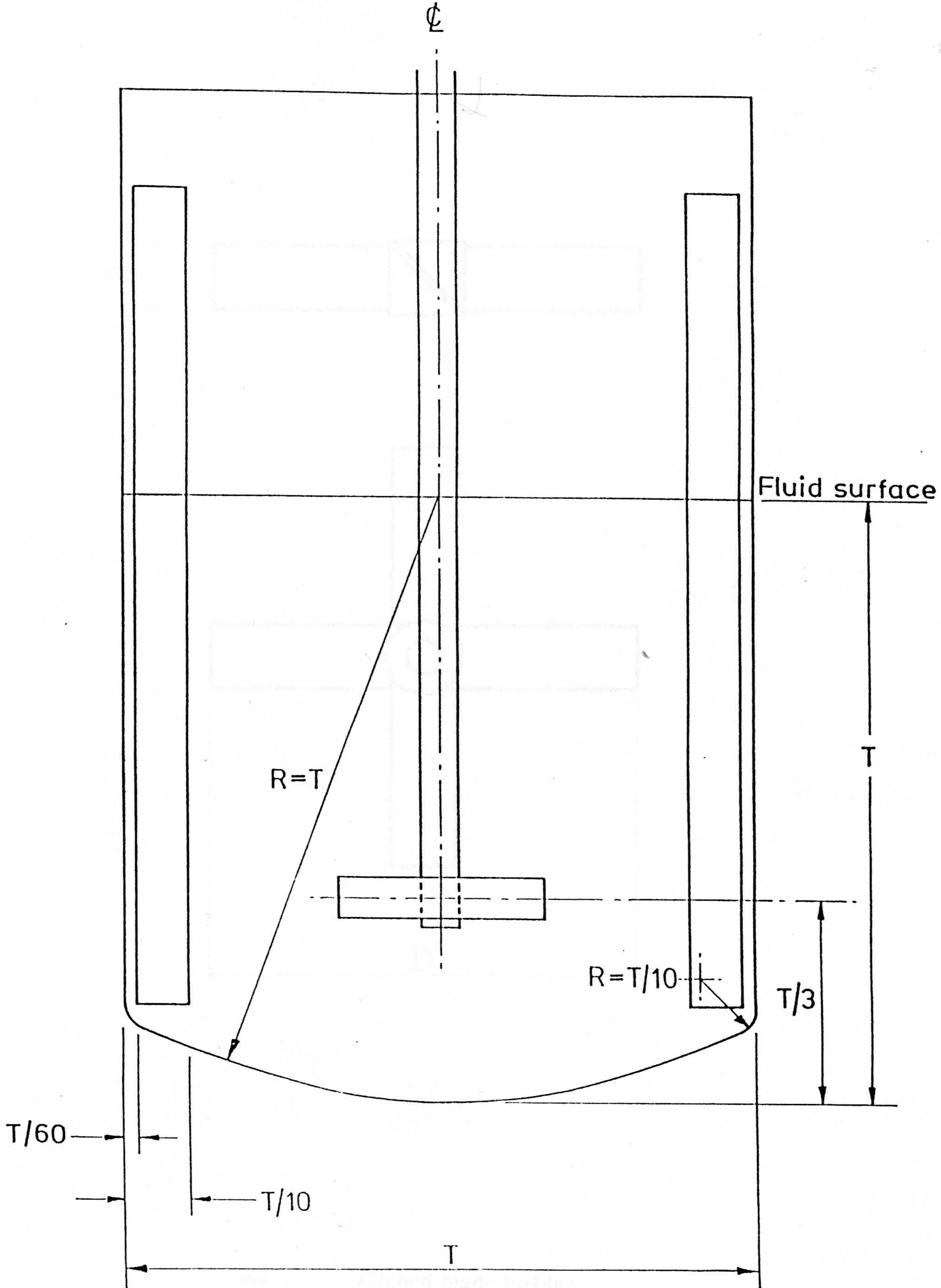
Selling Price (Capital Cost) versus Shaft Torque  
(from Rigden (1991))

| Selling Price<br>(k£) | Torque<br>(Nm) |
|-----------------------|----------------|
| 8                     | 250            |
| 12                    | 1000           |
| 18                    | 2000           |
| 30                    | 4800           |
| 50                    | 12000          |

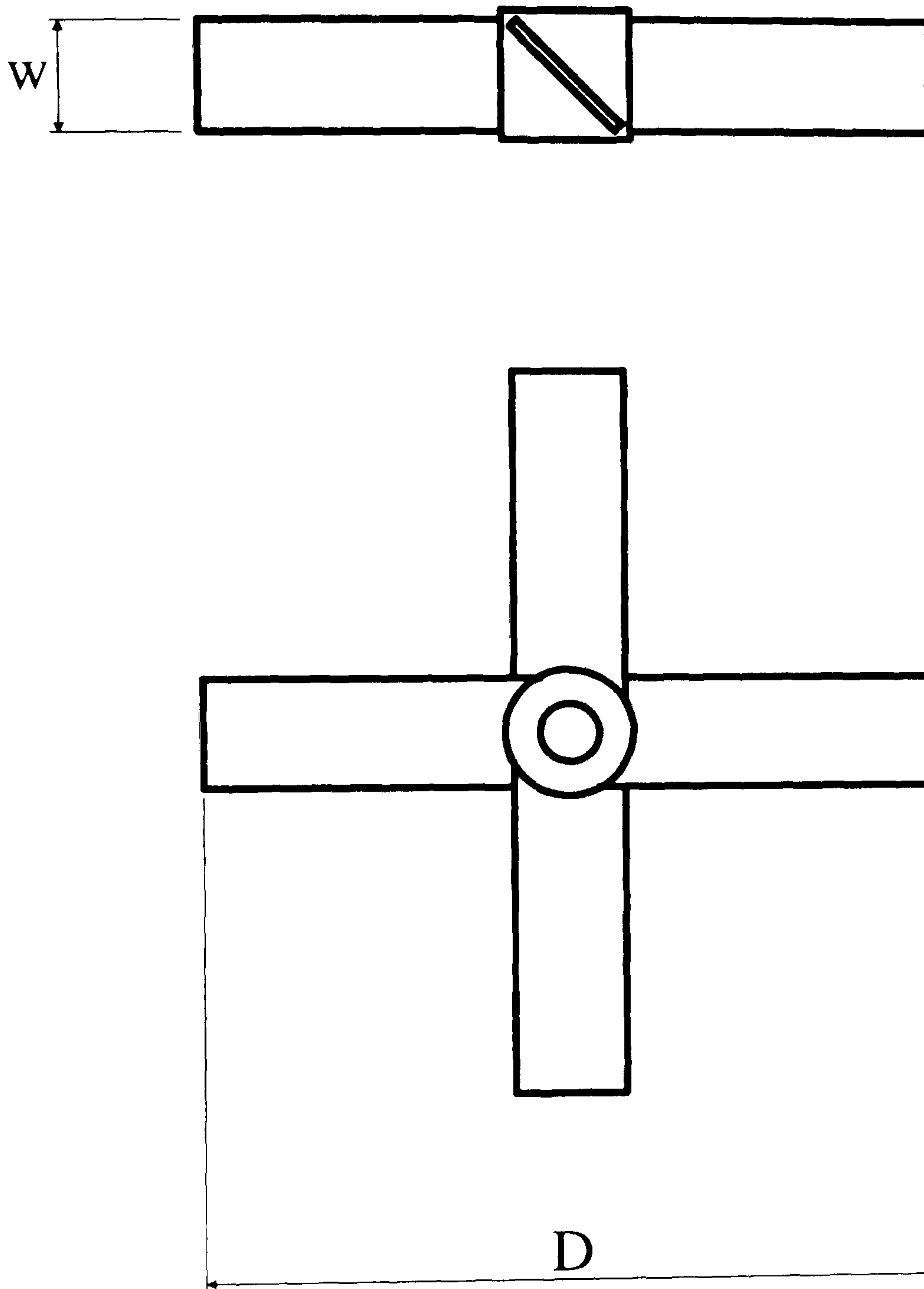
Table 7.2

Power Law Constants and Indices of Fluids in Economic Analysis

| Constant $K$<br>(Ns <sup><math>n</math></sup> m <sup>-2</sup> ) | Index $n$<br>(-) |
|-----------------------------------------------------------------|------------------|
| 2.00                                                            | 1.0              |
| 2.67                                                            | 0.9              |
| 3.55                                                            | 0.8              |
| 4.74                                                            | 0.7              |
| 6.31                                                            | 0.6              |
| 8.42                                                            | 0.5              |
| 11.22                                                           | 0.4              |

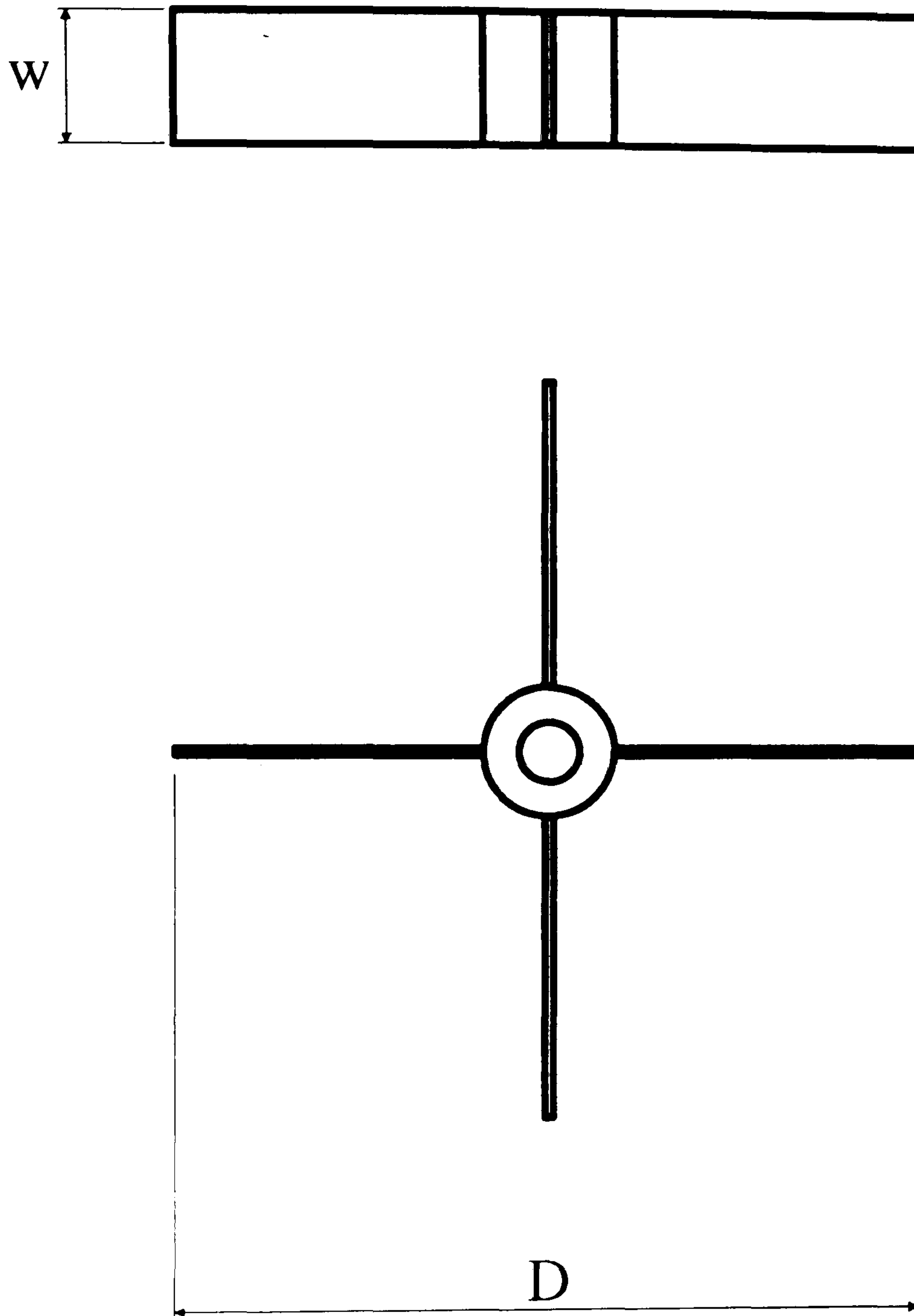


3.1 Vessel Arrangement



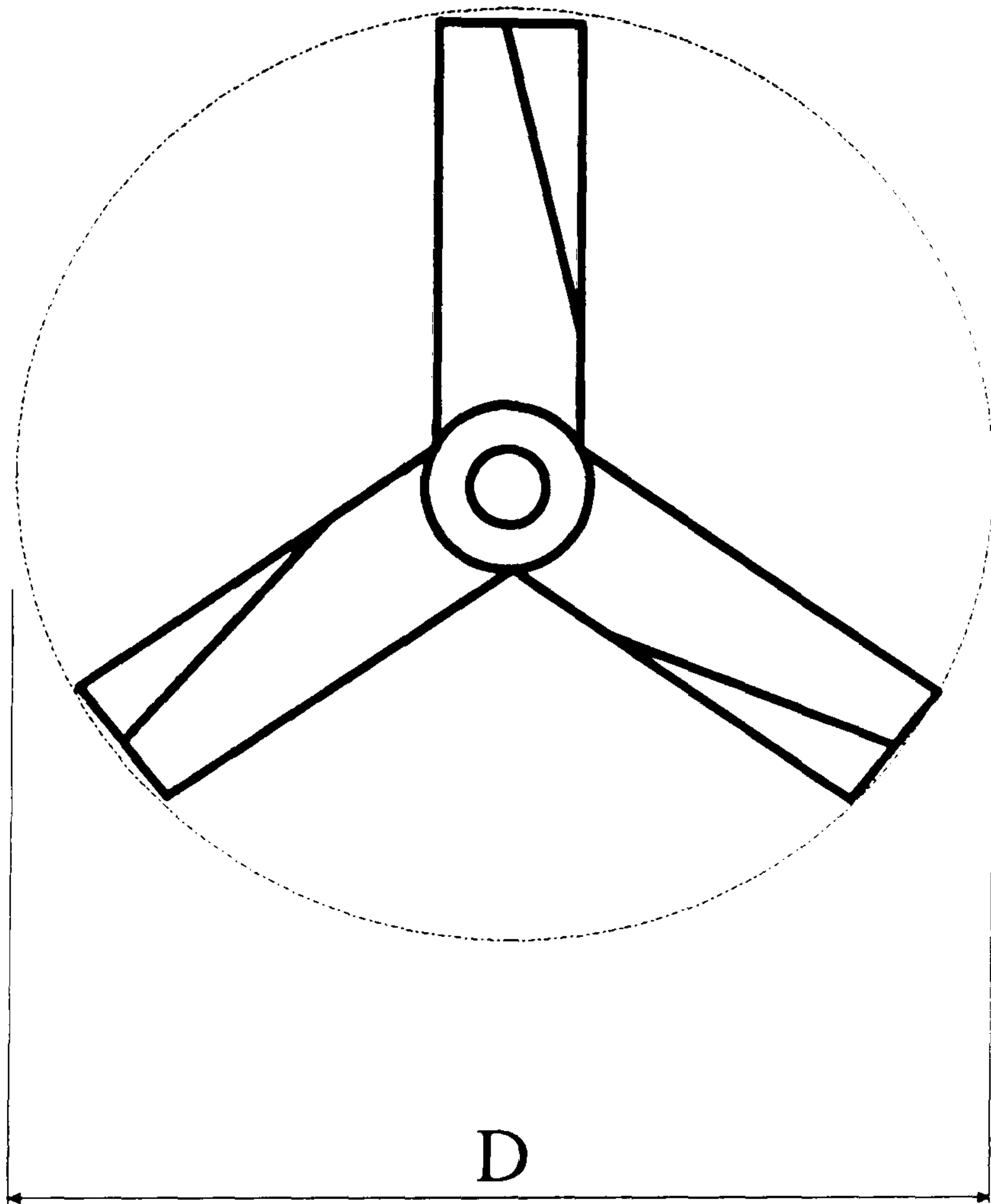
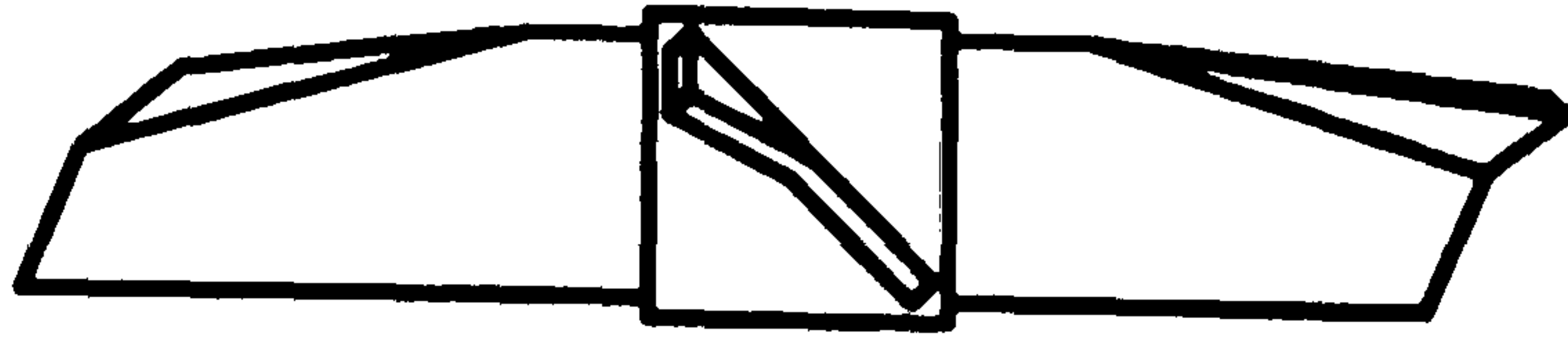
**3.2**

**Pitched blade turbine**



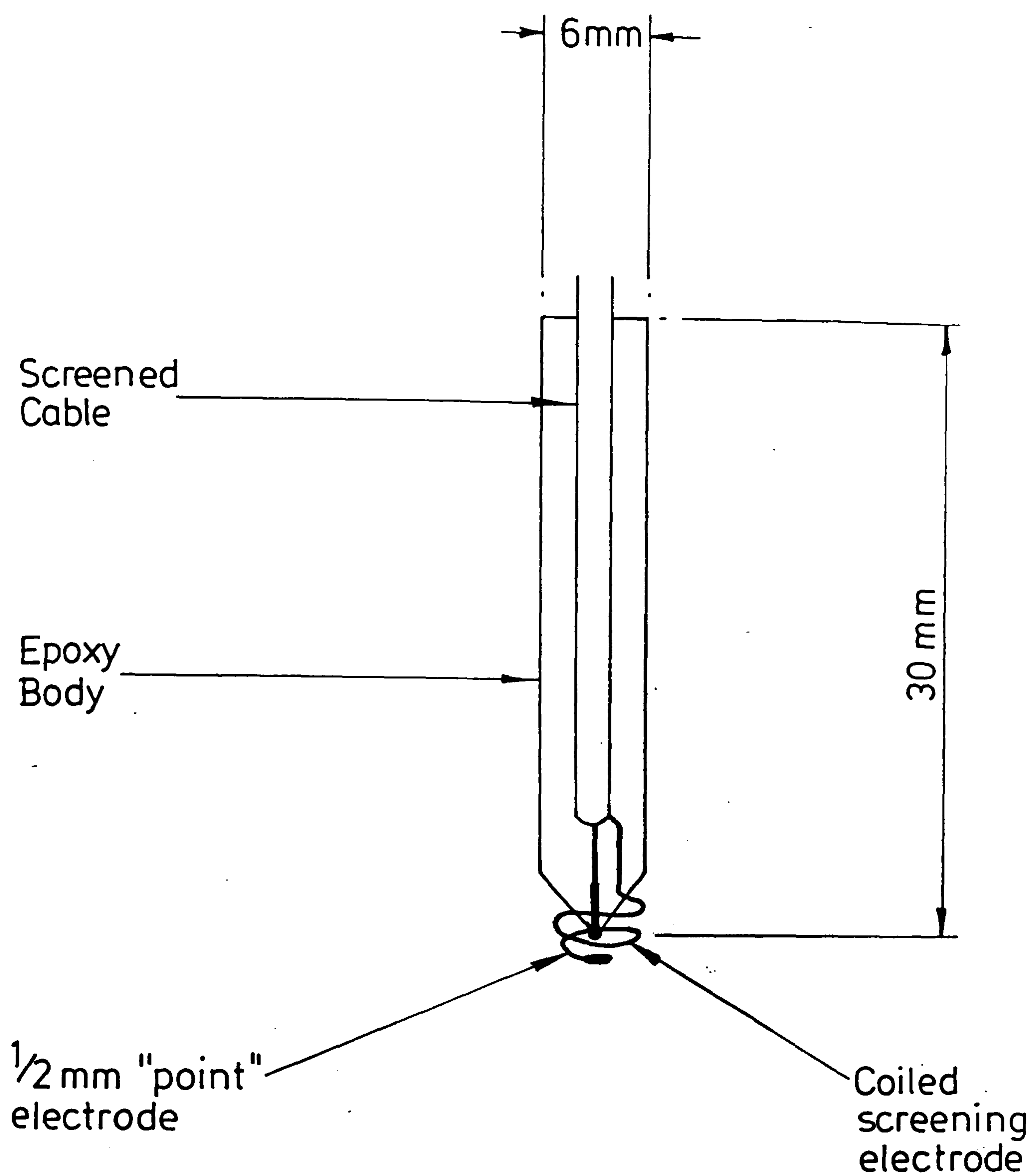
**3.3**

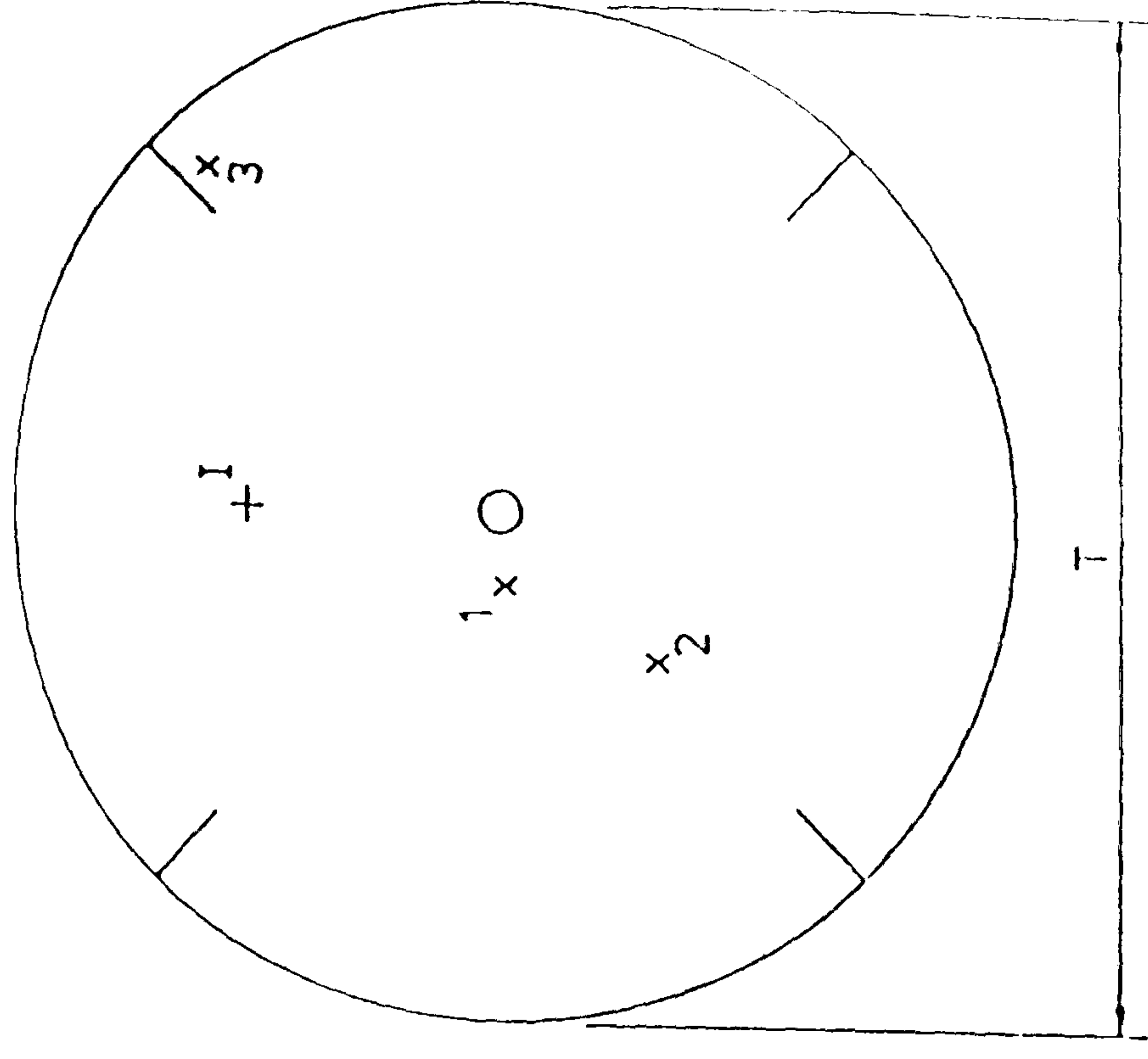
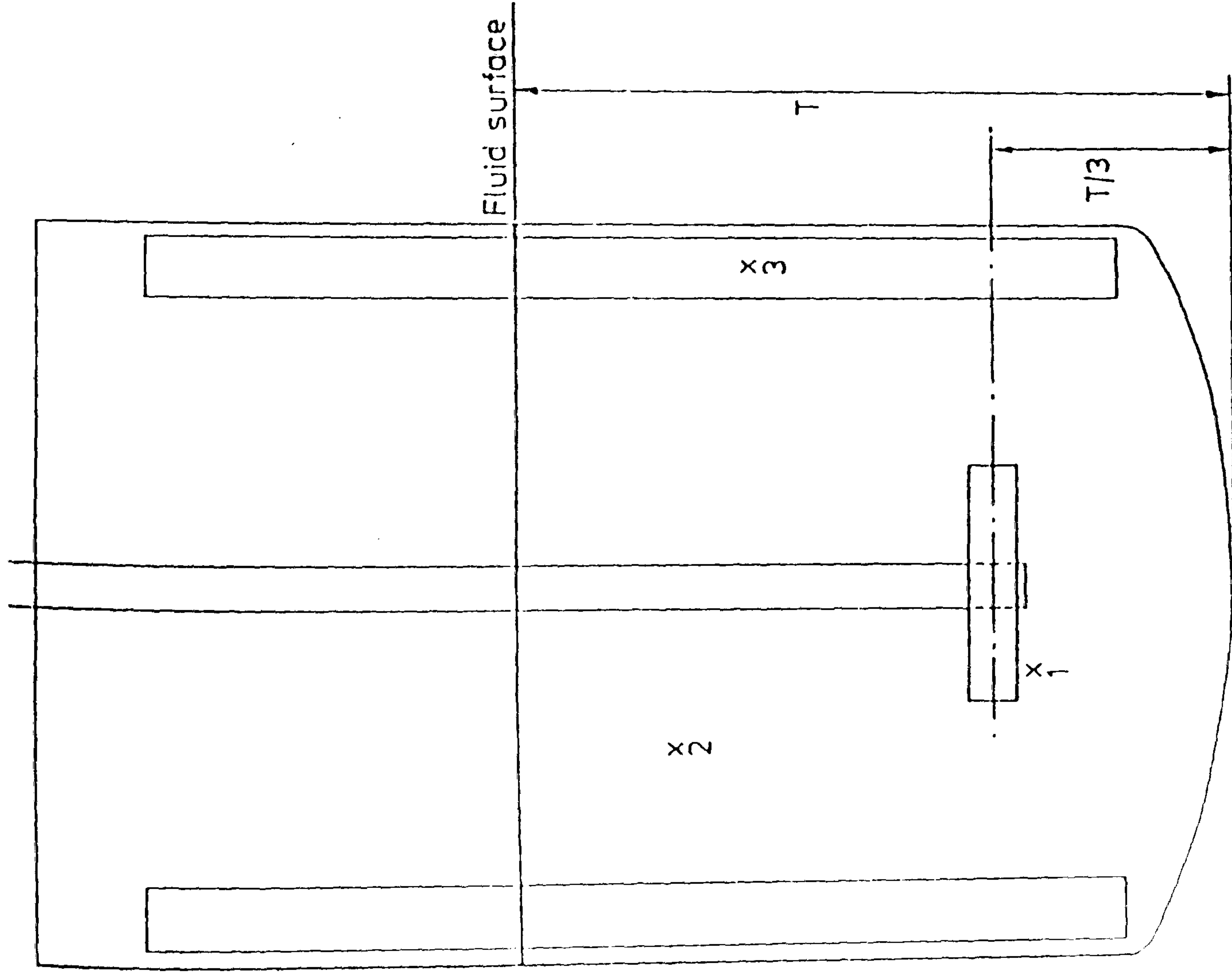
**Flat blade turbine**



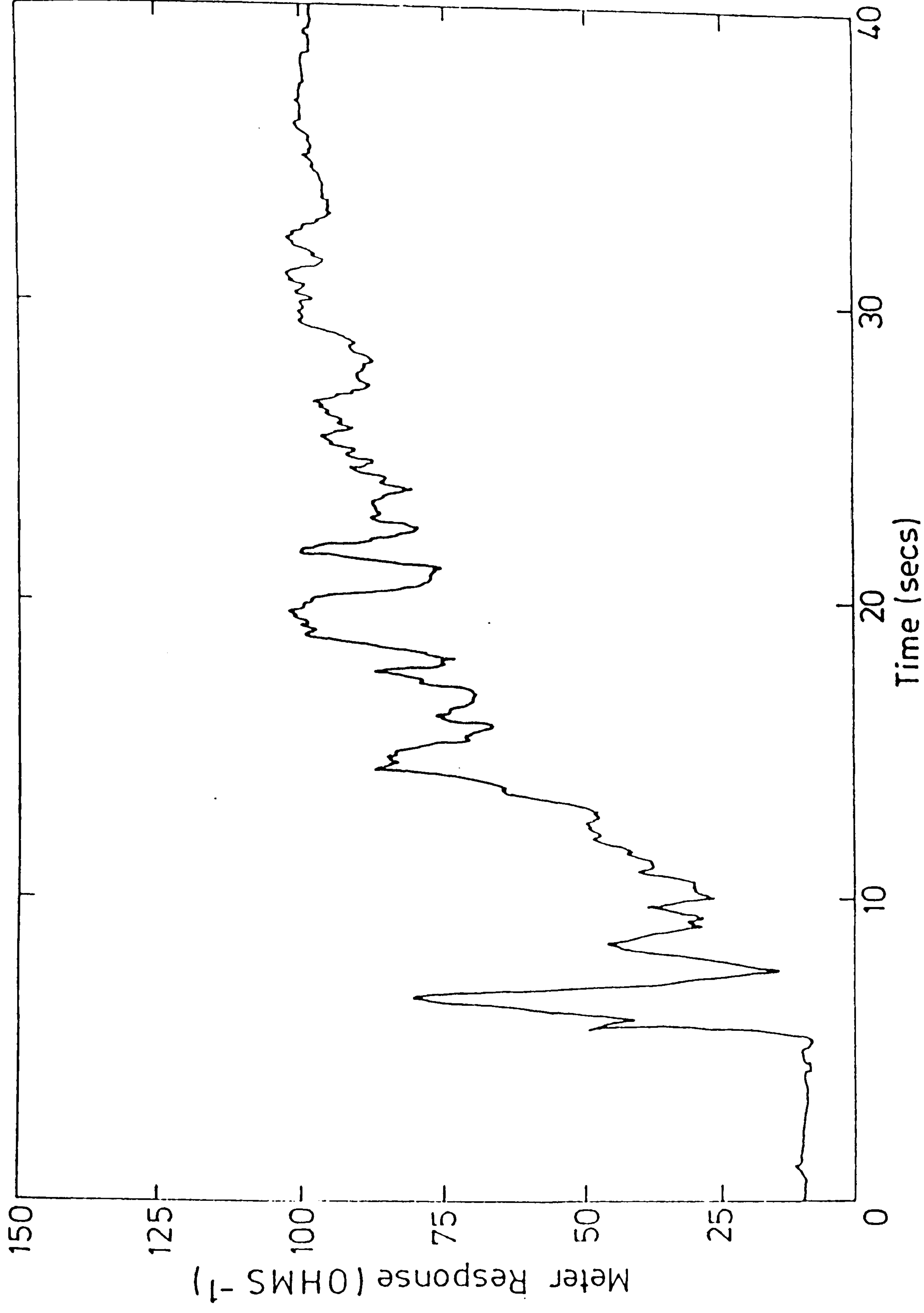
**3.4**

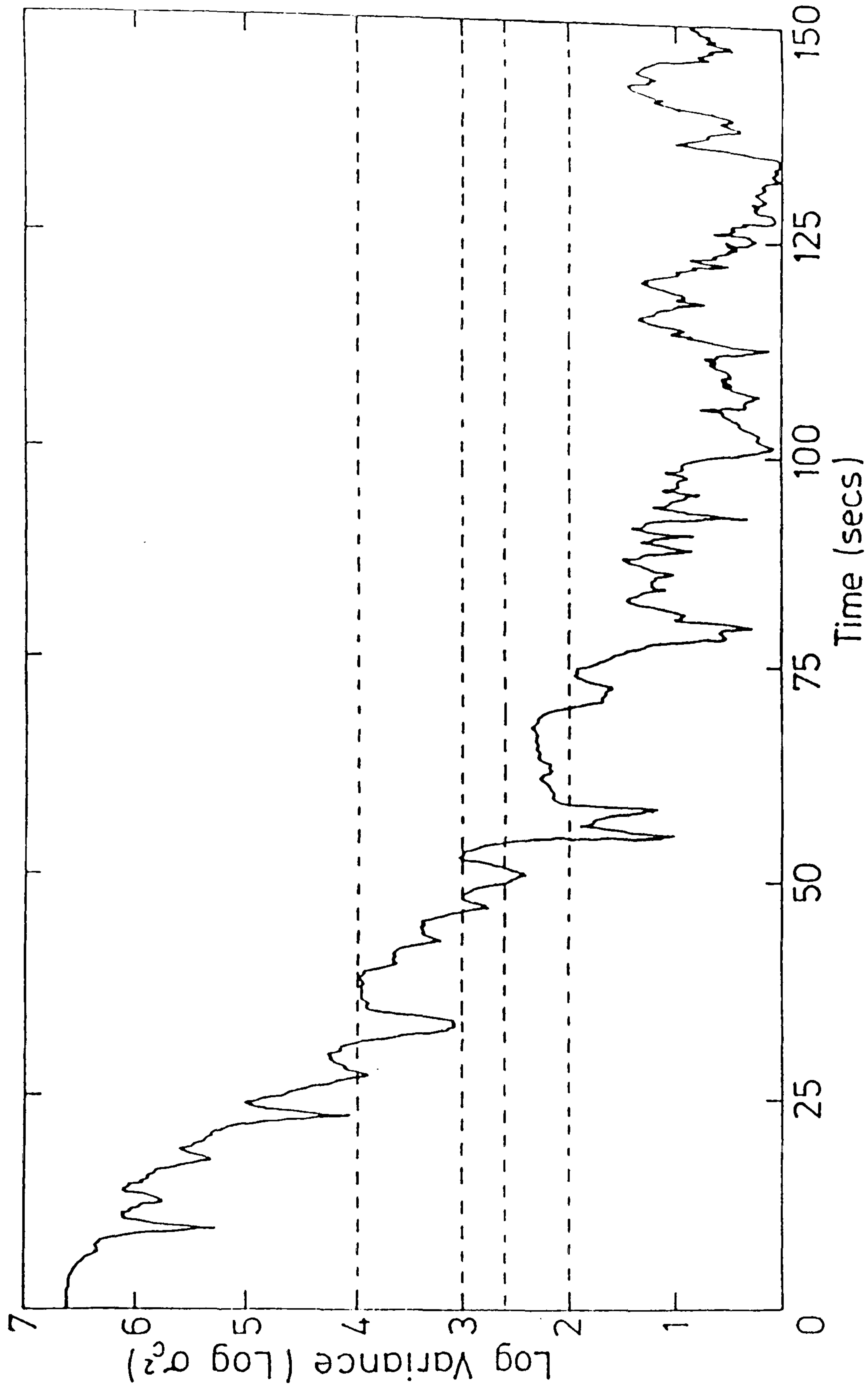
**Hydrofoil**





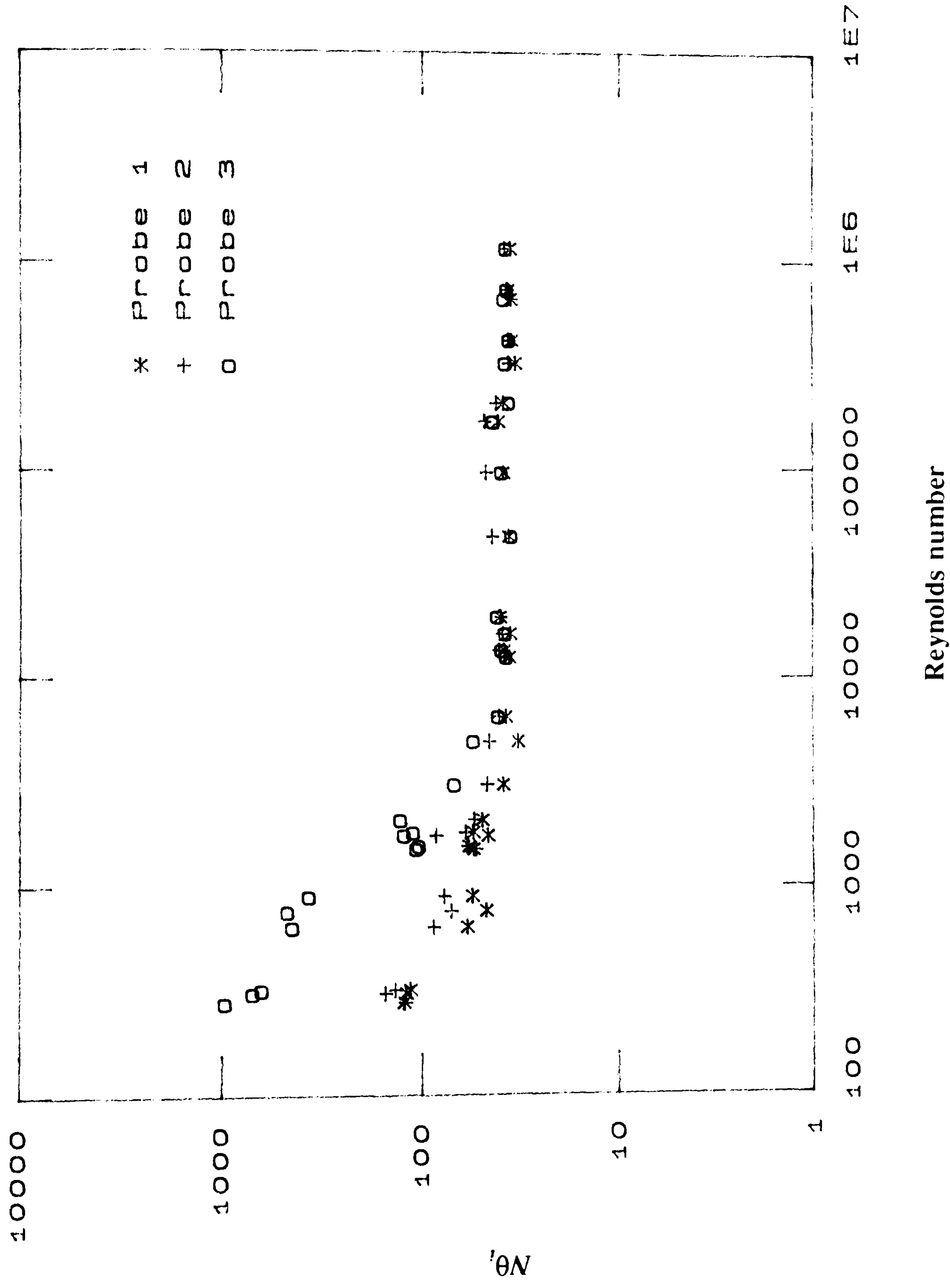
|          |                                                                                |
|----------|--------------------------------------------------------------------------------|
| Injector | 25mm above fluid surface<br>T/10 from shaft axis.                              |
| Probe 1  | T/50 below bottom of impeller.<br>T/8 from shaft axis.                         |
| Probe 2  | T/4.5 below fluid surface.<br>T/4.7 from shaft axis.                           |
| Probe 3  | T/3 below fluid surface.<br>T/2.2 from shaft axis, adjacent to baffle $\phi$ . |





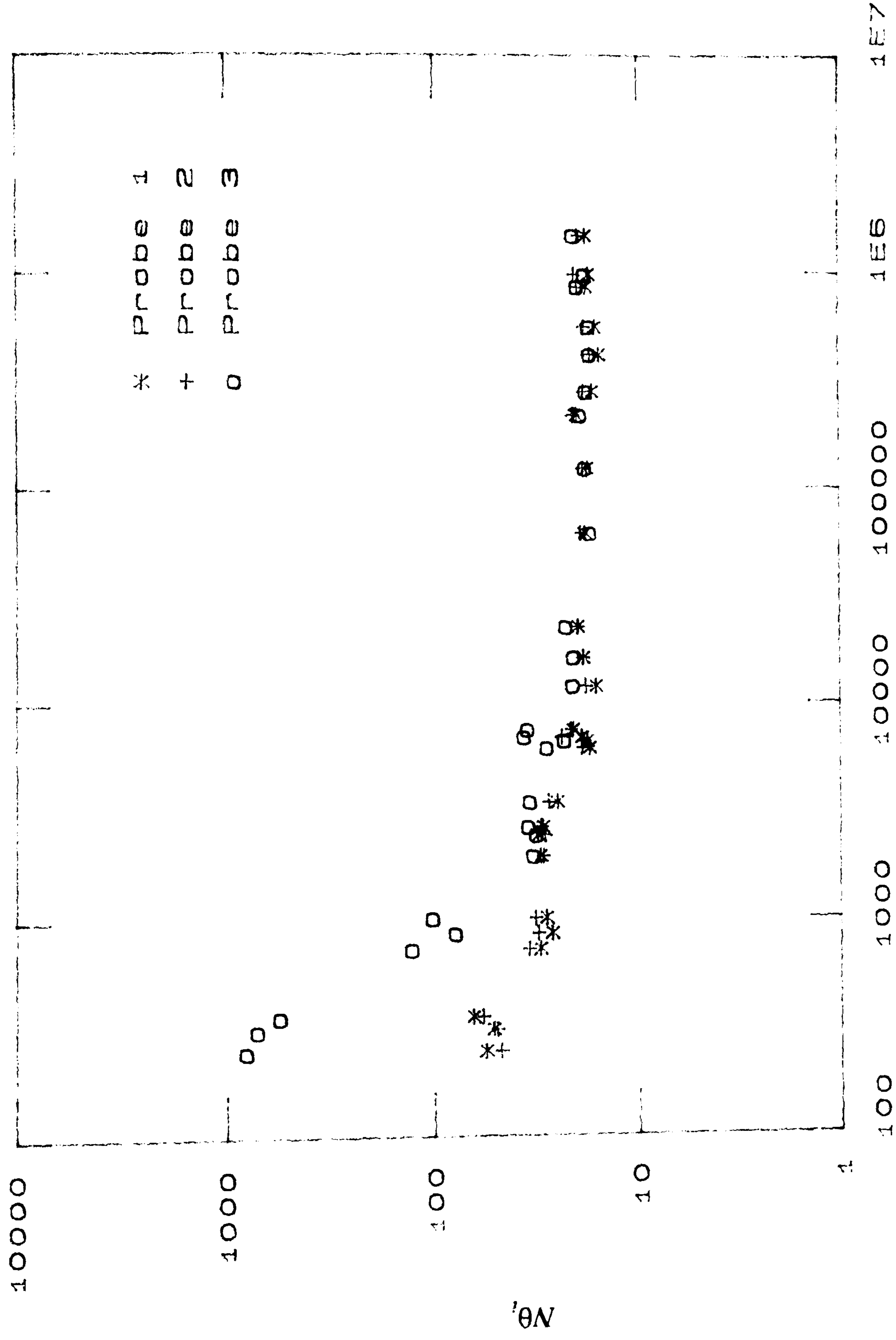
4.2 Typical Plot of  $\sigma_c^2$  versus Time

4.3 Plot of Individual Probe  $N\theta$  versus Reynolds number  
Newtonian data - T/3 Pitched blade turbine



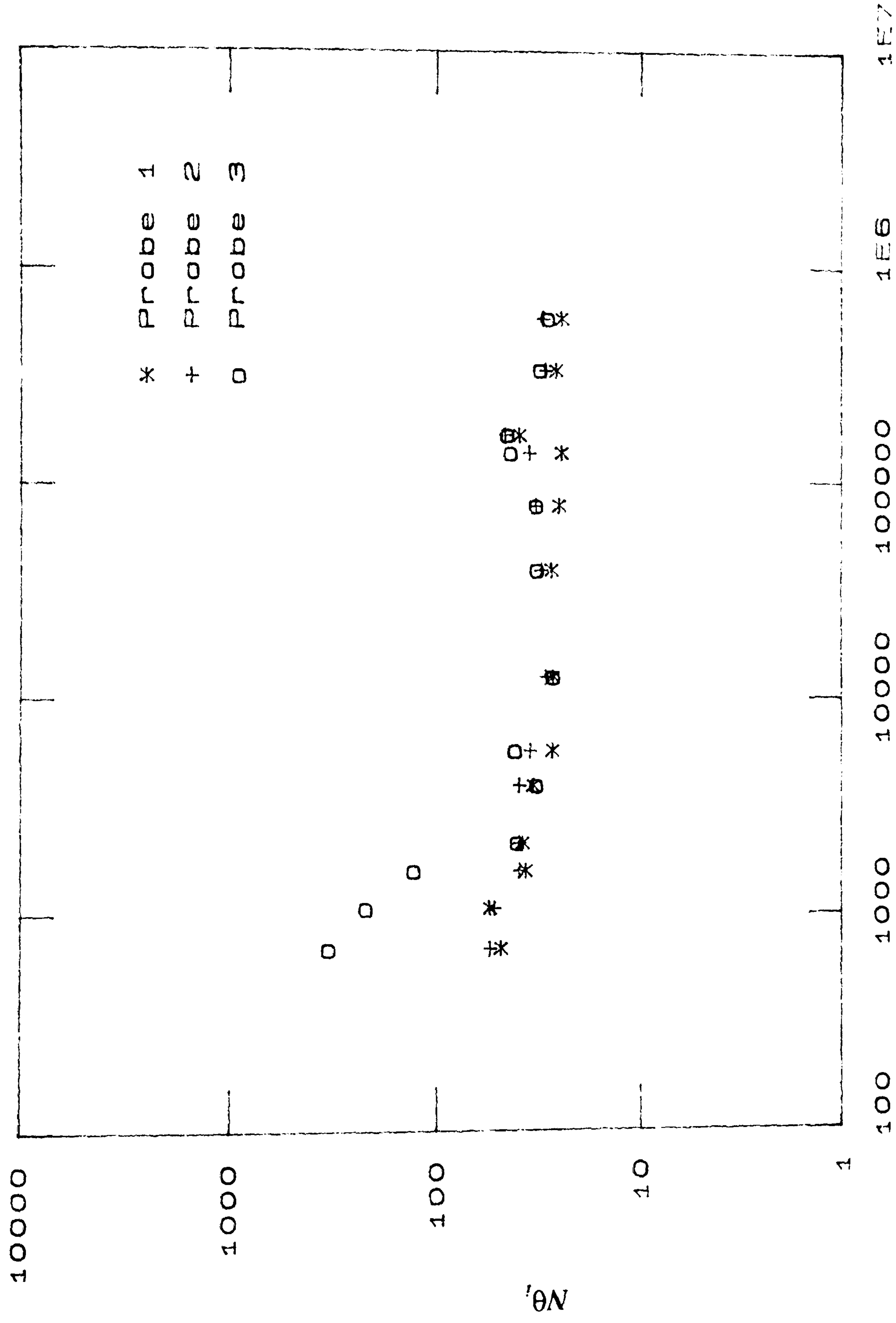
# 4.4

Plot of Individual Probe  $N\theta$  versus Reynolds number  
Newtonian data -  $T/2$  Pitched blade turbine



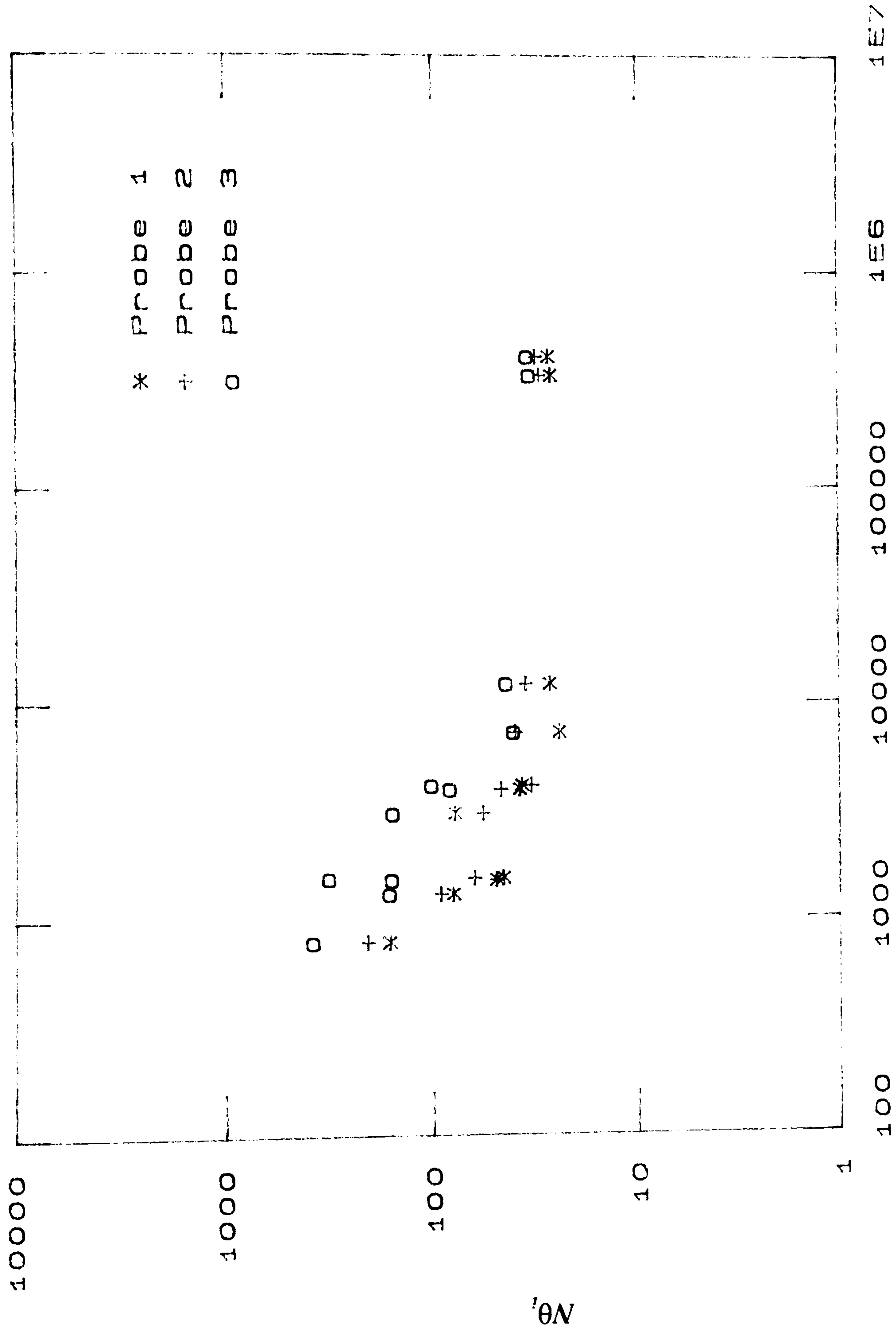
Reynolds number

4.5 Plot of Individual Probe  $N\theta$  versus Reynolds number  
Newtonian data - T/3 Flat blade turbine



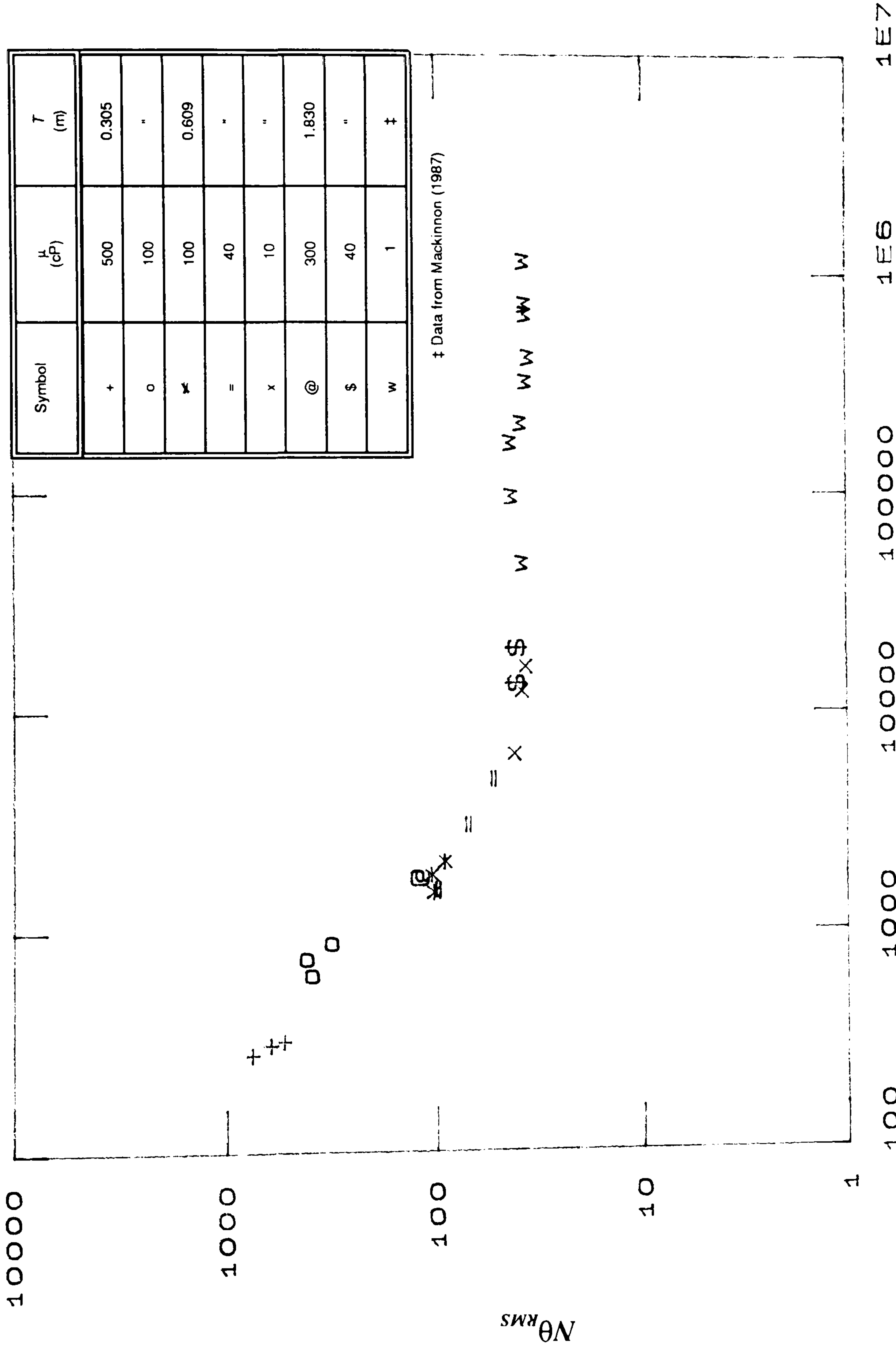
Reynolds number

Plot of Individual Probe  $N\theta$  versus Reynolds number  
Newtonian data - T/2 Lightnin A310 Hydrofoil



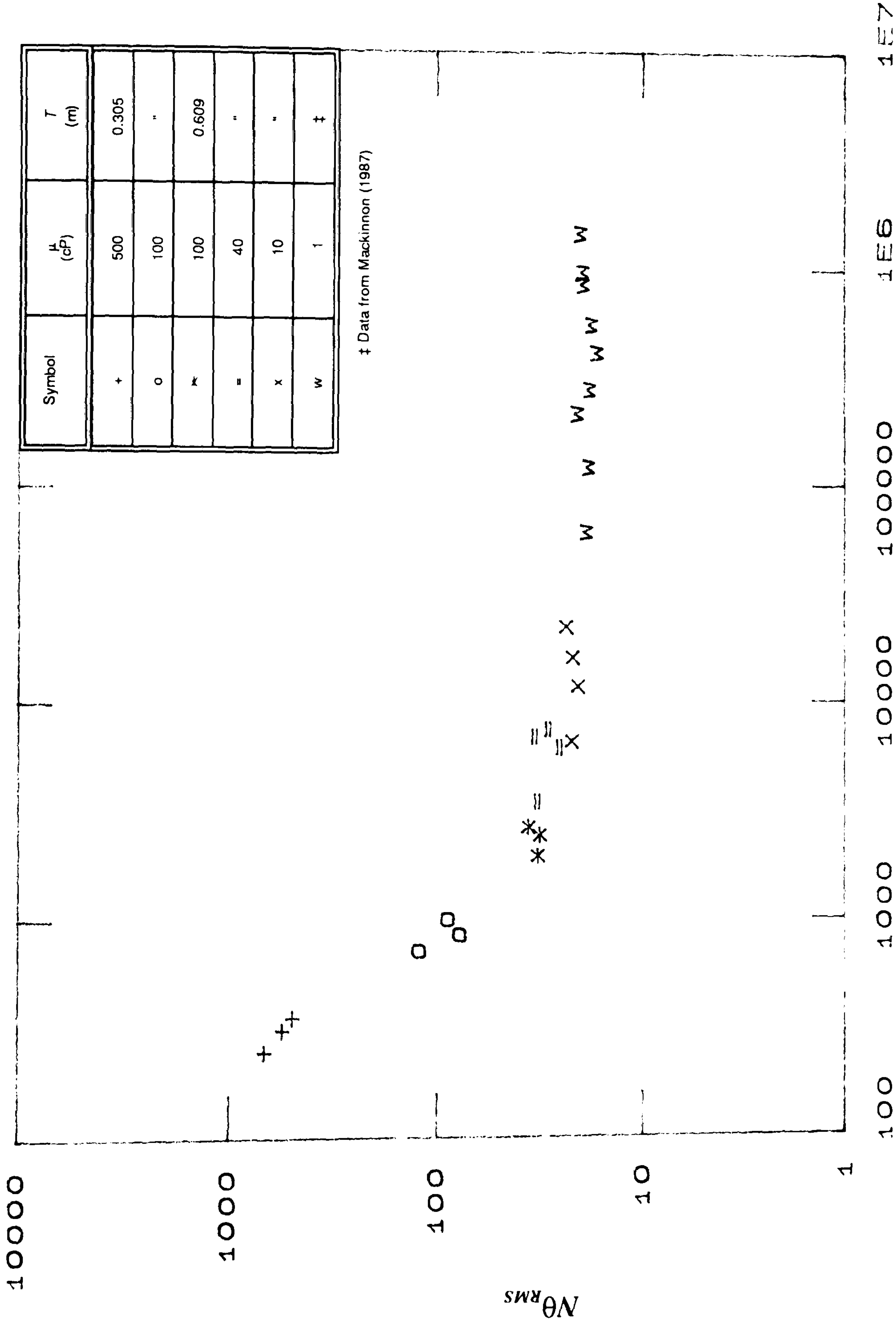
4.7

Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Newtonian data -  $T/3$  Pitched blade turbine



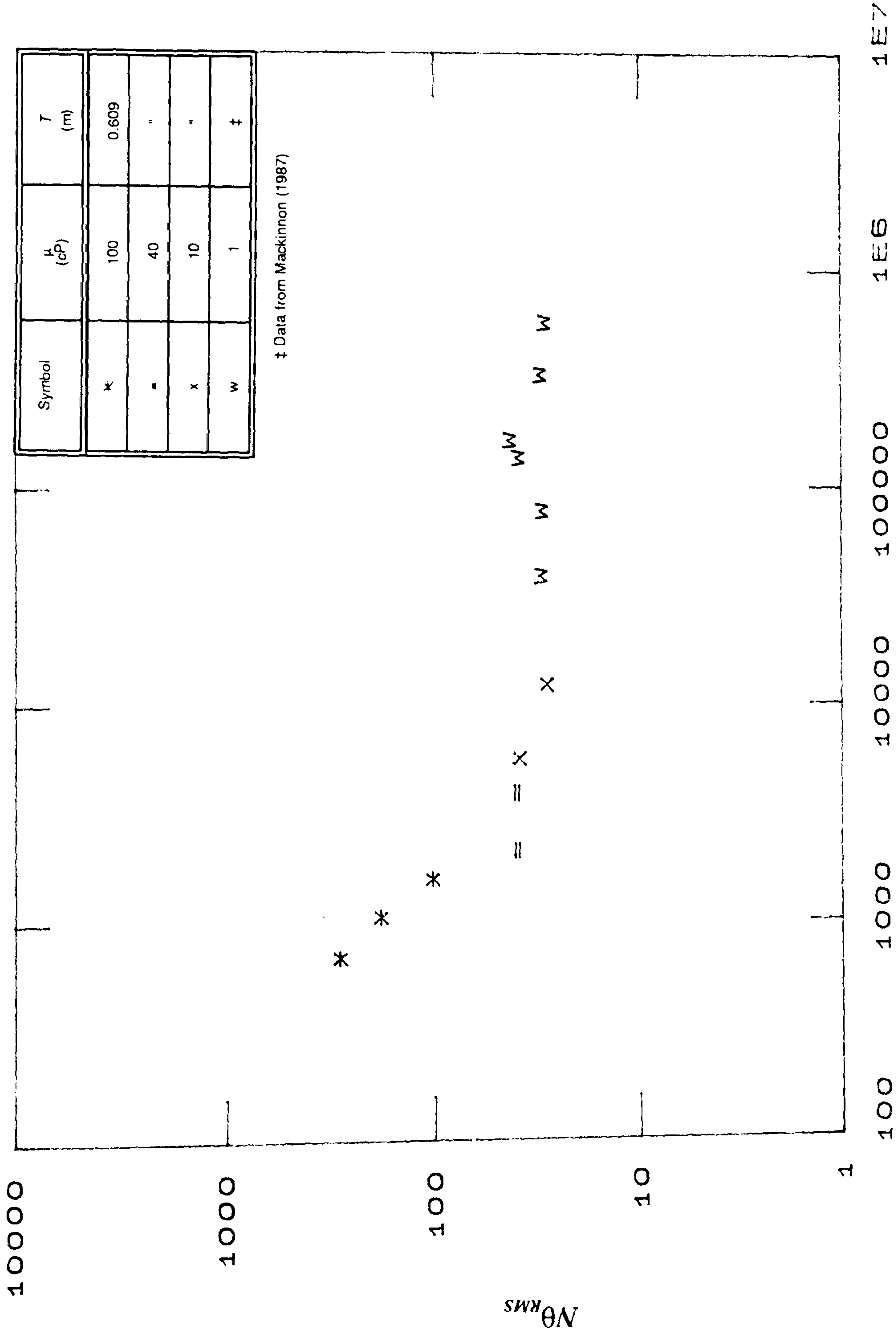
Reynolds number

4.8 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Newtonian data -  $T/2$  Pitched blade turbine



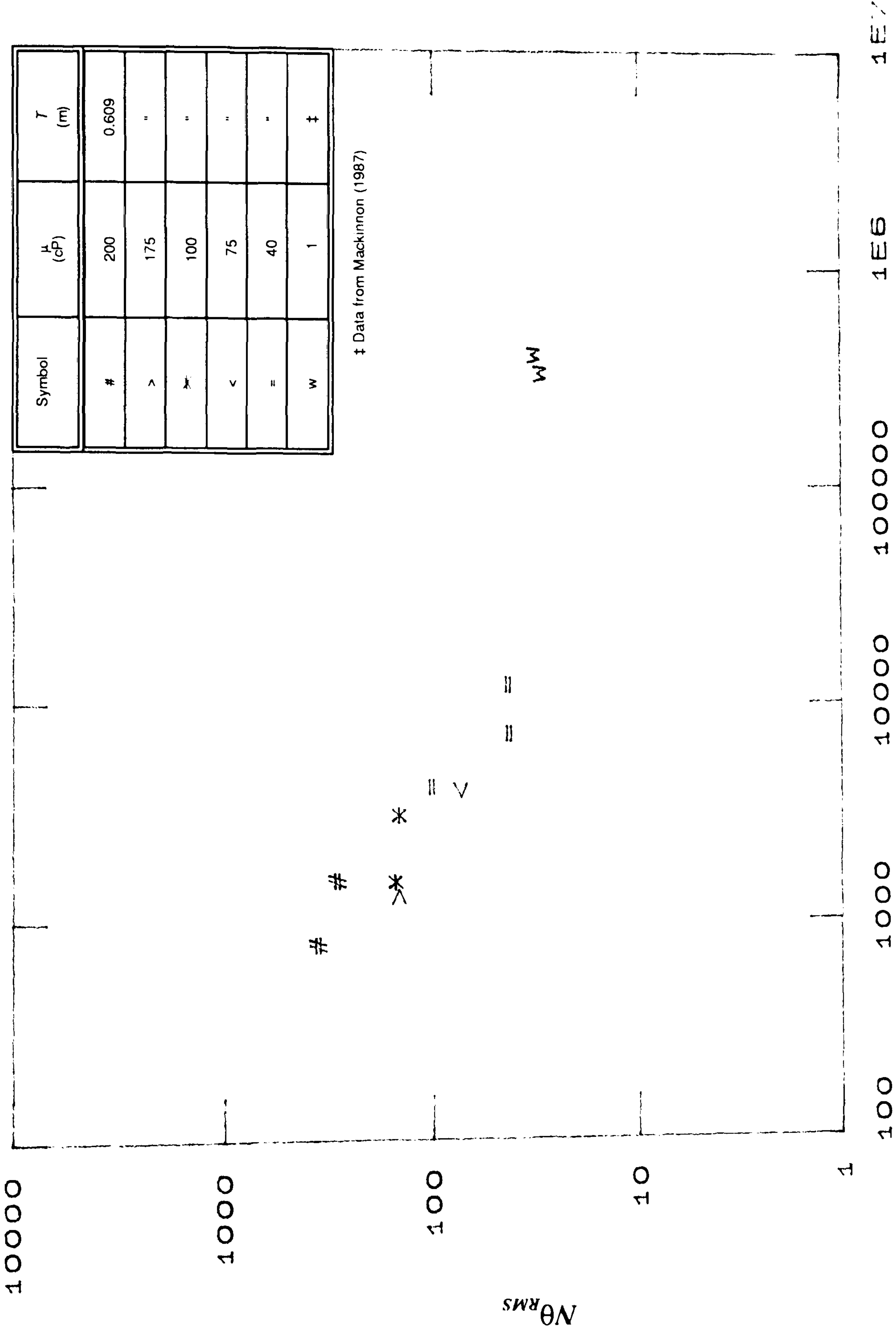
Reynolds number

Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Newtonian data -  $T/3$  Flat blade turbine



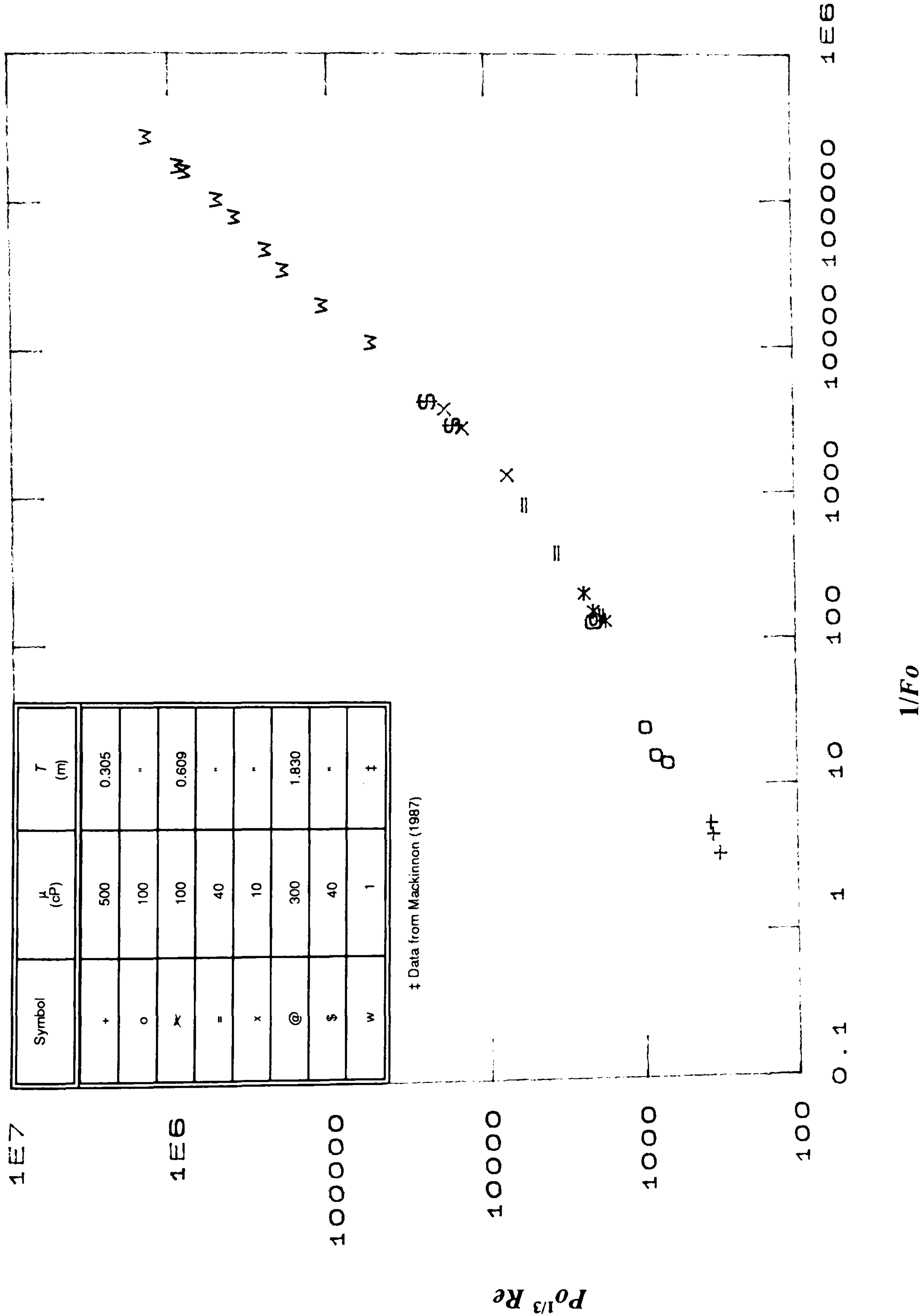
Reynolds number

4.10 Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Newtonian data - T/2 Lightnin A310 Hydrofoil

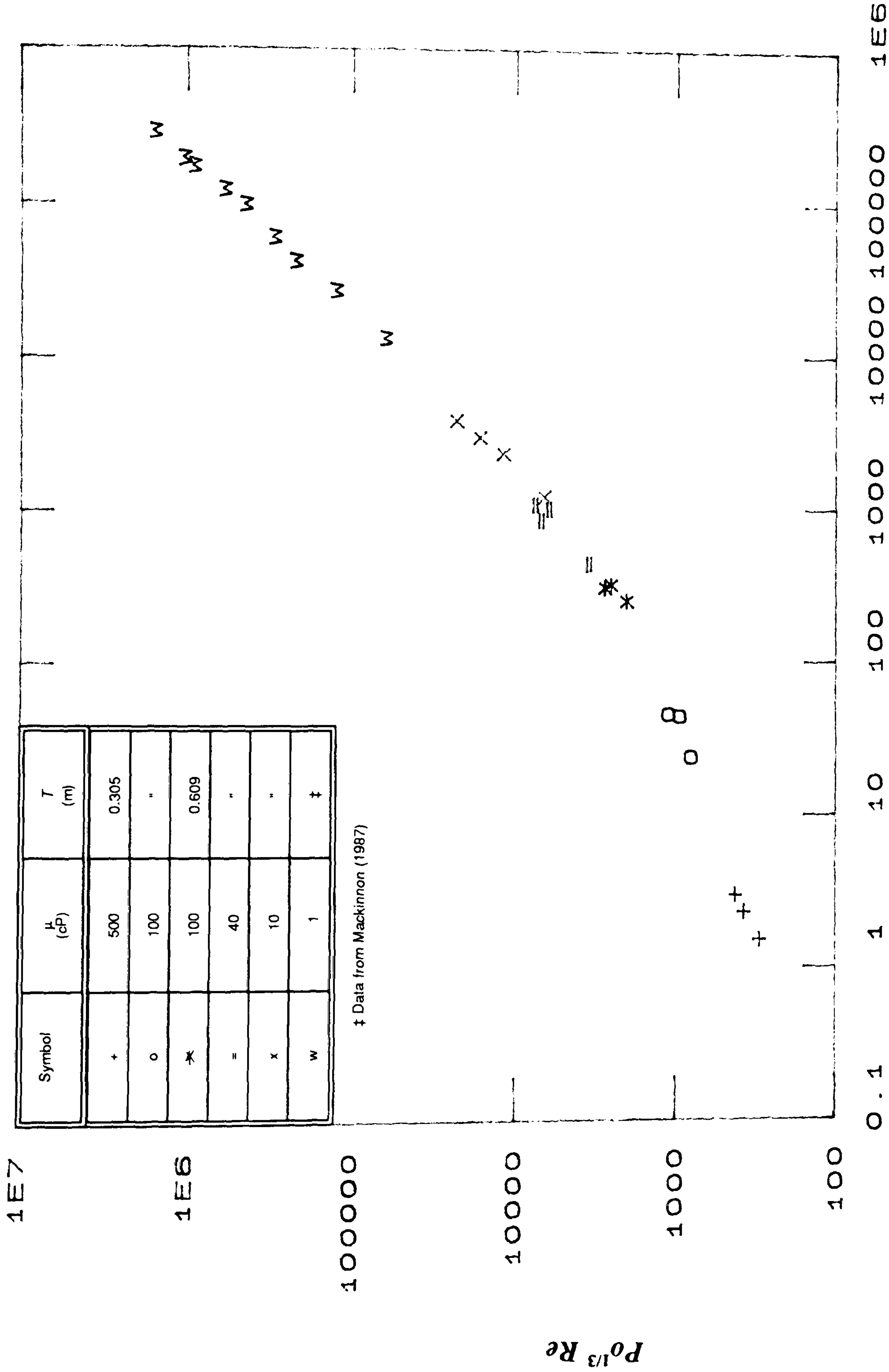


Reynolds number

4.11 Plot of Blend Time Correlation  
Newtonian data - T/3 Pitched blade turbine

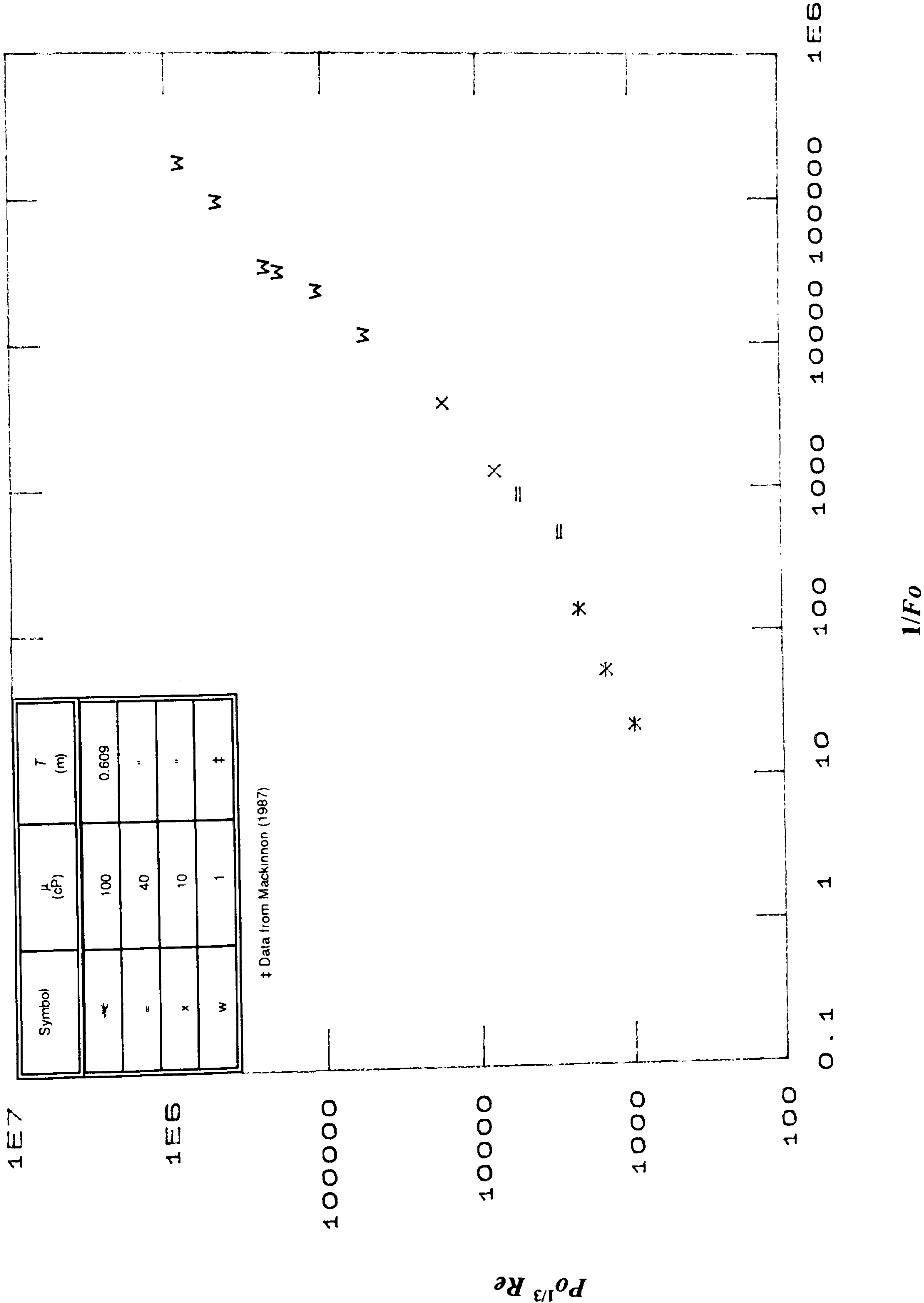


4.12 Plot of Blend Time Correlation  
Newtonian data -  $T/2$  Pitched blade turbine



4.13

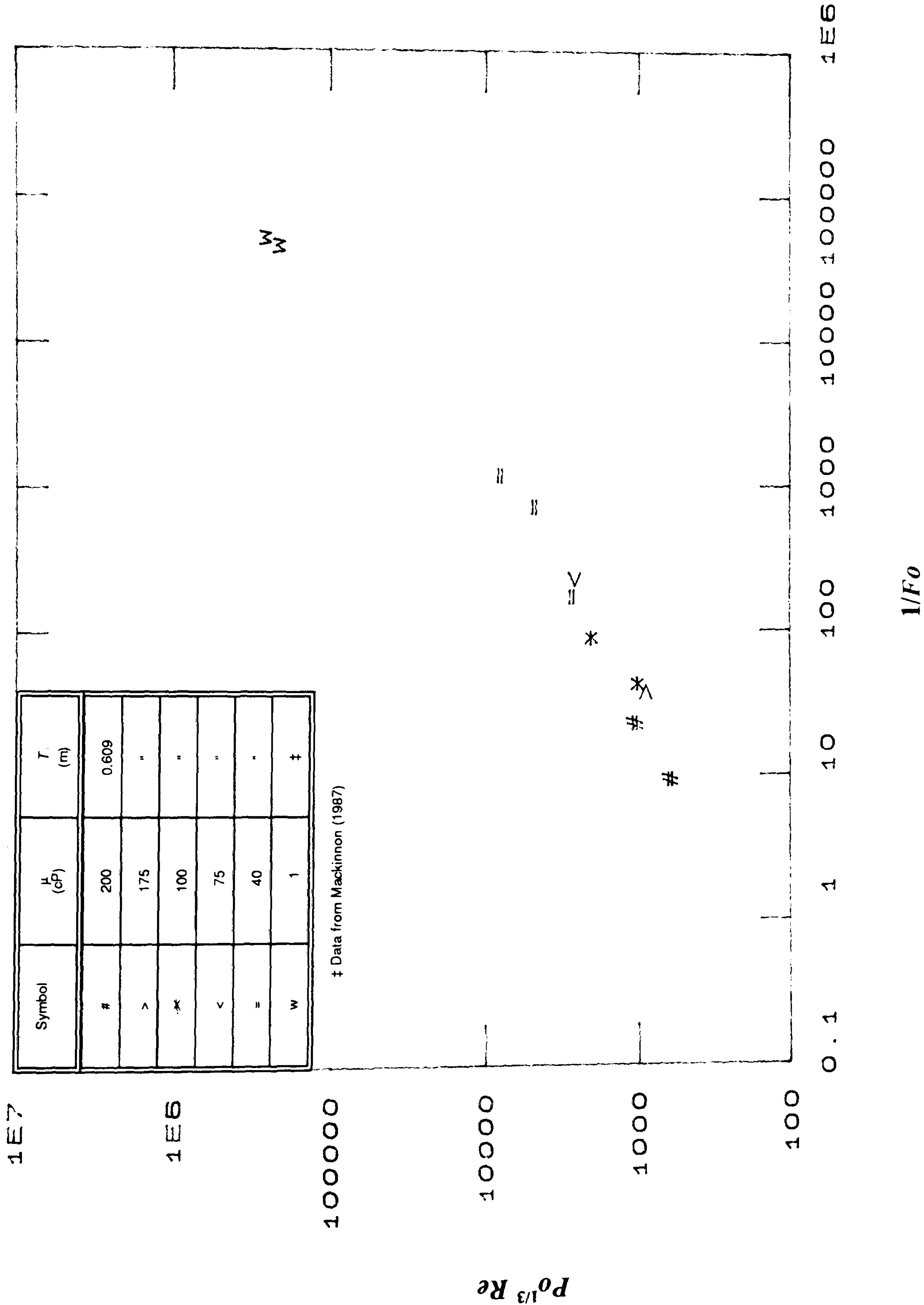
Plot of Blend Time Correlation  
 Newtonian data -  $T/3$  Flat blade turbine

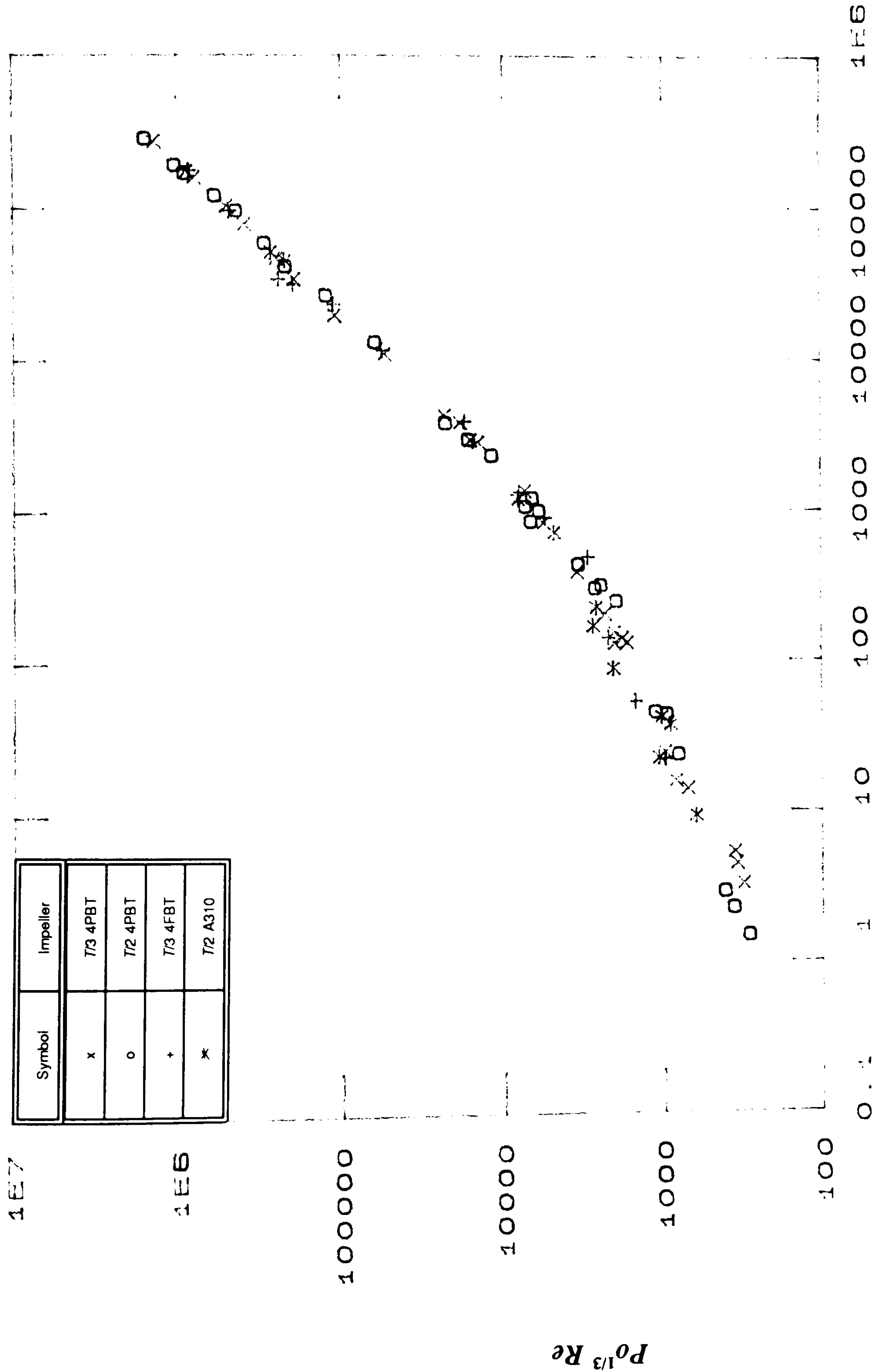


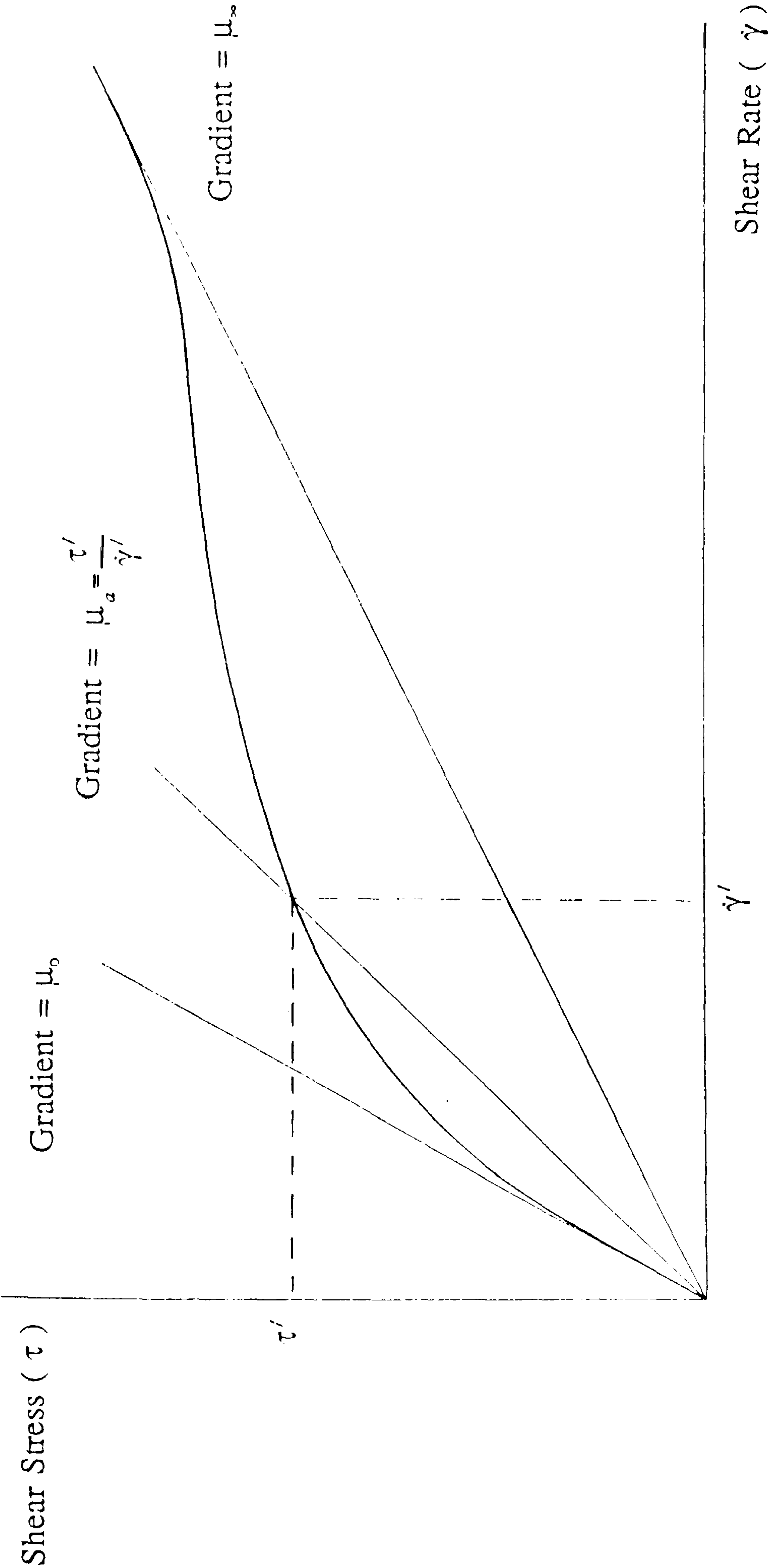
4.14

Plot of Blend Time Correlation

Newtonian data - T/2 Lightnin A310 Hydrofoil







$\mu_a$  is the apparent viscosity of the pseudoplastic fluid at a shear rate of  $\dot{\gamma}'$

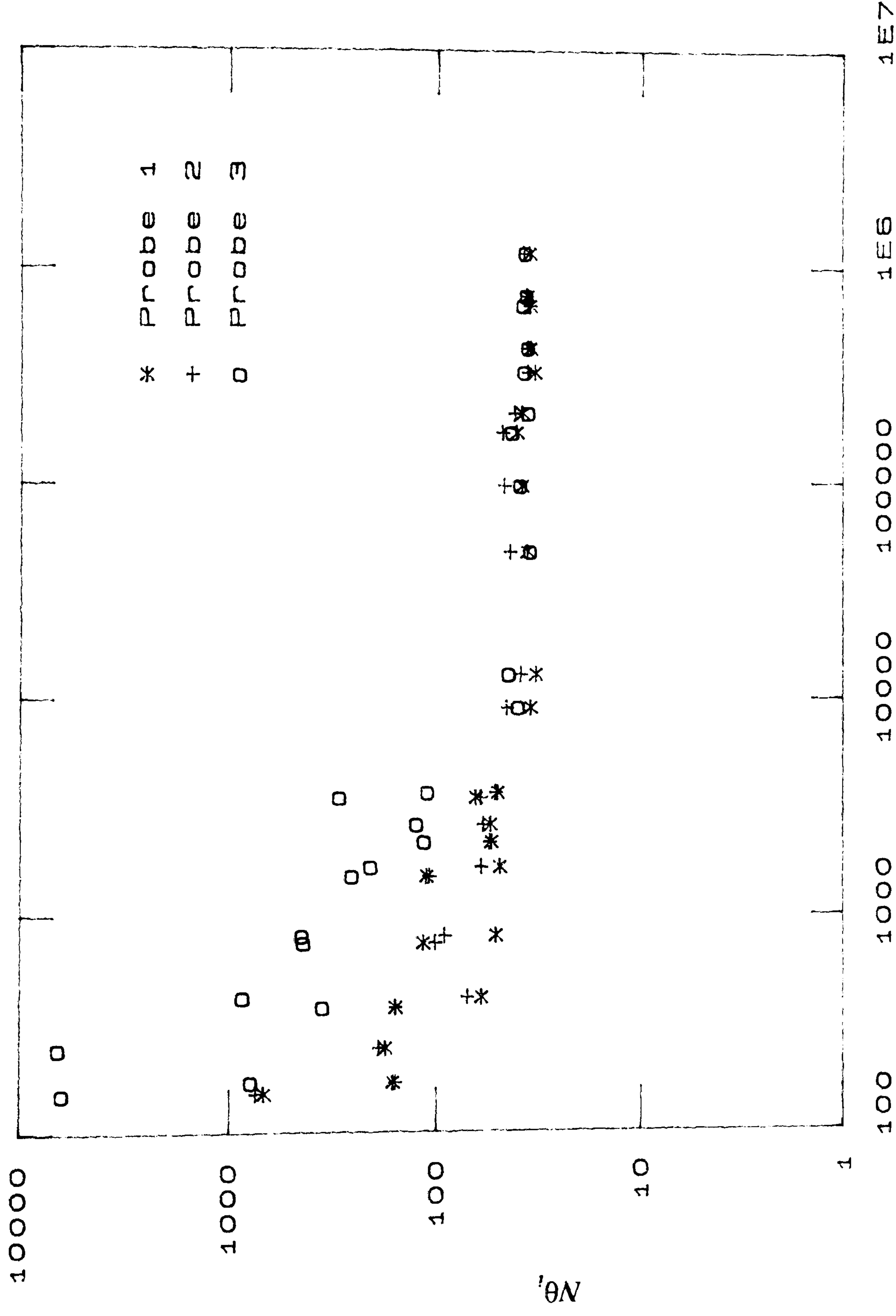
If the power law is used to describe the fluid rheology, then  $\mu_a = K\dot{\gamma}'^{(n-1)}$

### 5.1 Plot of Shear Stress versus Shear Rate for a Pseudo-plastic Fluid

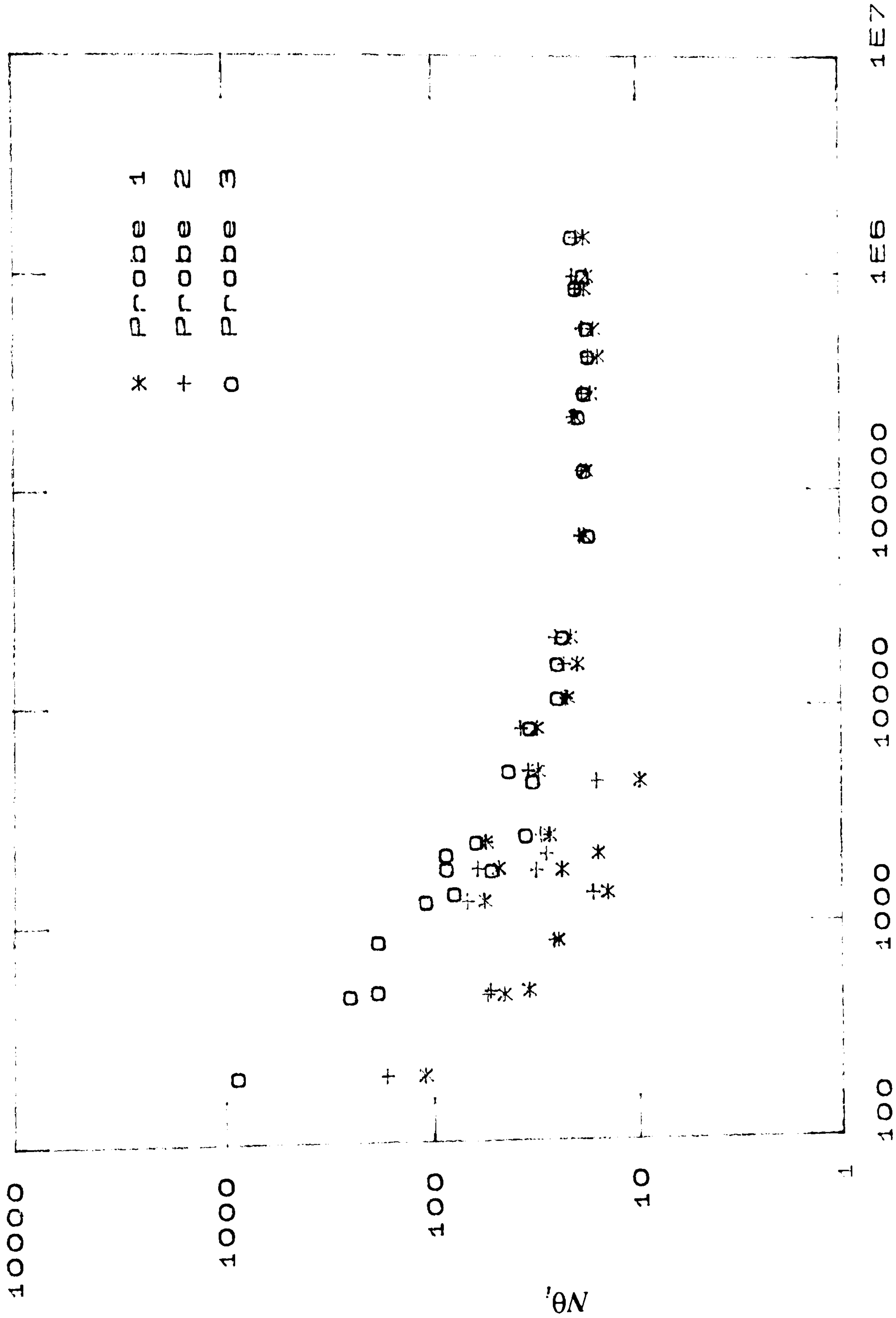


### 5.3

Plot of Individual Probe  $N\theta$  versus Reynolds number  
Pseudo-plastic data - T/3 Pitched blade turbine



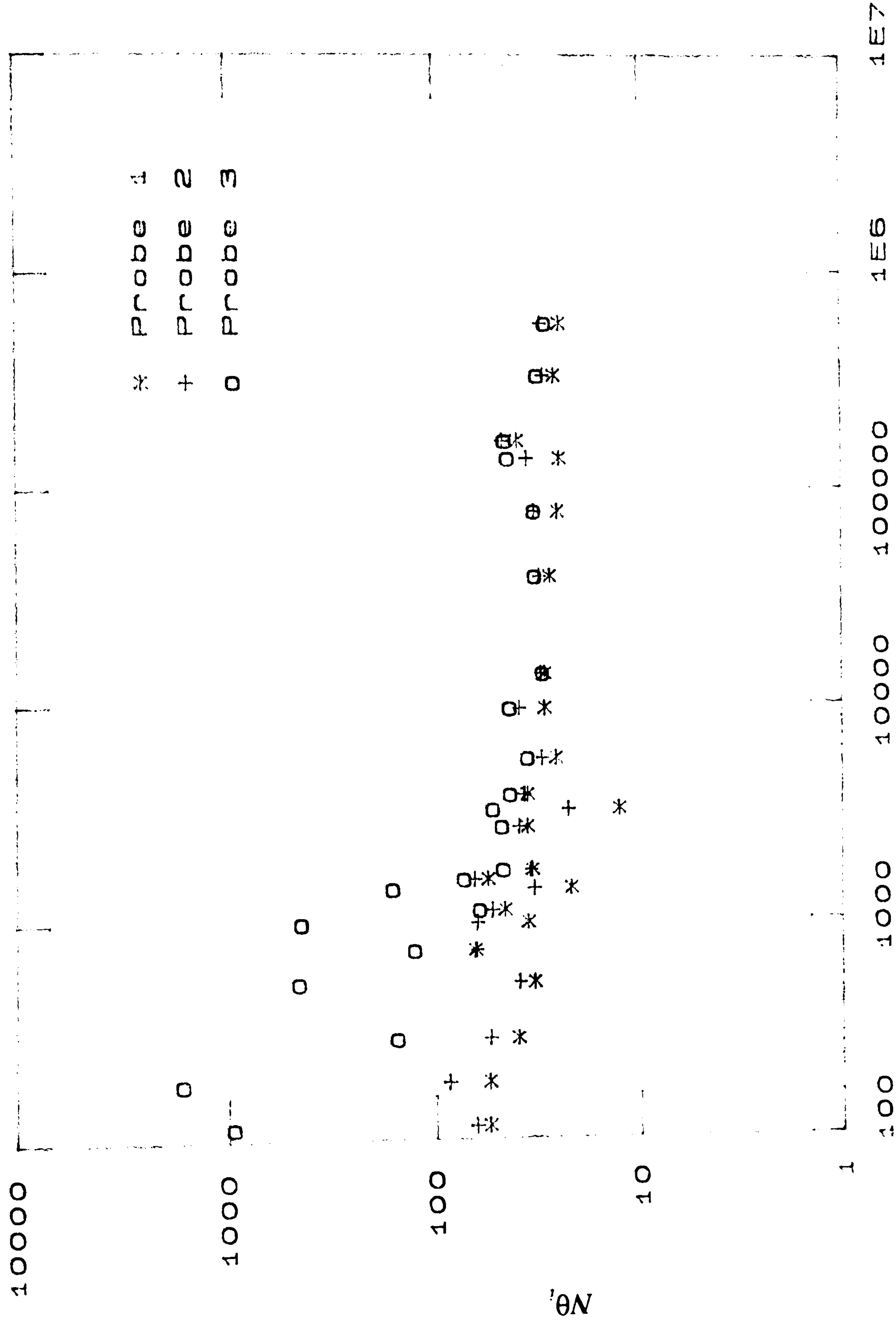
Plot of Individual Probe  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/2$  Pitched blade turbine



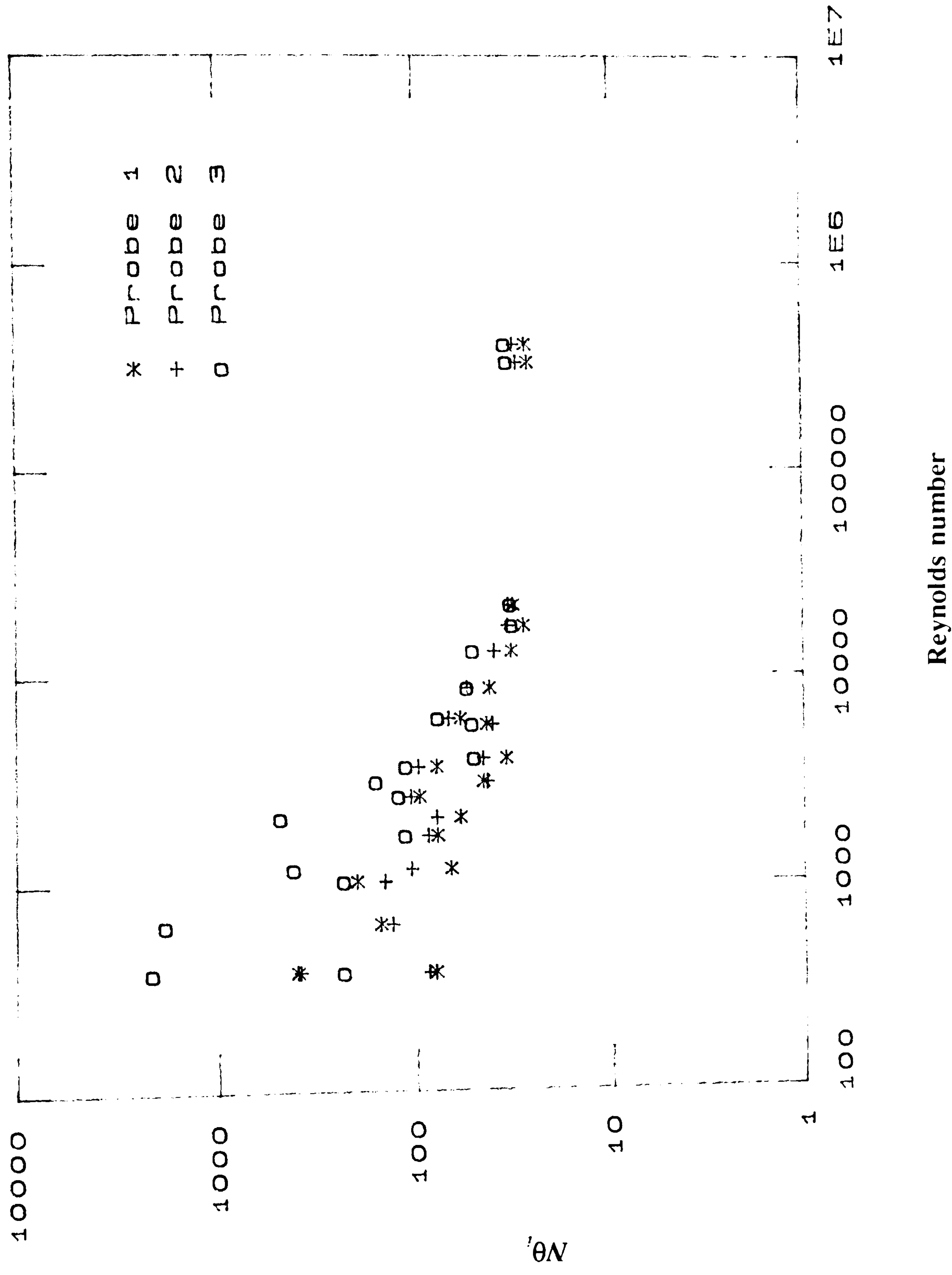
Reynolds number

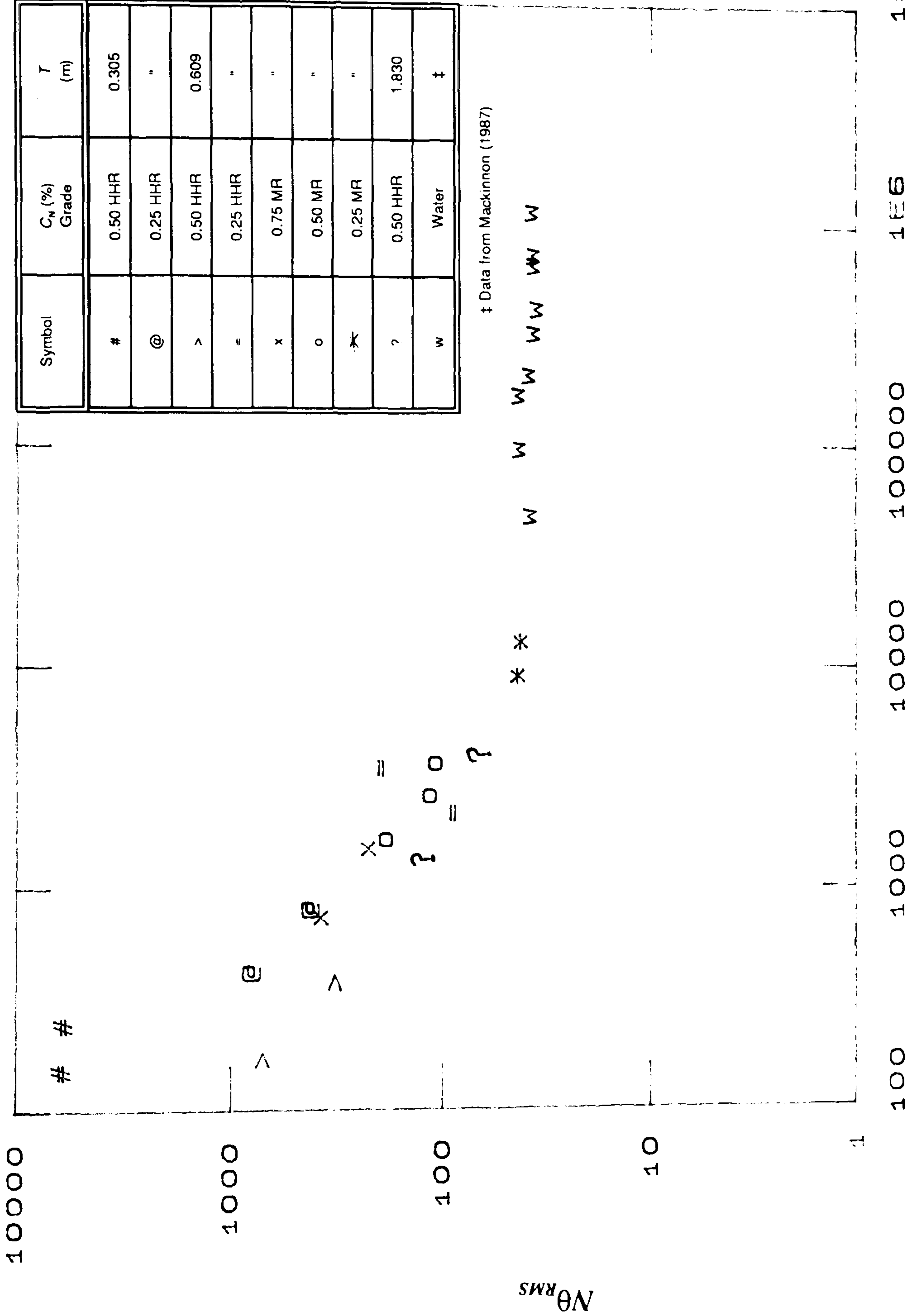
# 5.5

Plot of Individual Probe  $N\theta$  versus Reynolds number  
Pseudo-plastic data - T/3 Flat blade turbine



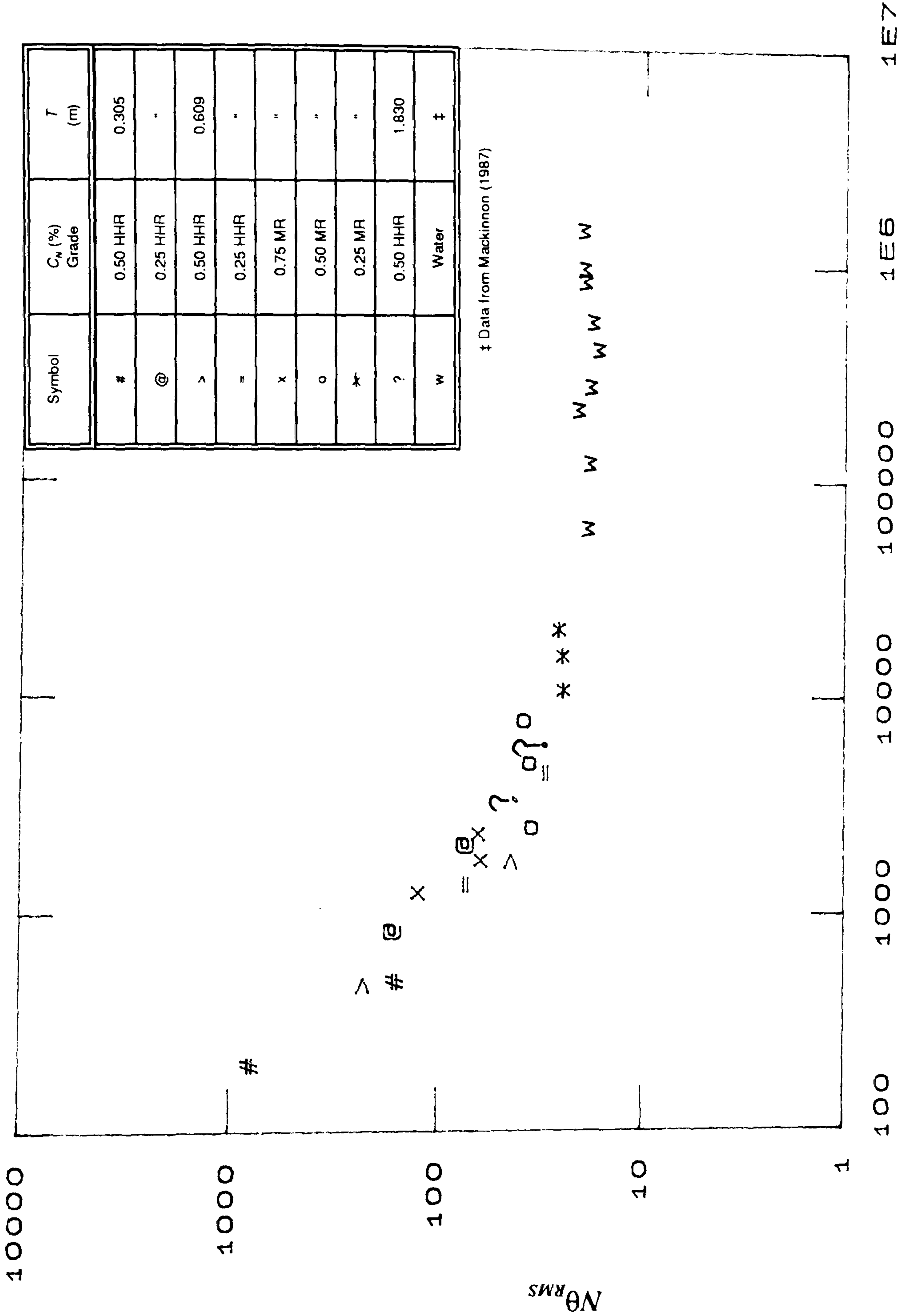
Reynolds number



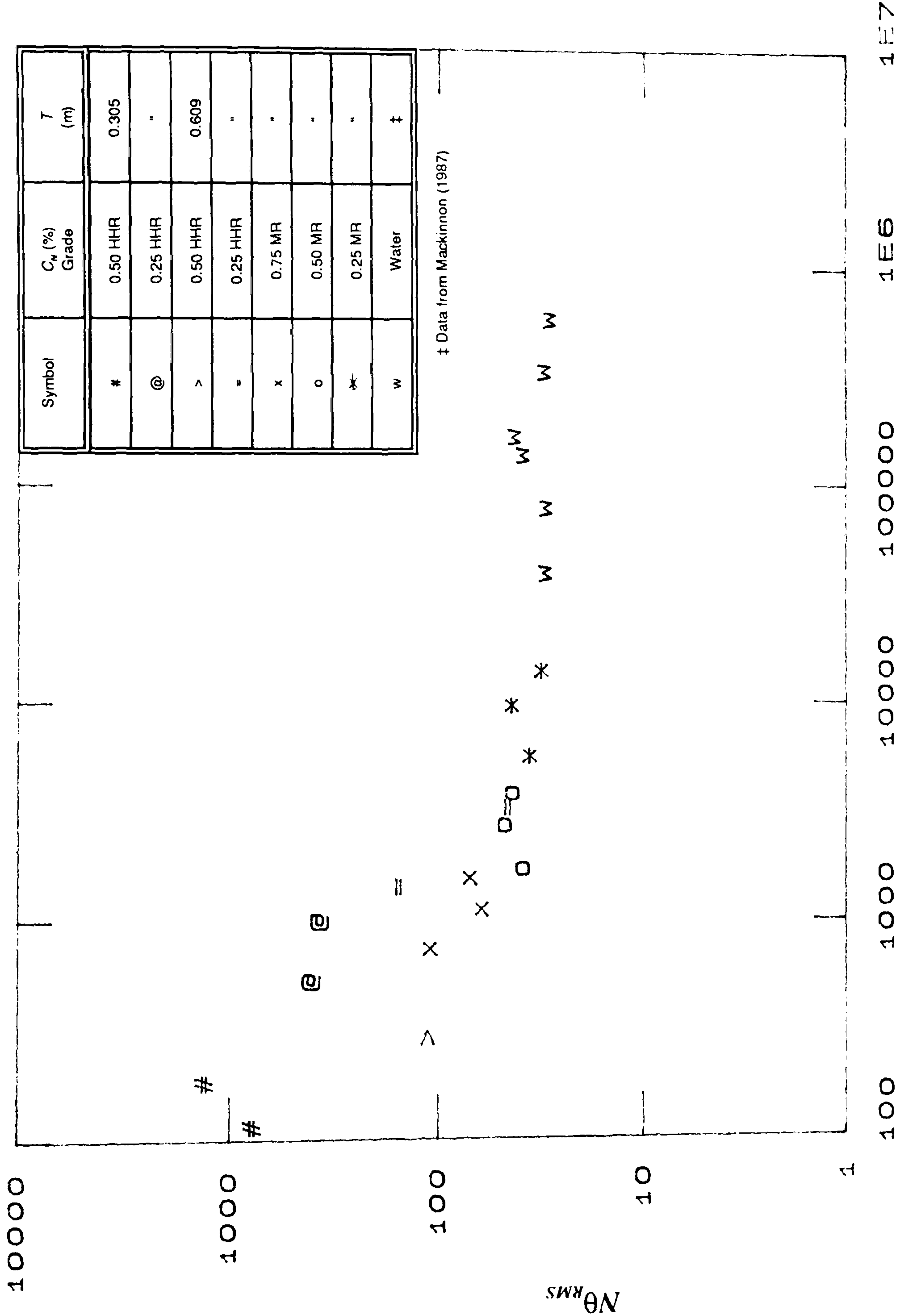


Reynolds number

Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data -  $T/2$  Pitched blade turbine



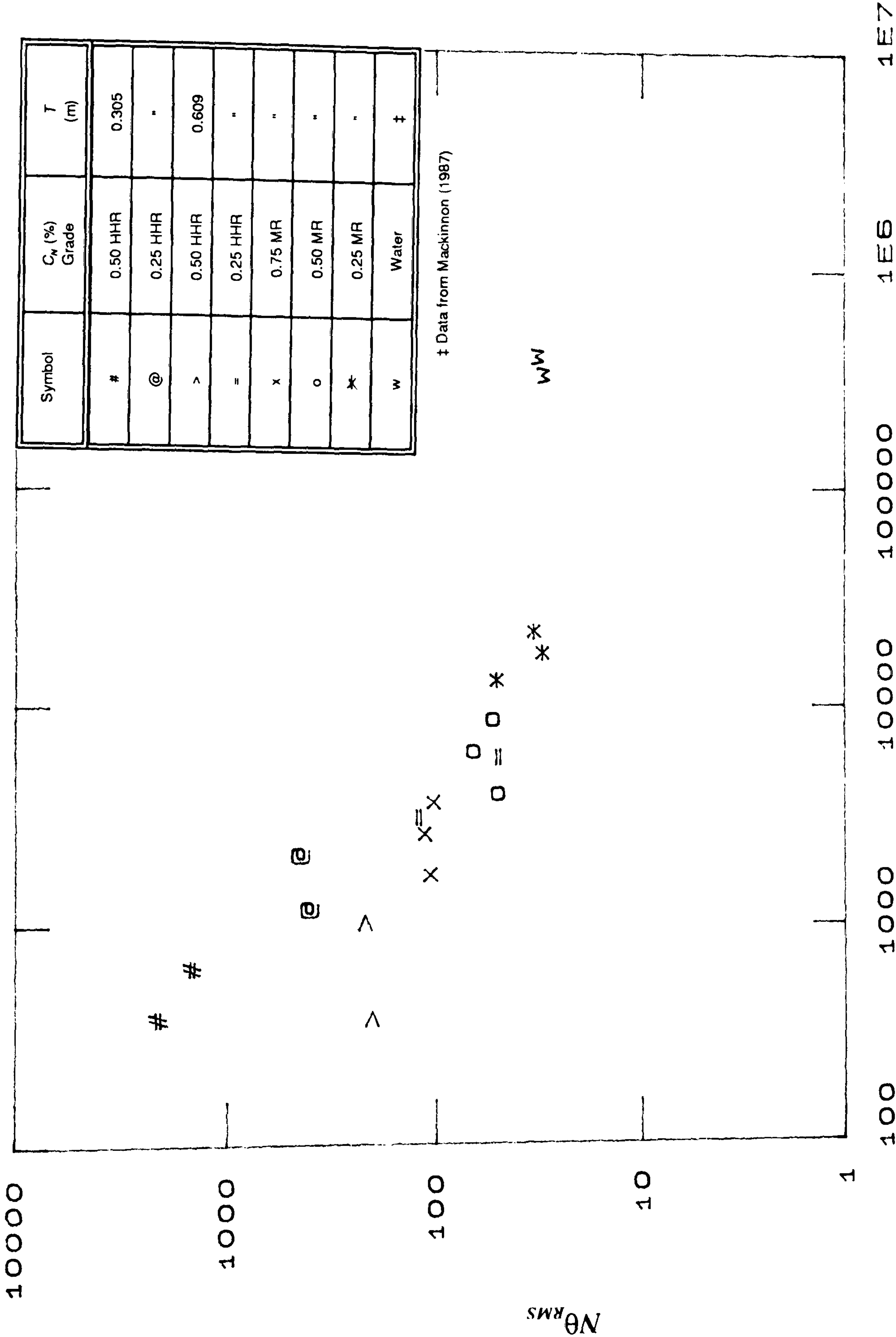
Reynolds number



Reynolds number

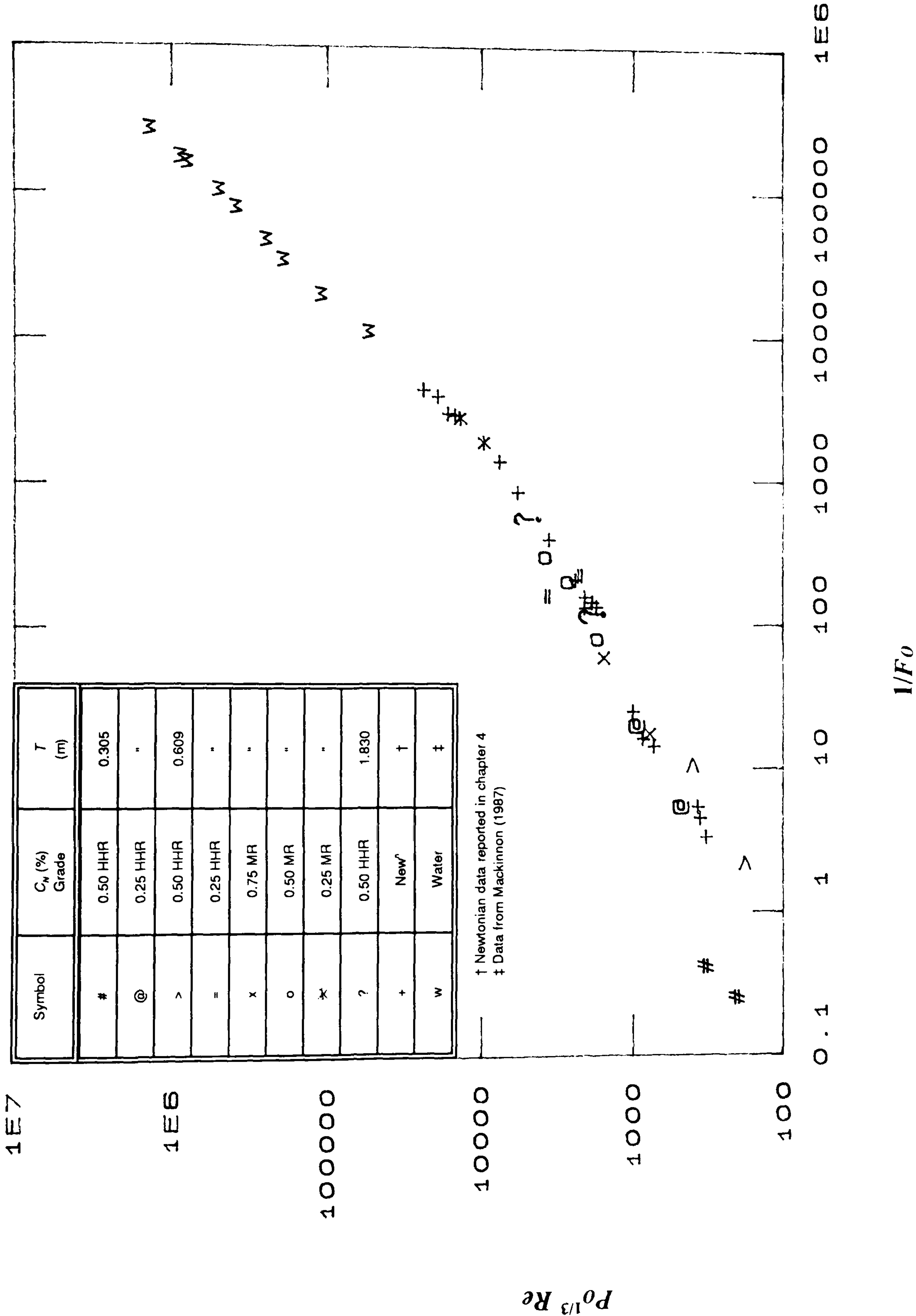
5.10

Plot of Root Mean Square  $N\theta$  versus Reynolds number  
Pseudo-plastic data - T/2 Lightnin A310 & Chemineer HE3 Hydrofoils

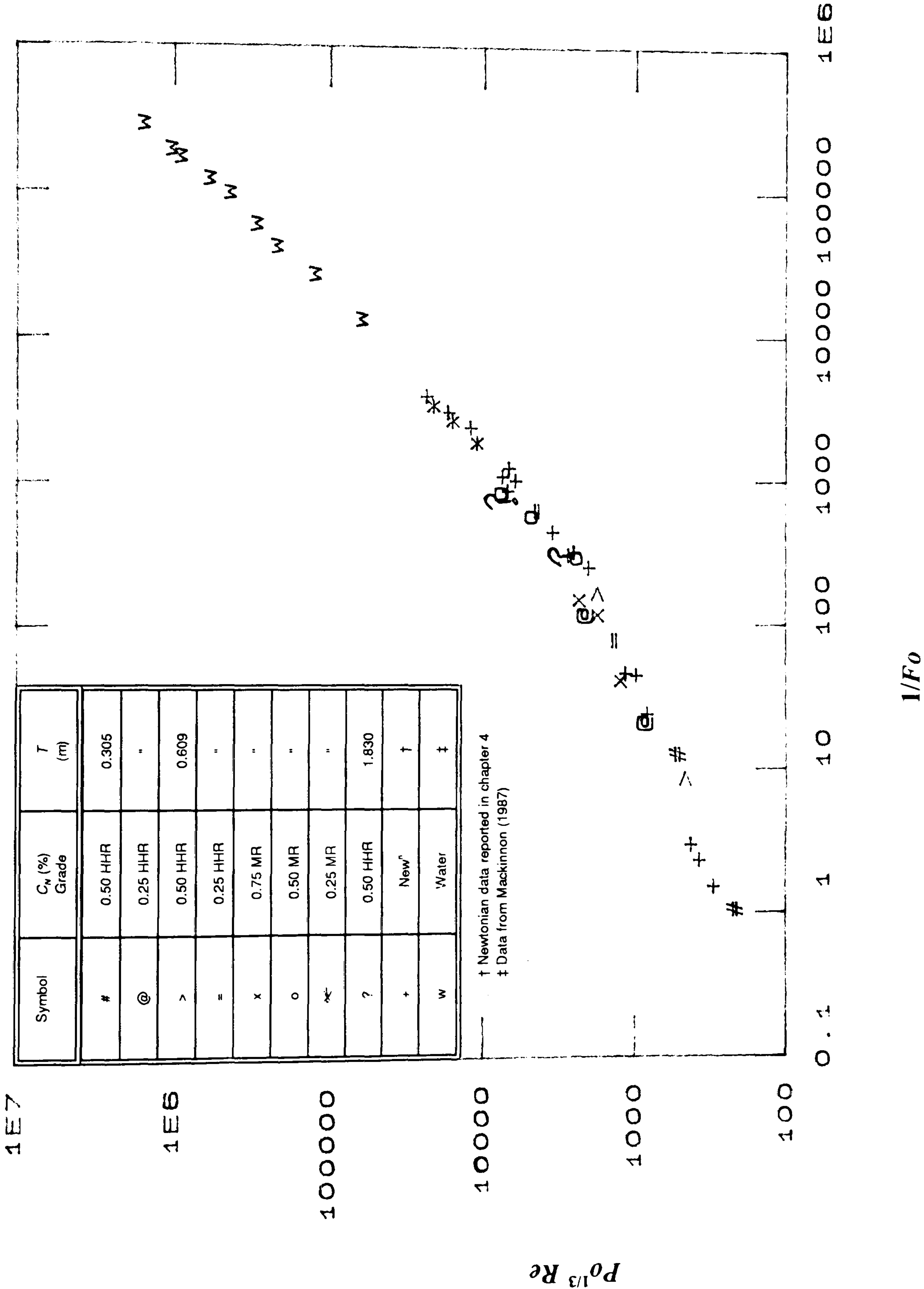


Reynolds number

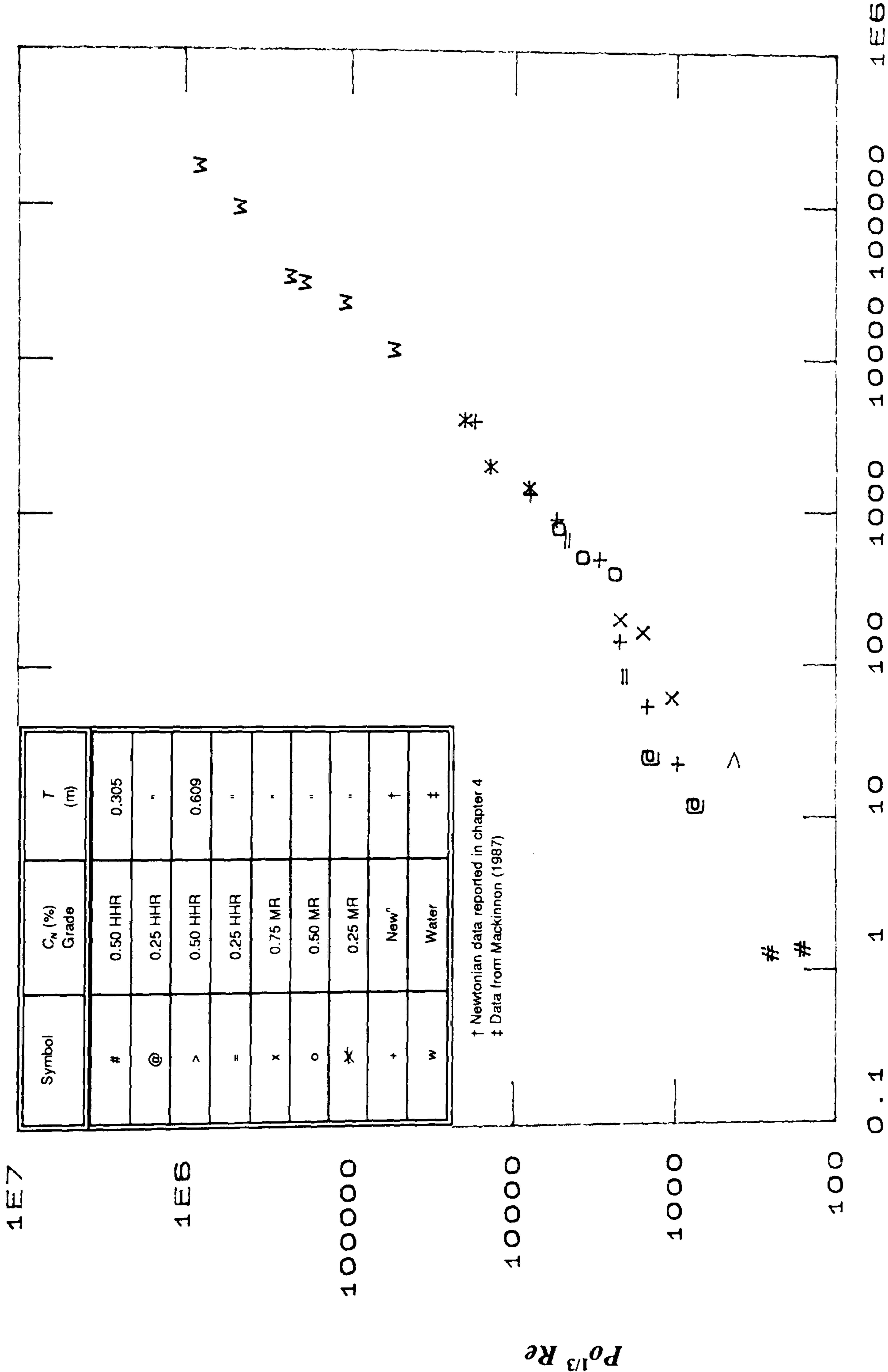
5.11
Plot of Blend Time Correlation
All data - T/3 Pitched blade turbine



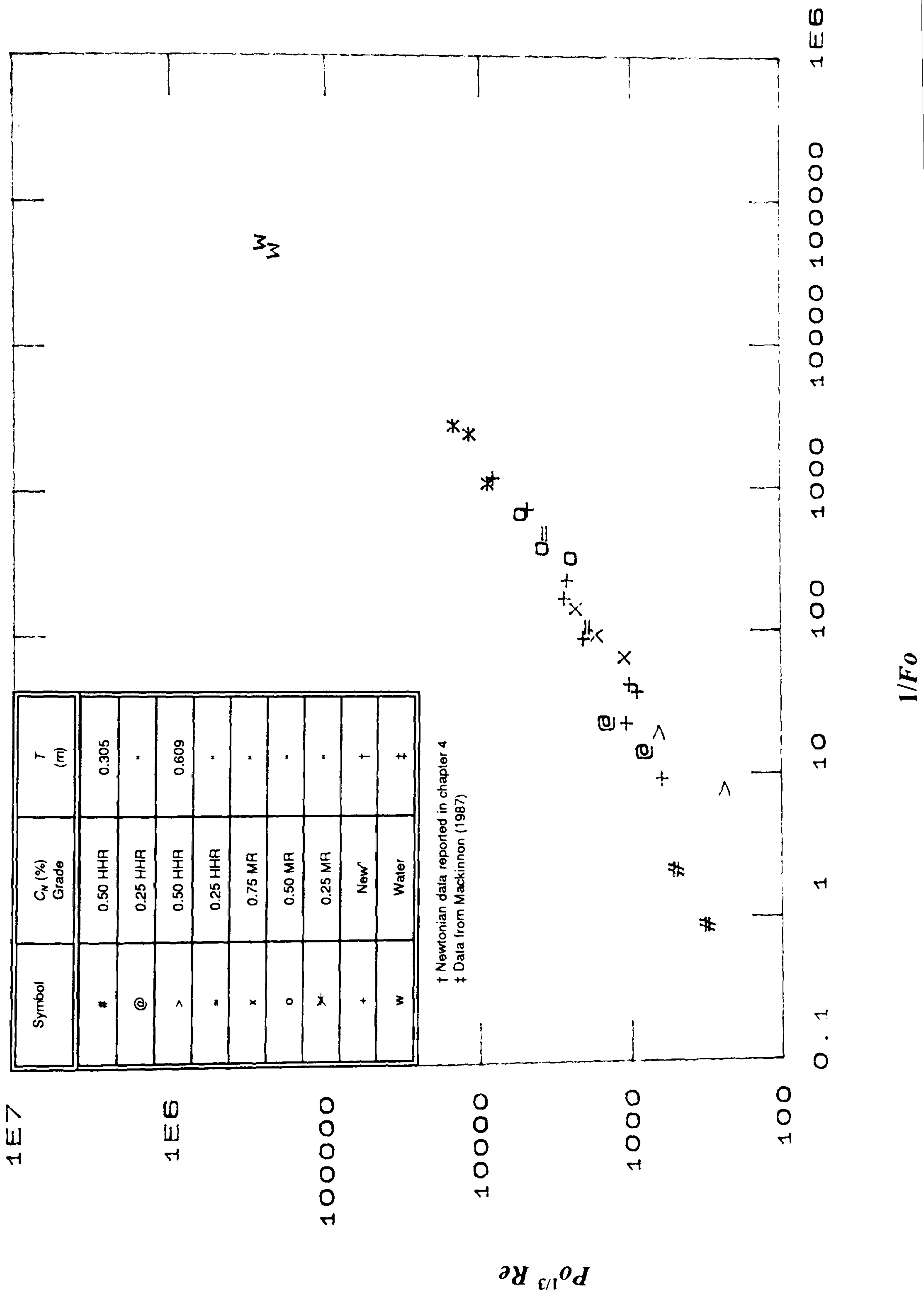
5.12 Plot of Blend Time Correlation  
All data -  $T/2$  Pitched blade turbine



5.13 Plot of Blend Time Correlation  
All data -  $T/3$  Flat blade turbine

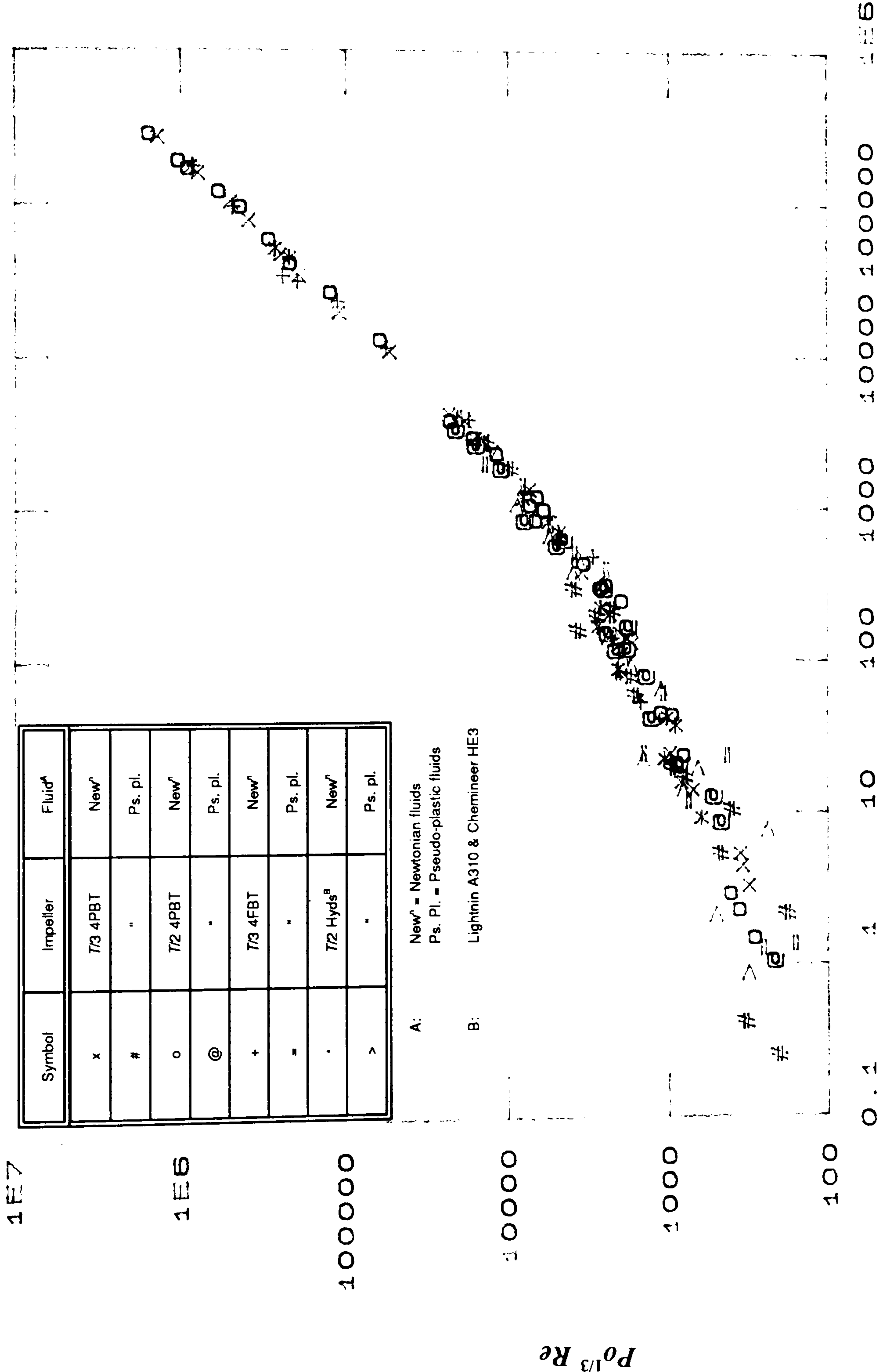


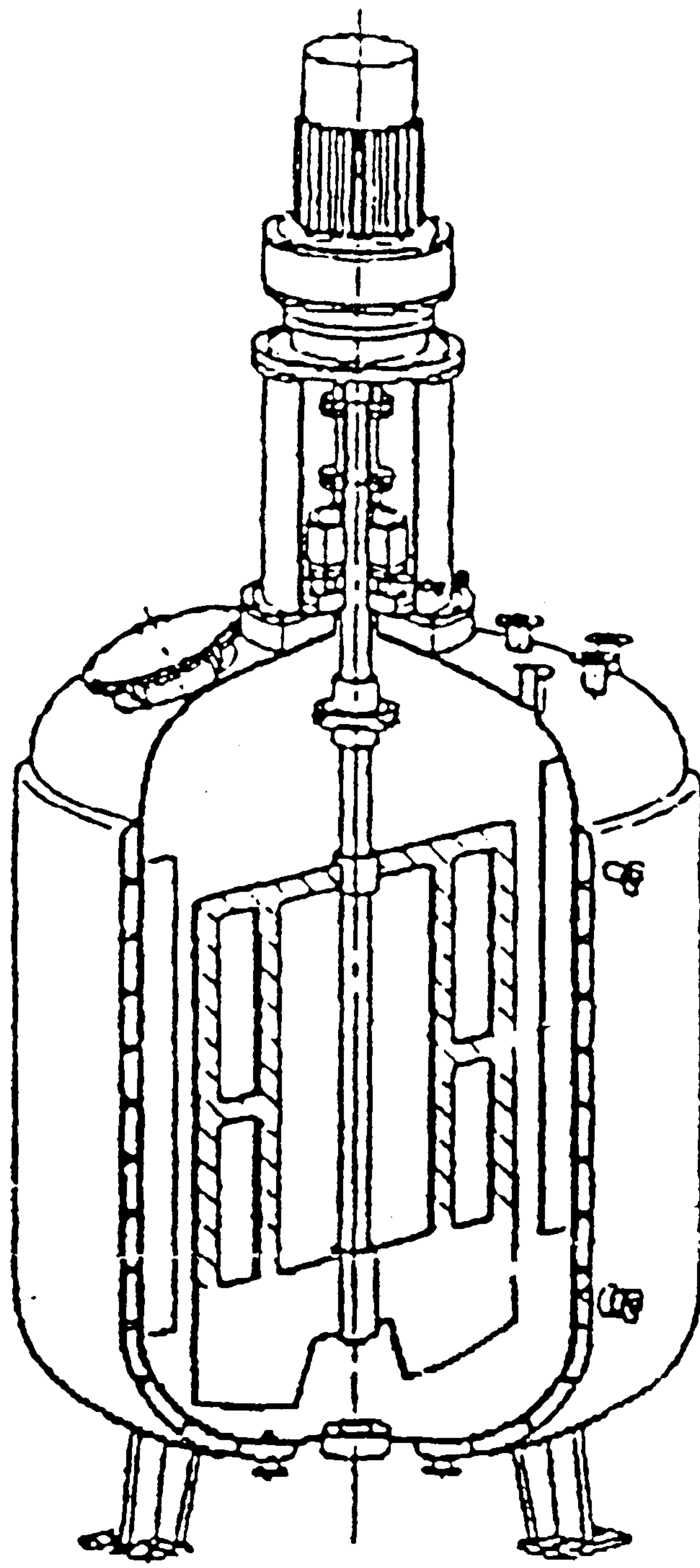
5.14 Plot of Blend Time Correlation  
All data - T/2 Lightnin A310 & Chemineer HE3 Hydrofoils



5.15

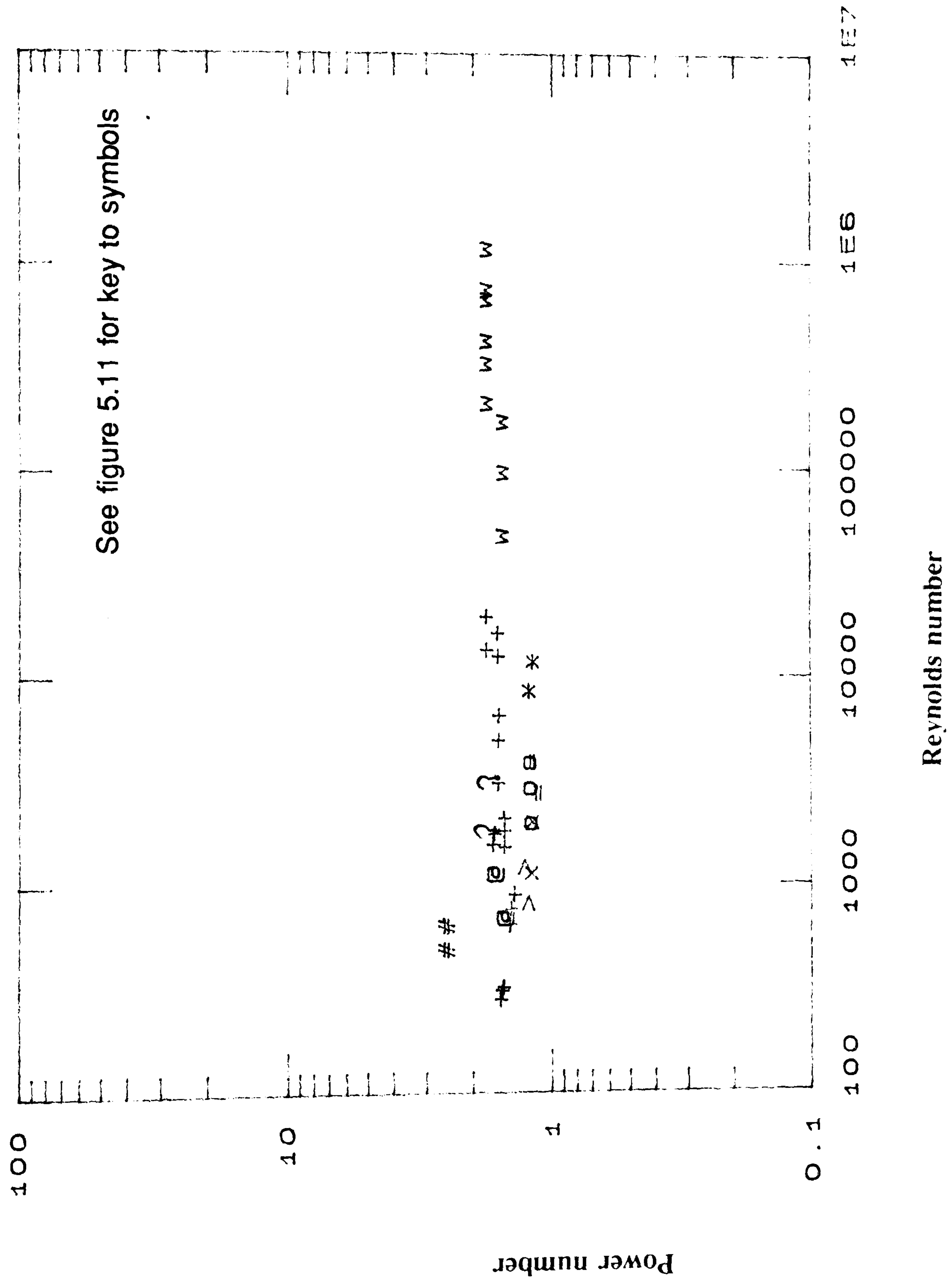
Plot of Blend Time Correlation  
Newtonian & Pseudo-plastic data - All impellers



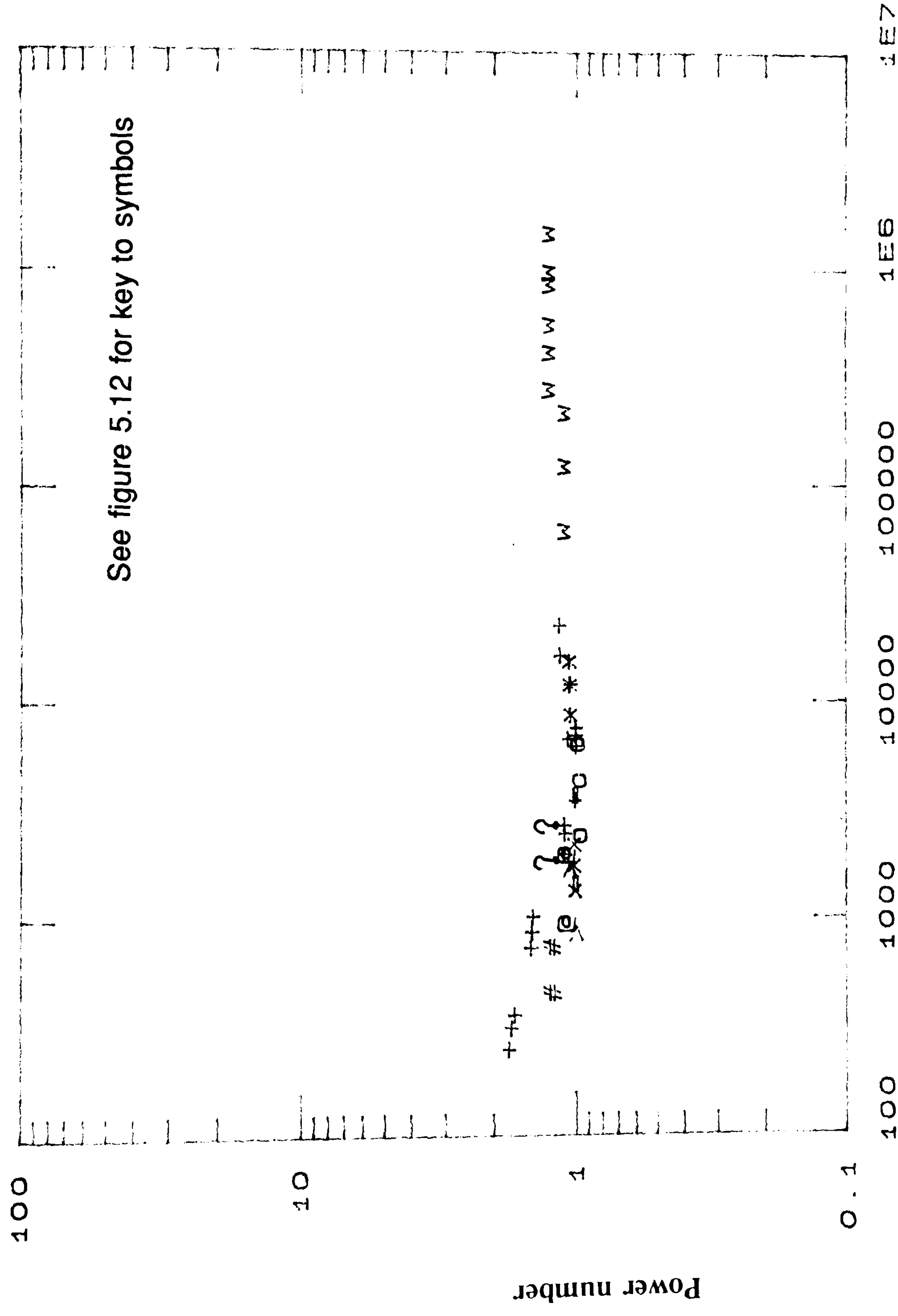


**5.16      “Max-Blend” Impeller by Sumitomo Heavy Industries**

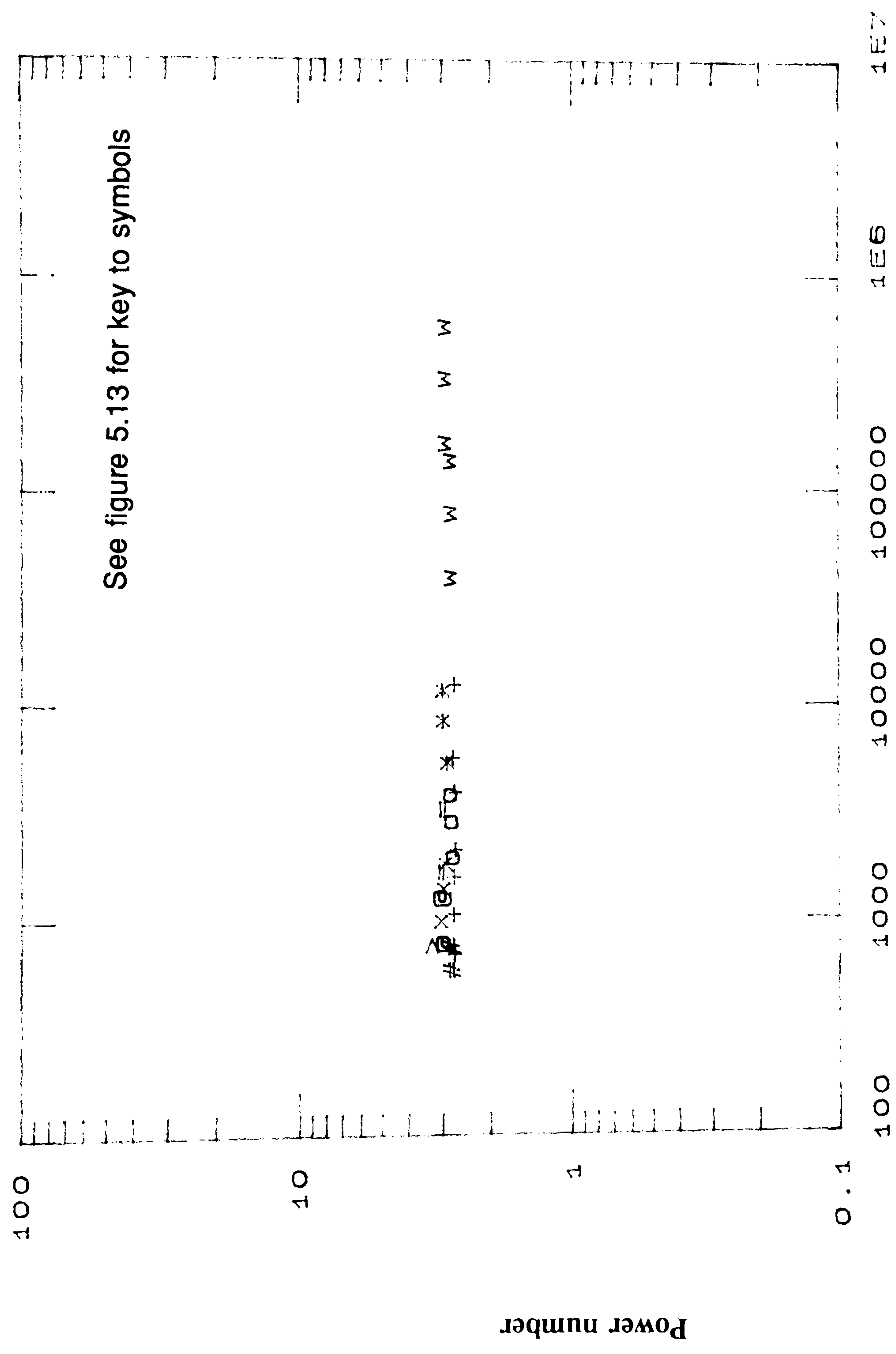
### 6.1 Plot of power number versus Reynolds number All data - $T/3$ Pitched blade turbine



## 6.2 Plot of power number versus Reynolds number



6.3 Plot of power number versus Reynolds number  
All data - T/3 Flat blade turbine

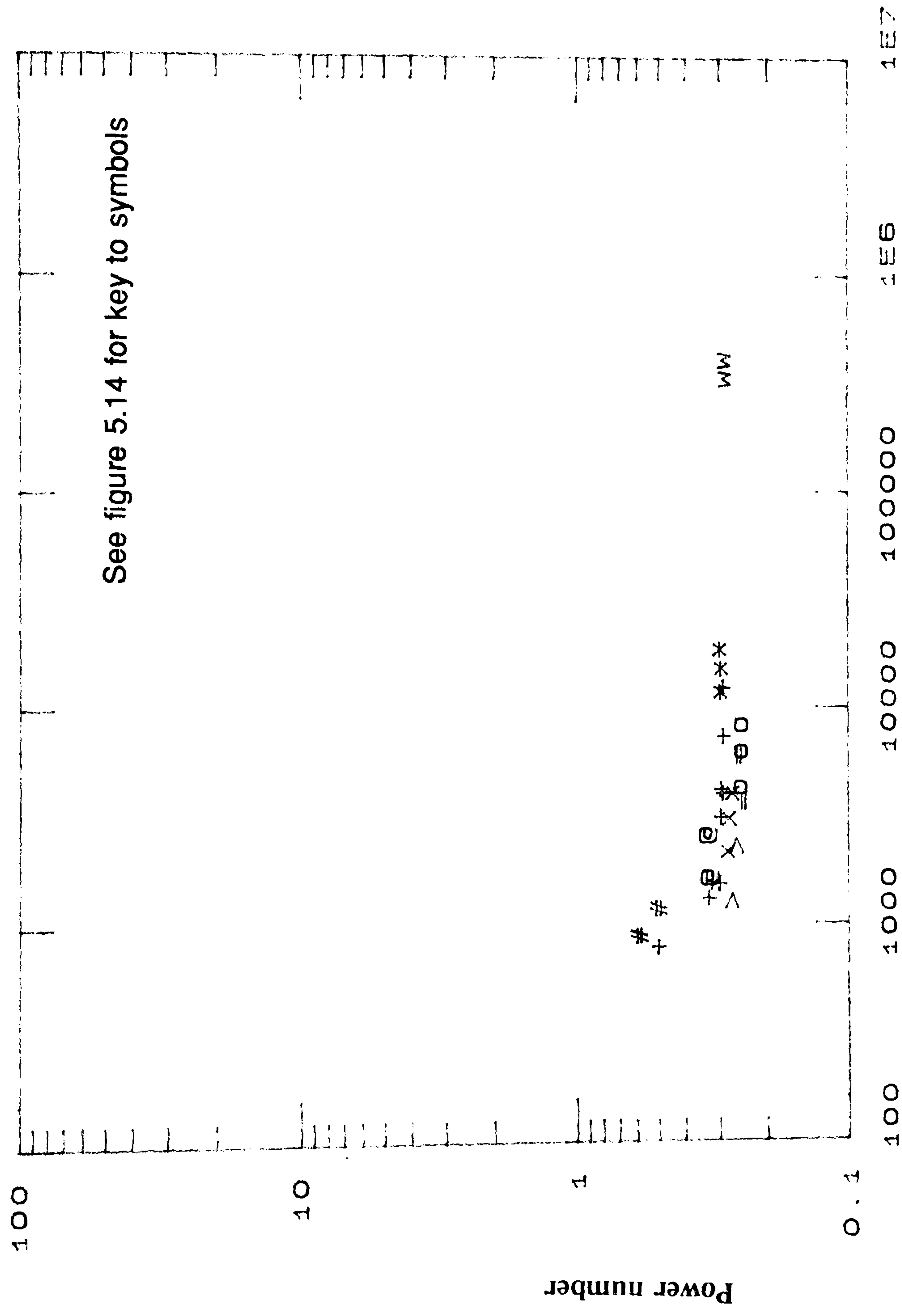


Reynolds number

## 6.4

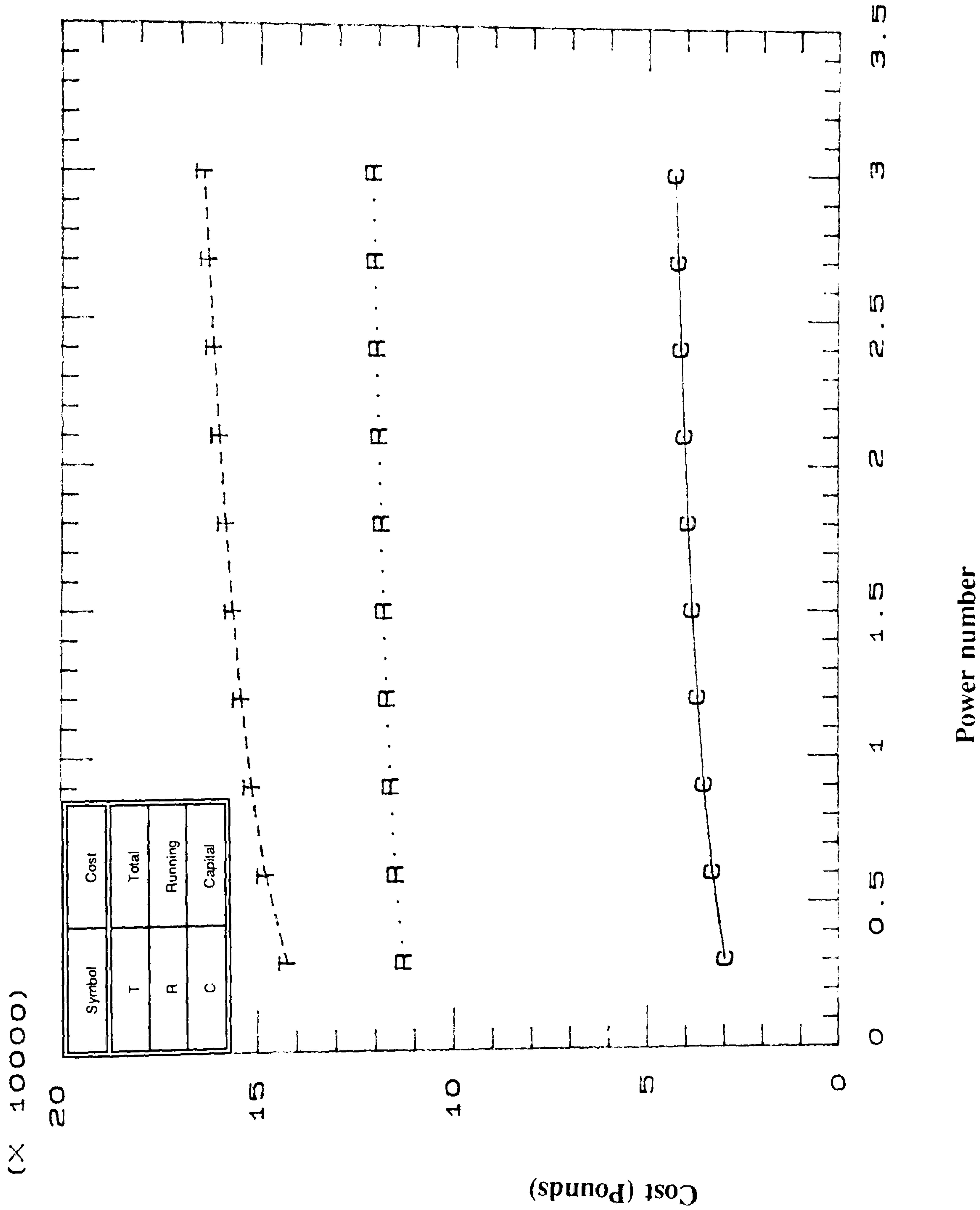
### Plot of power number versus Reynolds number

**All data - T/2 Lightning A310 & Chemineer HE3 Hydrofoils**

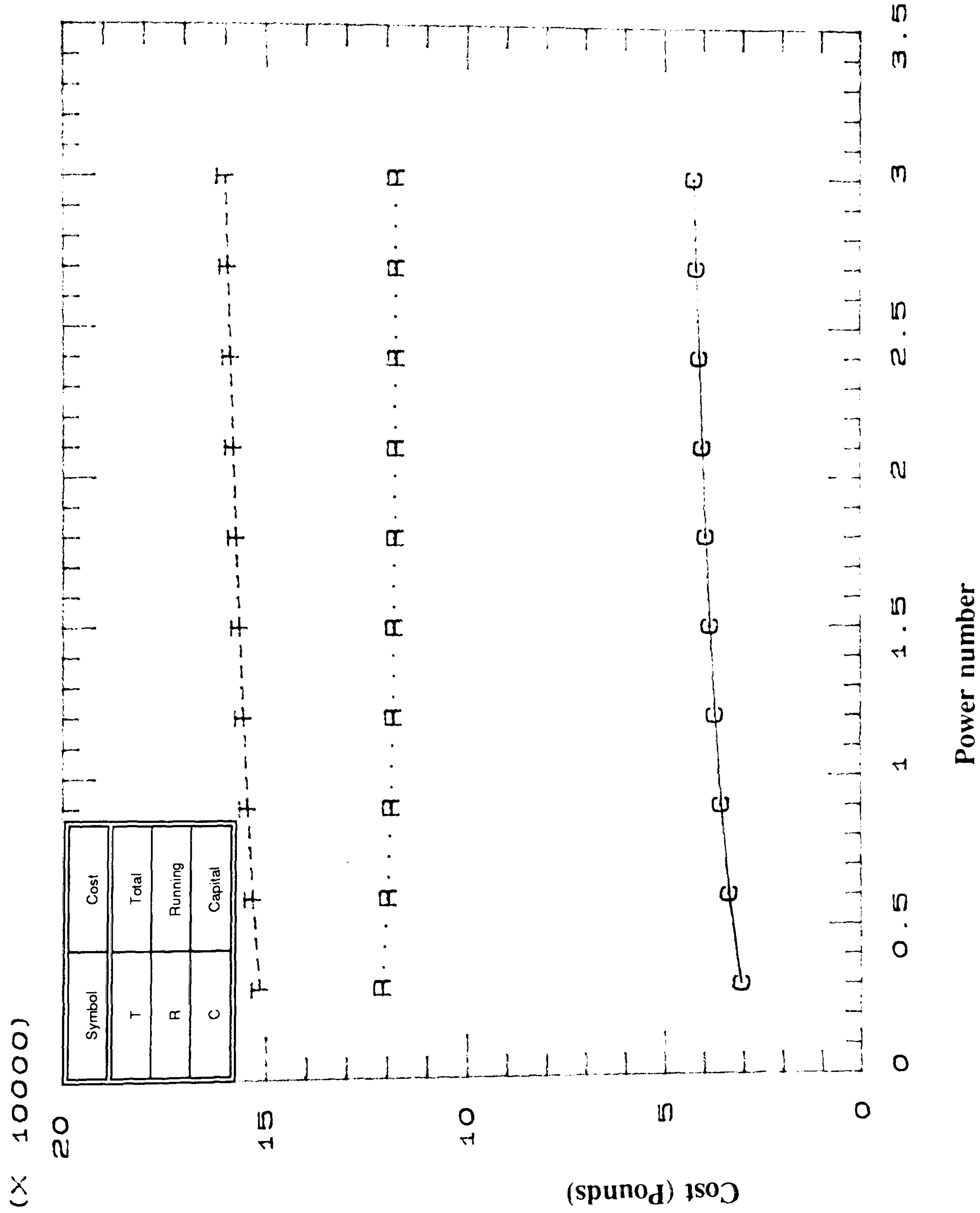


## Reynolds number

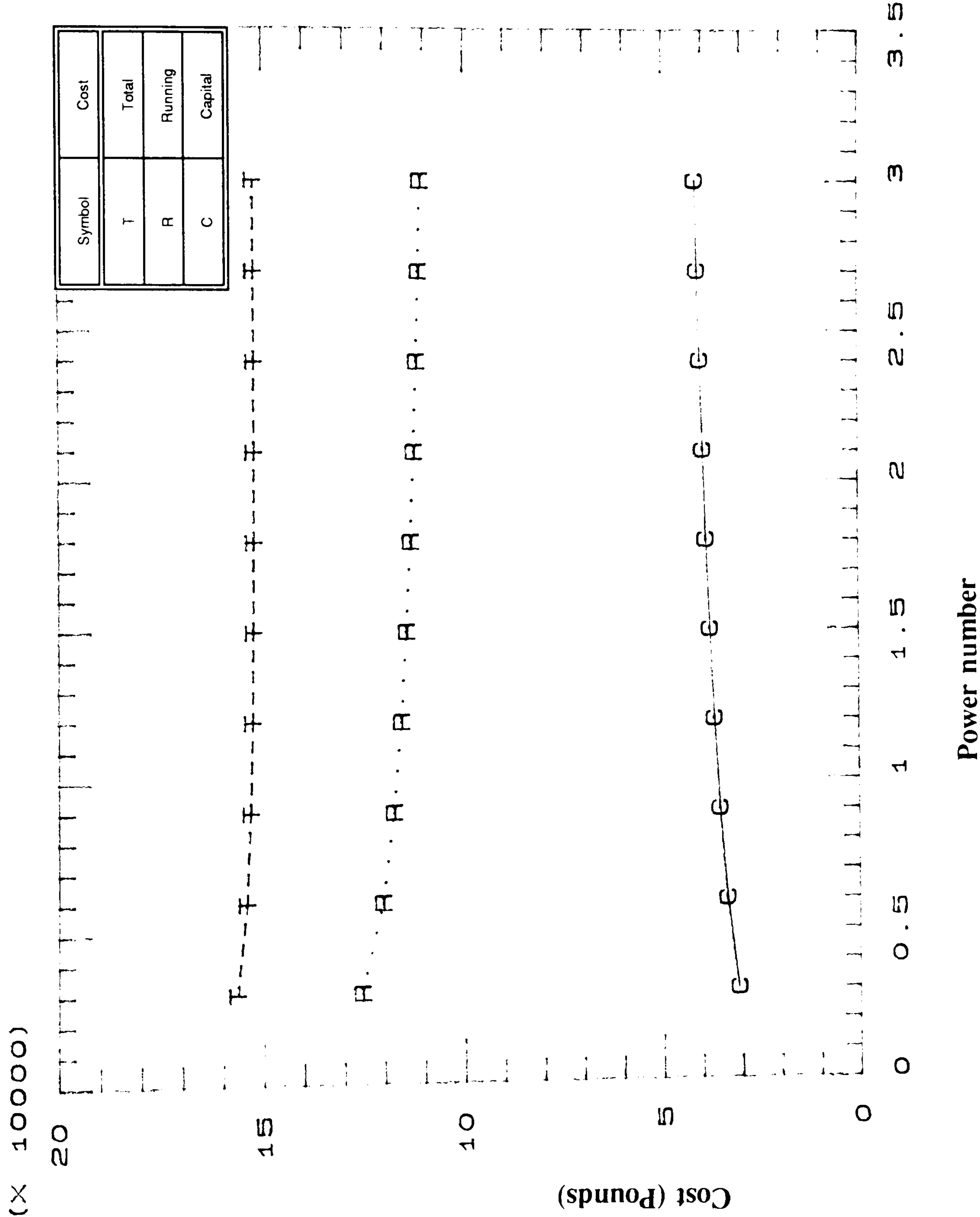
7.1 Plot of Costs versus power number -  $K = 2.00, n = 1.0$



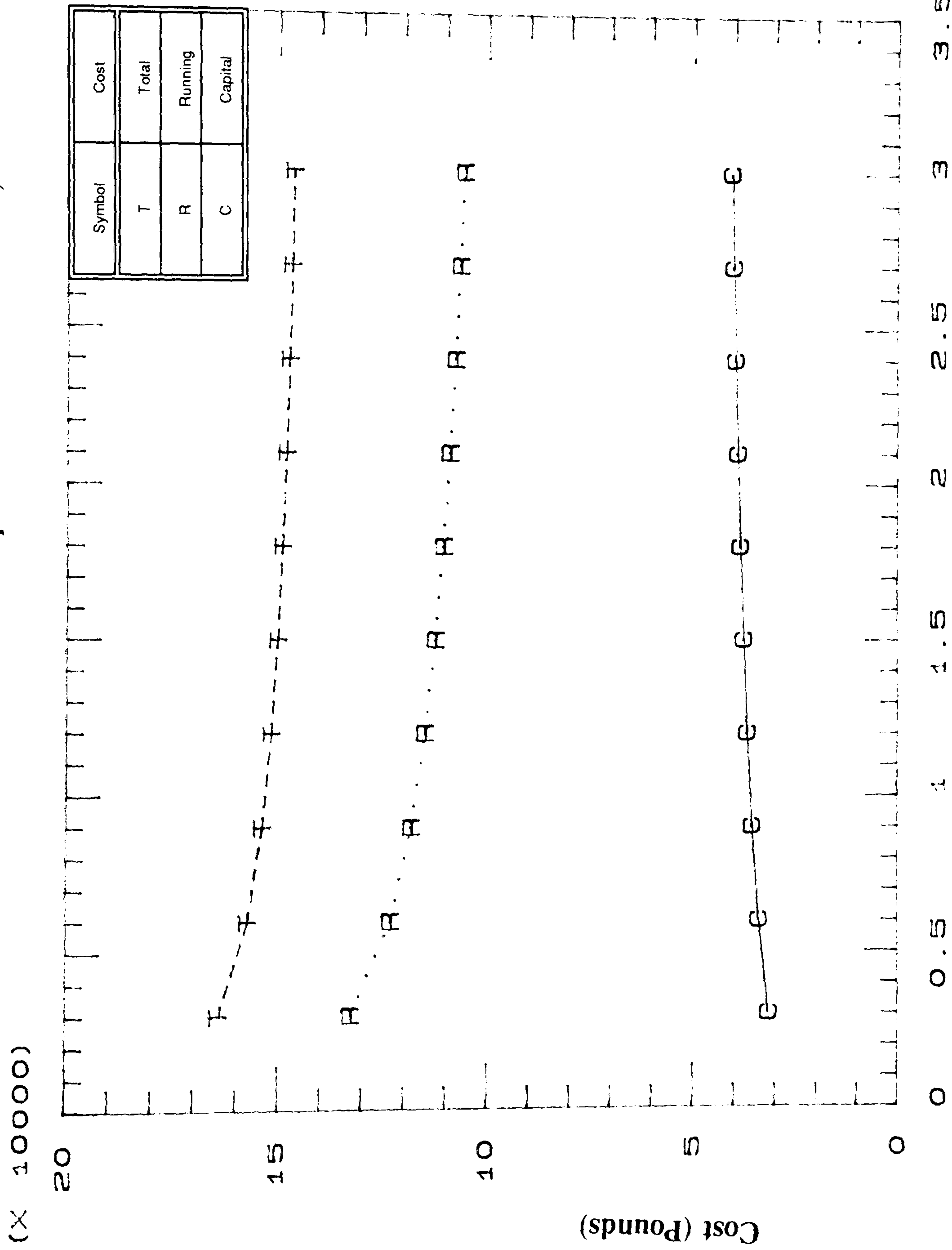
7.2 Plot of Costs versus power number -  $K = 2.67, n = 0.9$



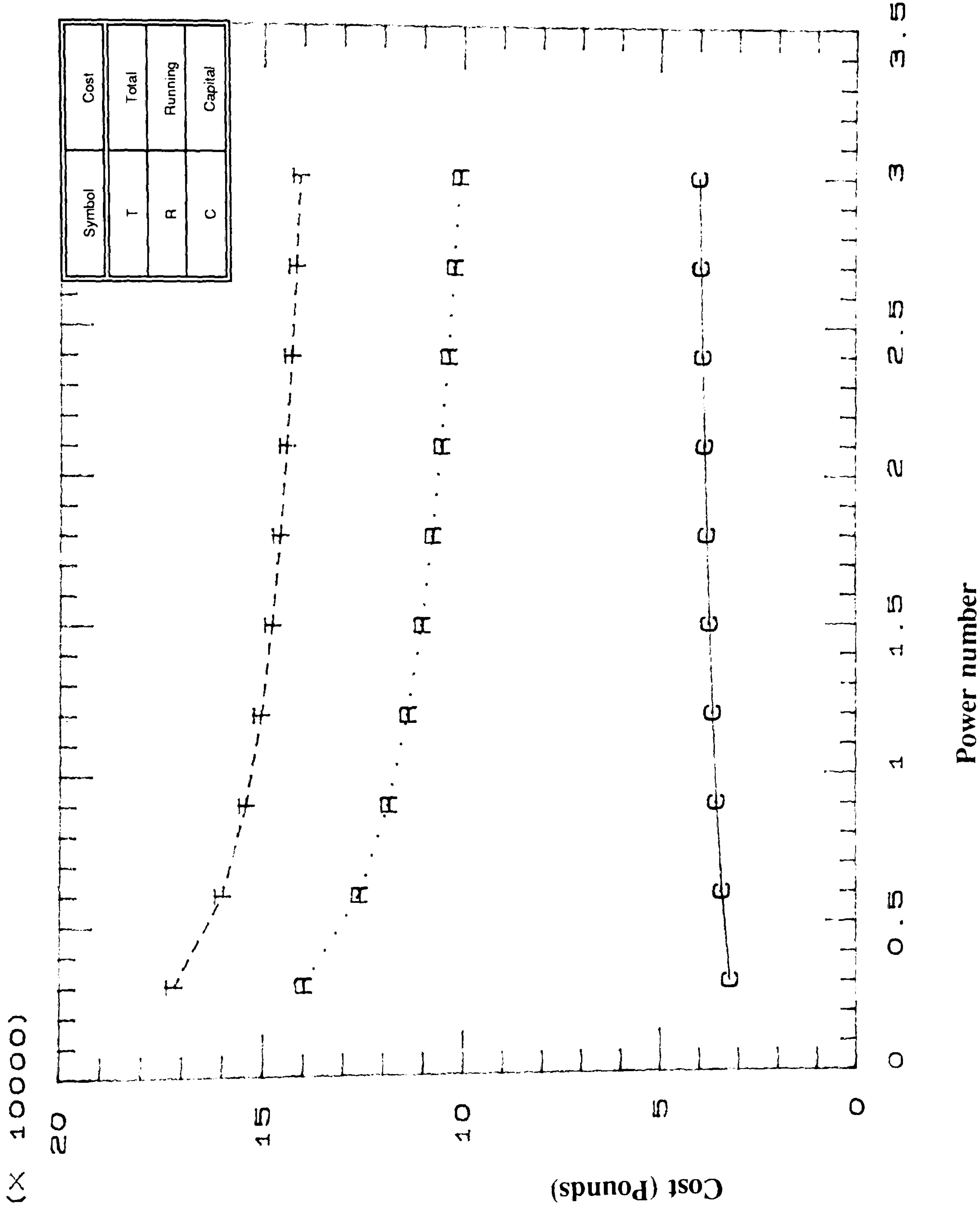
7.3 Plot of Costs versus power number -  $K = 3.55, n = 0.8$



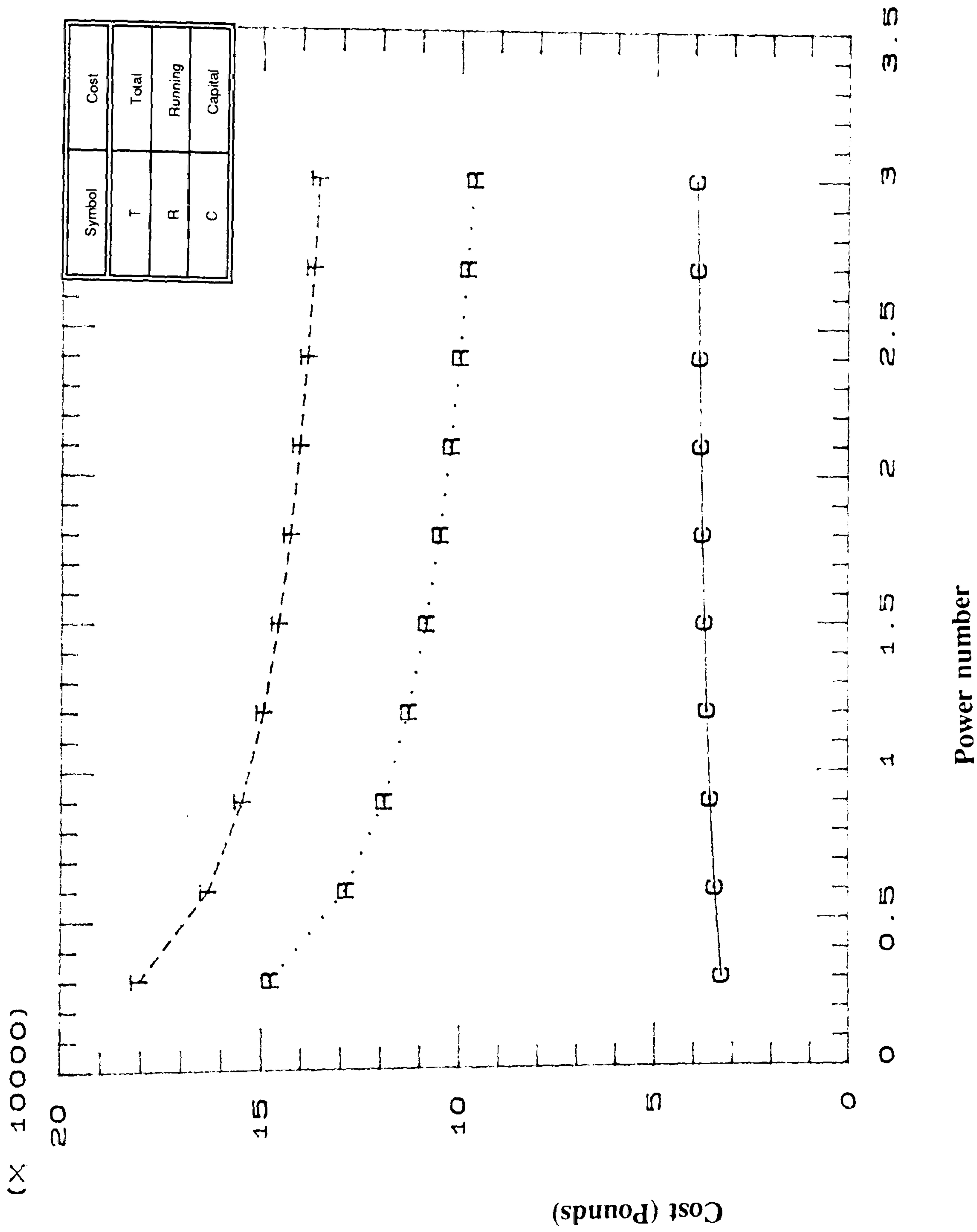
7.4 Plot of Costs versus power number -  $K = 4.74, n = 0.7$



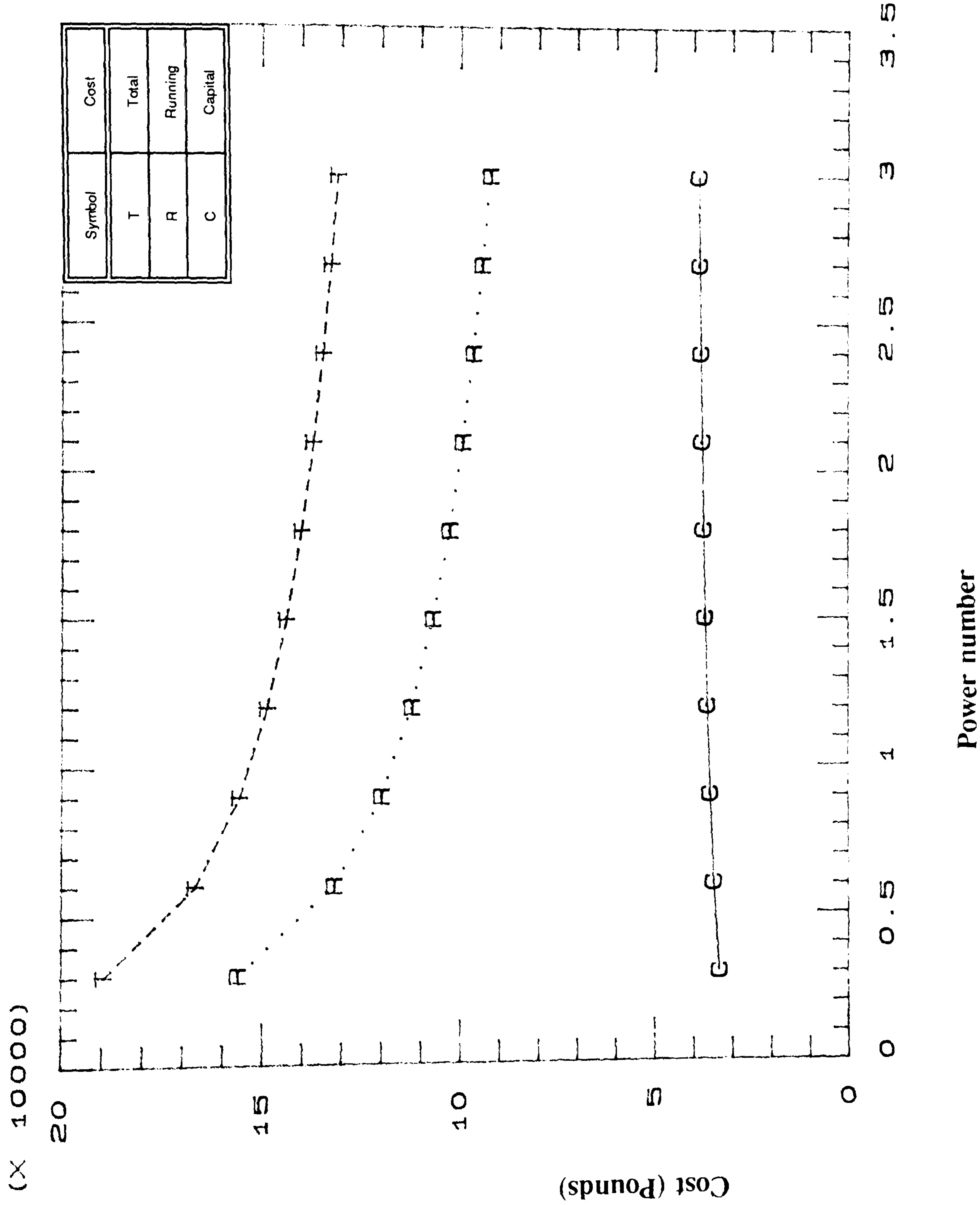
7.5 Plot of Costs versus power number -  $K = 6.31, n = 0.6$



7.6 Plot of Costs versus power number -  $K = 8.42, n = 0.5$



7.7 Plot of Costs versus power number -  $K = 11.22, n = 0.4$



## APPENDIX 1

### ECONOMIC ANALYSIS "tk-Solver" Equations

| VARIABLE SHEET                                                                                 |       |         |           |                    |
|------------------------------------------------------------------------------------------------|-------|---------|-----------|--------------------|
| St                                                                                             | Input | Name    | Output    | Unit               |
| Comment<br>Cost-Benefit Analysis Calculation<br>tk-Solver Program<br>Version 1.7      25/09/91 |       |         |           |                    |
|                                                                                                | 1000  | RHO     |           | kg m <sup>3</sup>  |
|                                                                                                | 5     | K       |           | Pa s <sup>n</sup>  |
|                                                                                                | .7    | NI      |           | -                  |
|                                                                                                | 3     | PO      |           | -                  |
| L                                                                                              |       | N       | .71857833 | s <sup>-1</sup>    |
|                                                                                                |       | D       | 2         | m                  |
|                                                                                                |       | RE      | 1385.0576 | -                  |
| L                                                                                              | .4    | RATIO   |           | D/T Ratio          |
|                                                                                                | 5     | T       |           | m                  |
|                                                                                                |       | V       | 91.314991 |                    |
|                                                                                                | 100   | TM      |           | s                  |
|                                                                                                |       | TAUw    | 38.911568 | N m <sup>-2</sup>  |
|                                                                                                |       | Rw      | 18.750483 | s <sup>-1</sup>    |
|                                                                                                |       | MUw     | 2.0752301 | Ns m <sup>-2</sup> |
|                                                                                                |       | OFO     | 120.46857 | -                  |
| L                                                                                              |       | P       | 35619.972 | W                  |
|                                                                                                |       | EPS     | .39007804 | W/kg               |
| L                                                                                              |       | TORK    | 7889.3204 | Nm                 |
| L                                                                                              |       | CAPTCO  | 37649.23  | Pound St.          |
|                                                                                                | 6     | UNITCOS |           | Pence/kWh          |
|                                                                                                | 8000  | HRSPYR  |           | Hours              |
|                                                                                                |       | POWCOS  | 17097.587 | Pound St.          |
|                                                                                                |       | MAINCO  | 1505.9692 | Pound St.          |
|                                                                                                |       | TOTFLO  | 1493676.1 | Pound St.          |
|                                                                                                | 11    | IR      |           | %                  |
|                                                                                                | 11    | LIFE    |           | Years              |
|                                                                                                |       | DISFAC  | .58000225 | -                  |
| L                                                                                              |       | DIRUCO  | 118691.15 | Pound St.          |
| L                                                                                              |       | TOCOST  | 156340.38 | Pound St.          |

| RULE SHEET |                                                                                    |
|------------|------------------------------------------------------------------------------------|
| S          | Rule                                                                               |
| *          | P=PO*(N <sup>3</sup> )*(D <sup>5</sup> )*RHO                                       |
| *          | TORK=(PO*(N <sup>2</sup> )*(D <sup>5</sup> )*RHO)/(2*PI())                         |
| *          | RE=RHO*N*(D <sup>2</sup> )/MUw                                                     |
| *          | OFO=(RHO*(T <sup>2</sup> ))/(TM*MUw)                                               |
| *          | TAUw=TORK/((T <sup>3</sup> )*1.622)                                                |
| *          | Rw=(TAUw/K) <sup>(1/NI)</sup>                                                      |
| *          | IF Rw < 400 THEN MUw=K*(Rw <sup>(NI-1)</sup> ) ELSE MUw=K*(400 <sup>(NI-1)</sup> ) |
| *          | RATIO=D/T                                                                          |
| *          | (PO <sup>(1/3)</sup> )*RE=182*(OFO <sup>0.5</sup> )                                |
| *          | V=0.165*((T/0.609) <sup>3</sup> )                                                  |
| *          | EPS=P/(RHO*V)                                                                      |
| *          | CAPTCO=488*(TORK <sup>0.4843</sup> )                                               |
| *          | POWCOS=HRSPYR*P*UNITCOS/100000                                                     |
| *          | MAINCO=0.04*CAPTCO                                                                 |
| *          | DISFAC=(1/((IR/100)*LIFE))*(1-(EXP(-(IR/100)*LIFE)))                               |
| *          | DIRUCO=(MAINCO+POWCOS)*LIFE*DISFAC                                                 |
| *          | TOTFLO=(POWCOS+DIRUCO)*LIFE                                                        |
| *          | TOCOST=CAPTCO+DIRUCO                                                               |