



Endogenous constraints on full productive capacity in a free-market economy

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Abstract

The full employment interest rate implicit in classical economic theory is 4½%, deduced by including the rate of normal profit in a simple macroeconomic model. By not fixing the interest rate at this optimum, Central Banks endogenously maintain excess productive capacity, cause unemployment, and encourage the exploitation of Labour by Capital.

Keywords Rate of normal profit · Classical theory · Full employment interest rate · Productive capacity · Central Banks

JEL Classification B0 · E0

1 Introduction

This short paper clarifies some concepts and extends the implications of the author's paper concerning the deduction of the full employment interest rate (Cole 2022). The original paper used an inventory-based model and incorporated the rate of normal profit, whilst assuming saving equals investment where firms are only making normal profit under perfect competition.

The rationale of the inventory-based model may require a more precise interpretation to reveal the logic behind it

(1) OUTPUT = INCOME income paid is for output value

(1a) UNSOLD OUTPUT = UNSPENT INCOME corollary

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(1b) UNSOLD OUTPUT \equiv REALISED INVENTORIES output not yet sold

(1c) UNSPENT INCOME \equiv LIQUIDITY PREFERENCE income not yet spent

from(1b), and (1c) and after iterative adjustments to income/output by firms,

(2) PLANNED INVENTORIES $< \Rightarrow$ LIQUIDITY PREFERENCE **in equilibrium**

where planned inventories are cost minimised by firms

(2a) Cost of maintaining planned inventories is inventory costs (TC).

Introducing the rate of normal profit is a means to an end, that end being the deduction of the full employment interest rate at which full productive capacity is realised in a free-market economy. The rate of normal profit is the inducement for savers to invest, earning interest and at least normal profit, instead of only interest. Whereas there is no dispute that the concept of the rate of normal profit exists, classical theory (Smith 1776), Ricardo (1817), neoclassical economics (Marshall 1890), (Wicksell 1898), and all subsequent orthodox economic theory, ignored it, because in a saving-investment cross, the inclusion of the rate of normal makes no difference to the result. However, once liquidity preference is introduced, there is a very different outcome.

2 Planned inventories

Planned inventories is the result of firms minimizing the joint costs of inventory renewal and finance costs, the latter being the sum of the interest rate i and the rate of normal profit n , i.e., $[i + n]$. The model used is a macroeconomic version of the original EOQ model (Harris 1913).

The cost of one aggregate inventory renewal of quantity Q is g (an indeterminate and obscure exogenous variable representing technological, institutional, and demographic conditions), Y is output (= income), and $Q^*/2$ is average inventories held at the cost of interest plus normal profit $[i + n]$

$$\text{Planned inventories } \frac{Q^*}{2} = \sqrt{\frac{gY}{2[i + n]}} \quad (1)$$

$$\text{Inventory costs (TC)} = \sqrt{2gY[i + n]}. \quad (2)$$

The square root function in planned inventories creates an economy of scale. The value of inventories, which include both capital goods and consumer goods, finished and part-finished, increases as more commercial and financial services

are expended on them before they reach final sale when unspent income is spent on unsold output.

3 Liquidity preference

Unfortunately, Keynes's (1936) liquidity preference suffers from several deficiencies (Viner 1936).

- (a) Keynes' liquidity preference theory is indeterminate like the classical theory of the rate of interest. This is because the liquidity preference curve itself shifts up and down with changes in the level of income.
- (b) The rate of interest influences and in turn is influenced by other important factors like the rate of saving, propensity to consume and marginal efficiency of capital, which Keynes's liquidity preference theory completely ignores.
- (c) It has been argued that the idea of hoarding has not been properly explained in Keynes' theory of interest. The factors that go to increase the propensity to hoard and the volume of hoarding are not sufficiently analysed and given their due place.
- (d) According to Keynes, the rate of interest is not a reward for waiting or saving. He forgets that saving or waiting is a necessary means to obtain funds for liquidity as Jacob Viner has pointed out, "there can be no liquidity without saving".

These deficiencies in Keynes's liquidity preference are rectified in the following manner:

Income is notated Y , the interest rate i , the rate of normal profit n , the proportion of income saved s , and liquidity preference L . Current saving is sY equal to investment I .

Liquidity preference represents a trade-off between current and future consumption. It is unspent income and the optimum amount held is equivalent to the expected future returns on investment (interest plus normal profit) summed to infinity, discounted to present value by the marginal efficiency of capital, [*mec*], being equal to the interest rate i

$$\text{Liquidity preference } L = \sum_{t=1}^{t=\infty} \frac{[i+n]sY}{[1+i]^t} \text{ where the interest rate } i \text{ is matched by [mec]}$$

$$\text{Liquidity preference } L = sY + \frac{nsY}{i}. \quad (3)$$

Liquidity preference, therefore, includes a stock equivalent to current saving (sY) destined for investment plus a stock of idle money which facilitates autonomous consumption and is the present value of normal profit on future investment nsY/i .

As income grows, both saving and idle money, which together make up liquidity preference, increase at the same rate. However, idle money is inversely related to the

interest rate i , because a lower interest rate implies less benefit in the future from saving and investment in the present.

As the interest rate rises, idle money decreases because of the higher returns on investment and the lesser relative benefit of the present value of normal profit; there is a shift from current consumption to future consumption as idle money is invested. And as the interest rate falls, idle money increases because of the lower returns on investment and the greater relative benefit of the present value of normal profit; there is a shift from future consumption to current consumption as more saving is held as idle money. By this means, consumption is smoothed between the present and the future.

4 Equilibrium

Equilibrium income is found when liquidity preference (3) equals planned inventories (1)

$$\text{Equilibrium income } Y = \frac{gi^2}{2s^2[i+n]^3}. \quad (4)$$

This entails that income/output Y is at its endogenous maximum Y^* (differentiate Y with respect to i and set equal to 0) only when the interest rate is double the rate of normal profit ($2n$), irrespective of the exogenous variables g and s . It is a backward-bending curve, with its upper leg becoming asymptotic to the vertical axis and the lower leg reaching zero after the optimum at $i=2n$ where there is full productive capacity and no involuntary unemployment.

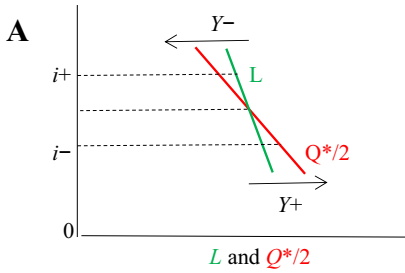
With a first glance at Eq. (4) for equilibrium income, it may seem puzzling how intersection of the two downward-sloping curves for planned inventories $Q^*/2$ (1) and liquidity preference L (3) can create a backward-bending curve for equilibrium income Y (4), with its apex at Y^* when the interest rate is $2n$. A proper graph cannot be drawn, because Y is initially unknown, but the sketches in Fig. 1A–C are informative.

Equilibrium income (4) now leads to equations for equilibrium liquidity preference L (=planned inventories), equilibrium saving sY (=investment I), equilibrium idle money nsY/i ($=L - sY$), and equilibrium inventory costs (TC)

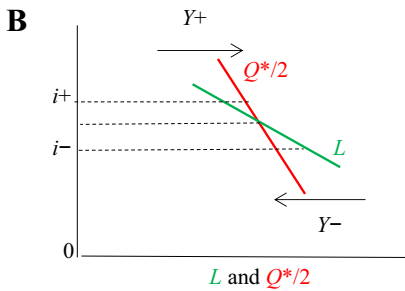
$$\text{Liquidity preference } L = \left[\frac{g}{2s} \right] \frac{i}{[i+n]^2} \text{ max. when } i = n \quad (5)$$

$$\text{Saving (= investment) } sY = \left[\frac{g}{2s} \right] \frac{i^2}{[i+n]^3} \text{ max. when } i = 2n \quad (6)$$

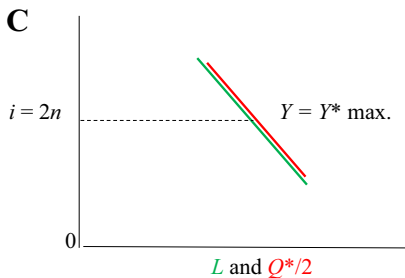
$$\text{Idle money } L - sY = \left[\frac{g}{2s} \right] \frac{in}{[i+n]^3} \text{ max. when } i = \frac{n}{2} \quad (7)$$



With higher interest rates, both curves shift left, becoming asymptotic to the i axis as Y decreases. With lower interest rates, both curves shift right with Y increasing, until they become tangential when $i = 2n$ and $Y = Y^*$ maximum.



With higher interest rates, both curves shift right, with Y increasing until they become tangential, when $i = 2n$ and $Y = Y^*$ maximum. With lower interest rates, both curves shift left with Y decreasing, eventually disappearing into the origin with $Y = 0$ and $i = 0$.



Firms adjust income/output in an iterative process until liquidity preference matches planned inventories. Income/output Y is at full productive capacity Y^* when planned inventories and liquidity preference are equal and their changes with respect to both interest rate i and income Y are equal. This only occurs when $i = 2n$.

Fig. 1 **A** With the interest rate above $2n$: L curve gradient $>$ $Q^*/2$ curve gradient. **B** With the interest rate below $2n$: $Q^*/2$ curve gradient $>$ L curve gradient. **C** With the interest rate at $2n$: $Q^*/2$ curve gradient $=$ L curve gradient (tangential) with respect to both i and Y

$$\text{Inventory costs (TC)} = \left[\frac{g}{2s} \right] \frac{2i}{[i+n]} \text{ positive function of } i \text{ with no max.} \quad (8)$$

These equations are all functions of $g/2s$ which is exogenous and can be ignored, but to graph them, the value of the rate of normal profit n must be deduced.

5 The rate of normal profit

Normal profit under perfect competition as defined by Marshall (1890) is when total revenue minus total costs is zero. After consumption and investment, the 'surplus' is idle money and residual cost is that of maintaining inventories (TC), so that

$$\begin{aligned} \text{Normal profit: idle money - inventory costs} &= 0 \text{ and firms are just breaking even} \\ \text{Normal profit when } (L - sY) - (TC) &= 0. \end{aligned}$$

The *rate* of normal profit is found when profit is normal as above, *and* investors have an equal expectation of the marginal efficiency of capital [*mec*] (equal to the interest rate), and hence saving/investment, rising or falling, i.e., from Eq. (6), $d^2(sY)/di^2 = 0$ being the same as $d^2I/di^2 = 0$.

Substituting Eqs. (7) and (8) into $(L - sY) - (TC)$

$$\text{Normal profit when } \frac{gin}{2s[i+n]^3} - \frac{gi}{s[i+n]} = 0.$$

Then, the interest rate i^\ddagger which coincides with normal profit is a function of the rate of normal profit n –

$$i^\ddagger = \sqrt{\frac{n}{2}} - n. \quad (9)$$

However, the *rate* of normal profit sufficient to induce savers to invest under all interest rates must be that which coincides, not only with normal profit, but also with where the change in saving/investment is minimized with respect to changes in the interest rate. The rate of normal profit is then an endogenous constant consistent with equimarginal risk of gain or loss

$$\text{Rate of normal profit when } \frac{d^2(sY)}{di^2} = 0 \text{ sY from Eq. (6),}$$

which is with an interest rate i^\ddagger that is again a function of the rate of normal profit

$$i^\ddagger = \left(2 \pm \sqrt{3} \right) n. \quad (10)$$

There are simultaneous equations [Eqs. (9) and (10)], to be solved to deduce the rate of normal profit

$$\text{The rate of normal profit } n = \frac{(2 \pm \sqrt{3})}{12} \approx 0.02233 \text{ or } 0.311. \quad (11)$$

There are two results, the first of $n=0.02233$ from *minimized* change in saving, the second of $n=0.311$ from *maximized* change in saving, so the latter can be discarded. The rate of normal is therefore the minimum value of 0.02233 or approximately $n \approx 2\frac{1}{4}\%$, endogenously determined

$$\text{The rate of normal profit } n = \frac{(2 \pm \sqrt{3})}{12} \approx 0.0225. \quad (12)$$

The equations for equilibrium liquidity preference (5) L (=planned inventories), equilibrium saving (6) sY (=investment I), equilibrium idle money (7) nsY/i ($=L - sY$), and equilibrium inventory costs (8) (TC) are graphed in Fig. 2, ignoring $g/2s$ which is common to all of them, as a macroeconomic template.

6 Keynes and Marx

Keynesian macroeconomic policy is to borrow idle money (Marx’s money hoards) from the wealthy and use it to finance public investment to make up for the lack of private sector investment. However, using deficit financing in this way only works

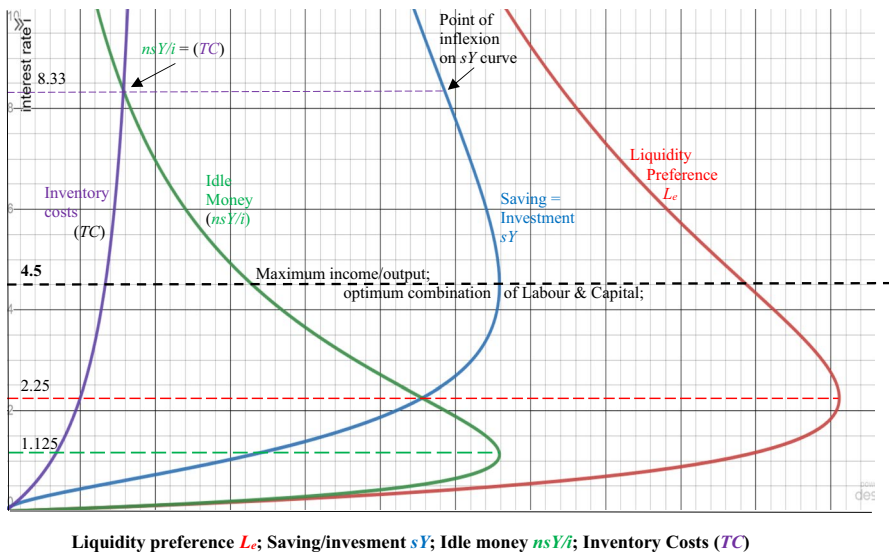


Fig. 2 The macroeconomic template

by pushing up interest rates to $4\frac{1}{2}\%$ from rates that were below that optimal level, thereby increasing investment and raising wages.

Keynesian policies do not work when interest rates are already at $4\frac{1}{2}\%$, because higher rates decrease investment and create unemployment, as shown by the macroeconomic template in Fig. 2. Therefore, the post-war consensus was a chimera based on the erroneous work of Keynes which was largely intended to discredit Marxist theory.

On the other hand, Keynes (1936) believed that there might be a full employment interest rate, but never deduced it. Therefore, although Keynes discredited Marx's claim that the capitalist system was doomed to failure, his remedy was flawed by ignoring the cost-minimizing behaviour of firms, by his deficient explanation of liquidity preference, and by his omission of the rate of normal profit which has been common to all macroeconomic theory, since Marshall (1890) failed to separate the variable interest rate from the constant and minimum rate of normal profit.

7 Productive capacity

The Central Bank sets the interest rate which determines liquidity preference as real money and is the base for credit money. Therefore, the money that a state creates will only be of any worth in so far as it reflects the value that is in circulation in the economy, in the form of the production and exchange of commodities (realised inventories), which Marx (1867) had warned.

Where this is not the case and the state creates money not based on liquidity preference, then this is a recipe for inflation and instability. Marxists point out that even in times of 'boom', the febrile global economy operates far below its productive capacity. They claim that this 'excess capacity' has become a hallmark symptom of a system that has long outlived its usefulness, and that even at its height, capitalism can only successfully utilize about 80–90% of its productive abilities. This falls to 70% or less in times of slump. In past recessions, the figure falls to as low as 40–50%. Printing money which is not based on liquidity preference using quantitative easing can stifle the operation of the market mechanism for long periods but leads to rampant inflation.

However, the author's original paper (Cole 2022) and this supplement have found that this 'excess capacity' arises, because the interest rate is not fixed by the Monetary Authority at $4\frac{1}{2}\%$, and with interest rates above or below $4\frac{1}{2}\%$, Capital is encouraged to exploit Labour and cause unemployment by limiting investment. As this continues, the wealthy becomes wealthier, and the poor becomes poorer. Neither Communism nor the halfway house of Socialism is the solution.

Productive capacity is deduced using Eq. (4) for equilibrium Y and maximum Y ($= Y^*$)

$$\text{Equilibrium income } Y = \frac{gi^2}{2s^2[i+n]^3} \text{ and } Y = Y^* \text{ max. when } i = 2n; \text{ therefore}$$

Maximum income $Y^* = \frac{2g}{27ns^2}$ for any values of exogenous variables g and s .

The ratio of income Y to maximum income Y^* is the proportion of productive capacity in use

$$\text{Productive capacity in use } \frac{Y}{Y^*} = \frac{27ni^2}{4[i+n]^3}$$

Subtracting productive capacity in use from unity gives the loss of productive capacity

$$\text{Loss of productive capacity} = 1 - \frac{Y}{Y^*} = 1 - \frac{27ni^2}{4[i+n]^3} \text{ and with } n = \frac{[2-3^{1/2}]}{12} \approx 0.0225$$

$$\text{Loss of productive capacity} \approx 1 - \frac{972i^2}{[4i+9]^3} \text{ with } i \text{ as percentage, } i\%. \quad (13)$$

Loss of productive capacity = 0 with no involuntary unemployment when $i = 4.5$.

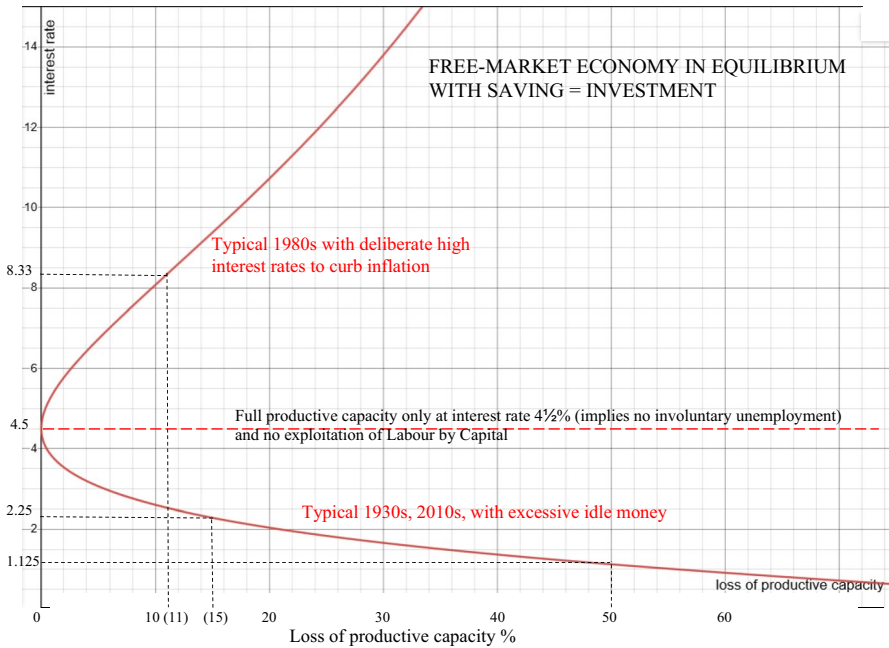


Fig. 3 Loss of productive capacity graph of Eq. (13)

The productive capacity graph is drawn in Fig. 3. It is backward-bending with its apex at an interest rate of $4\frac{1}{2}\%$, a reverse of the backward-bending saving/investment curve in Fig. 2.

8 Conclusion

The problem caused by the recent use of near zero interest rates is summed up by Lord William Hague (2022), former leader of the UK Conservative Party: *“I argued (in 2016) that eight years after the global financial crisis, central banks were behaving like doctors keeping their patients on a drip long after the emergency operation and with dangerous side effects. These were that savers were pushed into riskier assets, that house and share prices were driven ever higher, that inequality was exacerbated by poorer people being left out of such increases in paper wealth, that companies used borrowed money to buy back shares rather than find new productive investments and that “zombie” companies were kept afloat by artificially low borrowing costs. Worst of all, there would ultimately be a reckoning in which businesses and home buyers would be hit hard, after getting used to ultra-low rates for too long. The whole point of central banks being independent of government was that they could be brave enough to make people confront reality, not blow up a bubble of make-believe money to avoid immediate pain.”*

Modern Monetary Theory has not resolved the issue of equilibrium liquidity preference, which does not become perfectly elastic with interest rates at or below $2\frac{1}{4}\%$. Until MMT does so and accepts that the interest rate should be fixed at $4\frac{1}{2}\%$, the rich will become richer, whilst those less well-off will be financially squeezed, which is not the outcome desired by true free marketeers. There will be recurring financial and banking crises caused by inappropriate interest rate policies. Hence, Marxists will correctly continue to claim that Labour is always being exploited by Capital and that full productive capacity is invariably constrained.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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