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A rolling horizon approach for optimal management of microgrids under stochastic uncertainty

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Abstract

This work presents a Mixed Integer Linear Programming (MILP) approach based on a combination of a rolling horizon and stochastic programming formulation. The objective of the proposed formulation is the optimal management of the supply and demand of energy and heat in microgrids under uncertainty, in order to minimise the operational cost. Delays in the starting time of energy demands are allowed within a predefined time windows to tackle flexible demand profiles. This approach uses a scenario-based stochastic programming formulation. These scenarios consider uncertainty in the wind speed forecast, the processing time of the energy tasks and the overall heat demand, to take into account all possible scenarios related to the generation and demand of energy and heat. Nevertheless, embracing all external scenarios associated with wind speed prediction makes their consideration computationally intractable. Thus, updating input information (e.g., wind speed forecast) is required to guarantee good quality and practical solutions. Hence, the two-stage stochastic MILP formulation is introduced into a rolling horizon approach that periodically updates input information.

Keywords: energy planning; rolling horizon; stochastic programming; scheduling; mathematical programming; microgrid; MILP.

1. Introduction

Conventional power grids are based on centralised networks where power plants (i.e., nuclear or hydroelectric power stations) generate energy that is used at industrial and domestic levels. These grids were designed to take advantages of economies of scale. However, these kinds of grids have some disadvantages, including the energy losses in power transmission, the difficulty of supply to keep up with growing energy demand and the concern over environmental damage. One of the main causes that contribute to the negative environmental impact in centralised networks is the generation of energy using fuel-based sources. This negative environmental impact includes an increase in the pollution and the climate change. In this context, the European Union (EU) has set targets for 2020 regarding the climate change and sustainability. The EU's renewable energy directive sets a target of 20% of energy from renewable sources by 2020, as well as a reduction by 20% in GHC emissions (considering 1990 as a baseline) and improving the energy efficiency by 20% (European Commission, 2010).

There are significant progresses in the area of Energy Systems Engineering (ESE), by the enhancement of efficient and sustainable energy supply chains. ESE provides a methodological framework to obtain realistic solutions in the decision making of complex energy systems problems, by adapting a holistic system-based approach (Kopanos *et al.*, 2017). The main goals of managing energy systems focus on decreasing operational costs related to the use of these energy networks, reducing the environmental impact caused by the generation, transmission and use of energy and also satisfying the power requirements, subject to unpredicted disturbances.

One area of study of ESE is the appropriate management of microgrids. These kinds of grids are based on the decentralised networks that combine the energy generation systems with information between suppliers and consumers (Hodge et al., 2011). One of the challenges of these networks is the integration of renewable and non-renewable energy sources. These sources are typically located close to consumption points, reducing energy losses in its transmission. Moreover, the implementation of renewable energy represents a big opportunity in terms of sustainability. However, the use of natural sources has the disadvantage of intermittent and unpredictable energy generation, and currently, forecast techniques are far from reliable. Thus, one of the main weaknesses of the exploitation of renewable sources is the mismatch of the unpredictability of energy generation from renewable sources and demand. Therefore, the coordinated management of generation and demand is indispensable for the appropriate decision making in the management of microgrids.

Consequently, the implementation of electrical and thermal storage systems within the microgrid introduces more flexibility in its management. These systems allow alleviating the divergence between generation and demand, as well as to face the variability in generation and demand forecasts. Moreover, storage systems offer an extra degree of freedom to manage the adaptable energy demand according to time-based market prices, establishing operating flexibility to use times of low electricity prices and avoid peak prices, which may involve cost savings.

The behaviour of renewable energy and heat resources (e.g., wind speed, solar radiation) comprises the contemplation of uncertainty in microgrids, to guarantee good quality and useful management solutions in the decision making (Zio and Aven, 2011). In this context, microgrids can be altered by different types of uncertainty. These variations may affect the generation conditions (i.e., variability in the weather forecast), which may involve variations in the availability and generation capacity of renewable generators. Also, microgrids conditions can be altered by variability in demand, which affects the power and heat requirements. An overview of the open issues in the area of modelling, control and optimisation of energy networks in terms of generation, storage and distribution can be found in Soroush and Chmielewski (2013).

Therefore, multiple uncertain sources must be taken into account to ensure the feasibility and reliability of the obtained solutions. The most common types of uncertainty can be classified as follows:

(i) Internal sources, which considers fluctuations in process parameters inside the network.
(ii) External sources, which takes into account variability outside the considered network, like acquisition and selling prices, demand and availability of resources.
(iii) Other uncertainty sources, like measurements errors or strikes.

Notice that the consideration of uncertainty introduces more complexity in the decision making. Hence, the approaches to deal with uncertainty may be grouped into reactive and preventive methodologies (Liu and Ierapetritou, 2008):

- Reactive approaches are based on modifying an initial plan obtained by a deterministic approach, to adjust it to diverse alterations, or even updated data. Some useful reactive methodologies are the rolling horizon approach (Li and Ierapetritou, 2010) or the Model Predictive Control (MPC) technique (Oberdieck and Pistikopoulos, 2015).
- Preventive approaches consider all possible situations, which are usually known as scenarios. The aim of these approaches is to find a solution for all considered scenarios. Although these approaches have the advantage that the resulting solution is feasible for the considered scenarios, this solution could be too conservative, because all scenarios are considered. The most common preventive procedures are the stochastic programming (Ierapetritou *et al.*, 1996) and the robust optimisation (Li *et al.*, 2011). Stochastic optimisation techniques consider the uncertainty in the system and generate schedules that account for this uncertainty. Typical sources of uncertainty which are usually modelled by implementing stochastic programming include demands, supply of raw materials or processing times (Shapiro *et al.*, 2013). In the literature, there are several works focused on the application of this well-known technique (Sahinidis, 2004; Cui & Engell, 2010). Also, the stochastic programming can lead to rescheduling of tasks (Balasubramanian and Grossmann, 2002, Kopanos *et al.*, 2008). Moreover, Schildbach and Morari (2016) proposed a scenario-based stochastic MPC approach for the optimal management of the mid-term planning of a multi-echelon multi-product Supply Chain.

In this paper, a new discrete-time stochastic-based rolling horizon Mixed Integer Linear Programming (MILP) mathematical formulation is proposed to optimally manage a microgrid under uncertainty. This model will allow updating input information to react to any alteration from the nominal scenario, as well as to consider all scenarios that can take place. This work focuses on the scheduling of decisions within the microgrid.

The appropriate design and scheduling of microgrids are essential to ensure the optimal management of the network. The design of a microgrid considers the location and capacity of the elements to install (e.g., generators, storage systems) and their technical characteristics (Pruitt *et al.,* 2013). For instance, Asano *et al.* (2007) developed a model to design the number of the elements to be installed in a microgrid as well as their capacities, considering combined heat and power (CHP) systems under deterministic conditions. Koltsaklis *et al.*, (2014) formulated a mathematical model for the optimal design and operational planning of energy networks, considering CHP generators. This work applied Monte Carlo simulations to analyse the effect of uncertainty. Also, Zhou *et al.* (2013) presented a two-stage stochastic formulation for the optimal design and operation of distributed energy systems under uncertain conditions.

The scheduling of a microgrid at operational level focuses on the energy and heat generation, purchases and sales; as well as the storage and demand decisions at local level, given the elements that constitute the network (Manfren *et al.,* 2011; Mehleri *et al.,* 2012; Sou *et al.*, 2011). Different models have been developed to manage the use of electricity and heat. Regarding the generation management of energy and heat within a microgrid, Carrión and Arroyo (2006) developed a Mixed-Integer Linear Programming (MILP) formulation to minimise the operational cost of a microgrid considering the power requirements to be satisfied. Moreover, a mathematical model for the optimal management of the generation and storage levels that deterministically satisfies the energy demand by minimising the operational cost was presented by Zamarripa *et al.* (2011). Kopanos *et al.* (2013) developed an MILP for the energy generation scheduling on a residential microgrid to minimise the overall operational cost. The proposed formulation considered the energy generation planning through the exploitation of CHP systems. A review of the optimal design, scheduling and control of residential microgrids is presented in Liu *et al.*, (2013). Other mathematical formulations are based on a unit commitment problem. These kinds of mathematical formulations consider energy and heat generation restrictions, including generation limits, ramping limits and minimum up and down times. One challenge of these formulations is the fact that the consideration of these restrictions may involve non-linear constraints. The objective of unit commitment problems in the area of energy operations is to manage a set of generators in order to fulfil a pre-established demand. This is an extremely challenging optimisation problem due to the enormous number of possible combinations of the status (on/off) of the generators within a considered network (Bhardwaj *et al.*, 2012). This kind of formulations is applied to electricity and heat generation (Marcovecchio *et al.*, 2014). Hytowitz and Hedman (2015) proposed a stochastic-based formulation to consider the uncertainty related to the generation of electricity through the use of photovoltaic panels under uncertain conditions. Moreover, Zhang *et al.* (2013a) presented a scenario-based MPC in order to minimise the energy consumption of a building, considering heat, ventilation and air conditioning systems. Menon *et al.* (2016) proposed a mathematical formulation based on the exploitation of an MPC in order to manage the optimal control strategy for the electricity and heat within a multi-building network. Furthermore, microgrids can operate isolated or connected to the main power grid. Naraharisetti *et al.* (2011) developed an MILP formulation for the optimal scheduling of microgrids connected to the power grid, in order to evaluate energy policies. This connection can be used to export energy when there is an excess and to purchase energy when the sources of the microgrid are not able to satisfy the energy requirements.

Moreover, several mathematical formulations have been developed to deal with the exploitation of renewable sources. This decision making process aims to determine which generator to use at any time (Coroamă *et al.*, 2013; Chicco *et al.*, 2009; Xiao *et al.*, 2011; Ahadi *et al.*, 2016). Moreover, the management of renewable sources in remote networks has also been studied (Ranaboldo *et al.*, 2013; Akinyele and Rayudu, 2016).

Furthermore, there is an increase in the interest for the use and implementation of smart grids (Balta-Ozkan *et al.*, 2013; Xenias *et al.*, 2015). A smart grid is defined as a network able to incorporate the actions of the elements involving that network, including producers and consumers, to distribute electricity in an efficient, sustainable, profitable and safe way. In this area, Sun and Huang (2012) reviewed different optimisation methodologies for the energy management in smart homes. Also, Honarmand *et al.* (2014) and Bracco *et al.* (2015) presented mathematical formulations for the management of energy generation for transport purposes.

Otherwise, the management of the energy and heat demand represents an open issue in the area of Energy Systems Engineering. Regarding the smart houses management, Nistor *et al.* (2011) developed an MILP approach for the scheduling of tasks, which may be delayed according to the real-time electricity price, to achieve cost savings. Posteriorly, Rastegar *et al.* (2016) presented an MILP formulation to minimise the electricity payments of a smart home, by managing the human behaviour to better fulfil the energy requirements. This formulation considered both real-time energy prices and priority to perform the considered energy consumptions. Zhang *et al.* (2016) proposed a multi-objective optimisation approach to assess the trade-off between economics and environmental factors while scheduling energy tasks. In the industrial area, Kato *et al.* (2011) developed an MILP formulation to reduce the energy consumption cost, considering both energy generation and storage in a demand-based framework. Also, Zondervan *et al.* (2010) presented a Mixed Integer Non-Linear Programming (MINLP) to reduce the operational cost for process industries. This mathematical formulation determined the optimal schedule of tasks, considering both the availability and cost of electricity. Other works consider uncertain conditions. For instance, Mohammadi *et al.* (2014) presented a scenario-based optimisation approach for the energy operation management taking into account uncertainties in the given energy generation, consumption and market price. In the same area, Zakariazadeh *et al.* (2014) proposed an MILP formulation to address the schedule of microgrids including the demand management, by introducing flexible energy consumption tasks. The above problem has been modelled considering different scenarios.

Another significant challenge is the reduction of energy peak demands. This reduction involves a better exploitation of the existing power infrastructure. Peaks of power involve several disadvantages not only for the users, but also for the energy providers and for the grid. The network needs to withstand those peaks, introducing additional cost in the design of the grid. Also, electricity prices are based on peaks of demand, involving more expensive prices when peaks take place. Thereby, a peak-load shaving scheduling formulation was presented by Costanzo *et al.* (2011). Moreover, Della Vedova and Facchinetti (2012) proposed a scheduling framework to model the electrical availability, which involves reducing peaks of energy demand. Posteriorly, Zhang *et al.* (2013b) proposed an MILP formulation to manage the scheduling of energy and heat tasks within a smart building. This formulation considered the reduction of peaks by applying extra costs if electricity load from the power grid exceeds an established limit.

The management of energy and heat generation and demand has been analysed in a sequential way, by adjusting the process schedule to the price and availability of resources. For example, Nolde and Morari (2010) developed an MILP model for a steel plant. The objective of the proposed formulation is to minimise the overall energy cost by adjusting the energy consumption to its availability. The mathematical model introduces an extra penalisation in case of deviation from the contracted energy to the power supplier. Mitra *et al.* (2012) presented an MILP formulation to achieve operational cost savings, by adjusting the generation planning according to the electricity price. Hadera *et al.* (2015) proposed a mathematical formulation to minimise the total energy costs. The proposed formulation considers the demand response of the schedule of a steel plant, obtaining the optimal generation schedule as well as the energy purchases. In the area of smart houses, Mohsenian-Rad and León-García (2010) proposed a scheduling formulation considering domestic energy tasks, based on electricity prices.

Although the energy generation and demand have been studied, the coordinated scheduling of microgrids considering simultaneously both energy and heat generation and demand constitutes an open challenge. The formulation of a novel model to optimally manage a microgrid can be applied to achieve the optimal generation and schedule of tasks. This appropriate management may involve considerable benefits. These benefits may include cost savings, the reduction of energy peaks of demand and the reduction of the dependence on non-renewable sources. Hence, a discrete-time MILP mathematical model was presented by Silvente *et al.* (2012 and 2013) for the coordinated management of energy generation, storage and demand in a microgrid. Also, Silvente *et al.* (2015a) presented a comparison between discrete and hybrid-time MILP formulations to maximise the profit of a microgrid. More recently, Silvente and Papageorgiou (2017) presented an MILP formulation to manage the generation, storage and demand of energy and heat within a microgrid. This paper studied the impact of the delays in the starting time of tasks as well as the impact of eventual interruptions in the tasks.

Hence, electricity and heat generation models for the optimal management of microgrids have been reported in the last years. However, the management of microgrids contemplating the generation and demand of electricity and heat in a coordinated way constitutes a research gap. On the other side, rolling horizon strategies and stochastic formulations have been applied to a wide range of scheduling problems under uncertainty. However, the combination of both procedures to tackle uncertainty as well as their application to the specific area of the management of microgrids has not been fully-studied and still represents an open issue to the research community.

This work proposes a novel mathematical model to manage simultaneously energy and heat generation, purchases, sales, storage and tasks to minimise the cost of a microgrid by optimally adapting energy and heat generation and demand. The main novelty of this work is the combination of a rolling horizon approach and a stochastic formulation to optimally manage a microgrid under uncertainty. One of the main characteristics of this novel formulation is the flexibility in energy requirements, in terms of the starting time of energy consumptions. The energy requirements are limited by a time windows to perform a given energy consumption task, which can be delayed. Also, tasks may be interrupted. Another feature is the management of the uncertainty. This is given by the simultaneous implementation of reactive and proactive methodologies, corresponding to the rolling horizon approach and the stochastic programming. The aim of this methodology is to exploit the flexibility of the microgrid incorporating renewable energy sources. Then, the obtained solutions can be updated if new information is revealed or modified or even if an unpredicted event occurs in either energy and heat generation or demand. This methodology is presented through a case study to optimally manage a microgrid under uncertainty, considering energy and heat generation, purchases, sales, storage levels and tasks to be satisfied.

The work is organised as follows. In this section, the literature review on the energy management formulations and the application of rolling horizon and stochastic techniques have presented. Then, section 2 details the problem statement of the presented formulation. In section 3, the scheduling approach for the management of the microgrid under uncertainty is described in detail. Section 4 presents the main details of the case under study. Section 5 provides a discussion on the obtained results. Finally, section 6 presents the concluding remarks and future research directions are revealed.

2. Problem statement

The proposed mathematical formulation considers the generation and storage levels as well as the timing of tasks to be managed within a microgrid. The problem also considers electricity purchases and sales to the power grid, considering time-varying electricity prices. The goal is to minimise the operational cost.

The mathematical model includes electrical and heat generation, storage and demand balance constraints to describe their flows as well as technical equipment constraints, such as capacity constraints. Thus, the problem formulation is defined in terms of the items described below. Given:

(i) A scheduling horizon SH, which is divided into a number of equal-size time intervals $t \in T$, a prediction horizon PH and a control horizon CH.

(ii) A set of scenarios $s \in S$, which describes all considered uncertain profiles that can take place.

(iii) A set of electricity and heat generation sources, and their technical characteristics, such as capacities and efficiencies.

(iv) A set of electricity and heat storage systems, and their technical characteristics, including capacities, efficiencies, as well as charge and discharge limit rates.

(v) A set of houses $k \epsilon K$ within the microgrid.

(vi) A set of equipment units $j \epsilon J$ to perform the considered energy consumption tasks.

(vii) A set of energy consumption tasks $i \epsilon I$. A desirable (minimum) initial time of each task $Ts_{k,i}^{min}$ and its processing time $PT_{k,i,s}$ are given. The initial time $Ts_{k,i}$ of each task is delimited by a predefined time window, bounded by a minimum and maximum starting time ($Ts_{k,i}^{min}$ and $Ts_{k,i}^{max}$). If a task starts outside the given time window, an extra cost is applied.

(viii) A subset of operation periods $\theta \epsilon \Theta_{i,k}$ associated with a task a task i at home k. This subset indicates the periods of time in which a task i is active. This is used to allow different energy requirements as a function of the time (see Figure 1) and to allow eventual interruptions. Notice that the first operation period of a task corresponds to $\theta = 0$.

(ix) The task capacity profile $C_{i,\theta}$ (which can be constant or variable). Figure 1 represents consumption resource profile $C_{i,\theta}$ for a given task i.

(x) The overall heat demand of the microgrid, $H_{s,t}$.

(xi) The price of the electricity in the power grid, b_t.

(xii) The wind forecast speed, $v_{s,t}$.

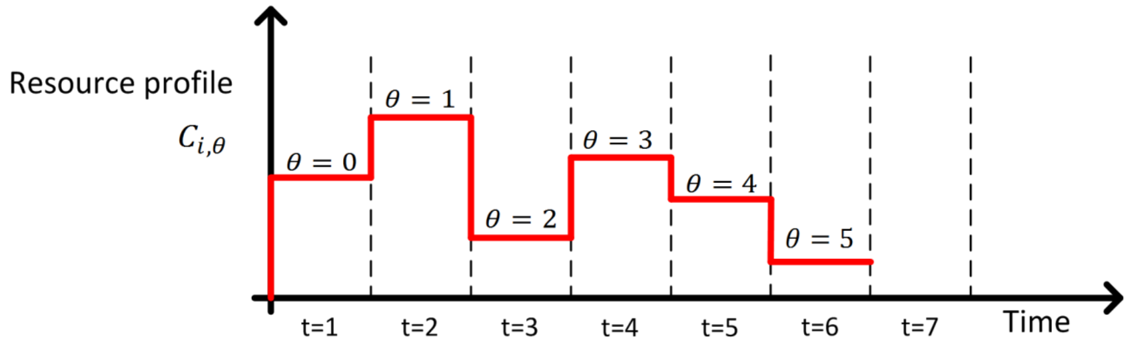

Figure 1. Time-varying resource profile for a task.

The goal is to minimise the operational cost of a microgrid, determining the energy and heat generation plan, the electricity purchases to the external power grid, the schedule of tasks, the electrical and thermal storage plan and the electricity to be exported to the power grid.

Furthermore, different tasks may be allocated in the same equipment unit. Thus, the sequence of operations is required to avoid any eventual overlap. Another feature is the fact that the consumption task profiles do not follow a constant profile, given by a set of task operation profiles θ.

The optimal management of the microgrid is based on the characteristics of each task (i.e., time windows, consumption task profile), renewable energy sources forecast (e.g., wind speed forecast) and time-varying electricity prices for both purchases and sales. Notice that electricity and heat generation is scheduled from generators, but also purchases and sales to the power grid are allowed. Furthermore, in order to reduce the peak electricity demand from the power grid, additional peak demand costs are applied when the overall demand is over an agreed threshold.

Moreover, the problem formulation has been introduced into a rolling horizon approach. This reactive approach is based on a scheduling formulation that solves iteratively the deterministic problem by moving forward the optimisation horizon in each iteration (Silvente *et al.*, 2015b). This approach assumes that input information is updated once uncertain parameters (e.g., wind speed forecast, energy and heat demands) are revealed. Notice that these parameters may be uncertain or not accurate enough. Then, the optimal schedule for the updated resulting scenario (and optimisation horizon) may be found. This methodology takes into account different time horizons (Figure 2):

- Scheduling horizon, corresponding to the overall time period to be optimised.
- Prediction horizon, where all information related to this time horizon are assumed to be known with certainty,
- Control horizon, in which decisions of the optimisation procedure are applied.

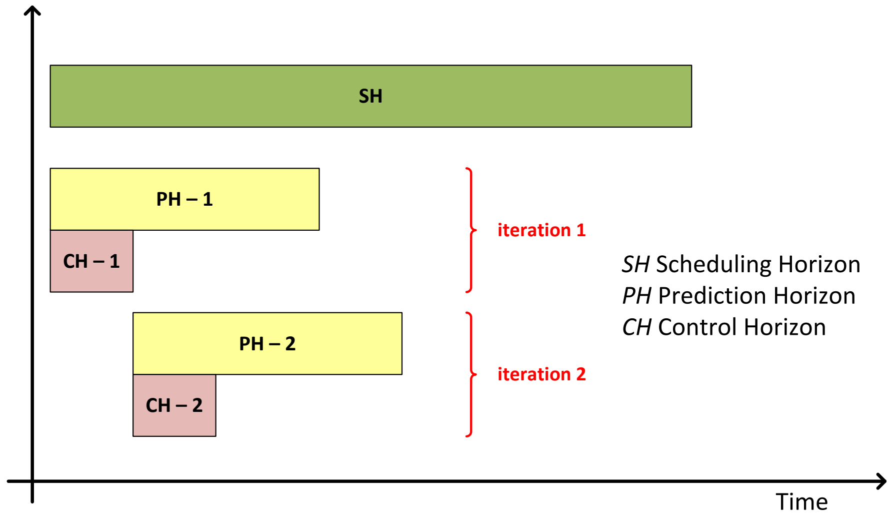

Figure 2. Time horizons associated with the rolling horizon scheme: scheduling, prediction and control horizon.

This formulation allows updating or modifying the status of the system to optimise the problem according to the current/updated data. This technique has been used to solve scheduling problems under uncertainty. For instance, Kopanos and Pistikopoulos (2014) presented a mathematical formulation for the management of a network by combining both rolling horizon approach and multi-parametric programming. More recently, Silvente *et al.* (2015b) presented a rolling horizon formulation to manage simultaneously the energy generation and demand within a microgrid, in order to maximise its profit.

The rolling horizon scheme is represented in Figure 3, which includes the time scales presented in Figure 2, and also links information of the past prediction horizon with the current prediction horizon. This approach can be applied as follows, and described in Figure 4 (Kopanos and Pistikopoulos, 2014):

- Firstly, the initial conditions of the system are established, as well as the durations of the scheduling, prediction and control horizons are defined.
- Then, the first prediction horizon period is solved. The values obtained through the optimisation process have to be fixed.
- At this time, uncertain parameters are updated. The scheduling problem has to be solved again, using information from the previous optimisation. This is given by the implementation of linking variables (see Section 3.11).
- If the new schedule corresponds to the final period of time, the procedure is completed. Otherwise, solutions obtained in the optimisation procedure for that iteration have to be fixed. Then, re-schedule and refresh the system until the final scheduling period.

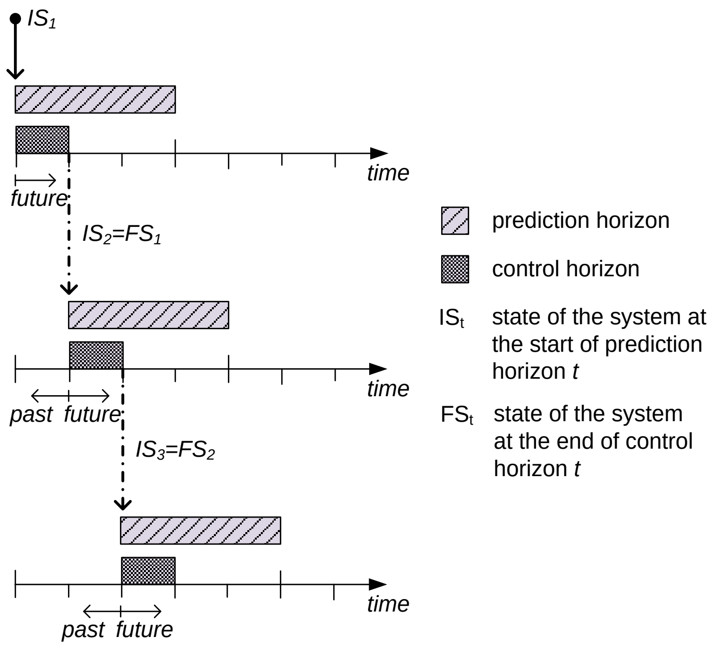

Figure 3. Reactive scheduling via a rolling horizon approach (Kopanos and Pistikopoulos, 2014).

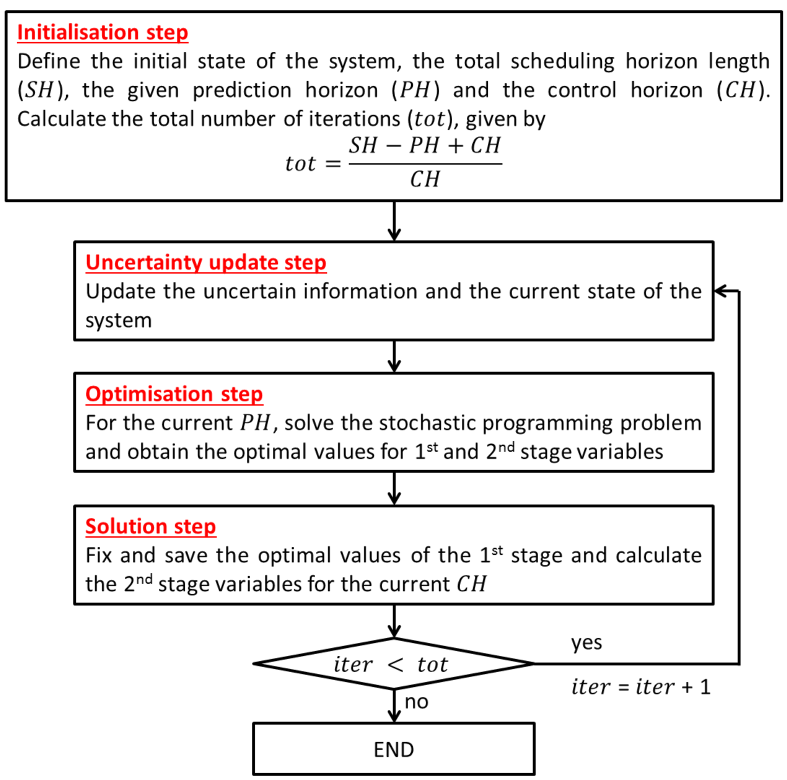

Figure 4. Algorithm for the rolling horizon approach.

The rolling horizon approach allows both updating and modifying the uncertain parameters, which can be associated with external and internal variabilities. The solution for each prediction horizon is optimal for this period of time. However, this solution could be suboptimal in practice when the overall scheduling horizon is considered. This occurs because future data outside the current prediction horizon is not considered when the optimisation procedure is performed. So, the duration of the prediction horizon should be suitable for obtaining good quality solutions. Therefore, the duration of the prediction horizon depends on the features of the problem under study (Kopanos and Pistikopoulos, 2014).

Moreover, the proposed rolling horizon approach is introduced into a two-stage stochastic formulation. This formulation contains two types of variables:

- First-stage variables, also known here-and-now variables, which do not depend on the scenario.
- Second-stage variables, also known wait-and-see variables, which will be different according to each scenario.

The decision making to minimise the cost of the microgrid involves:

(i) First-stage variables:
   a. The status (on/off) of each task i at home k in each time interval t, $W_{k,i,\theta,t}$ and $Z_{k,i,\theta,t}$, starting inside and outside the time window, respectively. Notice that the binary variable $W_{k,i,\theta,t}$ takes value 1 if task i at home k is active at time interval t for the operation period θ for tasks started inside the corresponding time window. Also, binary variable $Z_{k,i,\theta,t}$ is used for tasks started outside the established time window. These variables determine the schedule of consumption tasks within the microgrid.

(ii) Second-stage variables:
   a. The electric and thermal power $P_{s,t}^{B}$, $P_{s,t}^{CHP}$and $P_{s,t}^{W}$ to be generated through the boiler, CHP and wind turbines, respectively, in scenario s at time t.
   b. The electrical and heat storage levels, $S_{s,t}^{E}$ and $S_{s,t}^{T}$ respectively, in scenario s at the end of time t.
   c. The electrical charge and discharge rates ($S_{s,t}^{EC}$ and $S_{s,t}^{ED}$) and the heat charge and discharge rates ($S_{s,t}^{TC}$ and $S_{s,t}^{TD}$) in scenario s and time t.
   d. The amount of electric power to be imported from the power grid $Im_{s,t}$, and exported to the grid $Ex_{s,t}$ in scenario s and time t.
   e. The extra electric power purchases from grid $\gamma_{s,t}$ over a threshold β in scenario s at time t.

The proposed formulation corresponds to a discrete-time formulation, forcing tasks to start at the boundary of an interval. The computational effort to solve discrete-time formulations depends on the dimension of the problem under study. The duration of the intervals depends on the features of the problem. The effect of the length of the time interval was previously studied by Silvente *et al.* (2015a). Although short term time intervals may be used to obtain more accurate solutions, the problem could become unaffordable in terms of computational time. On the other hand, long-time intervals may lead to suboptimal solutions.

3. Mathematical model

The microgrid heat and electricity task scheduling problem is presented as a rolling horizon two-stage stochastic MILP model. The aim is to minimise the overall cost of the microgrid. The restrictions associated with the management of the microgrid are next detailed. In the presented formulation, $t \in TRH$ corresponds to the time intervals t included in the current prediction horizon, $i \epsilon I_k RH$ includes tasks i performed at home k included in the present prediction horizon and $\theta \in \Theta_{i,k} RH$ represents the task operation periods θ included in the current prediction horizon.

3.1. Generation and storage capacity constraints

The electric power $P^W_{s,t}$ generated by the wind turbines is calculated by applying equation (1). This constraint considers the number of wind turbines N_W, the air density ρ, the wind blade area A, the wind speed $v_{s,t}$ and the wind turbine efficiency η^w. The generation of electricity through wind turbines is constrained by the cut-in $\left(V^{cut-in}\right)$ and cut-out speed $(V^{cut-out})$. The cut-in and the cut-out speeds depend on the characteristic of the wind turbine. The cut-in speed is the minimum wind speed at which the wind turbine generates electricity according to its designated rated power; whereas the cut-out speed corresponds to the wind speed at which the wind turbine would be shut down to protect the turbine from any eventual damage (Villanueva and Feijóo. 2010). Thus, the wind turbine generates power only if the wind speed is above the cut-in speed and below the cut-out speed. Moreover, if the wind speed is above the nominal wind V^{nom}, the electricity generated corresponds to the maximum output, which corresponds to the generation level at the nominal wind speed (Zhang *et al.*, 2013).

$$P^W_{s,t} = \begin{cases} N_W \cdot 0.5 \cdot \rho \cdot A \cdot \eta^w \cdot min\left(v_{s,t}, V^{nom}\right)^3 & \forall s, t \epsilon TRH, V^{cut-in} \leq v_{s,t} \leq V^{cut-out} \\ 0 & \forall s, t \epsilon TRH, v_{s,t} < V^{cut-in}, v_{s,t} > V^{cut-out} \end{cases} \qquad (1)$$

The electric and thermal power generated by the CHP and the boiler generators in scenario s at time interval t ($P^{CHP}_{s,t}$ and $P^B_{s,t}$) should not exceed the designed generation capacity. This is given by constraints (2) and (3).

$$P^{CHP}_{s,t} \leq C^{CHP} \qquad \forall s, t \epsilon TRH \qquad (2)$$

$$P^B_{s,t} \leq C^B \qquad \forall s, t \epsilon TRH \qquad (3)$$

Also, the electricity and heat storage level in scenario s at time interval t ($S^E_{s,t}$ and $S^T_{s,t}$) should not exceed the maximum storage capacity, given by equations (4) and (5).

$$S^E_{s,t} \leq C^E \qquad \forall s, t \epsilon TRH \qquad (4)$$

$$S_{s,t}^{T} \leq C^{T} \qquad \forall s, t\epsilon TRH \qquad (5)$$

3.2. Electrical and thermal storage constraints

The electricity storage level in the electrical storage in scenario s at time interval t, $S_{s,t}^{E}$, is calculated through equation (6), considering the storage level at the previous period of time, $S_{s,t-1}^{E}$, and the electricity charged and discharged in the storage system. The efficiency of the charge and discharge of the electricity storage system η^{E}, has been considered. This efficiency involves electricity losses. Note that the electricity stored level at the beginning of the optimisation of the corresponding prediction horizon $S_{s,0}^{E}$ is equal to the electricity stored at the end of the optimisation procedure $S_{s,T}^{E}$, according to constraint (7). This corresponds to a terminal constraint used for the stability of the system. Moreover, electric power discharge and charge rates in scenario s at time t ($S_{s,t}^{ED}$ and $S_{s,t}^{EC}$) cannot exceed their limits (S^{EDmax} and S^{ECmax}). These rates are defined by the electrical storage characteristics. Notice that excessive discharge and charge rates would damage or even reduce the capacity of the storage system.

$$S_{s,t}^{E} = S_{s,t-1}^{E} + \delta \cdot \eta^{E} \cdot S_{s,t}^{EC} - \frac{\delta}{\eta^{E}} \cdot S_{s,t}^{ED} \qquad \forall s, t\epsilon TRH \qquad (6)$$

$$S_{s,0}^{E} = S_{s,T}^{E} \qquad \forall s \qquad (7)$$

$$S_{s,t}^{ED} \leq S^{EDmax} \qquad \forall s, t\epsilon TRH \qquad (8)$$

$$S_{s,t}^{EC} \leq S^{ECmax} \qquad \forall s, t\epsilon TRH \qquad (9)$$

Similarly, according to equation (10), heat storage level in the thermal storage system at time t in scenario s is given by the amount stored in the previous period $S_{s,t-1}^{T}$, and the heat charged and discharged in the heat storage system. The heat losses during the heat storage take into account the heat charge and discharge efficiency, η^{T}. According to constraint (11), the heat storage at the end of each optimisation period (i.e., prediction horizon) $S_{s,T}^{T}$ must return to its initial value $S_{s,0}^{T}$, avoiding any net heat accumulation. This constraint forces heat stored to return to the initial state at the end of the prediction horizon in scenario s. Moreover, thermal power discharge and charge rates in scenario s ($S_{s,t}^{TD}$ and $S_{s,t}^{TC}$) cannot surpass their limits (S^{TDmax} and S^{TCmax}), given by constraints (12) and (13).

$$S_{s,t}^{T} = S_{s,t-1}^{T} + \delta \cdot \eta^{T} \cdot S_{s,t}^{TC} - \frac{\delta}{\eta^{T}} \cdot S_{s,t}^{TD} \qquad \forall s, t\epsilon TRH \qquad (10)$$

$$S_{s,0}^{T} = S_{s,T}^{T} \qquad \forall s \qquad (11)$$

$$S_{s,t}^{TD} \leq S^{TDmax} \qquad \forall s, t\epsilon TRH \qquad (12)$$

$$S_{s,t}^{TC} \leq S^{TCmax} \qquad \forall s, t\epsilon TRH \qquad (13)$$

3.3. Starting and time of tasks

The initial time of a task could be within an established time window or outside the time window, according to constraint (14a). The time window for each task i is limited by a minimum and maximum starting time. Then, when a task starts inside the time window, the value of binary variable $W_{k,i,\theta,t}$ takes value 1 for the first value of the set θ. The first value of this set ($\theta = 0$) corresponds to the first time operation period. This binary variable corresponds to a first-stage variable, which does not depend on the scenario. Analogously, if a task starts after the time window, $Z_{k,i,\theta,t}$ takes value 1 in the starting time for the first value of the set θ. Note that if any task starts outside the current prediction horizon, $W_{k,i}^{o}$ or $Z_{k,i}^{o}$ takes value 1. So, all tasks are forced to start. If the demand side management is not considered, equation (14b) is applied instead of equation (14a). Constraint (14b) forces all tasks to start at the minimum starting time, not allowing any delay. Equation (15) is applied to ensure all tasks to be completed before the end of the overall scheduling horizon. This constraint avoids a task to start in a period of time where it cannot be finished.

$$\sum_{\substack{t\epsilon TRH \\ t\geq Ts_{k,i}^{min} \\ t\leq Ts_{k,i}^{max}}} W_{k,i,\theta,t} + \sum_{\substack{t\epsilon TRH \\ t>Ts_{k,i}^{max}}} Z_{k,i,\theta,t} + W_{k,i}^{o} + Z_{k,i}^{o} = 1 \qquad \forall k, i\epsilon I_k RH, \theta \in \Theta_{i,k} RH, \theta = 0 \qquad (14a)$$

$$\sum_{\substack{t\epsilon TRH \\ t=Ts_{k,i}^{min}}} W_{k,i,\theta,t} + W_{k,i}^{o} = 1 \qquad \forall k, i\epsilon I_k RH, \theta \in \Theta_{i,k} RH, \theta = 0 \qquad (14b)$$

$$\sum_{\substack{t\epsilon TRH \\ t>SH-\overline{PT}_{k,i}}} Z_{k,i,\theta,t} = 0 \qquad \forall k, i\epsilon I_k RH, \theta \in \Theta_{i,k} RH, \theta = 0 \qquad (15)$$

3.4. Penalties for delays in the starting time of tasks

In the first task operation period (which corresponds to $\theta = 0$), binary variables $W_{k,i,\theta,t}$ or $Z_{k,i,\theta,t}$ are active when a task i at home k starts at time t. These restrictions have been reformulated as a set of big-M constraints, according to equations (16) and (17), to determine the starting time of a task, $Ts_{k,i}$. Moreover, a penalisation $CPen_{k,i}$ is applied, considering the postponement in the starting time from the minimum starting time and the penalty cost of each task, according to equation (18). This set of equations is used to avoid unnecessary delays, since the solution of the current prediction horizon may lead to solutions where the solution can be to perform the task outside the current prediction horizon, minimising the solution of the current prediction horizon.

$$Ts_{k,i} \geq T_t - M \cdot \left(1 - W_{k,i,\theta,t} - Z_{k,i,\theta,t}\right) \qquad \forall k, i \epsilon I_k RH, \theta \in \Theta_{i,k} RH, \theta = 0, t \epsilon TRH \qquad (16)$$

$$Ts_{k,i} \leq T_t + M \cdot \left(1 - W_{k,i,\theta,t} - Z_{k,i,\theta,t}\right) \qquad \forall k, i \epsilon I_k RH, \theta \in \Theta_{i,k} RH, \theta = 0, t \epsilon TRH \qquad (17)$$

$$CPen_{k,i} = \left(Ts_{k,i} - Ts_{k,i}^{min}\right) \cdot \mu_{k,i} \qquad \forall k, i \epsilon I_k RH \qquad (18)$$

3.5. *Processing time and sequence of tasks*

Equation (19) forces all tasks to be completed. According to this equation, the summation of the active periods of time where the task takes place must be equal to the (roundup) processing time of the task. Notice that $\widetilde{W}_{k,i}$ and $\tilde{Z}_{k,i}$ are used to consider the processing time before the previous prediction horizon, inside and outside the time window, respectively. Equation (20) constrains the processing time of task i at home k to be performed outside the current prediction horizon, $\varphi_{k,i}$. This value is limited by its maximum value, given by the term $\tau_{k,i}$. Furthermore, equations (21) and (22) avoid any overlap in the task, establishing that only one operation period can be performed at time t. Note that $\overline{W}_{k,i,\theta,t}$ and $\bar{Z}_{k,i,\theta,t}$ are used to consider the status of any task in previous prediction horizons, inside and outside the time window, respectively. Moreover, equations (23) and (24) determine the sequence of the time operation periods within and outside the given time window, respectively. This set of two equations forces to follow the pre-established order in the consumption task profile. Furthermore, equation (25) is used to avoid any overlap in an equipment unit j able to perform more than one task i. Particularly, this equation forces that a task cannot start if the previous one has not finished (performed in the same equipment unit), not allowing neither an overlap nor a modification in the established task sequence.

$$\sum_{t \epsilon TRH} \sum_{\substack{\theta \in \Theta_{i,k} RH \\ \theta = 0}}^{\overline{PT}_{k,i}-1} W_{k,i,\theta,t} + \sum_{t \epsilon TRH} \sum_{\substack{\theta \in \Theta_{i,k} RH \\ \theta = 0}}^{\overline{PT}_{k,i}-1} Z_{k,i,\theta,t} + \widetilde{W}_{k,i} + \tilde{Z}_{k,i} + \varphi_{k,i} = \overline{PT}_{k,i} \qquad \forall k, i \epsilon I_k RH \qquad (19)$$

$$\varphi_{k,i} \leq \tau_{k,i} + 1 \qquad \forall k, i \epsilon I_k RH \qquad (20)$$

$$\sum_{\substack{\theta \in \Theta_{i,k} RH \\ \theta = 0}}^{\overline{PT}_{k,i}-1} \left(W_{k,i,\theta,t} + \overline{W}_{k,i,\theta,t} + Z_{k,i,\theta,t} + \bar{Z}_{k,i,\theta,t}\right) \leq 1 \qquad \forall k, i \epsilon I_k RH, t \epsilon TRH \qquad (21)$$

$$\sum_{t \epsilon TRH} \left(W_{k,i,\theta,t} + \overline{W}_{k,i,\theta,t} + Z_{k,i,\theta,t} + \bar{Z}_{k,i,\theta,t}\right) \leq 1 \qquad \forall k, i \epsilon I_k RH, \theta \in \Theta_{ik} RH \qquad (22)$$

$$\sum_{\substack{t' \epsilon TRH \\ t' \leq t}} \left(W_{k,i,\theta,t'} + \overline{W}_{k,i,\theta,t'} - W_{k,i,\theta+1,t'} \right) \geq 0 \qquad \forall k, i \epsilon I_k RH, \; \theta \in \Theta_{ik} RH, t \epsilon TRH \tag{23}$$

$$\sum_{\substack{t' \epsilon TRH \\ t' \leq t}} \left(Z_{k,i,\theta,t'} + \bar{Z}_{k,i,\theta,t'} - Z_{k,i,\theta+1,t'} \right) \geq 0 \qquad \forall k, i \epsilon I_k RH, \; \theta \in \Theta_{ik} RH, t \epsilon\, TRH \tag{24}$$

$$\sum_{\substack{t' \epsilon TRH \\ t' \leq t}} \left(W_{k,i,\theta,t'} + \overline{W}_{k,i,\theta,t'} + Z_{k,i,\theta,t'} + \bar{Z}_{k,i,\theta,t'} \right) \geq W_{k,i',\theta',t} + \overline{W}_{k,i',\theta',t} + Z_{k,i',\theta',t} + \bar{Z}_{k,i',\theta',t} \qquad \forall k, i \epsilon I_j, i' \epsilon I_j, i' > i, \; \theta \in \Theta_{ik} RH, \theta = \overline{PT}_{k,i} - 1, \; \theta' \in \Theta_{ik} RH, \theta' = 0, \; t \epsilon\, TRH \tag{25}$$

3.6. Interruption of tasks

The proposed formulation allows any eventual interruption in a task i. The following set of constraints is applied to manage interruptions. Hence, equation (26) is used to determine the potential status (on/off) of a task starting in the established time window. This equation considers the first and the last period of time where the task i is active, given by the minimum and maximum θ for each task. Thus, $\widehat{W}_{k,i,t}$ takes value 1 between the initial time and the calculated ending time. Observe that if a task i has started before the current prediction horizon, $W^o_{k,i}$ takes value 1, and 0 otherwise.

$$\widehat{W}_{k,i,t} = W^o_{k,i} + \sum_{\substack{t' \epsilon TRH \\ t' \leq t}} \left(W_{k,i,\theta,t'} + \overline{W}_{k,i,\theta,t'} \right) - \sum_{\substack{t' \epsilon TRH \\ t' < t}} W_{k,i,\theta',t'} \qquad \forall k, i \epsilon I_k RH, t \epsilon TRH, \; \theta \in \Theta_{i,k} RH, \theta = 0, \; \theta' \in \Theta_{i,k} RH, \theta' = \overline{PT}_{k,i} - 1 \tag{26}$$

Equations (27), (28) and (29) follow a general disjunctive programming (GDP) formulation, which is detailed in Appendix A.

Equation (27) establishes that if the potential status of a task is inactive ($\widehat{W}_{k,i,t} = 0$), there can be interruptions ($\lambda i^W_{k,i,t} = \lambda s^W_{k,i,t} = 0$). Equation (28) is applied to determine that an interruption is produced ($\lambda i^W_{k,i,t+1} = 1$) if a task is active at time t but inactive at time $t+1$ ($W_{k,i,\theta,t} = 1, W_{k,i,\theta,t+1} = 0$) if the potential status of a task is active ($\widehat{W}_{k,i,t} = 1$), which means that a task has started but it is not finished. This also avoids contemplating the end of a task as an interruption. Equation (29) is used to determine if a task continues interrupted at time $t+1$ ($\lambda s^W_{k,i,t+1} = 1$), for tasks which are inactive at t and at $t+1$ ($W_{k,i,\theta,t} = W_{k,i,\theta,t+1} = 0$), but its potential status is active ($\widehat{W}_{k,i,t} = 1$).

$$\widehat{W}_{k,i,t} \geq \lambda i^W_{k,i,t+1} + \lambda s^W_{k,i,t+1} \qquad \forall k, i \epsilon I_k RH, t \epsilon TRH \tag{27}$$

$$\sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta\le\overline{PT}_{k,i}}} W_{k,i,\theta,t+1} - \sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta<\overline{PT}_{k,i}}} W_{k,i,\theta,t} - \sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta<\overline{PT}_{k,i}}} \overline{W}_{k,i,\theta,t} - \widehat{W}_{k,i,t} + \lambda i^{W}_{k,i,t+1} + 1 \ge 0 \qquad \forall k, i\epsilon I_k RH, t\epsilon TRH \qquad (28)$$

$$\sum_{\substack{\theta\in\Theta_{ik}RH\\ \theta<\overline{PT}_{k,i}}} W_{k,i,\theta,t} + \sum_{\substack{\theta\in\Theta_{ik}RH\\ \theta<\overline{PT}_{k,i}}} \overline{W}_{k,i,\theta,t} + \sum_{\substack{\theta\in\Theta_{ik}RH\\ \theta\le\overline{PT}_{k,i}}} W_{k,i,\theta,t+1} - \widehat{W}_{k,i,t} + \lambda s^{W}_{k,i,t+1} \ge 0 \qquad \forall k, i\epsilon I_k RH, t\epsilon TRH \qquad (29)$$

Figure 5 explains the meaning of the following binary variables: $W_{k,i,\theta,t}$, $\widehat{W}_{k,i,t}$, $\lambda i^{W}_{k,i,t}$ and $\lambda s^{W}_{k,i,t}$. In this figure, a task whose processing time is 4 time intervals is represented. Thus, the task starts when $\theta = 0$ and finishes when $\theta = 5 - 1 = 4$. Hence, $\widehat{W}_{k,i,t} = 1$ during the first and the last time interval of the mentioned task. Also, observe that the figure remarks only the binary variables that are equal to 1.

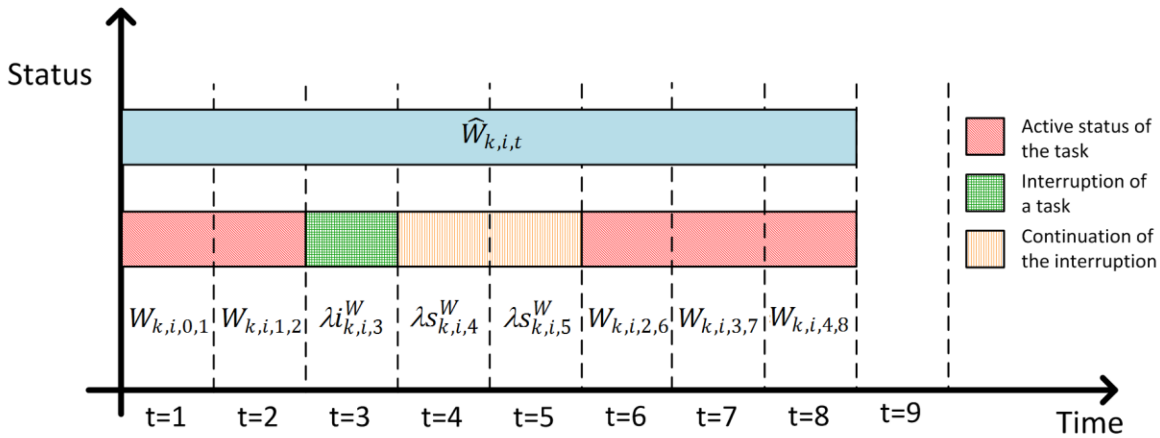

Figure 5. Explanation of binary variables involved in the interruption of tasks.

Equations (26) to (29) can be applied to those tasks that start outside the established time window, by implementing constraints (30) to (33).

$$\hat{Z}_{k,i,t} = Z^{o}_{k,i} + \sum_{\substack{t'\epsilon TRH\\ t'\le t\\ \theta=0}} \left(Z_{k,i,\theta,t'} + \bar{Z}_{k,i,\theta,t'}\right) - \sum_{\substack{t'\epsilon TRH\\ t'<t\\ \theta'=\overline{PT}_{k,i}}} Z_{k,i,\theta',t'} \qquad \forall k, i\epsilon I_k RH, t\epsilon TRH \qquad (30)$$

$$\hat{Z}_{k,i,t} \ge \lambda i^{Z}_{k,i,t+1} + \lambda s^{Z}_{k,i,t+1} \qquad \forall k, i\epsilon I_k RH, t\epsilon TRH \qquad (31)$$

$$\sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta\le\overline{PT}_{k,i}}} Z_{k,i,\theta,t+1} - \sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta<\overline{PT}_{k,i}}} Z_{k,i,\theta,t} - \sum_{\substack{\theta\in\Theta_{i,k}RH\\ \theta<\overline{PT}_{k,i}}} \bar{Z}_{k,i,\theta,t} - \hat{Z}_{k,i,t} + \lambda i^{Z}_{k,i,t+1} + 1 \ge 0 \qquad \forall k, i\epsilon I_k RH, t\epsilon TRH \qquad (32)$$

$$\sum_{\substack{\theta \in \Theta_{i,k}RH \\ \theta < \overline{PT}_{k,i}}} Z_{k,i,\theta,t} + \sum_{\substack{\theta \in \Theta_{i,k}RH \\ \theta < \overline{PT}_{k,i}}} \bar{Z}_{k,i,\theta,t} + \sum_{\substack{\theta \in \Theta_{i,k}RH \\ \theta \leq \overline{PT}_{k,i}}} Z_{k,i,\theta,t+1} - \hat{Z}_{k,i,t} + \lambda s^{Z}_{k,i,t+1} \geq 0 \qquad \forall k, i \epsilon I_k RH, t \epsilon TRH \qquad (33)$$

3.7. *Sequence of tasks*

A task i in equipment unit j cannot overlap another task i' in the same equipment unit j. Equation (34) avoids a task i' to start if the previous task i in the same equipment unit j has not finished.

$$\widehat{W}_{k,i,t} + \hat{Z}_{k,i,t} + W_{k,i,\theta,t} + Z_{k,i,\theta,t} \leq 1 \qquad \forall k, i \epsilon I_k RH, i' \epsilon I_k RH, i' > i, j, i \epsilon I_j, i' \epsilon I_j, t \epsilon TRH, \theta = 0 \qquad (34)$$

3.8. *Energy and heat balances*

Equation (35) determines the total electricity consumption at time interval t for each scenario s, for tasks performed within the established time window, which corresponds to the summation of the power consumption requirements from all tasks at different homes k. The electricity consumed at time period t considers the electricity supplied by the generators ($\delta \cdot P^{W}_{s,t}$ and $\delta \cdot P^{CHP}_{s,t}$), the energy received from the electrical storage ($\delta \cdot S^{ED}_{s,t}$), the electricity purchases from the power grid ($\delta \cdot Im_{s,t}$), the electricity sent to the electrical storage $\delta \cdot S^{EC}_{s,t}$ and electricity exported to the grid $\delta \cdot Ex_{s,t}$. The energy consumption is calculated considering the task operation periods θ (see Figure 1). Moreover, the use of electricity generated in the microgrid to perform a task i is not allowed when a task starts outside the established time window. Thus, according to constraint (36), electricity purchases to the grid are required to satisfy this particular energy requirement. The term $\delta \cdot Im^{Z}_{t,s}$ represents electricity purchases to the power grid to satisfy energy requirements of tasks started outside the time window.

$$\delta \cdot \sum_{k} \sum_{i \epsilon I_k RH} \left[\sum_{\substack{\theta \in \Theta_{i,k}RH \\ \theta \leq \overline{PT}_{k,i}}} C_{i,\theta} \cdot W_{k,i,\theta,t} - C_{i,\theta'} \cdot W_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i,s}\right) \right] = \delta \cdot \left(P^{W}_{s,t} + P^{CHP}_{s,t} + S^{ED}_{s,t} + Im_{s,t} - S^{EC}_{s,t} - Ex_{s,t}\right) \qquad \forall s, t \epsilon TRH, \theta' = \overline{PT}_{k,i} \qquad (35)$$

$$\delta \cdot \sum_{k} \sum_{i \epsilon I_k RH} \left[\sum_{\substack{\theta \in \Theta_{i,k}RH \\ \theta \leq \overline{PT}_{k,i}}} C_{i,\theta} \cdot W_{k,i,\theta,t} - C_{i,\theta'} \cdot Z_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i,s}\right) \right] = \delta \cdot Im^{Z}_{t,s} \qquad \forall s, t \epsilon TRH, \theta' = \overline{PT}_{k,i} \qquad (36)$$

Note that terms $C_{i,\theta'} \cdot W_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i,s}\right)$ and $C_{i,\theta'} \cdot Z_{k,i,\theta',t} \cdot \left(\overline{PT}_{k,i} - PT_{k,i,s}\right)$ in constraints (32) and (33) are used to calculate the exact energy consumption for energy consumption tasks that do not take place in the overall time interval. This allows improving the energy balance of the microgrid, due to an enhanced adjustment of the processing time of tasks. The implication of this term is explained in Figure 6. Although in this figure only binary variable $W_{k,i,\theta,t}$ is used, the same plot can be applied to binary variable $Z_{k,i,\theta,t}$.

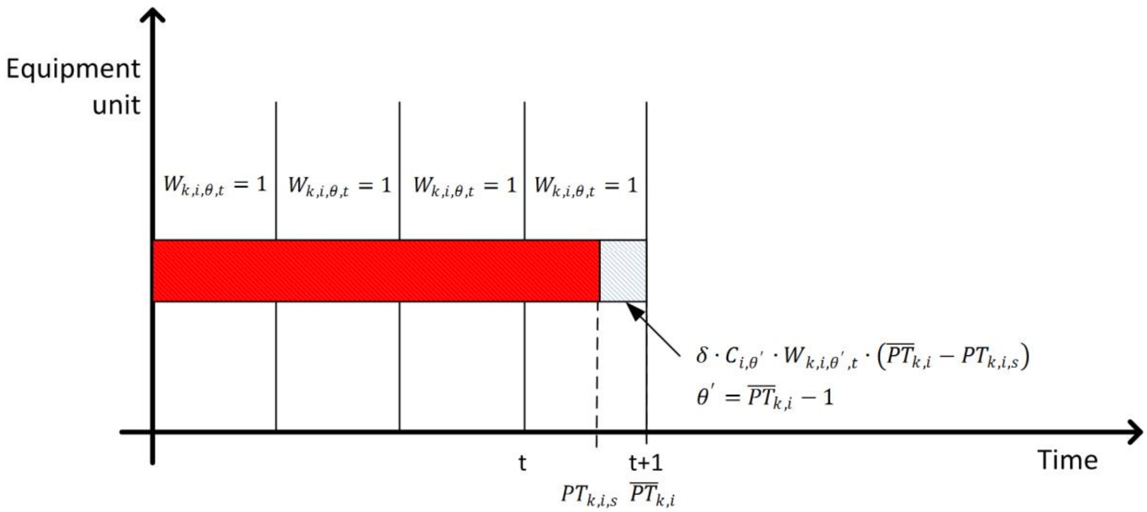

Figure 6. Schedule of an equipment unit affected by the processing time.

The heat consumed $\delta \cdot H_{s,t}$ at time interval t in scenario s considers the heat supplied by the considered CHP generator and boiler ($\delta \cdot P_{s,t}^{CHP}$ and $\delta \cdot P_{s,t}^{B}$) as well as the heat received from the thermal storage $\delta \cdot S_{s,t}^{TD}$, the heat transmitted to the thermal storage $\delta \cdot S_{s,t}^{TC}$, as well as any eventual unsatisfied heat demand $UH_{s,t}$.

$$\delta \cdot H_{s,t} = \delta \cdot \left(\alpha \cdot P_{s,t}^{CHP} + P_{s,t}^{B} + S_{s,t}^{TD} - S_{s,t}^{TC} + UH_{s,t}\right) \qquad \forall s, t \epsilon TRH \qquad (37)$$

3.9. *Peak of power demand*

One of the objectives of the management of the microgrid is to reduce the peak of power demand from the power grid. Thus, equation (38) is applied to achieve this goal. According to this equation, if electric power purchases from grid exceed an agreed threshold β at time t, the additional amount γ_t over threshold β is charged with an additional rate. On the contrary, if the electric power purchases are below the threshold β, the variable γ_t takes value 0, and the normal electricity price is applied.

$$\gamma_{s,t} \geq Im_{s,t} + Im_{t,s}^{Z} - \beta \qquad \forall s, t \epsilon TRH \qquad (38)$$

3.10. Objective function

The aim of the proposed mathematical formulation is to minimise the operational cost ϕ of the microgrid. This includes costs of generators and storage systems, purchases, penalties and revenues from electricity exported to the grid. Particularly, the following elements are considered in the objective function calculated in equation (39), considering the probability that a scenario takes place Pr_s. Notice that capital costs are independent of the schedule. Since the equipment capacities are predetermined, capital costs are not considered.

$\phi =$	Operational cost (to be minimised)
$\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} \frac{ng}{\alpha} \cdot P_{s,t}^{CHP}$	Operation and maintenance costs of the CHP generator
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} m^W \cdot P_{s,t}^W$	Operation and maintenance costs of the wind turbine
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} \frac{ng}{\eta^B} \cdot P_{s,t}^B$	Operation and maintenance costs of the boiler
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} m^E \cdot S_{s,t}^{ED}$	Electrical storage costs
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} m^T \cdot S_{s,t}^{TD}$	Thermal storage costs
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} b_t \cdot Im_{s,t}$	Cost of electricity imported from the grid
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} m^Z \cdot b_t \cdot Im_{s,t}^Z$	Cost of electricity imported from the grid for tasks starting outside the time windows
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} pk \cdot \gamma_{s,t}$	Revenues from electricity exported to the grid
$+\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} m^U \cdot UH_{s,t}$	Penalty cost for non-satisfying the heat demand
$-\delta \cdot \sum_{s} Pr_s \cdot \sum_{t \epsilon TRH} q \cdot Ex_{s,t}$	Revenues from electricity exported to the grid
$+\sum_{k} \sum_{i \epsilon I_k RH} \sum_{t \epsilon TRH} \mu i_i^W \cdot \lambda i_{k,i,t}^W$	Penalty cost if task starting inside the time windows is interrupted
$+\sum_{k} \sum_{i \epsilon I_k RH} \sum_{t \epsilon TRH} \mu s_i^W \cdot \lambda s_{k,i,t}^W$	Penalty cost if task starting inside the time windows remains interrupted
$+\sum_{k} \sum_{i \epsilon I_k RH} \sum_{t \epsilon TRH} \mu i_i^Z \cdot \lambda i_{k,i,t}^Z$	Penalty cost if task starting outside the time windows is interrupted
$+\sum_{k} \sum_{i \epsilon I_k RH} \sum_{t \epsilon TRH} \mu s_i^Z \cdot \lambda s_{k,i,t}^Z$	Penalty cost if task starting outside the time windows remains interrupted

$+\sum_{k}\sum_{i\epsilon I_k RH} CPen_{k,i}$ Penalties in case of deviation from the initial starting time (39)

3.11. Rolling horizon

The rolling horizon approach requires linking past events with the up-to-date status of the system. Thus, the following equations and variables are implemented to link past decisions with the current prediction horizon. Equations (40) and (41) update the information related to the initial time of task i at home k inside and outside the time window, respectively. These equations are used to consider the starting point of a given task outside the current prediction horizon. If $W_{k,i}^{o}$ (or $Z_{k,i}^{o}$) in iteration rh takes value 1, this task starts outside the current prediction horizon. Since all tasks must start, these two equations avoid infeasibilities for non-starting tasks. The binary parameter $W_{k,i}^{o}$ (or $Z_{k,i}^{o}$) takes into account the value of the binary variable $W_{k,i,\theta,t}$ (or $Z_{k,i,\theta,t}$), which takes value 1 for $\theta = 0$ if task i at home k starts at time t. The value of these binary variables for future decisions can vary in each iteration rh, but not for past decisions. Equations (42) and (43) update the processing time of task i at home k started before the current prediction horizon (PH) inside and outside the established time window respectively (i.e., periods of time where each task was active), which allows updating the remaining processing time of that tasks. Furthermore, equations (44) and (45) update the overall status of task i at home k, considering the task operation period θ, which is used to establish the consumption profile in a given task (notice that the consumption profile could not be constant).

$$\left(W_{k,i}^{o}\right)_{rh} = \left(W_{k,i}^{o}\right)_{rh-1} + \left(W_{k,i,\theta,t}\right)_{\substack{rh-1\\ \theta=0\\ t=T_t-CH+1}} \qquad \forall k, i\epsilon I_k RH \qquad (40)$$

$$\left(Z_{k,i}^{o}\right)_{rh} = \left(Z_{k,i}^{o}\right)_{rh-1} + \left(Z_{k,i,\theta,t}\right)_{\substack{rh-1\\ \theta=0\\ t=T_t-CH+1}} \qquad \forall k, i\epsilon I_k RH \qquad (41)$$

$$\left(\widetilde{W}_{k,i}\right)_{rh} = \left(\widetilde{W}_{k,i}\right)_{rh-1} + \left(\sum_{\theta\in\Theta_{i,k}RH} W_{k,i,\theta,t}\right)_{\substack{rh-1\\ t=T_t-CH+1}} \qquad \forall k, i\epsilon I_k RH \qquad (42)$$

$$\left(\tilde{Z}_{k,i}\right)_{rh} = \left(\tilde{Z}_{k,i}\right)_{rh-1} + \left(\sum_{\theta\in\Theta_{i,k}RH} Z_{k,i,\theta,t}\right)_{\substack{rh-1\\ t=T_t-CH+1}} \qquad \forall k, i\epsilon I_k RH \qquad (43)$$

$$\left(\overline{W}_{k,i,\theta,t}\right)_{rh} = \left(\overline{W}_{k,i,\theta,t}\right)_{rh-1} + \left(\sum_{\theta\in\Theta_{i,k}RH} W_{k,i,\theta,t'}\right)_{\substack{rh-1\\ t=T_t-CH+1}} \qquad \forall k, i\epsilon I_k RH, \theta\in\Theta_{i,k}RH \qquad (44)$$

$$(\bar{Z}_{k,i,\theta,t})_{rh} = (\bar{Z}_{k,i,\theta,t})_{rh-1} + \left(\sum_{\theta \in \Theta_{i,k}RH} Z_{k,i,\theta,t'}\right)_{t=T_t-CH+1}^{rh-1} \qquad \forall k, i \epsilon I_k RH, \theta \in \Theta_{i,k}RH \quad (45)$$

The electrical and heat storage levels of the preceding control horizon in each scenario s is connected to the initial electrical and heat storage levels of the current prediction horizon by applying equations (46) and (47), respectively. Also, the remaining processing time of task i at home k is also updated for each iteration, as calculated in equation (48). The current processing time is determined as the difference between initial processing time $PT0_{k,i,s}$ and the periods of time where this task was active ($\widetilde{W}_{k,i}$ and $\tilde{Z}_{k,i}$). Finally, equation (49) updates the maximum periods of time outside the current prediction horizon (PH) where a task i at home k can take place, in order to avoid any eventual unfeasibility for non-satisfying the completion of any task. The optimisation problem is iteratively solved as reported by the rolling horizon scheme. This approach will be used to update input information to the up-to-date available data.

$$S^E_{s,0} = \widehat{S^E}_{s,t+1} \qquad \forall s, t = T_t - CH + 1 \qquad (46)$$

$$S^T_{s,0} = \widehat{S^T}_{s,t+1} \qquad \forall s, t = T_t - CH + 1 \qquad (47)$$

$$PT_{k,i,s} = PT0_{k,i,s} - \widetilde{W}_{k,i} - \tilde{Z}_{k,i} \qquad \forall k, i \epsilon I_k RH, s \qquad (48)$$

$$\tau_{k,i} = SH - T_t \cdot CH - PH - 1 \qquad \forall k, i \epsilon I_k RH \qquad (49)$$

Here it should be noted that not all tasks are included in each iteration. For instance, tasks already completed are not considered in the current iteration. Thus, tasks that could start or have started but are not fully completed at the time the current iteration starts are considered by the algorithm. Notice that a task i is unfinished in iteration rh if $(PT_{k,i,s})_{rh} > 0$. As a consequence, tasks are not taken into account if the minimum starting time $Ts^{min}_{k,i}$ is located after the current prediction horizon.

4. Illustrative example

The scheduling problem includes electricity and heat generation, purchases, sales, storage and schedule of tasks of a microgrid to be optimised. Figure 7 represents a scheme of the considered microgrid.

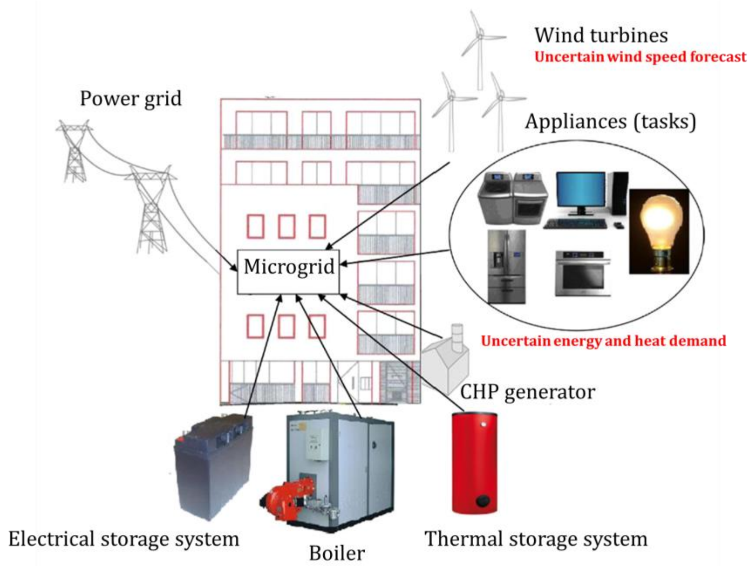

Figure 7. Schematic diagram of the microgrid (Silvente *et al.*, 2017).

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Equal-size time intervals are considered. Input information and decisions related to electricity and heat generation and demand are given for every 30 min, which corresponds to the duration of each time interval as well as the length of the control horizon ($CH = 1$ time interval). The total scheduling horizon considered is 24 h. The length of the time intervals is 30 min. Prediction Horizons (PHs) of 2, 4 and 6 hours have been considered. Input data related to uncertain parameters (e.g., electricity and heat demands, wind forecast) are refreshed at the beginning of time intervals.

The case study considers a microgrid formed of 1, 5 and 10 homes. The technical information and costs are detailed below:

- Wind turbines. The capacity of the wind turbine corresponds to 10 kW_e. The maintenance cost is 0.005 £/kWh_e. The cut-in speed is 5.0 m/s, whereas the cut-out wind is 25.0 m/s, and the nominal wind speed is 12.0 m/s. The power coefficient is 47%, and the blade diameter is 4.0 m.
- One common CHP generator. The electrical efficiency is assumed to be 35%. Heat to power ratio is considered to be equal to 1.3. Natural gas cost is 0.027 £/kWh.
- One common boiler.
- One common electrical storage unit. Charge and discharge efficiencies are 95%. Charge and discharge limits correspond to 0.333 kW_e. Maintenance cost is 0.005 £/kWh_e.
- One common thermal storage unit. Charge and discharge efficiencies are 98%. Charge and discharge limits correspond to 0.667 kW_{th}. Maintenance cost is 0.001 £/kWh_{th}.
- The connection the power grid, which allows importing and exporting electricity. The real-time electricity price is collected from Balancing Mechanism Reporting System. Additionally,

an extra cost of 0.05 $£/kWh_e$ is applied when electric power demand per home from the grid is over the agreed threshold. Electricity may also be exported to the power grid with 0.01 $£/kWh_e$.

The number of considered wind turbines for the different homes, as well as the maximum capacities for the CHP, boiler, electrical storage, thermal storage and the threshold is shown in Table 1.

Table 1. Physical constraints and capacities.

Element	1 home	5 homes	10 homes
Number of wind turbines, N_W	1	4	7
CHP capacity, P^{CHP} (kW)	1.2	4.8	8.4
Boiler capacity, P^B (kW)	2.8	11.2	19.6
Electrical storage capacity, C^E (kWh)	0.5	2.0	3.5
Thermal storage capacity, C^T (kWh)	0.7	2.8	4.9
Threshold, β (kW)	3.0	12.0	21.0

Furthermore, 12 equipment units j per home k are considered to operate 16 different tasks i. These tasks allow a degree of flexibility in their starting time, accepting some delays from the established minimum starting time $Ts_{k,i}^{min}$. Also, all tasks except $i1$ and $i2$, have constant power consumption. Data related to power requirements to perform a task, minimum and maximum starting time (constituting the time window) and processing time of tasks can be found in Table 2. The power requirements for the non-constant profile tasks, corresponding to tasks $i1$ and $i2$, is presented in Table 3.

Table 2. Power requirements, earliest and latest starting time and processing time of tasks.

Task i	Equipment unit j	Power, $C_{i,\theta}$ (kW)	Minimum starting time, $Ts_{k,i}^{min}$ (h)	Maximum starting time, $Ts_{k,i}^{max}$ (h)	Processing time, $PT_{k,i,s}$ (h)
$i1$	$j1$	Variable	0.5	7.0	2.0
$i2$	$j2$	Variable	1.0	4.0	1.5
$i3$	$j3$	2.50	5.0	9.0	1.2
$i4$	$j4$	3.00	0.0	4.5	1.0
$i5$	$j5$	5.00	10.0	14.5	0.7
$i6$	$j6$	1.70	0.0	2.0	0.5
$i7$	$j7$	0.84	10.0	10.5	6.0
$i8$	$j8$	0.10	9.0	16.0	2.3
$i9$	$j9$	0.30	9.0	13.5	3.0
$i10$	$j10$	1.20	0.0	9.5	0.8
$i11$	$j11$	0.30	0.0	24.0	24.0
$i12$	$j12$	3.50	10.0	14.0	3.1
$i13$	$j3$	2.50	10.5	15.5	1.8
$i14$	$j6$	1.70	6.5	12.0	0.9
$i15$	$j9$	0.30	16.5	21.0	3.4
$i16$	$j12$	3.50	17.5	22.0	1.5

Table 3. Variable power requirements.

Operation period θ	Task $i1$	Task $i2$
$\theta 0$	1.80	2.15
$\theta 1$	0.22	0.21
$\theta 2$	1.80	0.45
$\theta 3$	0.22	

Data related to heat demand, wind forecast speed as well as electricity prices are presented in Figures B1, B2 and B3 respectively in Appendix B. Also, penalty costs for interrupting or for remaining interrupted a task i started inside or outside the time window is presented in Table B1 in Appendix B. This table also contains the penalty cost from deviations from the minimum starting time of task i at home k. Furthermore, a penalty cost of 0.2 $£/kWh_{th}$ is applied when the heat demand is not satisfied. If any task i starts outside the time window, an extra cost factor m^{Z} is applied, which takes value 1.50. This involves increasing the price of the purchases of electricity to the power grid by 50%.

Different scenarios have been taken into account. These scenarios include variations in the nominal value of the expected wind speed, processing time of tasks and overall heat demand. This is introduced in the model in order to embrace all possible situations. Particularly, three different situations have been considered in each time interval for the wind speed $v_{s,t}$, processing time $PT_{k,i,s}$ and heat demand $H_{s,t}$. Each situation for each considers input parameter considers three different situations, which corresponds to a low, medium and high value. This corresponds to the 80%, 100% and 120% of the nominal value, respectively. Also, each scenario has an associated occurrence probability. The probability of each scenario and the values (low, medium and high) associated with the considered scenarios can be found in Table B2 in Appendix B.

Notice that the seasonality may affect the operations of the microgrid, since the generation and the demand may be affected by the conditions of the season. Furthermore, another factor that may influence the management of the microgrid is the location, which can modify the availability and requirements of energy. In our case, the proposed illustrative example evaluates one selected day type, in order to highlight to potential benefits of using a rolling horizon stochastic formulation to deal with uncertainty in the management of microgrids.

5. Results and discussion

5.1. Monte Carlo simulation

Monte Carlo simulation technique has been used to assess the impact of uncertain parameters associated with the microgrid (e.g., wind turbine speed, overall heat demand and processing time of a task) in the objective function. Monte Carlo simulation has been implemented in GAMS. This simulation requires an iterative algorithm. Firstly, the model is solved under predicted conditions. Once the model has been solved, the binary decisions (i.e., schedule of tasks), has been fixed. Then, the model has been solved iteratively, considering variations in input parameters. Particularly, the model has been solved for 300 iterations, considering the following Normal distributions (Table 4). The normal probability distribution is commonly applied to natural phenomena and for observations where the mean value converge in a central point and are symmetric about this point. This continuous probability distribution is characterised by a mean value and a standard deviation. There are other representations, such as binomial and Poisson distributions, which are applied for discrete distributions.

Table 4. Characteristics of the considered probability distributions.

Parameter	Probability distribution	Mean value	Standard deviation
Wind speed	Normal distribution	Nominal wind speed	0.50·Nominal wind speed
Heat demand	Normal distribution	Nominal heat demand	0.30·Nominal heat demand
Processing time	Normal distribution	Nominal processing time	0.30·Nominal processing time

Notice that binary variables are fixed. So, although variability in the processing time of tasks is applied, the minimum and maximum values of the processing time under uncertainty correspond to the boundaries of the time intervals of the nominal processing time.

Figure 8 shows the expected cost for 1 house after running different iterations and the cost for the deterministic problem. The value of the objective function is worsened by 19.6% when variability in the above mentioned parameters are applied. So, the variability associated with the microgrid requires managing the uncertainty to obtain good and quality practical solutions. Thus, the rolling horizon stochastic formulation constitutes one way to manage the uncertainty.

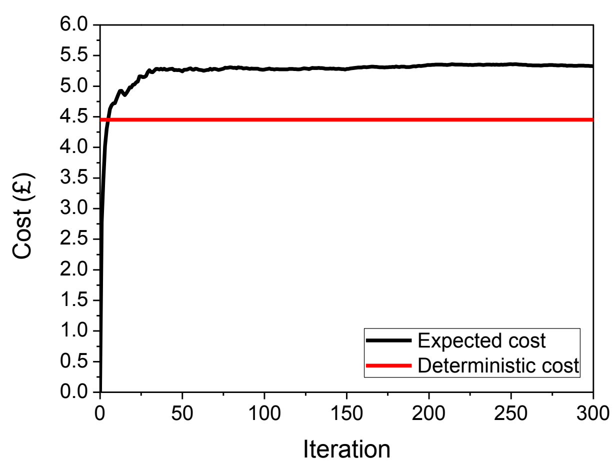

Figure 8. Results of the Monte Carlo simulation.

5.2. *Rolling horizon stochastic approach*

The solution of the model stipulates the optimal schedule for electricity and heat generation, purchases, sales, storage and consumption that minimises the cost of the microgrid. The combination of the rolling horizon and stochastic programming used to tackle uncertainty, has allowed updating input data and considering uncertain profiles. This update includes wind speed, the processing time of tasks and the overall heat demand. The results next presented consider a single home for a prediction horizon of 4 h for the most common scenario, corresponding to medium values of wind forecast, processing time of tasks and overall heat demand levels, which corresponds to scenario $s14$. Figures 9a and 9b show the electrical power generation, purchases and sales; and the heat generation profiles, respectively. The electricity and heat storage level profiles are presented in Figure 9c. Also, Figure 9d plots the daily schedule of tasks. Some energy tasks have been delayed, applying a penalty cost for these deviations from the initial starting time. These delays correspond to tasks $i4$ (3.0 h), $i5$ (3.0 h), $i12$ (0.5 h), $i13$ (10.0 h) and $i16$ (2.0 h). The mentioned tasks have a relatively low penalty for delaying their initial starting time. Moreover, tasks $i3$, $i7$, $i10$, $i12$, $i15$ and $i16$ are interrupted. This disruption of the task is produced because is more economical rather than purchasing electricity to the grid. The interested reader can be referred to Silvente and Papageorgiou (2017) for more detail explanations related to the effect of the delays in the starting time of tasks as well as in their interruption.

Notice that the intervals where tasks are active (i.e., consuming electricity) are the same for all scenarios, since the binary variable determining electricity consumption is a first-stage variable. Therefore, the active time intervals do not depend on the scenario. This is a big advantage of this methodology since different scenarios have been considered simultaneously. Thus, this reactive and proactive approach allows updating information, to react to alterations from the initial conditions, such as modifications in the power availability (i.e., wind speed), processing tasks of tasks and in the overall heat demand. Furthermore, different durations of the prediction horizon have been taken into account. This has been done to analyse and compare the obtained solutions by considering different durations of the prediction horizon. Therefore, predictions horizons of 2, 4, 6 and 8 h have been contemplated. The control horizon for these situations has been established in 0.5 h, which corresponds to 1 time interval. Also, the perfect information case has been considered. This hypothetical situation corresponds to the case where the duration of the prediction horizon is equivalent to the overall scheduling horizon (24 h), assuming that all input parameters are certain. However, this is not a realistic situation, since the system may be affected by variability.

Table 5 contains the electric power generation, purchases and sales levels as well as thermal power generation level for one home considering different durations of the prediction horizons and the perfect information case. Based on the obtained results, there is a substantial reduction on the dependence to the connected grid, in terms of electricity purchases, when the duration of the prediction horizon is increased. The differences in the imported electricity according to the duration of the prediction horizon are due to the fact that as the duration of the prediction horizon increase, more information is received in the system to proceed to solve the optimisation problem. As the availability of future data is reduced when short prediction horizons are considered, there is no data of future power requirements. Consequently, due to this lack of future information, electricity is sold to the power grid instead of been stored. Accordingly, the overall cost decreases for longer prediction horizons. Thus, the use of accurate forecast techniques will allow improving the prediction of future events, increasing the duration of the prediction horizon and improving the quality of the obtained solution.

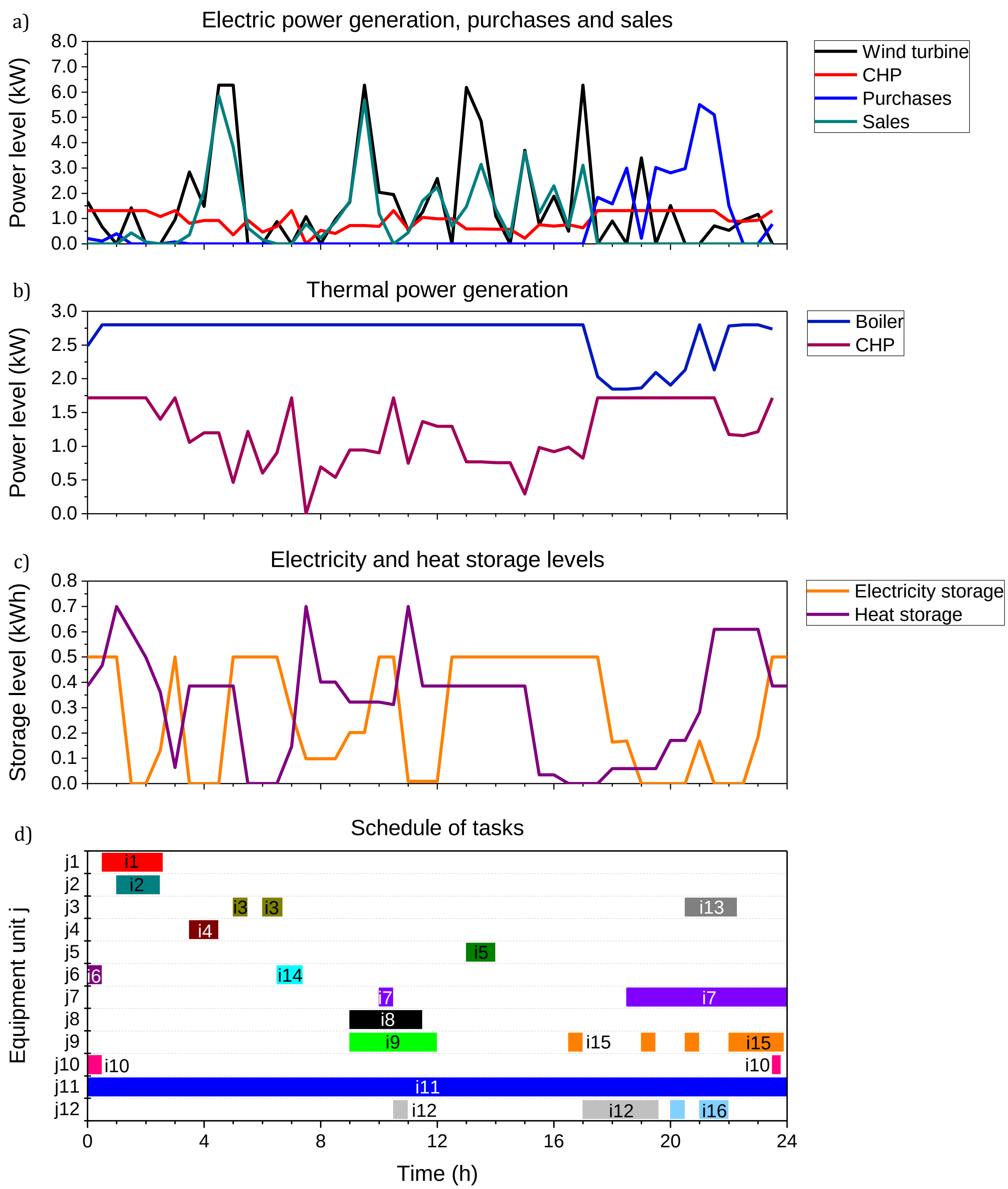

Figure 9. a) Electric power generation, purchases and sales for a given scenario; b) Thermal power generation profile for a given scenario; c) Electricity and heat storage level profiles for a given scenario; d) Schedule of tasks for 1 home and prediction horizon of 4 hours.

Table 5. Electricity and heat generation, purchases and sales for 1 home considering different prediction horizons for 1 home.

Key Performance Indicator	$PH = 24h$	$PH = 8h$	$PH = 6h$	$PH = 4h$	$PH = 2h$
Electricity via wind turbine (kWh)	37.6	37.6	37.6	37.6	37.6
Electricity via CHP (kWh)	21.5	20.9	21.3	22.4	21.6
Electricity purchases (kWh)	3.1	6.1	10.3	14.6	19.7
Electricity sales (kWh)	10.5	13.0	17.7	23.1	28.2
Heat via boiler (kWh)	64.9	65.6	65.0	63.7	64.7
Heat via CHP (kWh)	27.9	27.2	27.7	29.1	28.1

The duration of the prediction horizon also affects the delays associated with each task. Table 6 summarises the delays according to the duration of the prediction horizon. According to expectations, longer prediction horizons involve less delay in the starting time of tasks. Shorter prediction horizons involve more delays. This is because in each iteration, the optimisation procedure obtains the optimal solution for the mentioned iteration. Thus, postponing some tasks (applying a penalty) is more economical that perform them, due to the generation costs involved in the process. Naturally, longer prediction horizons also try to find the optimal results for the considered iterations, but receiving more information, such as complete information of the time window in which the task should be carried out. Therefore, delays are reduced compared with shorter time horizons.

Table 6. Delays for different prediction horizons for 1 home.

Tasks	Delays (h) for prediction horizons				
	$PH = 24h$	$PH = 8h$	$PH = 6h$	$PH = 4h$	$PH = 2h$
$i4$	3.0	3.5	3.5	3.0	22.0
$i5$	3.0	3.0	3.0	3.0	3.0
$i7$	0.0	0.0	0.0	0.0	8.0
$i12$	0.5	2.0	0.5	0.5	0.0
$i13$	1.0	0.0	7.5	10.0	11.5
$i15$	0.0	0.0	0.0	0.0	1.5
$i16$	0.0	0.0	0.5	2.0	4.5
Total delays (h)	7.5	8.5	15.0	18.5	50.5

Obviously, the duration of the prediction horizon also affects the length of the interruption of tasks. Thus, longer prediction horizons implicate a reduction in the duration of the interruptions. However, the time interruptions for the shortest prediction horizon are reduced. One reason that explains this is that previous delays do not allow interrupting some tasks. For example, tasks $i7$, $i15$ and $i16$ were postponed, which involve less interruptions to complete tasks within the scheduling horizon. Table 7 displays the interruptions for different prediction horizons for one home.

Table 7. Interruptions for different prediction horizons for 1 home.

Tasks	Time interruptions (h) for prediction horizons				
	$PH = 24h$	$PH = 8h$	$PH = 6h$	$PH = 4h$	$PH = 2h$
$i3$	0.0	4.0	11.5	14.0	3.5
$i5$	0.0	0.5	0.0	0.0	0.0
$i7$	8.0	8.0	8.0	8.0	1.0
$i9$	0.0	0.0	0.0	0.0	8.5
$i10$	0.0	0.0	0.0	23.5	23.5
$i12$	2.5	1.0	4.0	6.0	8.5
$i13$	0.0	5.0	0.0	0.0	0.0
$i15$	3.5	3.5	3.5	4.0	0.0
$i16$	1.0	1.0	0.5	0.5	0.0
Time interrupted (h)	15.0	23.0	27.5	56.0	45.0

The implementation of the rolling horizon may involve constant variations in the expected schedule since the problem is solved iteratively. This is produced because input data is refreshed and new information is received in the optimisation procedure in each iteration. The interested reader can be referred to Silvente *et al*. (2015b) for more detail explanations. To highlight these alterations, Figure 10 shows the changes produced in the schedule of tasks $i15$ and $i16$. The expected schedule considering different iterations as well as the final schedule is included. Notice that the schedule before the current prediction horizon cannot be modified since this corresponds to past events.

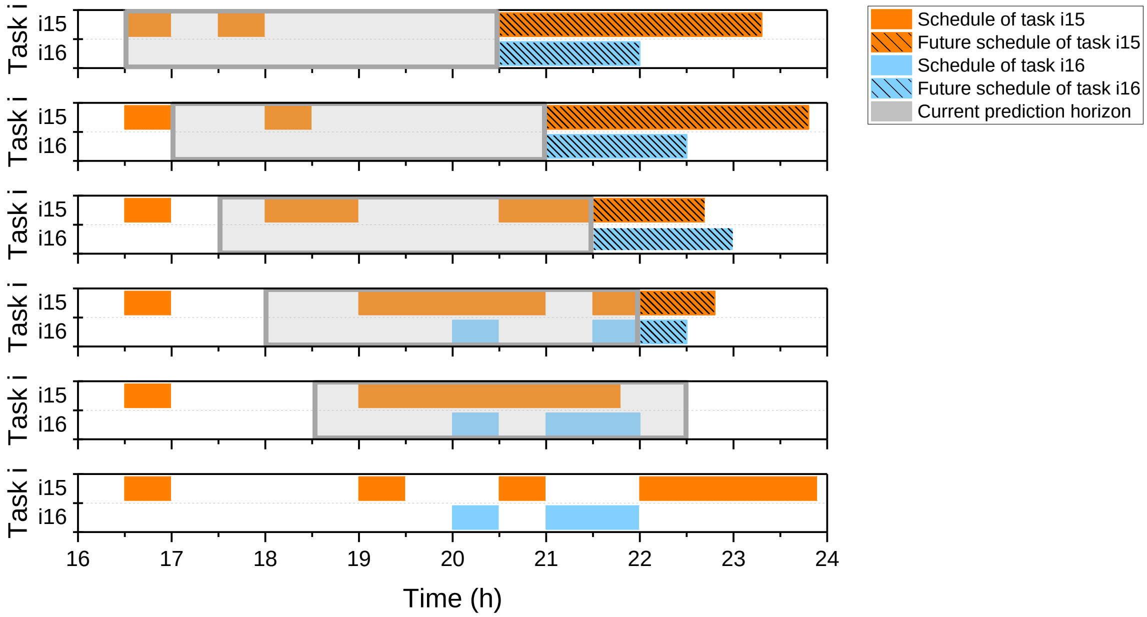

Figure 10. Evolution of scheduled tasks for 1 home and prediction horizon of 4 hours for different iterations. The last schedule corresponds to the final-implemented schedule.

Table 8 shows the value of the overall cost of the microgrid (objective function), considering different durations of the prediction horizon as well as considering different number of homes within the microgrid. As expected, longer prediction horizon involves a reduction in the operational cost and closes the difference from the perfect information (PH=24 h) solution. The optimal daily cost is reduced by 2.3%, 8.1% and 8.8% for prediction horizons of 4, 6 and 8 hours respectively, compared with the Monte Carlo simulation. This highlights the importance of selecting an appropriate duration of the prediction horizon.

The MILP formulation has been implemented in GAMS 24.7 and solved using CPLEX 12.6, in an Intel® Xenon® CPU E5-1650 v3 @ 3.50GHz, with 32GB of installed memory (RAM). Table 8 also contains the main model statistics data obtained for each iteration during the optimisation procedure, considering different prediction horizons as well as different number of houses. For the same number of houses, the number of equations and continuous and discrete variables augments when the prediction horizon increases. This involves an increase in the computational effort to solve the problem, since more decisions have to be taken.

Table 8. Daily total cost of the microgrid considering different prediction horizons and number of homes and model statistics.

Number of houses	Daily total cost (£)	Prediction horizon	Number of equations	Continuous variables	Discrete variables	Resource time (CPU, s)
1 house	6.31	$PH = 2\,h$	66,081	68,570	66,400	1.4
	5.20	$PH = 4\,h$	68,505	70,470	67,520	3.4
	4.89	$PH = 6\,h$	71,266	72,565	68,136	4.1
	4.85	$PH = 8\,h$	74,154	74,628	68,704	11.9

	4.82	$PH = 24\,h$	105,222	96,556	76,800	409.6
5 houses	45.04	$PH = 2\,h$	323,705	337,122	332,000	11.3
	41.68	$PH = 4\,h$	329,345	341,006	337,600	13.9
	37.93	$PH = 6\,h$	336,670	345,865	340,680	19.9
	37.47	$PH = 8\,h$	344,630	350,564	343,520	22.7
	37.46	$PH = 24\,h$	448,130	410,092	384,000	582.6
10 houses	116.43	$PH = 2\,h$	645,735	672,812	664,000	40.9
	115.82	$PH = 4\,h$	655,395	679,176	675,200	71.3
	109.93	$PH = 6\,h$	668,425	687,490	681,360	130.9
	109.90	$PH = 8\,h$	682,725	695,747	687,040	234.5
	100.45	$PH = 24\,h$	876,765	802,172	768,000	3600.0

6. Concluding remarks

This work addresses the coordinated management of electricity and heat generation, purchases, sales, storage and consumption within a microgrid. Several tasks have been scheduled, considering consumption profiles, time windows to execute the consumption, eventual interruptions, time-varying grid electricity prices and peak demand penalisations.

Monte Carlo simulations have been carried out to analyse the effect of the uncertainty on the objective function. According to the obtained results, the presence of uncertainty may increase the cost of the microgrid by 19.6%, compared with the situation in which only deterministic conditions are considered. Thus, techniques to handle uncertainty are required to manage the microgrid. A rolling horizon two-stage stochastic MILP formulation has been presented for an enhanced management of microgrid under uncertainty. A combination of reactive and proactive approaches has been used to cope with uncertainty associated with the electricity and heat generation as well as their demand. This methodology allows updating input information, to react to deviations from the nominal plan, which allows adapting electricity and heat generation, purchases, storage levels, consumption and sales to the current upload parameters. The mathematical model has been tested for different length of prediction horizons. The application of this methodology has allowed improving the optimal solution up to 8.8% compared with the Monte Carlo simulation. As a conclusion, longer prediction horizons favour obtaining enhanced solutions, under the assumption of precise demand and weather predictions.

The proposed formulation has been applied to manage a case study of a microgrid. This may be applied as the basis for managing problems involving more advanced complexity. Moreover, the mathematical model can be extended by considering more details in the operations of the microgrid, such as the consideration of start-up profiles when a task is started or interrupted.

Nomenclature

Indices and sets

i	set of tasks
j	set of equipment units
k	set of homes in the microgrid
t	set of time intervals
θ	set of task operation periods
s	set of scenarios
rh	set of iteration of the rolling horizon approach
$i\epsilon I_j$	subset of tasks i performed in equipment unit j
$i\epsilon I_k RH$	subset of tasks i performed at home k included in the current prediction horizon
$j\epsilon J_k$	subset of equipment units j available at home k
$t \in TRH$	subset of time intervals t included in the current prediction horizon
$\theta \in \Theta_{i,k} RH$	subset of task operation periods included θ in the current prediction horizon

Parameters

A	wind generator blade area (m^2)
b_t	electricity buying price from the grid at time t (£/kWh_e)
$C_{i,\theta}$	power consumption capacity of task i at operation period θ (kW_e)
C^{CHP}	CHP generator capacity (kW_e)
C^W	wind generator capacity (kW_e)
C^B	boiler capacity (kW_{th})
C^E	electrical storage capacity (kWh_e)
C^T	thermal storage capacity (kWh_{th})
$H_{s,t}$	heat demand in scenario s at time t (kW_{th})
m^E	cost per unit input (maintenance) for electrical storage unit (£/kWh_e)
m^T	cost per unit input (maintenance) for thermal storage unit (£/kWh_{th})
m^W	wind generator maintenance cost (£/kWh_e)
m^Z	extra cost factor due to starting the task outside the time window
ng	price of natural gas (£/kWh)
N_W	number of wind turbines
pk	difference between peak and base electricity demand price from the grid (£/kWh_e)
Pr_s	probability of scenario s
P_t^W	electric power from wind generator at time t (kW_e)
$PT0_{k,i,s}$	initial processing time of task i at home k in scenario s
$PT_{k,i,s}$	remaining processing time of task i at home k in scenario s
$\overline{PT}_{k,i}$	roundup value of the processing time of task i at home k
q	electricity selling price to grid (£/kWh_e)
$S_{s,0}^E$	initial state of electrical storage in scenario s (kWh_e)
$\widehat{S^E}_{s,t}$	linking variable determining the electrical storage level at the end of interval t in the current prediction horizon
S^{ECmax}	electric power storage charge limit (kW_e)
S^{EDmax}	electric power storage discharge limit (kW_e)
$S_{s,0}^T$	initial state of thermal storage in scenario s (kWh_{th})
$\widehat{S^T}_{s,t}$	linking variable determining the thermal storage level at the end of interval t in the current prediction horizon
S^{TCmax}	thermal storage charge limit in scenario s (kW_{th})
S^{TDmax}	thermal storage discharge limit (kW_{th})
SH	scheduling horizon (h)

$Ts_{k,i}^{max}$ latest starting time of task i at home k
$Ts_{k,i}^{min}$ earliest starting time of task i at home k
$v_{s,t}$ wind speed in scenario s at time t (m/s)
V^{nom} nominal wind speed (m/s)
V^{cut-in} cut in wind speed (m/s)
$V^{cut-out}$ cut out wind speed (m/s)
$W_{k,i}^{o}$ 1 if task i at home k starts outside the current prediction horizon but within the time window, 0 otherwise
$\widetilde{W}_{k,i}$ processing time of task i started within the time window at home k before the current prediction horizon
$\overline{W}_{k,i,\theta,t}$ 1 if task i at home k is active at time t starting within the time window in previous prediction horizons, 0 otherwise
$Z_{k,i}^{o}$ 1 if task i at home k starts outside the current prediction horizon and outside the time window, 0 otherwise
$\tilde{Z}_{k,i}$ processing time of task i started outside the time window at home k before the current prediction horizon
$\bar{Z}_{k,i,\theta,t}$ 1 if task i at home k is active at time t starting outside the time window in previous prediction horizons, 0 otherwise
α CHP heat-to-power ratio
β agreed electric power peak demand threshold from the grid (kW_e)
δ time interval duration (h)
ρ air density (kg/m^3)
η^B boiler efficiency
η^{CHP} CHP generator electrical efficiency
η^E electrical storage charge/discharge efficiency
η^T thermal storage charge/discharge efficiency
η^W wind generator efficiency
$\mu_{k,i}$ penalty cost for delays in the starting time of task i at home k $(£/h)$
μh penalty cost for unsatisfied heat demand $(£/kWh_{th})$
μi_i^W penalty cost for interrupting task i started within the time window (£)
μs_i^W penalty cost for remaining interrupted task i started within the time window (£)
μi_i^Z penalty cost for interrupting task i started outside the time window (£)
μs_i^Z penalty cost for remaining interrupted task i started outside the time window (£)

Continuous variables
$CPen_{k,i}$ penalty cost for delays in the starting time of task i at home k (£)
$Ex_{s,t}$ electric power exported to the grid in scenario s at time t (kW_e)
$Im_{s,t}$ electric power imported from the grid in scenario s at time t (kW_e)
$P_{s,t}^{B}$ thermal power from boiler in scenario s at time t (kW_{th})
$P_{s,t}^{CHP}$ electric power from CHP generator in scenario s at time t (kW_e)
$S_{s,t}^{E}$ electricity in storage in scenario s at time t (kWh_e)
$S_{s,t}^{EC}$ electric power storage charge rate in scenario s at time t (kW_e)
$S_{s,t}^{ED}$ electric power storage discharge rate in scenario s at time t (kW_e)
$S_{s,t}^{T}$ heat in storage in scenario s at time t (kWh_{th})
$S_{s,t}^{TC}$ thermal power storage charge rate in scenario s at time t (kW_{th})
$S_{s,t}^{TD}$ thermal power storage discharge rate in scenario s at time t (kW_{th})

$Ts_{k,i}$	starting time of task i at home k (h)
T_t	time corresponding to time interval
$UH_{s,t}$	unsatisfied heat demand in scenario s at time t (kW_{th})
$\gamma_{s,t}$	extra electricity load from the grid over the threshold β in scenario s at time t (kW_e)
$\tau_{k,i}$	maximum processing time of task i at home k that can be performed outside the current prediction horizon
$\varphi_{k,i}$	processing time of task i at home k to be performed outside the current prediction horizon
ϕ	daily cost of the microgrid to be minimised (£)

Binary variables

$W_{k,i,\theta,t}$	1 if task i at home k is active in operation period θ at time t starting within the time window, 0 otherwise
$Z_{k,i,\theta,t}$	1 if task i at home k is active in operation period θ at time t starting outside the time window, 0 otherwise
$\widehat{W}_{k,i,t}$	1 if the potential task i at home k is active at time t starting within the time window, 0 otherwise
$\hat{Z}_{k,i,t}$	1 if the potential task i at home k is active at time t starting outside the time window, 0 otherwise
$\lambda i^{W}_{k,i,t}$	1 if task i at home k starting within the time window interrupted at time t, 0 otherwise
$\lambda s^{W}_{k,i,t}$	1 if task i at home k starting within the time window remains interrupted at time t, 0 otherwise
$\lambda i^{Z}_{k,i,t}$	1 if task i at home k starting outside the time window is interrupted at time t, 0 otherwise
$\lambda s^{Z}_{k,i,t}$	1 if task i at home k starting outside the time window remains interrupted at time t, 0 otherwise

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Appendix A. Mathematical model

Equations (27), (28) and (29) are applied to determine any eventual in a task. The formulation of these equations is based on a general disjunctive programming (GDP) formulation. In particular:

(i) If the potential status of a task is inactive, the variable $\widehat{W}_{k,i,t}$ takes value 0. This means that at time t, the task has not started or has finished. Consequently, if $\widehat{W}_{k,i,t} = 0$, there is neither interruption ($\lambda i^{W}_{k,i,t} = 0$) nor remaining interruptions ($\lambda s^{W}_{k,i,t} = 0$), because the task is not active.

$$\neg \widehat{W}_{k,i,t} \Rightarrow \neg \lambda i^{W}_{k,i,t} \wedge \neg \lambda s^{W}_{k,i,t}$$

(ii) If the potential status of a task is active, the variable $\widehat{W}_{k,i,t}$ takes value 1. This means that at time t, the task has started but has not finished. So, if $\widehat{W}_{k,i,t} = 1$, the status of the task is active at time t ($W_{k,i,\theta,t} = 1$, $\overline{W}_{k,i,\theta,t} = 1$) and is not active at time $t+1$ ($W_{k,i,\theta,t+1} = 0$), an interruption is produced ($\lambda i^{W}_{k,i,t+1} = 1$).

$$\widehat{W}_{k,i,t} \wedge W_{k,i,\theta,t} \wedge \neg W_{k,i,\theta,t+1} \Rightarrow \lambda i^{W}_{k,i,t}$$

(iii) Furthermore, if $\widehat{W}_{k,i,t} = 1$, and the status of the task is not active neither at time t nor at time $t+1$ ($W_{k,i,\theta,t} = \overline{W}_{k,i,\theta,t} = W_{k,i,\theta,t+1} = 0$), this involves that the task remains interrupted ($\lambda s^{W}_{k,i,t+1} = 1$).

$$\widehat{W}_{k,i,t} \wedge \neg W_{k,i,\theta,t} \wedge \neg W_{k,i,\theta,t+1} \Rightarrow \lambda s^{W}_{k,i,t+1}$$

Appendix B. Input data

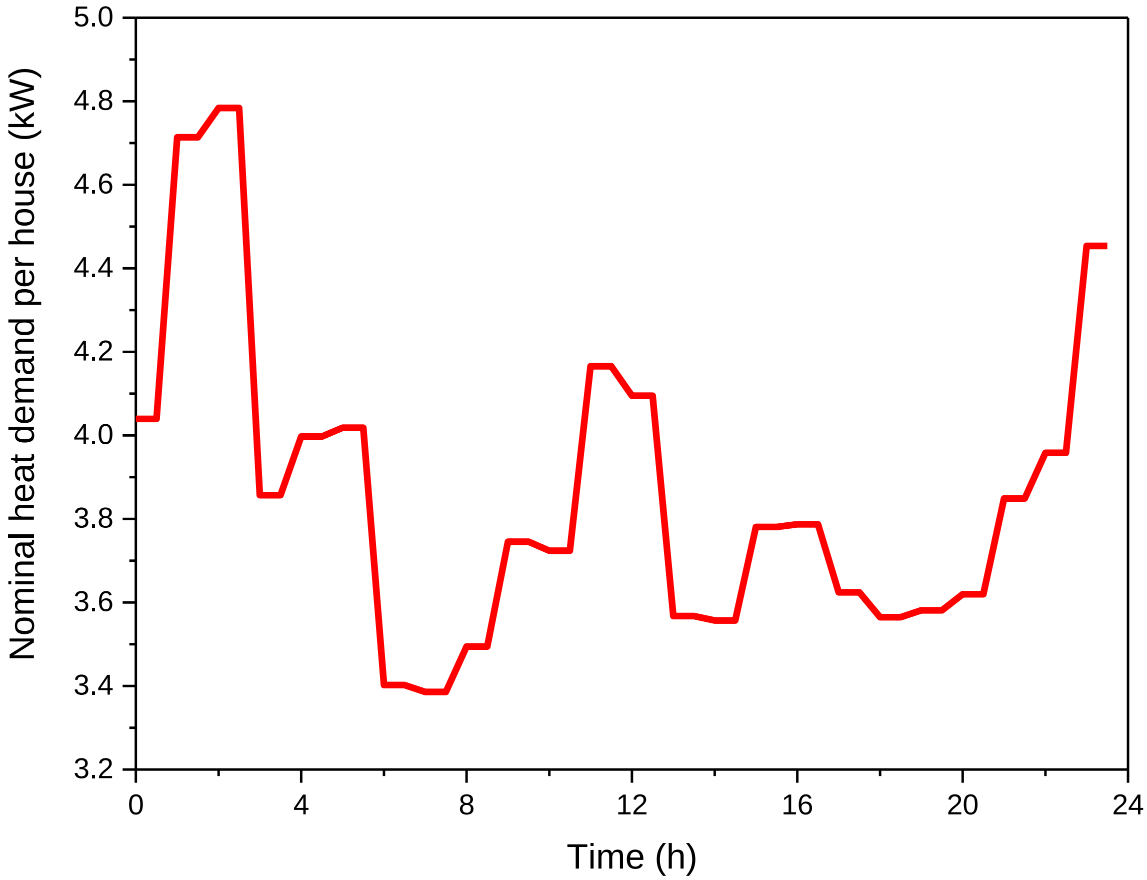

Figure B1. Nominal heat demand per house.

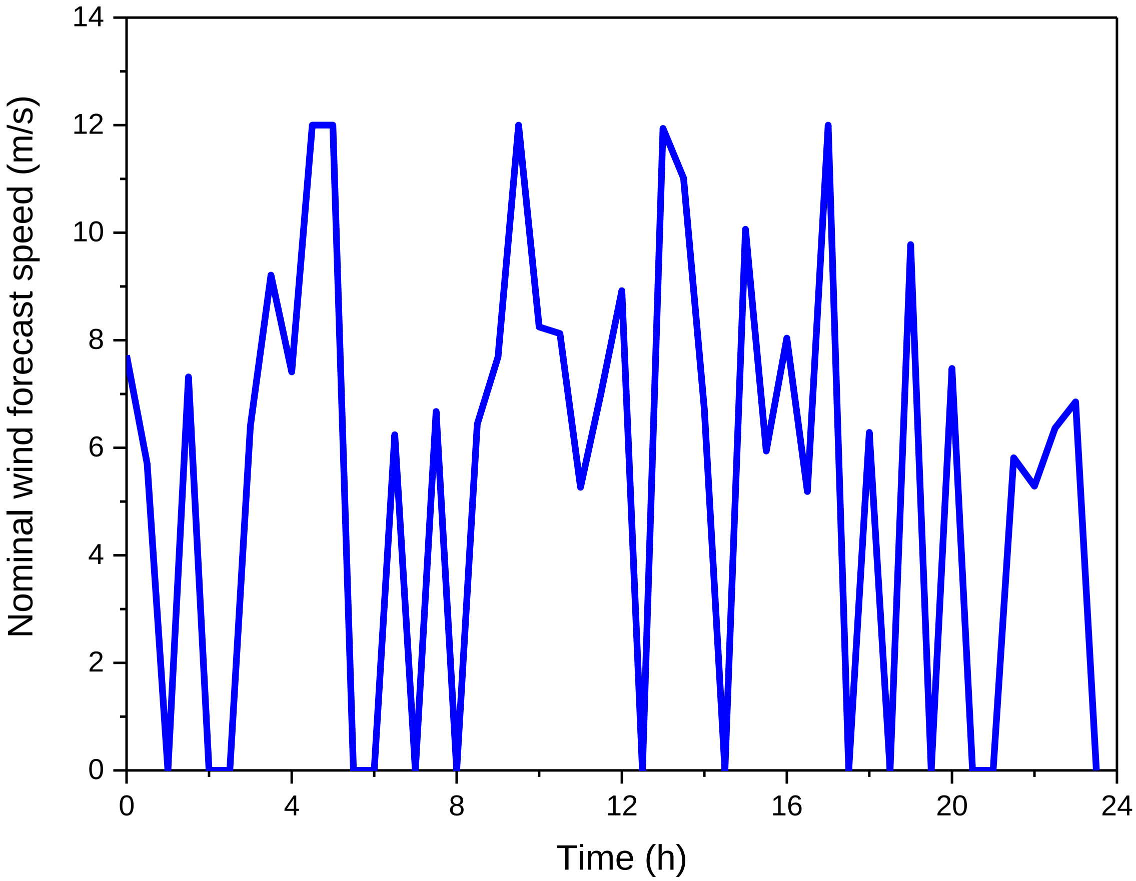

Figure B2. Nominal wind forecast speed.

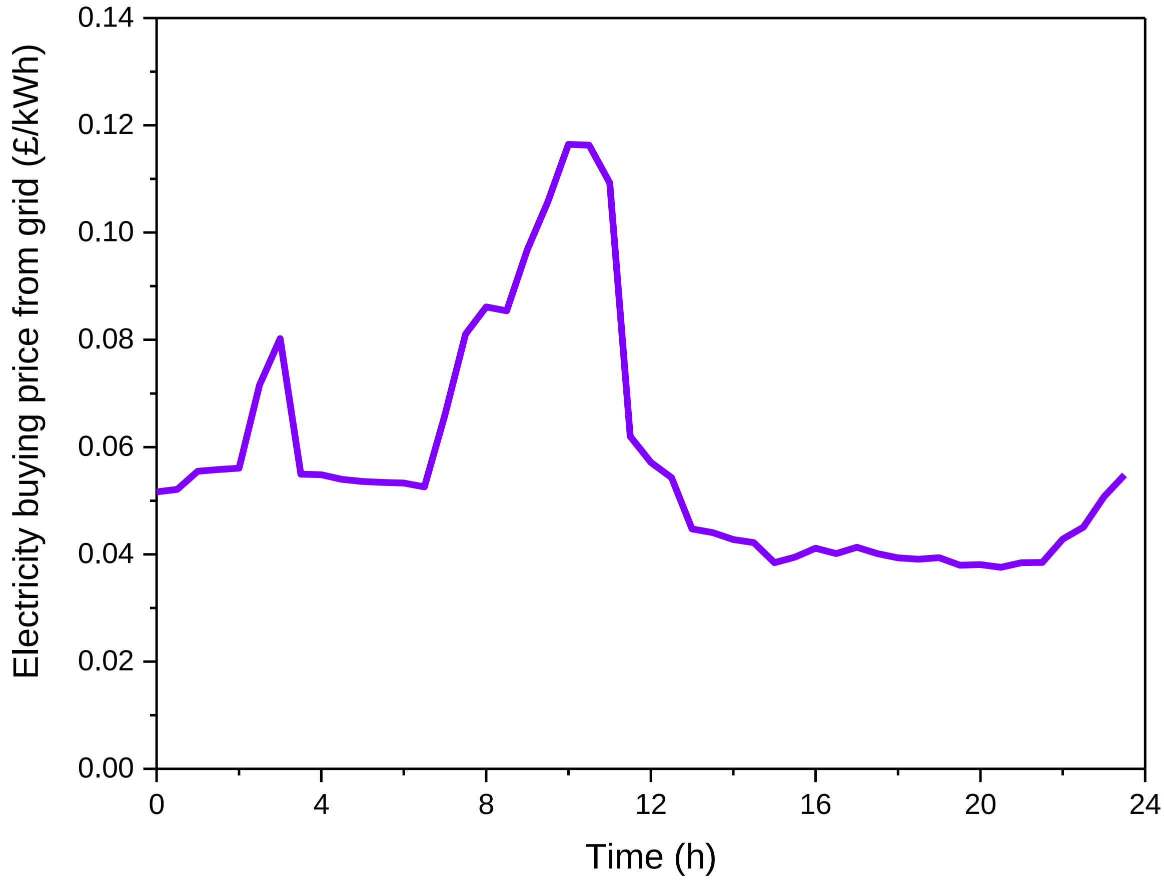

Figure B3. Electricity buying price from the power grid.

Table B1. Penalty costs.

Task i	Penalty cost for interruption a task i				Penalty cost from deviation to the target, $\mu_{k,i}$ (£/h)
	Within the time window		Outside the time window		
	Interruption, μi_i^W (£)	Remain interrupted, μs_i^W (£)	Interruption, μi_i^Z (£)	Remain interrupted, μs_i^Z (£)	
$i1$	0.0500	0.0050	0.5000	0.0500	0.0500
$i2$	0.0100	0.0020	0.1000	0.0200	0.0100
$i3$	0.0300	0.0010	0.3000	0.0100	0.0200
$i4$	0.0800	0.0900	0.8000	0.9000	0.0080
$i5$	0.0100	0.0010	0.1000	0.0100	0.0100
$i6$	0.0100	0.0010	0.1000	0.0100	0.0200
$i7$	0.0100	0.0010	0.1000	0.0100	0.0060
$i8$	0.0200	0.0070	0.2000	0.0700	0.0040
$i9$	0.0100	0.0010	0.1000	0.0100	0.0090
$i10$	0.0100	0.0010	0.1000	0.0100	0.1000
$i11$	0.1000	0.1000	0.1000	1.0000	0.1000

$i12$	0.0200	0.0100	0.2000	0.1000	0.0200
$i13$	0.0500	0.0010	0.5000	0.0100	0.0080
$i14$	0.0100	0.0020	0.1000	0.0200	0.0100
$i15$	0.0001	0.0001	0.0010	0.0010	0.0300
$i16$	0.0010	0.0001	0.0100	0.0010	0.0100

Table B2. Information associated with the different scenarios.

Scenario s	Probability, Pr_s	Wind forecast level	Processing time level	Heat demand level
$s1$	0.001	Low	Low	Low
$s2$	0.050	Low	Low	Medium
$s3$	0.002	Low	Low	High
$s4$	0.010	Low	Medium	Low
$s5$	0.100	Low	Medium	Medium
$s6$	0.010	Low	Medium	High
$s7$	0.020	Low	High	Low
$s8$	0.050	Low	High	Medium
$s9$	0.020	Low	High	High
$s10$	0.010	Medium	Low	Low
$s11$	0.050	Medium	Low	Medium
$s12$	0.010	Medium	Low	High
$s13$	0.050	Medium	Medium	Low
$s14$	0.250	Medium	Medium	Medium
$s15$	0.050	Medium	Medium	High
$s16$	0.010	Medium	High	Low
$s17$	0.050	Medium	High	Medium
$s18$	0.010	Medium	High	High
$s19$	0.020	High	Low	Low
$s20$	0.050	High	Low	Medium
$s21$	0.004	High	Low	High
$s22$	0.010	High	Medium	Low
$s23$	0.100	High	Medium	Medium
$s24$	0.010	High	Medium	High
$s25$	0.001	High	High	Low
$s26$	0.050	High	High	Medium
$s27$	0.002	High	High	High

A rolling horizon approach for optimal management of microgrids under stochastic uncertainty

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