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CRANFIELD

The effect of the sweepback of Delta wings on the performance of an aircraft at supersonic speeds

-by-

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-SUMMARY-

The variation with sweepback of the total drag of an aircraft in level flight at supersonic speeds is calculated. It is shown that sweepback is not uniformly beneficial, but that in general the optimum amount of sweepback depends on the design speed and altitude.

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1. Introduction

A considerable amount of evidence has been produced in recent months on the variation with sweep-back of lift and drag of an aerofoil under supersonic conditions. It appeared desirable to work out some concrete examples to find out what the data obtained implied in terms of the performance of an aircraft at supersonic speeds. A complete survey appeared impracticable, in view of the number of parameters involved. However, the set of diagrams produced in the present paper will at least serve to give an idea of the general trend of the results. In choosing the characteristics of the hypothetical aircraft under investigation, it was assumed that the main plane is the predominent component of the aircraft. This was done merely in order to underline the effect of sweepback, and without reference to the undecided issue of 'All wing' versus 'Nearly all body' aircraft at supersonic speeds. Similarly, in order to bring out the effect of the induced drag which becomes negligible at very low wing loadings, the wing loading was taken to be as high as possible under realistic conditions. It should be borne in mind, however, that while a wing loading of 40 lb./sq.ft. is probably too high a figure for landing, especially at very low aspect ratios, the wing loading in level flight will be rather higher than acceptable for landing in any case.

No specific assumptions were made on the type of controls used, since the effect of these on the drag in level flight may be considered to be of second order of magnitude only.

2. Assumptions and procedures

The following characteristics were assumed for the hypothetical aircraft under investigation.

Wing loading : $w = 40$ lb/sq.ft.

Thickness-chord ratio : $t/c = 0.06$ (constant along the span).

Aerofoil section : Double-wedge.

Position of maximum thickness :

(i) At 0.50 of the chord aft of the leading edge.

(ii) At 0.25 of the chord aft of the leading edge.

Planform : Triangular (Delta wing).

The aspect ratio then varies linearly with the tangent of the apex semi-angle, $A = 4 \tan \gamma$.

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The skin friction drag coefficient C_{D_F} of the wing was taken to be 0.005 throughout, while the data for the wave drag coefficient at zero incidence, C_{D_W}, were taken from ref.1. Figs. 1 and 2, which were obtained by computation from expressions given in ref.1 show the relative value of the profile drag coefficient calculated by the three dimensional theory of ref.1, compared with the corresponding values obtained by Ackeret's two dimensional theory ('strip-theory').

The wave drag coefficient and the skin friction drag coefficient together make up the profile drag coefficient C_{D_P} of the wing which is approximately independent of incidence, $C_{D_P} = C_{D_F} + C_{D_W}$. To account for the drag due to fuselage and control surfaces, C_{D_P} was multiplied by $\frac{4}{3} = 1.33$, so that the total parasitic ('non-induced') drag C_{D_0} equals $C_{D_0} = 1.33\ C_{D_P} = 1.33\ (C_{D_F} + C_{D_W})$. All the above drag coefficients are based on wing gross area, as under subsonic conditions. It was assumed that the aircraft is streamlined, so that form drag can be neglected, or otherwise is so small that it may be assumed to be included in the term $1.33\ C_{D_F}$.

When the aeroplane flies at positive incidence, the drag is increased by the addition of 'induced drag'. As in subsonic flow, the induced drag coefficient is proportional to the square of the lift coefficient, $C_{D_i} \propto C_L^2$, or $C_{D_i} = K\,C_L^2$, say. The contribution of the other aircraft components to both lift and induced drag will be neglected. It will be seen that for the calculation of our curves, only the value of K (i.e., neither the value of C_{D_i} nor of C_L, as depending on incidence) is used explicitly, so that the effect of the other components in this connection is likely to be even less important than might appear at first sight.

The values of K were taken from ref.2. For infinite aspect ratio (no sweepback), we have $K_\infty = \frac{\sqrt{M^2 - 1}}{4}$, as confirmed by Ackeret's theory. The value of $K \tan \mu$ is plotted in Fig.4. It will be seen that both in Figs. 1 and 2 and in Fig.4, the plotted quantities depend only on the parameter $\lambda = \cot \mu \tan \gamma = \frac{A\sqrt{M^2 - 1}}{4}$, where μ is the Mach angle, $\operatorname{cosec} \mu = M$, A is the aspect ratio of the wing, and γ the apex semi-angle.

The total drag D in level flight was then calculate by using the formulae $C_D = C_{D_0} + C_{D_i} = C_{D_0} + KC_L^2$, $D = \frac{1}{2} C_D \rho V^2 S$, $W = L = \frac{1}{2} C_L \rho V^2 S$ (where W is the all up weight, L ithe lift, ρ the air density, V the free stream velocity, and S the gross wing area), just as in conventional performance calculations. In Figs.5 - 12, the ratio of total drag (or thrust required) and of weight is plotted against aspect ratio (or apex semi-angle) for altitudes 10000', 40000', 60000', 80000', and for speeds corresponding to Mach numbers 1.2 and 2 at those altitudes.

3. Discussion of results

As Figs. 1 and 2 show, the wave drag of a Delta wing of given area tends to 0 for very small aspect ratios, but for moderate aspect ratios it rises over and above the value obtained for the non-sweptback wing. This increase in drag is rather more pronounced on Fig.2 (max. thickness at 0.25 of the chord) than on Fig.1 (max. thickness at 0.5 of the chord). It is interesting to note that the type of variation of the curve in Fig.1 is much the same as that of the corresponding curve obtained in ref.3 for a diamond shaped aerofoil (max. thickness at 0.5 of the chord), as shown in Fig.3. This tends to underline the importance of the sweepback of the leading edge, and of the position of maximum thickness, compared with which the sweepback or sweepforward of the trailing edge appears to be relatively irrelevant.

Fig.4 shows that $K \tan \mu$ retains its two dimensional value as long as the leading edges of the wing are outside the Mach cone issuing from the apex ($\lambda > 1$). However, as λ decreases below 1, the value of $K \tan \mu$ decreases at first, and then rises again, tending to infinity, as λ tends to 0. Thus, the trends of variation of wave drag and induced drag with varying angle of sweepback are opposed to one another, and it is interesting to see how the variation of the total drag is affected by these diverging tendencies. In general, the induced drag will be the more important the lower the speed and the higher the altitude.

At low altitudes (Figs. 5 and 9) the induced drag is negligible. Its importance becomes apparent at 40000' (Figs. 6 and 10) and after than increases rapidly. As a result there is then a distinct optimum apex semi-angle (or aspect ratio) for which the drag is a minimum, this angle being in the region of $\gamma = 20^{\circ}$ for $M = 1.2$ and just above $\gamma = 10^{\circ}$ for $M = 2.0$ at an altitude of 40000'. At still higher altitudes one effect of the induced drag is to flatten the curve considerably, except for the increase for very small aspect ratios. Thus, the drag - weight ratio for $M = 1.2$ has a minimum for an apex angle of about 75°, but the reduction in drag achieved by that amount of sweepback compared with non-sweptback conditions is small.

The effect of the reduction of lift with decreasing aspect ratio can be seen from the angles of incidence required for level flight which are quoted in Figs. 6 - 8. While no published evidence appears to be available on this point, it is likely, however -as in subsonic flow - that for small aspect ratios the aircraft will remain unstalled even at fairly high incidences.

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It will be seen that under the conditions of Figs. 9 - 11, the total drag round about an aspect ratio of six is higher for $M = 1.2$ than for $M = 2.0$, so that the drag actually decreases with increasing speed. The reason for this is that as the speed increases, the Mach angle decreases, so that $\lambda = \cot \mu \tan \gamma$ for a given aerofoil increases. Now for a maximum thickness position at 0.25 of the chord, the value of the wave drag coefficient for $\lambda = 1$ is more than twice its two dimensional value. And for the cases under consideration, the rate of decrease of the drag coefficient as λ varies from 1 upwards is more rapid than the rate of increase of the V^2 term in the expression for the total drag.

In conclusion, it appears that a large angle of sweepback is not uniformly beneficial for the performance of an aircraft at supersonic speeds. While it is likely that Delta wings or wings of similar shape will be adopted in any case for supersonic aircraft, for reasons of stability, the actual optimum amount of sweepback can be determined only as a function of the height and speed to which the aircraft is designed.

List of References

No.	Author	Title etc.
1	A.E. Puckett	Supersonic wave drag of thin aerofoils, Journal of the Aeronautical Sciences, Vol.13, 1946.
2	A. Robinson	Lift and drag of a flat Delta wing at supersonic speeds, R.A.E. Tech. Note No. Aero. 1791, 1946.
3	A. Robinson	The wave drag of diamond shaped aerofoils at zero incidence, R.A.E., Tech. Note No.Aero. 1784, 1946.

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WAVE DRAG OF A DELTA WING

FIG.1.

POSITION OF MAX. THICKNESS
0·5 CHORD FROM LEADING EDGE.

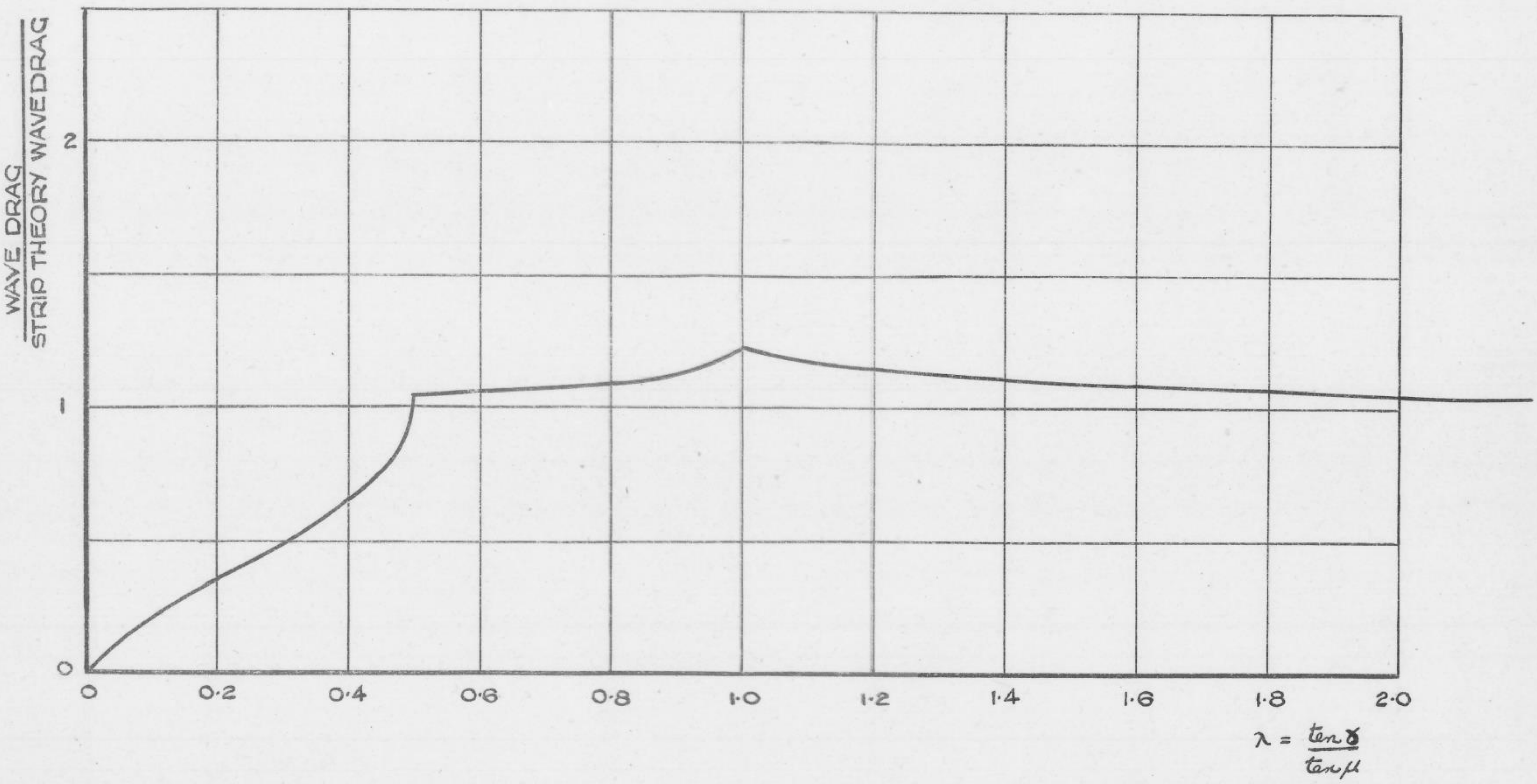
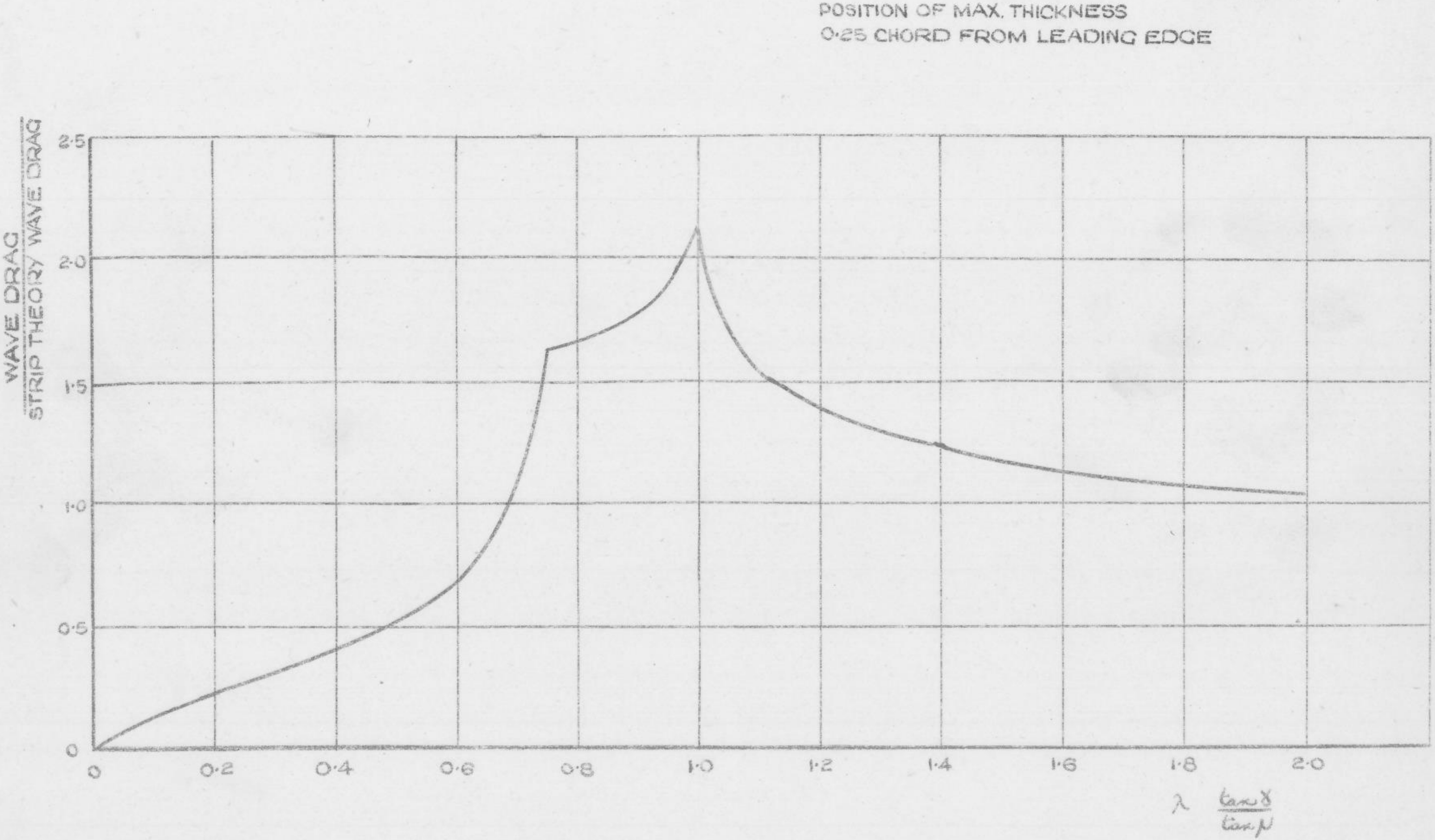


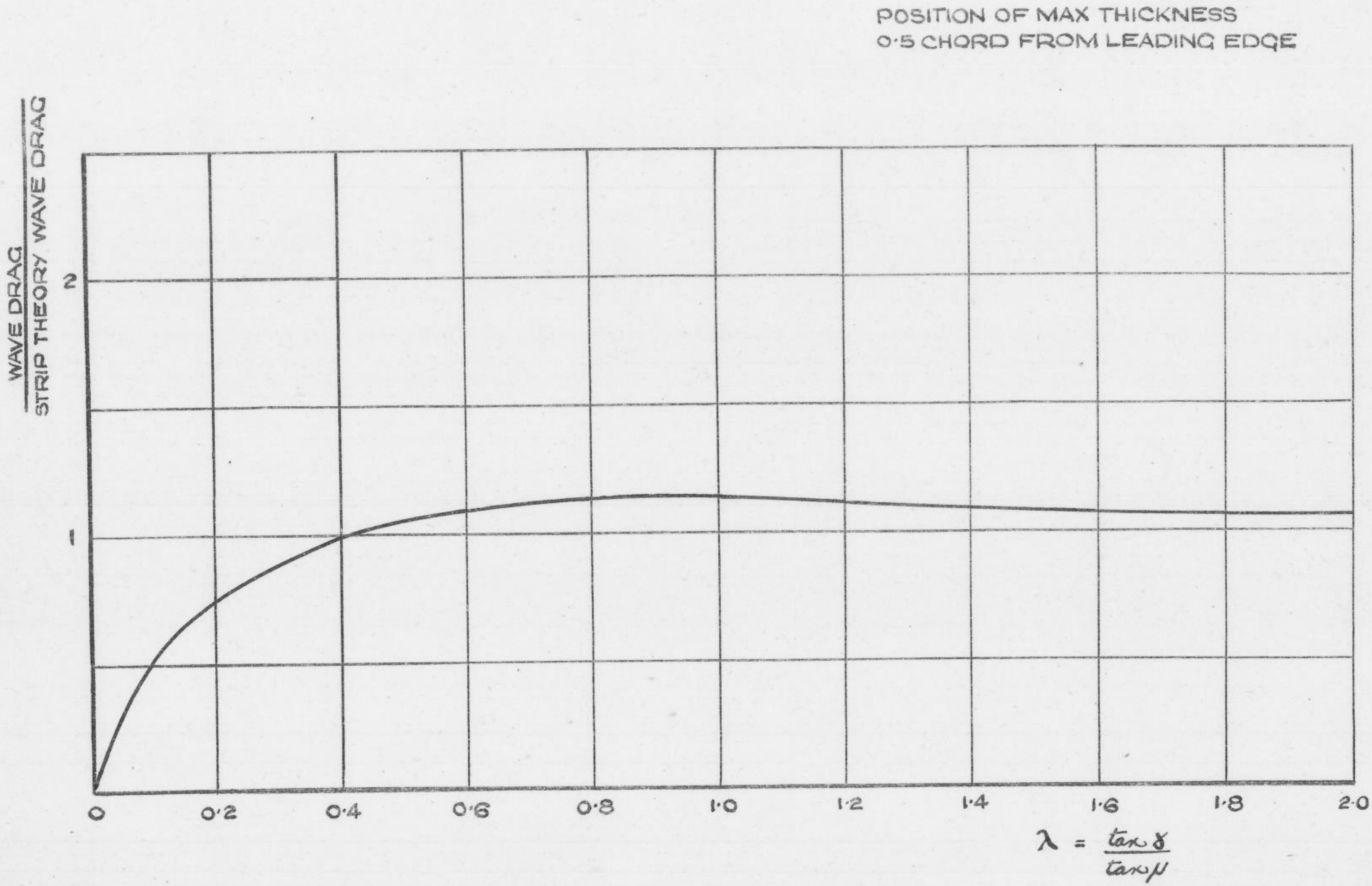
FIG. 2.



WAVE DRAG OF A DELTA WING

FIG.3.

WAVE DRAG OF A DIAMOND SHAPED WING



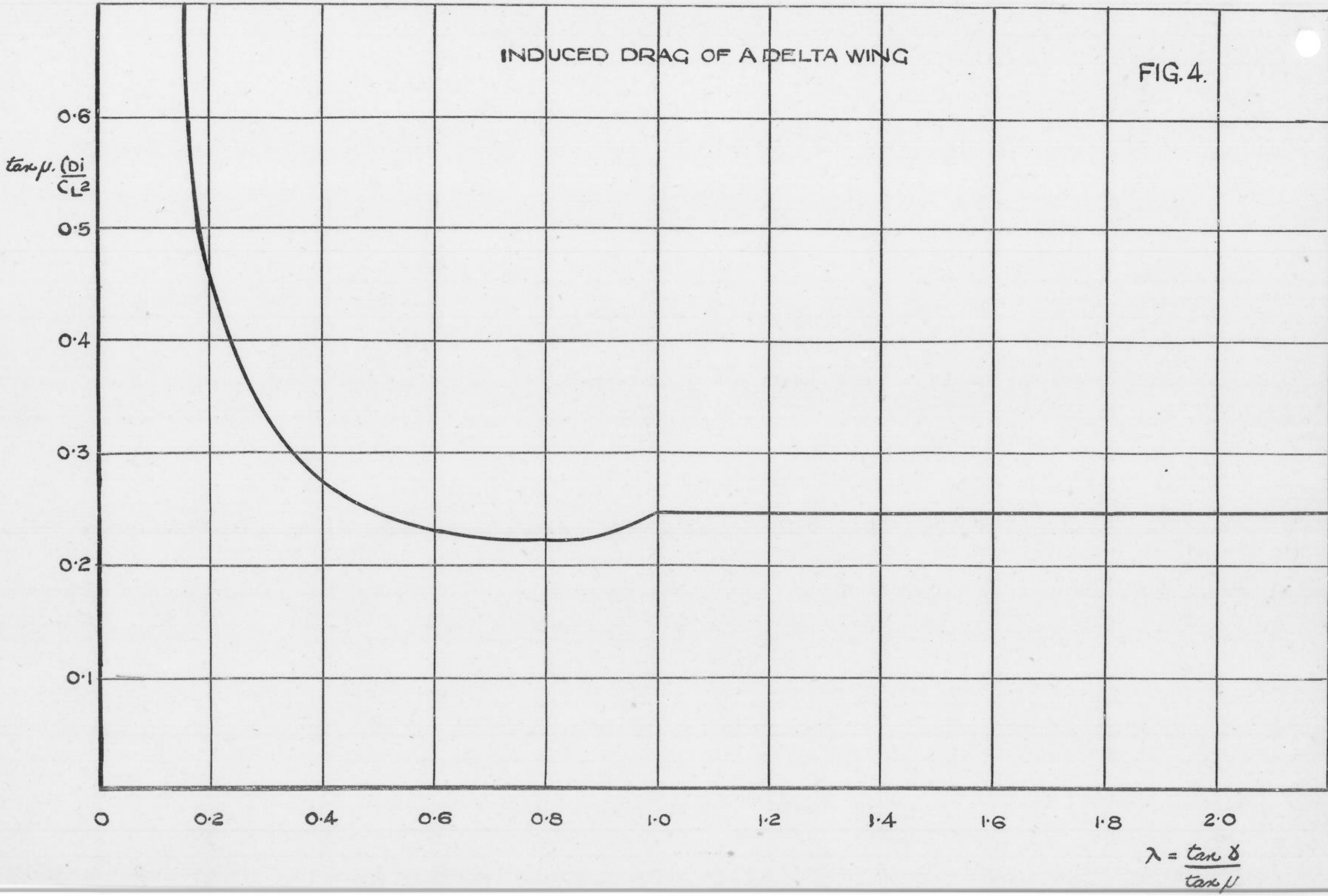


FIG. 4.

INDUCED DRAG OF A DELTA WING

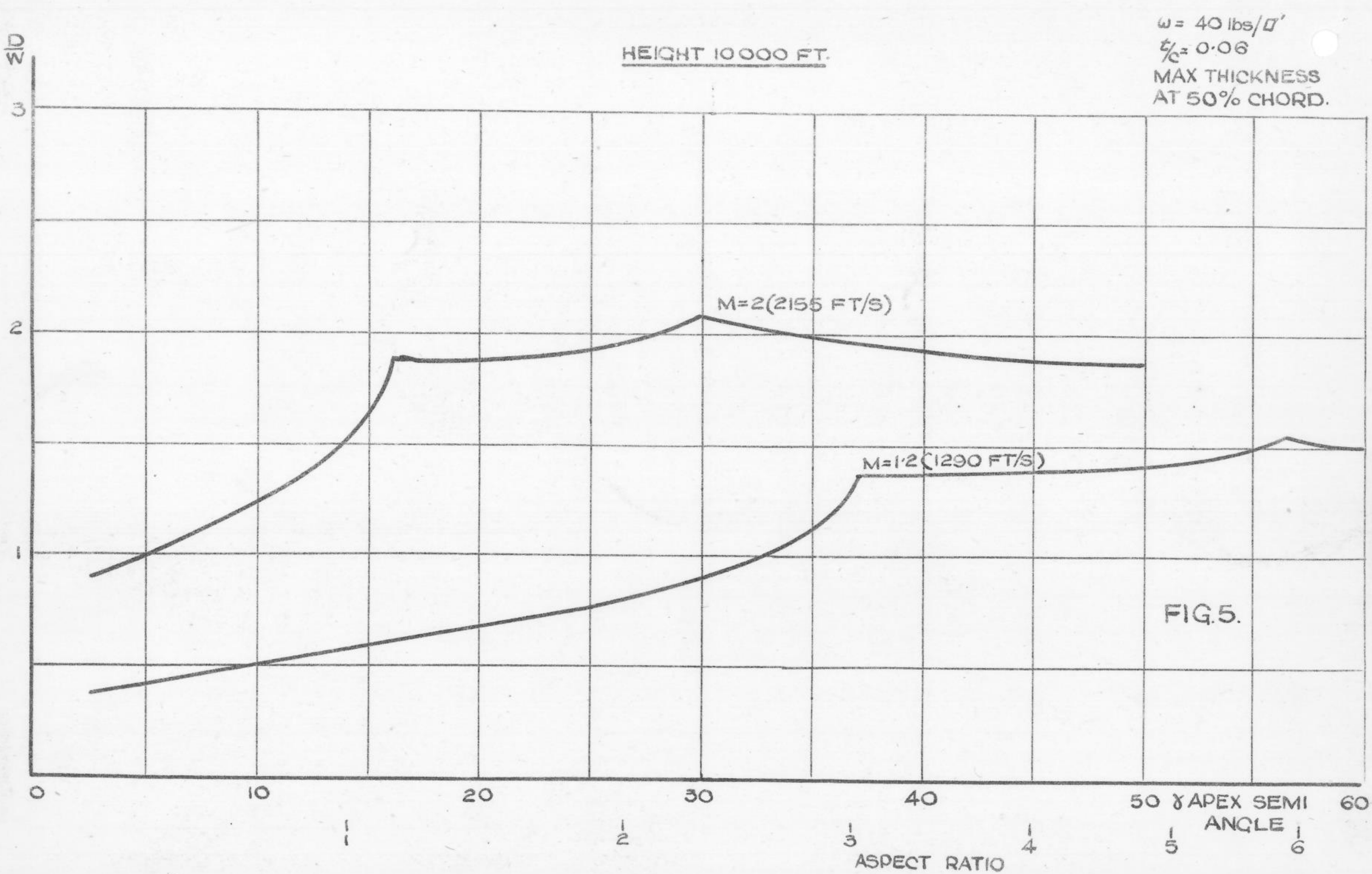

HEIGHT 10 000 FT.
ω = 40 lbs/□'
t/c = 0·06
MAX THICKNESS
AT 50% CHORD.
D/W
3
2
1
M=2(2155 FT/S)
M=1·2(1290 FT/S)
0
10
20
30
40
50
60
γ APEX SEMI ANGLE
1
2
3
4
5
6
ASPECT RATIO

FIG.5.

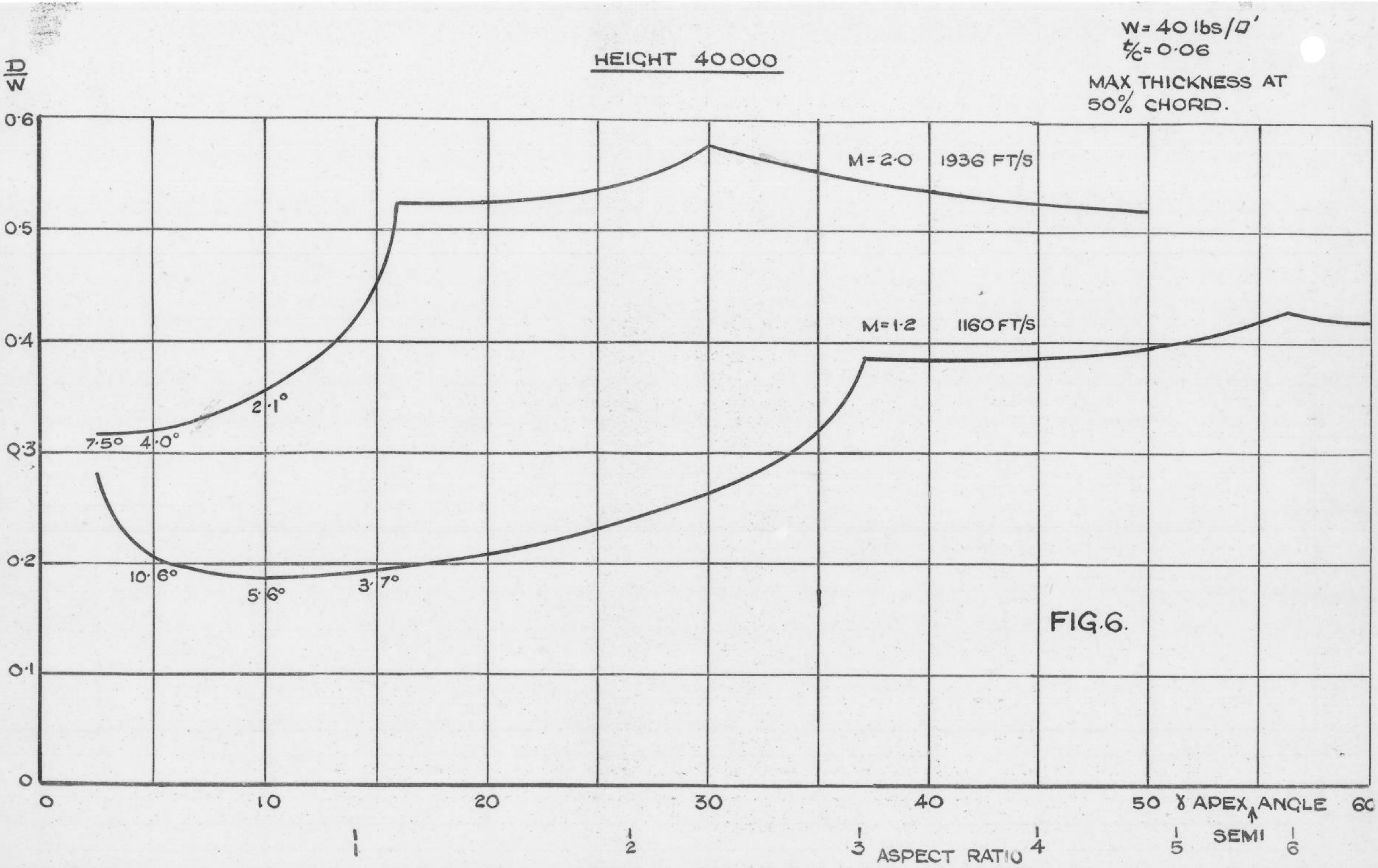

HEIGHT 40000
W = 40 lbs/□'
t/c = 0·06
MAX THICKNESS AT
50% CHORD.
D/W
0·6
0·5
0·4
0·3
0·2
0·1
0
10
20
30
40
50
60
γ APEX ANGLE
SEMI
1
2
3
4
5
6
ASPECT RATIO
M=2·0 1936 FT/S
M=1·2 1160 FT/S
7·5° 4·0°
2·1°
10·6°
5·6°
3·7°

FIG.6.

HEIGHT 60 000 FT.

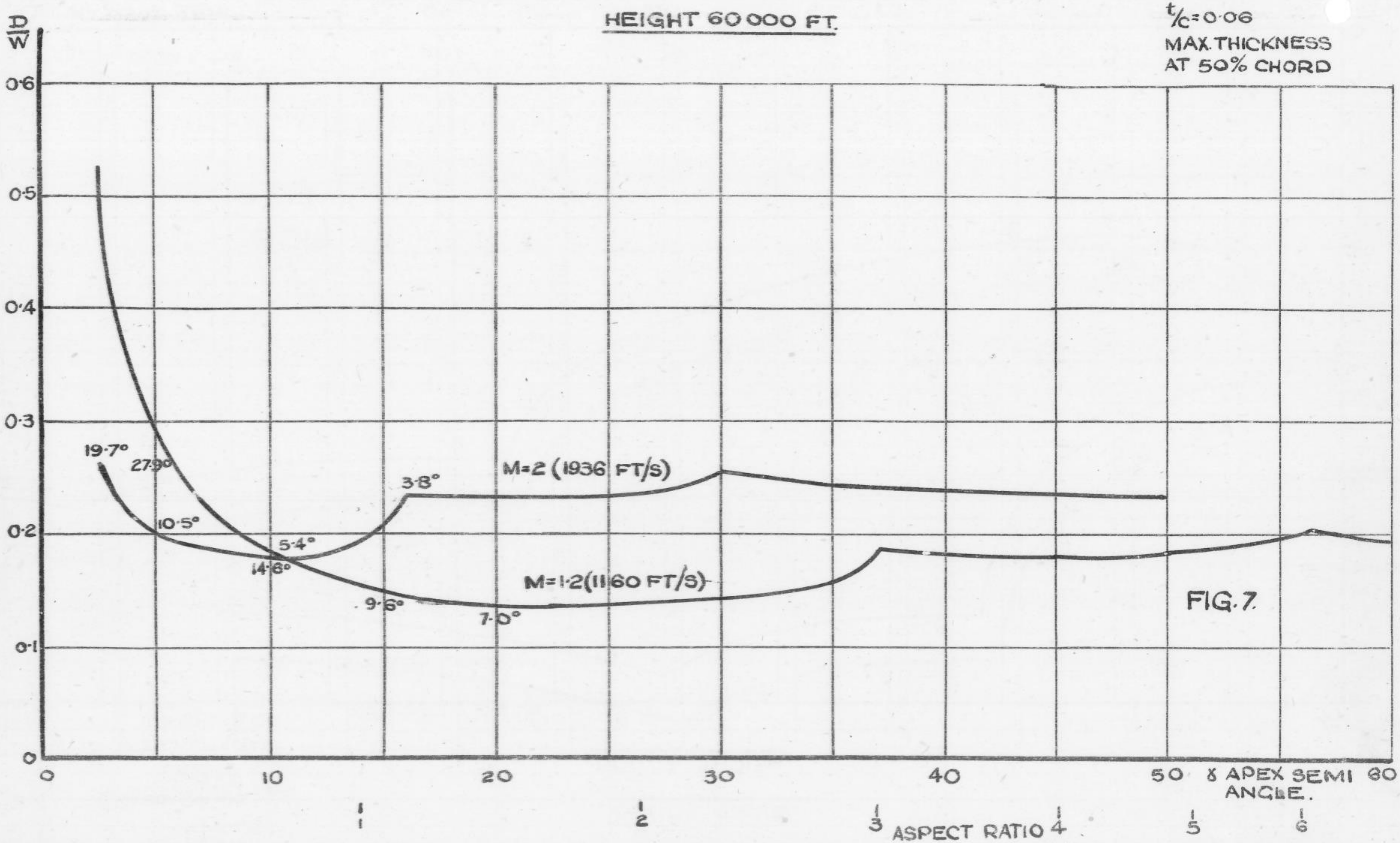


FIG. 7.

HEIGHT 80 000 FT

ω = 40 lbs/□'
t/c = 0·06
MAX THICKNESS
IS AT 50% CHORD

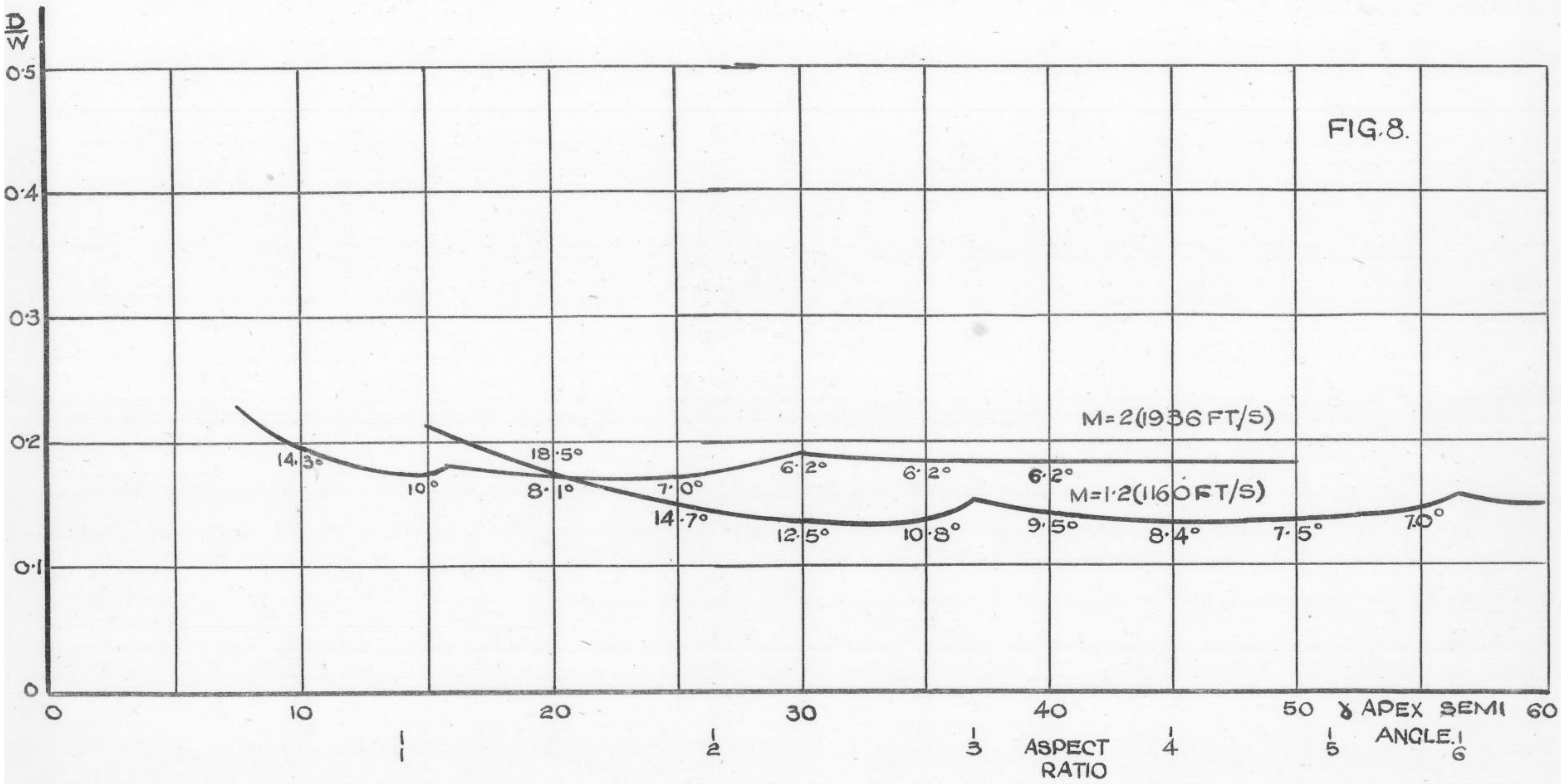


FIG. 8.

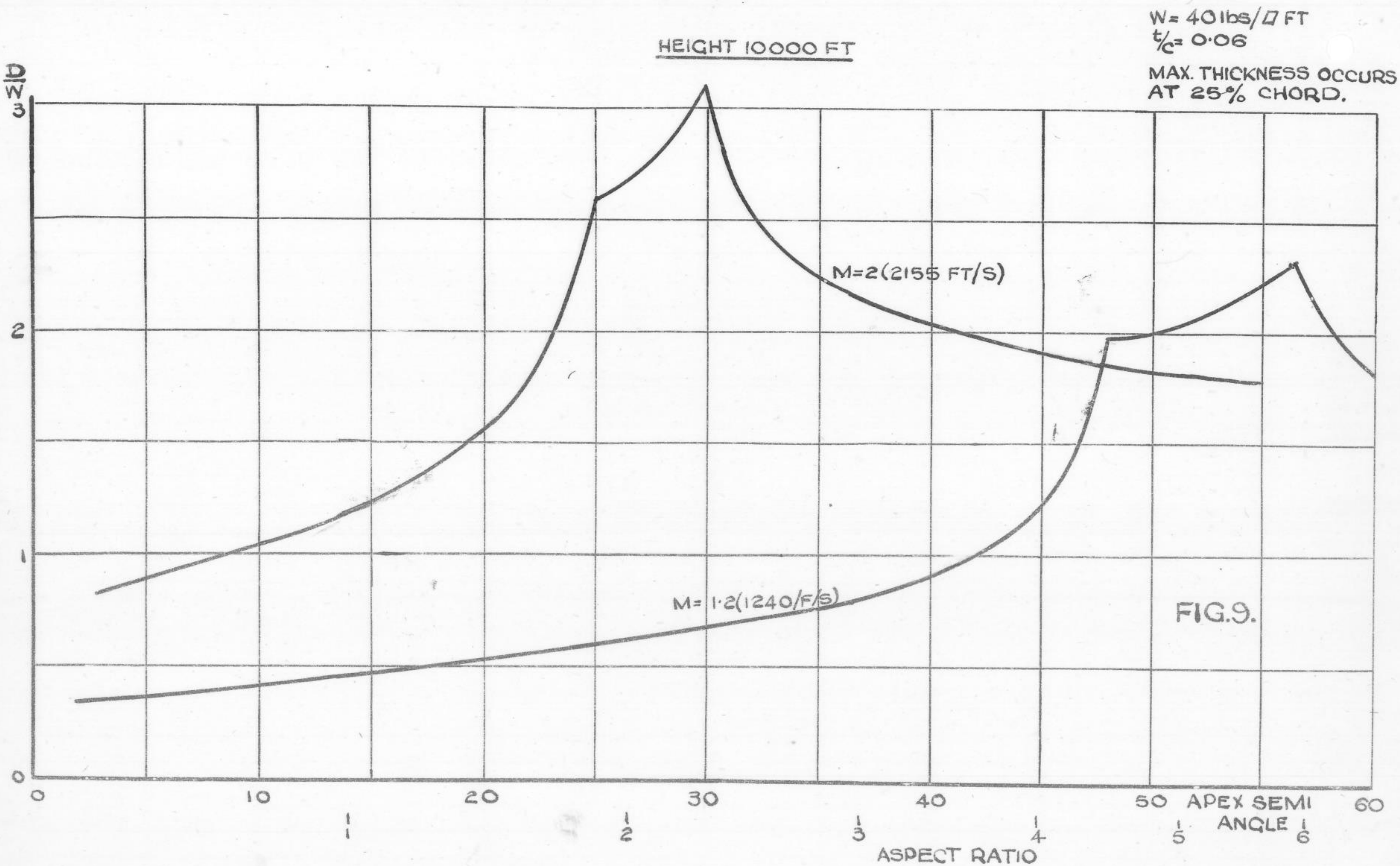

HEIGHT 10000 FT
W= 40 lbs/□ FT
t/c= 0·06
MAX. THICKNESS OCCURS AT 25% CHORD.
D/W
0
1
2
3
M=2(2155 FT/S)
M= 1·2(1240/F/S)
0
10
20
30
40
50
60
APEX SEMI ANGLE
1
2
3
4
5
6
ASPECT RATIO

FIG.9.

HEIGHT 40 000 FT

W = 40 lbs/□ FT
$t/c = 0{\cdot}06$
MAX. THICKNESS IS AT 25% CHORD.

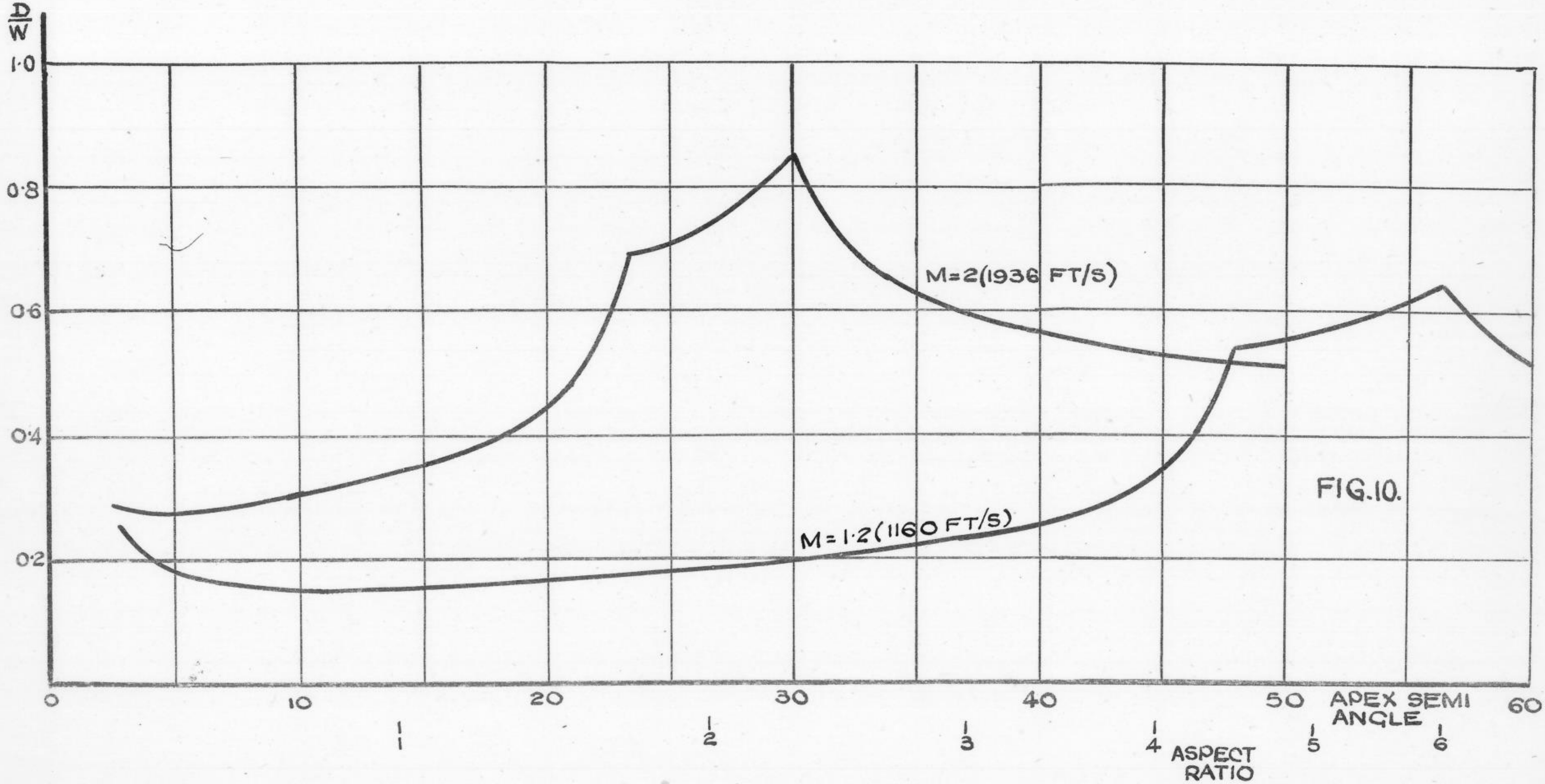


FIG.10.

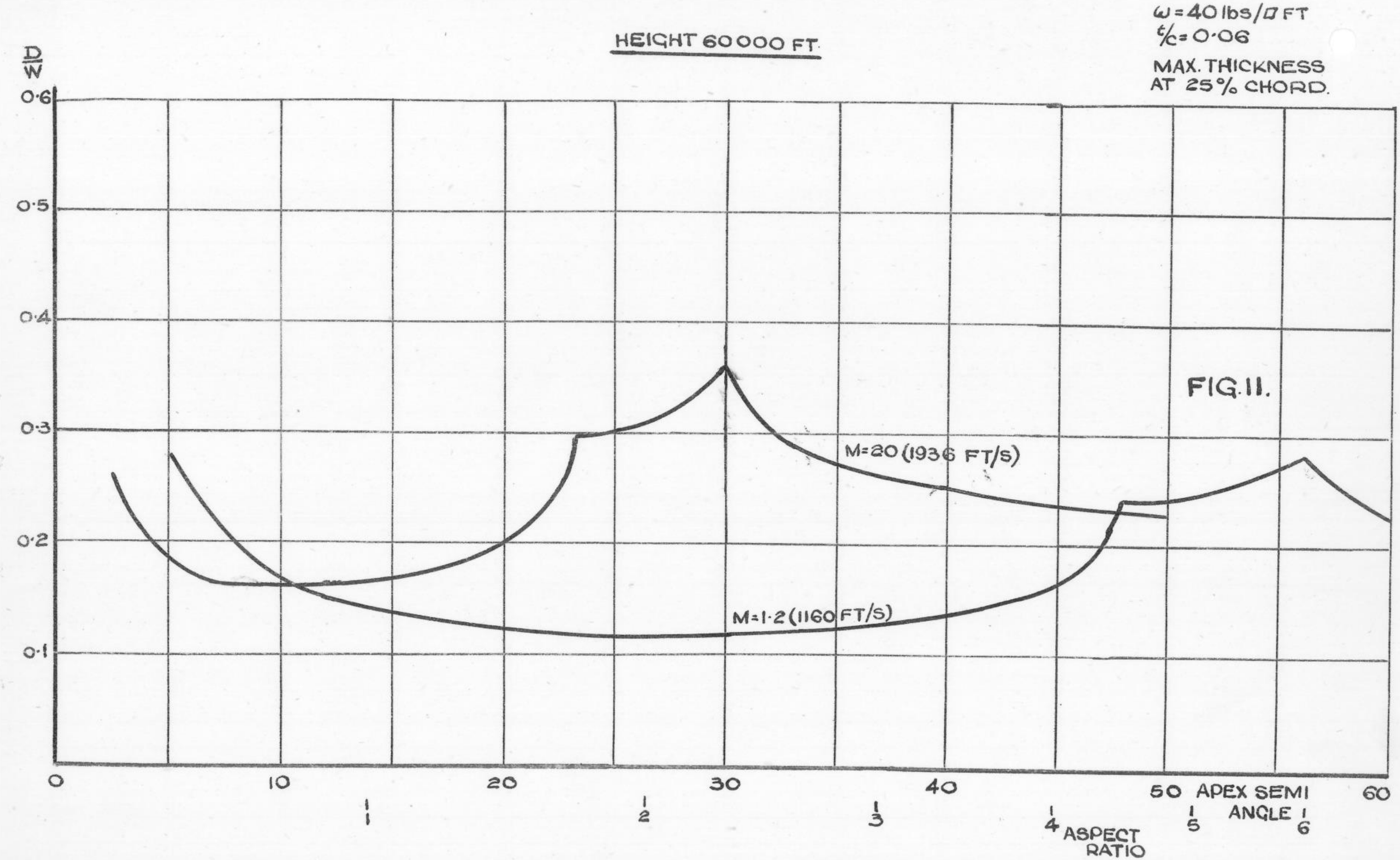

HEIGHT 60 000 FT.
ω = 40 lbs/□ FT
t/c = 0·06
MAX. THICKNESS
AT 25% CHORD.
D/W
0·6
0·5
0·4
0·3
0·2
0·1
0
10
20
30
40
50
60
APEX SEMI ANGLE
1
2
3
4
5
6
ASPECT RATIO
M=2·0 (1936 FT/S)
M=1·2 (1160 FT/S)

FIG. 11.

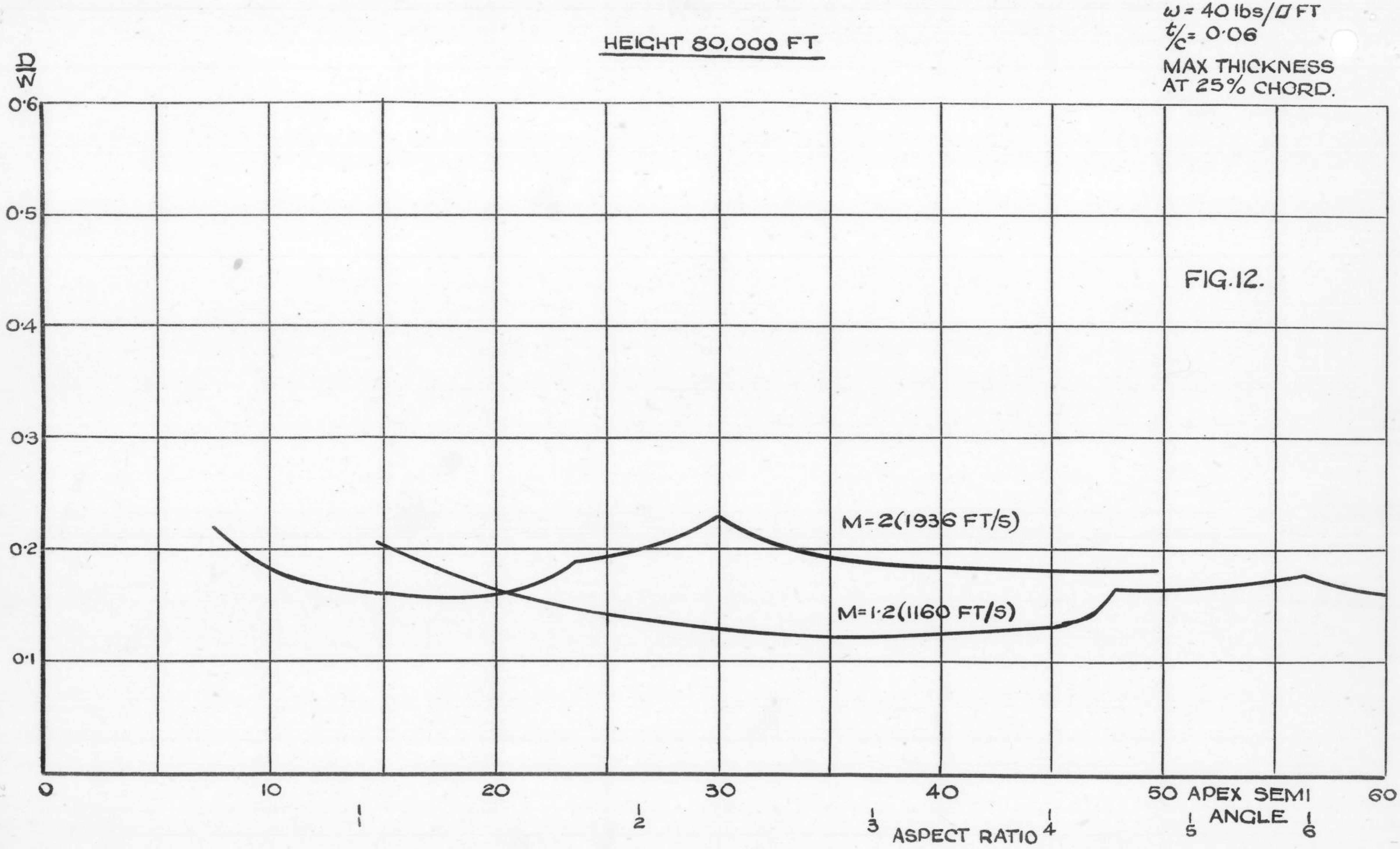

HEIGHT 80,000 FT
ω = 40 lbs/□ FT
t/c = 0·06
MAX THICKNESS
AT 25% CHORD.
D/W
0·6
0·5
0·4
0·3
0·2
0·1
0
10
20
30
40
50 APEX SEMI ANGLE
60
1
2
3
ASPECT RATIO
4
5
6
M=2(1936 FT/S)
M=1·2(1160 FT/S)

FIG.12.