

# Multiple Model $\mathcal{L}_1$ Adaptive Fault-Tolerant Control of Small Unmanned Aerial Vehicles

Toufik Souanef<sup>1</sup>

<sup>1</sup>Cranfield University, United Kingdom. Email: toufik.souanef@cranfield.ac.uk

## ABSTRACT

This paper presents a method for fault-tolerant control of small fixed-wing Unmanned Aerial Vehicles (UAVs). The proposed design is based on multiple-model  $\mathcal{L}_1$  adaptive control. The controller is composed of a nominal reference model and a set of suboptimal reference models. The nominal model is the one with desired dynamics that are optimal regarding some specific criteria. In a suboptimal model the performance criteria are reduced, it is designed to ensure system robustness in the presence of critical failures. The controller was tested in simulations and it was shown that the multiple model  $\mathcal{L}_1$  adaptive controller stabilizes the system in case of inversion of the control input, while the  $\mathcal{L}_1$  adaptive controller with a single nominal model fails.

## INTRODUCTION

Unmanned Aerial Vehicles (UAVs), commonly known as drones and referred to as Remotely Piloted Aircraft (RPA) by the International Civil Aviation Organisation (ICAO), are aircraft without a human pilot aboard. According to the assigned missions or to their size, there are many different classes of UAVs (Valavanis and Vachtsevanos, 2015). This work focuses on small fixed-wing UAVs that is, with wingspans less than 2 metres and payload smaller than 2 kg. Small UAVs are gaining growing interest because of their low cost, high manoeuvrability, and simple maintenance. They are used for a wide range of military and civilian tasks (Austin, 2011). The operation of UAVs, especially in urban environments, needs a high degree of safety and reliability. However, small UAVs are generally built with low-cost components and materials, which increases the probability

24 of occurrence of faults and failures. For that reason, the design of fault-tolerant control systems is  
25 required (Blanke et al., 2006; Ducard, 2009; Patton, 1997; Zhang and Jiang, 2008).

26 Fault-tolerant control is defined as a system that possesses the ability to accommodate failures  
27 automatically (Zhang and Jiang, 2008). Fault-tolerant control systems are classified as either  
28 passive or active (Hwang et al., 2009; Rotondo, 2017). Passive fault-tolerant control is based on  
29 robust control while assuming the worst case conditions (Amin and Hasan, 2019; Benosman, 2011;  
30 Edwards et al., 2000; Wang, 2010; Yang et al., 2001). Nevertheless, the designed controllers tend to  
31 be conservative from performance viewpoint (Jiang and Yu, 2012). Active fault-tolerant controllers  
32 are composed of a fault detection scheme and a supervision module. On the basis of the information  
33 of the former, the supervision module may decide how to reconfigure the controller (Abbaspour  
34 et al., 2020; Amin and Hasan, 2019; Rotondo, 2017). However, applying such advanced control  
35 systems for small UAVs is difficult, because of their limited computing resources.

36 A compromise between the two approaches is adaptive control, which is based on the reconfig-  
37 uration of the controller parameters without involving an explicit fault detection module (Bodson,  
38 2003; Ma et al., 2020; Nian et al., 2020; Tao, 2004; Xue et al., 2020; Yang et al., 2014).

39 A crucial aspect in applying adaptive control techniques to real-world systems is the transient  
40 response guarantee, in the absence of which, overly poor tracking behaviour can occur before  
41 ideal asymptotic convergence takes place (Zang and Bitmead, 1990). Moreover, the transient  
42 performance improvement cannot be achieved through high-gain feedback, which will degrade  
43 the robustness of the closed-loop system. However, most adaptive control methods focus on the  
44 asymptotic performance, providing no transient performance guarantee without resorting to high-  
45 gain feedback. One solution to this issue is based on  $\mathcal{L}_1$  adaptive control (Hovakimyan and Cao,  
46 2010). The adaptive control architecture decouples the estimation loop from the control loop  
47 through the introduction of a low-pass filter. As a result, arbitrarily fast adaptation can be used  
48 without sacrificing system robustness. The benefit of  $\mathcal{L}_1$  adaptive control is its capacity for fast and  
49 robust adaptation that leads to desired transient performance for both system signals, inputs and  
50 outputs. These characteristics make it suitable for systems with unknown dynamics and subject to

51 possible faults and external disturbances, such as small UAVs.

52 Despite the excellent performance of  $\mathcal{L}_1$  adaptive control, for fault-tolerant control (Ackerman  
53 et al., 2017; Ahmadi et al., 2019; Dobrokhodov et al., 2013; Mühlegg et al., 2015; Patel et al.,  
54 2009; Sørensen and Breivik, 2015; Tian et al., 2020; Zhou et al., 2019), it is still true that when  
55 the uncertainties induced by disturbances, faults or failures are too large, they may reduce the  
56 performance of the controller or even make the system unstable. Actually, if a fault or a failure occurs  
57 on the system, the unknown parameters may go outside the predefined sets of the control design,  
58 which may lead to poor system performance or more critically to system instability. Furthermore,  
59 when a fault affects the system actuators it reduces their capabilities and, if the nominal performance  
60 of the system is maintained, the actuators will work beyond their nominal set point, which might  
61 lead to severe failures that cannot be compensated by a fault-tolerant controller. Therefore, it is not  
62 reasonable to maintain the same desired performance of the system, because after a fault or a failure  
63 it is not possible to recover the nominal performance. This is especially true for non-redundant  
64 systems such as low-cost UAVs.

65 The proposed solution is based on the application of the multiple model  $\mathcal{L}_1$  adaptive controller  
66 (Souanef and Fichter, 2015). The key idea is to design an  $\mathcal{L}_1$  adaptive controller with a nominal  
67 reference model and a set of suboptimal reference models. The nominal model is the model with  
68 desired dynamics that are optimal regarding some specific criteria. A suboptimal model does not  
69 necessarily verify these specifications. It is designed to ensure system robustness in the presence of  
70 large uncertainties. This multiple-model  $\mathcal{L}_1$  adaptive control design can expand the performance  
71 of the  $\mathcal{L}_1$  adaptive control schemes to effectively deal with plant hard failures such as the inversion  
72 of the control direction (a long-standing issue that is difficult for a single-model adaptive controller  
73 to deal with) which may be caused by uncertain system structural damage and component (actuator  
74 or sensor) failures.

75 A similar approach for performance degradation based on multiple model control was presented  
76 in (Jiang and Zhang, 2006; Zhang and Jiang, 2003). The design was made under the assumption that  
77 the model of the plant has no uncertainties, which is not realistic, especially for post-fault systems.

78 Furthermore, only actuator faults were addressed while structural faults were not considered.

79 The main contributions of this paper are:

- 80 • Development of a method for adaptive fault-tolerant control based on an  $\mathcal{L}_1$  adaptive  
81 controller with a nominal reference model and a set of suboptimal reference models, so as  
82 to avoid system instability in the presence of hard faults/failures.
- 83 • Extension of the method proposed in (Souanef and Fichter, 2015) to Multi-Input Multi-  
84 Output (MIMO) systems.

## 85 NOTATION

86 Throughout the paper,  $\|\cdot\|$  denotes the 2-norm and  $\|\cdot\|_\infty$  denotes the infinity norm of a vector or  
87 a matrix. The notation  $\|\xi\|_{\mathcal{L}_\infty}$  denotes the  $\mathcal{L}_\infty$ -norm of the vector  $\xi(t)$ . For a stable proper transfer  
88 matrix  $G(s)$ ,  $\|G(s)\|_{\mathcal{L}_1}$  denotes its  $\mathcal{L}_1$ -norm.  $\mathbb{R}^n$  denotes the n-dimensional real vector space.  
89  $\mathbb{I}$  denotes an identity matrix of appropriate dimensions. Boldface notation is used for matrices,  
90 vectors, and tensors; italics are for for all variables and lower-case Greek letters; and Roman for all  
91 numerals, upper-case Greek characters, and mathematical operators.

## 92 PROBLEM FORMULATION

93 For control design, the dynamic model of an aircraft can be formulated as the following class  
94 of MIMO systems (Lavretsky and Wise, 2013)

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_p \mathbf{x}(t) + \mathbf{B}_p \mathbf{u}_p(t) + \mathbf{f}(t, \mathbf{x}), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ \mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t), \end{aligned} \tag{1}$$

96 where  $\mathbf{A}_p = \mathbf{A} + \Delta \mathbf{A} \in \mathbb{R}^{n \times n}$  is an unknown matrix,  $\mathbf{A} \in \mathbb{R}^{n \times n}$  is a known matrix,  $\Delta \mathbf{A} \in \mathbb{R}^{n \times n}$   
97 an unknown matrix of the system dynamics,  $\mathbf{B}_p = \mathbf{B}(\mathbb{I}_m + \Delta \mathbf{B}) \in \mathbb{R}^{n \times m}$  is an unknown matrix,  
98  $\mathbf{B} \in \mathbb{R}^{n \times m}$  is a known matrix,  $\Delta \mathbf{B} \in \mathbb{R}^{m \times m}$  is an unknown matrix of the control input uncertainties,  
99  $\mathbf{C} \in \mathbb{R}^{m \times n}$  is a known matrix,  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector which is assumed to be available through  
100 measurement,  $\mathbf{u}_p(t) \in \mathbb{R}^m$  is the control input vector  $\mathbf{y}(t) \in \mathbb{R}^m$  is the output vector and  $\mathbf{f}(t, \mathbf{x}) \in \mathbb{R}^n$   
101 is a vector of unknown nonlinear functions.

102 Now consider the control

$$103 \quad \mathbf{u}_p(t) = \mathbf{u}(t) + \mathbf{K}_l \mathbf{x}(t), \quad (2)$$

104 where  $\mathbf{K}_l \in \mathbb{R}^{m \times n}$  is a gain matrix that defines  $\mathbf{A}_m = \mathbf{A} + \mathbf{B}\mathbf{K}_l$ , where  $\mathbf{A}_m \in \mathbb{R}$  is a Hurwitz matrix  
 105 that defines the desired dynamics of the system. The resulting system to be controlled by the  
 106 adaptive control is

$$107 \quad \dot{\mathbf{x}}(t) = \mathbf{A}_m \mathbf{x}(t) + \mathbf{B}\omega \mathbf{u}(t) + \tilde{\mathbf{f}}(t, \mathbf{x}), \quad (3)$$

108 where  $\omega = \mathbb{I}_m + \Delta \mathbf{B}$  and  $\tilde{\mathbf{f}}(t, \mathbf{x}) = \Delta \mathbf{A} \mathbf{x}(t) + (\omega - \mathbb{I}_m) \mathbf{K}_l \mathbf{x}(t) + \mathbf{f}(t, \mathbf{x})$ . Assuming  $\tilde{\mathbf{f}}(t, \mathbf{x}) = \mathbf{B}(\theta^\top \mathbf{x}(t) +$   
 109  $\eta_m(t)) + \eta_u(t)$ , the system in (3) can be parametrised as follows

$$110 \quad \dot{\mathbf{x}}(t) = \mathbf{A}_m \mathbf{x}(t) + \mathbf{B}(\omega \mathbf{u}(t) + \theta^\top \mathbf{x}(t) + \eta_m(t)) + \eta_u(t), \quad (4)$$

111 where  $\theta^\top \in \mathbb{R}^{m \times n}$  is a matrix of constant unknown parameters representing model uncertainties,  
 112  $\eta_m(t) \in \mathbb{R}^m$  is an unknown matched disturbance, and  $\eta_u(t) \in \mathbb{R}^n$  is an unknown unmatched  
 113 disturbance.

114 **Assumption 1.** The unknown model parameters are bounded, i.e.,  $\theta \in \Theta$ , where  $\Theta$  is a known  
 115 compact convex set. The system input gain matrix  $\omega$  is assumed to be an unknown (non-singular)  
 116 strictly row-diagonally dominant matrix with  $\text{sgn}(\omega_{ii})$  known. Also, it is assumed that there exists  
 117 a known compact convex set  $\Omega$  such that  $\omega \in \Omega \subset \mathbb{R}^{m \times m}$ .

**Assumption 2.** The non-linear function  $\eta_m(t)$  is uniformly bounded, i.e., there exist unknown  
 real constant  $L_m > 0$ , such that for all  $t \geq 0$  the following bound hold:

$$\|\eta_m(t)\| \leq L_m.$$

**Assumption 3.** There exist unknown real constant  $L_u > 0$ , such that for all  $t \geq 0$  the following

bound hold

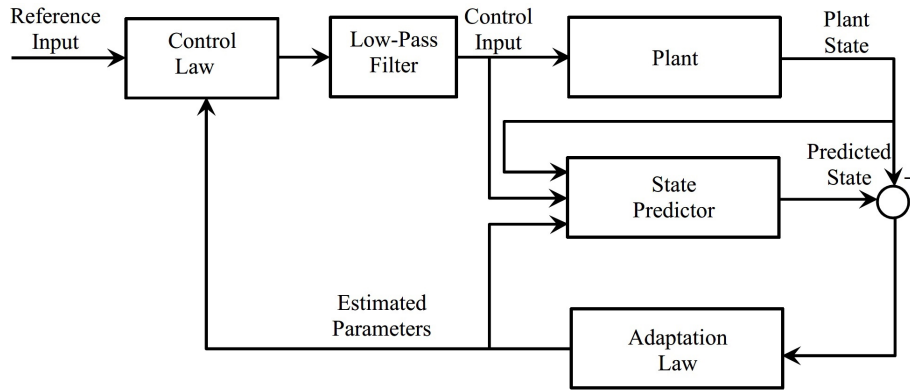
$$\|\eta_u(t)\| \leq L_u.$$

118 **Remark 1.** Assumptions 2 and 3 are acceptable for real systems, given that a superior bound  
 119 of disturbances, which the system may hold without being broken, is usually known from technical  
 120 specifications or engineering insights.

121 The objective is to design a state-feedback controller to ensure that the output of the system  
 122 tracks a given piecewise continuous bounded reference signal  $r(t)$  and consequently maintain the  
 123 stability of the control system despite the presence of faults and/or external disturbances.

## 124 $\mathcal{L}_1$ ADAPTIVE CONTROL

125 We consider the architecture of the  $\mathcal{L}_1$  adaptive controller which is composed of the state  
 126 predictor, the adaptation law and the control law (Figure 1).



**Fig. 1.** Block diagram of the  $\mathcal{L}_1$  adaptive controller.

127 The state predictor is defined as

$$128 \quad \hat{\mathbf{x}}(t) = \mathbf{A}_m \hat{\mathbf{x}}(t) + \mathbf{B}(\hat{\omega}(t)\mathbf{u}(t) + \hat{\theta}^\top(t)\mathbf{x}(t) + \hat{\eta}_m(t)) + \hat{\eta}_u(t), \quad (5)$$

129 where  $\hat{\mathbf{x}}(t)$  is the predicted state and  $\hat{\theta}(t)$ ,  $\hat{\omega}(t)$ ,  $\hat{\eta}_m(t)$ , and  $\hat{\eta}_u(t)$  are the estimates of the unknown  
 130 system parameters and disturbances.

131 The sliding surface is defined as

$$132 \quad \sigma(t) = \lambda \tilde{\mathbf{x}}(t), \quad (6)$$

133 where  $\tilde{\mathbf{x}}(t) = \hat{\mathbf{x}}(t) - \mathbf{x}(t)$  is the state estimation error and  $\lambda \in \mathbb{R}^{m \times n}$  is a constant arbitrary matrix,  
 134 chosen such that  $\lambda \mathbf{B}$  is non-singular and the coefficients  $\lambda(i, j) : i = 1..n; j = 1..m$  form a stable  
 135 hyperplane.

136 The estimation of the matched disturbance  $\eta_m(t)$  is defined by

$$137 \quad \hat{\eta}_m(t) = \begin{cases} -(\lambda \mathbf{B})^{-1} (\lambda \mathbf{A}_m \tilde{\mathbf{x}}(t) + \rho \sigma(t)) - \hat{L}_m(t) \frac{\mathbf{B}^\top \lambda^\top \sigma(t)}{\|\mathbf{B}^\top \lambda^\top \sigma(t)\|}, & \text{if } \sigma(t) \neq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

138 where  $\rho > 0$  is arbitrary and the estimated bound  $\hat{L}_m(t)$  is given by

$$139 \quad \hat{L}_m(t) = \Gamma \|\sigma^\top(t) \lambda \mathbf{B}\|, \quad (8)$$

140 where  $\Gamma \in \mathbb{R}^+$  is the adaptation rate.

141 The estimation of the unmatched disturbance  $\eta_u(t)$  is defined by

$$142 \quad \hat{\eta}_u(t) = \begin{cases} -\hat{L}_u(t) \frac{\lambda^\top \sigma(t)}{\|\lambda^\top \sigma(t)\|}, & \text{if } \sigma(t) \neq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

143 where the estimated bound  $\hat{L}_u(t)$  is computed by

$$144 \quad \hat{L}_u(t) = \Gamma \|\sigma^\top(t) \lambda\|, \quad (10)$$

145 The input gain matrix  $\omega$  and unknown parameters matrix  $\theta$  are estimated by

$$\begin{aligned}
 \dot{\hat{\omega}}(t) &= -\Gamma \text{Proj}(\hat{\omega}(t), \mathbf{u}(t) \sigma^\top(t) \lambda \mathbf{B})^\top, \\
 \dot{\hat{\theta}}(t) &= -\Gamma \text{Proj}(\hat{\theta}(t), \mathbf{x}(t) \sigma^\top(t) \lambda \mathbf{B}).
 \end{aligned}
 \tag{11}$$

147 The control law is given by

$$\mathbf{u}(s) = -\mathbf{K} \mathbf{D}(s) \left( \hat{v}_1(s) + \hat{v}_2(s) - \mathbf{K}_g \mathbf{r}(s) \right),
 \tag{12}$$

149 where  $\mathbf{D}(s)$  is an  $m \times m$  proper transfer matrix;  $\mathbf{K} \in \mathbb{R}^{m \times m}$ ;  $\mathbf{K}_g = -(\mathbf{C} \mathbf{A}_m^{-1} \mathbf{B})^{-1}$  is the pre-filter  
 150 of the MIMO control law;  $\hat{v}_1(s)$  is the Laplace transformation of  $\hat{v}_1(t) = \hat{\theta}^\top(t) \mathbf{x}(t) + \hat{\omega}(t) \mathbf{u}(t)$ ;  
 151  $\mathbf{H}_m(s) = \mathbf{C}(s\mathbb{I} - \mathbf{A}_m)^{-1} \mathbf{B}$ ;  $\mathbf{H}_0(s) = \mathbf{C}(s\mathbb{I} - \mathbf{A}_m)^{-1}$ ; and  $\hat{v}_2 = \hat{\eta}_m(t) + \mathbf{H}_m^{-1}(s) \mathbf{H}_0(s) \hat{\eta}_u(s)$ .

The design of  $\mathbf{D}(s)$  and  $\mathbf{K}$  should lead to a strictly proper and stable filter transfer matrix

$$\mathbf{F}(s) = \omega \mathbf{K} \mathbf{D}(s) (\mathbb{I} + \omega \mathbf{K} \mathbf{D}(s))^{-1},$$

152 with static gain  $\mathbf{F}(0) = \mathbb{I}$ .

153 Let

$$\begin{aligned}
 L &= \max_{\theta \in \Theta} \|\theta\|_1, \quad \mathbf{H}(s) = (s\mathbb{I} - \mathbf{A}_m)^{-1} \mathbf{B}, \\
 \mathbf{G}(s) &= \mathbf{H}(s) (\mathbb{I} - \mathbf{F}(s)).
 \end{aligned}
 \tag{13}$$

155 The  $\mathcal{L}_1$  adaptive controller is subject to the  $\mathcal{L}_1$  norm condition

$$\|\mathbf{G}(s)\|_{\mathcal{L}_1} L < 1.
 \tag{14}$$

157 Moreover, the design of  $\mathbf{F}(s)$  needs to ensure that

$$\mathbf{G}_u(s) = (s\mathbb{I} - \mathbf{A}_m)^{-1} - \mathbf{F}(s) \mathbf{H}(s) \mathbf{H}_m^{-1}(s) \mathbf{H}_0(s),
 \tag{15}$$

159 is proper and stable. Furthermore, since the transfer matrix  $\mathbf{G}_u(s)$  is proper and stable it has an  $\mathcal{L}_1$   
 160 norm (Hovakimyan and Cao, 2010).

161 **Remark 2.** It has been shown in (Souanef et al., 2015) and (Souanef, 2019) that the adaptation  
 162 laws of the external disturbances in equations (7) and (9) use the estimated bounds from equations (8)  
 163 and (10). This relaxes the assumption that the bounds of the external disturbances are known, which  
 164 is required in  $\mathcal{L}_1$  adaptive control based on projection-type adaptive laws (Cao and Hovakimyan,  
 165 2008).

166 **Remark 3.** If a fault or failure occurs on the system, the unknown parameters may go outside  
 167 the predefined sets. Therefore, the stability conditions in (14) and (15) may become not satisfied.  
 168 Hence, it is necessary to maintain system stability and a minimum of good performance, this is done  
 169 through the design of a set of suboptimal models which become effective when large uncertainties  
 170 appear on the plant.

## 171 MULTIPLE MODEL $\mathcal{L}_1$ ADAPTIVE CONTROL OF MIMO SYSTEMS

172 Considering probable faults scenario, a set of plant parameterisations, based on multiple models,  
 173 is arranged, and the objective is that the satisfactory controller is selected automatically to deal  
 174 with every situation. This means that the model which is the best match of the plant is selected.

175 The desired performance of each model is made through the design of the pair  $(\mathbf{A}_{m(i)}, \mathbf{B}_i)$ , for  
 176  $i \in \mathcal{I}$ .

177 The system in (1) can consequently be parameterised as follows

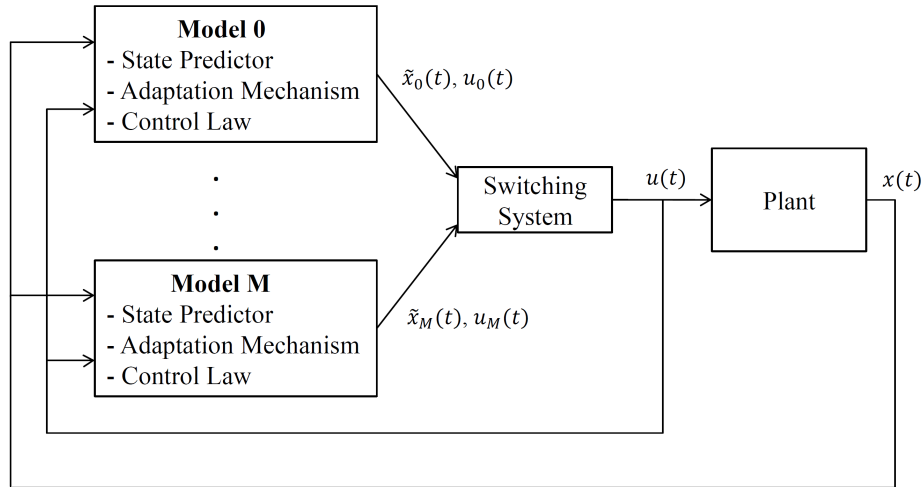
$$178 \quad \dot{\mathbf{x}}(t) = \mathbf{A}_{m(i)}\mathbf{x}(t) + \mathbf{B}_i(\omega_i\mathbf{u}(t) + \theta_i^\top \mathbf{x}(t) + \eta_{m(i)}(t)) + \eta_{u(i)}(t), \quad (16)$$

179 where  $\mathbf{A}_{m(i)} \in \mathbb{R}^{n \times n}$  are known Hurwitz matrices that define the desired dynamics of the system  
 180  $\mathbf{B}_i \in \mathbb{R}^{n \times m}$  are the desired input matrices,  $\omega_i \in \mathbb{R}^{m \times m}$  are unknown constant matrices representing  
 181 the system input gain,  $\theta_i^\top \in \mathbb{R}^{m \times n}$  are matrices of constant unknown parameters representing model  
 182 uncertainties,  $\eta_{m(i)}(t) \in \mathbb{R}^m$  are unknown matched disturbances, and  $\eta_{u(i)}(t) \in \mathbb{R}^n$  are unknown  
 183 unmatched disturbances.

184 **Assumption 4.** The system input gain matrices  $\omega_i$  are assumed to be unknown (non-singular)  
 185 strictly row-diagonally dominant matrices with known signs of diagonals. Also, it is assumed that  
 186 the unknown parameters are bounded, i.e.,  $\theta_i \in \Theta_i$ , where  $\Theta_i$  are known compact convex sets.  
 187 Furthermore, the functions  $\eta_{m(i)}$  and  $\eta_{u(i)}$  are uniformly bounded, i.e., there exist unknown real  
 188 constants  $L_{m(i)} > 0$  and  $L_{u(i)} > 0$ , such that for all  $t \geq 0$   $|\eta_{m(i)}(t)| \leq L_{m(i)}$  and  $\|\eta_{u(i)}(t)\| \leq L_{u(i)}$ .

### 189 Controller Design

190 The multiple model  $\mathcal{L}_1$  adaptive controller, as shown in Figure 2, is composed of a set of state  
 191 predictors, a set of adaptation laws, a set of control laws and a control input selector (switching  
 system). The state predictors are defined by



192 **Fig. 2.** Block diagram of the multiple model  $\mathcal{L}_1$  adaptive controller.

$$193 \quad \dot{\hat{\mathbf{x}}}_i(t) = \mathbf{A}_{m(i)} \hat{\mathbf{x}}_i(t) + \mathbf{B}_i (\hat{\omega}_i(t) \mathbf{u}(t) + \hat{\theta}_i^\top(t) \mathbf{x}(t) + \hat{\eta}_{m(i)}(t)) + \hat{\eta}_{u(i)}(t), \quad (17)$$

194 where  $\hat{\mathbf{x}}_i(t)$  are the predicted states and,  $\hat{\theta}_i(t)$ ,  $\hat{\omega}_i(t)$ ,  $\hat{\eta}_{m(i)}(t)$ , and  $\hat{\eta}_{u(i)}(t)$  are the estimates of the  
 195 unknown system parameters and external disturbances. The initial state of the state predictor is  
 196 equal to the plant state at switching time  $t_k$ :

$$197 \quad \hat{\mathbf{x}}(t_k) = \mathbf{x}(t_k). \quad (18)$$

198 The sliding surfaces are given by

$$199 \quad \sigma_i(t) = \lambda_i \tilde{\mathbf{x}}_i(t), \quad (19)$$

200 where  $\tilde{\mathbf{x}}_i(t) = \hat{\mathbf{x}}_i(t) - \mathbf{x}(t)$  are the state estimation errors and  $\lambda_i \in \mathbb{R}^{m \times n}$  are constant arbitrary  
 201 matrices, chosen such that  $\lambda_i \mathbf{B}_i$  are non-singular and the coefficients  $\lambda_i(k, j) : k = 1 \dots n; j = 1 \dots m$   
 202 form a stable hyperplane.

203 The adaptation laws are given by

$$204 \quad \begin{aligned} \dot{\hat{\omega}}_i(t) &= -\Gamma_i \text{Proj}(\mathbf{u}(t) \sigma_i^\top \lambda_i \mathbf{B}_i)^\top, \\ \dot{\hat{\theta}}_i(t) &= -\Gamma_i \text{Proj}(\mathbf{x}(t) \sigma_i^\top \lambda_i \mathbf{B}_i), \\ \hat{\eta}_{m(i)}(t) &= \begin{cases} -(\lambda_i \mathbf{B}_i)^{-1} (\lambda_i \mathbf{A}_{m(i)} \tilde{\mathbf{x}}_i(t) + \rho_i \sigma_i) - \hat{L}_{m(i)}(t) \frac{\mathbf{B}_i^\top \lambda_i^\top \sigma_i}{\|\mathbf{B}_i^\top \lambda_i^\top \sigma_i\|} & \text{if } \sigma_i \neq 0, \\ 0 & \text{if not,} \end{cases} \\ \hat{\eta}_{u(i)}(t) &= \begin{cases} -\hat{L}_{u(i)}(t) \frac{\lambda_i^\top \sigma_i}{\|\lambda_i^\top \sigma_i\|} & \text{if } \sigma_i \neq 0, \\ 0 & \text{if not,} \end{cases} \\ \dot{\hat{L}}_{m(i)}(t) &= \Gamma_i \|\sigma_i^\top \lambda_i \mathbf{B}_i\|, \\ \dot{\hat{L}}_{u(i)}(t) &= \Gamma_i \|\sigma_i^\top \lambda_i\|, \end{aligned} \quad (20)$$

205 where  $\rho_i > 0$  are arbitrary and  $\Gamma_i \in \mathbb{R}^+$  are the adaptation rates.

Let

$$\mathbf{H}_{m(i)}(s) = \mathbf{C}_i (s\mathbb{I} - \mathbf{A}_{m(i)})^{-1} \mathbf{B}_i \text{ and } \mathbf{H}_{0(i)}(s) = \mathbf{C}_i (s\mathbb{I} - \mathbf{A}_{m(i)})^{-1}.$$

206 The control laws are given by

$$207 \quad \mathbf{u}_i(s) = -\mathbf{K}_i \mathbf{D}_i(s) \left( \mathbf{K}_{g(i)} \mathbf{r}(s) - \hat{v}_{(i)}(s) \right), \quad (21)$$

208 where  $\hat{v}_{(i)}(s) = \hat{v}_{1(i)}(s) + \hat{v}_{2(i)}(s)\hat{\eta}_{u(i)}(s)$ ,  $\hat{v}_{1(i)}(s)$  are the Laplace transformations of  $\hat{v}_{1(i)}(t) =$   
 209  $\hat{\theta}_i^\top(t)\mathbf{x}(t) + \hat{\omega}_i(t)\mathbf{u}_i(t)$ ,  $\hat{v}_{2(i)}(s) = \hat{\eta}_{m(i)}(s) + \mathbf{H}_{m(i)}^{-1}(s)\mathbf{H}_{0(i)}(s)\hat{\eta}_{u(i)}(s)$ ,  $\mathbf{K}_{g(i)} = -(\mathbf{C}_i\mathbf{A}_{m(i)}^{-1}\mathbf{B}_i)^{-1}$  are  
 210 the pre-filters of the MIMO control laws,  $\mathbf{D}_i(s)$  are  $m \times m$  strictly proper transfer matrices and  
 211  $\mathbf{K}_i \in \mathbb{R}^{m \times m}$ .

212 Let  $\mathbf{B}_i^\dagger = (\mathbf{B}_i^\top \mathbf{B}_i)^{-1} \mathbf{B}_i^\top$  be the pseudo-inverse of  $\mathbf{B}_i$ , considering that  $\mathbf{B}_i$  has full column rank,  
 213 then  $\hat{v}_{2(i)}(t) = \hat{\eta}_{m(i)}(t) + \mathbf{B}_i^\dagger \hat{\eta}_{u(i)}(t)$ .

214 For analysis purposes, without loss of generality, it is assumed that the control laws use the  
 215 same filter parameters.  $\mathbf{D}_i(s)$  are chosen  $\mathbf{D}_i(s) = \frac{\mathbf{D}_0(s)}{s}$  and  $\mathbf{K}_i = 1$ , where  $\mathbf{D}_0(s)$  is a proper stable  
 216 transfer matrix.

217 Therefore, the control laws can be written as

$$218 \quad \mathbf{u}_i(s) = \frac{\mathbf{D}_0(s)}{s} \left( \mathbf{K}_{g(i)} \mathbf{r}(s) - \hat{v}_{(i)}(s) \right). \quad (22)$$

219 The switching logic is defined by

$$220 \quad \min_{i \in \mathcal{I}} \left\{ J_i = c_1 \|\tilde{\mathbf{x}}_i\|^2 + c_2 \int_0^t e^{-c_3(t-\tau)} \|\tilde{\mathbf{x}}_i(\tau)\|^2 d\tau \right\}, \quad (23)$$

221 where  $c_1$ ,  $c_2$  and  $c_3$  are arbitrary positive real. The model that minimises the criterion becomes the  
 222 selected model and its output is applied to the plant.

223 **Remark 4.** For practical implementation the discrete-time version of the switching logic in  
 224 (23) is given by

$$225 \quad \min_{i \in \mathcal{I}} \left\{ J_i = c_1 \|\tilde{\mathbf{x}}_i(kT)\|^2 + \frac{c_2}{c_3 kT + 1} \sum_{j=0}^k \|\tilde{\mathbf{x}}_i(jT)\|^2 \right\}, \quad (24)$$

226 where  $T$  is the sampling period.

227 **Remark 5.** It is assumed in this work that the switching is arbitrary, i.e., not dwell time or average  
 228 dwell time. To prevent arbitrarily fast switching, a non-zero waiting time  $T_{min} > 0$  is introduced after  
 229 every switching. By the end of the waiting period  $T_{min}$ , the controller corresponding to the model  
 230 with the minimum index is chosen (switched) to control the plant Narendra and Balakrishnan

231 (1997). Note that  $T_{min}$  is different from the dwell time  $\tau$ , and a switching signal has a dwell  
232 time  $\tau > 0$ , used for waiting another controller being activated, if the switching times satisfy  
233  $t_{k+1} - t_k \geq \tau, \forall k > 0$  (Liberzon, 2003).

234 **Remark 6.** It is a common practice to stop the control switching when  $J_i(t) \leq \epsilon \forall i = 1, 2, \dots, N$ ,  
235 for some pre-chosen, arbitrary and small  $\epsilon > 0$  Tan et al. (2017a).

236 **Remark 7.** It is quiet understood that both traditional projection-based adaptive law (Cao and  
237 Hovakimyan, 2008) and the piecewise-constant adaptive law (Cao and Hovakimyan, 2009) can be  
238 applied to the design of the multiple model  $\mathcal{L}_1$  adaptive controller. The advantage of the sliding  
239 mode adaptation law is that it permits to estimate the upper bounds of the external disturbances  
240 which makes the system more robust.

## 241 **Controller Analysis**

242 In this section, the performance of the  $\mathcal{L}_1$  adaptive controller is analysed. More specifically it  
243 is shown that:

- 244 • The reference models resulting from perfect knowledge of the uncertainties and a corre-  
245 sponding non-adaptive controller are stable, subject to some conditions involving the filter  
246  $D_0(s)$ .
- 247 • The prediction errors, i.e. the errors between the states of the plant and those of the state  
248 predictors, are bounded.
- 249 • The differences between the states/input of the system and those of the reference systems  
250 are proportional to the prediction error

251 For a switching system, it is not straightforward to compute the  $\mathcal{L}_1$  norm condition in (14) and  
252 (15). Actually, for Linear Time Invariant (LTI) systems, the  $\mathcal{L}_1$  norm is readily computed from  
253 the impulse response. However, for a switched system, the impulse response is time dependent  
254 (switching signal-dependent), and computing the  $\mathcal{L}_1$  norm is not as straightforward as in the LTI  
255 case.

256 A similar approach to (Snyder, 2019) is applied here. It is based on a new method of analysing

257  $\mathcal{L}_1$  adaptive control. This approach results in necessary and sufficient conditions provided in the  
 258 form of linear matrix inequalities (LMIs).

259 *Reference Models Analysis*

260 The reference system with the nominal parameters is defined as follows

$$261 \begin{aligned} \dot{\mathbf{x}}_r(t) &= \mathbf{A}_{m(i)} \mathbf{x}_r(t) + \mathbf{B}_i (\omega_i \mathbf{u}_r(t) + \theta_i^\top \mathbf{x}_r(t) + \eta_{m(i)}(t)) + \eta_{u(i)}(t), \quad \mathbf{x}_r(0) = \mathbf{x}_0 \\ \mathbf{u}_r(s) &= - \frac{\mathbf{D}_0(s)}{s} (\nu_{r(i)}(s) - \mathbf{K}_g(i) \mathbf{r}(s)) \end{aligned} \quad (25)$$

262 where  $\nu_{r(i)}(s) = \nu_{1(i)}(s) + \nu_{2(i)}(s)$ ,  $\nu_{1(i)}(s)$  are the Laplace transformations of  $\nu_{1(i)}(t) = \theta_i^\top \mathbf{x}_r(t) + \omega_i(t) \mathbf{u}_r(t)$   
 263 and  $\nu_{2(i)}(s) = \eta_{m(i)}(s) + \phi_i(s) \eta_{u(i)}(s)$ , with  $\phi_i(s) = \mathbf{H}_{m(i)}^{-1}(s) \mathbf{H}_{0(i)}(s)$ .

264 Alternatively we can write  $\nu_{2(i)}(t) = \eta_{m(i)}(t) + \mathbf{B}_i^\dagger \eta_{u(i)}(t)$ .

265 **Remark 8.** It should be noted that the reference control law is not implementable, since it  
 266 depends on the unknown parameters and it is used only for analysis purposes.

267 Letting  $(\mathbf{A}_f, \mathbf{B}_f, \mathbf{C}_f, \mathbf{D}_f)$  be a minimal realisation of  $\mathbf{D}_0(s)$  with  $n_f$  states, the reference system  
 268 dynamics can be written in state-space form

$$269 \begin{aligned} \begin{bmatrix} \dot{\mathbf{x}}_r(t) \\ \dot{\mathbf{x}}_{f_1}(t) \\ \dot{\mathbf{x}}_{I_1}(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} \mathbf{A}_{m(i)} + \mathbf{B}_i \theta_i^\top & 0 & -\mathbf{B}_i \omega_i \\ \mathbf{B}_f \theta_i^\top & \mathbf{A}_f & \mathbf{B}_f \omega_i \\ \mathbf{D}_f \theta_i^\top & \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix}}_{\bar{\mathbf{A}}_i} \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_{f_1}(t) \\ \mathbf{x}_{I_1}(t) \end{bmatrix} \\ &+ \underbrace{\begin{bmatrix} \mathbf{B}_i \\ \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix}}_{\bar{\mathbf{B}}_i} \nu_{2(i)}(t) - \begin{bmatrix} 0 \\ \mathbf{B}_f \mathbf{K}_g i \\ \mathbf{D}_f \mathbf{K}_g i \end{bmatrix} r(t) \end{aligned} \quad (26)$$

$$\begin{aligned} [u_r(t)] &= \underbrace{\begin{bmatrix} 0 & 0 & -\mathbb{I} \end{bmatrix}}_{\bar{\mathbf{C}}} \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_{f_1}(t) \\ \mathbf{x}_{I_1}(t) \end{bmatrix}, \quad \begin{bmatrix} \mathbf{x}_r(0) \\ \mathbf{x}_{f_1}(0) \\ \mathbf{x}_{I_1}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

270 where  $\mathbf{x}_{f_1}$ ,  $\mathbf{x}_{I_1}$  are the states of  $\mathbf{D}_0(s)$  and the integrator respectively.

271 The reference control law can be rewritten in more compact form as

$$\begin{aligned}
 \dot{\bar{\mathbf{x}}}(t) &= \bar{\mathbf{A}}_i \bar{\mathbf{x}}(t) + \bar{\mathbf{B}}_i v_{2(i)}(t) + \bar{\mathbf{E}}_i r(t), \quad \bar{\mathbf{x}}(0) = \bar{\mathbf{x}}_0, \\
 \mathbf{u}_r(t) &= \bar{\mathbf{C}} \bar{\mathbf{x}}(t),
 \end{aligned}
 \tag{27}$$

273 where  $\bar{\mathbf{x}}^\top(t) \triangleq [\mathbf{x}_r^\top(t), \mathbf{x}_{f_1}^\top(t), \mathbf{x}_I^\top(t)]$ .

**Lemma 2** Give an arbitrary matrix  $\mathbf{Q} = \mathbf{Q}^\top > 0$ , if there exists a constant symmetric matrix  $\mathbf{P} > 0$  verifying

$$\bar{\mathbf{A}}_i^\top \mathbf{P} + \mathbf{P} \bar{\mathbf{A}}_i \leq -\mathbf{Q}, \quad \forall \theta_i \in \Theta_i \text{ and } \forall \omega_i \in \Omega_i,$$

274 then the Lyapunov function  $V = \bar{\mathbf{x}}^\top \bar{\mathbf{P}} \bar{\mathbf{x}}$  guarantees the stability of the switching reference systems  
 275 in (27).

276 This fact is straightforward from the converse Lyapunov theorem for LTI systems.

### 277 *Transient Performance and Steady-State Performance*

278 In the following Lemma, it is stated that the prediction errors  $\tilde{\mathbf{x}}_i(t)$  and the estimation errors of  
 279 the unknown parameters are bounded.

280 **Lemma 3** The following bound holds for the norm of the prediction error  $\forall i \in \mathcal{I}$

$$\|\tilde{\mathbf{x}}_i\|_{\mathcal{L}_\infty} \leq \delta,
 \tag{28}$$

282 where  $\delta > 0$  is an arbitrary small real.

283 Furthermore, the prediction errors  $\tilde{\mathbf{x}}_i(t)$  converge asymptotically to zero, i.e.,

$$\lim_{t \rightarrow \infty} \tilde{\mathbf{x}}_i(t) = 0.
 \tag{29}$$

285 **Proof.** The proof is omitted here because of lack of space. Interested readers are referred to

286 Souanef (2019).

**Theorem.** There exist positive constants  $\kappa_2$  and  $\kappa_3$  such that, for each model  $i$  the error between the actual system and the reference system is bounded by

$$\begin{aligned}\|\mathbf{x}_r(t) - \mathbf{x}(t)\| &\leq \kappa_2 \\ \|\mathbf{u}_r(t) - \mathbf{u}(t)\| &\leq \kappa_3.\end{aligned}$$

287 Furthermore, if the closed-loop system is stable then

$$288 \quad \lim_{t \rightarrow \infty} \|\mathbf{x}_r(t) - \mathbf{x}(t)\| = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} \|\mathbf{u}_r(t) - \mathbf{u}(t)\| \quad (30)$$

289 **Proof.** In this section, the dependence of the parameters on  $(t)$  is dropped unless it is not clear  
290 from the context.

291 From (22) it can be written

$$292 \quad \mathbf{u}(s) = -\frac{D_0(s)}{s} \left( \omega_i \mathbf{u}(s) + v_i(s) + \tilde{v}_i(s) - \mathbf{K}g_i \mathbf{r}(s) \right), \quad (31)$$

293 where  $\tilde{v}_{(i)}(s) = \tilde{v}_{1(i)}(s) + \tilde{v}_{2(i)}(s)$ ,  $\tilde{v}_{1(i)}(s)$  are the Laplace transformations of  $\tilde{v}_{1(i)} = \tilde{\theta}_i^\top \mathbf{x}(t) +$   
294  $\tilde{\omega}_i(t) \mathbf{u}(t)$  and  $\tilde{v}_{2(i)}(s) = \tilde{\eta}_{u(i)}(s) + \phi_i(s) \tilde{\eta}_{u(i)}(s)$ .

295 Consequently, the closed-loop systems (16) and (31) can be written as follows

$$\begin{aligned}296 \quad \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}}_{f_1} \\ \dot{\mathbf{x}}_{I_1} \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{m(i)} + \mathbf{B}_i \theta_i^\top & 0 & -\mathbf{B}_i \omega_i \\ \mathbf{B}_f \theta_i^\top & \mathbf{A}_f & \mathbf{B}_f \omega_i \\ \mathbf{D}_f \theta_i^\top & \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_{f_1} \\ \mathbf{x}_{I_1} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_i \\ \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix} v_{2(i)} \\ &+ \begin{bmatrix} 0 \\ \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix} \tilde{v}_i - \begin{bmatrix} 0 \\ \mathbf{B}_f K g_i \\ \mathbf{D}_f K g_i \end{bmatrix} \mathbf{r}.\end{aligned} \quad (32)$$

297 The error between the state of the reference system and the actual plant,  $e = x_r - x$ , can be

298 expressed as

$$299 \begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{x}}_{f_1} \\ \dot{\mathbf{x}}_{I_1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{m(i)} + \mathbf{B}_i \theta_i^\top & 0 & -\mathbf{B}_i \omega_i \\ \mathbf{B}_f \theta_i^\top & \mathbf{A}_f & \mathbf{B}_f \omega_i \\ \mathbf{D}_f \theta_i^\top & \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix} \begin{bmatrix} e \\ x_{f_1} \\ x_{I_1} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_i \\ \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix} \tilde{\mathbf{v}}_i. \quad (33)$$

300 The control error can also be formulated as follows

$$301 \mathbf{e}_u = \mathbf{u}_r - \mathbf{u} = \begin{bmatrix} 0 & 0 & -\mathbb{I} \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \mathbf{x}_{f_1} \\ \mathbf{x}_{I_1} \end{bmatrix}. \quad (34)$$

302 From (16) and (17), the prediction error dynamics can be written as

$$303 \dot{\tilde{\mathbf{x}}}_i = \mathbf{A}_{m(i)} \tilde{\mathbf{x}}_i + \mathbf{B}_i (\tilde{\omega}_i \mathbf{u} + \tilde{\theta}_i^\top \mathbf{x} + \tilde{\eta}_{m(i)}) + \tilde{\eta}_{u(i)}. \quad (35)$$

304 Thus

$$305 \tilde{\mathbf{v}}_i = \mathbf{B}_i^\dagger (\dot{\tilde{\mathbf{x}}}_i - \mathbf{A}_{m(i)} \tilde{\mathbf{x}}_i). \quad (36)$$

306 Passing  $\mathbf{B}_i^\dagger \dot{\tilde{\mathbf{x}}}_i$  through the filter  $(s\mathbb{I} + D_0(s)\omega_i)^{-1} D_0(s)$ , we can write

$$307 \begin{bmatrix} \dot{\mathbf{x}}_{f_2} \\ \dot{\mathbf{x}}_{I_2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_f & \mathbf{B}_f \omega_i \\ \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix} \begin{bmatrix} x_{f_2} \\ x_{I_2} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix} \mathbf{B}_i^\dagger \tilde{\mathbf{x}}_i. \quad (37)$$

Applying this to the error dynamics in (33) we have

$$\begin{aligned}
 \begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{x}}_{f_1} \\ \dot{\mathbf{x}}_{I_1} \\ \dot{\mathbf{x}}_{f_2} \\ \dot{\mathbf{x}}_{I_2} \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{m(i)} + \mathbf{B}_i \theta_i^\top & 0 & -\mathbf{B}_i \omega_i & -\mathbf{B}_i \mathbf{C}_f & -\mathbf{B}_i \mathbf{D}_f \omega_i \\ \mathbf{B}_f \theta_i^\top & \mathbf{A}_f & \mathbf{B}_f \omega_i & 0 & 0 \\ \mathbf{D}_f \theta_i^\top & \mathbf{C}_f & \mathbf{D}_f \omega_i & 0 & 0 \\ 0 & 0 & 0 & \mathbf{A}_f & \mathbf{B}_f \omega_i \\ 0 & 0 & 0 & \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \mathbf{x}_{f_1} \\ \mathbf{x}_{I_1} \\ \mathbf{x}_{f_2} \\ \mathbf{x}_{I_2} \end{bmatrix} \\
 &+ \begin{bmatrix} -\mathbf{D}_f \mathbf{B}_i^\dagger \\ -\mathbf{B}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \\ -\mathbf{D}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \\ -\mathbf{B}_f \mathbf{B}_i^\dagger \\ -\mathbf{D}_f \mathbf{B}_i^\dagger \end{bmatrix} \tilde{\mathbf{x}},
 \end{aligned} \tag{38}$$

and

$$\mathbf{e}_u = \begin{bmatrix} 0 & 0 & -\mathbb{I} & -\mathbf{C}_f & -\mathbf{D}_f \omega_i \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \mathbf{x}_{f_1} \\ \mathbf{x}_{I_1} \\ \mathbf{x}_{f_2} \\ \mathbf{x}_{I_2} \end{bmatrix} + \begin{bmatrix} -\mathbf{D}_f \mathbf{B}_i^\dagger \end{bmatrix} \tilde{\mathbf{x}}. \tag{39}$$

Letting

$$\begin{aligned}
 \bar{\mathbf{H}}_i &= \begin{bmatrix} -\mathbf{B}_i \mathbf{C}_f & -\mathbf{B}_i \mathbf{D}_f \omega_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{\mathbf{J}}_i = \begin{bmatrix} -\mathbf{D}_f \mathbf{B}_i^\dagger \\ -\mathbf{B}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \\ -\mathbf{D}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \end{bmatrix}, \\
 \bar{\mathbf{G}}_i &= \begin{bmatrix} -\mathbf{B}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \\ -\mathbf{D}_f \mathbf{B}_i^\dagger \mathbf{A}_{m(i)} \end{bmatrix}, \quad \bar{\mathbf{L}}_i = \begin{bmatrix} 0 & \mathbf{C}_f & \mathbf{D}_f \omega_i \end{bmatrix},
 \end{aligned}$$

312 it follows from (38) and (39) that

$$313 \begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\bar{\mathbf{x}}}_{f_2} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{A}}_i & \bar{\mathbf{H}}_i \\ 0 & \bar{\mathbf{F}}_i \end{bmatrix} \begin{bmatrix} \bar{\mathbf{e}} \\ \bar{\mathbf{x}}_{f_2} \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{J}}_i \\ \bar{\mathbf{G}}_i \end{bmatrix} \tilde{\mathbf{x}}, \quad (40)$$

314 and

$$315 \mathbf{e}_u = \begin{bmatrix} \bar{\mathbf{C}} & \bar{\mathbf{L}}_i \end{bmatrix} \begin{bmatrix} \bar{\mathbf{e}} \\ \bar{\mathbf{x}}_{f_2} \end{bmatrix} + \begin{bmatrix} -\mathbf{D}_f \mathbf{B}_i^\dagger \end{bmatrix} \tilde{\mathbf{x}}, \quad (41)$$

316 where  $\bar{\mathbf{e}} = [\mathbf{e}^\top, \mathbf{x}_{f_1}^\top, \mathbf{x}_{I_1}^\top]^\top$  and  $\bar{\mathbf{x}}_{f_2} = [\mathbf{x}_{f_2}^\top, \mathbf{x}_{I_2}^\top]^\top$ .

317 Note that the reference system is stable and the filter represented by  $\bar{\mathbf{F}}_i$  is a subsystem of the  
318 reference system when  $\theta = 0$ . Therefore, from Lemma 1, there exists positive definite matrices  
319  $\mathbf{Q}_i(\omega_i) > 0$  such that for all  $\omega_i \in \Omega$ ,

$$320 \bar{\mathbf{F}}_i^\top \bar{\mathbf{Q}}_i + \bar{\mathbf{Q}}_i \bar{\mathbf{F}}_i \leq -\mathbb{I}. \quad (42)$$

321 Let  $\bar{V}_i(t) = \bar{\mathbf{x}}_{f_2}^\top \bar{\mathbf{Q}}_i \bar{\mathbf{x}}_{f_2}$ , where  $V_i(0) = 0$ . Differentiating along the system trajectories it follows that

$$\begin{aligned} \dot{V}_i &= \bar{\mathbf{x}}_{f_2}^\top (\bar{\mathbf{F}}_i^\top \bar{\mathbf{Q}}_i + \bar{\mathbf{Q}}_i \bar{\mathbf{F}}_i) \bar{\mathbf{x}}_{f_2} + 2\bar{\mathbf{x}}_{f_2}^\top \bar{\mathbf{Q}}_i \bar{\mathbf{G}}_i \tilde{\mathbf{x}} \\ &\leq -\|\bar{\mathbf{x}}_{f_2}\|^2 + 2\|\bar{\mathbf{x}}_{f_2}\| \beta_F \|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty} \\ &\leq -\|\bar{\mathbf{x}}_{f_2}\|^2 + \beta_F^2 \|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty}^2 \end{aligned} \quad (43)$$

322 where the last line follows from square completion and  $\beta_F = \sqrt{n} \max_{i \in \mathcal{I}} \|\bar{\mathbf{Q}}_i \bar{\mathbf{G}}_i\|$ .

323 By integrating it is straightforward to show that the following bound holds for  $\bar{\mathbf{x}}_{f_2}$

$$324 \|\bar{\mathbf{x}}_{f_2}\|_{\mathcal{L}_\infty} \leq \kappa_1, \quad (44)$$

325 where  $\kappa_1 = \sqrt{n} \max_{i \in \mathcal{I}} \|\bar{\mathbf{Q}}_i \bar{\mathbf{G}}_i\| \delta$  and  $\delta$  is the upper bound of  $\tilde{\mathbf{x}}_i$  defined in Lemma 2.

326 We now define the Lyapunov functions  $\bar{W}_i = \bar{\mathbf{e}}^\top \bar{\mathbf{P}}_i \bar{\mathbf{e}}$ . Differentiating along the system trajectories

328 it follows that

$$\begin{aligned}
\dot{W}_i &= \bar{\mathbf{e}}^\top (\bar{\mathbf{A}}_i^\top \bar{\mathbf{P}}_i + \bar{\mathbf{P}}_i \bar{\mathbf{A}}_i) \bar{\mathbf{e}} + 2\bar{\mathbf{e}}^\top \bar{\mathbf{P}}_i \bar{\mathbf{H}}_i \bar{\mathbf{x}}_{f_2} + 2\bar{\mathbf{e}}^\top \bar{\mathbf{P}}_i \bar{\mathbf{J}}_i \tilde{\mathbf{x}} \\
&\leq -\|\bar{\mathbf{e}}\|^2 + 2\|\bar{\mathbf{e}}\|\beta_{\bar{\mathbf{e}}}\|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty} \\
&\leq -\|\bar{\mathbf{e}}\|^2 + \beta_{\bar{\mathbf{e}}}^2\|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty}^2,
\end{aligned} \tag{45}$$

329 where  $\beta_e = \left( \kappa_1 \max_{i \in \mathcal{I}} \|\bar{\mathbf{P}}_i \bar{\mathbf{H}}_i\| + \sqrt{n} \max_{i \in \mathcal{I}} \|\bar{\mathbf{P}}_i \bar{\mathbf{J}}_i\| \right)$ .

330 Therefore, the following bound holds

$$331 \quad \|\bar{\mathbf{e}}\|_{\mathcal{L}_\infty} \leq \kappa_2, \tag{46}$$

332 where  $\kappa_2 = \left( \kappa_1 \max_{i \in \mathcal{I}} \|\bar{\mathbf{P}}_i \bar{\mathbf{H}}_i\| + \sqrt{n} \max_{i \in \mathcal{I}} \|\bar{\mathbf{P}}_i \bar{\mathbf{J}}_i\| \right) \delta$ .

333 Furthermore

$$\begin{aligned}
\|e_u\|_{\mathcal{L}_\infty} &\leq \|\bar{\mathbf{C}}\| \|\bar{\mathbf{e}}\|_{\mathcal{L}_\infty} + \|\bar{\mathbf{L}}_i\| \|\bar{\mathbf{x}}_{f_2}\|_{\mathcal{L}_\infty} + \|\mathbf{D}_f \mathbf{B}_i^\dagger\| \|\tilde{\mathbf{x}}\|_{\mathcal{L}_\infty}, \\
&\leq \kappa_3,
\end{aligned} \tag{47}$$

334 where  $\kappa_3 = \|\bar{\mathbf{C}}\| \kappa_2 + \left( \max_{i \in \mathcal{I}} \|\bar{\mathbf{L}}_i\| + \max_{i \in \mathcal{I}} \|\mathbf{D}_f \mathbf{B}_i^\dagger\| \right) \delta$ .

335 This completes the proof. □

### 336 UAV LATERAL-DIRECTIONAL CONTROL IN CASE OF INVERSION OF THE COMMANDS

337 A critical situation in flight control systems is that in case of structural damage of the aircraft,  
338 the control direction can be inverted. In fact, if an aircraft suffers damage, a control surface may  
339 generate a totally opposite angular acceleration, which means the actuation signs will be changed  
340 (Liu et al., 2010; Tan et al., 2017b). The inversion of the sign of the control direction can also  
341 result from actuators or software faults. This situation cannot be handled by  $\mathcal{L}_1$  adaptive controller  
342 with a single model. Actually, a conservative condition in adaptive control is that the sign of input  
343 vector must be known and should not change (Ioannou and Sun, 2012).  
344  
345

## 346 Controller Design

347 The lateral-directional equations of motion of a fixed-wing aircraft are described by the set of  
 348 states  $(\beta, p, r, \phi)$ , where  $\beta$  is the sideslip angle,  $p$  is the roll rate,  $r$  is the yaw rate and  $\phi$  is the roll  
 349 angle. The control inputs are the aileron deflection  $\delta_a$  and the rudder deflection  $\delta_r$ .

350 The objective is to design a control input  $u = [\delta_a, \delta_r]^\top$  to enable tracking of the roll command  
 351  $\phi_c$  and the sideslip angle command  $\beta_c$ .

352 From (Stevens and Lewis, 2003), the linearised model of the lateral-directional dynamics of a  
 353 fixed-wing aircraft can be written in matrix form as follows

$$\begin{aligned}
 \underbrace{\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix}}_{\dot{\mathbf{x}}} &= \underbrace{\begin{bmatrix} \frac{Y_\beta}{V_a} & \frac{Y_p}{V_a} & \frac{Y_r}{V_a} - 1 & \frac{g}{V_a} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{\mathbf{A}_p} \underbrace{\begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix}}_{\mathbf{x}} \\
 &+ \underbrace{\begin{bmatrix} \frac{Y_{\delta_a}}{V_a} & \frac{Y_{\delta_r}}{V_a} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}_p} \underbrace{\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}}_{\mathbf{u}},
 \end{aligned} \tag{48}$$

355 where  $(Y_\beta, Y_p, Y_r, Y_{\delta_a}, Y_{\delta_r})$ ,  $(L_\beta, L_p, L_r, L_{\delta_a}, L_{\delta_r})$  and  $(N_\beta, N_p, N_r, N_{\delta_a}, N_{\delta_r})$  are the lateral-directional  
 356 stability derivatives,  $V_a$  is the trimmed airspeed and  $g$  is the gravity. It should be recalled that the  
 357 stability derivatives cannot be measured, and they vary depending on flight conditions.

358 Taking the external disturbances and the model uncertainties into account, the system in (48)  
 359 can be extended as follows

$$\dot{\mathbf{x}} = \mathbf{A}_p \mathbf{x} + \mathbf{B}_p \mathbf{u} + \mathbf{f}(t, \mathbf{x}). \tag{49}$$

The system with its nominal desired dynamics can be parameterised to become similar to the class of MIMO systems in (16) defined by

$$\dot{\mathbf{x}} = \mathbf{A}_{m(0)}\mathbf{x} + \mathbf{B}_0(\omega_0\mathbf{u} + \theta_0^\top\mathbf{x} + \eta_{m(0)}) + \eta_{u(0)}(t).$$

A second model for the case of inversion of the sign of the aileron command is given by

$$\dot{\mathbf{x}} = \mathbf{A}_{m(1)}\mathbf{x} + \mathbf{B}_0\beta_1(\omega_1\mathbf{u} + \theta_1^\top\mathbf{x} + \eta_{m(1)}) + \eta_{u(1)}(t),$$

361 where  $\beta_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ .

A third model for the case of inversion of the sign of the rudder command is given by

$$\dot{\mathbf{x}} = \mathbf{A}_{m(2)}\mathbf{x} + \mathbf{B}_0\beta_2(\omega_2\mathbf{u} + \theta_2^\top\mathbf{x} + \eta_{m(2)}) + \eta_{u(2)}(t),$$

362 where  $\beta_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ .

A fourth model for the case of inversion of both the signs of the aileron and the rudder commands is given by

$$\dot{\mathbf{x}} = \mathbf{A}_{m(3)}\mathbf{x} + \mathbf{B}_0\beta_3(\omega_3\mathbf{u} + \theta_3^\top\mathbf{x} + \eta_{m(3)}) + \eta_{u(3)}(t),$$

363 where  $\beta_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ .

364 The input matrix  $B_0$  was taken to be the same for both models.

### 365 **Simulation Results**

366 In order to show the efficiency of the multiple model controller, simulations were first made  
367 using only the nominal controller, i.e., the  $\mathcal{L}_1$  adaptive controller with one model. Two situations

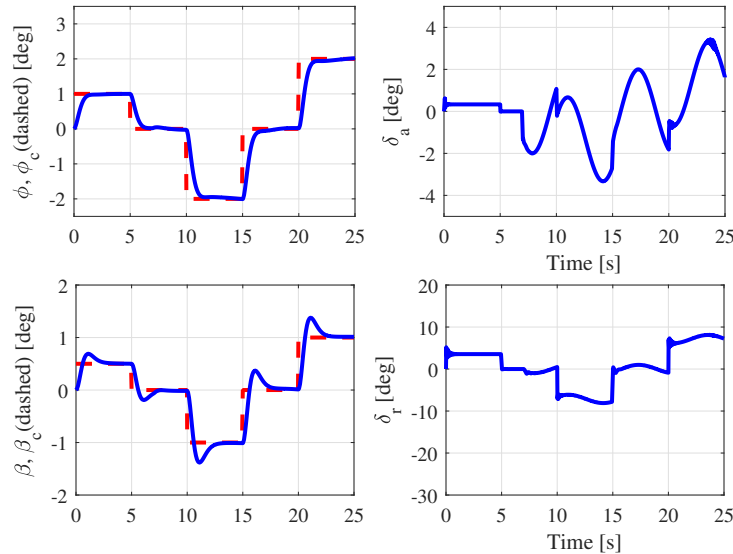
368 were considered in this case:

- 369 1. Control inputs loss of effectiveness of 50% without the inversion of the commands;
- 370 2. Control inputs loss of effectiveness of 50% with the inversion of the sign of the aileron
- 371 command.

372 Furthermore, the following uncertainties were added to the plant at simulation time  $t = 7$  s:

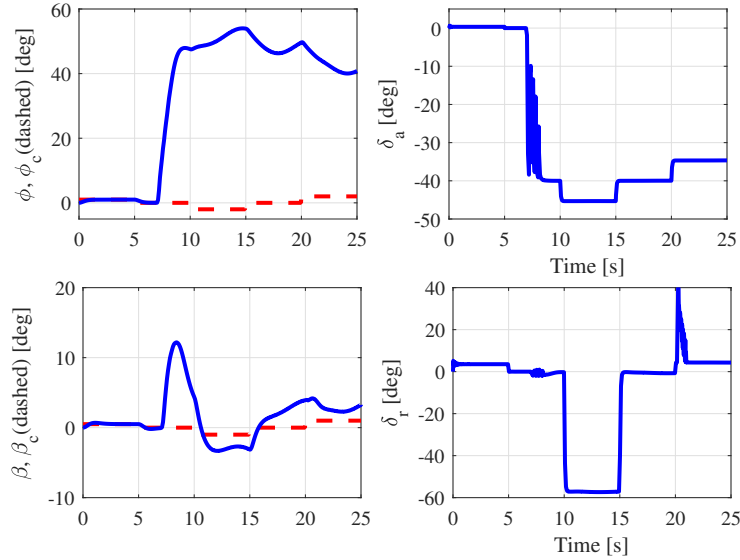
- 373 • Linear-in-state unknown parameters;
- 374 • Matched disturbance  $d_m = \sin(2\pi t)$  deg.

375 Simulation results for the nominal  $\mathcal{L}_1$  adaptive controller, without inversion of actuation signs,  
376 are shown in Figure 3. As expected, the system has good performance in the presence of uncer-  
377 tainties. The aileron command  $\delta_a$  and the rudder command  $\delta_r$  are within acceptable limits.



**Fig. 3.** Closed-loop tracking performance of the nominal controller without inversion of the sign of the commands.

378 In the second scenario of loss of effectiveness of 50% with the inversion of the sign of the  
379 aileron command, the system with only the nominal controller has become unstable as it can be  
380 observed in Figure 4. This is a direct consequence of the fact the adaptation laws (7)-(11) cannot  
381 match the correct control direction when it is inverted.



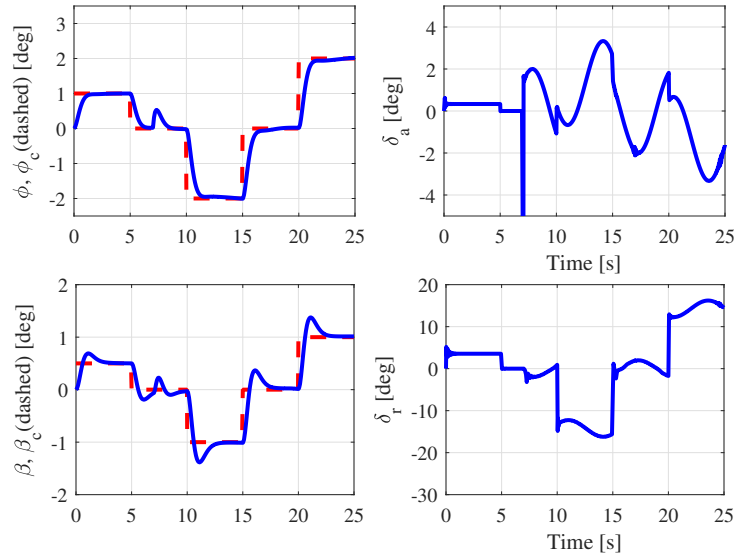
**Fig. 4.** Closed-loop tracking performance of the nominal controller with inversion of the sign of the aileron.

382 Next, the multiple model controller was applied. The tuning parameters and the desired  
 383 dynamics of the suboptimal controller were the same as the nominal controller. Three situations  
 384 were considered in simulations:

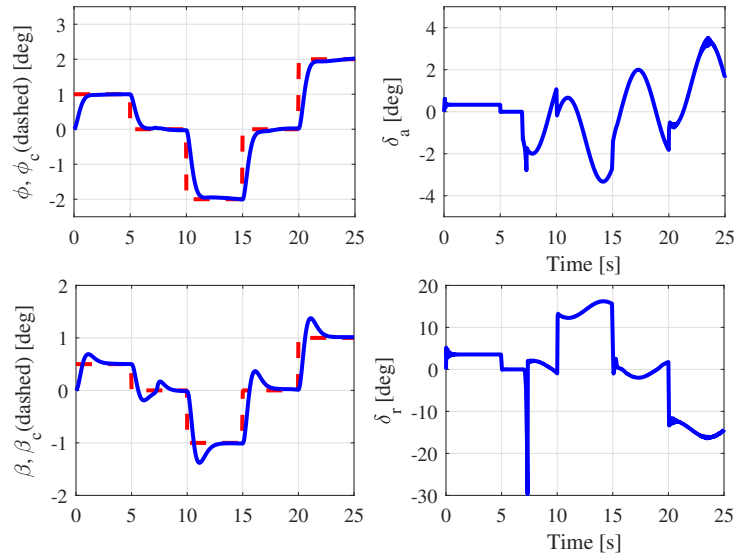
- 385 1. Control inputs loss of effectiveness of 50% with the inversion of the sign of the aileron  
 386 command;
- 387 2. Control inputs loss of effectiveness of 50% with the inversion of the sign of the rudder  
 388 command;
- 389 3. Control input loss of effectiveness of 50% with the of both the signs of the aileron and the  
 390 rudder commands.

391 Moreover, the same uncertainties as in previous simulations were added to the plant. The failures  
 392 were introduced at simulation time  $t = 7$  s.

393 The simulation results in the case of inversion of the sign of aileron command, the rudder  
 394 command, and both the ailerons and the rudder commands, are shown in Figure 5, Figure 6 and  
 395 Figure 7, respectively. In each situation, the system has stayed stable and has shown a good tracking  
 396 performance. The aileron command  $\delta_a$  and the rudder command  $\delta_r$  were within acceptable limits.



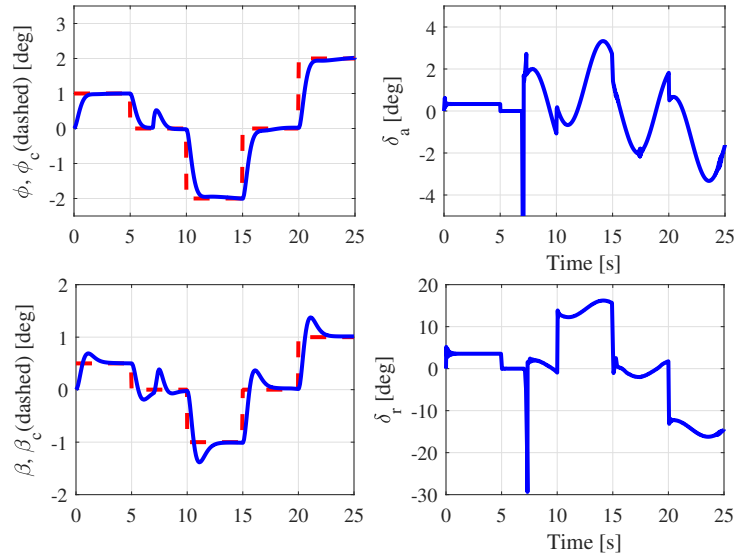
**Fig. 5.** Closed-loop tracking performance of the multiple model controller with inversion of the sign of the aileron.



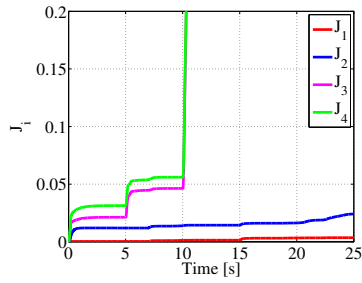
**Fig. 6.** Closed-loop tracking performance of the multiple model controller with inversion of the sign of the rudder.

397 Furthermore, as it is shown on Figure 8, the matching model to each failure case corresponds  
 398 to the minimum cost function defined in (23).

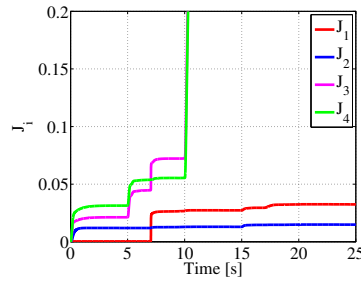
399 These simulations conclude that the application of the multiple model  $\mathcal{L}_1$  adaptive controller is  
 400 justified in case of structural damages or faults that lead to inversion of the sign of the control input  
 401 of flight systems.



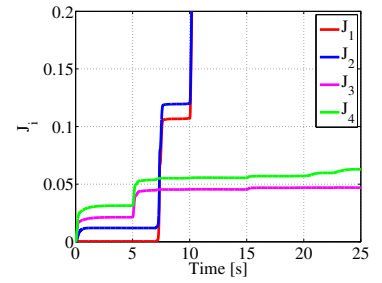
**Fig. 7.** Closed-loop tracking performance of the multiple model controller with inversion of the sign of both the aileron and the rudder.



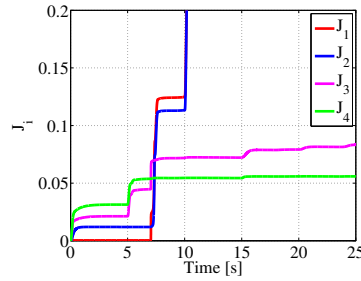
(a) 1st failure case.



(b) 2nd failure case.



(c) 3rd failure case.



(d) 4th failure case.

**Fig. 8.** Cost Functions

## SUMMARY

In this paper, an approach for  $\mathcal{L}_1$  adaptive fault-tolerant control MIMO systems is proposed. The aim is the fault-tolerant control of small fixed-wing UAVs in the presence of critical failures.

405 The design is based on a nominal model for a fault-free plant and a set of suboptimal models for the  
406 plant under failures. The switching between the models is based on a simple quadratic criterion.

407 The main advantage of this approach is that it allows a larger class of uncertainties and faults to  
408 be considered and can achieve better accommodation and preserve system integrity. Simulations  
409 have shown that the multiple model  $\mathcal{L}_1$  adaptive has stabilised the system in case of inversion of  
410 the control input, while the controller with a single model has failed.

## 411 **PRACTICAL APPLICATIONS**

412 Small drones or Unmanned Aerial Vehicles (UAVs) that is, with wingspans less than 2 metres  
413 and payload smaller than 2 kg, are generally built based on commercial Radio Controlled (RC)  
414 airplane. Small UAVs are gaining growing interest because of their low cost, high manoeuvrability  
415 and simple maintenance. Autonomy, although relative because they are still operated under human  
416 supervision, is the main feature of small UAVs compared to RC airplane. Autonomy has been made  
417 possible through the development of advanced autopilot (flight control) systems. They are used for a  
418 wide range of military and civilian tasks, such as: inspection, detection, transportation, monitoring,  
419 search and rescue, photography, imaging, mapping, intelligence, surveillance, reconnaissance,  
420 agriculture, entertainment etc. However, small UAVs are generally built with low-cost components  
421 and materials, which increases the probability of occurrence of faults and failures. The proposed  
422 flight control solution permits to maintain the UAVs in flight in the presence of hard failure, while  
423 other approaches fail. Therefore, the mission can be safely terminated, either automatically, or  
424 manually. Alternatively, it it can also be decided to complete the operation in degraded (limited)  
425 mode. Therefore, this solution is recommended for the operation of drones, especially in urban  
426 environments that needs a high degree of safety and reliability.

## 427 **DATA AVAILABILITY STATEMENT**

428 The data that support the findings of this study are available from the author upon reasonable  
429 request.

## ACKNOWLEDGEMENTS

I would like to express my gratitude to my PhD supervisor Pr. Walter Fichter, University of Stuttgart, for his guidance and detailed advice through the elaboration of this research. I would also like to express my appreciation to my thesis referee Professor Florian Holzapfel for his insights and comments.

## REFERENCES

- Abbaspour, A., Mokhtari, S., Sargolzaei, A., and Yen, K. K. (2020). “A survey on active fault-tolerant control systems.” *Electronics*, 9(9), 1513.
- Ackerman, K. A., Xargay, E., Choe, R., Hovakimyan, N., Cotting, M. C., Jeffrey, R. B., Blackstun, M. P., Fulkerson, T. P., Lau, T. R., and Stephens, S. S. (2017). “Evaluation of an  $\mathcal{L}_1$  adaptive flight control law on calspan’s variable-stability learjet.” *Journal of Guidance, Control, and Dynamics*, 40(4), 1051–1060.
- Ahmadi, K., Asadi, D., and Pazooki, F. (2019). “Nonlinear  $\mathcal{L}_1$  adaptive control of an airplane with structural damage.” *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 233(1), 341–353.
- Amin, A. A. and Hasan, K. M. (2019). “A review of fault tolerant control systems: advancements and applications.” *Measurement*, 143, 58–68.
- Austin, R. (2011). *Unmanned aircraft systems: UAVs design, development and deployment*, Vol. 54. John Wiley & Sons.
- Benosman, M. (2011). “Passive fault tolerant control.” *Robust Control Theory Appl.*
- Blanke, M., Kinnaert, M., Lunze, J., Staroswiecki, M., and Schröder, J. (2006). *Diagnosis and fault-tolerant control*, Vol. 2. Springer.
- Bodson, M. (2003). “Reconfigurable nonlinear autopilot.” *Journal of Guidance, Control, and Dynamics*, 26(5), 719–727.

454 Cao, C. and Hovakimyan, N. (2008). “Design and analysis of a novel  $\mathcal{L}_1$  adaptive control architec-  
455 ture with guaranteed transient performance.” *IEEE Transactions on Automatic Control*, 53(2),  
456 586–591.

457 Cao, C. and Hovakimyan, N. (2009). “ $\mathcal{L}_1$  adaptive output-feedback controller for non-strictly-  
458 positive-real reference systems: missile longitudinal autopilot design.” *Journal of guidance,*  
459 *control, and dynamics*, 32(3), 717–726.

460 Dobrokhodov, V., Xargay, E., Hovakimyan, N., Kaminer, I., Cao, C., and Gregory, I. M. (2013).  
461 “Multicriteria analysis of an  $\mathcal{L}_1$  adaptive flight control system.” *Proceedings of the Institution of*  
462 *Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 227(4), 413–427.

463 Ducard, G. J. (2009). *Fault-tolerant flight control and guidance systems: Practical methods for*  
464 *small unmanned aerial vehicles*. Springer.

465 Edwards, C., Spurgeon, S. K., and Akoachere, A. (2000). “A sliding mode static output feedback  
466 controller based on linear matrix inequalities applied to an aircraft system.” *Journal of Dynamic*  
467 *Systems, Measurement, and Control*, 122(4), 656–663.

468 Hovakimyan, N. and Cao, C. (2010).  *$\mathcal{L}_1$  adaptive control theory: Guaranteed robustness with fast*  
469 *adaptation*, Vol. 21. Siam.

470 Hwang, I., Kim, S., Kim, Y., and Seah, C. E. (2009). “A survey of fault detection, isolation, and  
471 reconfiguration methods.” *IEEE transactions on control systems technology*, 18(3), 636–653.

472 Ioannou, P. A. and Sun, J. (2012). *Robust adaptive control*. Courier Dover Publications.

473 Jiang, J. and Yu, X. (2012). “Fault-tolerant control systems: A comparative study between active  
474 and passive approaches.” *Annual Reviews in control*, 36(1), 60–72.

475 Jiang, J. and Zhang, Y. (2006). “Accepting performance degradation in fault-tolerant control system  
476 design.” *Control Systems Technology, IEEE Transactions on*, 14(2), 284–292.

477 Lavretsky, E. and Wise, K. A. (2013). “Robust adaptive control.” *Robust and Adaptive Control*,  
478 Springer, 317–353.

479 Liberzon, D. (2003). *Switching in systems and control*. Springer Science & Business Media.

480 Liu, Y., Tao, G., and Joshi, S. M. (2010). “Modeling and model reference adaptive control of aircraft  
481 with asymmetric damage.” *Journal of Guidance, Control, and Dynamics*, 33(5), 1500–1517.

482 Ma, Y., Jiang, B., Tao, G., and Badihi, H. (2020). “Minimum-eigenvalue-based fault-tolerant  
483 adaptive dynamic control for spacecraft.” *Journal of Guidance, Control, and Dynamics*, 43(9),  
484 1764–1771.

485 Mühlegg, M., Niermeyer, P., Falconi, G. P., and Holzapfel, F. (2015). “ $\mathcal{L}_1$  fault tolerant adaptive  
486 control of a hexacopter with control degradation.” *2015 IEEE Conference on Control Applications*  
487 *(CCA)*, IEEE, 750–755.

488 Narendra, K. S. and Balakrishnan, J. (1997). “Adaptive control using multiple models.” *IEEE*  
489 *transactions on automatic control*, 42(2), 171–187.

490 Nian, X., Chen, W., Chu, X., and Xu, Z. (2020). “Robust adaptive fault estimation and fault tolerant  
491 control for quadrotor attitude systems.” *International Journal of Control*, 93(3), 725–737.

492 Patel, V. V., Cao, C., Hovakimyan, N., Wise, K. A., and Lavretsky, E. (2009). “ $\mathcal{L}_1$  adaptive controller  
493 for tailless unstable aircraft in the presence of unknown actuator failures.” *International Journal*  
494 *of Control*, 82(4), 705–720.

495 Patton, R. J. (1997). “Fault-tolerant control: the 1997 situation.” *IFAC Proceedings Volumes*,  
496 30(18), 1029–1051.

497 Rotondo, D. (2017). *Advances in gain-scheduling and fault tolerant control techniques*. Springer.

498 Snyder, S. (2019). “L1 adaptive control within learn-to-fly.” Ph.D. thesis, University of Illinois at  
499 Urbana-Champaign, University of Illinois at Urbana-Champaign.

500 Sørensen, M. E. N. and Breivik, M. (2015). “Unmanned aerial vehicle fault-tolerant control  
501 by combined  $\mathcal{L}_1$  adaptive backstepping and fault-dependent control allocation.” *2015 IEEE  
502 Conference on Control Applications (CCA)*, IEEE, 1880–1886.

503 Souanef, T. (2019). *Adaptive Guidance and Control of Small Unmanned Aerial Vehicles*. Shaker  
504 Verlag, Germany.

505 Souanef, T., Boubakir, A., and Fichter, W. (2015). “ $\mathcal{L}_1$  adaptive control of systems with disturbances  
506 of unknown bounds.” *Advances in Aerospace Guidance, Navigation and Control*, Springer, 151–  
507 165.

508 Souanef, T. and Fichter, W. (2015). “Fault tolerant  $\mathcal{L}_1$  adaptive control based on degraded models.”  
509 *Advances in Aerospace Guidance, Navigation and Control*, Springer, 135–149.

510 Stevens, B. L. and Lewis, F. L. (2003). *Aircraft Control and Simulation*. Wiley, New York.

511 Tan, C., Tao, G., Qi, R., and Yang, H. (2017a). “A direct MRAC based multivariable multiple-model  
512 switching control scheme.” *Automatica*, 84, 190–198.

513 Tan, C., Tao, G., Yang, H., and Xu, F. (2017b). “A multiple-model adaptive control scheme  
514 for multivariable systems with uncertain actuation signs.” *2017 American Control Conference  
515 (ACC)*, IEEE, 1121–1126.

516 Tao, G. (2004). *Adaptive control of systems with actuator failures*. Springer.

517 Tian, S., Wang, J., Lin, D., and Pei, P. (2020). “ $\mathcal{L}_1$  adaptive control design of a helicopter in vertical  
518 flight.” *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace  
519 Engineering*, 234(14), 2089–2099.

520 Valavanis, K. P. and Vachtsevanos, G. J. (2015). *Handbook of Unmanned Aerial Vehicles*. Springer.

521 Wang, J. (2010). “Robust and nonlinear control literature survey (no. 19).” *International Journal  
522 of Robust and Nonlinear Control*, 20(12), 1427–1430.

- 523 Xue, Y., Zhen, Z., Yang, L., and Wen, L. (2020). “Adaptive fault-tolerant control for carrier-based  
524 uav with actuator failures.” *Aerospace Science and Technology*, 107, 106227.
- 525 Yang, F., Zhang, H., Jiang, B., and Liu, X. (2014). “Adaptive reconfigurable control of systems with  
526 time-varying delay against unknown actuator faults.” *International Journal of Adaptive Control  
527 and Signal Processing*, 28(11), 1206–1226.
- 528 Yang, G.-H., Wang, J. L., and Soh, Y. C. (2001). “Reliable controller design for linear systems.”  
529 *Automatica*, 37(5), 717–725.
- 530 Zang, Z. and Bitmead, R. R. (1990). “Transient bounds for adaptive control systems.” *29th IEEE  
531 Conference on Decision and Control*, IEEE, 2724–2729.
- 532 Zhang, Y. and Jiang, J. (2003). “Fault tolerant control system design with explicit consideration  
533 of performance degradation.” *IEEE Transactions on Aerospace and Electronic Systems*, 39(3),  
534 838–848.
- 535 Zhang, Y. and Jiang, J. (2008). “Bibliographical review on reconfigurable fault-tolerant control  
536 systems.” *Annual Reviews in Control*, 32(2), 229–252.
- 537 Zhou, Y., Liu, H., Guo, H., and Duan, X. (2019). “ $\mathcal{L}_1$  adaptive dynamic inversion attitude control  
538 for unmanned aerial vehicle with actuator failures.” *Proceedings of the Institution of Mechanical  
539 Engineers, Part G: Journal of Aerospace Engineering*, 233(11), 4129–4140.

540  
541  
542  
543  
544  
545  
546  
547  
548  
549  
550  
551  
552  
553

## List of Figures

1	Block diagram of the $\mathcal{L}_1$ adaptive controller. . . . .	6
2	Block diagram of the multiple model $\mathcal{L}_1$ adaptive controller. . . . .	10
3	Closed-loop tracking performance of the nominal controller without inversion of the sign of the commands. . . . .	23
4	Closed-loop tracking performance of the nominal controller with inversion of the sign of the aileron. . . . .	24
5	Closed-loop tracking performance of the multiple model controller with inversion of the sign of the aileron. . . . .	25
6	Closed-loop tracking performance of the multiple model controller with inversion of the sign of the rudder. . . . .	25
7	Closed-loop tracking performance of the multiple model controller with inversion of the sign of both the aileron and the rudder. . . . .	26
8	Cost Functions . . . . .	26

# Multiple model L1 adaptive fault-tolerant control of small unmanned aerial vehicles

Souanef, Toufik

2024-01-01

Attribution-NonCommercial-NoDerivatives 4.0 International

---

Souanef T. (2024) Multiple model L1 adaptive fault-tolerant control of small unmanned aerial vehicles. *Journal of Aerospace Engineering*, Volume 37, Issue 1, January 2024

<https://doi.org/10.1061/JAEEZ.ASENG-4427>

*Downloaded from CERES Research Repository, Cranfield University*