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The Characteristics of a Two-dimensional Supersonic Sail

- by -

E. A. Boyd, M.A.

of the

Department of Aerodynamics

SUMMARY

The two-dimensional supersonic sail is analysed using Busemann's second-order theory. It is found to have a universal shape, which is part of a Sici spiral. Aerodynamic characteristics are calculated for a few sails. The tension in sails flying at Mach numbers of 2 and 3 at altitudes of 20,000 and 70,000 feet is large and suggests that wire sails will be needed for flight under these conditions.

CONTENTS

LIST OF SYMBOLS

B	total length of sail
c	chord of sail
C_D	drag coefficient
C_L	lift coefficient
C_M	moment coefficient
$C_p = p - p_\infty / q_\infty$	pressure coefficient
C_1, C_2	coefficients in Busemann's second order approximation
D	total drag
L	total lift
M	moment about leading edge
M_∞	free-stream Mach number
p	pressure
q_∞	free-stream dynamic pressure
s	arc length of sail
T	tension in sail per unit span
x, y	cartesian co-ordinates
α	incidence of sail
θ	local angle of sail

Suffixes

L	leading edge or lower surface
T	trailing edge
u	upper surface
∞	free-stream conditions

1. Introduction

The two-dimensional supersonic sail is investigated using simple wave theory in the form of Busemann's second-order approximation. This limits the sails considered to those which turn the flow through an angle attainable by an attached oblique shock wave. For a particular deflection the Mach number must be high enough to maintain wholly supersonic flow over the sail. For a particular Mach number the deflection must not be too large. The largest trailing-edge angle used in the calculations below is 40°.

It might be argued that the inviscid problem considered here is not likely to be a realistic one because the continuous compression on the lower surface of the sail would cause separation of the boundary layer. On this point we have the experimental work of Johannesen (Ref. 2) on two-dimensional supersonic flow over concave surfaces. He concluded that the flow outside the boundary layer was in good agreement with the inviscid flow if the boundary layer was turbulent. If the boundary layer was laminar, it separated, a shock wave originated at the point of separation, and there was a considerable deviation from the inviscid flow. On the assumption that unseparated flow is possible over the sail, the results of the present analysis should be of some relevance.

Numerical results are included to give estimates of the tension likely to occur in the sail.

2. Sail geometry

If the two-dimensional supersonic sail is a tight one, such that the maximum deflection concave to the stream can be attained by an attached oblique shock, the pressure distribution on the surface of the sail can be calculated by simple wave theory. Using Busemann's second order approximation we have

$$C_p = C_1 \theta + C_2 \theta^2 \tag{1}$$

where θ is the local angle of the sail (see Fig. 1) and

$$C_1 = \frac{2}{(M_\infty^2 - 1)^{\frac{1}{2}}} \tag{2}$$

$$C_2 = \frac{(M_\infty^2 - 2)^2 + \gamma M_\infty^4}{2(M_\infty^2 - 1)^2} \tag{3}$$

The pressure difference across the sail is given by

$$C_{p_L} - C_{p_u} = 2\, C_1\, \theta = k\theta \tag{4}$$

where

$$k = \frac{4}{\sqrt{M_\infty^2 - 1}} \tag{5}$$

The pressure difference across each element ds of the sail is in equilibrium with the tension in the sail so that

$$(p_L - p_u) \cos\theta \; ds = d(T \sin\theta)$$

i.e.

$$p_L - p_u = T \frac{d\theta}{ds} \; . \tag{6}$$

$$\therefore \qquad k\, q_\infty\, \theta = T \frac{d\theta}{ds} \; . \tag{7}$$

Equation (7) can be integrated to give the arc length of the sail, s, measured from the leading edge,

$$\frac{k\, q_\infty}{T} s = \int_{\theta_L}^{\theta} \frac{d\theta'}{\theta'} = \log \frac{\theta}{\theta_L} \tag{8}$$

The total length, B, of the sail is given by

$$\frac{k\, q_\infty}{T} B = \log \frac{\theta_T}{\theta_L} \; . \tag{9}$$

The parametric equations of the sail are

$$\frac{k\, q_\infty}{T} x = \int_{\theta_L}^{\theta} \frac{\cos\theta' \, d\theta'}{\theta'} \tag{10}$$

$$= \mathrm{Ci}\,(\theta) - \mathrm{Ci}\,(\theta_L)$$

and

$$\frac{k\, q_\infty}{T} y = \int_{\theta_L}^{\theta} \frac{\sin\theta' \, d\theta'}{\theta'} \tag{11}$$

$$= \mathrm{Si}\,(\theta) - \mathrm{Si}\,(\theta_L)$$

where Si (θ) and Ci (θ) are the sine and cosine integrals, respectively.

Thus the two-dimensional supersonic sail forms part of a Sici spiral, (Ref. 1).

The chord, c, of the sail is given by

$$\frac{k\,q_\infty}{T}\,c = \sqrt{\left(\frac{k\,q_\infty}{T}\,x_T\right)^2 + \left(\frac{k\,q_\infty}{T}\,y_T\right)^2} \qquad (12)$$

and its geometrical angle of attack, α, by

$$\alpha = \tan^{-1}\left\{\left(\frac{k\,q_\infty}{T}\,y_T\right)\Big/\left(\frac{k\,q_\infty}{T}\,x_T\right)\right\} \qquad (13)$$

3. Aerodynamic characteristics

The total lift, drag, and leading-edge moment of the two-dimensional sail are determined directly by the attaching forces and their angles at the leading and trailing edges because the tension in the sail is uniform. Thus

$$L = T\,(\sin\theta_T - \sin\theta_L) \qquad (14)$$

$$D = T\,(\cos\theta_L - \cos\theta_T) \qquad (15)$$

$$M = T\,(y_T\cos\theta_T - x_T\sin\theta_T) \qquad (16)$$

The corresponding coefficients can be expressed as

$$C_L = \frac{L}{q_\infty c} = k\,\frac{(\sin\theta_T - \sin\theta_L)}{(k\,q_\infty c/_T)} \qquad (17)$$

$$C_D = \frac{D}{q_\infty\,c} = k\,\frac{(\cos\theta_L - \cos\theta_T)}{(k\,q_\infty\,c/_T)} \qquad (18)$$

$$C_M = \frac{M}{q_\infty\,c^2} = k\,\frac{\left(\frac{k\,q_\infty y_T}{T}\cos\theta_T - \frac{k\,q_\infty\,x_T}{T}\sin\theta_T\right)}{(k\,q_\infty\,c/_T)^2} \qquad \ldots\ldots \quad (19)$$

and

$$\frac{C_L}{C_D} = \cot\tfrac{1}{2}(\theta_L + \theta_T) \qquad (20)$$

where C_D is the wave drag coefficient since the frictional drag has not been evaluated here.

4. Results

Fig. 2 shows the shape of the supersonic sail for a leading-edge angle of 4°. The local sail angle is indicated on the curve, the largest angle plotted being 40°. It is known that for large trailing-edge angles simple wave theory will not give an accurate description. The leading-edge angle chosen is arbitrary because the sail shape shown is a universal one. If a larger leading-edge angle is required the sail co-ordinates can be calculated immediately by transferring the origin to the appropriate point on the curve. The sail may be terminated of course at any angle up to 40° depending on the aerodynamic characteristics required.

The aerodynamic characteristics have been calculated for sails with three particular values of leading-edge angle, namely $\theta_L = 4^{\circ}, 10^{\circ}$, and 20°, and for trailing-edge angles up to 40°. Where necessary a particular sail is indicated in the graphs by a number pair which refers to (θ_L, θ_T). Fig. 3 shows the overall lift and drag coefficients obtainable and Fig. 5 the moment coefficient about the leading-edge. Fig. 4 clearly indicates the high lift-drag ratios (neglecting friction) obtainable with tight sails with small leading-edge angles, a fact which is easily deduced from equation (20), and at the same time defines the limited C_L range obtainable from any one sail.

Table 1 compares the tensions in three particular sails flying at Mach numbers of 2 and 3 at altitudes of 20,000 and 70,000 ft. The tension in the (4, 10) sail is relatively high because it is too tight. In the limit of a flat plate the tension is infinite. Moving from a (4, 20) to a (10, 30), or a (4, 30) to a (10, 30), sail makes the sail loading about half as much again and approximately doubles the tension. From the figures quoted even at 70,000 ft. it would seem, taking into account the loss of strength of the materials because of the high temperatures experienced, that the sails will need to be wire ones.

5. Conclusions

The universal shape of the two-dimensional supersonic sail is drawn in Fig. 2 for a leading-edge angle of 4° and a trailing-edge angle of 40°. The geometry of any other sail can be found by interpolation.

Figures are included to show the values of C_L, C_D, C_M, and C_L/C_D for a range of sails.

Table 1 summarises the results of a few particular examples. Only wire sails are likely to support the tensions calculated, remembering that the sail material will lose strength at the high temperatures which will be met.

6. References

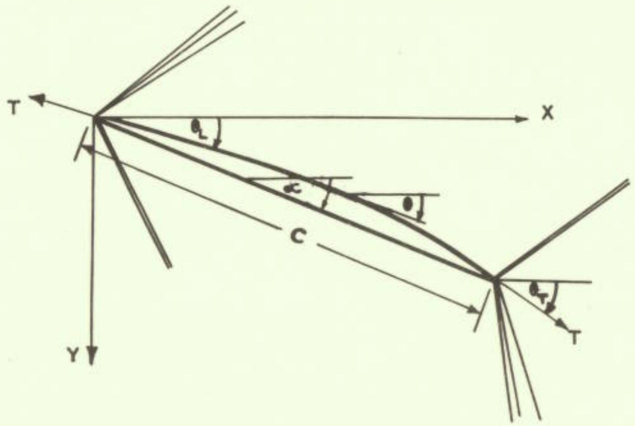
1.	Jahnke, E., Emde, F.	Tables of functions. Dover Publications, 1945.
2.	Johannesen, N.H.	Phil. Mag., series VII, vol. 43, 1952, pp 568 - 580.

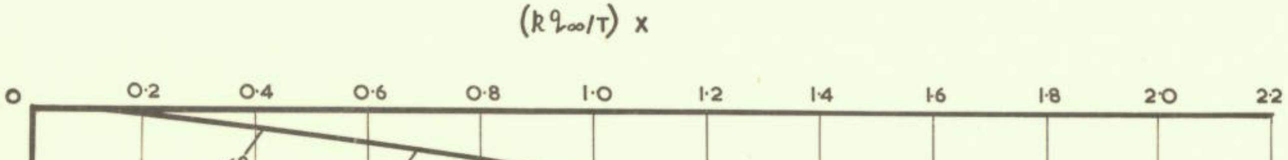
Height ft.	Mach No.	Sail (θ_L, θ_T)	Load/unit area lb./sq.ft.	Tension lb./ft.
20,000	2	(4,10)	723	69,600
		(4,20)	1070	39,400
		(10,20)	1530	91,000
	3	(4,10)	996	96,000
		(4,30)	1670	42,400
		(10,30)	2620	80,000
70,000	2	(4,10)	44	4,200
		(4,20)	65	2,390
		(10,20)	93	5,520
	3	(4,10)	94	9,080
		(4,30)	176	3,990
		(10,30)	246	7,520

Note: Each sail considered has a chord of 10 ft.

TABLE 1. COMPARISON OF SAIL LOADINGS

FIG. 1. SAIL CO-ORDINATES.





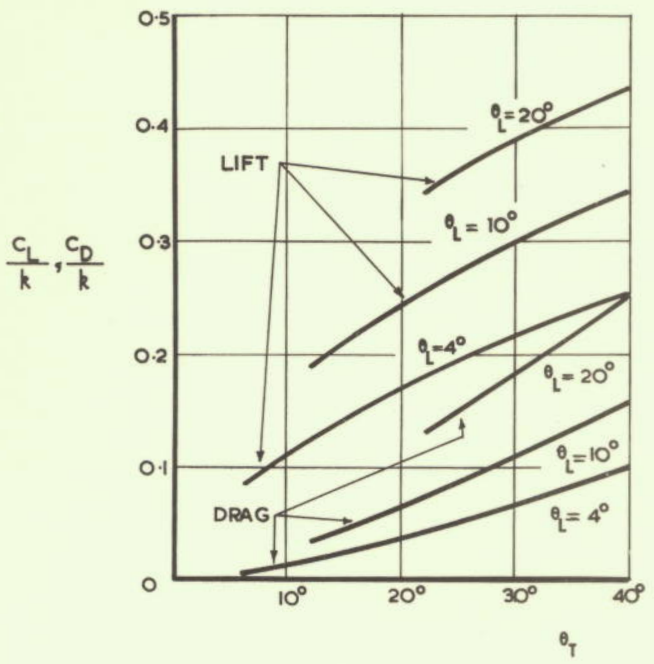


FIG 3 LIFT and DRAG COEFFICIENTS.

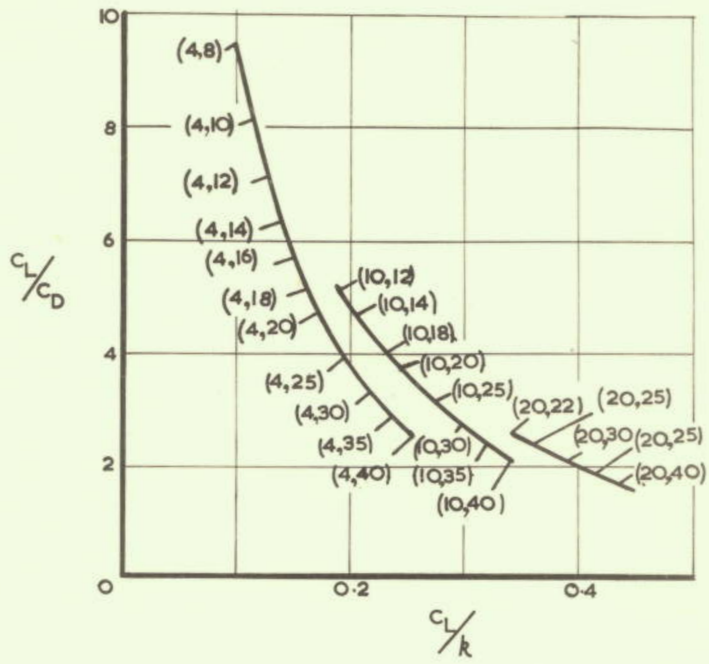
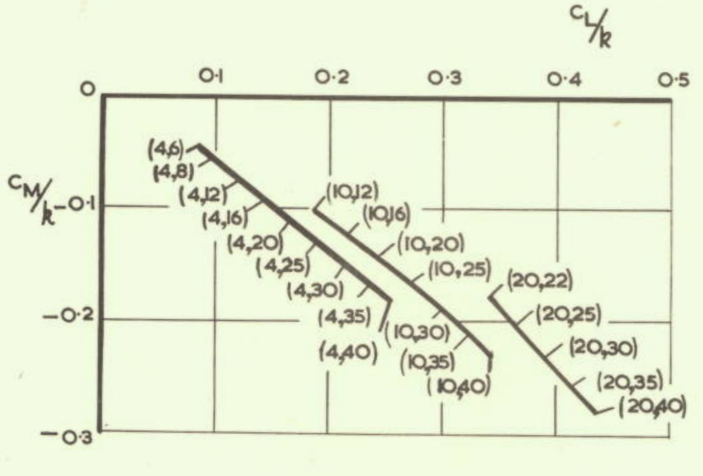


FIG. 4. LIFT—DRAG RATIOS.



NUMBER PAIRS REFER TO (θ_L, θ_T).

FIG. 5. MOMENT COEFFICIENT ABOUT LEADING EDGE.