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An improved wavelet-ARIMA approach for forecasting metal prices

Thomas Kriechbaumer[a], Andrew Angus[b], David Parsons[a], Monica Rivas Casado[a*]

[a]School of Applied Sciences, Cranfield University, Cranfield, MK43 0AL, United Kingdom
[b]School of Management, Cranfield University, Cranfield, MK43 0AL, United Kingdom
*Corresponding author: m.rivas-casado@cranfield.ac.uk, Tel.:01234750111 ext2706

Abstract

Metal price forecasts support estimates of future profits from metal exploration and mining and inform purchasing, selling and other day-to-day activities in the metals industry. Past research has shown that cyclical behaviour is a dominant characteristic of metal prices. Wavelet analysis enables to capture this cyclicality by decomposing a time series into its frequency and time domain. This study assesses the usefulness of an improved combined wavelet-autoregressive integrated moving average (ARIMA) approach for forecasting monthly prices of aluminium, copper, lead and zinc. The performance of ARIMA models in forecasting metal prices is demonstrated to be increased substantially through a wavelet-based multiresolution analysis (MRA) prior to ARIMA model fitting. The approach demonstrated in this paper is novel because it identifies the optimal combination of the wavelet transform type, wavelet function and the number of decomposition levels used in the MRA and thereby increases the forecast accuracy significantly. The results showed that, on average, the proposed framework has the potential to increase the accuracy of one month ahead forecasts by \$53/tonne for aluminium, \$126/tonne for copper, \$50/tonne for lead and \$51/tonne for zinc, relative to classic ARIMA models. This highlights the importance of taking into account cyclicality when forecasting metal prices.

Keywords: *Forecast, Wavelet analysis, Multiresolution analysis, Metal prices*

1. Introduction

Metal price forecasts support forward planning and investment decisions in metal producing and processing industries, such as mining, refining and fabrication (Watkins and McAleer, 2004). Dooley and Lenihan (2005) argued that metal price tends to be the major factor causing variability in revenues from mining operations. Therefore, accurate price forecasts are essential to assess the economic viability of metal exploration and mining activities. Moreover, the volatile and cyclical behaviour of international metal prices strongly affects the economic stability of nations whose exports are dominated by metals (Labys et al., 2000; Radetzki, 2008; Watkins and McAleer, 2004). To give some examples, aluminium accounts for almost 40% of the total value of exports from Mozambique and Tajikistan, and copper has a share of almost 70% of the total value of exports from Zambia (UNCTAD, 2011). The ability to accurately forecast primary commodity prices can support budgetary planning (Dehn, 2000) and the development of stabilisation policies in these countries (Cashin et al., 2002; Deaton, 1999; Deaton and Miller, 1995).

Metal prices are the result of complex market dynamics and stochastic economic processes, which makes price forecasting difficult (Labys, 2006). Dooley and Lenihan (2005) used autoregressive integrated moving average (ARIMA) and lagged forward price models to forecast monthly lead and zinc cash prices. They found that ARIMA models perform marginally better and they are a useful tool for mining companies to predict metal prices. Labys (2006) used a structural time series model to forecast monthly prices of copper, lead, tin, zinc and other primary commodities. He emphasised the importance of correctly accounting for cyclicality in modelling and forecasting primary commodity prices. Much evidence suggests that cyclical behaviour is a dominant characteristic of metal prices (Cashin et al., 2002; Davutyan and Roberts, 1994; Labys et al., 1998; Roberts, 2009). Labys et al. (1998) found short-term cycles of durations of less than twelve months in monthly prices of aluminium, copper, lead, zinc and other metals and encouraged producers, consumers and traders to re-examine their price forecasting methods based on this finding.

A relatively novel technique to capture cyclicality in time series is wavelet analysis. In contrast to the traditional Fourier analysis, which transforms a time series into its frequency domain, the wavelet transform decomposes a time series into its frequency *and time* domain. Thereby, variations of different frequencies in a time series are not only identified but also localised. This feature has been used for several purposes in resource economics, for instance, to identify cycles in primary commodity prices (Davidson et al., 1998; Naccache, 2011), to examine co-movement between primary commodity prices (Conner and Rossiter, 2005; Tonn et al., 2010) and to analyse the relationship between crude oil prices and a range of macroeconomic variables (Aguiar-Conraria and Soares, 2011; Jammazi and Aloui, 2010; Naccache, 2011). Davidson et al. (1998) argued that wavelet analysis may help to forecast commodity price movements. Also Ramsey (1999) found that wavelets have the potential to increase the predictive power of time series methods. These findings and the above mentioned results of Dooley and Lenihan (2005) provide the motivation for combining wavelets with ARIMA models to forecast monthly base metal prices.

Schlüter and Deuschle (2010) identified and tested several ways that wavelets could support time series forecasting. For crude oil spot price predictions one week ahead, they found that the highest forecast accuracy is achieved if wavelets are used in a multiresolution analysis (MRA) that decomposes the time series into a smooth series representing a trend and a number of detail series describing fluctuations of known approximate frequency around this trend. In a second step, these subseries are extended using time series methods and, thirdly, summed to obtain the forecast of the original time series. The wavelet-ARIMA model applied in this study follows this approach. This technique can be more accurate than directly forecasting the original series, since the subseries tend to have a more stable variance and typically no outliers (Shafie-khah et al., 2011).

Attempts to improve the predictive power of MRA-based forecasting have focused primarily on the choice of a technique to forecast the detail and smooth series. Several statistical forecasting techniques have been tried to this end, such as ARIMA models (Conejo et al., 2005; Fernandez, 2007, 2008), a combination of ARIMA and generalized autoregressive conditional heteroskedasticity (GARCH) models (Tan et al., 2010), spline and trigonometric extrapolation (Yousefi et al., 2005) or support vector machines (Pahasa and Theera-Umpon, 2007). Much research has also concentrated on the combination of artificial neural networks with wavelets (e.g. Amina et al., 2012; Amjady and Keynia, 2008; Aussem and Murtagh, 1997; Bashir and El-Hawary, 2009; Catalão et al., 2011; Chen et al., 2010; Jammazi and Aloui, 2012). However, in the vast majority of studies on MRA-based forecasting, the configuration of the MRA has been either neglected or chosen based on assumptions rather than on empirical evidence.

The MRA involves the choice of a wavelet transform type, a wavelet function and the number of decomposition levels taken into account. With few exceptions (Jammazi and Aloui, 2012; Murtagh et al., 2004; Nguyen and Nabney, 2010), the discrete wavelet transform (DWT) is the most frequently used wavelet transform type in wavelet-based forecasting, especially in the context of resource and energy economics (Amjady and Keynia, 2008; Bashir and El-Hawary, 2009; Catalão et al., 2011; Conejo et al., 2005; Fernandez, 2007, 2008; Mandal et al., in press; Nowotarski et al., 2013; Shafie-khah et al., 2011; Tan et al., 2010; Voronin and Partanen, in press; Zhang and Tan, 2013). Most authors do not discuss this choice or the possibility of using other transform types, such as the maximum overlap DWT (MODWT). Also the wavelet function choice is rarely discussed if wavelets are used for forecasting. Several authors state that they chose a specific wavelet function because it offers "*an appropriate trade-off between wave-length and smoothness*" without further specifying how the choice relates to the time series characteristics or how it affects the forecast accuracy (Amjady and Keynia, 2008; Conejo et al., 2005; Shafie-khah et al., 2011; Tan et al., 2010, Voronin and Partanen, in press). A few authors assessed a limited selection of wavelet functions and chose the best performing alternative (Aggarwal et al., 2008; Rocha Reis and Alves da Silva, 2005; Yousefi et al., 2005, Nowotarski et al., 2013).

In the context of electricity price forecasting, Conejo et al. (2005) recommended the use of short wavelets because the larger support interval of longer wavelet functions might corrupt the prediction. On the other hand, Gencay et al. (2001) found that longer wavelets approximate an ideal band-pass filter better than short wavelet functions. Moreover, they suggested that the shape of the wavelet function should resemble that of the time series to be decomposed, for example the Haar wavelet function is suitable for decomposing data appearing to be constructed of piecewise constant functions, while wavelet functions of higher order should be used for smoother time series. In practice, this recommendation can only roughly guide the choice of a wavelet function as the number of wavelet functions is high and many wavelets are similar in shape. According to Crowley (2007), the choice of a wavelet function and the number of decomposition levels depends on the type and frequency of the variations that the analyst aims to capture in a time series. Different numbers of decomposition levels have found application in MRA-based forecasting, for example three levels (Conejo et al., 2005), five levels (Yousefi et al., 2005) and up to seven levels of decomposition (Fernandez, 2007). However, it is unclear which combination of wavelet transform, wavelet function and number of decomposition levels performs best for a specific time series and whether these decisions have a significant effect on the accuracy of MRA-based forecasting.

The aim of this paper is to contribute to the improvement of metal price forecasting by assessing the usefulness of a combined wavelet-ARIMA model for predicting monthly base metal prices. The objectives are to:

- assess the effect of the wavelet transform type, namely the DWT and the MODWT, the wavelet function and the number of decomposition levels on the predictive power of the wavelet-ARIMA forecasting technique;
- calibrate the wavelet-ARIMA model by identifying the combination of these choices that achieves the highest predictive power for forecasting monthly cash prices of aluminium, copper, lead and zinc up to twelve months ahead; and
- quantify the predictive power of the proposed forecasting technique and compare it with that achieved with normal ARIMA models without prior MRA and a naïve modelling approach.

To the authors' best knowledge, this is the first study to statistically test the effect of the MRA configuration on time series forecasts considering a large set of more than 400 different configurations and a large number of simulated forecasts to enable accurate quantification of the forecast error and robust statistical testing. Moreover, this is the first study that applies MRA-based forecasting to metal prices.

This paper has five sections: Section 2 outlines the methods chosen to identify the optimal MRA configuration for forecasting metal prices, to validate the wavelet-ARIMA model and to quantify its predictive power. The results are shown in Section 3. Section 4 discusses the results in relation to previous papers applying wavelet-based forecasting and identifies future research topics. Conclusions are given in Section 5.

2. Methods

2.1 Data

This study forecast time series of the monthly nominal cash prices of aluminium, copper, lead and zinc traded on the London Metal Exchange (LME) in US$/tonne. Following Dooley and Lenihan (2005) and Labys (2006), monthly data were chosen to capture the timespans relevant for short-term planning of companies in the metals industry. The LME was favoured over other metal exchanges as it is the major market for pricing non-ferrous metals worldwide (Watkins and McAleer, 2004). The data were provided by the United Nations Conference on Trade and Development (UNCTAD, 2012). Each time series analysed comprised 628 observations that cover the period from January 1960 to April 2012. Prior to forecasting, the natural logarithm of the prices was taken. This led to a more stable variance throughout the sample, thus could be assumed to increase the accuracy of the forecasts (Lütkepohl and Xu, 2012).

2.2 Introduction to wavelet theory

A wavelet is a highly localised function with a defined number of oscillations that last through a certain time period and fade away about zero. These features enable wavelets to analyse non-stationary time series, a main distinction from the trigonometric functions used in classic Fourier analysis. Each wavelet 'family' is derived from a 'mother' wavelet ψ by expansion ("dilation") and translation to yield daughter wavelets $\psi_{u,s}(t)$,

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right), \tag{1}$$

where ψ is the mother wavelet function, translated by the location index u, which indicates its position in the time domain, and dilated by the scale index s, which describes the width of a daughter wavelet. The shape of the wavelet function depends on the wavelet family, for example Haar wavelets are discrete, square shaped and symmetric, Daublets have asymmetric shape and symmlets as well as Coiflets are almost symmetric (see Figure 1). The width of a mother wavelet is defined by its "order" (Crowley, 2007). Mathematical definitions of the wavelets used in this study are given in Gencay et al. (2001).

Through a wavelet transform, a time series is expressed in its time-frequency representation by projecting a set of daughter wavelets onto the time series under study. This results in wavelet coefficients. Large coefficients occur where the shape of the time series $x(t)$ is similar to that of the respective daughter wavelet $\psi_{u,s}(t)$. This reveals not only the existence of certain frequencies in a time series but also the location where the frequencies occur. Depending on the number of wavelet coefficients produced, three forms of wavelet transforms can be distinguished: continuous wavelet transform (CWT), DWT and MODWT. In the CWT, wavelet coefficients are produced by continuously dilating and translating the mother wavelet, so that wavelet coefficients $W\{x\}(u,s)$ are calculated for all possible scales and times:

$$W\{x\}(u,s) = \int_{-\infty}^{\infty} x(t)\psi_{u,s}(t)dt\ . \tag{2}$$

The DWT produces only the minimal number of coefficients necessary to reconstruct the original function $x(t)$. This reduction is achieved by discretising the parameters u and s, so that $u = k2^{-j}$, and $s = 2^{-j}$, where j and k are integers. The respective decomposition level is defined by j, where $j = 1, \ldots J$ and J is the number of decomposition levels produced. The MODWT contains all possible shifted versions of the DWT (Nason, 2008); it applies the same reduction of coefficients as the DWT in the scale dimension, but produces the maximum number of coefficients in the time dimension, so that $u = k$ for the MODWT. While the DWT is restricted to dyadic time series ($T = 2^J$, where T is the length of the time series), the MODWT can handle time series of any length (Gencay et al., 2001). Moreover, the MODWT is shift invariant and produces smoother approximations to the original time series (Jammazi and Aloui, 2012). As financial data are inherently discrete, only the DWT and MODWT are considered in this study.

In an MRA, wavelet transforms are used to decompose a time series into a smooth series A_J, consisting of smooth coefficients $a_{J,k}$, and a set of detail series D_j, consisting of detail coefficients $d_{j,k}$. In this study, the MRA was implemented using Mallat's pyramid algorithm for fast implementation of discrete wavelet transforms (Mallat, 1989). The procedure consists of a decomposition and a reconstruction stage (see Figure 2). In the decomposition, the original time series X is filtered using a high-pass filter H, which is based on a daughter wavelet, and its counterpart low-pass filter L, called scaling filter. In the DWT, both filter outputs are subsampled to half their original length to yield wavelet coefficients cD_j and scaling coefficients cA_j. In contrast, the MODWT does *not* include this subsampling step (Gencay et al., 2001). In the reconstruction stage, the series of wavelet and scaling coefficients are upscaled by inserting zeros between the coefficients and convolved with their respective reconstruction filters *H'* and *L'* to obtain detail *D* and smooth *A* representations of the input time series. This single-resolution analysis can be extended to a *multi*-level decomposition by repeating the procedure using the smooth series as new input data (see Figure 3).

The smooth series is the main component of the MRA and can be described as a filtered, de-noised version of the original time series (Crowley, 2007). The detail series capture fluctuations in the original series around this smooth series. Each detail series represents a certain decomposition level *j*, which indicates the time scale of the fluctuations. For monthly time series data, these are: D_1 (2-4 months), D_2 (4-8 months), D_3 (8-16 months), D_4 (16-32 months), D_5 (2.7-5.3 years), D_6 (5.3-10.6 years), D_7 (10.6-21.3 years), D_8 (21.3-42.6 years) and D_9 (more than 42.6 years). The original time series can be reconstructed by summing the coefficients of the smooth series A_J and the coefficients of all detail series D_j obtained in the MRA:

$$x_t = \sum_{j=1}^{J} a_{J,t} + d_{j,t} \ . \tag{3}$$

For a detailed introduction to wavelet analysis see for example Gencay et al. (2001).

2.3 Wavelet-ARIMA forecasting framework

The wavelet-ARIMA forecasting framework involves three steps:

1. Through a wavelet-based MRA the original time series is decomposed into a smooth series and a set of detail series.
2. The set of detail series and the smooth series are forecast independently through ARIMA models.
3. The extended smooth and detail series are summed to obtain the forecast of the original time series.

The first step involves deciding upon a wavelet transform type, a wavelet function and the number of decomposition levels *J*. As the DWT requires dyadic time series, the time series analysed were all reduced to 512 observations. This enabled the consideration of nine decomposition levels, where $J = 1$ means that the time series is decomposed to one smooth and one detail series and $J = 9$ means that one smooth series and nine detail series are considered.

Based on the findings by Dooley and Lenihan (2005; see Section 1), ARIMA models were chosen to extend the detail and smooth series resulting from the MRA. The automatic ARIMA model fitting algorithm introduced by Hyndman and Khandakar (2008) allows that a large number of ARIMA models can be fitted within a short time and overcomes subjectivity in ARIMA model fitting. Using this algorithm, a large number of forecasts could be simulated within reasonable time (see Section 2.4) which revealed the error characteristics of different wavelet models and allowed for quantification and robust statistical comparison of their predictive power. To reduce the computing time of the automatic ARIMA model fitting algorithm, the maximum number of the autoregressive and moving average parameters taken into account in ARIMA model fitting was limited to five in both model calibration and validation. To examine the effect of this restriction on the achieved forecast accuracies, the model validation was also carried out with the maximum number of parameters set to 15. Based on the findings by Yousefi et al. (2005), who used a similar wavelet-based technique to predict oil prices, the sample size for fitting ARIMA models to the detail and smooth series was fixed to 100 months.

2.4 Framework calibration

The wavelet functions taken into account in this study were Daubechies extremal phase ("Daublets"), Daubechies least asymmetric ("symmlets"), best localised and Coiflet wavelets with a range of randomly selected orders, resulting in a total of 25 different wavelets. To enable the consideration of all nine decomposition levels mentioned previously, all 512 time series values were used to compute detail and smooth coefficients. Considering all possible combinations of the two wavelet transform types, the 25 wavelet functions and the possible nine decomposition levels, 450 different configurations of the wavelet-ARIMA approach were obtained. With each of these configurations, out-of-sample forecasts were produced for all possible periods in the time series. The time series length of 512 observations and the sample size of 100 observations for ARIMA model fitting allowed the prediction of a total of 401 to 412 periods with the exact number depending on the forecast horizon. This large number of simulations allowed for robust statistical testing of the sensitivity of the wavelet-ARIMA models to the MRA configuration. Forecasts were made one to twelve months ahead (see Figure 4). The relatively long forecast horizons of up to twelve months were included in the analysis in order to reveal the sensitivity of the forecast accuracy to the forecast horizon. For each horizon, the configuration of the wavelet-ARIMA model that achieved the highest predictive power was identified. The predictive power was expressed as mean absolute error (MAE),

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|x_t - \hat{x}_t\right|, \tag{4}$$

where n is the number of periods forecast, x_t is the actual price at time t and $\hat{x}_t$ is the predicted price at time t. To assess the susceptibility of the wavelet-ARIMA models to large forecast errors, the analysis was also conducted with the root mean square error (RMSE) as error measure:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t - \hat{x}_t)^2}\ . \tag{5}$$

Moreover, the absolute forecast error (AE) of the assessed models was plotted against time to reveal their reliability during different periods in the time series

2.5 Framework validation

The wavelet-ARIMA model with the lowest MAE was validated by predicting all possible periods in the time series. In contrast to the framework calibration, only those price values *prior* to the point in time t at which the prediction was made were used to compute the detail and smooth coefficients in the MRA and future prices were assumed to be unknown. This corresponds to a forecasting problem in practice. To obtain detail and smooth coefficients up to t, the time series had to be extended artificially, which is referred to as "padding". This is necessary as the detail and smooth coefficients at t depend not only on the time series value at t but also on some values before and after t (Gencay et al., 2001). The extension of the time series was achieved by symmetrically reflecting the time series at its boundary. This is a standard padding technique provided by common software programs for wavelet analysis, such as the Wavelet Toolbox in MATLAB (2011) or the package "wavelets" in R (R Development Core Team, 2008).

The predictive power of the proposed wavelet-ARIMA model was compared with that of a normal ARIMA model applied directly to the metal price time series and a naïve model. As in the ARIMA models for the subseries in the wavelet-ARIMA approach, the normal ARIMA model was estimated using the automatic model fitting algorithm by Hyndman and Khandakar (2008). Naïve models are very simple forecasting methods against which the performance of more sophisticated techniques can be compared. The naïve model applied in this study used the last available observation as the future forecast, i.e. this month's price was the forecast for all future months (Makridakis et al., 1998). The differences between the forecast errors of the wavelet-ARIMA model calibration and validation, the ARIMA and the naïve model were tested through Diebold-Mariano tests (Diebold and Mariano, 1995).

All steps of the analysis were conducted using the statistical program R (R Development Core Team, 2008), version 2.15.1 with the packages "wavelets" for the MRA and "forecast" for the automated ARIMA model fitting.

3. RESULTS

3.1 Framework calibration

The results showed that the forecast accuracy achieved with the wavelet-ARIMA approach was highly sensitive to the wavelet transform type, wavelet function and the number of decomposition levels. This was the case for the results obtained both with the MAE and the RMSE. For aluminium price forecasts, the difference between the MAE achieved with the best and the worst performing wavelet-ARIMA model configuration was as high as $63/tonne for one month ahead and $266/tonne for twelve months ahead forecasts. For one month ahead forecasts of copper, lead and zinc prices, the differences were $158, $64 and $77/tonne, respectively. For twelve months ahead forecasts, the MAE with the best performing model configuration was $1,826, $934 and $490/tonne lower than that of the worst configuration for copper, lead and zinc, respectively. Figure 5 illustrates the sensitivity of the wavelet-ARIMA approach to the configuration of the MRA for one step ahead copper price forecasts. It can be seen that suboptimal configurations of the MRA can lead to wavelet-ARIMA models with lower predictive power than ARIMA and naïve models. The other forecast horizons and metals analysed showed a similar picture.

The highest forecast accuracies were achieved using the MODWT as the transform type. For all metals and forecast horizons analysed, the lowest MAE was obtained with MODWT-based wavelet-ARIMA models (see Table 1). Also with the RMSE as error measure, the highest predictive power was mostly achieved with the MODWT (see Table 2). Diebold-Mariano tests were conducted to compare the accuracy of the best performing MODWT-based model with that of the best performing DWT-based model. The results showed that the absolute forecast errors of the MODWT-based model were significantly lower in five out of the twelve forecast horizons for aluminium and copper, and in six and seven cases for lead and zinc, respectively (p-value<0.05). Moreover, the MODWT-based wavelet-ARIMA approach was found to be less sensitive to the wavelet function choice than the DWT-based approach. For one step ahead copper price forecasts, the standard deviation of the MAEs achieved with different wavelet functions was considerably higher for the DWT than for the MODWT (Figure 6), indicating that the latter was less sensitive to the wavelet function choice. A similar picture can be expected for other forecast horizons and metals.

The calibrated wavelet-ARIMA model achieved high forecast accuracies during most periods of the metal price series analysed. However, relatively high errors were obtained during periods characterised by sudden price changes (see Figure 7). Spikes in forecast errors occurred in 1988 and 2004/2005 for aluminium, during 2008/2009 for copper and during 1988/89 for zinc price forecasts. Note that the forecast errors for all metals showed spikes in the period from 2006 to 2008 if longer forecast horizons were chosen. The forecast error of the ARIMA model showed increased fluctuations from 2005/2006 onwards. With the exception of aluminium, these stronger fluctuations appeared also in the wavelet-ARIMA forecast errors. Figure 7 also provides a good comparison of the forecast accuracies achieved with the wavelet-ARIMA and the normal ARIMA model.

3.2 Framework validation

Table 1 shows the configurations of the wavelet-ARIMA model with the highest predictive power for the respective metals and forecast horizons as well as the MAE that it achieved in the model calibration. Table 2 shows the equivalent for the RMSE as forecast error measure. The accuracies obtained in the model validation, and those obtained with ARIMA and naïve models are also shown. The forecasting methods analysed showed decreasing accuracies with increasing forecast horizons. In the calibration, the wavelet-ARIMA model achieved the highest predictive power (both for MAE and RMSE) for all metals and forecast horizons analysed. The results for the Diebold-Mariano test showed that the absolute errors achieved by the wavelet-ARIMA model were significantly lower than that achieved by the ARIMA and the naïve model (p-value<0.05). The predictive power of normal ARIMA models did not differ significantly from that of the naïve model (p-value>0.05). These findings were consistent for all forecast horizons and metals analysed. However, the accuracies achieved in the validation of the wavelet-ARIMA models were significantly lower than those achieved in the calibration (p-value<0.05) and were not higher than those of the ARIMA and the naïve models.

Figure 8 illustrates the accuracies obtained with the wavelet-ARIMA, ARIMA and naïve models in predicting monthly aluminium, copper, lead and zinc prices from May 2011 to April 2012. The wavelet-ARIMA forecasts were generated using the configurations shown in Table 1 for the respective metals and forecast horizons. Both the ARIMA and the naïve model overestimated most of the price values and did not predict the metal price decrease occurring in this period. In contrast, the wavelet-ARIMA models predicted this downward trend for all four metals, both in the framework calibration and validation. The results obtained in the calibration showed that for all metals, the first five predicted price movements corresponded to the actual movements, i.e. price increases and decreases from one month to the other were predicted correctly. With the exception of aluminium, the price movement predictions of the wavelet-ARIMA model were frequently erroneous for forecast horizons longer than six months. The price movements predicted in the wavelet-ARIMA model validation resembled those obtained in the model calibration. For aluminium, the first five price movements from April to September 2011 were forecast correctly in the wavelet-ARIMA validation. For copper and zinc, four, and for lead, three out of the first five price movements were forecast correctly. Although these forecasts provide more accurate information than those of ARIMA and naïve models, the high predictive power achieved in the wavelet-ARIMA model calibration could not be confirmed in the model validation. The accuracy of the wavelet-ARIMA models was further improved by increasing the maximum number of ARIMA model parameters. In contrast, this did not affect the forecasts obtained with normal ARIMA models. If a higher number of ARIMA model parameters was allowed to forecast copper prices from May 2011 to April 2012, the first six price movements were predicted correctly both in the calibration and the validation of the wavelet-ARIMA model for this period (see Figure 9). The forecast obtained in the framework validation captured the cyclical price behaviour well.

4. DISCUSSION

This study showed that wavelet-ARIMA models have the potential to achieve relatively high accuracies in predicting selected metal prices. By capturing the cyclical behaviour of metal prices, wavelet analysis can considerably improve classic ARIMA models. The results showed that, on average, the proposed wavelet-ARIMA technique has the potential to increase the accuracy of one month ahead forecasts by $53/tonne for aluminium, $126/tonne for copper, $50/tonne for lead and $51/tonne for zinc, relative to classic ARIMA models (see Table 1). The superiority of wavelet-ARIMA models over traditional ARIMA models has also been shown by Conejo et al. (2005) for electricity prices and Fernandez (2007, 2008) for shipments data of the US metal and material manufacturing industry. The novelty of this study consists in showing that the achievement of high forecast accuracies with wavelet-ARIMA models requires optimal combination of wavelet transform type, wavelet function and number of decomposition levels in the wavelet-based MRA.

In the context of resource and energy economics, the DWT is the most frequently used transform type in wavelet-based forecasting. In contrast, the findings of this study indicated that the MODWT is superior in MRA-based forecasting. In forecasting monthly aluminium, copper, lead and zinc prices, it outperformed the DWT-based wavelet-ARIMA approach. The results on the sensitivity of the MODWT-based approach to the wavelet function choice were consistent with those by Gencay et al. (2001). They found that the wavelet coefficients resulting from the MODWT are less sensitive to the wavelet function choice than those resulting from the DWT. This feature of the MODWT is transmitted to the detail and smooth coefficients and finally to the forecast errors obtained in the MODWT-based wavelet-ARIMA approach. The results suggested that the optimal number of decomposition levels considered in the wavelet-ARIMA model increases with increasing forecast horizons. For shorter horizons of one to four months, one or two decomposition levels yielded the most accurate forecasts if the MAE was used as error measure (see Table 1). Longer horizons of five to twelve months required three to six levels. This was consistent for all four metals analysed. A possible explanation is that higher numbers of decomposition levels in the MRA yield detail series of higher levels, which explicitly capture fluctuations of longer time scales in the time series. Such longer-term fluctuations might be especially relevant for longer-term forecasts. The effects of the transform type, wavelet function and number of decomposition levels used were similar for all four metal price series analysed. This supports the argument that the optimal configuration of the wavelet-based forecasting approach depends on the characteristics of the time series analysed (Gencay et al., 2001).

While for the majority of the periods forecast the wavelet-ARIMA model achieved high predictive power, sudden price shocks pose a limitation to this forecasting technique. The model could not predict the sharp price decrease and subsequent increase occurring in base metal prices during 2008/09, which might have been triggered by the economic crisis. Another period of low forecast accuracy is the sharp increase of aluminium, copper and zinc prices in 1988/89. This price upswing was triggered by historically low metal inventories and a rising worldwide demand for these metals (USGS, 1999). Knowing these limitations of the wavelet-ARIMA approach enables the identification of suitable complementary forecasting models. Moreover, it shows that relying solely on time series methods, and thus only on the past characteristics of a time series to predict future prices involves some remaining risk.

Conejo et al. (2005) found that the wavelet-ARIMA approach has higher predictive power than normal ARIMA models when forecasting hourly electricity prices. For monthly base metal prices this result was also obtained in the wavelet-ARIMA model calibration. Although the results obtained in the model validation appeared to be more useful in predicting short-term price trends and movements than ARIMA and naïve models, the validation could not confirm the high accuracies achieved when the model was calibrated. In the validation, future prices were assumed to be unknown, so that the time series had to be extended artificially to compute detail and smooth coefficients up to the prediction time. This causes so-called boundary distortions, i.e. the information obtained from the wavelet transform is corrupted at the start and the end of the time series. Boundary distortions can affect the forecasts in two ways (Rocha Reis and Alves da Silva, 2005). Firstly, depending on the sample used for fitting the forecasting model, the model parameters can be affected by boundary distortions at the beginning and the end of the time series. Secondly, boundary distortions at the end of the time series corrupt the detail and smooth coefficients used to generate the forecast. Therefore, the high accuracies achieved in the wavelet-ARIMA model calibration could be realised by identifying a padding technique that minimised boundary distortions. Standard padding techniques are symmetric reflection, periodic extension and zero-padding (Crowley, 2007). In the first, the time series is extended by reflecting the time series at its boundaries, as done in this study for the wavelet-ARIMA model validation. Periodic extension means that the end of the time series is extended by the values occurring at the beginning of the series, i.e. the series is assumed to be a circle. Finally, zero-padding involves extending the beginning and the end of the series with zeros. Rocha Reis and Alves da Silva (2005) suggested appending forecast values at the right boundary of the time series. For electricity load forecasts, they found that well-performing forecasting models that do not involve MRA can be further improved if the predictions are used for padding in MRA-based forecasting. They concluded that the additional gain in accuracy made the effort of identifying the best MRA-independent forecasting technique worthwhile. How well this technique reduces boundary distortions depends on the accuracy of the MRA-independent technique's forecast (Rocha Reis and Alves da Silva, 2005). Future research will focus on minimising boundary distortions in order to fully exploit the large potential of wavelet-ARIMA models for forecasting metal prices. Already, the wavelet-ARIMA technique enables the prediction of short-term trends and metal price movements over a forecast horizon of less than one year. Thus, it can support investment, purchasing and selling decisions that are affected by metal prices.

In contrast, the confidence in classic ARIMA models for forecasting monthly base metal prices raised by the findings of Dooley and Lenihan (2005) could not be confirmed. The performance of the ARIMA models was not better than that of the naïve model. Dooley and Lenihan (2005) assessed the forecast accuracy of ARIMA models based on the prediction of a single period. The study at hand involved forecasts of a large sample of periods. This revealed that the performance of ARIMA models varies strongly, especially since 2005/06 (see Figure 7), which leads to a low predictive power on average.

5. CONCLUSIONS

This study showed that wavelet-ARIMA models are a promising technique for forecasting metal prices with high accuracy. The configuration of the wavelet-based

MRA significantly affects the predictive power of the approach. Although the DWT is the most frequently used wavelet transform type in MRA-based forecasting, the MODWT was found to achieve more accurate metal price forecasts. Normal ARIMA models were shown to be rather unsuitable for predicting monthly base metal prices. Their performance is not significantly different from that of a naïve model. The findings of this study highlight the importance of taking into account cyclicality when forecasting metal prices.

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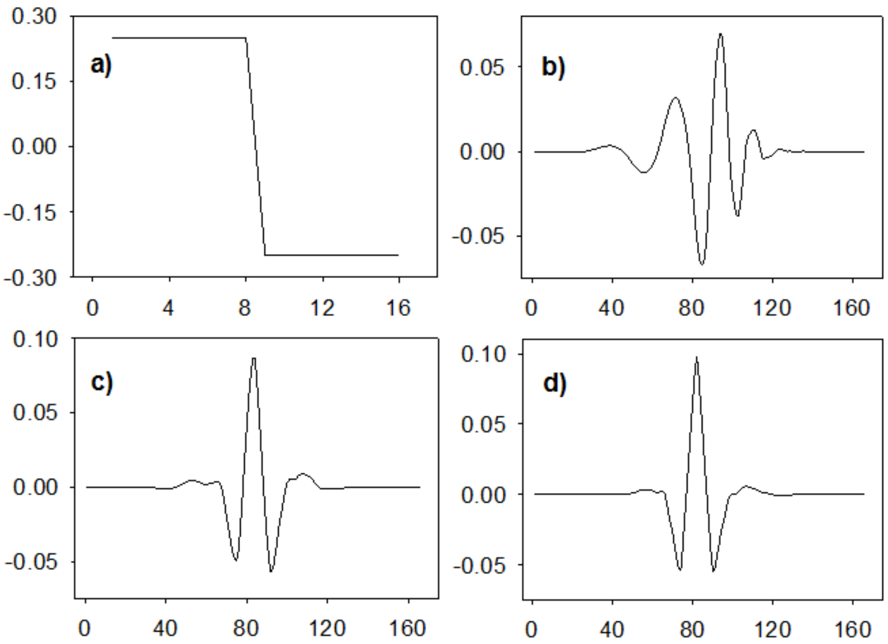

Figure 1. Shapes of different wavelets: a) Haar wavelet, b) Daublet of order 12, c) symmlet of order 12, d) Coiflet of order 12.

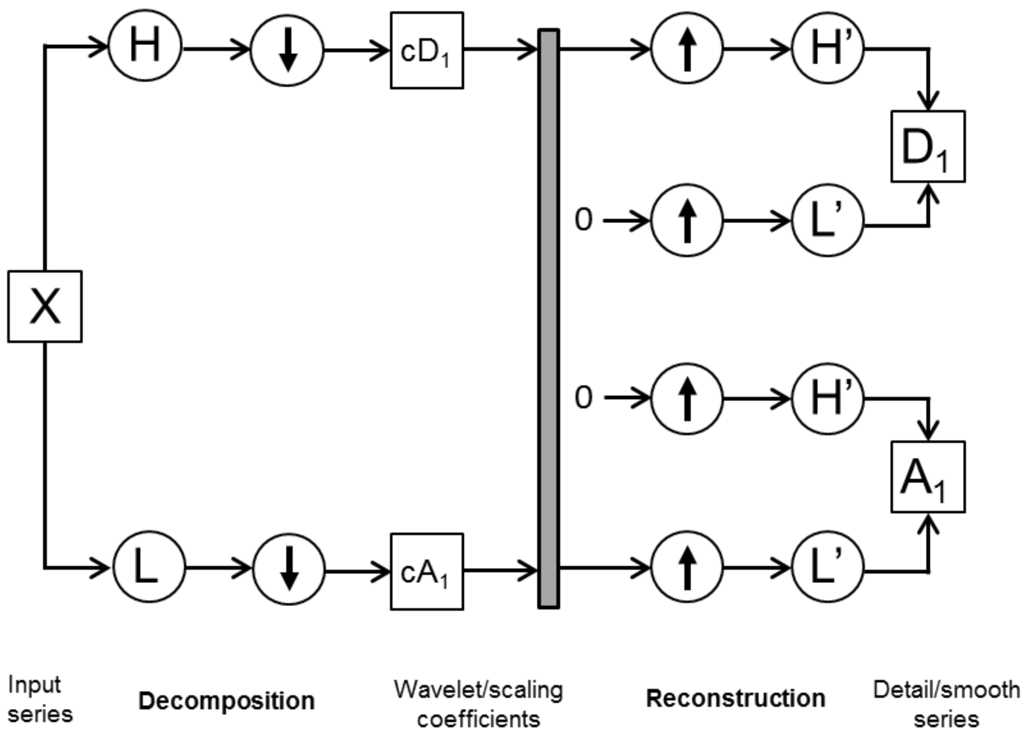

Figure 2. Single-resolution analysis based on Mallat's pyramid algorithm (modified from Rocha Reis and Alves da Silva, 2005). X, H, L, cD_1, cA_1, H', L', D_1 and A_1 stand for input time series, high-level decomposition filter, low-level decomposition filter, wavelet coefficients of decomposition level 1, scaling coefficients of decomposition

level 1, high-level reconstruction filter, low-level reconstruction filter, detail series of decomposition level 1 and smooth series of decomposition level 1, respectively.

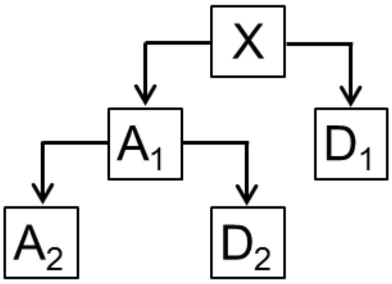

Figure 3. Multiresolution analysis scheme (modified from Rocha Reis and Alves da Silva, 2005). X, A_1, D_1, A_2 and D_2 stand for input time series, smooth series of decomposition level 1, detail series of decomposition level 1, smooth series of decomposition level 2 and detail series of decomposition level 2, respectively.

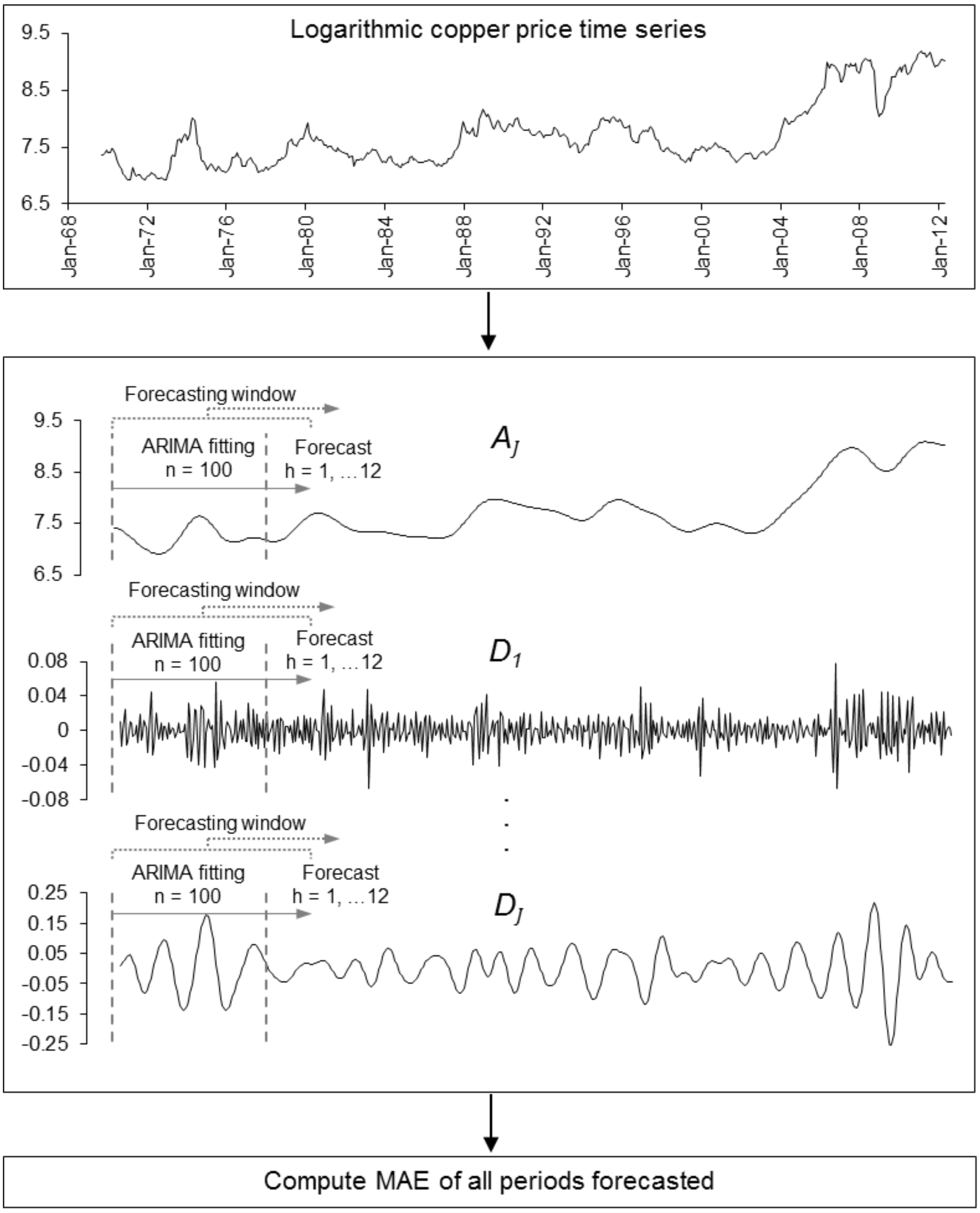

Figure 4. Framework calibration. The forecasting window is shifted to the right 412 times until the end of the time series is reached, thereby computing forecasts and forecast errors for 401 to 412 periods, depending on the forecast horizon. The summation of the smooth and detail series forecasts for each period is not shown in the figure. The multiresolution analysis shown was conducted with the maximum overlap discrete wavelet transform, the symmlet wavelet of order 8 and four levels of decomposition. A, D, J, n, h and MAE stand for smooth series, detail series, highest decomposition level, sample size, forecast horizon and mean absolute error, respectively.

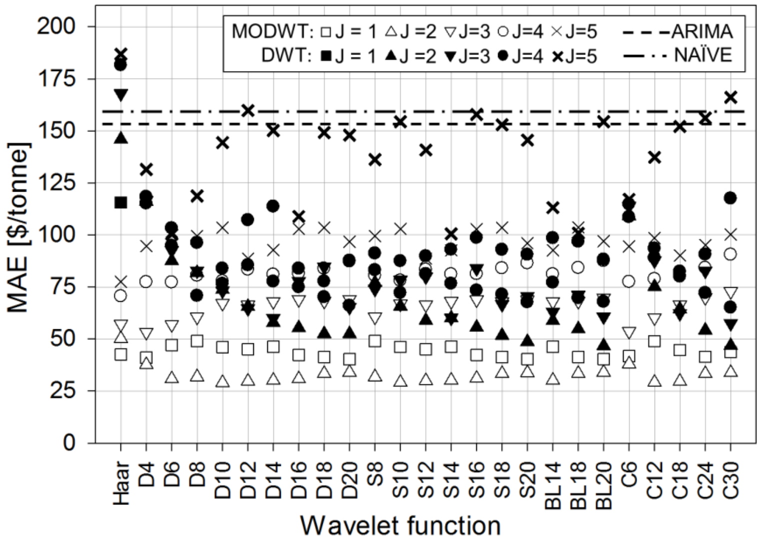

Figure 5. Mean absolute forecast error (MAE) achieved with wavelet-ARIMA models of different configurations, shown for one step ahead copper price forecasts. MODWT, DWT, J, D, S, BL and C stand for maximum overlap discrete wavelet transform, discrete wavelet transform, number of decomposition levels, Daublets, symmlets, best localised and Coiflet wavelets, respectively. The MAEs achieved with six to nine decomposition levels are not shown, as they are nearly identical to those obtained with five decomposition levels.

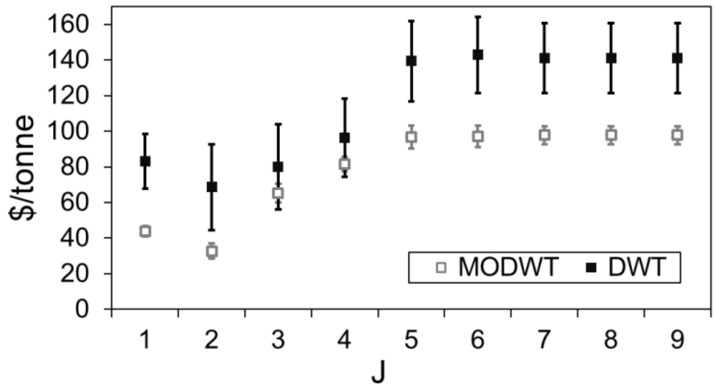

Figure 6. Average mean absolute forecast error achieved with 25 different wavelet functions, shown for one step ahead wavelet-ARIMA copper price forecasts. The length of the whiskers corresponds to the standard deviation. MODWT, DWT and J stand for maximum overlap discrete wavelet transform, discrete wavelet transform and number of decomposition levels, respectively.

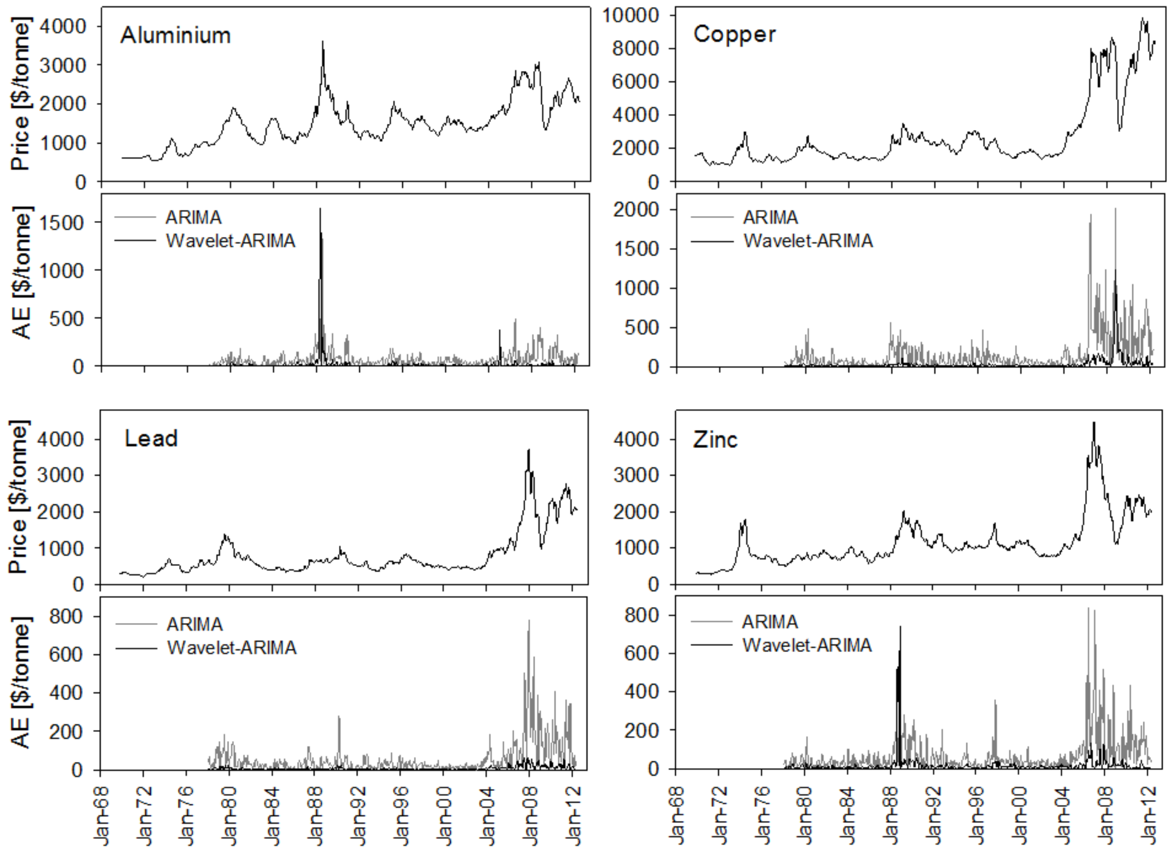

Figure 7. Evolvement of absolute forecast error (AE) over time, shown for one step ahead forecasts of monthly metal prices.

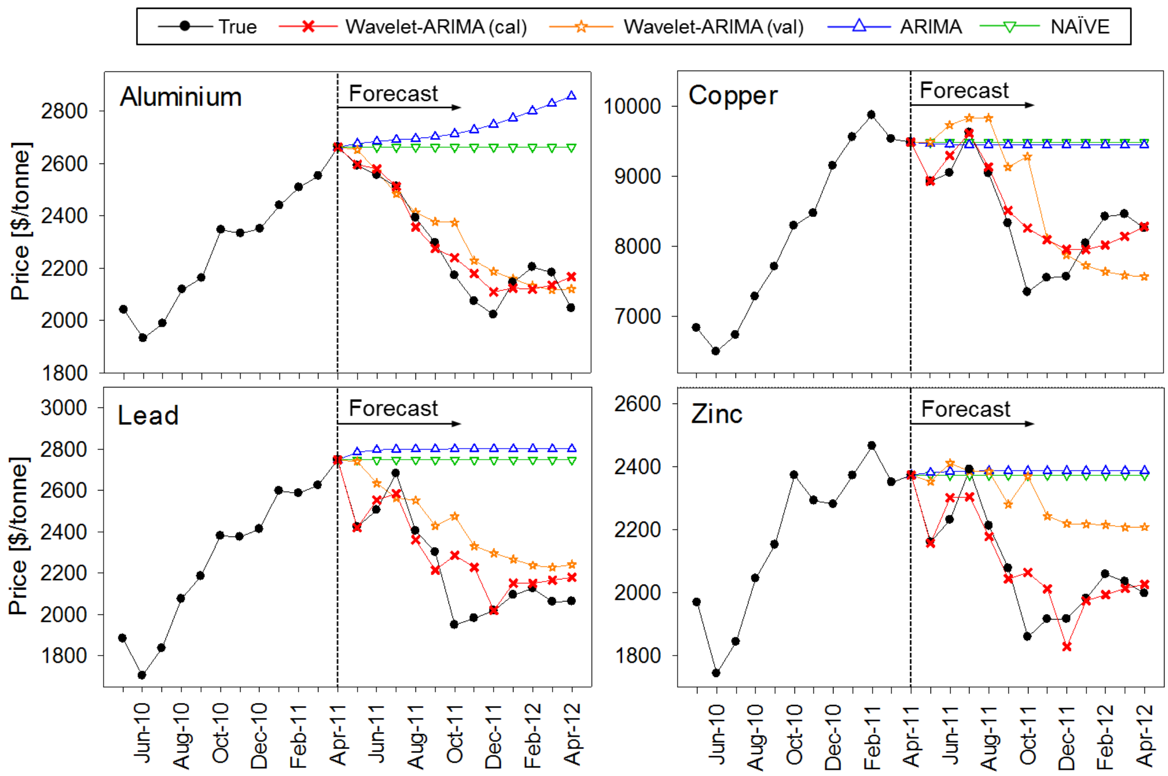

Figure 8. Price forecasts for May 2011 to April 2012. Cal and val stand for calibration and validation, respectively.

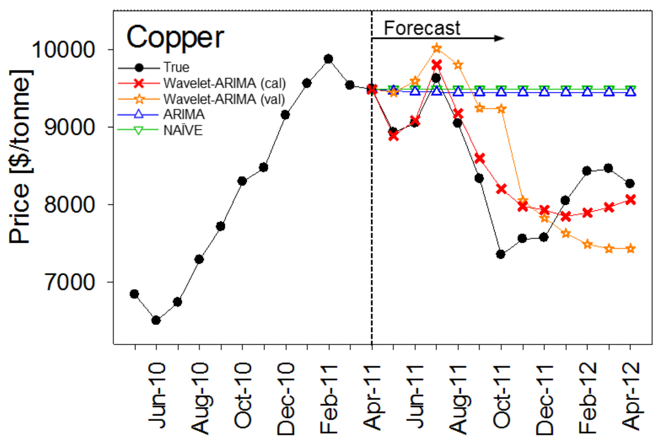

Figure 9. Copper price forecasts for May 2011 to April 2012 with the maximum number of ARIMA model parameters set to 15. Cal and val stand for calibration and validation, respectively.

Table 1. Mean absolute forecast error (MAE) achieved by the best performing wavelet-ARIMA model, the ARIMA and the naïve model. MRA, h, wf, J, MODWT, D, S and C stand for multiresolution analysis, forecast horizon, wavelet function, number of decomposition levels, maximum overlap discrete wavelet transform, Daublet, symmlet and Coiflet wavelet, respectively.

	Wavelet-ARIMA			ARIMA	NAÏVE
	MRA configuration	**Calibration**	**Validation**		
h	transform/wf/J	MAE	MAE	MAE	MAE
Aluminium					
1	MODWT/Haar/1	21.24	81.67	73.84	72.90
2	MODWT/D12/2	44.12	156.92	118.67	114.66
3	MODWT/S16/2	62.47	209.77	152.74	146.09
4	MODWT/S16/2	79.22	251.76	187.26	177.61
5	MODWT/Haar/4	91.14	266.56	218.61	206.57
6	MODWT/Haar/4	94.41	299.66	250.14	233.60
7	MODWT/Haar/4	94.54	327.06	280.38	257.65
8	MODWT/Haar/4	104.61	349.36	306.04	278.19
9	MODWT/Haar/4	116.48	371.55	328.89	297.04
10	MODWT/Haar/4	126.34	397.00	350.98	314.36
11	MODWT/Haar/4	134.60	419.91	371.27	330.58
12	MODWT/C6/4	139.27	418.12	391.56	345.35
Copper					
1	MODWT/D10/2	29.05	164.30	154.56	159.97
2	MODWT/D12/2	72.94	346.13	270.39	263.33
3	MODWT/D6/2	107.42	470.80	364.64	344.38
4	MODWT/C12/2	159.59	551.13	448.64	414.76
5	MODWT/C6/3	206.65	595.00	511.18	467.53
6	MODWT/Haar/4	220.33	631.99	576.43	519.34
7	MODWT/Haar/4	221.36	686.79	634.41	565.58
8	MODWT/D6/3	234.74	780.55	688.69	602.44
9	MODWT/Haar/4	273.63	783.91	729.00	631.39
10	MODWT/Haar/4	295.11	829.14	770.58	652.27
11	MODWT/Haar/4	307.06	868.23	816.70	675.77
12	MODWT/Haar/4	309.74	902.37	877.38	712.78
Lead					
1	MODWT/S12/2	6.99	55.46	56.59	55.30
2	MODWT/D12/2	19.76	110.25	92.62	89.48
3	MODWT/S12/2	30.24	148.71	117.41	113.75
4	MODWT/S12/2	43.77	176.66	145.16	134.95
5	MODWT/C6/3	65.35	207.58	170.96	155.26
6	MODWT/Haar/3	67.46	220.34	195.56	173.17
7	MODWT/Haar/4	70.53	237.83	218.66	191.02
8	MODWT/Haar/4	79.23	246.77	239.57	209.86
9	MODWT/Haar/4	82.97	259.88	257.29	224.38
10	MODWT/Haar/4	90.09	274.41	274.16	237.17
11	MODWT/Haar/4	99.64	287.21	291.63	246.95
12	MODWT/Haar/4	108.85	298.26	312.54	257.27
Zinc					
1	MODWT/Haar/1	17.04	73.10	68.06	68.94
2	MODWT/D14/2	35.84	151.68	115.90	113.40
3	MODWT/D8/2	43.45	200.52	152.94	144.30
4	MODWT/S18/2	55.17	235.98	185.16	174.30
5	MODWT/D6/3	71.66	260.33	219.16	198.10
6	MODWT/D6/3	83.25	294.32	252.84	218.19
7	MODWT/D6/3	92.27	329.16	288.74	239.18
8	MODWT/D6/3	98.81	359.53	323.73	258.04

9	MODWT/C6/6	116.29	334.79	356.60	274.34
10	MODWT/C6/5	124.02	341.91	392.81	293.49
11	MODWT/Haar/6	134.86	393.08	434.46	310.32
12	MODWT/Haar/6	133.97	403.73	478.12	327.39

Table 2: Root mean square forecast error (RMSE) achieved by the best performing wavelet-ARIMA model, the ARIMA and the naïve model. MRA, h, wf, J, MODWT, DWT, D, S and C stand for multiresolution analysis, forecast horizon, wavelet function, number of decomposition levels, maximum overlap discrete wavelet transform, discrete wavelet transform, Daublet, symmlet and Coiflet wavelet, respectively.

	Wavelet-ARIMA			**ARIMA**	**NAÏVE**
	MRA configuration	**Calibration**	**Validation**		
h	transform/wf/J	RMSE	RMSE	RMSE	RMSE
Aluminium					
1	MODWT/Haar/5	63.40	403.07	123.39	114.11
2	MODWT/Haar/5	101.45	606.81	190.20	170.81
3	MODWT/Haar/5	112.55	787.55	249.17	216.78
4	MODWT/Haar/5	137.24	907.36	307.90	259.67
5	DWT/D8/6	148.36	1626.97	355.33	296.06
6	DWT/D8/6	154.29	1673.65	404.47	331.54
7	MODWT/Haar/4	158.65	1124.76	454.38	362.06
8	MODWT/Haar/4	166.90	1182.94	505.92	387.52
9	MODWT/Haar/4	181.40	1238.73	556.46	410.02
10	MODWT/Haar/4	193.40	1295.05	603.56	428.33
11	MODWT/Haar/4	198.80	1346.75	655.14	444.91
12	MODWT/Haar/4	200.75	1386.83	724.42	462.12
Copper					
1	MODWT/D4/1	78.34	280.45	285.34	286.68
2	MODWT/S10/2	189.44	608.56	493.20	472.12
3	MODWT/Haar/3	244.86	751.48	664.57	616.22
4	MODWT/Haar/3	318.95	880.74	815.35	733.54
5	DWT/S10/4	352.40	1176.58	948.58	831.50
6	DWT/S10/4	390.42	1282.98	1083.11	919.44
7	MODWT/Haar/4	414.76	1124.76	1201.75	992.14
8	MODWT/D6/3	444.13	1256.34	1325.09	1051.33
9	MODWT/Haar/4	508.05	1238.73	1416.03	1084.85
10	MODWT/D4/5	539.71	1181.97	1490.57	1106.18
11	MODWT/D4/4	566.21	1298.45	1556.38	1127.32
12	MODWT/Haar/4	571.68	1386.83	1664.82	1163.72
Lead					
1	MODWT/S12/2	12.25	134.95	106.24	105.07
2	MODWT/D12/2	36.82	245.07	177.60	166.61
3	MODWT/D4/2	58.76	303.70	224.93	206.76
4	MODWT/C6/3	110.71	368.12	270.13	242.03
5	MODWT/C6/3	142.31	420.42	319.70	276.06
6	DWT/C12/5	148.82	699.03	370.06	310.23
7	MODWT/Haar/4	149.25	538.75	423.26	344.08
8	MODWT/Haar/4	149.51	574.30	481.14	376.43
9	MODWT/Haar/4	153.07	615.16	534.40	402.45
10	MODWT/Haar/4	168.00	660.37	587.49	423.74
11	MODWT/Haar/4	196.10	699.39	639.63	441.89
12	MODWT/Haar/4	209.82	734.89	694.73	458.62
Zinc					
1	MODWT/Haar/1	55.55	124.85	117.56	114.90
2	MODWT/Haar/2	87.16	211.60	205.42	186.01

3	MODWT/D6/3	100.74	306.06	276.73	236.25
4	MODWT/D6/3	126.09	364.44	340.09	275.29
5	MODWT/D6/3	143.09	420.18	401.22	310.25
6	MODWT/S12/3	162.74	454.32	468.15	346.14
7	MODWT/Haar/4	169.04	538.75	554.39	384.22
8	MODWT/Haar/4	178.69	574.30	670.43	425.04
9	MODWT/Haar/4	201.66	615.16	799.88	462.21
10	MODWT/Haar/4	219.07	660.37	929.31	494.94
11	MODWT/Haar/6	228.83	676.09	1063.37	525.40
12	MODWT/Haar/6	229.73	699.10	1212.13	554.94

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Kriechbaumer, Thomas

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