SUMMARY

The purpose of this note is to review some current theories of ignition in stagnant and flowing mixtures, and to present an alternative theory to describe the process that occurs in a turbojet combustion chamber when the engine is being relighted after a flame-out at high altitudes. This theory leads to an expression of the form

\[ I = f \left( \frac{E^{0.25} P^{0.5} T^{2.5}}{U_{ref} \left( \frac{\Delta P}{ft} \right)^{0.5}} \right) \]

in which relighting performance is related to the energy in the spark, the operating variables of pressure, temperature and velocity, and the 'pressure loss factor' of the flame tube. It is suggested that the above expression serves to indicate the relative importance of the various factors influencing the ignition process in practical combustion chambers, and could prove useful as a correlating parameter for ignition data obtained from such systems.
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IGNITION THEORIES

Introduction

The precise mechanism whereby an inflammable mixture is caused to ignite has for a long time been the subject of speculation and experiment and has undergone several reinterpretations in the light of various new theories. Most of these theories are based on the idea that the transient ignition source, usually an electrical spark, must supply to the combustible mixture sufficient energy to create a critically small volume of hot gas which just satisfies the necessary and sufficient condition for propagation, namely, that the rate of heat generation equals the rate of heat loss. The various theories tend to fall into two groups; those in which diffusion is considered to be the most important process in flame initiation, and those in which thermal mechanisms are considered more important. At the present time the most highly developed theories are those of Lewis and von Elbe, Fenn, Swett and Yang. Useful reviews have been provided by Bradford and Finch, Wigg and Hazard.

Lewis and von Elbe

These workers proceed from the assumption that practically the entire energy of a spark discharge becomes thermal in nature within a very short time. The size of the inflamed volume produced by the spark determines whether or not the flame will propagate. If the size is too small, the temperature gradient between the gas in the core and the outer unburnt gas will be steeper than in a steady-state combustion wave, and the rate of heat liberation in the zone of reaction will be insufficient to compensate for
the heat loss to the outer zone of unburned gas. The temperature falls, reaction ceases, and the incipient combustion wave is extinguished.

In order to make a quantitative estimation of the energy required for ignition, a model for the combustion wave is proposed which postulates the existence, just ahead of the flame, of a belt of gas whose enthalpy is greater than the surrounding burned and unburned gases. An element of gas being overrun by the wave acts first as a heat sink, absorbing energy until the temperature is high enough for chemical reaction to start. The element then becomes a source, losing heat to the unburned side of the wave. Thus heat is continuously borrowed from, and repaid to, a fund which is described as the 'excess enthalpy' of the wave.

In the case of a spherically growing flame, the wave area, and hence the total amount of excess enthalpy, continuously increases. Clearly, the ignition source can furnish only an initial amount of this energy, and after the flame has reached a certain size the excess enthalpy is supplied from the heat content of the burned gas. On this basis, it is seen that the source must supply a minimum energy $E_i$, which enables the flame to grow to a critical diameter beyond which the wave can propagate 'under its own power'. These arguments lead to the following expression for minimum ignition energy

$$E_i = \pi d^2 \left( \frac{k}{S_L} \right) (T_b - T)$$

where
- $k =$ thermal conductivity of unburned mixture
- $S_L = $ flame speed
- $T_b = $ temperature of burned gas
- $T = $ initial gas temperature
- $d = $ critical diameter for propagation, assumed equal to quenching distance.
Lewis and von Elbe checked this equation against their own test data and those of Calcote et al. and found fairly good agreement. They accepted this as evidence that the theory, though very simplified, was basically correct. Since then, however, serious doubts have been cast on the whole concept of 'excess enthalpy', notably by Friedman and Burke and also by Spalding. From an examination of all the available evidence, Linnett concluded that the experimental data do not support the view that an excess enthalpy hump is necessary for flame propagation. However, although its derivation is suspect, the above equation does permit fairly accurate quantitative prediction of minimum ignition energies in stagnant mixtures where data on burning velocity and quenching distance are available.

Fenn

Fenn considers a homogeneous volume element heated to flame temperature by the source of ignition, and of such size that the rate of heat generation by a second order chemical reaction exactly equals the rate of heat loss by conduction. From this model Fenn derives the following expression for minimum ignition energy

$$\log \left( \frac{E \frac{N_o^3}{T_b - T}}{p_o} \right) = L + 2 \log T - 2 \log p_o \frac{0.65 \alpha T_b \text{ (lean)}}{R T_b}$$

where

- $N_o$ = mole fraction of oxygen
- $p_o$ = gas pressure
- $R$ = gas constant
- $\alpha$ = constant
- $L = \text{undetermined constant}$

Fenn found that this equation gave a reasonable correlation of experimental data with $\alpha = 16$. 
Yang

The most thorough treatment of ignition in stagnant mixtures is probably that due to Yang, who has proposed a non-adiabatic theory of ignition which takes into account the time-space distribution of energy of the ignition source. Three types of source are considered: plane, line and point, and separate expressions for minimum ignition energy are derived for each type. For a line source it is found that

$$E_l = 16.\ell \cdot \rho \cdot c_v \left[ \frac{q \cdot M_{cr} \cdot (1-x_R)^n}{16 \cdot K(x_\rho)^v} \right] ^{\frac{n-1}{n}}$$

where $\ell$ = length of line source

$\rho$ = gas density

$c_v$ = mean specific heat at constant volume

$q$ = energy of reaction

$M_{cr}$ = critical mass flow rate

$x_R$ = fuel concentration at source

$x$ = average fuel concentration

$k$ = constant in expression for temperature profile

$v$ = reaction order

$n$ = constant

This theory has yet to be fully compared with experiment, but preliminary indications are that it may meet with more success than previous theories.

Swett

Swett's theory is of practical importance because it is intended to account for the ignition effects observed in flowing mixtures. Essentially the theory is very similar to that of Fenn, except that it is based on the
idea that only a portion of the discharge length is important in the ignition process and that heat loss by thermal conduction is negligible compared with heat loss by eddy diffusion. It leads to the following expression for minimum ignition energy.

\[ E_i = \frac{8\pi U t_s J c_p (T_b - T)^{3/2} \exp(2E/\sqrt{RT_b})}{H^{3/2}x_f x_o \rho a^2 \log(1+2Ut_s/d_q)} - f(\sqrt{u^2}) \]

where

- \( U \) = local gas velocity
- \( t_s \) = duration of discharge
- \( J \) = mechanical equivalent of heat
- \( c_p \) = specific heat of gas at constant pressure
- \( H \) = heat of combustion of fuel
- \( x_f \) = molar concentration of fuel
- \( x_o \) = molar concentration of oxygen
- \( \rho \) = gas density
- \( E \) = activation energy
- \( f(u^2) \) = function of turbulence intensity
- \( d_q \) = quenching distance
- \( a \) = constant

The above relationship is substantiated by a limited amount of test data, all of which were obtained from minimum ignition energy measurements at the optimum fuel-air ratio. Unfortunately, like other formulae, it relies on a knowledge of various properties of the mixture that are unknown or difficult to determine. In fact the major difficulty in applying all the above theories to any practical ignition problem lies in assigning reasonable values to properties such as flame temperature and quenching distance. Usually the uncertainties in these values are so great that the usefulness
of the theories in predicting ignition energies or in providing worthwhile relationships between the important variables is very limited. From the combustion engineer's viewpoint what is required is a relationship between minimum ignition energy, or some other criterion of combustion performance, and the size and pressure loss of the combustion system and its operating conditions of pressure, temperature, and air mass flow. The following analysis attempts to provide a relationship of this kind.

**ALTERNATIVE THEORY**

The process of initiating combustion in a flowing fuel-air mixture is visualized as follows: Passage of the spark creates a small, roughly spherical, volume of gas in which the temperature is initially very high but falls rapidly as the hot kernel grows in size and as heat is diffused outwards into the surrounding unburned gas. The effect of this outward flow of heat is to initiate chemical reactions in the layer of fresh mixture adjacent to the hot kernel, and these reactions may be so vigorous that within a very short time the rate of heat generation by chemical reaction may exceed the rate of heat loss by turbulent diffusion. If this happens the spark kernel will continue to expand and steady burning conditions will soon be established. However, if for any reason the rate of heat release by chemical reaction at the surface of the kernel is less than the rate of heat loss by diffusion, the temperature within the kernel will continue to fall until the reactions cease altogether.

As in previous theories, of critical importance to the ignition process is the size to which the spark kernel has grown by the time its temperature has fallen to the normal flame temperature of the mixture. The criterion
for successful ignition is that at this point the rate of heat release in the thin reaction zone surrounding the heated zone should exceed the rate of heat loss from the volume. Now the rate of heat release per unit surface area of the flame front is dependent on the diameter of the volume, whereas the rate of heat loss per unit surface area is not. Thus equating these rates gives a value for the minimum diameter of spark kernel, below which the flame front will fail to propagate. This model for the ignition process is similar to that employed by Swett\(^3\), the essential difference being that whereas Swett's theory contemplates heat generation throughout the entire spark volume, the present theory is based on the assumption that the heat release occurs entirely in the relatively thin shell surrounding this volume, i.e. in the flame front itself. This approach is believed to be more realistic because it is difficult to conceive how heat generated at the centre of the kernel can compensate for heat loss at the boundary, especially since any flow of heat from the centre to the surface is opposed by the radial inward flow of burned products from the flame front.

Considering first the general case of spark ignition in a flowing combustible mixture, at the point in time when the spark kernel has reached its critical diameter the rate of heat release at the surface may be expressed as follows:

\[
\text{rate of heat release} = \rho \cdot \pi d^2 \cdot S_T \cdot f \cdot H.
\]

where \(\rho\) = density of unburned gas

\(d\) = critical diameter of spark kernel, assumed spherical.

\(S_T\) = turbulent burning velocity of mixture

\(f\) = fuel-air ratio by weight

\(H\) = lower calorific value of fuel.
The rate of mixing between the spark kernel and the gas stream is given by the product of the eddy diffusivity, the mixing area and the density gradient. If it is assumed that the eddy diffusivity is proportional to the product of a mixing length, \( \ell \), and the turbulent velocity in the gas stream, then

\[
\text{mixing rate} = (\text{eddy diffusivity})(\text{mixing area})(\text{density gradient})
\]

\[
\text{mixing rate} = \alpha (\ell \sqrt{u^2})(\rho/\ell)
\]

Assuming that \( \sqrt{u^2} \) is \( U \), then mixing rate \( \propto \rho \cdot d^2 \cdot U \) and rate of heat loss

\[
= C_1 \cdot \rho \cdot \pi \cdot d^2 \cdot U \cdot c_p \cdot \Delta T
\]

where \( U = \text{flow velocity} \)

\( \sqrt{u^2} = \text{mean fluctuation velocity} \)

\( c_p = \text{gas specific heat at constant pressure} \)

\( \Delta T = \text{temperature rise due to combustion} \)

For the ignition condition (1) equals (2)

i.e. \( \rho \cdot \pi d^2 \cdot S_T \cdot f.H. = C_1 \cdot \rho \cdot \pi d^2 \cdot U \cdot c_p \cdot \Delta T \)

or

\[ S_T = C_1 U \]  
(3)

since \( fH = c_p \cdot \Delta T \), by definition.

According to Damkohler\textsuperscript{12}, turbulent burning velocity may be related to laminar flow speed by an equation of the form

\[ S_T \propto S_L \cdot \text{Re}^{0.5} \]

i.e. \[ S_T \propto S_L \left( \frac{U \cdot d \cdot p}{\eta} \right)^{0.5} \]

or

\[ S_T \propto \frac{S_L \cdot (U d \cdot p)}{\eta^{0.5}} \]  
(4)

since \( \rho \propto \frac{P}{T} \) and \( \eta \propto T^{0.7} \)
where \( P \) = gas pressure

\( T \) = initial gas temperature

\( \eta \) = gas viscosity

\( Re \) = Reynolds number, based on kernel diameter

Substituting (4) into (3) gives

\[
S_L \left( \frac{U d P}{m} \right)^{0.5} \propto U
\]

which, on rearranging, gives

\[
d \propto \frac{U^{1.7}}{P S_L^{2.5}}
\]  

(5)

The variation of laminar flame speed with pressure and temperature may be expressed as

\[
S_L = S_{Lo} \cdot P^{n-2/2} (T/288)^x
\]  

(6)

where \( S_{Lo} \) = laminar flame speed for mixture at standard atmospheric pressure and temperature \((288 \, ^\circ K)\)

\( n \) = reaction order

\( x \) = temperature exponent

Substituting (6) into (5) gives

\[
d \propto \frac{U}{P^{n-1} T^{2x - 1.7} S_{Lo}^{2}}
\]  

(7)

Equations (5) and (7) relate the minimum critical size of spark kernel to the velocity, pressure and temperature of the unburned mixture, and to the laminar flame speed. The effect of fuel-air ratio on minimum critical size is embodied in the term for laminar flame speed. Thus the minimum critical
size is smallest for mixture strengths close to stoichiometric and increases for weaker and richer mixtures.

For propane-air mixtures \( n = 2 \) and \( x \) is approximately \( 1.35 \) over the range \( 290\,^\circ\text{K} \) to \( 420\,^\circ\text{K} \). Thus for propane (7) becomes

\[
d = \frac{U}{PT S_{LO}}
\]  

(8)

The heat required to raise the temperature of the critical sphere of diameter \( d \) to the flame temperature must be furnished from an external source. It is reasonable, therefore, to assume that the minimum energy requirement for ignition should be proportional to the heat required to raise this critical volume to the burned gas temperature, or

\[
E_i = C_2 \frac{\pi}{6} d^3 c_p \rho \Delta T
\]  

(9)

Substitution of \( d \) from (7) into (9) gives

\[
E_i \propto \frac{c_p \Delta T U^3}{P^{3n-4} T \sigma_{6x-4.1} S_{LO}}
\]  

(10)

The above equation predicts a marked dependence of minimum ignition energy on fuel-air ratio though the term \((\text{laminar flame speed})^6\) which is borne out by experience. However, for any given fuel-air ratio, \( \Delta T \), the temperature rise due to combustion, is constant, as also is \( S_{LO} \) and \( c_p \), and (10) becomes

\[
E_i \propto \frac{U^3}{P^{3n-4} T \sigma_{6x-4.1}}
\]

For \( n = 2 \), \( x = 1.35 \), we have

\[
E_i \propto \frac{U^3}{P^2 T^4}
\]  

(11)
As far as pressure is concerned, the relationship $E_i \propto P^{-2}$ is amply confirmed by the results of many workers/Blanc et al\textsuperscript{13}, Swett\textsuperscript{14} and Penn\textsuperscript{2} and is illustrated in figure 7. Unfortunately very few data are available on the effects of velocity and temperature. Swett found that minimum ignition energy increased only linearly with velocity, whereas a very much stronger dependence on velocity is evident from the data of Straight, Fletcher and Foster\textsuperscript{15}, which are reproduced in figure 1. The influence of temperature on minimum ignition energy has been examined by Penn\textsuperscript{2}. From his tabulated results a relationship can be calculated of the form $E_i \propto T^{-b}$, with values of $b$ for n-heptane, iso octane, n-pentane and propane of 3.8, 4.4, 3.7, and 3.0 respectively. These values, although very approximate, fall within the order of temperature dependence predicted by equations (10) and (11).

Application of theory to gas turbine ignition

It is clear from equation (3) that successful ignition depends upon the attainment of a certain minimum value of the ratio $S_T/U$ by the time the spark kernel has expanded to its critical diameter. Thus the ratio $S_T/U$ represents a realistic parameter for the analysis of the influence of flow variables and various design factors on the ignition performance of gas turbine combustion chambers. Denoting ignition performance by $I$, we have

$$I = f\left(\frac{S_T}{U}\right) \quad (12)$$

Now turbulent burning velocity may be related to laminar flame speed by a general expression of the form

$$S_T \propto S_L Re^a$$
It can readily be shown that

\[ U = U_{\text{ref}} \left( \frac{\Delta P_{\text{ft}}}{q_{\text{ref}}} \right)^{0.5} \]  

(14)

where

- \( U_{\text{ref}} \) = chamber reference velocity
- \( \Delta P_{\text{ft}} \) = flame-tube pressure drop
- \( q_{\text{ref}} \) = dynamic head based on chamber inlet pressure and temperature, and \( V_{\text{ref}} \).

Substitution of (13) and (14) into (12) leads to

\[ I = f \left[ \frac{S_{\text{L}} p^{n-2/2} T^{x} \left( \frac{P d}{T^{0.75}} \right)^{a}}{U^{1-a} \left( \frac{\Delta P_{\text{ft}}}{q_{\text{ref}}} \right)^{0.5(1-a)}} \right] \]  

(15)

It has been demonstrated elsewhere\(^{16}\) that combustion efficiency data obtained during low pressure tests on several types of combustion chamber could be satisfactorily correlated by a parameter based on a burning velocity model and assuming \( a = 0.43 \). This compares very favourably with the value of 0.4 obtained in separate experiments by Damkohler\(^{12}\) and Delbourg\(^{17}\).

Inserting into (15) \( a = 0.43, n = 2 \) and \( x = 1.4 \) (i.e. assuming that kerosine flame speed has the same temperature dependence as propane) leads to

\[ I = f \left[ \frac{A P T^{2.5}}{M \left( \frac{\Delta P_{\text{ft}}}{d_{\text{ref}}} \right)^{0.75} \left( \frac{P d}{T^{0.75}} \right)^{0.75}} \right] \]  

(16)

but \( d = \left( \frac{E}{\rho c_p \Delta T} \right)^{1/3} \) from equation (9),
and, for any given fuel-air ratio, \( c_p \Delta T = \text{constant} \),

hence \( d = \left( \frac{E}{p} \right)^{1/3} \) \( \text{(17)} \)

Substituting \( d \) from (17) into (16) we have, for any given fuel-air ratio

\[
I = f \left[ \frac{E^{0.25} A P^{1.5} T^{1.5}}{M \left( \frac{\Delta P}{\Delta t} \right)^{0.5}} \right] \quad \text{(18)}
\]

or, using normal combustion chamber nomenclature,

\[
I = f \left[ \frac{E^{0.25} A_{ref} P_2^{1.5} T_2^{1.5}}{M \left( \frac{\Delta P}{\Delta t} \right)^{0.5}} \right] \quad \text{(19)}
\]

where \( E \) = effective thermal component of total ignition energy, joules.

\( A_{ref} \) = maximum cross sectional area of combustor, in\(^2\).

\( \left( \frac{\Delta P}{\Delta t} \right) \) = flame-tube pressure loss factor

\( M \) = chamber air mass flow, lb/sec

\( P_2 \) = inlet air pressure, p.s.i.a.

\( T_2 \) = inlet air temperature, °K

Equation (19) may be written in an alternative form, by substituting \( M = \rho A U \), as

\[
I = f \left[ \frac{E^{0.25} P_2^{0.5} T_2^{2.5}}{U_{ref} \left( \frac{\Delta P}{\Delta t} \right)^{0.5}} \right] \quad \text{(20)}
\]
Discussion

Equations (19) and (20) provide quantitative relationships between the main variables influencing the ignition process. It should be appreciated, however, that they have no significance under conditions where ignition performance is limited by stability considerations, or through inadequate flame-tube interconnection.

Although the general form of equations (19) and (20) shows broad agreement with practical experience in regard to the effects of pressure, temperature and velocity, there are very few experimental data against which to check the numerical accuracy of the exponents derived for each variable. However, confirmation for the relationship between pressure and velocity is provided in the extensive data reported by Foster and co-workers\textsuperscript{15, 18} from numerous tests carried out on a single turbojet combustor. In all these tests, which covered a wide range of fuels, it was found that the data could be satisfactorily correlated by a parameter $U/P^{0.5}$, in exact agreement with (20). A typical correlation is reproduced in figure 1 which also provides support for equation (20) in demonstrating how a relatively small increase in velocity necessitates a large increase in ignition energy.

It is of interest to compare equation (19) with the following parameter for the correlation of combustion efficiency data,\textsuperscript{16} which was derived from a burning velocity model based on equation (13), also with $a = 0.43$.

$$\eta_c = f \left[ \frac{P_2^{1.75} A_{ref} D_{ref}^{0.75} T_r}{\exp \frac{\Delta P_{ref}}{300 q_{ref}}} \right]$$

(21)
Of special significance is the lower pressure exponent in the ignition parameter. This is fully consistent with all experimental observations on turbojet combustors, but hitherto this difference has never been satisfactorily explained and has sometimes been attributed to the effects of mixing. In the present analysis the lower pressure exponent emerges as a direct result of the model assumed for the ignition process.

It is also of interest to compare in equations (19) and (21) the role of flame-tube pressure loss factor, which has a controlling effect on the level of turbulence in the primary-zone air flow. Although in both cases turbulence has a beneficial effect on flame speed, in the ignition parameter this effect is outweighed by the adverse effect of turbulence in increasing the rate of heat loss from the spark kernel. From an ignition viewpoint, therefore, a reduction in flame-tube pressure loss should have a beneficial effect on performance.

The most significant factor influencing ignition performance, however, is velocity. Some idea of its importance can be gained from equation (20) which demonstrates that a 10 percent reduction in velocity is roughly equivalent to a 50 percent increase in spark energy. Equation (20) also stresses the beneficial effect of air temperature, but under altitude relighting conditions this is a function of altitude only and lies outside the province of chamber design. In any case, with liquid fuels the effect of an increase in air temperature is difficult to assess, since it also increases the rate of fuel evaporation and thereby changes the mixture strength in the vicinity of the igniter in a way that is virtually impossible to predict.
In so far as it is possible to check equations (19) and (20) against actual experimental data the level of agreement is remarkably good. It is suggested, therefore, that these equations could prove useful in assessing the relative importance of the various factors influencing ignition performance in practical combustion chambers and in the correlation of ignition data obtained from such systems.

REFERENCES


Fig. 1. Minimum spark energy requirement as a function of inlet air pressure and velocity. (ref.15)