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# FLIGHT TECHNIQUES FOR THE MEASUREMENT OF STABILITY DERIVATIVES AND AIRCRAFT RESPONSE

by

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# THE COLLEGE OF AERONAUTICS

## DEPARTMENT OF FLIGHT

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### SUMMARY

A method of obtaining aircraft frequency-response from transient response data by Fourier analysis is currently being investigated. This report describes progress that has been made between the commencement of the contract (1st December, 1964) and the time of writing (October, 1965). The dynamic response characteristics of a Hawker Siddeley 'Dove' aircraft are being determined from flight measurements using the Fourier method of analysis. At present attention is centred on the longitudinal response as the short-period mode of the aircraft is well damped and should be defined by a simple transfer function. The aircraft transient responses to various pilot-applied control inputs are recorded and the Fourier analysis of these transients is being carried out on a Ferranti Pegasus digital computer.

Three development flights have been completed to date, and some preliminary results have been obtained, although the detailed analysis of the flight data is awaiting the incorporation of the instrumentation calibration into the computer programme. This last procedure will speed up the analysis of future data.

### INTRODUCTION

The method of obtaining the frequency response from the transient response, as used in the current programme, is by Fourier analysis. The advantage of this method is that the frequency-response is determined without knowledge of the equations of motion (or the transfer function) relating the output to the input. Also Fourier analysis automatically includes structural responses, providing that the instrumentation is accurate enough, whereas in the so-called 'curve-fitting' methods these can only be included if provision is made for them in the assumed form of the transfer function.

## TEST EQUIPMENT

### Aircraft Instrumentation

The test aircraft is a Hawker Siddeley 'Dove' (but the intention is to continue the work on a Morane-Saulnier M.S.760 'Paris' Ia turbojet aircraft). The 'Dove' aircraft is fitted with a nose boom on which are mounted wind-vanes for sensing sideslip and angle-of-attack. These vanes are fitted with Penny and Giles inductive pick-offs. Pitch, roll and yaw rates are sensed by Smith's RGS/1 rate gyros of 0-10 deg./sec. range, and have temperature-compensated fluid damping. Normal and lateral accelerations are sensed by R.A.E. accelerometers with inductive pick-offs and eddy current damping. The normal accelerometer range is ±0.5 g. The elevator deflection is sensed by a Penny and Giles rectilinear potentiometer.

The rate gyros and linear accelerometers are mounted on a machined plate which is situated on the axis of symmetry of the aircraft at a mean C.G. of $.33\bar{c}$. All sensors were aligned to within 0.10 degrees with respect to aircraft axes. This was done by using a precision spirit level on the datum points of the wing main-spar in the fuselage and ensuring by the aid of a theodolite, that these datum points were on a level parallel to the wing tips. The machined mounting plate for the transducer package was then aligned to the main-spar datums. The same process was employed in the longitudinal sense using the datum pegs on the fuselage side.

### Instrumentation Calibration

All sensing instruments were statically and dynamically calibrated. The static calibrations of the rate gyros and the accelerometer were carried out on a rate table and whirling arm respectively. The wind vanes and the control surface deflection potentiometers were statically calibrated in position on the aircraft with the aid of protractor jigs. In the case of the wind vanes, the zero positions with respect to aircraft axes were determined with the aid of a theodolite.

The dynamic calibration of each sensor and its associated electronics was carried out using a compound pendulum with displacement and velocity pick-offs. The galvanometer trace recorder, which is used for recording flight data was used to record the instrumentation calibration data.

Examples of the instrumentation calibrations are shown in figures 1(a) to 1(c).

Data Recording is done by a continuous trace recorder, the signals to which are conditioned by active filters and recorded by the galvanometers in the recorder both before and after filtering. This makes it possible to readily see the amount of 'noise' present in the system.

Operating in parallel with the continuous trace recorder is the College of Aeronautics digital data instrumentation system (CADDIS). In this system the signals from the transducers are converted from analogue to digital form (pulse code modulation) and recorded on a time-sharing basis on the channels of a magnetic tape recorder. For ease of handling, the signals from the transducers are scaled to between 0 and 1 Volt by means of simple attenuators. Also, because the transducers have inductive pick-offs, demodulators are interposed between the pick-offs and a multiplex unit which is responsible for sequencing the time-sharing channels and for the frequency of data sampling. The signals then pass through an analogue digital converter before being stored in digital form on the airborne tape recorder. The master time reference for the sequential pulse generators is provided by a 120 kc/s. crystal clock. The magnetic tape has sixteen channels of which five are available for direct analogue recording and one for speech commentaries. Of the primary channels, one is for a parity signal, one for synchronization and the remaining eight are used for the digital data outputs. For the aircraft longitudinal mode, only four of these channels are used, the quantities recorded being elevator deflection, pitch rate, normal acceleration and incidence. They are sampled 150 times per second. A detailed description of the airborne digital data instrumentation system is contained in reference 1.

Data Processing of the trace records, is done on a Benson-Lehner 'Oscar' trace reader. The trace records are sampled at one-tenth second intervals and scaled and linearised according to the static calibrations of the instrumentation. As a result of this process, the data is reproduced in punched paper tape form ready for analysis.

In the case of the magnetic tape data, the tape from the airborne recorder is replayed on a ground installation in 'real-time' (7½ ins./sec.) so that the start and finish points of each run can be ascertained using the pilot's commentary as a guide. From this 'real-time' replay, a 'quick-look' record is produced from a digital-analogue converter synchronised with the channel selector and feeding a ten channel sample and hold circuit, the outputs of which are fed to a ten channel, galvanometer trace recorder. This record is then used to determine the exact start and stop positions of the part of each run to be analysed. The magnetic tape is then again replayed at a speed reduced by a factor of 32 (15/64 ins./sec.) so as to permit asynchronous operation with the paper tape punch. The need to reduce the magnetic tape speed makes it desirable to use the most efficient paper tape format possible and so the standard binary coded decimal tape form has been abandoned in favour of retaining the data as an 8 bit binary number. The punched paper data tapes are then fed into a calibration subroutine on the 'Pegasus' digital computer. This programme scales the data according to the instrumentation static calibrations and also converts the data from binary form to decimal form ready for analysis. A detailed description of the magnetic tape processing installation is found in reference 2.

## Method of Analysis

The Fourier method of analysis of reference 3 has been used to write a machine code programme for use on the 'Pegasus' digital computer. The basic principle of this method lies in the approximation of a step function by a Fourier series, and in the approximation of a transient by a series of step functions. The frequency-response function for an arbitrary input is then obtained by expressing the arbitrary input and the resultant response each in terms of a series of step functions and hence in the form of trigonometric series, which are readily calculable on the computer. An outline of the method is given in Appendix 1.

A block diagram of the complete recording, calibration and analysis process is shown in figure 3.

## Results and Discussion

A typical galvonometer trace record, taken on the second flight, is shown in figure 4. This trace shows the angle-of-attack, pitch rate and normal acceleration responses to a 'doublet' input of the elevator. Other inputs used were square wave, triangular wave and impulse elevator deflctions. The transient response traces show very little 'noise' effect as a result of a comparison of the filtered and unfiltered signals. The main source of 'noise' on the first two flights was due to engine vibration affecting the galvonometer trace recorder. The trace recorder mounting has since been modified and improved traces were obtained on the third flight.

The amplitude ratios of the frequency-responses of the angle-of-attack, pitch rate and normal acceleration to the 'doublet' elevator input of figure 4 are shown in figures 5(a) to 5(c). Also shown in figure 5 is the spread of results obtained from the responses to the various other elevator inputs. As may be seen from the elevator 'doublet' frequency-responses there are a number of peaks appearing in the curves, some at almost regular intervals of frequency. These peaks are due to the lack of harmonic content of the input at these frequencies and so it is necessary to apply various types of input or inputs of different duration in order to cover these regions of uncertainty and to ensure that no legitimate secondary peak or other significant characteristic exists in that particular range of frequencies. Halving the duration of square wave or triangular wave inputs, for example, doubles the spacing between frequencies of zero harmonic content. An ideal input, of course, would have equal harmonic content at all frequencies, but in practice the duration of the input must be sufficient to ensure accurate measurement of the response and consequently, the input transform will closely approach or reach zero at some frequency in the range considered. A continuous frequency sweep type input has been tried on the third flight in an attempt to excite all frequencies in the range under consideration more evenly, but this data is still being analysed.

The choice of too large a sampling time-interval of the transient is also a cause of scatter of results in the frequency domain. In this case the scatter usually diverges rapidly with increasing frequency due to the characteristics of the Fourier transform and this is not readily apparent from the results of figure 5 as the scatter is fairly uniform over the frequency range considered. These results shown in figure 5 are for a sampling time-interval of 1/10 second whereas the magnetic-tape data, yet to be analysed, is sampled at 1/150 second. Additional data tapes are also being prepared from the same magnetic-tape at sampling rates varying from 1/150 second to approximately 1/20 second so that the effect of the sampling time interval on the frequency-response can be studied.

The end conditions of the trace records should have little effect on the frequency-response function provided that the response has reached a steady value. This is because in the method of analysis, the derivatives of the input and transient response functions have been used in the Fourier transform intervals and hence the values of the intervals outside the time interval of the integration are zero. However, the pilot had experienced some difficulty in returning the aircraft to the trimmed condition at the end of each manoeuvre and so a simple gunsight, with collimating lens to avoid parallex effects was mounted above the instrument panel. This was used on the third flight and not only provided the pilot with an attitude reference with respect to the horizon but also enabled him to assess more accurately the amplitudes of the aircraft responses.

The 'Dove' is a highly damped aircraft longitudinally and from the results of figure 5 it is seen that the natural frequency of the short period mode of the aircraft occurs at a frequency between 2 and 3 rad/sec. Although the data had been analysed up to a frequency of 25 rad/sec., the scatter of these initial results precluded any detection of fuselage structural frequencies.

## Conclusions

A method of obtaining aircraft frequency-response from transient flight data by Fourier analysis has been developed. Transient data for the longitudinal responses of a 'Dove' aircraft have been recorded on both trace recorder and digital magnetic tape systems operating in parallel. Trace records from the second flight have been analysed and some tentative frequency response curves obtained. These curves show a large scatter and future work will be directed towards reducing this. Further development of the computer programme will be necessary before a comparison can be made between these curves and the curves from the magnetic tape record.

## Future work

Future work will be applied principally towards the development of the present Fourier analysis programme, and the associated instrumentation calibration sub-routines, to the point where repeatable longitudinal frequency-response curves can be obtained as an almost completely automated process. The effect of the following variables on the repeatability of frequency-response curves will be studied:

(a) Length of data record.
(b) End conditions of record.
(c) Shape of control input.
(d) Data sampling rate.

The effect of 'noise' on the sampled records will be determined by a comparison of the frequency responses obtained from the filtered and unfiltered trace records.

An analysis of instrument errors will be made to assess whether an instrumentation system with a higher potential accuracy is necessary for consistent results.

When sufficiently developed, the complete process will be applied to obtaining frequency-response functions for the lateral modes of an aircraft. Transfer functions and aerodynamic derivatives, or groups of derivatives, can then be obtained and compared with those obtained by other methods.

The flying programme will be continued initially on the 'Dove' aircraft, but as the process is developed and refined it will be continued using the 'Paris' turbojet aircraft.

## LIST OF SYMBOLS

| | |
|---|---|
| $a(\omega)$ | Real part of $\frac{\text{OUTPUT}}{\text{ARBITRARY INPUT}}(i\omega)$ |
| $(\omega)$ | Phase difference between output and input. |
| $b(\omega)$ | Imaginary part of $\frac{\text{OUTPUT}}{\text{ARBITRARY INPUT}}(i\omega)$ |
| $\Delta t$ | Increment in $x(t)$. |
| $\Delta\theta$ | Increment in $\theta(t)$. |
| $F(i\omega)$ | Frequency spectrum. |
| $h(\omega)$ | Real part of $\frac{\text{OUTPUT}}{\text{STEP}}(i\omega)$ |
| $k(\omega)$ | Imaginary part of $\frac{\text{OUTPUT}}{\text{STEP}}(i\omega)$ |
| $l(\omega)$ | Real part of $\frac{\text{INPUT}}{\text{STEP}}(i\omega)$ |
| $m(i\omega)$ | Imaginary part of $\frac{\text{INPUT}}{\text{STEP}}(i\omega)$ |
| $q$ | Positive even integer. |
| $R(\omega)$ | Amplitude ratio of output to input. |
| $r$ | Positive odd integer. |
| $\theta_i(t)$ | Arbitrary input. |
| $\omega$ | Angular frequency in radians/second. |
| $x(t)$ | Transient response. |

# LIST OF REFERENCES

1. M.A. Perry and R.G. Thory
"An Airborne Data Recording System"
British Communications and Electronics.
September, 1962.

2. M.A. Perry and R.G. Thory
"CADDIS - Digital Data Handling System for Flight Testing"
British Communications and Electronics.
September, 1964.

3. E. Rushton and M.G. Medhurst
"Routine Computing Method for Approximating to the Frequency Response from the Transient Response"
R.A.E. Tech. Note I.A.P. 1083.
July, 1959.

## Appendix 1

## FREQUENCY-RESPONSE FUNCTION FOR AN ARBITRARY INPUT

A complete transient response may be expressed as a series of steps as shown in figure 2 . The frequency-response function is then expressed in series form as :

$$F(i\omega) = Re^{i\alpha} = \Delta x_1 e^{-i\omega\frac{\Delta t}{2}} + \Delta x_3 e^{-i\omega\frac{3\Delta t}{2}} + \; \ldots\ldots \qquad (1)$$

With the frequency expressed in deg./sec. for convenience, the real and imaginary parts of the frequency-response function are :

$$\left.\begin{aligned} R_e &= R\cos\alpha = \sum_{r=1}^{q-1} \Delta x_r \cos\left(\frac{180}{\pi} r\omega\frac{\Delta t}{2}\right), \; r \;\; \text{odd} \\ Im &= R\sin\alpha = -\sum_{r=1}^{q-1} \Delta x_r \sin\left(\frac{180}{\pi} r\omega\frac{\Delta t}{2}\right), \; r \;\; \text{odd} \end{aligned}\right] \qquad (2)$$

The frequency-response function for an arbitrary input may then be found by writing :

$$\begin{aligned} F(i\omega) &= \frac{\text{OUTPUT}}{\text{ARBITRARY INPUT}}(i\omega) \\ &= \frac{\text{OUTPUT}}{\text{STEP INPUT}}(i\omega) \bigg/ \frac{\text{ARBITRARY INPUT}}{\text{STEP INPUT}}(i\omega) \end{aligned} \qquad (3)$$

or, expressed in real and imaginary parts :

$$F(i\omega) = \frac{h(\omega) + i\,k(\omega)}{l(\omega) + i\,m(\omega)} \qquad (4)$$

wherein,

$$\left.\begin{aligned} h(\omega) &= \sum_{r=1}^{q-1} \Delta x_r \cos\left(r \cdot \frac{180}{\pi} \cdot \omega \cdot \frac{\Delta t}{2}\right) \\ k(\omega) &= -\sum_{r=1}^{q-1} \Delta x_r \sin\left(r \cdot \frac{180}{\pi} \cdot \omega \cdot \frac{\Delta t}{2}\right) \\ l(\omega) &= \sum_{r=1}^{q-1} \Delta \theta_r \cos\left(r \cdot \frac{180}{\pi} \cdot \omega \cdot \frac{\Delta t}{2}\right) \\ m(\omega) &= -\sum_{r=1}^{q-1} \Delta \theta_r \sin\left(r \cdot \frac{180}{\pi} \cdot \omega \cdot \frac{\Delta t}{2}\right) \end{aligned}\right] \tag{5}$$

where r is odd.

Rationalising equation (4) and expressing it in real and imaginary parts yields :

$$F(i\omega) = a(\omega) + b(\omega) \tag{6}$$

where,

$$\left.\begin{aligned} a(\omega) &= \frac{hl + km}{l^2 + m^2} \\ b(\omega) &= \frac{kl - hm}{l^2 + m^2} \end{aligned}\right] \tag{7}$$

Then the required amplitude ratio and phase difference of output to input are :

$$\left.\begin{aligned} R(\omega) &= (a^2 + b^2)^{\frac{1}{2}} \\ \alpha(\omega) &= \tan^{-1}\left(\frac{b}{a}\right) \end{aligned}\right] \tag{8}$$

Thus equations (8) represent the frequency-response of a system to an arbitrary input.

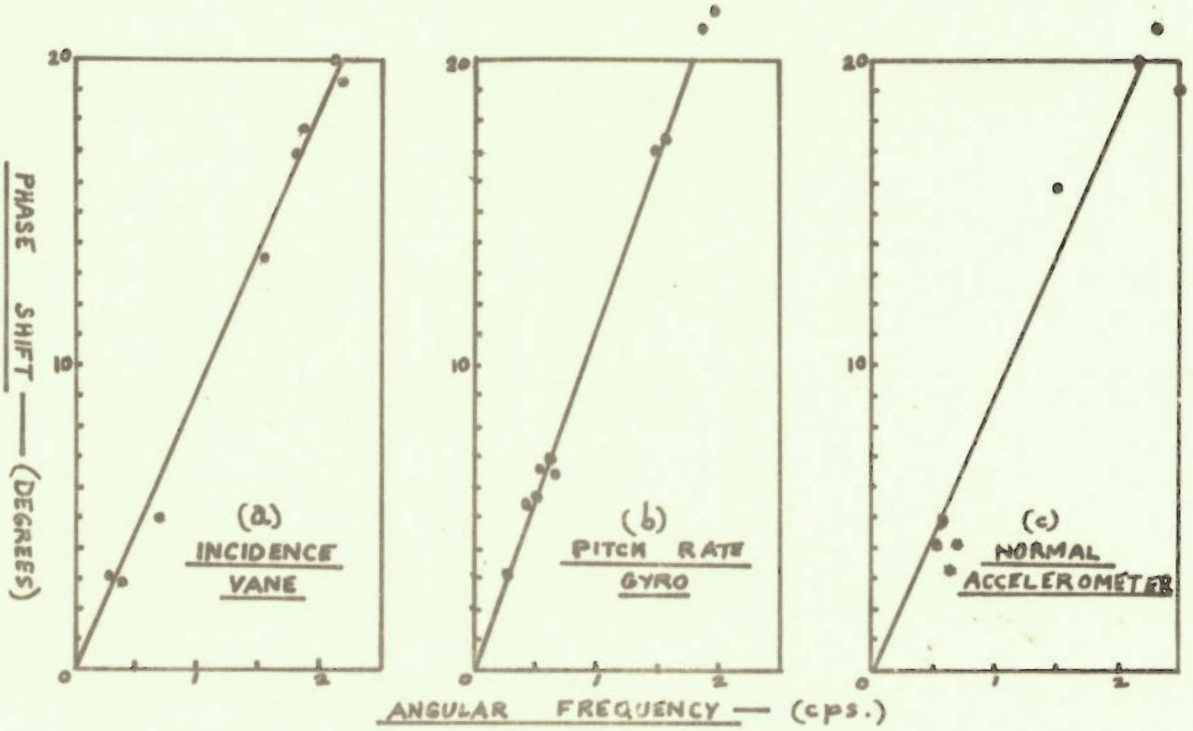

FIG. 1. INSTRUMENTATION CALIBRATIONS.

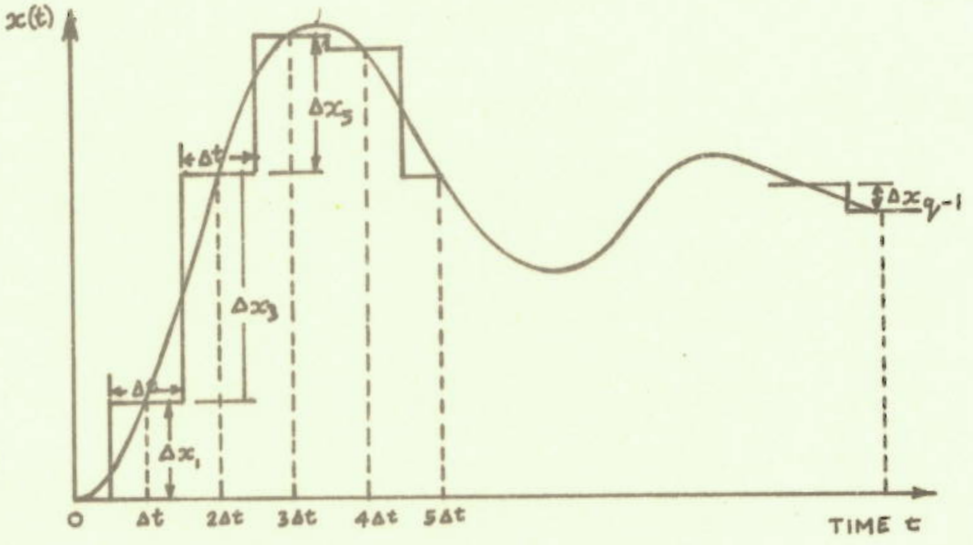

FIG. 2. APPROXIMATION OF TRANSIENT RESPONSE, $x(t)$, BY A SERIES OF INCREMENTAL STEPS.

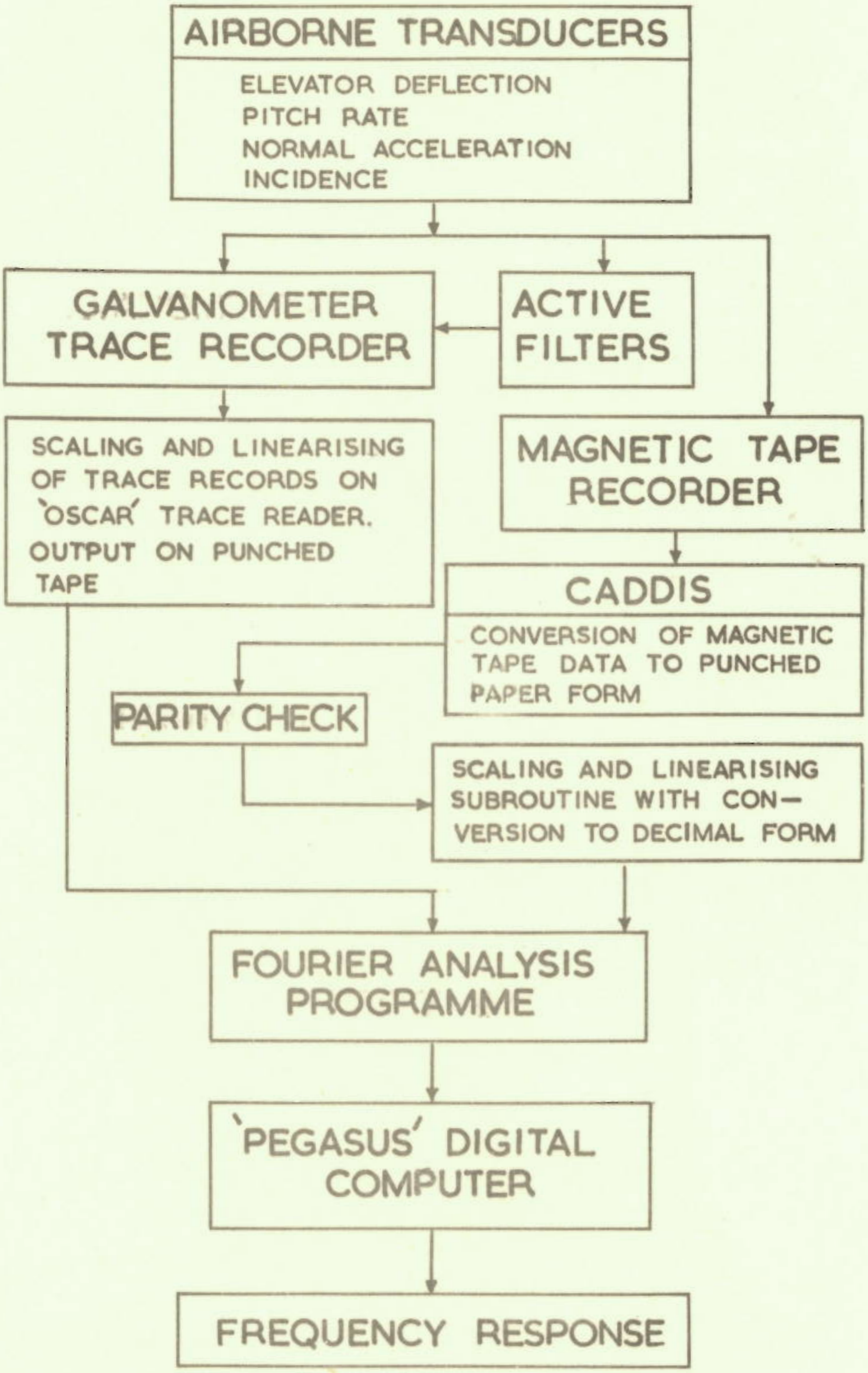

FIG.3. BLOCK DIAGRAM OF DATA RECORDING PROCESSING AND ANALYSIS

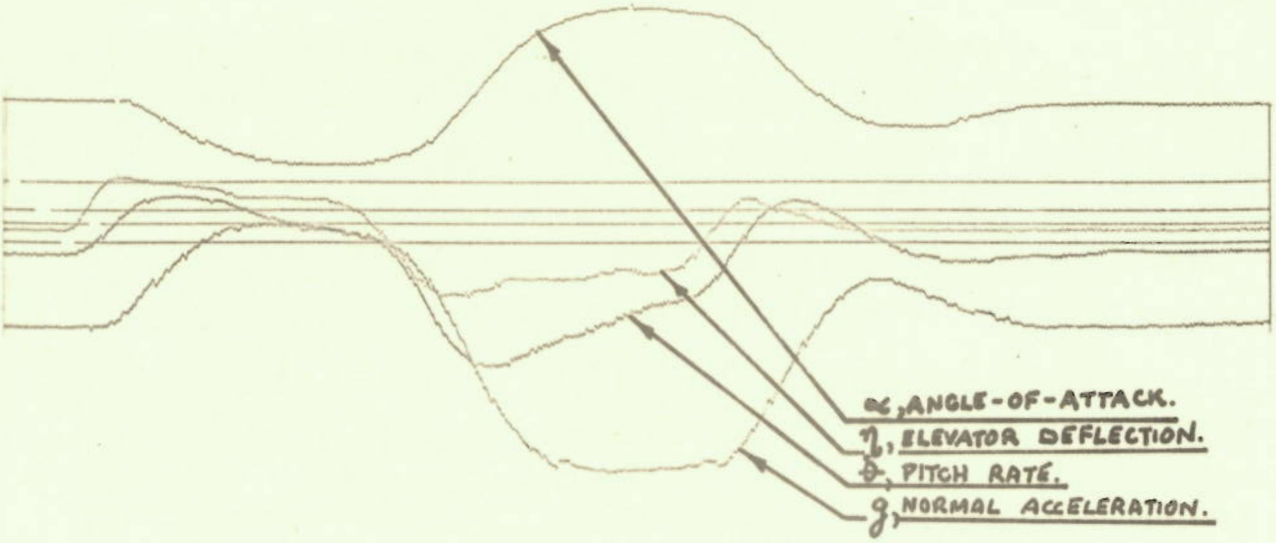

FIG. 4. AIRCRAFT TRANSIENT RESPONSE TO 'DOUBLET' ELEVATOR INPUT. (FLIGHT 2, RUN 8 REF.)

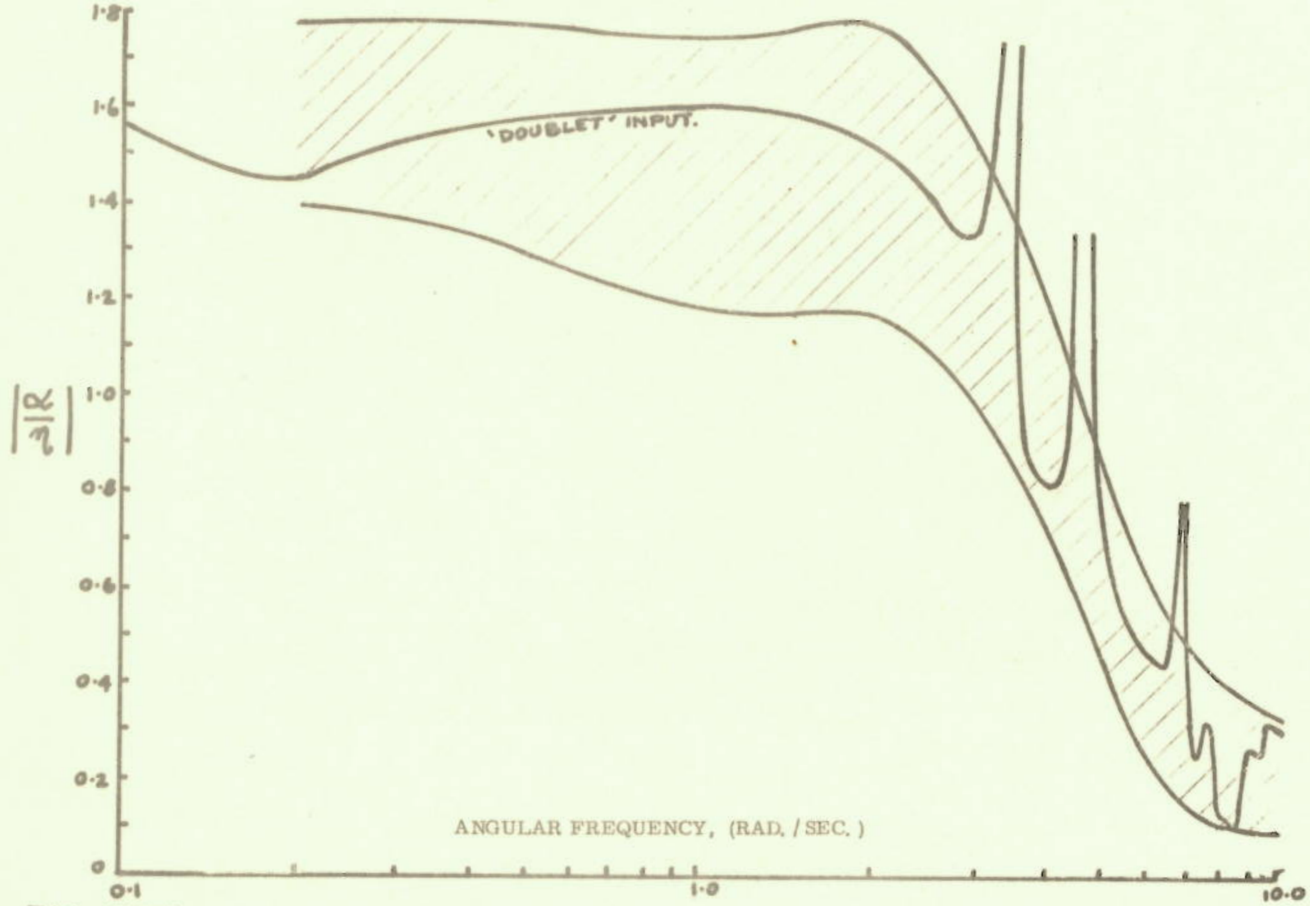

FIG. 5(a).

AMPLITUDE RATIO OF ANGLE OF ATTACK FREQUENCY RESPONSE TO ELEVATOR INPUT

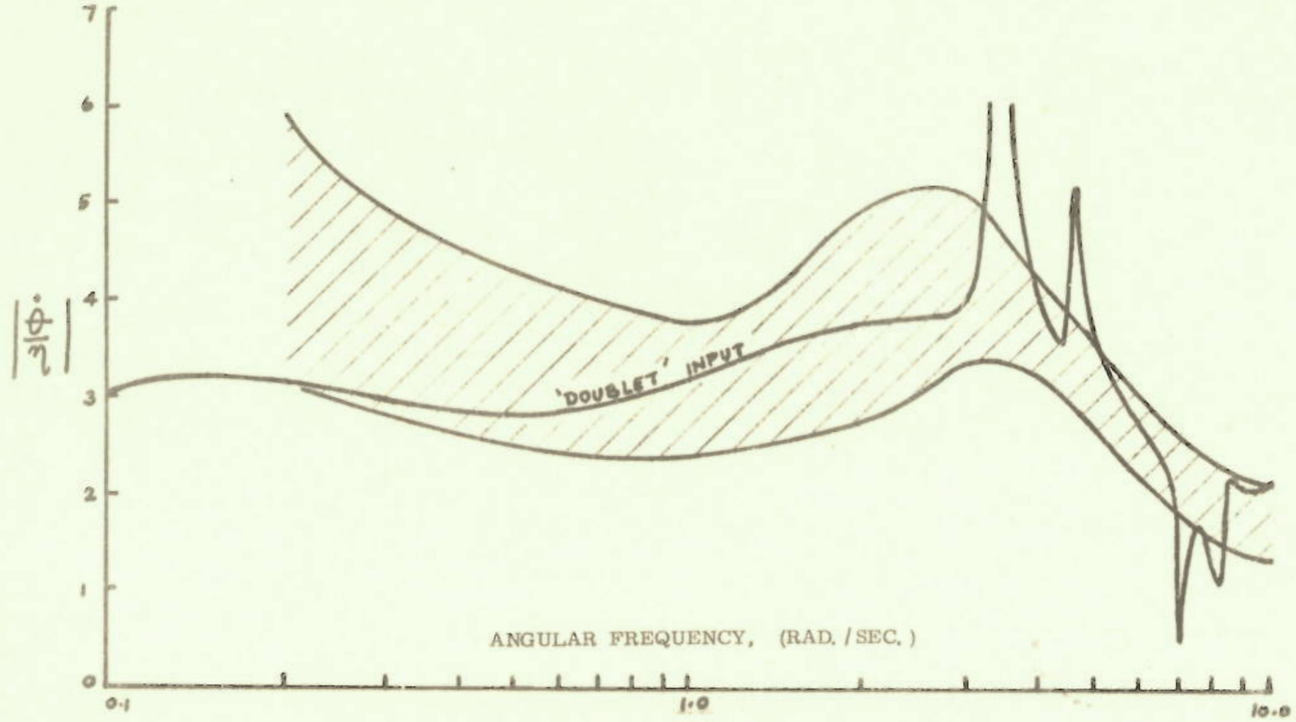

FIG. 5(b) AMPLITUDE RATIO OF PITCH RATE FREQUENCY RESPONSE TO ELEVATOR INPUT

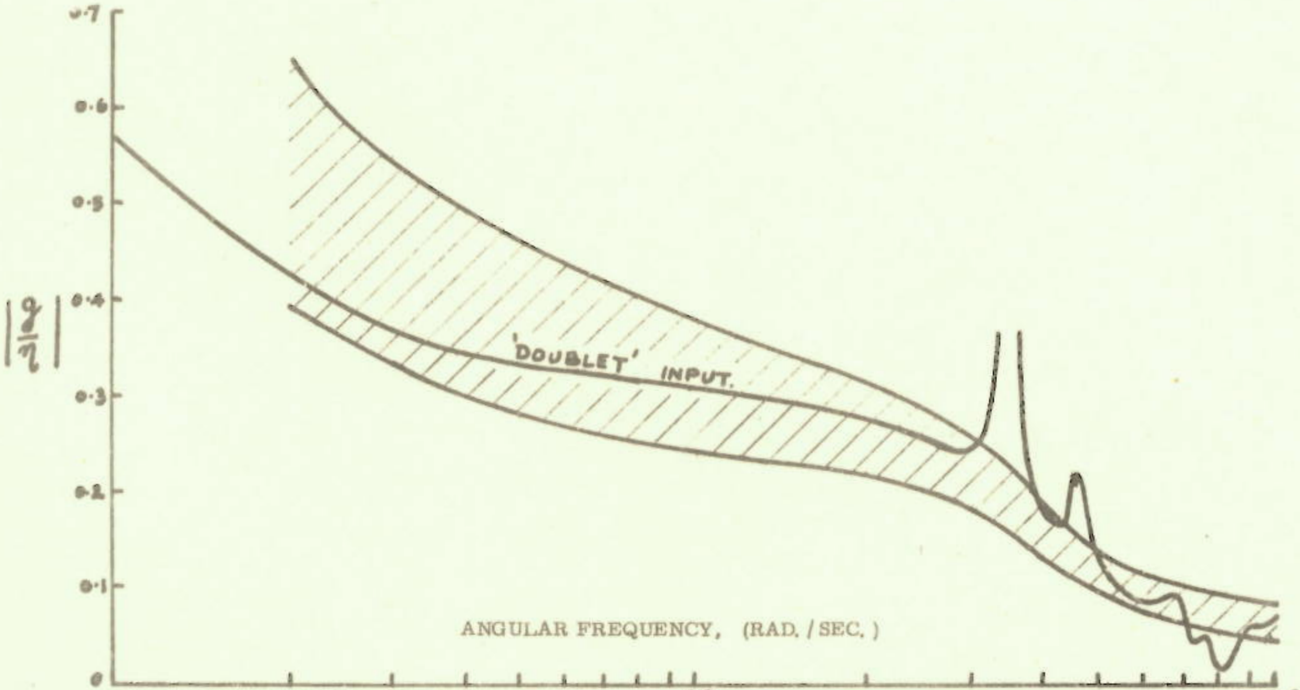

FIG. 5(c). AMPLITUDE RATIO OF NORMAL ACCELERATION FREQUENCY RESPONSE TO ELEVATOR INPUT