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The Busemann correction to the characteristics of the two-dimensional hypersonic sail

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SUMMARY

The two-dimensional hypersonic sail is examined using the Newton-Busemann pressure law. The results are compared with those of Daskin and Feldman (1958) who used the empirical modified Newtonian pressure law. It is found that for a given chord length of sail a corrected sail will give a specified lift for a smaller tension in the sail.

At a flight Mach number of 10 at 100,000 ft. the tension in one particular sail considered could be supported with a working stress of about 20 tons/in^2.

CONTENTS

LIST OF SYMBOLS

B	total length of sail
C	chord of sail
C_D	drag coefficient
C_L	lift coefficient
C_p	pressure coefficient
C_{pmax}	value of C_p behind normal shock
D	drag
L	lift
M_∞	free-stream Mach number
p	pressure
p_o	stagnation pressure behind normal shock
q_∞	dynamic pressure, $\frac{1}{2}\rho_\infty U^2$
s	length along sail
T	tension per unit span
U	free-stream speed
x	longitudinal co-ordinate
y	normal co-ordinate
α	angle of attack
θ	local angle of sail
ρ	density
σ	local sail angle, see Fig. 1b

Subscripts

T	trailing edge
L	leading edge
∞	conditions far upstream of sail

1. Introduction

Daskin and Feldman (1958) investigated the characteristics of the two-dimensional hypersonic sail using the empirical Newtonian pressure law. As Hayes and Probstein (1959) point out, a rational theory of hypersonic flow should include the centrifugal correction of Busemann. Here the two-dimensional hypersonic sail is re-examined using the complete Newtonian pressure law, and the results compared with those of Daskin and Feldman.

The results obtained by including the centrifugal correction for a body of convex curvature are less accurate than those obtained by neglecting the correction. In the theory this is a result of the singularity which occurs when the pressure on the body falls to zero, and, as Freeman (1960) has pointed out, invalidates the assumption of the closeness of the shock wave to the body. However the present problem deals with a body of concave curvature (lower surface of the sail) and in this case the full centrifugal correction should apply.

2. The sail equation

The tw-dimensional hypersonic sail is assumed to be in a Newtonian flow. Accordingly we assume that a thin shock layer coincides with the surface of the sail and that there is no friction between the layer and the sail.

Fluid which enters the shock layer is assumed to flow along streamlines with its velocity unchanged downstream of the shock. Because of the curvature of the shock layer the pressure on the under-surface of the sail, concave to the oncoming stream, is increased by a pressure difference across the layer due to the centrifugal effect. The pressure on the under-surface of the sail is then given by the Newton-Busemann pressure law, (see Figs. 1a, 1b for notation),

$$\frac{C_p}{C_{pmax}} = \sin^2\theta + \sin\theta \, \frac{d\theta(y)}{dy} \int_{\theta_L}^{\theta(y)} \cos\sigma(y') \, dy' . \quad (1)$$

In the limit of very high Mach number we may assume that the pressure on the reverse side of the sail is zero. The pressure on the front is given by equation (1), where the left hand side takes the simpler form p/p_o, thus

$$\frac{p}{p_o} = \sin^2\theta + \sin\theta \, \frac{d\theta}{dy} \int_{\theta_L}^{\theta} \cos\sigma \, dy' . \quad (2)$$

Neglecting the mass of the sail, each element of the sail is in equilibrium under the forces due to the pressure difference across the sail and the two-dimensional tension in the sail. Thus

$$p \cos\theta \; ds = T \, d(\sin\theta)$$

or

$$p = T \frac{d\theta}{ds} \tag{3}$$

Substituting (2) in (3) gives the basic equilibrium equation of the sail

$$\sin^2\theta + \sin\theta \frac{d\theta}{dy} \int_{\theta_L}^{\theta} \cos\sigma \; dy' = \frac{T}{p_o} \frac{d\theta}{ds} . \tag{4}$$

which may be written

$$\sin^2\theta + \frac{d\theta}{ds} \int_{\theta_L}^{\theta} \sin\sigma \cos\sigma \; ds' = \frac{T}{p_o} \frac{d\theta}{ds} \tag{5}$$

or

$$\frac{ds}{d\theta} \sin^2\theta + \int_{\theta_L}^{\theta} \sin\sigma \cos\sigma \frac{ds'}{d\sigma} \, d\sigma = \frac{T}{p_o} . \tag{6}$$

3. Sail geometry

Put in (6)

$$\phi(\theta) = \frac{ds}{d\theta} \sin^2\theta . \tag{7}$$

$$\phi(\theta) + \int_{\theta_L}^{\theta} \cot\sigma \; \phi(\sigma) \, d\sigma = \frac{T}{p_o} . \tag{8}$$

Differentiating (8) with respect to θ yields

$$\phi'(\theta) + \cot\theta \; \phi(\theta) = 0 \tag{9}$$

which has the solution

$$\phi(\theta) = A \operatorname{cosec}\theta \tag{10}$$

where A is arbitrary constant.

But, at $\theta = \theta_L$, $\phi(\theta_L) = {}^T/p_o$ from equation (8), so that

$$A = \frac{T}{p_o} \sin \theta_L \tag{11}$$

From (7), (10), and (11)

$$\frac{ds}{d\theta} = \frac{T}{p_o} \cdot \frac{\sin \theta_L}{\sin^3 \theta} \tag{12}$$

The arc length, s, of the sail measured from the leading edge is given by

$$\frac{p_o s}{T} = \sin \theta_L \int_{\theta_L}^{\theta} \frac{d\theta}{\sin^3 \theta}$$

$$\frac{p_o s}{T} = \frac{1}{2} \left(\cot \theta_L - \cot \theta \frac{\sin \theta_L}{\sin \theta} + \sin \theta_L \ln \frac{\tan \frac{1}{2}\theta}{\tan \frac{1}{2}\theta_L}\right) \tag{13}$$

The total length of the sail, B, is

$$\frac{p_o B}{T} = \frac{1}{2} \left(\cot \theta_L - \cot \theta_T \frac{\sin \theta_L}{\sin \theta_T} + \sin \theta_L \ln \frac{\tan \frac{1}{2}\theta_T}{\tan \frac{1}{2}\theta_L}\right) \tag{14}$$

where θ_T is the trailing-edge angle.

The parametric equations of the sail follow simply.

$$\frac{dx}{d\theta} = \cos\theta \frac{ds}{d\theta} = \frac{T}{p_o} \sin \theta_L \cdot \frac{\cos\theta}{\sin^3\theta} \text{ , from (14).}$$

$$\frac{p_o x}{T} = \frac{1}{2} \left(\frac{1}{\sin\theta_L} - \frac{1}{\sin\theta} \cdot \frac{\sin\theta_L}{\sin\theta}\right) \tag{15}$$

$$\frac{dy}{d\theta} = \sin\theta \frac{ds}{d\theta} = \frac{T}{p_o} \frac{\sin\theta_L}{\sin^2\theta} \text{ , from (14).}$$

$$\frac{p_o y}{T} = \cos\theta_L - \cos\theta \frac{\sin\theta_L}{\sin\theta} \qquad (16)$$

Eliminating θ between (15) and (16) gives for the equation of the sail

$$- x \sin\theta_L = \frac{1}{2} \frac{p_o y^2}{T} - y \cos\theta_L \, . \qquad (17)$$

The shape of the sail is a parabolic cylinder with the vertex of the parabola pointed downstream, as was noted by Hayes and Probstein (1959).

The co-ordinates of the trailing-edge are

$$\frac{p_o x_T}{T} = \frac{1}{2} \left(\frac{1}{\sin\theta_L} - \frac{1}{\sin\theta_T} \frac{\sin\theta_L}{\sin\theta_T} \right) \qquad (18)$$

$$\frac{p_o y_T}{T} = \cos\theta_L - \cos\theta_T \frac{\sin\theta_L}{\sin\theta_T} \qquad (19)$$

and the chord, c, of the sail is

$$\frac{p_o c}{T} = \sqrt{\left(\frac{p_o x_T}{T}\right)^2 + \left(\frac{p_o y_T}{T}\right)^2} \qquad (20)$$

while the geometrical angle of attack, α , is given by

$$\alpha = \tan^{-1} \left[\frac{p_o y_T}{T} \Big/ \frac{p_o x_T}{T} \right] . \qquad (21)$$

4. Aerodynamic characteristics

The tension in the sail is uniform so that the simplest method of finding the total lift and drag is to consider only the attaching forces and their angles at the leading and trailing edges. Then it follows that

$$L = T(\sin\theta_T - \sin\theta_L) . \tag{22}$$

$$D = T(\cos\theta_L - \cos\theta_T) . \tag{23}$$

and the corresponding coefficients are

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U^2 c} = \frac{p_o}{q_\infty} \frac{(\sin\theta_T - \sin\theta_L)}{p_o c/T} \tag{24}$$

$$C_D = \frac{D}{\frac{1}{2}\rho_\infty U^2 c} = \frac{p_o}{q_\infty} \frac{(\cos\theta_L - \cos\theta_T)}{p_o c/T} \tag{25}$$

and

$$\frac{C_L}{C_D} = \frac{L}{D} = \frac{\sin\theta_T - \sin\theta_L}{\cos\theta_L - \cos\theta_T} \tag{26}$$

$$= \cot\tfrac{1}{2}(\theta_L + \theta_T) .$$

The lift-drag ratio is given by the same result for both the corrected and uncorrected sail. Clearly lift-drag ratios in excess of unity are achieved if $\theta_L + \theta_T < 90^\circ$; in other words with tight sails with small leading edge angles.

For infinite Mach number, and a specific heat ratio of 1.4, $p_o/q_\infty = 1.839$. For Mach numbers above 4, p_o/q_∞ rapidly approaches this limiting value. See Daskin and Feldman (1958), Table 1.

5. Results

In Fig. 2 the sail shape given by the complete pressure law is compared with that found by Daskin and Feldman (1958) using the uncorrected Newtonian pressure law. The corrected sail is much shorter and tighter than the uncorrected one when both are used at the same tension.

Fig. 3 compares the aerodynamic characteristics of the two sails with $\theta_L = 10^o$. For a given trailing-edge angle, θ_T, the corrected sail offers a higher lift and drag for a smaller incidence.

A better comparison of the performance of the two sails is got by requiring a given lift from a given chord length of sail. The two sails will not work at the same incidence, for at a given incidence the corrected sail always gives a higher lift, Fig. 4. For a given lift the corrected sail will work at a lower incidence. Above a certain value of lift it is necessary to increase the leading-edge angle, θ_L.

When the lifts given by a corrected and an uncorrected sail are equal, Fig. 6 shows that the value of $p_o c/T$ is greater for the corrected sail. In other words, if the chord, c, is the same for both sails, there is a smaller tension in the corrected sail. When the values of $p_o c/T$ in Fig. 6 are greater for an uncorrected sail than for a corrected one, it will be found in Fig. 4 that the uncorrected sail is no longer able to yield as much lift as the corrected sail.

Fig. 5 shows the increase of drag with incidence. Similar conclusions to those in the last paragraph can be drawn from the drag results.

To estimate the magnitude of the tension in one of these sails consider a sail with $\theta_L = 10^o$, $\theta_T = 20^o$, at a flight Mach number of 10 at 100,000 ft. The sail loading is found to be 240 lb/ft^2. The tension per foot span in a sail of 10 ft. chord, would be 14,500 lb. A sail 0.05 in. thick of woven high tensile steel wire could support such a tension, with a stress of about 20 tons/in^2.

6. Conclusions

A corrected sail is much shorter and tighter than an uncorrected one, when there is the same tension in the sails. For any given incidence there is a corrected sail which will give a higher lift and drag than an uncorrected one. For given leading-edge and trailing-edge angles the corrected sail gives higher lift and drag at a smaller incidence. For a fixed chord length of sail yielding a specified lift there will be less tension in a corrected sail.

Flight at a Mach number of 10 at 100,000 ft. would require a sail ($\theta_L = 10^o$, $\theta_T = 20^o$) 0.05 in. thick of woven high tensile steel wire, working at a stress of about 20 tons/in^2.

7. References

1. Daskin, W., Feldman, L. — Journal Aero. Sciences, Vol. 25, 1958, pp 53 - 55.
2. Freeman, N. — Journal of Fluid Mechanics, Vol. 8, 1960, pp 109.
3. Hayes, W.D., Probstein, R.F. — Hypersonic flow theory, Academic Press, 1959.

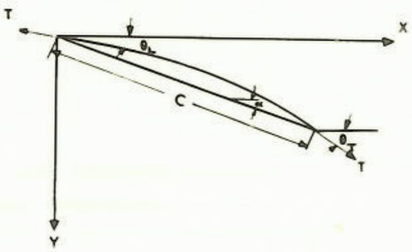

FIG Ia SAIL NOMENCLATURE

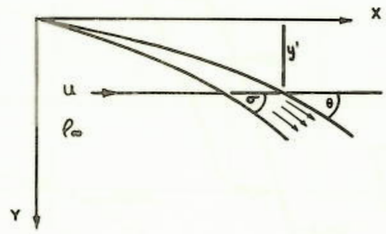

FIG. Ib. SHOCK LAYER.

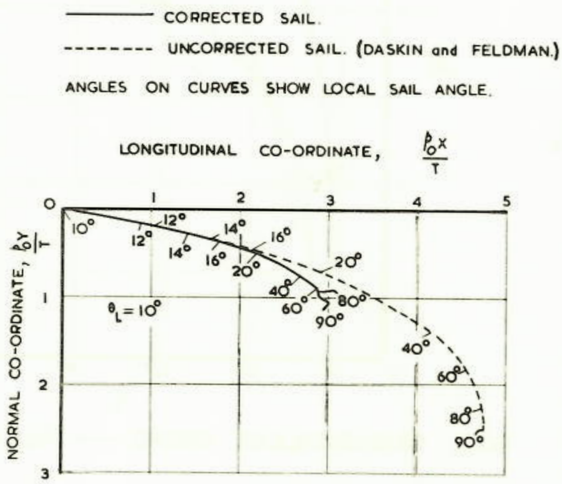

FIG.2. SAIL SHAPE.

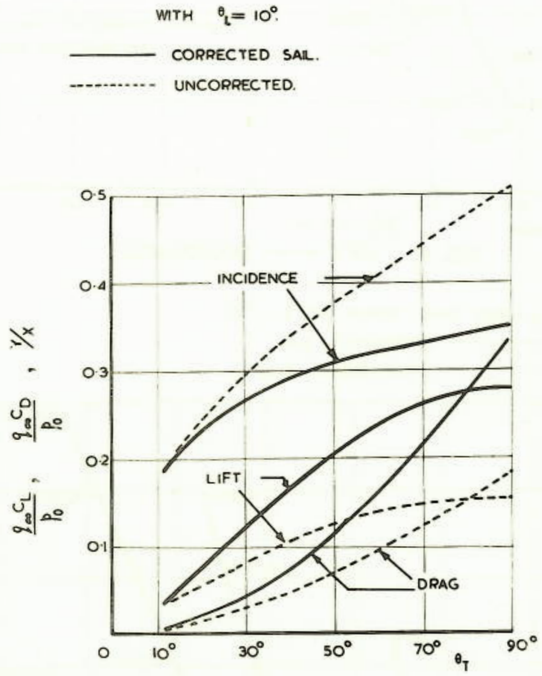

FIG. 3. AERODYNAMIC CHARACTERISTICS OF HYPERSONIC SAILS.

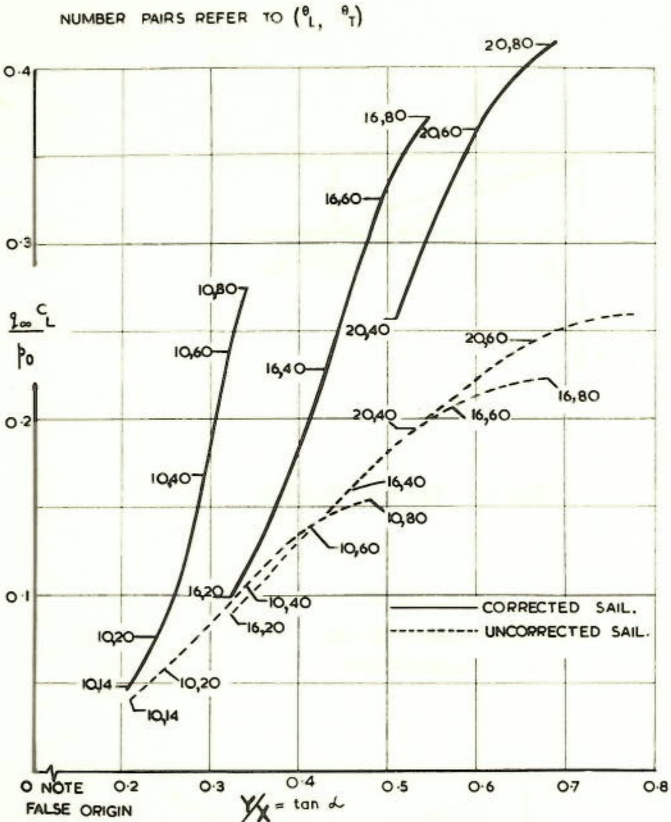

FIG. 4. LIFT —— INCIDENCE

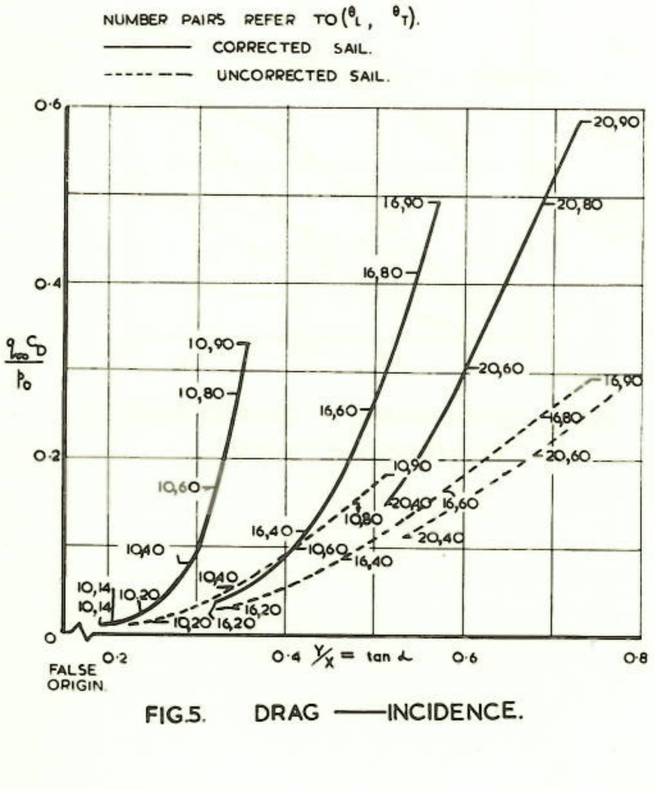

FIG.5. DRAG ——INCIDENCE.

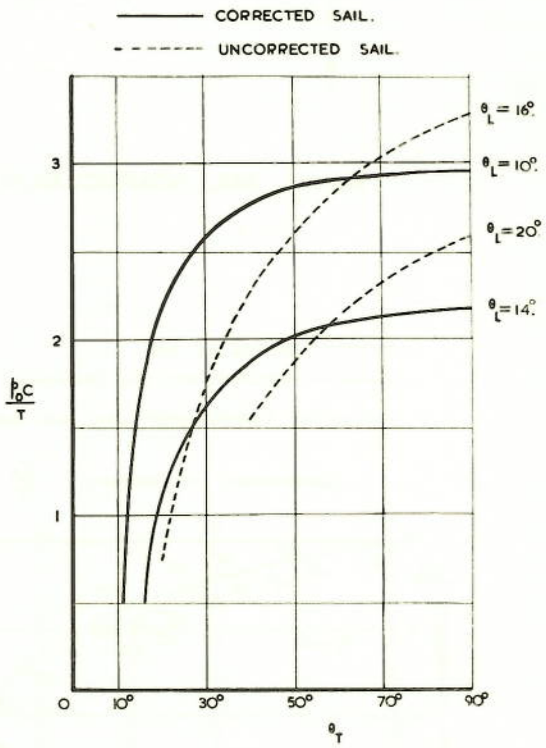

FIG.6. DIMENSIONLESS CHORD ——TAIL ANGLE.