REPORT NO. 144

December, 1960.

THE COLLEGE OF AERONAUTICS

CRANFIELD

# Base Pressure at Supersonic Speeds in the Presence of a Supersonic Jet

- by -

Flt. Lt. A. H. Craven, M.Sc., Ph.D., D.C.Ae., R.A.F.
(Royal Air Force Technical College, Henlow)

D. H. Chester, D.C.Ae.,

and

Lt. B. H. Graham, B.S., D.C.Ae., U.S.N.

## SUMMARY

The effects on base pressure of jet Mach number, free stream Reynolds number and jet to base diameter ratio have been investigated experimentally.

It was found that, for jet stagnation pressures greater than that required for the nozzle to reach its design Mach number, an increase of jet Mach number reduced the base pressure. Similarly the base pressure increased with increase of the ratio of jet diameter to base diameter and, at nigh jet stagnation pressures, base pressures higher than free stream static pressure were found. The base pressure was independent of free stream Reynolds numbers greater than $2 \times 10^6$ per foot but increased with reduction of Reynolds number below $2 \times 10^6$ per foot.

Unsteady wave patterns were found when the jet Mach number did not differ markedly from the free stream Mach number and the jet had just reached its design conditions.

---

* Part of this work was submitted by D.H.C. and B.H.G. in partial fulfilment of the requirements for the Diploma of the College of Aeronautics.

## LIST OF SYMBOLS

| | |
|---|---|
| $c_{p_B}$ | base pressure coefficient |
| $d$ | radius of control element |
| $d_B$ | diameter of base |
| $d_J$ | jet diameter |
| $l$ | distance of the plane of the velocity traverse from the base |
| $M$ | Mach number |
| $M_J$ | jet Mach number |
| $p$ | static pressure |
| $p_B$ | base pressure |
| $P_1$ | stagnation pressure of free stream |
| $P_J$ | jet stagnation pressure |
| $r$ | radial distance from the jet centre line |
| $R_B$ | base radius |
| $R_J$ | jet radius |
| $u$ | streamwise velocity |
| $u_J$ | velocity at jet exit |
| $v$ | velocity normal to the free stream direction |
| $x$ | distance downstream of the base |
| $\rho$ | density |
| $\rho_J$ | density at the jet exit |

Suffices

| | |
|---|---|
| $_1$ | free stream in plane of the base |
| $_2$ | in the plane of the velocity traverse |

CONTENTS

## 1. Introduction

The problem of predicting the base pressure on a body of revolution from which a jet issues is in principle possible using an extended form of the analogous two-dimensional method developed by Cortright (1) which in two dimensions shows reasonable agreement with experimental evidence. Reid and Hastings (2) point out that such a treatment of the axisymmetric problem would be extremely laborious as the shapes of the streamlines enclosing the flow in the vicinity of the base have to be calculated by iterative methods using axisymmetric characteristics. When one resorts to experiment to determine base pressure we find, following Reid and Hastings, that six variables are important. These are the jet design Mach number $M_J$ and free stream Mach number $M_1$, the jet divergence angle and the shape of the afterbody upstream of the base, the ratio of jet diameter to base diameter $d_J/d_B$ and the ratio of the jet and free stream stagnation pressures $P_J/P_1$.

In their experiments Reid and Hastings use a cylindrical afterbody in the centre of which is one of a series of conical nozzles designed to give a jet with design Mach number 2.0. The base diameter is kept constant and the jet diameter and nozzle divergence angle are varied. The free stream Mach number and Reynolds number were respectively 2.0 and $32.4 \times 10^5$ per foot throughout. The effect of jet stagnation pressure ratio on base pressure is found to fall into three phases. As $P_J/P_1$ is increased $p_B/p_1$ first rises and then falls rapidly until it is considerably less than the value corresponding to a sealed base. Thereafter $p_B/p_1$ increases steadily with $P_J/P_1$. These three phases are shown to correspond to fundamental changes of flow pattern in the base region. Increasing the ratio of jet diameter to free stream diameter in the case when jet and free stream Mach numbers are the same

(i) increases the maximum base pressure at the end of the first phase and increases the jet stagnation pressure ratio where the maximum occurs

(ii) decreases the value of the stagnation pressure ratio at which the minimum base pressure occurs. The minimum base pressure is independent of $P_J/P_1$.

In the third phase at a given value of $P_J/P_1$, $p_B/p_1$ increases with increase of $d_J/d_B$, increases markedly when the jet design Mach number is reduced from 2.0 to 1.0, and increases very slightly with increase of nozzle divergence angle.

One section of reference 2 is devoted to a method of correlating data on annular base pressures. It is expected that when the ratio of $p_B/p_1$ for the annular base to $p_B/p_1$ for the analogous two-dimensional case is plotted against $d_J/d_b$, all the points would lie on or close to a unique curve. If this were the case the base pressure in axisymmetric flow would be immediately determined from the two-dimensional base pressure in similar conditions. The method correlates the results of reference 2 reasonably well but is less successful when the results of other workers (3,4) are included.

Bromm and O'Donnell (3) have investigated the effect on the base pressure of nozzle divergence angle and a limited range of jet design Mach number. They conclude that at high jet pressure ratios the base pressure tends to become independent of jet pressure ratio and that nozzle divergence angle has, in general, a larger effect than jet Mach number. Increasing the free stream Mach number increases the base pressure for a given jet pressure ratio. Baughman and Kochendorfer (4) have investigated the effects of a jet issuing from conical afterbodies at two free stream Mach numbers.

The experiments described in this paper were designed to extend the base pressure studies reported in reference 8 to supersonic speeds, to supplement the results of Reid and Hastings, and in particular to investigate

(i) the effect of differences between the free stream and jet design Mach numbers upon the base pressure of a cylindrical body for which $d_J/d_B$ is small.

(ii) the effect of variation of free stream Reynolds number

(iii) the effect of variations of jet design Mach number and the ratio of jet diameter to base diameter on the base pressure on a modified conical boat-tail.

A momentum analysis is used to relate the base pressure to the velocity distribution in the wake and the jet and stream conditions.

## 2. The Wind Tunnel, Models and Instrumentation

### 2.1. The wind tunnel

The experiments were performed in the College of Aeronautics 9 in. x 9 in. supersonic tunnel. The stagnation pressure in the tunnel can be adjusted between 3 in. and 30 in. of mercury and controlled accurately. Such adjustment allowed a variation of free stream Reynolds number between $3.5 \times 10^5$ and $40 \times 10^5$ per foot. The square working section was formed by a pair of symmetrical liners designed to give a flow of Mach number $M_1 = 2.0$ in the working section.

The model was supported on a pipe of 2 inches diameter, which also supplied the compressed air jet. It passed from the settling chamber of the tunnel through the throat into the working section. This pipe also carried the pressure tubes from the model to the manometer bank. The reduction of the original throat area due to the presence of the pipe increased the working section Mach number measured at the position of the base from 2.02 to 2.05.

Dry compressed air was available for the jet with a maximum stagnation pressure of 80 lb. per sq. in. gauge.

Free stream static pressure was measured at a tapping in the tunnel wall immediately opposite the base of the cylindrical afterbody or alternatively just upstream of the shoulder of the conical afterbody.

### 2.2. The models

In the experiments to determine the effect of Reynolds number and jet design Mach number on the base pressure of a cylindrical afterbody, for which $d_J/d_B$ is small, a single model with interchangeable nozzles was used. This consisted of a right cylinder two inches in diameter attached to the supply pipe by an airtight screw coupling, which allowed the model to be rotated about its centre line. The inside of the model was bored out to the same diameter as the supply pipe to form a plenum chamber for the jet, which issued from the centre of the base through a convergent-divergent nozzle. The base had six pressure tappings at equal spacing along a radius. The nozzles, which had a common exit diameter ($d_J$ = 0.375"), were in the form of internally contoured plugs, with common external dimensions, which fitted into the hollow interior of the model (Fig. 1). The internal shaping of the nozzles was roughed out on a lathe and finished by hand using a tool shaped to the required contour.

The ordinates for the nozzle were based on calculations by Clippinger (5). All the nozzles were designed to give uniform parallel flow in the exit plane at their design Mach numbers.

In the tests to determine the effect of changing the ratio of jet to base diameter, four models were used; one for each of the four jet Mach numbers, the nozzle being integral with the model. Initially the body was cylindrical and 2 inches in diameter with six pressure tappings along a base radius. To change the jet to base diameter ratio, a conical ($9^o$) boat-tail was cut on the model which ended in a short parallel section of the required diameter. As the base diameter was reduced so, of course, was the number of base pressure tappings until in the last tests, for $d_J/d_B = 0.8$ only one 0.5 mm. hypodermic tube could be accommodated in the base (Fig. 2). The body was connected to the supply pipe and the nozzles were shaped internally in the manner described in the previous paragraph.

## 2.3. Instrumentation

In all the tests the tunnel stagnation pressure, the jet and tunnel static pressures, and the lower jet stagnation pressures were measured on mercury manometers. The higher jet stagnation pressures were measured on a Bourdon gauge. The base pressures were measured on butyl phthalate manometers using the tunnel static pressure as the reference pressure.

Traversing gear employing separate pitot and static tubes was used. The outside and inside diameters of both tubes were respectively 1 mm. and 0.5 mm. The static tube, following the geometry recommended by Holder, North and Chinneck (6), had a conical nose and four $\frac{1}{4}$ mm. holes evenly spaced circumferentially 8 mm. aft of the nose.

Flow visualisation was by a schlieren system. Photographs of shock patterns were taken using continuous lighting and sparks of 1/5 microsecond duration derived from a barium titanate capacitor with a stainless steel electrode.

## 3. The scope of the tests

All the tests were performed with a free stream Mach number of 2.05.

(i) The effects of jet Mach number were investigated on a cylindrical afterbody for the case $d_J/d_B = 0.1875$. The jet design Mach numbers were 1.0, 1.40, 2.06, 2.51, 2.98, 3.50 and 4.00. The tests were performed at a free stream Reynolds number of approximately $34 \times 10^5$ per foot. The boundary layer on the afterbody was turbulent.

(ii) The effect of changing the free stream Reynolds number in the range $3.5 \times 10^5$ to $40 \times 10^5$ per foot was investigated on the cylindrical base for which $d_J/d_B = 0.1875$. The effect of changing the jet Mach number was also investigated to a limited extent, nozzles giving $M_J = 1.0$, 2.06, 2.98 and 4.00 being used.

(iii) The effect of changing the jet to base diameter was investigated on a conical afterbody with a short parallel end section. The ratio $d_J/d_B$ was varied in the stages 0.3, 0.4, 0.6 and 0.8. Four jet Mach numbers $M_J = 1.0$, 2.02, 3.00 and 3.97 were used. The free stream Reynolds numbers were $24 \times 10^5$ and $36 \times 10^5$ per foot.

(iv) The flow in the mixing region was examined in all the experiments by optical methods. In the experiments of the first paragraph above some velocity traverses were made in various planes downstream of the base.

## 4. Test procedure

### 4.1. Preliminary tests

It was realised at the outset that considerable non-uniformities in the flow could result from the presence of a circular pipe along the centre line of a two-dimensional convergent-divergent nozzle. Some preliminary tests were therefore performed to assess the degree of non-uniformity.

Firstly, on a reduced scale, a cylindrical tube was passed through the throat and working section of a 2" and 2" induced flow tunnel running at a Mach number 2.0. No additional shocks were visible and the variation of static pressure along the working section was not altered.

In the 9" x 9" tunnel the model was situated so that the base was approximately one inch downstream of the forward limit of the constant static pressure region of the working section. Again the wall static pressure varied only in the region where it was affected by the expansion from the base. No additional shocks appeared in the working section upstream of the base. With no jet issuing, the base pressure was measured at various tunnel stagnation pressures to cover the range of Reynolds numbers of the tests. The base pressure was independent of radial and azimuthal position and agreed well with the results obtained by Chapman (7) (Fig. 3). It was concluded that the degree of non-uniformity was negligibly small. Subsequent velocity traverses confirmed this conclusion.

## 4.2. Base pressure measurements

The tunnel stagnation pressure was set at the value corresponding to the desired free stream Reynolds number. Once set this pressure was maintained automatically. Measurements of base pressure, tunnel and jet static pressures were made with no jet flow and at a series of increasing values of the jet stagnation pressure up to 80 lb/sq.in. The jet stagnation pressure was then reduced in steps to zero, the manometers again being read at each step. The procedure was repeated at another value of tunnel stagnation pressure and/or with another jet nozzle.

# 5. Results

## 5.1. The effect of jet design Mach number $M_J$ on the base pressure

On the base of a cylindrical afterbody, the radius of which is large compared with the radius of the jet ($d_J/d_B$ = 0.1875) the effect of increasing the jet design Mach number $M_J$ is to increase the base pressure for a given ratio of jet stagnation pressure to tunnel stagnation pressure $P_J/P_1$ (Fig. 4) for $P_J/P_1$ less than the value giving a minimum value of $p_B/p_1$. Above this value an increase of $M_J$ causes a decrease of $p_B/p_1$. The variation with $M_J$ of the maximum base pressure at the end of the base bleed phase is very small. The maximum remains constant for values of $M_J$ not greater than $M_1$ (i.e. $M_J$ = 1.0, 1.4 and 2.0) but decreases slightly with increase of $M_J$ above 2.0. The value of $P_J/P_1$ at which this maximum occurs is independent of jet design Mach number. The value of $P_J/P_1$ giving a minimum base pressure increases with increase of jet design Mach number. The value of the minimum base pressure is approximately independent of jet design Mach number for $M_J > 2.0$ and decreases very slightly as $M_J$ changes from 1.0 to 2.0

In the case of the conical afterbody the same general trends are found (Fig. 5). The maximum base pressure and the values of $P_J/P_1$ are independent of jet design Mach number. The minimum base pressure increases slightly with $M_J$.

There is no significant radial variation of base pressure in any of the tests.

5.2. <u>The effect of free stream Reynolds number</u>

The curves of $p_B/p_1$ against $P_J/P_1$ for a given jet design Mach number and ratio of jet to base diameter are independent of free stream Reynolds number provided this is greater than $20 \times 10^5$ per foot (Fig. 7). For the cylindrical body ($d_J/d_B = 0.1875$) and $M_J = 1.0, 2.0, 3.0$ a reduction of free stream Reynolds number below $20 \times 10^5$ causes an increase of $p_B/p_1$ for a given value of $P_J/P_1$. In the case of $M_J = 4.0$, $p_B/p_1$ is at first increased by a decrease of free stream Reynolds number but for $R = 3.5 \times 10^5$ and $6.2 \times 10^5$ the base pressure is reduced below the value for R greater than $20 \times 10^5$. It is noted that the base pressure is sensibly independent of Reynolds number for low values of R.

5.3. <u>The effect of changing the ratio of jet diameter to base diameter (Fig. 6)</u>

An increase of the jet to base diameter ratio increases the base pressure for any jet stagnation pressure ratio and given jet design Mach number. There is a slight variation of this general result in the case of $M_J = 4.0$ and $d_J/d_B = 0.8$ (Fig. 6d). At high jet stagnation pressures the base pressure becomes greater than the free stream static pressure; a condition giving a base thrust instead of the more usual base drag.

## 6. Discussion

### 6.1. Accuracy of the results

The base pressure and tunnel and jet static pressures could be measured to within 0.02" of butyl phthalate and mercury respectively. At the higher Reynolds numbers (i.e. tunnel static pressure greater than $1\frac{1}{2}$ inches of mercury) this accuracy of reading corresponds to a 3 per cent possible error in $p_B/p_1$. At the low Reynolds number this can increase to 10 per cent.

The tunnel and jet stagnation pressures could be held constant while base pressure readings were being taken, and the error in reading was 0.02 in. of mercury, or at the higher jet stagnation pressure 0.1 lb/sq.in. (i.e. 0.2" Hg.). The overall error is therefore expected to be no greater than 5 per cent for Reynolds numbers greater than $20 \times 10^5$, increasing to 20 per cent at a Reynolds number of $3.5 \times 10^5$.

No corrections were applied to the static pressure readings obtained in the wake traverses.

### 6.2. The flow pattern (Figs. 8 and 9)

In all the test irrespective of jet design Mach number and jet to base diameter ratio the variation of base pressure with increase of jet stagnation pressure shows the same main features described by Reid and Hastings (2). For zero jet flow there is a closed circulating flow near the base, in which the air entrained by the stream is replaced by air forced forward by the pressure rise across the trailing shock (Fig. 8a). The value of base pressure coefficient found with a supersonic external flow at the same Reynolds number (-0.18) compares well with that found in previous tests (Ref. 8) with subsonic external flow (-0.165).

When there is a very small jet mass flow (i.e. $P_J$ very small; the base bleed phase) the base pressure rises with increase of jet stagnation pressure. This stage (very slow jet and supersonic stream) corresponds to the case in Ref. 8 of a supersonic jet and slow stream when the jet stagnation pressure parameter J is approximately four. Increasing $P_J$ in the present tests corresponds to a reduction of J in those tests. The base pressure increases corresponding to the reduction in base drag coefficient found in Ref. 8. In this base bleed phase the static pressure in the jet at exit is less than that of the flow in the circulating region and thus the very low speed jet flow is deflected along the base and is entrained into the free stream (Fig. 8b). A small vortex is probably formed at the jet lip. The circulating region moves away from the base and results in an increase of base pressure. As the jet stagnation pressure rises the jet flow penetrates further into the circulating region, the lip vortex grows, but the jet flow is still entrained into the free stream via the base. At a specific value of $P_J/P_1$ (0.10 in the present experiments) the jet air is no longer turned back to the base and penetrates, by reason of its increased momentum and pressure, the circulating region. This latter region moves back towards the base outside the lip vortex, which remains. (Fig. 8c). This vortex movement corresponds to the attainment of maximum value of the base pressure and is the end of the base bleed phase. Under these conditions the flow everywhere in the nozzle is subsonic. A further increase in jet stagnation pressure produces sonic conditions at the throat of the jet nozzle.

The slight reduction in the maximum base pressure when the jet design Mach number is increased can be explained in terms of the changes in the jet mass flux. Since the nozzle throat diameter is successively reduced as the design Mach number is increased, so the jet mass flux, for a given stagnation pressure ratio, is also reduced. Thus a smaller proportion of the air entrained by the main flow originates in the jet and a larger proportion must come from the circulating region thereby reducing the base pressure.

As the jet stagnation pressure is raised above $P_J/P_1 = 0.10$ the base pressure falls, the rate of decrease being reduced by an increase of jet design Mach number. Almost immediately the jet flow becomes sonic at the throat, and when the jet stagnation pressure is increased a little more, a normal shock is formed in the nozzle and subsonic flow exists at the exit. As soon as the nozzle runs full (i.e. the exit flow is at the design Mach number) the characteristic diamond pattern is noticed which extends from the nozzle exit to the end of the circulating region and terminates in a normal shock (Fig. 8d). When the nozzle runs full the jet static pressure is still somewhat lower than the base pressure. A further slight increase of the jet stagnation pressure is needed to equalise the jet static and base pressures. Subsequently the base pressure reaches a minimum value, and the corresponding value of $P_J/P_1$ rises with increase in jet Mach number.

As soon as the base pressure and jet static pressure equalise, and before the base pressure reaches its minimum value, very short duration spark photographs show the existence of unsteady spherical waves in the region between the jet and the shock which trails into the main stream from the end of the circulating region (Fig. 9a(i) and b(i)). These waves appear strongly for jets of design Mach number less than or equal to 2 and weakly for $M_J = 2.5$. They were barely noticeable for $M_J$ greater than 2.5 and became more closely spaced as $M_J$ increased. In other words they exist when the difference between the Mach numbers of the stream and the now subsonic jet is not very different from unity. It is also noted that the waves are strong enough to break the smooth curve of the trailing shock into discrete almost straight sections. It is suggested that a moving equilibrium pattern is set up in order to satisfy the pressure boundary condition on the vortex sheet streaming from the boundary layer on the outside surface of the body and results in the jet having a wavy-like external boundary. The external flow is therefore at its maximum velocity in the hollows and a shock is formed at each crest as a result of the compression.

As the jet stagnation pressure approaches that giving a minimum in base pressure the unsteady shocks weaken and a steady second trailing shock appears downstream of the first. The latter originates at the edge of the normal shock in the jet, which moves downstream and shrinks with increase of jet stagnation pressure (Fig. 8e). This second shock appears to be the final state of development of the unsteady shocks noted at the lower jet stagnation pressures. The appearance of this steady second shock coincides with the minimum in base pressure (Fig. 9a(ii) and b(ii)) and the appearance of an expansion at the jet lip. The jet static pressure at exit is greater than the base pressure and the jet flow expands on leaving the nozzle causing the circulating region to contract with a consequent increase in base pressure. The increase in $p_B$ in this phase is very slight for $M_J$ greater than 3.0 and for $d_J/d_B = 0.1875$ (Fig. 4), but is more marked as the ratio of jet to body diameter is increased (Figs. 5 and 6).

Wake traverses and shadowgraph studies behind the cylindrical afterbody ($d_J/d_B = 0.1875$) show that the jet does not expand in size greatly downstream of the nozzle exit even though at the high jet stagnation pressures the jet static pressure is considerably higher than the base pressure. This must be the result of constraining action by the free stream since Johannesen (9) found that a jet at $M_J = 1.4$ exhausting into a still atmosphere spread to several times its exit diameter and maintained its definition to at least one hundred jet diameters from the exit. In the present tests the jet became indistinguishable from the wake, at four body diameters (i.e. twenty two jet diameters) from the exit.

## 7. The effect of free stream Reynolds number

In the case of the sealed base reduction of the free stream Reynolds number below $20 \times 10^5$ per foot causes a fall in the base pressure (i.e. a rise in the base pressure coefficient $c_{p_B}$) which rises again as the Reynolds number is reduced below $9 \times 10^5$ per foot (Fig. 3). The "step" near $20 \times 10^5$ per foot is attributed by Chapman (7) to a change of state of the vortical layers shed from the body from turbulent to laminar. With jet flow from a comparatively large throat the effect is reversed, the base pressure rising with reduction of Reynolds number below $20 \times 10^5$ (Fig. 7). It is suggested that the presence of the jet keeps the vortical layers turbulent.

As the jet Mach number rises above 3 the variation of base pressure with Reynolds number again tends to the pattern described by Chapman for the sealed base (Fig. 7e). For given stagnation conditions and nozzle exit area the mass flux in the jet is proportional to

$$\frac{M_J}{\left[1 + \frac{\gamma - 1}{2} M_J^2\right]^{\gamma + 1/2(\gamma - 1)}}$$

and the momentum flux is proportional to

$$\frac{M_J^2}{\left[1 + \frac{\gamma - 1}{2} M_J^2\right]^{\gamma/\gamma - 1}}$$

It is seen immediately that as the jet Mach number is raised above 3 the values of mass and momentum flux decrease sharply and conditions rapidly approach the limiting case of the sealed base.

8. The effect of jet to base diameter ratio on base pressure (Figs. 6,11)

The results reported here are for the base pressure on a base preceded by a conical body on which the boundary layer is comparatively thick (i.e. $\delta = 0\ (d_B - d_J)$). For these reasons the results are not comparable with those of Reid and Hastings. The present results for a cylindrical body for which $d_J/d_B = 0.1875$, $M_1 = 2.05$ and $M_J = 2.0$ agree reasonably well with those of Reid and Hastings for $d_J/d_B = 0.2$ (Fig. 10). Reid and Hastings measure their static pressure $p_1$ on the afterbody just upstream of the base. During the present tests it was found that this pressure was sensitive to the jet conditions particularly for the larger values of $d_J/d_B$. A reference pressure, such as the static pressure at the tunnel wall in the plane of the base, is therefore more desirable.

The main effect of increasing the ratio $d_J/d_B$ for a given jet Mach number and jet stagnation pressure is to increase the value of base pressure. This is to be expected since the expansion downstream of the nozzle exit can affect more of the base region. Fig. 11a shows clearly that for $d_J/d_B = 0.3$ the jet is surrounded and separated from the main stream by a slow moving wake. As the jet diameter is increased this wake becomes thinner and faster. Furthermore the shape of the jet affects the mainstream flow. For $d_J/d_B = 0.3$ there is only one shock trailing from the mixing region, while at $P_J/P_1 = 10.6$ a second weak shock appears. When $d_J/d_B = 0.6$ and $0.8$ there are three such shocks, the downstream ones weakening with increase of jet stagnation pressure. Comparable state^ with the Mach number 2 jet (Fig. 11b) show five trailing shocks at low jet stagnation pressures and one at high stagnation pressure. This last picture shows a state of base thrust instead of the more usual base drag. (It must be stressed that this base thrust does not include the forces on the conical sides of the afterbody). In this case the jet expands rapidly as it leaves the nozzle and grows to a diameter much larger than the base.

## 9. Comparison between theory and experiment (Fig. 10)

A momentum analysis given in the appendix to this paper shows that the base pressure can be expressed simply in terms of the jet and stream conditions, the base and jet radii, and the velocity profile in the wake taken sufficiently far downstream for the static pressure to be constant. The expression for the ratio of base to free stream static pressure is

$$\frac{p_B}{p_1} = \frac{R_B^2}{R_B^2 - R_J^2} - \frac{R_J^2}{R_B^2 - R_J^2} \cdot \frac{p_J}{p_1}\left[1 + \gamma M_J^2 \left(1 - \frac{u_1}{u_J}\right)\right] + \frac{2\gamma}{R_B^2 - R_J^2} \int_0^\infty r M_2^2(r) \left(1 - \frac{u_1}{u_2(r)}\right) dr$$

Velocity traverses were made at a distance of four body diameters behind the cylindrical afterbody $\left(d_J/d_B = 0.1875\right)$ for the jet Mach number 1.4, 2.0 and 4.0. The profiles obtained were integrated according to the above expression, giving values of $p_B/p_1$ which were within ten per cent of the experimental values. Traverses at one and two diameters downstream showed marked variations in static pressure. The values of base pressure calculated from these traverses were seriously different from the measured values.

## 10. Conclusions

1. As the jet stagnation pressure is increased the base pressure at first rises and then falls sharply to a value considerably less than that for zero jet flow. Further increase of jet stagnation pressure causes an increase in base pressure, the rate of increase decreasing with increase of jet Mach number.

2. The base pressure increases with increase of the ratio of jet diameter to base diameter. At high jet stagnation pressures the base pressure can be greater than the free stream static pressure giving a base thrust rather than the more usual base drag.

3. The base pressure in the presence of a jet is independent of free stream Reynolds number when the latter is greater than $2 \times 10^6$ per foot. Below this value, the base pressure rises with reduction of Reynolds number for $M_J$ between 1.0 and 3.0. For $M_J = 4.0$ the change in

base pressure with Reynolds number obeys the same trends as for a sealed base.

## 11. Acknowledgements

The authors wish to express their gratitude to Mr. G. M. Lilley for his advice and encouragement throughout the study, to Mr. S. H. Lilley for the design and erection of the experimental equipment and to Mr. H. Stanton for making the afterbodies and nozzles.

## 12. References

1. Cortright, E.M. Some aerodynamic considerations of nozzle - afterbody combinations. Aeronautical Engineering Review, vol.15, 1956.

2. Reid, J., Hastings, R.C. The effect of a central jet on the base pressure of a cylindrical afterbody in a supersonic stream. R.A.E. Report Aero.2621, 1959.

3. Bromm, A.F., O'Donnel, R.M. Investigation at supersonic speeds of the effect of jet Mach number and divergence angle of the nozzle upon the pressure of the base annulus of a body of revolution. N.A.C.A. Res.Mem. L54I16, 1954.

4. Baughman, L.E., Kochendorfer, F.D. Jet effects on base pressures of conical afterbodies at Mach 1.91 and 3.12. N.A.C.A. Res.Mem. E57E06, 1957.

5. Clippinger, R.F. Supersonic axially symmetric nozzles. B.R.L. Report 794, 1951.

6. Holder, D.W., North, R.J., Chinneck, A. Experiments with static tubes in a supersonic airstream. A.R.C. R & M. 2782, 1953.

7. Chapman, D. An analysis of base pressure at supersonic speeds and comparison with experiment. N.A.C.A. Tech.Note 2137, 1950.

8. Craven, A.H. Base pressure at subsonic speeds in the presence of a supersonic jet. College of Aeronautics Report 129, 1960.

9. Johannesen, N.H. The mixing of free axially symmetric jets of Mach number 1.4. A.R.C. Report 18,968, 1959. and Further results on the mixing of free axially symmetric jets of Mach number 1.4. A.R.C. Report 20,981, 1959.

## APPENDIX

### The base pressure in terms of the velocity in the wake

Consider the flow into and out of the region ABCDEF (Fig. 12). BC coincides with the base and DE is sufficiently far from the centre-line for the axial component of velocity to be approximately equal to the freestream velocity $u_1$. EF is sufficiently far from the base for the element to include completely the circulating region and the shock system.

The momentum equation in the axial direction is, ignoring the boundary layers on the body and in the jet nozzle,

$$\underset{AB}{\int_0^{R_J}} 2\pi r\,(p_J + \rho_J u_J^2)\,dr + \underset{BC}{\int_{R_J}^{R_B}} 2\pi r\, p_B\, dr + \underset{CD}{\int_{R_B}^{d}} 2\pi r\,(p_1 + \rho_1 u_1^2)dr$$

$$= \underset{FE}{\int_0^{d}} 2\pi r(p_2 + \rho_2 u_2^2)dr + 2\pi d \underset{DE}{\int_0^{l}} \rho v u\, dx \qquad \ldots\ldots \quad (1)$$

or since $\rho_J u_J^2 = \gamma p_J M_J^2$ etc.,

$$p_J (1 + \gamma M_J^2)\, R_J^2 + (d^2 - R_B^2)\, p_1 (1 + \gamma M_1^2) + 2\int_{R_J}^{R_B} r\, p_B\, dr$$

$$= 2 \underset{x = l}{\int_0^{d}} r\, p_2(1 + \gamma M_2^2)dr + 2d \underset{r = d}{\int_0^{l}} \rho v u\, dx \qquad ;\ldots\ldots \quad (2)$$

Now $p_B$ is sensibly constant across the base and we assume that the control box is taken large enough so that

(i) along EF ($x = l$) the static pressure is constant
(ii) along DE ($r = d$) $u \approx u_1$.

Thus (2) becomes

$$p_B (R_B^2 - R_J^2) = p_1 d^2 + 2\gamma p_1 \int_{o}^{d} r M_2^2 dr + 2 du_1 \int_{o}^{1} \rho v \, dx = \rho_J R_J^2 \cdot$$

$$\qquad x = 1 \qquad\qquad r = d$$

$$(1 + \gamma M_J^2) - (d^2 - R_B^2) p_1 (1 + \gamma M_1^2) \qquad \ldots\ldots \quad (3)$$

The equation of continuity for the motion is

$$\int_{o}^{R_J} 2\pi r \rho_J u_J dr + \int_{R_B}^{d} 2\pi r \rho_1 u_1 dr = \int_{o, x=1}^{d} 2\pi r \rho_2 u_2 dr + 2\pi d \int_{c, r=d}^{1} \rho v \, dx$$

$$\text{or} \quad 2d \int_{o, r=d}^{1} \rho v \, dx = \rho_J u_J R_J^2 + (d^2 - R_B^2) \rho_1 u_1 - 2 \int_{o, x=1}^{d} r \rho_2 u_2 dr \qquad (4)$$

Eliminating v between (3) and (4)

$$p_B (P_B^2 - R_J^2) = p_1 R_B^2 - R_J^2 p_J \left[ 1 + \gamma M_J^2 \left( 1 - \frac{u_1}{u_J} \right) \right] + 2\gamma p_1 \int_{o, x=1}^{d} r M_2^2(r) \left[ 1 - \frac{u_1}{u_2(r)} \right] dr$$

or

$$\frac{p_B}{p_1} = \frac{R_B^2}{R_B^2 - R_J^2} - \frac{R_J^2}{R_B^2 - R_J^2} \cdot \frac{p_J}{p_1} \left[ 1 + \gamma M_J^2 \left( 1 - \frac{u_1}{u_J} \right) \right] + \frac{2\gamma}{R_B^2 - R_J^2} \cdot$$

$$\int_{o, x=1}^{\infty} r M_2^2(r) \left[ 1 - \frac{u_1}{u_2(r)} \right] dr \qquad (6)$$

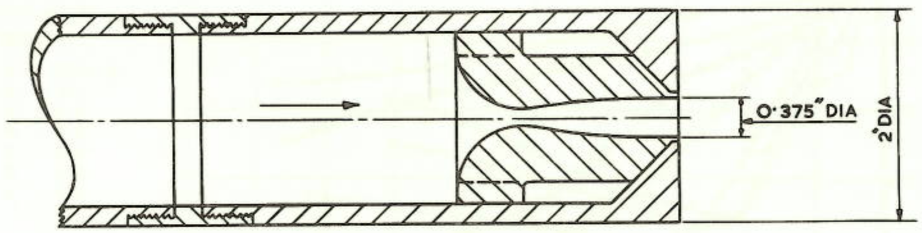



FIG. 1. CYLINDRICAL BODY - JET MACH NUMBER AND FREE STREAM RENOLDS NUMBER TESTS

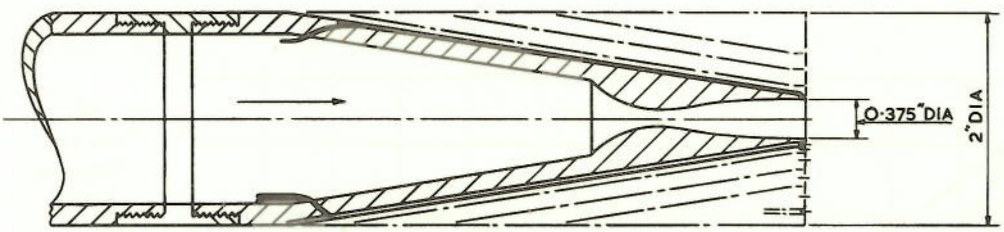



FIG. 2. CONICAL BODY - JET TO BASE DIAMETER TESTS

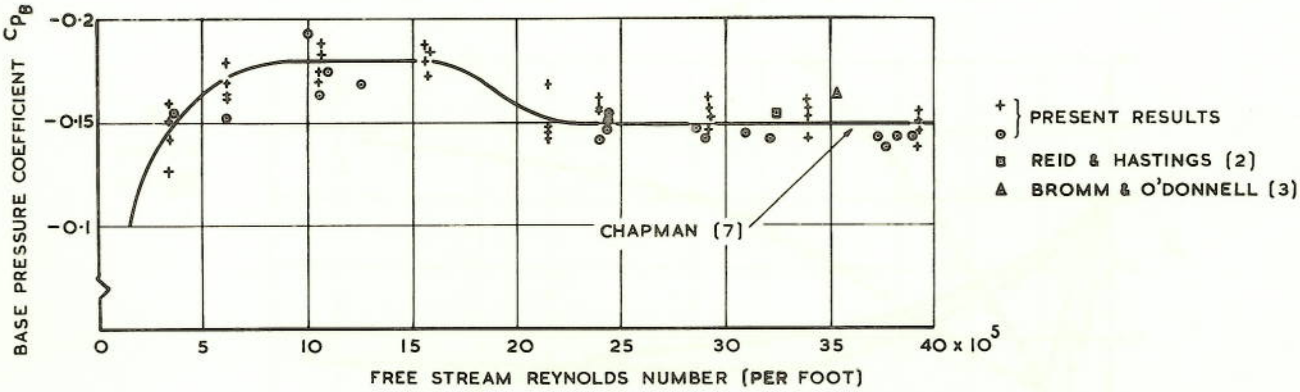



FIG. 3. VARIATION OF BASE PRESSURE COEFFICIENTS WITH FREE STREAM REYNOLDS NUMBER FOR A SEALED BASE ($M_1$ = 2)

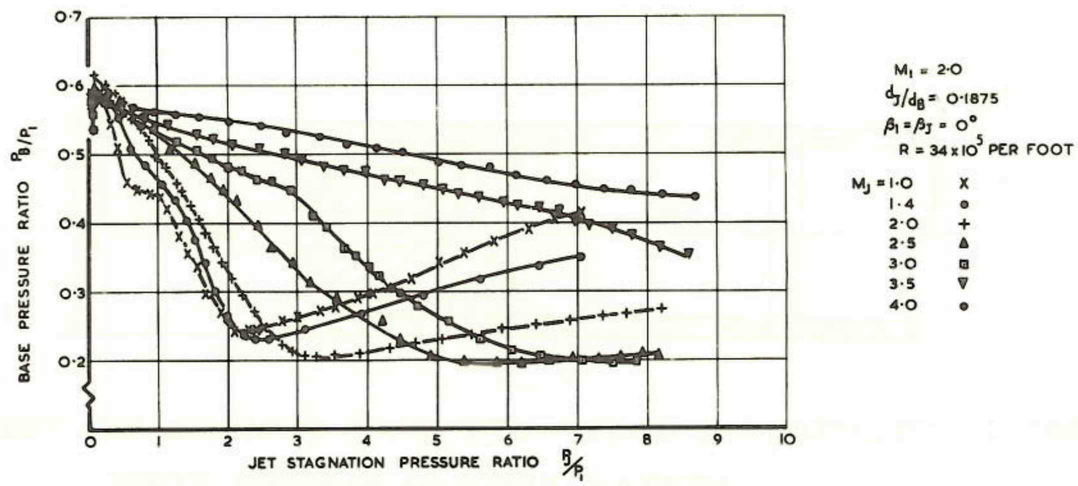



FIG. 4. EFFECT OF JET DESIGN MACH NUMBER ON BASE PRESSURE

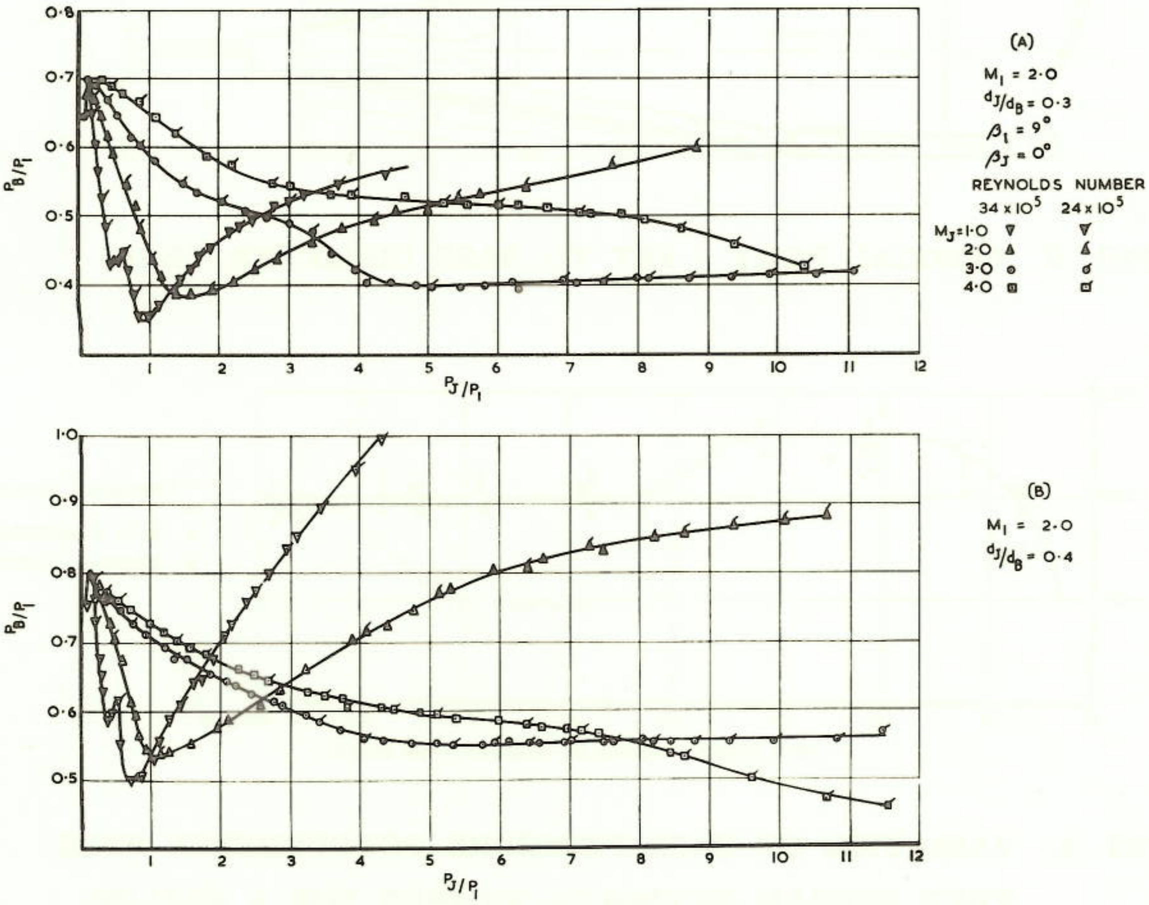



FIG. 5. EFFECT OF JET MACH NUMBER ON BASE PRESSURE ON A CONICAL AFTERBODY

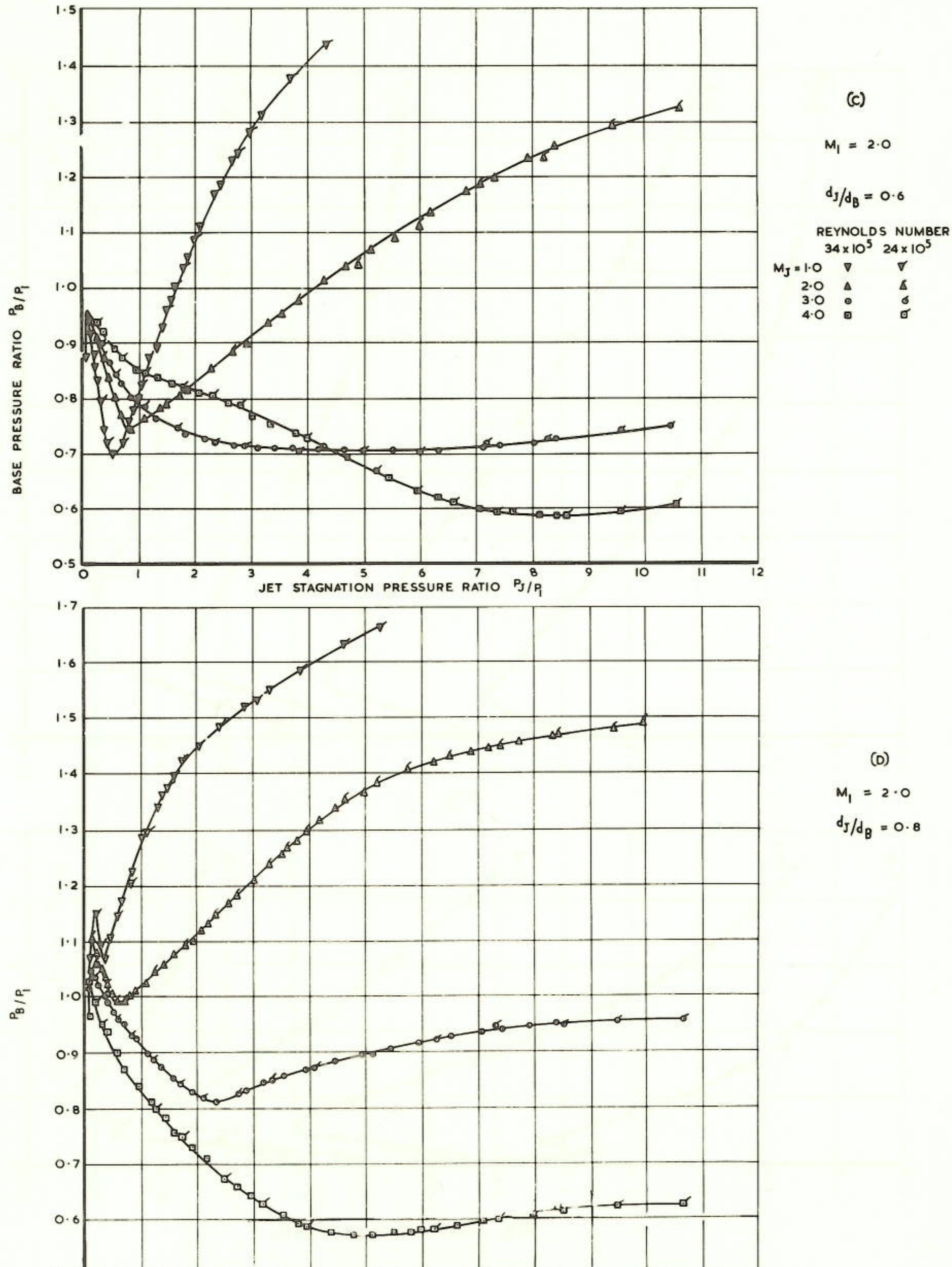



FIG. 5. EFFECT OF JET MACH NUMBER ON BASE PRESSURE ON A CONICAL AFTERBODY

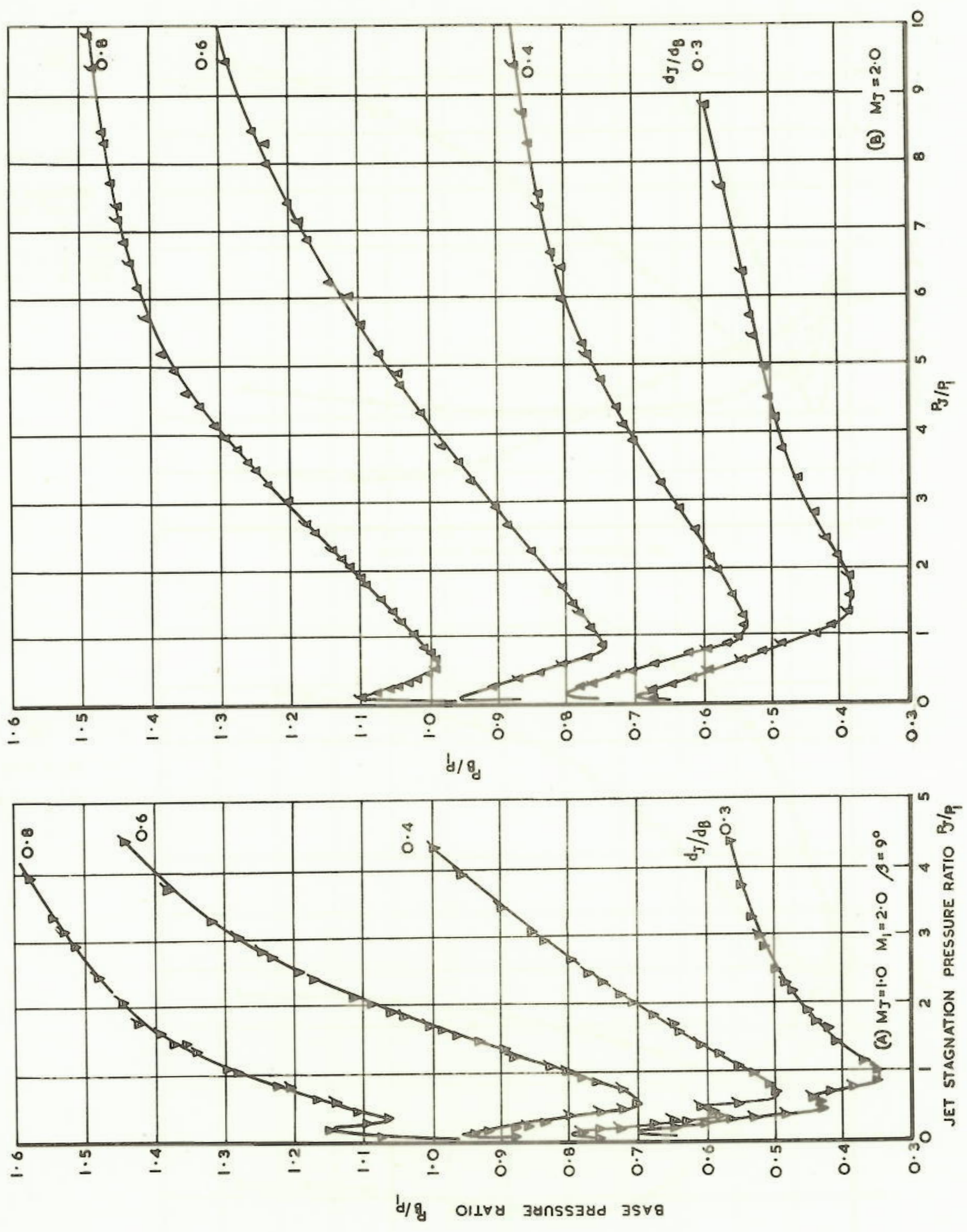



FIG. 6. EFFECT OF JET TO BASE DIAMETER RATIO ON BASE PRESSURE

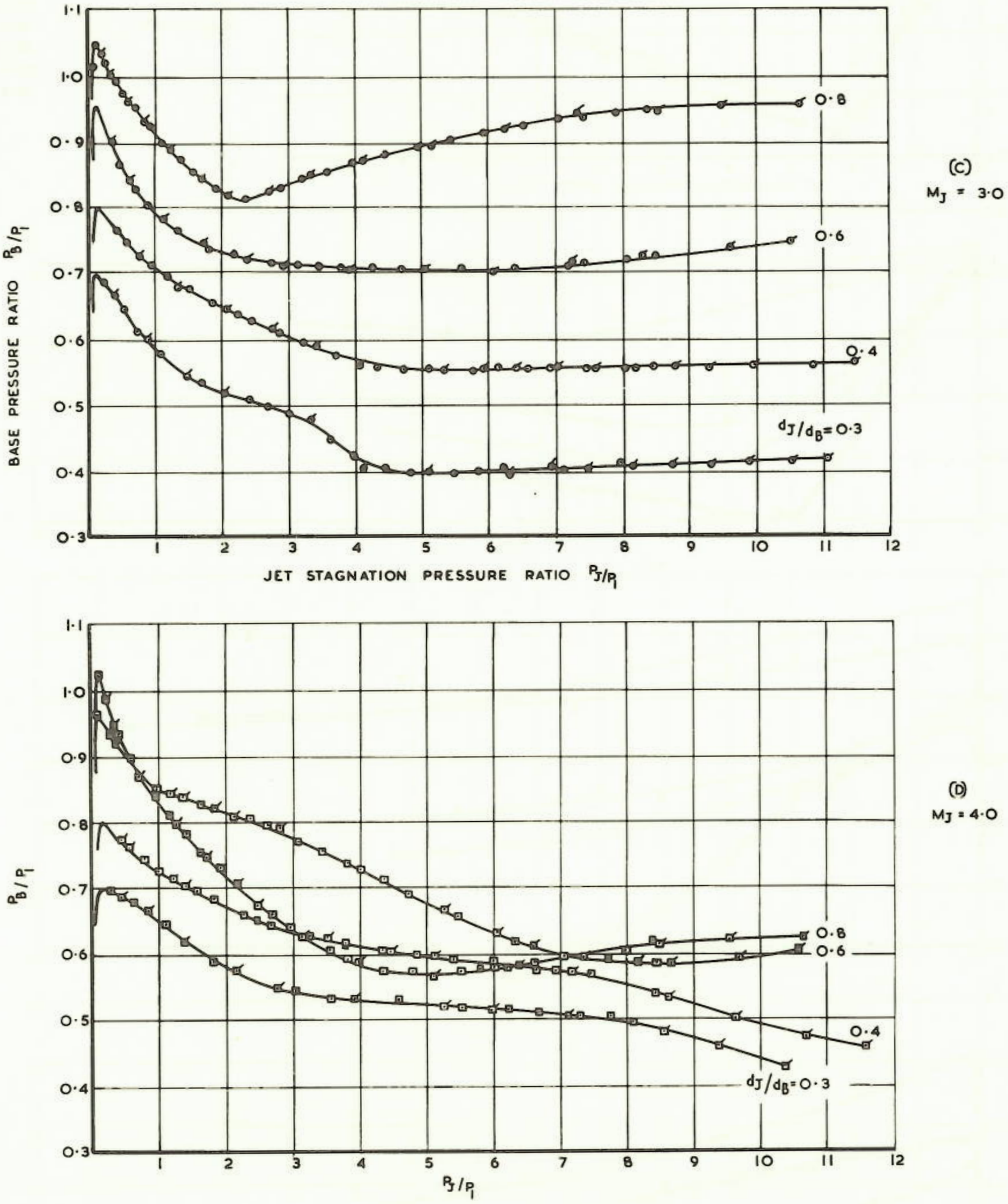



FIG. 6. EFFECT OF JET TO BASE DIAMETER RATIO ON BASE PRESSURE

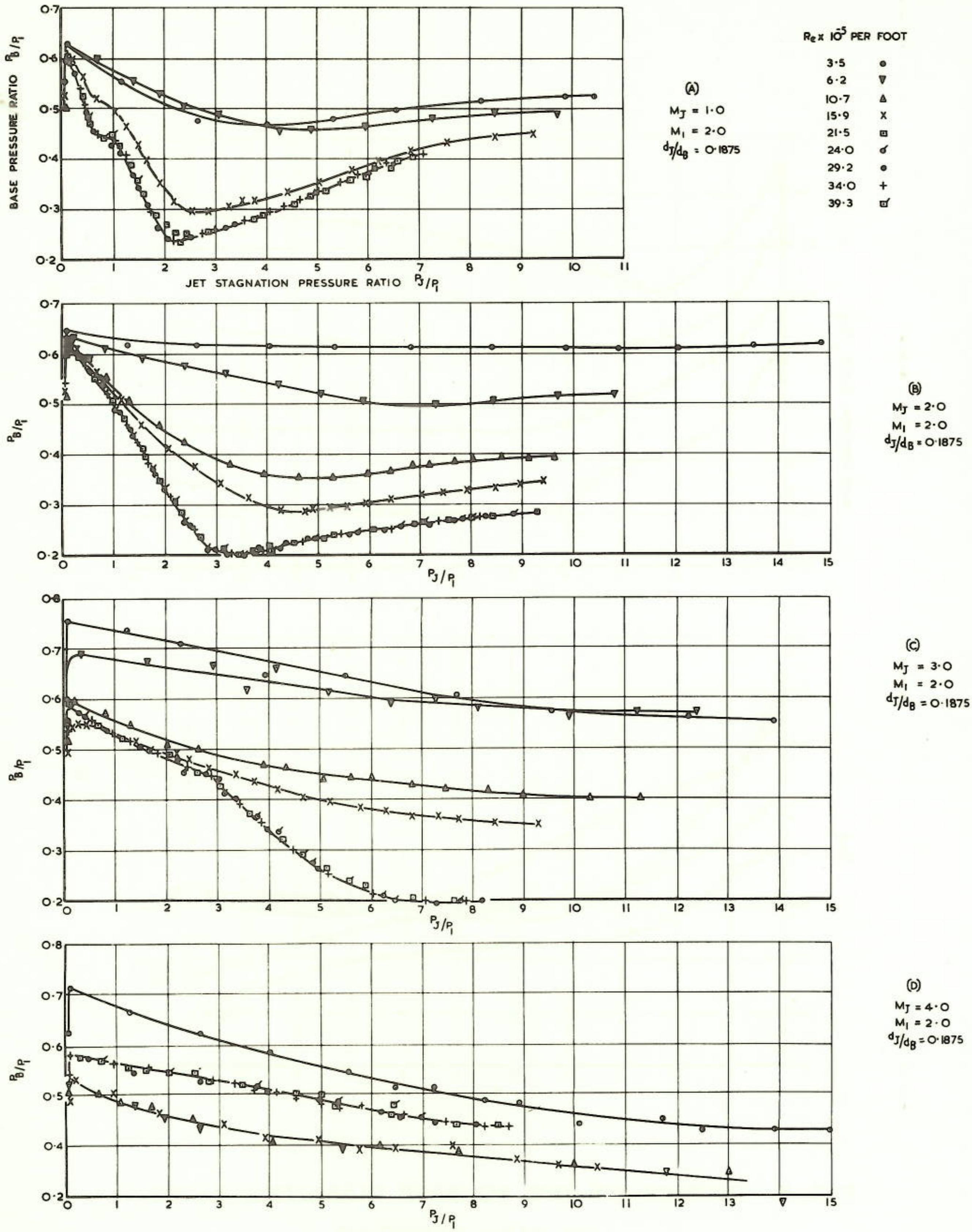



FIG. 7. VARIATION OF BASE PRESSURE WITH FREE STREAM REYNOLDS NUMBER

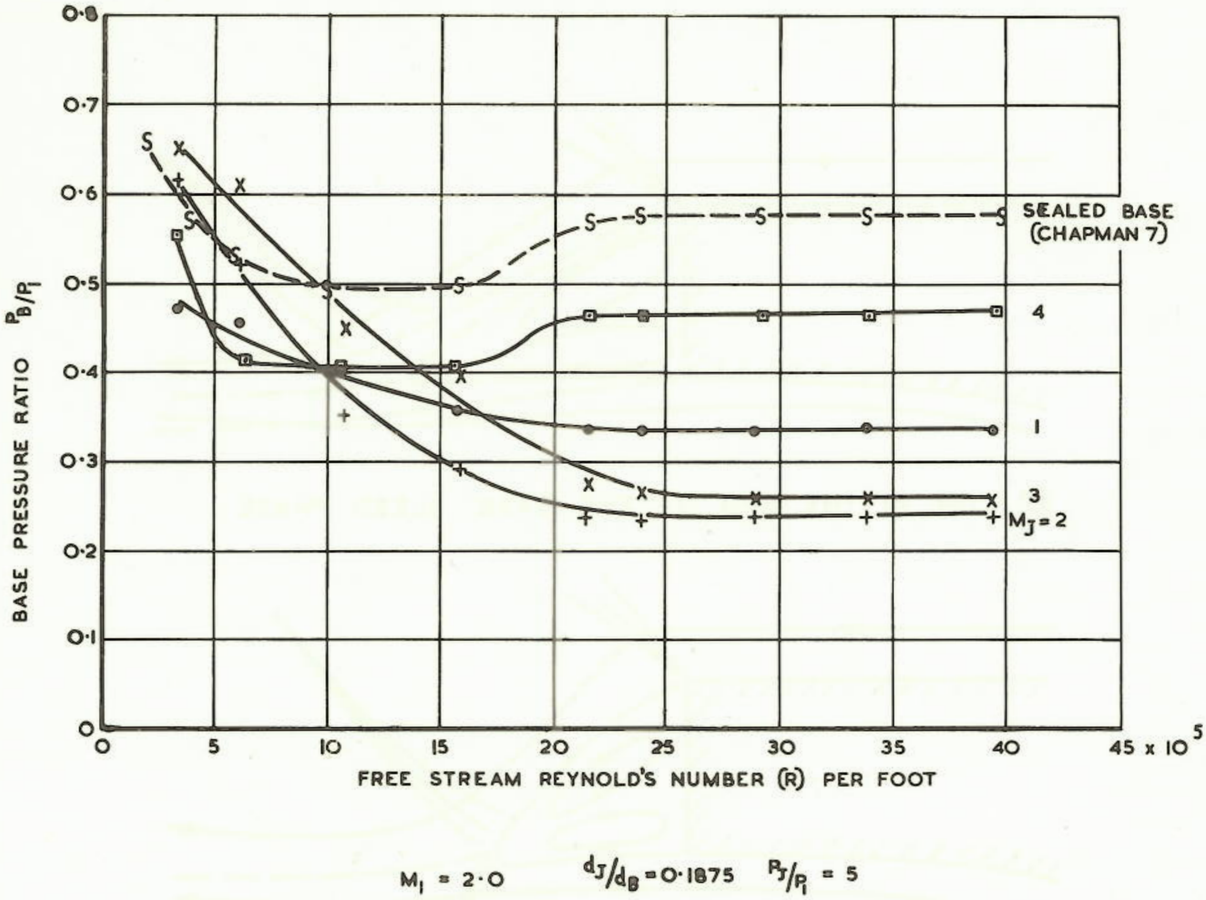



FIG. 7 (e) TYPICAL VARIATION OF BASE PRESSURE WITH REYNOLDS NUMBER

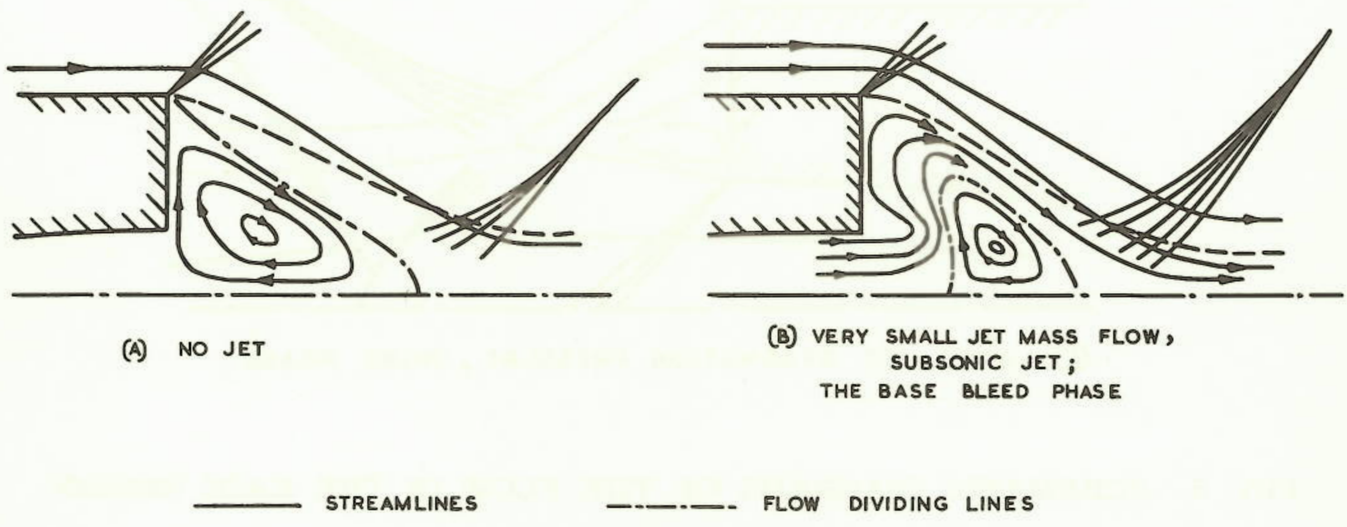
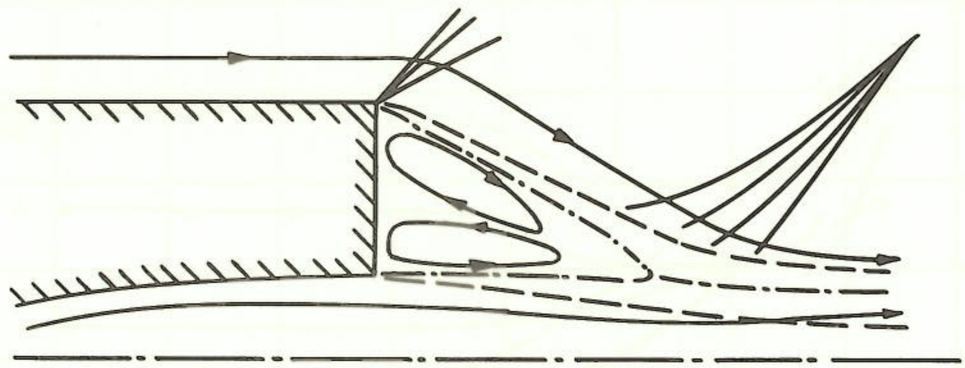
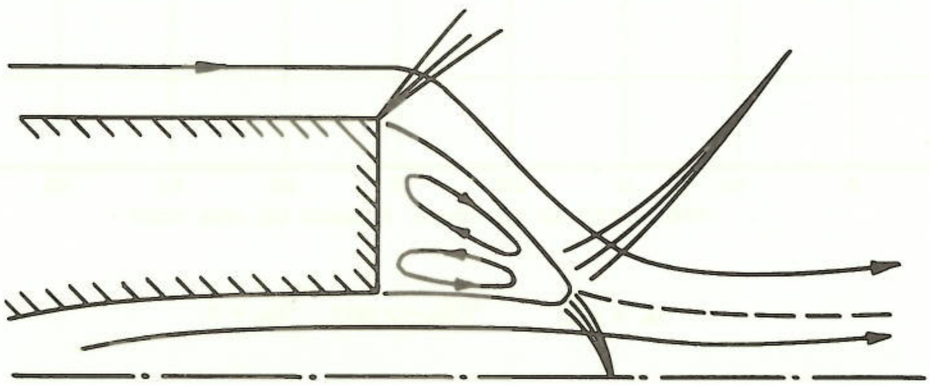


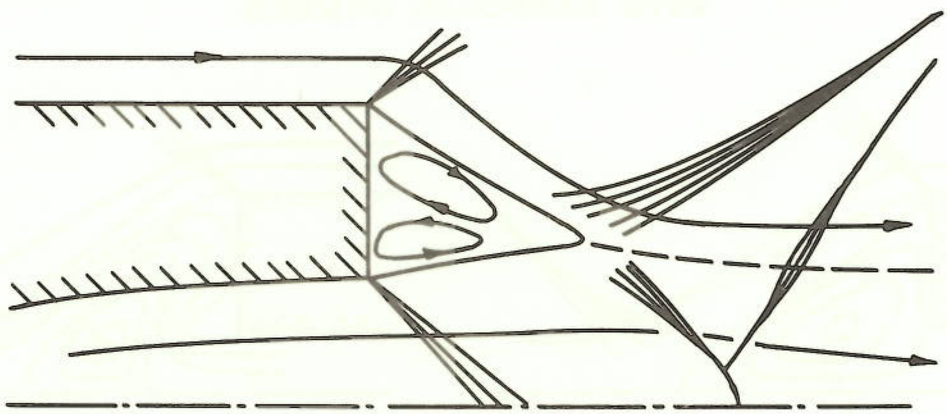

FIG. 8. SCHEMATIC DIAGRAMS OF THE FLOW IN THE BASE REGION



(C) FLOW AT THE END OF THE BASE BLEED PHASE

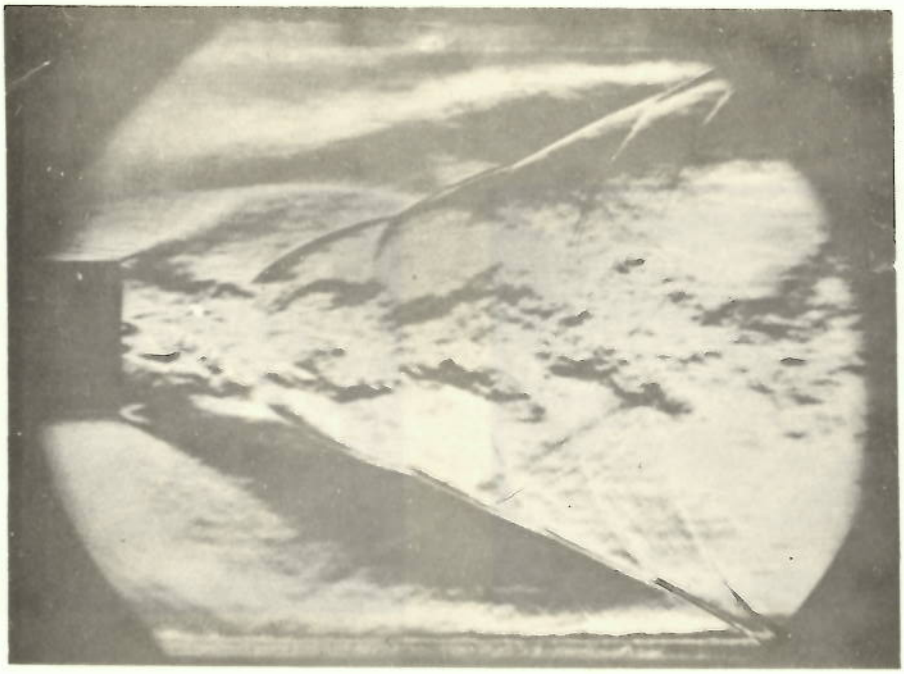


(D) NOZZLE RUNNING FULL, $p_B = p_1$ ; SECOND PHASE

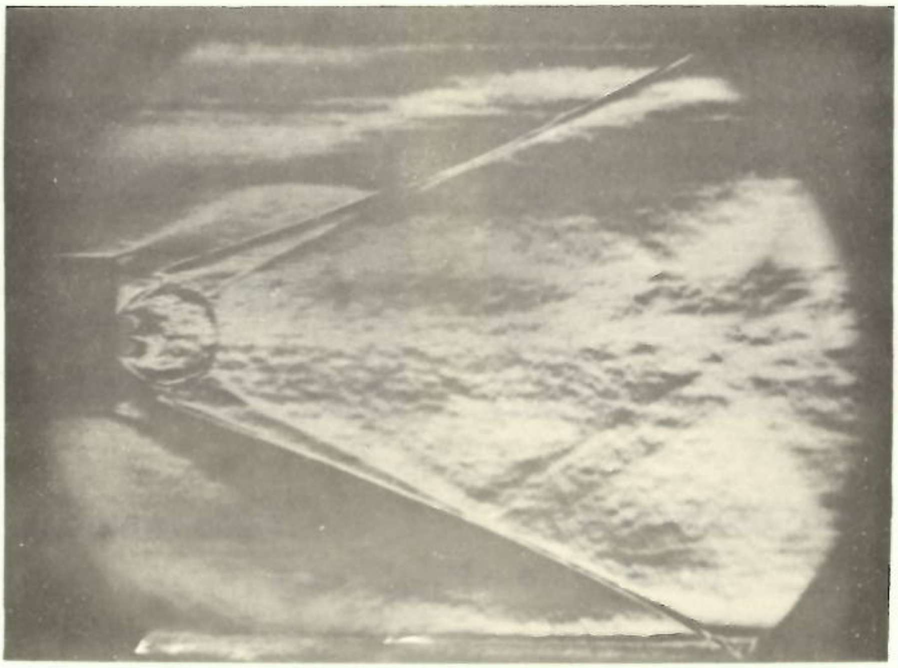


(E) LARGE JET STAGNATION PRESSURE, THIRD PHASE

FIG. 8. SCHEMATIC DIAGRAMS OF THE FLOW IN THE BASE REGION



(i) $M_J = 1.0$ ; $P_{J/P_1} = 1.2$



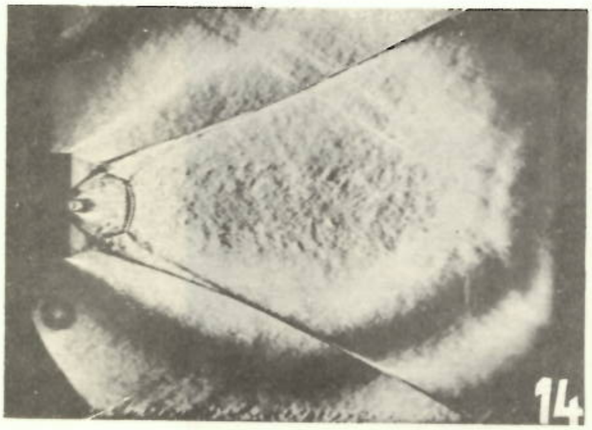
(ii) $M_J = 1.0$ ; $P_{J/P_1} = 4.1$

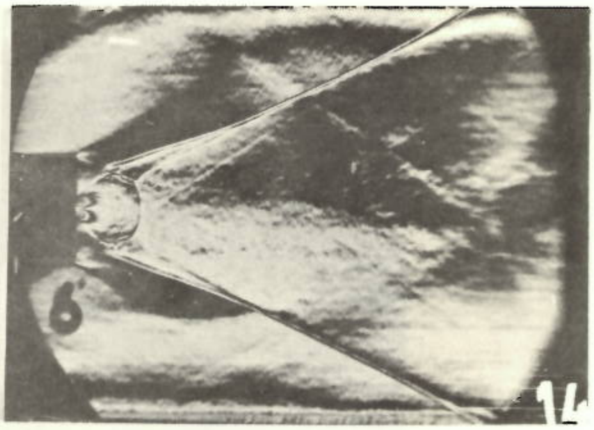
FIG. 9a. FLOW IN THE MIXING REGION

SLIT HORIZONTAL

SLIT VERTICAL

(i) $M_J = 1.4$ ; $P_J / P_1 = 1.56$





SLIT HORIZONTAL

SLIT VERTICAL

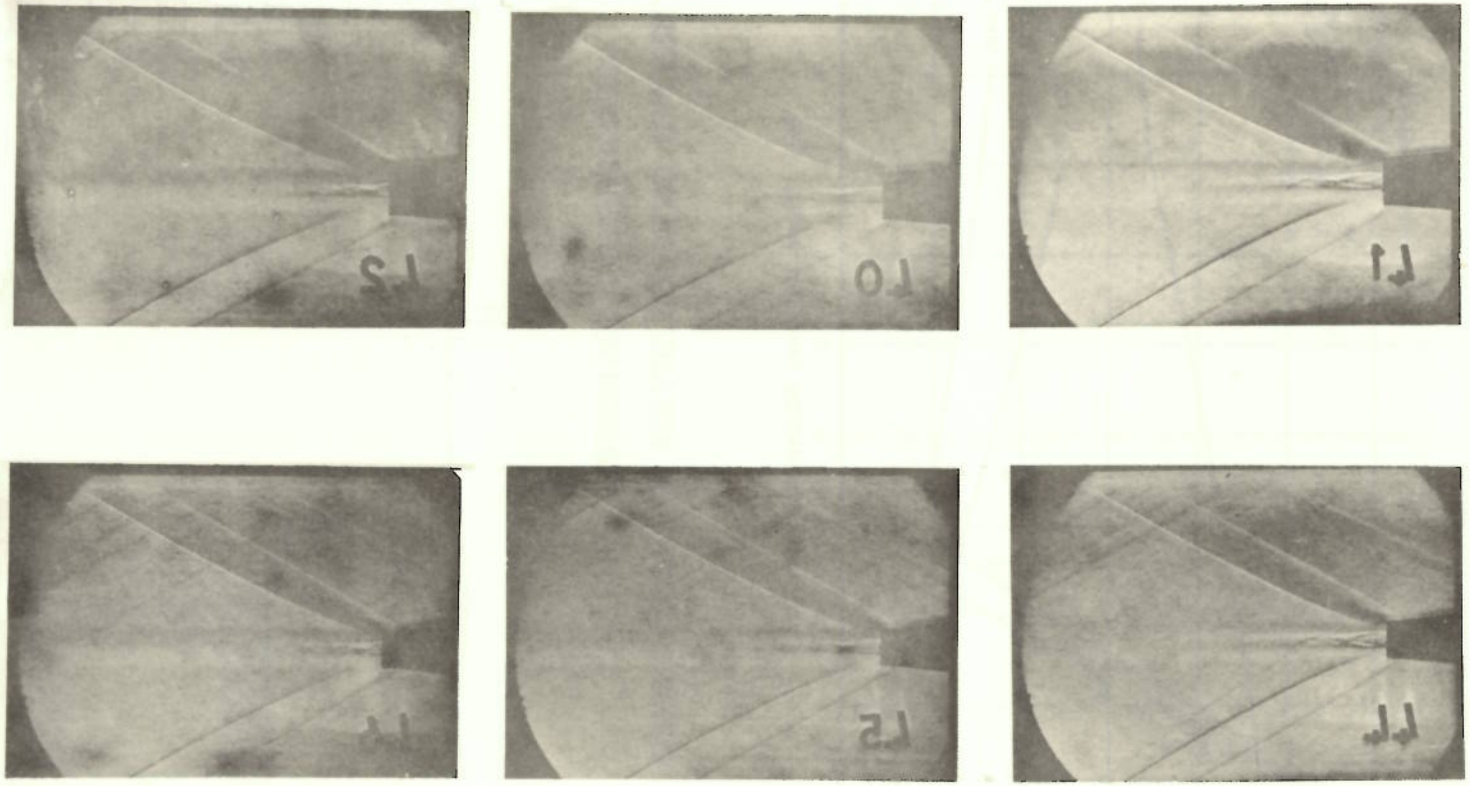
(ii) $M_J = 1.4$ ; $P_J / P_1 = 4.1$

FIG. 9b. FLOW IN THE MIXING REGION

$d_J/d_B$

0.3

0.4



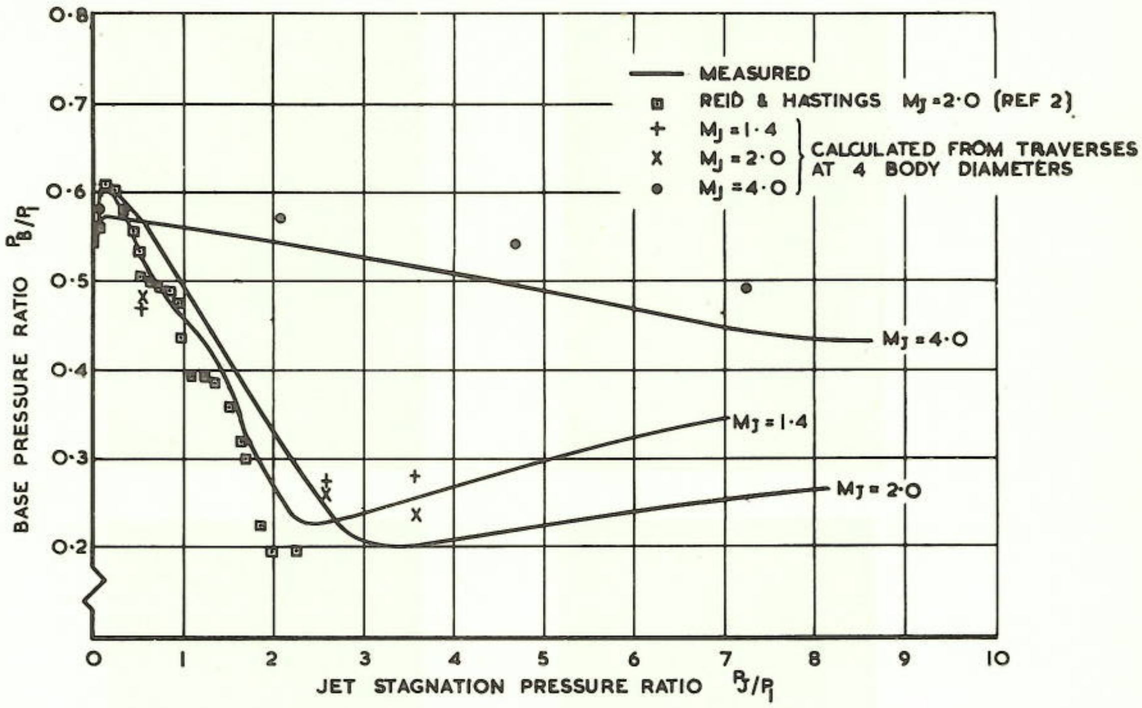



FIG. 10. COMPARISON OF MEASURED AND CALCULATED VALUES OF BASE PRESSURE COEFFICIENT

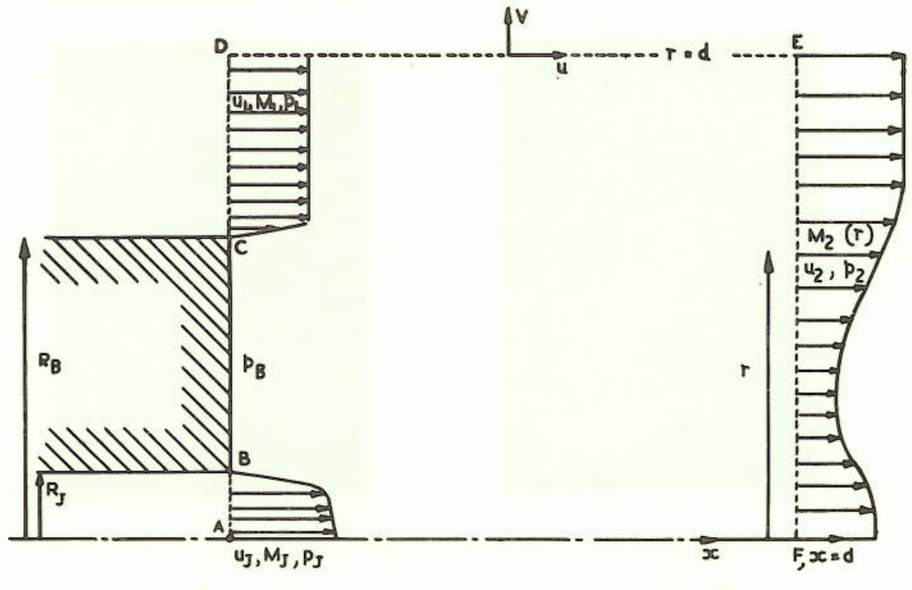
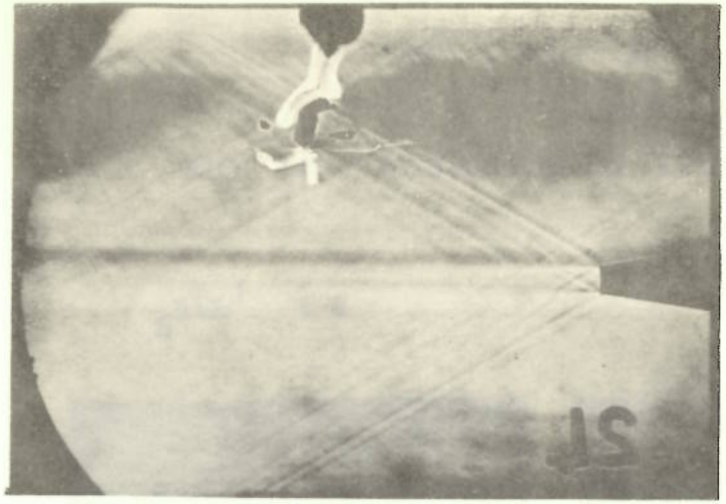



FIG. 12. THE REGION CONSIDERED IN THE MOMENTUM ANALYSIS

$P_J/P_1$

0.6



9.8

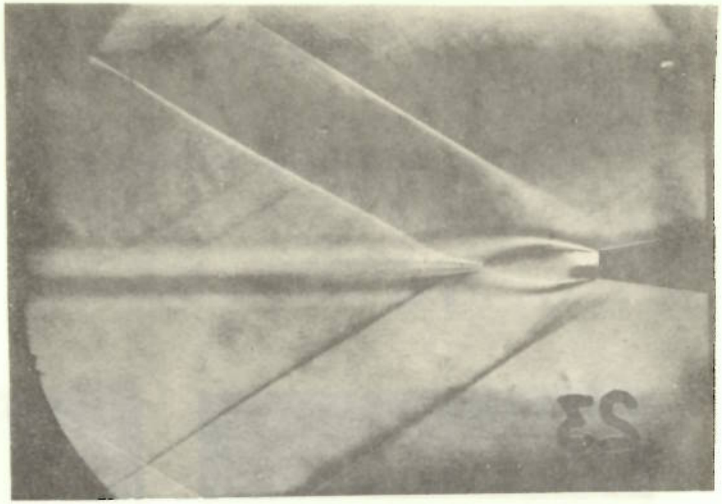
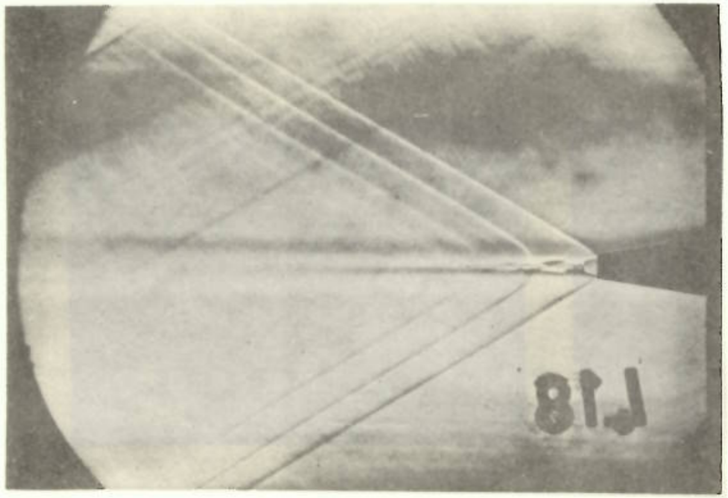


FIG. 11b. CONICAL AFTERBODY; $d_J/d_B$ = 0.8, $M_J$ = 2.0, $M_1$ = 2.0, $R = 24 \times 10^5$ per foot

$P_J/P_1$

4.6



7.2

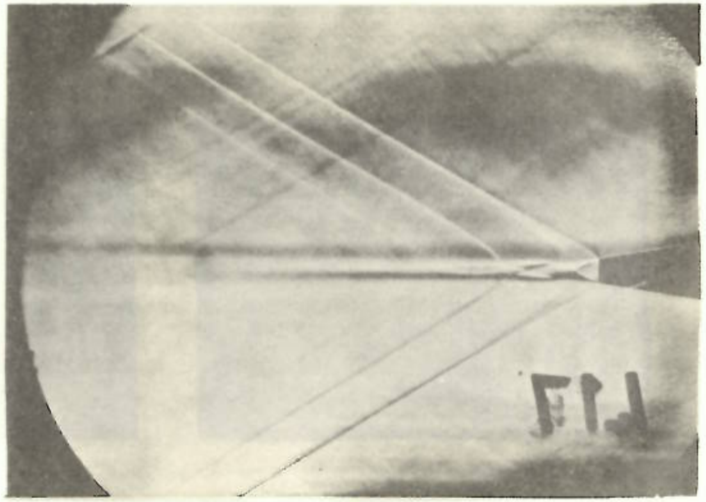


FIG. 11c. CONICAL AFTERBODY; $M_J$ = 4.0, $M_1$ = 2.0, $R = 24 \times 10^5$ per foot

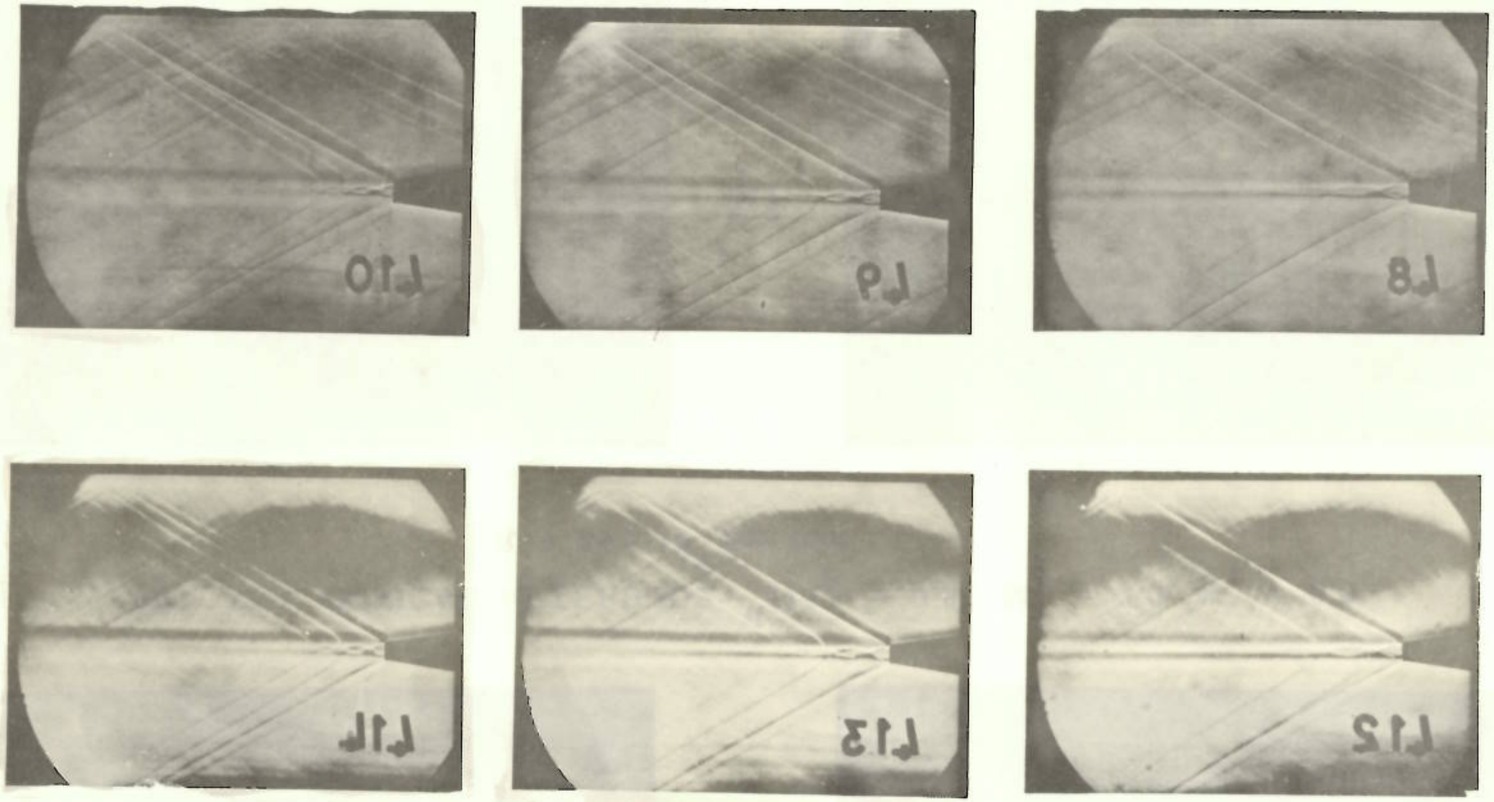



FIG. 11a. VARIATION OF FLOW PATTERN WITH JET TO BASE DIAMETER RATIO

$M_J$ = 4.0 $M_1$ = 2.0, R = 24 x $10^5$ per foot