Survey

Allocation of quality control stations in multistage manufacturing systems

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Allocation of quality control stations in multistage manufacturing systems

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Abstract

The allocation of quality control stations (AQC) in multi-stage manufacturing systems has been studied extensively over the last fifty years. The objective of this paper is to review the existing approaches, propose a classification of the available models in terms of the type of manufacturing system that they refer to and the applied solution methods, and then examine the effectiveness of the inspection strategies by developing appropriate generalised algorithm and software tool. The review firstly revealed that the inspection allocation problem has been studied comprehensively using variety of analytical and Monte Carlo simulation methods rather than combination of both simulation techniques in a simulation-optimisation framework. Secondly a large proportion of the papers focus on several work stations representing part of a manufacturing line without attempting to solve the global optimisation problem which lead to solutions based on complete enumeration that are known to be computationally ineffective when the number of workstations increase. The developed simulation program demonstrated that methods determining the position of inspection by using complete enumeration method (EM) are of limited use in the majority of manufacturing situations when the number of workstations exceeds eighteen. This led to the development of a heuristic algorithm the performance of which was compared with the complete enumeration algorithm. It was found that heuristic method can derive an acceptable solution significantly faster. At present authors continue to develop heuristic algorithms for the AQC problem and metaheuristics using biologically inspired techniques.

Keywords: Manufacturing; Quality; Resources Allocation; Inspection Optimisation.

1. Introduction

The quality management policies in the majority of companies evolve continuously over a number of years by focusing on quality issues that are critical at any given instant of time. This approach usually focuses on particular critical operations and does not take into account the need for global analysis of quality problems within a manufacturing system. As a result quality policies

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tend not fully utilise the available financial, human and equipment resources. Furthermore, in the current economic situation, the reduction of waste becomes of paramount importance because the increase in the product cost affects the overall competitiveness of the products. This paper targets specifically waste resulting from unidentified defective items being processed unnecessarily during manufacturing operations. The solution of inspection effort allocation issues needs to adopt the corresponding utility strategies. Such strategies aim to allocate an economically appropriate level of inspection effort by striking a balance among the different cost components connected with inspection, scrap, repair and replacement due to quality failure, and/or the warranty penalty in the case where a nonconforming product has been shipped to customers. Inspection-oriented strategy focuses on optimization that minimizes the expected total manufacturing cost, maximizes the quality and is capable of delivering the demanded quantities of the product. The expected total cost consists of the manufacturing cost, inspection cost, internal failure cost and external failure cost.

The general problem of allocation of quality control stations (AQCs) in manufacturing systems can be divided into two sub-problem categories aiming to answer the following research questions: (1) If the requested quality level, production volume and product costs are fixed what is the optimal number of inspection stations for a manufacturing process? (2) If the number of inspection stations is fixed for a particular process where the optimal places to position are them in order to optimise the quality, cost, and delivery manufacturing attributes?

The objective of this paper is to present an overview on how these questions have been answered and to examine the inspection strategies by developing the appropriate algorithm and software tool.

2. Inspection allocation problem

2.1. Problem background

The procedure of making decisions of whether or not to inspect a final or semi-finished product at every processing workstation is shown schematically in Figure 1. It is assumed that if inspection is performed after every workstation, then the scrap and rework costs will stay at a minimum level. These savings have to be considered against the inspection costs which include equipment, staff, time, shop floor space and increase the number of works in process. Therefore, if these in-process inspections are performed too often unnecessary, costs will incur. In a practical setting usually the expectation is that manufacturing processes at each work station are capable of achieving the required quality tolerances for 99.97% of the items which is the 3σ level when the process is on target Montgomery (1997).
It is possible that a shift of up to 1.5σ from the target quality level may occur undetected subject to the quality procedure which may lead to drop in quality level down to 93.3% and when such situation occurs our inspection stations have to be positioned in a way that will minimise the overall lost of profit. If the shift is larger than that we consider that the problem is not any longer a quality monitoring but a manufacturing issue that has to be fixed by different means.

The purpose of the inspection allocation strategy is to allocate an economically appropriate level of inspection activity by determining the correct balance among different cost components indicated above.

2.2. Structure of the line

Products are often processed through multi-stage production systems, where incoming raw materials are transformed into the finished products in a chain of different processing stages. There are several types of production systems such as: (i) serial systems; (ii) assembly systems; and (iii) non-serial systems. In a serial production system, the raw materials pass through a sequence of processing workstation to the final products (see Shiau (2002) and VanVolsem et al. (2007)) whereas in an assembly system, at a certain stage the product may be fixed or assembled with products from other processing lines (see Penn and Raviv (2007) and Deliman and Feldman (1996)). A system that is neither serial nor non assembly falls into category of non-serial system, (see Taneja and Viswanadham (1994) and Emmons and Rabinowitz (2002)). However, it is more difficult to determine undetected defects in assembly system than in serial line. The difficulty arises due to assembly stages at which multiple serial lines join to form a single serial line. At such assembly stages, the number of undetected defective output flow items of the assembly stage entering the assembly line stage depends on the proportion of defective items leaving all series lines to that assembly stage.

2.3. Inspection time

In the reviewed papers, inspection time plays a major role to the total manufacturing costs. Longer inspection times obviously strain the inspection capacity which may cause increased inspection errors. Saxena et al. (1990) explained this by using a simulation model to examine the performance of five inspection stations on the basis of job completion time in serial production systems under different operating conditions. They found that inspection time was the most influential factor for the selection of a particular heuristics rule. The vast majority of the papers were assumed that the inspection time for each inspection station can be represented by the inspection cost (see Shiau (2002), Shiau et al. (2007), Penn and Raviv (2007), Shiau (2003a) and Shiau (2003b)), just to name a few. Indeed only one paper in the optimisation techniques (Lee and Unnikrishnan (1998)) used inspection time as constraint to the objective function of the total cost. They found that by considering inspection time as constraint, the number of inspection plans can be reduced.

2.4. Repair of defects

As illustrated in Figure 1, during the inspection, once an item does not confirm to the specifications certain actions will be taken to repair, replace or simply scrap it. However, defective
products, when allowed to pass through the production stream, might cause time delay and become costly to repair at a later stage of operation. To account for this in a model, some researchers have assigned a reparability level for defects. For example Hsu (1984) and Chen et al. (1998) have assumed deterministic assignment of reparability; and Narahari and Khan (1996) and Barad (1990) have adopted a probabilistic approach which assumes that a defect is repairable with a given probability. The repair occurs only when every nonconforming predefined quality of a product is larger than the specification limit. In the absence of both repairable and replacement, the production volume in the model will shrink as a result of inspection. In real life without replacement or repair larger lot sizes have to be introduced in order to meet production plans and to avoid delivery delays.

2.5. Nonconforming products

Products may become nonconforming because of improper performance of a processing operation. The chance that a unit will become nonconforming at a given stage is referred to as the nonconforming processing rate for the stage, and may be constant or variable, and may alternate between an acceptable level and an out-of control level. A given processing stage may cause a single type of nonconformity or multiple types. In the inspection station allocation, the majority of the reviewed papers have assumed that each workstation may have a specific probability of producing defective parts. Products considered being nonconforming and subsequently removed from the production flow may have some or no salvage value at all. The salvage value represents the revenue generated by selling the rejected items as scrap of lower grade products (see Eppen and Hurst (1974)). They assumed that a unit rejected by inspection, whether good or defective, is always removed because it incurs a salvage cost which might be negative.

2.6. Inspection capability

During the inspection operation two types of error may be generated by the inspection procedure: type-I error and type- II error. The type-I error refers to as a rejection a good items and is also known as the producer risk, whereas type- II error refers to as the acceptance of a non-conformance items and is also known as the consumer risk. The type II error is usually more serious. Not all the authors have considered both types of error. Rebello et al. (1995) and Tannock (1997) have only considered one of the two types and other authors have simply assumed a perfect inspection (e.g., Narahari and Khan (1996) and Penn and Raviv (2008)).

3. Modelling features

An inspection quality allocation problem is usually solved through an optimization formulation. In this section, types of cost components have been considered will be discussed followed by the solution approach proposed by the researchers. Table 1 presents a summary of the classifications where each publication is represented by the first author’s name followed by a two-digit publication year in order to conserve space. In order to recognise the type of manufacturing and inspection system a publication considered, the information under different categories will be selected. For example, Peters and Williams (1984) considered a serial system, inspection is therefore assumed to be error free. In their paper, inspection cost was assumed to be fixed and variable. Some of the publications appear in multiple categories because they consider multiple scenarios in a production system. For example, Penn and Raviv (2007), has used the dynamic programming technique and branch and bound method to solve their model.

3.1. Cost components

The manufacturing cost of a product is one of the major factors under consideration. The usual requirement is that the products to be manufactured at an acceptable quality level and minimum costs. Perhaps for that reason, the majority of the researchers chose to focus on specific cost components related to quality failures (internal failure cost and external failure cost). The internal failure costs incur inside the company, such as the costs of reworking, scrapping and replacement. The external failure costs incur after the goods are shipped to customers, for example the costs of
replacement, repairing, and quality loss. In Table 1, some papers consider all cost components, whereas others consider only a subset. If a paper does not appear under a specific category, it means that the particular cost is not considered by the paper. Examples of these include Narahari and Khan (1996), who assumed the inspection cost to be negligible so that it does not appear in either one of the categories, and Tannock and Saelem (2007), who only considered a fixed inspection cost and no variable inspection cost.

The vast majority of the reviewed papers have not considered in their models all items of external failure cost. They only represented them as aggregated (penalty cost). The penalty cost is usually associated with the final production of undetected nonconforming items that reach the customer. Some other researchers did not consider at all any item of external failure cost (see Tannock and Saelem (2007) and Barad (1990)), whereas a few papers are considered at most other items of external failure cost in their models such as quality loss, replacement cost and repair cost (see Shiaw (2002) and Shiaw (2007)). The repair cost occurs only when every nonconforming predefined quality of a product is larger than specification limit otherwise the replacement cost will take place.

Other costs included are inspection cost and manufacturing cost. Inspection cost occurs only when an inspection station is located after workstation; otherwise manufacturing cost will take place. Inspection cost is a sum of the fixed cost and the variable cost. The fixed cost is a sum of the costs connected with test-equipment installation, setup, etc. The variable cost is the total number of conforming parts and the number of defective parts produced at inspection station multiplied by the inspection cost for carrying out of 100% item-by-item. The vast majority of the researchers used a linear function for the variable inspection cost. The manufacturing cost is thereby assumed to be a sum of the material cost, overhead cost and setup cost.

<table>
<thead>
<tr>
<th>System features</th>
<th>Category</th>
<th>Publications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production line</td>
<td>Serial</td>
<td>Lindsay (64), Pruazan (67), Eppen (74), Balluo (82), Petters (84), Shiaw (02), Lee (98), Shiaw (07), Barad (90), Raz (00), VanVolsem (07), Shiaw (03a), Shiaw (03b), Kogan (02), Rebsto (95), Tayi (88), Raghavachari (91), Rau (05), Jewkes (95), White (69), Park (88), Enrick (75), Kang (90), Yum (87), Bai (96), White (65), Ballou (85), Chengalur (92), Hsu (84), Raz (00), Finkelshein (05), Sadegheii (07), Taneja (94), Freiesleben (06), Kake (04), Rot (91), Langner (02), Galante (07), Chen (98), Yao (99), Kim (08), Jang (02), Taneja (94) Tannock (95), Tannock (97), Lee (96), Siemiatkowski (06), Tannock (07), Crostack (05), Saxena (90)</td>
</tr>
<tr>
<td>Assembly</td>
<td>Non-serial</td>
<td>Penn (08), Hadjinicola (03), Penn (07), Gunter (85), Chen (99), Raz (87), Delimans (96), Valenzuela (04), Gardner (95), Estrop (92), Clark (99), Emmons (02), Rau (05), Taneja (94), Narahari (96), Rau et al. (05)</td>
</tr>
<tr>
<td>Inspection time</td>
<td>Limited</td>
<td>Lee (98), Shiaw (03a), Shiaw (03b), Shiaw (02), Park (88), Bai (96), White (69), Ballou (85), Chengalur (92), Penn (08), Penn (07), Gunter (85), Taneja (94), Shiaw (07), Jang (02), Rau et al. (05)</td>
</tr>
<tr>
<td>AOQL</td>
<td>Rate of inspection</td>
<td>Rebsto (95), Hadjinicola (03)</td>
</tr>
<tr>
<td>Type I and II error</td>
<td>Type I and II error</td>
<td>Lindsay (64), Taneja (94)</td>
</tr>
<tr>
<td>Inspection capability</td>
<td>Free of error</td>
<td>Kakade (04)</td>
</tr>
</tbody>
</table>

| Type II error | Type II error | Lub (95), Delimans (96), Tannock (97) |
| Type II error | Free of error | Kogan (02), Tayi (88), Raghavachari (91), Emmons (02), Barad (90), White (65), VanVolsem (07), Jewkes (95), Park (88), Hadjinicola (03), Narahari (96), Lindsay (64), White (69), Pruazan (67), Penn (08), Penn (07), Hsu (84), Finkelshein (05), Gunter (85), Sadegheii (07), |
3.2. Solution approaches

In the inspection allocation problems, the most common treatment that the models are developed with objective of minimising the expected total cost per unit produced. The total cost includes some or all of the following costs: internal failure cost and external failure cost, inspection cost, and manufacturing cost. Table 1(continued) shows these costs regarding to each paper. However, not all the papers try to minimise the total cost. As shown in Table 2, a few papers (see for example Rebello et al. (1995) and Valenzuela et al. (2004)) have decided to maximise the production capacity. This usually occurs when an inspection scheduling problem and the allocation problem is concurrently considered (Mandrolì et al. (2006)). Constraints that were used by the researchers in the optimization of an inspection are mostly related to the characteristics of the manufacturing system such as the structure of the system, the type of defect and the type of inspection. As shown in Table 1 not all the surveyed papers have addressed the constraints. Limited number of inspection station has considered by most of the researchers, and some other authors have addressed average of outgoing quality level (AOQL) and rate of inspection.

Table 1-Classification of models according to system features (continued)

<table>
<thead>
<tr>
<th>System features</th>
<th>Category</th>
<th>Publications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Internal failure cost</strong></td>
<td>Rework/Repair</td>
<td>Eppen (74), Lee (98), Shiau (03a), Kogon (02), Rebello (95), Rau (05), Barad (90), VanVolsem (07), Jewkes (95), Park (88), Enrick (75), Hadjiminica (03), Kang (90), Yum (87), Narahari (96), Bai (96), White (69), Raz (00), Finkelstein (05), Taneja (94), Shiau (07), Freiesleben (06), Kakade (04), Raz (91), Langner (02), Galante (07), Chen (99), Deliman (96), Chen (98), Yao (99), Kim (08), Jang (02), Petters (84), Valenzuela (04), Tannock (95), Tannock (97), Tannock (07), Crostack (05), Saxena (90), Clark (99), Rau et al. (05)</td>
</tr>
<tr>
<td><strong>Replace</strong></td>
<td></td>
<td>Shiau (03a), Shiau (03b), Shiau (02), Rebello (95), Barad (90), VanVolsem (07), White (69), Yum (87), White (65), Hsu (84), Clark (99)</td>
</tr>
<tr>
<td><strong>Scrap</strong></td>
<td></td>
<td>Eppen (74), Lee (98), Shiau (03a), Shiau (03b), Shiau (02), Kogon (02), Rebello (95), Balluo (82), Tayi (88), Raghavachari (91), Rau (05), Barad (90), Park (88), Hadjiminica (03), Kang (90), Yum (87), Narahari (96), Lindsay (64), White (69), Chengalur (92), Hsu (84), Raz (00), Finkelstein (05), Gunter (85), Sadegheh (07), Taneja (94), Shiau (07), Freiesleben (96), Raz (91), Langner (02), Galante (07), Chen (99), Raz (87), Deliman (96), Kim (08), Petters (84), Tannock (07), Gardner (95), Estrop (92), Clark (99), Lee (96), Tannock (95), Tannock (97), Siemiatkowski (06), Crostack (05), Saxena (90), Rau et al. (05)</td>
</tr>
<tr>
<td><strong>External failure cost</strong></td>
<td>Replacement</td>
<td>Lee (98), Shiau (07), Shiau (02), Shiau (03b)</td>
</tr>
<tr>
<td><strong>Repair</strong></td>
<td></td>
<td>Lee (98), Shiau (07), Shiau (02), Shiau (03b)</td>
</tr>
<tr>
<td><strong>Quality loss</strong></td>
<td>Penalty</td>
<td>Shiau (07), Shiau (02), Shiau (03b)</td>
</tr>
<tr>
<td><strong>Penalty</strong></td>
<td></td>
<td>Petters (84), Balluo (82), Penn (07), Raz (00), VanVolsem (07), Rau (05), Kogon (02), Rebello (95), Tayi (88), Raghavachari (91), Rau (05), Jewkes (95), Kang (90), Bai (96), White (69), Pruzan (67), Balluo (85), Penn (07), Chengalur (92), Penn (08), Raz (00), Finkelstein (05), Taneja (94), Kakade (04), Raz (91), Galante (07), Deliman (96), Chen (98), Yao (99), Valenzuela (04), Rau et al. (05)</td>
</tr>
<tr>
<td><strong>Inspection cost</strong></td>
<td>Fixed</td>
<td>Balluo (82), Tayi (88), Park (88), Balluo (85), Chengalur (92), Raz (00), Chen (99), Jang (02), Tannock (07), Gardner (95), Estrop (92), Clark (99)</td>
</tr>
<tr>
<td><strong>Variable</strong></td>
<td></td>
<td>Rebello (95), Rau (05), Emmons (02), Barad (90), VanVolsem (07), Jewkes (95), Park (88), Enrick (75), Hadjiminica (03), Yum (87), Lindsay (64), White (65), Hsu (84), Sadegheh (07), Taneja (94), Shiau (07), Kakade (04), Raz (91), Langner (02), Galante (07), Raz (87), Deliman (96), Chen (98), Yao (99), Valenzuela (04), Lee (96), Tannock (95), Tannock (97), Siemiatkowski (06)</td>
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</table>
The most common approach used by researchers was the dynamic programming (DP) technique because of the multistage arrangement of a manufacturing system that is well described by stages and states of the DP models (Mandrol et al. (2006)). Some previous publications presented an interesting remark. For example, Lee and Unnikrishnan (1998) and Shiau (2002, 2007) have pointed out that the DP approach employed in previous methodologies becomes quite impractical as the set of possible combinations grows exponentially. However, they do not provide any material evidence to prove their remark. Nonlinear programming also has been studied extensively because of the nature of the inspection allocation problem function in which some of the decision variables can only have integer values for example whether or not to inspect at workstation and the serial number of the inspection stations. Other techniques included genetic algorithm and simulated annealing (see Table 2 for more detail).

In surveys conducted by Raz (1986), Menipaz (1978) and Mandrol et al. (2006) the numerical optimisation techniques have been identified as important. Therefore, they did not cover simulation publications which are related to allocation of inspection station problem. In this paper, Monte Carlo simulation methods have been extensively studied. The majority of simulation papers considered in this review focused on simple processes in order to simplify their models. For example, a single process issue was investigated by Clark and Tannock (1999) and Estrop et al. (1992). Most of the simulation papers attempted to answer the second of the research questions. They examined the performance of inspection station allocation through heuristics rules on the basis of the parameters considered in serial production systems under different operating conditions (see for example Saxena et al. (1990), Lee and Chen (1996), Siemiatkowski and Przybylski (2006) and Gardner et al. (1995)).

The review has concluded that the following mathematical methods were used out of 62 papers 29% dynamic programming, 22% nonlinear programming, 10% genetic algorithm, 6% branch and bound, 6% Markov decision, 5% simulated annealing, 3% zero one Integer programming and 2% each of linear programming, Tabu search and expert system. Also the review has shown that 16% out of 63 papers have used Monte Carlo simulation technique.

<table>
<thead>
<tr>
<th>System Features</th>
<th>Category</th>
<th>Publication</th>
</tr>
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<tbody>
<tr>
<td>Solution Optimisation Dynamic</td>
<td>Eppen (74), Lindsay (64), Puzran (67), Petters (84), White (69), Enrick (75), Bai (96), White (65), Penn (08), Hsu (84), Raz (00), Finkelshtein (05), Gunter (85), Yao (99), Chen (98), Chengalur (92), Raghavachari (91), Penn (07)</td>
<td></td>
</tr>
<tr>
<td>Programming</td>
<td>Ballou (85), Narahari (96), Jawkes (95), Lee (98), Hadjinicola (03), Shiu (03a), Shiu (03b), Shiu (02), Barad (90), Rau et al. (05), Rau (05), Ballou (82), Emmons (02), Kogan (02)</td>
<td></td>
</tr>
<tr>
<td>Genetic Algorithm</td>
<td>Galante (07), Freiesleben (06), Shiu (07), Taneja (94), Sadegheii (07), Van Volsem (07)</td>
<td></td>
</tr>
<tr>
<td>Branch and Bound</td>
<td>Penn (07), Langner (02), Raz (91), Raz (87)</td>
<td></td>
</tr>
<tr>
<td>Markov Decision</td>
<td>Jang (02), Kim (08), Tayi (88), Deliman (96)</td>
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</tr>
<tr>
<td>Simulated Annealing</td>
<td>Chen (99), Kakade (04), Sadegheii (07)</td>
<td></td>
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<tr>
<td>Zero One Integer</td>
<td>Park (88), Yum (87)</td>
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</tbody>
</table>
Programming
Tabu Search
Expert System
Linear Programming
Objective Function
Minimum Total Cost
Maximum Profit
Monte Carlo Simulation Technique

Valenzuela (04)
Kang (90)
Rebello (95)
Epen (74), Lindsay (64), Shiau (03a), Shiau (03b), Shiau (02), Tayi (88), Raghavachari (91), Rai et al. (05), Rai (05), Burad (90), VanVolsem (07), Jawes (95), Narahari (96), Bai (96), Ballard (85), Chengalur (92), Raz (00), Taneja (94), Shiau (07), Freiesleben (06), Kakade (04), Raz (91), Langner (02), Galante (07), Chen (99), Chen (98), Yao (99), Jang (02), Kim (08)
Rebello (95), Hadjinicola (03), Penn (08), Penn (07), Valenzuela (04)
Tannock (95), Tannock (97), Lee (96), Siemiatkowski (06) Tannock (07), Saxena (90), Gardner (95), Clark (99), Estrop (92), Crostack (05)

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</thead>
<tbody>
<tr>
<td>4. Model formulation</td>
<td></td>
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<tr>
<td>4.1. Model description</td>
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</table>
To examine the inspection allocation problem, a serial multistage manufacturing system has been studied. Figure 2 illustrates the characteristics of the type of multistage system under consideration as follows:
1. The system is considered to be made up of 10 workstations arranged serially and parts are entering the system in batches.
2. Each workstation has a specific probability of producing defective parts.
3. A 100% inspection screen is applied to all parts processed in workstation if an inspection station is performed after it in the sequence.
4. The system has limited number of inspection stations (e.g. five stations). Each inspection station can be assigned to perform inspection operation for one or more workstations.
5. The selection of an inspection station is subjected to a time constraint.
6. Rework items may be incurring defects in the subsequent reworking process.
7. Only one final product is considered in the system.
8. Nonconforming items can either be scrapped or sent for rework. At each inspection station there is exists a specific probability of selecting nonconforming items for rework.
9. Two types of inspection errors are considered in the system. A type I error involves the classification of a conforming unit (CU) as a nonconforming unit (NCU), and a type II error means the classification of an NCU as a CU.

![Figure 2 Serial manufacturing processes with inspection stations](image)

4.1. Description of indices
- m: Refers to the inspection station assigned;
K: Refers to workstation.

4.2. Description of inputs

n: Number of workstations in the system;
Q: Number of parts entering the system;
q: Number of inspection stations in the manufacturing system;
\( \alpha_m \): Probability that the \( m \) inspection operation incorrectly classifies a conforming unit (CU) as a nonconforming unit (NCU);
\( \beta_m \): Probability that the \( m \) inspection station incorrectly classifies a NCU as a CU;
\( \Delta_m \): Probability of repairing a defective unit at the \( m \) inspection station;
\( E_m \): Probability of repairing parts incurring a defect on subsequent processing at the \( m \) inspection station;
\( D_{km} \): Direct cost of material to repair a defect part;
P: Final sale price of each unit is sold;
\( Z_k \): Probability of a nonconforming of part processing at the \( k \) workstation;
Y: Direct cost of material to repair a defect at the customer’s end;
\( IT_m \): Processing inspection time of parts at \( m \) inspection station;
\( g_k \): Multiplicative fraction of the manufacturing cost expressed as rework cost;
\( x_k \): Unit scraping cost at the \( k \) workstation;
W: Percentage of parts replaced at the customer’s end;
\( I_{km} \): Unit inspection cost at \( m \) inspection station;
\( V_k \): Unit manufacturing cost at the \( k \) workstation;
\( fn_m \): Unit inspection processing time at \( m \) inspection station.

4.3. Description of variables

\( NG_k \): Number of conforming parts leaving the \( k \) workstation;
\( ND_k \): Number of defective parts leaving the \( k \) workstation;
\( RSC_{km} \): Rework scraped cost of parts on subsequent processing at \( k \) workstation;
\( NR_{km} \): Number of parts sent back for rework to the previous workstation \( k \);
\( MC_k \): Total manufacturing cost for parts at the \( k \) workstation;
\( IT_{km} \): Total inspection cost at \( m \) inspection station;
\( NS_k \): Number of parts scraped at the \( k \) workstation;
\( NRS_{km} \): Number of parts discarded after reworking processing;
\( RC_{km} \): Rework cost at the \( k \) workstation;
TC: Total cost of manufacturing parts.

4.4. Work flow analysis

Flow constraints consider the number of conforming parts departure a workstation or inspection location, and the number of defective parts entering a following workstation or inspection location. The following equation is represented the first stage.

\[ NG_1 = Q(1 - Z_1) \]  \hspace{1cm} (1)

For all other stations the equation is defined recursively as follows:

\[ NG_k = [NG_{k-1} - (1 - \alpha_{k-1}) + ND_{k-1} \beta_{k-1} + NR_{k-1}] \cdot (1 - Z_k) \]  \hspace{1cm} (2)

The number of defective parts produced at first a workstation is: \( ND_1 = Q \cdot Z_1 \)  \hspace{1cm} (3)

For all other stations, it is:

\[ ND_k = [NG_{k-1} \cdot (1 - \alpha_{k-1}) + (ND_{k-1} \cdot (1 - \beta_{k-1})) + NR_{k-1}] \cdot Z_k \]  \hspace{1cm} (4)
The number of parts classified as defective by inspection station $m$ after workstation $k$ but can be repaired is given by:

$$NR_k = NG_k \ast \alpha_{km} + ND_k (1 - \beta_{km})$$  

(5)

5. Cost model analysis

5.1. Expected manufacturing cost

This cost is assumed to be a sum of the material cost, overhead cost, and setup cost. The number of parts processed at station $k$ is the sum of the number of parts correctly classified as conforming parts flowing into station $k$ from the earlier process station and the number of defective parts incorrectly classified as conforming parts flowing into station $k$ from the previous process station. Hence, the manufacturing cost ($MC_k$) is defined as follows:

$$MC_k = [NG_{k-1} \ast (1 - \alpha_{k-1}) + ND_{k-1} \ast \beta_{k-1}] \ast V_k$$  

(6)

In the case where no inspection station is performed, $\alpha_{k-1} = 0$ and $\beta_{k-1} = 1$. Thus:

$$MC_k = [NG_{k-1} + ND_{k-1}] \ast V_k$$  

(7)

5.2. Expected inspection cost

Inspection cost is consists of sum of the fixed cost and variable cost. The fixed cost is sum of the costs connected with test-equipment installation, setup, calibration, etc. The variable cost is the total number of conforming parts and the number of defective parts produced at station $k$ multiplied by the inspection cost for carrying out of 100% item-by-item inspection of incoming batches. Therefore, the total inspection cost is given by:

$$TIC_{km} = FC_{km} + [(NG_k + ND_k) \ast I_{km}]$$  

(8)

5.3. Internal failure cost

The internal failure cost is the sum of reworking cost and scrap cost.

- **Reworking cost**
  
  This is the cost of reworking a nonconforming part identified at an inspection station. At each inspection station the nonconforming parts can be scrapped, sent back for repair or incorrectly classified as conforming parts. The number of parts as nonconforming but repairable are given by:

$$NR_{km} = [NG \ast \alpha_{km} + ND (1 - \beta_{km})] \ast A_k$$  

(9)

Then the rework cost is:

$$RC_{km} = (NR_{km}) \ast (g_k \ast V_k)$$  

(10)

The rework parts may be incurred defects on subsequent processing as they did in the original process.

$$NRS_{km} = NR_{km} \ast E_k$$  

(11)

- **Scrap cost**
  
  This expression represents the number of non-repairable items produced at $k$ station on detection subsequent $m$ inspection stage.

$$NS_k = NG_k \ast \alpha_{km} + ND_k (1 - \beta_{km})$$  

(12)

The scrap cost is:

$$SC_{km} = NS_k \ast x_k$$  

(13)

Also the scrap cost that may result from a subsequent reworking process is given by:
\[ RSC_{km} = NRS_{km} \cdot x_k \] (14)

Then the internal failure cost is given by:

\[ IFC_{km} = SC_{km} + RSC_{km} + RC_{km} \] (15)

5.4. External failure cost

This is the cost incurred after the products have been sold to customers. Examples include the cost of replacement and repair. External failure cost (EFC), is the sum of the product of the number of defective parts replaced at the customer’s end (W * ND_k), the sale price (P) of the part and the sum of the product of the number of defective parts repaired at the customer’s end (1 - W) and the direct cost of materials to repair a defective unit (Y).

\[ EFC = W \cdot ND_k \cdot P + (1 - W) \cdot ND_k \cdot Y \] (16)

The inspection screen is applied to all parts processed in workstation if an inspection operation is performed, otherwise:

\[ I_{km} = \alpha_k = \Delta_n = 0 \text{ and } \beta_k = 1.0 \]

The total cost (TC) of processing and inspection of Q parts in an n-stage serial manufacturing system is given by the following equation:

\[ TC = \sum_{i=1}^{n} (IFC_i + MC_i + IC_{km} + EFC) \] (17)

The sum of the total cost of processing and inspecting the parts produced in the manufacturing system is expressed as the total system cost TC. The objective function for the inspection station allocation and assignment problem for a manufacturing system producing parts is expressed as follows:

Minimise \[ TC = \sum_{i=1}^{n} TC \] (18)

In this paper, the objective function is constrained by time of inspection to reduce number of inspection plans and to maintain the nominal production rate. General time equation involves inspecting all parts plus set-up time at inspection station m in the manufacturing system. The constraints are the following:

\[ IT_m \leq [NG_k + ND_k] \cdot fn_m + tss_m \] (19)

\[ \sum_{i=1}^{n} q_i \leq NI \] (20)

Equation (20) shows that there is limited number of inspection stations.

The allocation of quality inspection station problem grows exponentially with the number of workstations. For example, in a 21 workstation, there are more than 2,000,000 (M=2^n) ways to locate inspection positions and select a capable inspection station. Assume C consists of all assignment location combinations, then:

\[ C = [(X_{11}, ..., X_{i1}, ..., X_{n1}), ..., (X_{1m}, ..., X_{km}, ..., X_{nm}), ..., (X_{1M}, ..., X_{kM}, ..., X_{nM})] \]

If workstation k should be screened by an inspection station m, then \( X_{km} = 1 \), otherwise \( X_{km} = 0 \). A possible allocation plan among the lowest expected sum cost can then be determined after reconsidering every assignment location combination. As shown in Figure 3 the processing time to solve the problem exponentially grows with increased number of workstation (WS). The experimental data were approximated using exponential regression model which has shown correlation \( r = 0.99 \) and the coefficient of the determination is \( r^2 = 0.99 \) see equation (21).

\[ Time = 0.0003 \cdot e^{(0.07 \cdot WS)} \] (21)

For example, using equation (21), the duration of computation for 20 workstations is expected to be 180 hours. Therefore, it is impractical to allocate inspection places by using EM. In order to find a
solution when the number of workstation increases, a heuristic method was developed which will be introduced in the following section.

![Graph showing the duration of computational time in relation to the number of workstations](image)

**Figure 3** Schematic showing the duration of computational time in relation to the number of workstations

### 6. Heuristic method

The heuristic method (HM) aims to determine the inspection plan with the lowest total cost. The objective function of the total cost in this particular case is constrained by inspection time. The proposed method is named Heuristic Method Time Constraint (HMTC). In HMTC the number of items inspected is multiplied by the inspection processing time allocated to each item which should be less than or equal to the inspection time assigned for the inspection station being considered, (see equation 19). This is done in order to maintain the nominal production rate and to reduce the number of feasible plans to be evaluated.

In the heuristic method, every assigned location in C is reviewed to determine the inspection plans that have to be assigned for one workstation. The inspection plan will be considered when \( X_{km} = 1 \) and the sum of row elements of each of the generated inspection station plans match the number of inspection stations required. For example, if an inspection plan in the assignment location combination is denoted by 011100110, it means that it has 5 inspection stations, assuming the available number of inspection stations is limited to 5. This inspection plan will be considered, because the number of inspection stations matches the number of inspection stations required. If the condition is not met, then the inspection plan will be rejected because it is unnecessary to check the inspection plans for unsatisfactory assignment location combination. The above approach will avoid the unnecessary processing. The heuristic method idea is to minimise the cost by early identification of a non-conforming products in the manufacturing processes. The following steps describe the heuristic procedure:

**Step1** Generate the set of assignment location combinations C, for a multistage manufacturing system.

**Step2** Based on limited inspection stations, decide the number of inspection stations required.

**Step3** Check whether the inspection plans match the number of inspection stations available. If yes go to step 4, otherwise go back to step 2.

**Step4** Calculate the inspection time for the inspection station (IT_{in}) in the inspection plan. If (IT_{in}) less than or equal to the inspection time assigned for the inspection station being considered, go to step 5. Otherwise this inspection plan will not be considered, go back to step 3.

**Step5** According to the order of workstations, if \( X_{km} = 1 \) calculate total inspection cost, internal failure cost and manufacturing cost. If \( X_{km} = 0 \), calculate total a manufacturing cost and external failure cost. Go to step 6.

**Step6** Calculate the total inspection cost plan. Go to step7.

**Step7** Check whether all inspection stations have been taken into account. If yes go to step 8, otherwise go back to step 4.

**Step8** Check whether all inspection plans are considered. If yes go to step 9, otherwise go back to step 3.

**Step9** Determine the inspection plan that has the lowest cost.
7. Case study and discussion

The following simulation experiment was conducted when measuring the time used by the heuristic method to find the best positions for inspection stations. A multistage manufacturing system model with 10 workstations with different parameters arranged in a serial manner was used to allocate 5 inspection stations as shown in Table 3. The batch size used in the experiment was 100. The parameters were randomly generated using a uniform random number generator to evaluate the processing time efficiency. It was found that the heuristic method (HM) and the one with time constraint (HMTC) have better processing time efficiency than that in the EM.

Table 3- Performance parameters

<table>
<thead>
<tr>
<th>Performance parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>fnm</td>
<td>4-7</td>
</tr>
<tr>
<td>Vh</td>
<td>85-180</td>
</tr>
<tr>
<td>Zk</td>
<td>0.09-0.18</td>
</tr>
<tr>
<td>lam</td>
<td>60-120</td>
</tr>
<tr>
<td>a</td>
<td>0.03-0.07</td>
</tr>
<tr>
<td>B</td>
<td>0.03-0.07</td>
</tr>
<tr>
<td>A</td>
<td>0.05-0.09</td>
</tr>
<tr>
<td>Xk</td>
<td>85-120</td>
</tr>
</tbody>
</table>

Table 4 shows that HMTC can reduce the number of inspection plans down to 88.57% whereas HM can reduce them down to 75.39%. Table 4 shows that the HMTC and HM can produce the optimal solution. However, the cost deviation respectively is only 0.03 and 0.01.

On the other hand, the HM still has a bigger savings (75.63%) in terms of processing time than that of the HMTC. Therefore, it can be concluded that the HM can be applied when multistage manufacturing system incorporates more workstations.

Table 4 - Performance of the HM in comparison to EM

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of inspection plans considered</th>
<th>Time processing (seconds)</th>
<th>Reducing inspection plans</th>
<th>Saving Time %</th>
<th>Total cost</th>
<th>Cost deviation (100)</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM</td>
<td>252</td>
<td>31.2</td>
<td>75.39%</td>
<td>75.63%</td>
<td>367126</td>
<td>[A/B - 1]100</td>
<td>HM</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>HMTC</td>
<td>117</td>
<td>36.1</td>
<td>88.57%</td>
<td>71.80%</td>
<td>362619</td>
<td>B</td>
<td>0.01</td>
</tr>
<tr>
<td>EM</td>
<td>1024</td>
<td>128</td>
<td></td>
<td>88.57%</td>
<td>355589</td>
<td>HMTC</td>
<td></td>
</tr>
</tbody>
</table>

8. Conclusions

1. There is a need for a generalised (composite) model addressing all the similar inspection allocation scenarios. The model can serve as a metaheuristic that will be used to select an appropriate heuristic or algorithm for the solution of a particular of allocation of quality control stations (AQC) problem.
2. The simulation experiment has shown that computational time increases significantly when the number of workstations is more than 20.
3. The majority of the papers reviewed in this research analytically consider models with limited number of work stations between 3 and 5 which is insufficient in practical situations.
4. The review shown that the inspection allocation problem has been studied comprehensively using variety of analytical and Monte Carlo simulation methods rather than combination of both simulation techniques in a simulation-optimisation framework.

References


Kogan, K., & Raz, T. (2002). Optimal allocation of inspection effort over a finite planning horizon, IIE Transactions, 34, 515-527.


