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Skin Temperatures and Heat Transfer over Wedge Wings at Extreme Speeds of Flight

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SUMMARY

By paying special attention to the wing design and altitude of flight, it is possible to ensure that the highest temperature reached at the leading edge of the wing of an aircraft, in level flight at speeds of the order of the circling velocity, need be no more than about 1000°C. Formulae and charts are presented to enable the actual skin temperature close to the nose to be predicted, for a wedge-shaped wing, in terms of skin thickness and conductivity.

Contents

Notation Used in Main Text

L — wing loading

L_m — measure of L in kg/sq.m (1 kg/sq.m = 0.205 lb/sq.ft)

Q — constant chosen so that $Q/\sqrt{x}$ approximates to rate of heat transfer per unit surface area from boundary layer close to leading edge

Q_{max} — maximum of Q with respect to flight speed (achieved for $q_1^2 = \frac{2}{3} g R$)

$Q(\delta)$ — value of Q relative to that for flat plate moving parallel to itself, at same speed, and having same surface pressure

R — earth's radius

T — surface temperature

T_{LE} — value of T at wing leading-edge

T_{max} — maximum of T_{LE} with respect to flight speed (achieved for $q_1^2 = \frac{2}{3} g R$)

d — skin thickness

d_m — measure of d in cms.

f — non-dimensional temperature defined by equation (1), and evaluated in fig.2

g — gravitational acceleration

k — thermal conductivity of skin

k_m — measure of k in cals/cm. sec. deg.C. (1 cal/ cm. sec.deg.C = 242 B. Th. U/ft. hr. deg. F)

ℓ — characteristic length of surface affected by conductivity, and defined by equation (2).

p_δ surface pressure on underside of wedge aerofoil, taken as constant, (measured in atmospheres)

q_1 speed of flight (measured in Km./sec.)

x distance from leading edge

x_m measure of x in metres

δ inclination of undersurface of wedge to direction of flight (positive if forward facing)

ε_o emissivity of external surface of underside of wing

ε_i rate of nett heat loss by radiation per unit area of interior surface of underside of wing $\div \sigma T^4$

σ Stefan's Constant ($= 5.7 \times 10^{-8}$ watts/sq.m. $(\text{deg.C})^4$,

$= 1.73 \times 10^{-9}$ B.Th.U/sq.ft.hr $(\text{deg.F})^4$.)

1. Introduction

At speeds of flight usual for present-day aircraft it is well-known that due to aerodynamic heating, the wing surface of an aircraft - at least near its nose - reaches the "thermometer" temperature whose value depends only on the speed of flight. If the same were true at such speeds as would, for example, be involved in flight of satellite vehicles, the consequences would be profound, because the corresponding thermometer temperatures are of the order of 10,000°C and more. Fortunately, it is not true, and the present report discusses one limit to the maximum temperature reached close to the wing leading-edge, in sustained level flight at such speeds, as imposed by the action of conduction of heat within and along the skin. There may well be other and indeed lower limits, but the one considered can be established with a fair amount of certainty, even if it represents an unnecessarily pessimistic view of the intensity of kinetic heating.

The results we deduce are relevant only to a wing of wedge section (constructed as a hollow shell with a thin skin), not because of any particular merit that this section may have, but because the boundary layer flow about it may be most easily deduced for flight at such extreme speeds as those we have in mind. In the same way, the aircraft might well not be designed as a "flying-wing" so that the temperature reached by the fuselage would also present a problem of some importance. However, as we shall show that the wing loading

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must be low, and the wing thickness (or wedge angle) can be without penalty very large, it is likely that an all-wing design might after all be involved.

We first of all establish (in section 2) how we may determine the limit to the skin temperature, and also the rate of heat transfer at extreme speeds. Details of this latter determination are given in Appendix I, and as the flow in the boundary layer is affected by the intense shock wave developed by a wedge at incidence, it is necessary also to discuss the relevant shock wave conditions: this is done in Appendix II. We next suggest (in section 3) measures which may be adopted to reduce the maximum skin temperature found by these means; and finally, by way of illustration, we calculate numerical values of the temperature reached by an aircraft, designed to include these measures, and flying at the particular speed at which kinetic heating effects are most severe. This speed is incidentally shown to be equal to just over 80% of the circling velocity (namely, 6.5 Km/sec.). It is the intention to show that particular attention to the flight plan and overall design can greatly simplify the problems of kinetic heating. Nose temperatures of no more than around 1000°C are all that need be involved. Downstream of the nose temperatures will of course become progressively smaller still, but it is not our object to determine anything other than a limit to the maximum values.

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2. The Deduction of a Limit to Skin Temperature.

Supposing that there are no regions of turbulent flow, and that steady conditions prevail, the highest temperature will be reached at the nose of the body or wing surface, where the heat transfer is greatest. As mentioned in the introduction, in what follows we shall be dealing expressly with a wedge shaped wing (fig.1a) constructed as a hollow shell. The surface pressures in inviscid flow over such a wing are constant on the upper, lower and base surfaces, and the usual approach of boundary layer theory shows that at the nose, the rate of heat transfer due to kinetic heating is infinite, unless the local surface temperature has the particular "thermometer" value. If this approach were indeed valid, then it would follow that a non-conducting skin would instantaneously heat up to the thermometer temperature characteristic of the flight condition.

However, the argument which leads to this deduction can be faulted on two counts. In the first place, the rate of heat transfer is not likely to be infinite at the nose, and the theory which predicts such a singularity breaks down in this vicinity. For a start it is well-known that the simplified form of the Prandtl equations of motion and of energy, usually invoked in boundary layer analysis, are inapplicable in this region, and the full Navier Stokes equations are to be preferred, though they have proved intractable. Quite apart from this, there are modifications to the usual solutions to account

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for the self-induced pressure field of the boundary layer, and the proximity of the leading edge shock wave, and also to account for the existance of a velocity of slip and a temperature jump at the surface. All of these factors are known to produce effects of increasing importance towards the leading-edge, which might well determine a maximum rate of heat transfer. Whilst orders of magnitude of this maximum rate can be estimated by existing analyses, it is not considered that sufficient is known as yet to predict the value with any certainty.

In what follows we shall treat the usual approximations to the rate of kinetic heating as valid right up to the leading-edge, and although this is plainly wrong, it does err on the pessimistic side in providing a local over-estimate of heat flux. But even then it does not follow that the nose will heat up to the "thermometer temperature", because the second assumption we mentiohed in connection with this deduction can also be faulted. This is that the skin is a non-conductor of heat. Whilst this may be an adequate assumption in dealing with the surface temperature distribution downstream, it can certainly lead to important errors in the vicinity of the nose, where - if the skin were really non-conducting, there would exist severe longitudinal temperature gradients; even in a medium of small conductivity these gradients would imply an appreciable longitudinal rate of heat flux along the skin, which would tend to "cool" the nose at the expense of the structure downstream.

The problem of the action of conductivity in limiting the nose temperature has been discussed elsewhere[1]. The assumptions of this analysis are that:

(i) The skin is of uniform thickness d, which is small compared with the characteristic length ℓ of the region affected by conductivity (to be defined later)

(ii) The rate of kinetic heating per unit area to the external surface is $Q/x^{1/2}$, where x is the distance from the nose, and Q may be taken as a constant over the length ℓ. (The value of Q may be different on upper and lower wing surfaces).

(iii) The exterior surface loses by radiation a nett heat flux equal to $\varepsilon_o \sigma T^4$ where σ is Stefan's constant, and T the surface temperature; and the interior surface suffers a nett loss of heat by radiation equal to $\varepsilon_i \sigma T^4$

(iv) The interior of the shell formed by the wing skin is non-conducting.

Then it follows that

$$T = \left[\frac{Q^4}{(\varepsilon_o+\varepsilon_i)^3 \sigma^3 k d}\right]^{1/13} f\left(\frac{x}{\ell}\right) \qquad (1)$$

where ℓ is the characteristic length and

$$\ell = \left[\frac{k^4 d^4}{(\varepsilon_o+\varepsilon_i)\sigma Q^3}\right]^{2/13} \qquad (2)$$

These relations can be deduced on the basis of dimensional homogeneity, and the function f satisfies a non-linear second-order differential equation. The function f is shown in fig.2, which represents the result of a more refined method of solution of the differential equation involvel than that given in the reference work. The evaluation of the maximum temperature which - as will be seen from fig.2 - is attained at the nose ($x = 0$), is possible once we have allotted a suitable value to the constant, Q. As the heat transfer at, or close to, the nose depends only on the local value of the surface temperature, and not on its downstream variation, it is possible to evaluate Q by reference to calculations of the heat transfer to surfaces with constant surface temperature, which fortunately is the condition most usually studied. This temperature is placed equal to T_{LE} and then the relation

$$T_{LE} = 1.15 \left[\frac{Q^4}{(\varepsilon_o + \varepsilon_i)^3 \sigma^3 kd} \right]^{1/13} \qquad (3)$$

obtained by noting that $f(0) = 1.15$, is an implicit equation for T_{LE}.

At extreme flight Mach numbers it was shown in ref.2 that Q is nearly independent of the surface temperature - unless it has improbably high values. It is only flight at extreme speeds which interests us in the present discussion, so that the results of this reference work will be invoked. They are relevant to the laminar boundary layer on a surface at uniform pressure p_δ moving at velocity

q_1, and inclined at only a small angle of incidence to the flow, and they show that

$$\frac{Q}{x^{1/2}} = 44\, q_1^2 \left(\frac{p_\delta}{x_m}\right)^{1/2} \text{Kw/sq m.}$$

where q_1 is in Km/sec., p_δ in atmospheres and x_m in metres. This dimensional form of the result demands no assumption about the atmospheric conditions prevailing as will be clear from the analysis of Appendix I, which generalises the previous results so as to make them applicable to forward facing surfaces inclined at a finite angle δ to the direction of motion. We find in fact in this Appendix that if dissociation effects are ignored

$$Q = 44\, Q(\delta) q_1^2 p_\delta^{1/2} \quad \text{Kw/(m.)}^{3/2} \qquad (4)$$

where the function $Q(\delta)$ is shown in fig.3 for all angles δ below that for shock wave detachment at high speeds. This is taken to be about 50°, which is higher than the figure usually quoted: however the figure used is based on an allowance for the change in the specific heats of air at the elevated temperatures existing behind an intense shock wave. The precise form of these allowances is detailed in Appendix II, and a comparison of the variation of $Q(\delta)$ obtained by assuming the specific heats constant is provided in fig.3. In this matter it may also be pointed out that some investigators have suggested that the effects of radiation from behind an intense shockwave itself are such that it can be treated as virtually an isothermal process, in which event the corresponding form of the function $Q(\delta)$ approximates simply to $Q(\delta) = \cos^2\delta$, also shown in fig.3.

It will be noticed that all these assumptions lead to the deduction that the heat transfer reduces with increasing surface deflection, δ. The adopted assumptions of the present work suggest in fact a reduction intermediate between the others. This reduction over a forward facing surface may appear unreasonable unless it is recalled that the comparison is based on surfaces subjected to equal pressure (p_δ) . An inclined surface will only preserve this same pressure if the altitude of flight is higher, and the speed of flow outside the boundary layer is lower. If the altitude were kept the same, then of course the inclined surface would be subjected to a much higher rate of heat transfer. For all rearward facing surfaces, the arguments of ref. 2 remain unaffected and the value of $Q(\delta)$ can be consequently taken as unity for $\delta \leqslant 0$.

The above discussion was prefixed by the statements that there were to be no regions of turbulent flow, and that steady conditions prevail. Whether or not steady conditions prevail, will largely depend on the flight plan: in all consequent assessments we shall take the wing to be on an aircraft in steady level flight. But it must be pointed out that in most other conditions of interest, the rate of heat transfer at the nose is so large that steady conditions would at least be quickly reached in the region affected by conductivity.

Again, in regard to the implied possibility that higher rates of kinetic heating - and with them, higher surface temperatures, might be reached in regions of turbulent flow, if present, it should be noted that over a forward facing inclined surface the flow outside the boundary layer has a relatively small Mach Number (of order unity) and a very high temperature (of order M_1^2 times that of the ambient air, if M_1 is the flight Mach Number). Thus, the ratio of surface to local static temperature is in all cases of interest small (and much less than unity). Almost certainly this would suggest that the rate of heat transfer is sufficiently large to stabilise the boundary layer in all disturbances; research in this field[3] has already deduced this to be true at least for two-dimensional wave type disturbances in such conditions, whatever the Reynolds Number of the flow.

3. Measures for the Reduction of the Maximum Skin Temperature.

Equations (3) and (4) above provide a basis for the calculation of the maximum skin temperature. Without introducing any numerical values, they permit us to deduce qualitatively those measures which will reduce its value.

For instance, increase of skin conductivity evidently lowers the maximum temperature reached and this might conceivably affect the choice of material used, though the author is not competent to discuss all the issues that such a measure might involve. Moreover it must be remembered that the conductivity of materials at elevated temperatures can be very different from its value at ordinary temperatures: for most pure metals it is lower, and for alloys higher. For example all steels or ferrous substances at high temperatures tend to have about the same conductivity, though the high alloys are poor conductors in other circumstances. On the other hand, it must also be recalled that the use of special conducting material would be necessary only over the length of the order ℓ (the characteristic length defined by equation (2)). In most examples of interest the numerical value of ℓ is not more than a few centimetres;thus the opportunity for choice of material is probably wider than might at first sight appear. This important fact must also be remembered in many other connections.

Evidently it also pays to make the skin thickness as large as possible, again over the length affected significantly by conductivity. However, if the skin thickness d becomes appreciable compared with ℓ we cannot expect the analysis leading to the results under consideration to remain valid, as this violates one of the simplifying assumptions of the method. This point will be returned to at a later stage, but the general conclusion is always valid that the thicker the skin, the smaller will be the temperature.

Again the emissivity of the outer surface should be made as high as possible, to radiate as much heat away as possible. This is well-known as a desirable feature of design. It equally pays to radiate heat away from the inner surface, provided it is not reflected or radiated back to it again. Now in relation to the particular wing under consideration, provided the wedge angle of the wing is not so large that the upper surface is forward facing, the pressure on this surface will be very small, and as a result so also will be the heat input to it by kinetic heating. Thus one can afford to radiate heat to it from the underside, which is suffering severe kinetic heating. All surfaces being "black" half of the heat emitted by the inner surface of the underside is returned from the facing surface, while half is radiated away from the outer surface of the upper side. The temperature of the upper side becomes $(1/\sqrt[4]{2})$ times that of the under side, and the value of $\varepsilon_i = 0.5$. (If any of the surfaces are not "black", then $\varepsilon_i < 1/2$.) We see from (3) that allowing a transfer of heat by radiation of this kind is equivalent in effect to over a threefold increase in thickness in alleviating the maximum temperature. In case it is undesirable to heat any particular part of the wing interior in this way, it would be a simple matter to provide it with a reflecting surface by way of insulation.

Even greater reductions can be obtained by projecting the underside of the wing forward of the other so that radiation from both its sides can be obtained, uninhibited by any re-emission and absorption (fig. 1b); in this way with black surfaces it would be appropriate to take $\varepsilon_o = \varepsilon_i = 1$. This may not appear too undesirable a modification if it is again remembered that the projection need be only of short length, to achieve the desired effect.

The only other measure open for us to adopt, to decrease the temperature, is to decrease the rate of heat transfer from the boundary layer, so far as is possible. Now equation (4) shows that at a fixed speed of flight this means decreasing the surface pressure, or increasing the lower surface inclination (δ). For a wing of the type considered, in which the upper surface pressure will be small compared with that of the underside at extreme Mach Numbers, the lift loading will be ($p_\delta \cos\delta$), whilst if the wing loading is L, say, the lift loading required for level flight is

$$L\left(1 - \frac{q_1^2}{gR}\right) = p_\delta \cos\delta \qquad (5)$$

R being the radius of the earth. Hence we see that reduction of surface pressure, and so of temperature, is really a matter of reducing the wing loading.

Equations (4) and (5) together show that the angle of inclination (or incidence) of the underside may be increased with advantage provided that $\left[Q(\delta)/\sqrt{\cos\delta}\right]$ is thereby decreased. The least value of this function is probably attained for a value in excess of the angle δ required for shockwave detachment; but as the analysis quoted is not applicable in this range, we can only say with assurance that increasing the altitude of flight, up to the highest value reconcilable with the maintainance of an attached shock at the wedge leading edge will reduce the rate of heat transfer, and so be of advantage. Such large wing incidences as are thereby involved have another advantage, in that it is possible to use quite large wedge angles for the wing section, without making any appreciable difference to the heat flux to the upper surface, which will be to all intents and purposes exposed to a near-vacuum.

To sum up, the measures to be adopted to reduce the maximum temperature are:

(i) To provide an adequate skin thickness;
(ii) To use a material of high conductivity;
(iii) To project the underside forward of the upper;

} over a length of the underside of the order of the characteristic length, ℓ;

(iv) To ensure that all surfaces have a good emissivity, even if this implies allowing the upper side of the wing to be heated by the underside;

(v) To ensure that the wedge angle of the wing is not large enough to cause the upper surface pressure to be appreciable compared with that existing over the underside;

(vi) To allow flight to take place at an altitude at least as high as that compatible with the existence of an attached shock wave at the leading-edge;

(vii) To design the aircraft with a small wing loading.

4. Some Numerical Values

It is instructive to see how the application of the measures just discussed lead to relatively modest values of temperature rise even in the most extreme conditions of flight.

We see in fact from relations (4) and (5) that the heat transfer will be highest at the speed at which

$$q_1^2\left(1 - \frac{q_1^2}{gR}\right)^{1/2}$$

is maximum, i.e. at $q_1^2 = 2/3\ g\ R$. This corresponds to 6.5 Km/sec., and might typically imply a flight Mach Number of about 20. The relative reduction in nose temperature at other speeds, supposing that the altitude is adjusted to give a constant lift coefficient, is shown in fig.4.

We take the incidence as being that giving a maximum deflection shock at these extreme speeds, so that from (4) and (5), in dimensional terms,

$$Q_{max} = 6.85\ L_m^{1/2}\ \text{Kw/m}^{3/2}\ (L_m \text{ in Kg/sq.m.}) \qquad (6)$$

The altitude giving this rate of heat transfer will be such that the lift coefficient equal to 1.23 and the dynamic head is 0.271 times the wing loading. Thus if the wing loading were 100 Kg/sq.m (20.5 lb/sq.ft), the indicated air speed would be only 20 metres/sec. (45 m.p.h.). The relative air density would be about 10^{-5}, implying a height in the region of 80 Km (50 miles). The structural advantages resulting from a flight path restrained to employ such low indicated air speeds, need no amplification.

Assuming the underside surface projects forward of the upper one, and all surfaces are black (so that $\varepsilon_o = \varepsilon_i = 1$), we have from (6), in (3), that at this critical speed the nose temperature reaches its highest value of

$$T_{max} = 624\left(\frac{L_m^2}{k_m d_m}\right)^{1/13} \quad \text{deg.C.absolute} \qquad (7)$$

where L_m is in kg/sq.m as before, k_m is in cals/cm.sec. deg.C, and

d_m in cms. An alinement chart for a rapid interpretation of this result is given in fig.5. From this one may quickly be persuaded that nose temperatures in this, the most extreme condition of flight likely to be encountered, can still be of reasonable proportions. Figures of around 1000°C are all that need be involved. Remember, too, that these figures are probably based on a pessimistic estimate of the heat transfer in the neighbourhood of the nose.

One reservation must be made at this stage, however. It will be recalled that the above result has been obtained on the assumption that (d/ℓ) is small. A few specimen exact calculations (for a projecting lip bevelled at the nose to form a wedge, as shown in fig. 1c) have indicated that where the value of d/ℓ is about unity, the actual nose temperature is higher, by nearly 15%, than that which would be predicted by (7). This may not sound too serious an error until it is realised that, to obtain the correct answer by equation (7), we should have to substitute in place of d_m a figure for the "effective" skin thickness which is only one fifth of the actual thickness. Greater skin thicknesses than ℓ appear to produce little or no further reduction in temperature, so that the effective skin thickness is never likely to exceed $1/5\,\ell$, no matter how thick the skin is made.

In view of this is is safer in practice to use the relation obtained from equations (2) (3) and (6),

$$T_{max} = 1.15 \left[\frac{Q^2_{max}}{(\varepsilon_o + \varepsilon_i)\sigma k (d/\ell)} \right]^{1/5}$$

$$= 288 \left[\frac{L_m}{k_m (d/\ell)} \right]^{1/5} \quad \text{deg.C.absolute}$$

which gives T_{max} in terms of an effective value of d/ℓ rather than d. A nomogram to interpret maximum temperatures on this basis is given in fig.6. The chart of fig.5 may then subsequently be used to determine d, and so also ℓ, relevant to the initial choice of (d/ℓ). But it will be appreciated that the value of d so found is an "effective" thickness, and the actual skin thickness required will be considerably larger if the initial value of (d/ℓ) has been chosen as around 0.1.

In many structural considerations, the temperature gradient and the duration of time during which severe heating occurs are of interest. From equations (1) and (2) and by references to fig.2, it is possible to show that the maximum temperature gradient is

$$\left(\frac{\partial T}{\partial x}\right)_{max} = 0.11 \frac{T_{max}}{\ell}$$

and those measures which reduce the nose temperature will also reduce the temperature gradient.So far as the duration of the period of high temperature is concerned, it all depends on the flight plan. A time scale is suggested in fig.4 for a glide path at constant path at constant lift coefficient. Once again an advantage of using a high incidence appears, this time in that the L/D ratio being low, a relatively rapid decelleration is achieved.

Conclusions

(i) The actual rate of heat transfer to the surface at the nose of a wedge wing may well be finite; but even if it is assumed to go to infinity (like $1/\sqrt{x}$) as predicted by boundary layer theories, then the actual surface temperature is considerably below the thermometer temperature at the nose, due to conduction of heat along and within the skin.

(ii) Equations (3) and (4) of the main text together enable the temperature at the nose of the wing to be estimated at high flight speeds

(iii) The temperature at the nose is the maximum reached by the surface; higher values might be reached if turbulent flow existed, but this seems unlikely due to the stabilising effect of the high rates of heat transfer on the laminar flow.

(iv) An estimate is made (in Appendix I) of the rate of heat transfer to an inclined wedge aerofoil at extreme speeds, taking account of the conditions of flow behind the intense nose shock wave (Appendix II). If the surface pressure and speed of flight are kept constant, increasing the wing incidence (i.e. increasing the altitude of flight) serves to decrease heat transfer.

(v) This is true at least up to incidences which would cause the shock wave to become detached from the nose of the wing. Due to the change in specific heats of air at elevated temperatures the maximum deflection through a shock wave may be greater than that usually suggested (Appendix II).

(vi) Certain measures which reduce the maximum temperature reached by the surface (at the nose) are detailed in section 3, and summarised at the end of this section.

(vii) The nose temperature will reach its highest value at a speed equal to $\sqrt{\frac{2}{3}}$ the circling velocity (i.e. at 6.5 Km/sec). The minimum temperature at other speeds of flight is shown in fig.4.

(viii) For flight at this speed, charts are given in figs. 5 and 6 to enable the maximum temperature to be determined. The temperatures involved need only be around about 1000°C if appropriate but not unreasonable measures are taken to ensure this.

(ix) In section 4 it is noted that there is a limit to the reduction which can be made to the nose temperature by increasing the skin thickness.

(x) The above conclusions are based on a neglect of dissociation, and its presence will affect both the shock wave conditions and the boundary layer heat transfer properties. In at least the latter connection there appears at the moment to be no adequate way of taking it into account.

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Notation used in Appendices

C		Sutherland's constant $\left(\mu \propto T^{3/2}\Big/T+C\right)$
F		shear stress at wall
$F(\delta)$		shear stress relative to that of uninclined plate with same surface pressure and moving at same speed
H		enthalpy, or total heat
M	=	q/a, local Mach number
Q		heat flux to wall
$Q(\delta)$		heat flux relative to that of uninclined plate with same surface pressure and moving at same speed
R_x	=	$(\rho qx/\mu)$, local Reynolds number
$(R_x)_o$	=	$\rho_o a_o x/\mu_o$
T		temperature
a		speed of sound
c	=	C/T
c_f	=	$F/\frac{1}{2}\rho_\delta q_\delta^2$, local skin friction coefficient
c_p	=	dH/dT, specific heat at constant pressure
$\bar{c}_p$	=	$(H-H_1)\Big/(T-T_1)$
k		thermal conductivity
k_h	=	$Q/\frac{1}{2}\rho_\delta q_\delta^3$, local heat transfer coefficient
m		molecular weight
p		pressure
q		gas speed
u		component of gas velocity perpendicular to shock
v		" " " " parallel to shock
Γ	=	$mc_p/m_1 c_{p_1}$
γ		ratio of specific heats
δ		angular deflection of flow through shock

δ_{max} maximum value of δ with respect to ψ

μ viscosity

ρ density

$\sigma = kc_p/\mu$, Prandtl number

ψ angle between shock wave and free stream direction

ψ_{max} value of ψ corresponding to maximum deflection

ω power index in relation $\mu \propto T^{\omega}$

Subscripts

'w' denotes wall conditions

'δ' denotes conditions at the outside of the boundary layer

'o' denotes some standard atmospheric datum, say I.C.A.N. sea-level.

'1' denotes conditions upstream of shock-wave

'2' denotes conditions downstream of shock-wave.

(N.B. In example considered of wedge shaped aerofoil, subscripts 'δ' and '2' are interchangeable).

APPENDIX I

Conditions in Boundary Layer of an Inclined Plane Surface producing an Intense Attached Shockwave

Supposing that the air in the boundary layer behaves as a perfect gas and has constant molecular weights, Prandtl number and specific heats, any non-dimensional expression of a surface property (such as skin friction or heat transfer coefficient) can be expressed as a function of

$$R_{x_\delta} = \frac{\rho_\delta q_\delta x}{\mu_\delta}$$

where 'δ' denotes conditions outside the boundary layer, and also in terms of the parameters

$$\frac{q_\delta^2}{c_{p_\delta} T_\delta} = (\gamma_\delta - 1)M_\delta^2 \ , \quad \sigma_\delta = \frac{c_{p\delta} k_\delta}{\mu_\delta} \ , \quad \frac{T_W}{T_\delta} \ , \text{ and } \ \omega \ \left(\text{or } \frac{C}{T_\delta} = c_\delta\right) .$$

where T_W is the (constant) wall temperature, and ω is the power index in the viscosity temperature variation $(\mu \propto T^{\omega})$. The pressure gradient is taken to be zero.

In the boundary layer condition under consideration the temperature outside the boundary layer will be a quantity such that from (2.12) of Appendix II

$$\frac{T_\delta}{T_1} = \frac{T_2}{T_1} = O\,(M_1^2 \psi^2)$$

As the shockwave is 'intense', $M_1^2\psi^2$ is indefinitely large; we shall suppose however that T_W remains bounded and of order T_1 as $M_1^2\psi^2 \to \infty$, - which is plainly an assumption of reality, because surface temperatures of the order of the large stagnation values are of little practical interest. Consequently

$$\frac{T_W}{T_\delta} = O\left(\frac{1}{M_1^2\psi^2}\right)$$

and may, in the presence of intense shockwaves, be taken as zero.

Further $\frac{C}{T_\delta}$ will likewise be small which implies that we may take $\omega = \frac{1}{2}$ (i.e. $\mu \propto T^{\frac{1}{2}}$ as is characteristic of high temperatures). Also in general, the temperature within the boundary layer will be of the large magnitude, T_δ (except, in the limit, at infinitesimal distances from the surface). Thus the specific heats and Prandtl number, which have an asymptotic value at elevated temperatures, can indeed be treated as constant across the boundary layer.

Hence it follows that the assumptions implicit in the expression of the boundary layer properties at the beginning of this Appendix are applicable in the limiting condition $M_1^2 \psi^2 \to \infty$, if we take

$$\frac{T_W}{T_\delta} = 0, \quad \omega = \tfrac{1}{2} \ (\text{or } c_\delta = 0), \quad M_\delta = M_2 , \qquad (1.1)$$

together with the numerical values relevant to elevated temperatures given in ref. 2 as

$$\gamma_\delta = \frac{9}{7} = 1.286, \quad \sigma_\delta = \frac{18}{23} = 0.782 \quad \ldots\ldots(1.2)$$

Solutions for the 'flat plate' boundary layer, with precisely the values of T_W/T_δ, ω and σ given by (1.1) and (1.2), do not appear to have been worked, although closely corresponding solutions are known. It would not be inappropriate therefore to use one of the various interpolating expressions which have been revised. For instance, Young[4] suggests

$$c_f = \frac{F}{\frac{1}{2}\rho_\delta q_\delta^2} = 0.664 \left[0.45+0.55\,\frac{T_W}{T_\delta} + 0.09(\gamma_\delta - 1)M_\delta^2\,\sigma^{\frac{1}{2}}\right]^{-(1-\omega)/2} \sqrt{\frac{\mu_\delta}{\rho_\delta q_\delta x}} \quad \ldots\ldots\ldots\ldots(1.3)$$

so that in the present example,

$$c_f = \frac{1.71}{\sqrt{M_2(R_x)_\delta}} \left(1 + \frac{19.8}{M_2^2}\right)^{-1/4} \quad \ldots\ldots\ldots\ldots(1.4)$$

Similarly Crocco[5] has suggested that, if we define

$$k_h = \frac{Q}{\frac{1}{2}\rho_\delta q_\delta^3}$$

then

$$\frac{k_h}{c_f} = \frac{1}{2\sigma^{1/6}} \left[1 + \frac{2}{\sigma^{\frac{1}{2}}(\gamma_\delta - 1)M_\delta^2}\left(1 - \frac{T_w}{T_\delta}\right)\right] \ldots\ldots(1.5)$$

which simplifies in the present instance to

$$\left.\begin{aligned} \frac{k_h}{c_f} &= 0.52\left(1 + \frac{7.91}{M_2^2}\right) \\ \text{or} \quad k_h &= \frac{0.89}{\sqrt{M_2(R_x)_\delta}}\left(1 + \frac{7.91}{M_2^2}\right)\bigg/\left(1 + \frac{19.8}{M_2^2}\right)^{-1/4} \end{aligned}\right\} \ldots(1.6)$$

It is interesting to note that for large values of M_2 (i.e. small surface inclinations - a condition for which of course the simplifications made above are actually inapplicable), equations (1.4) and (1.6) yield

$$c_f = \frac{1.71}{\sqrt{M_\delta(R_x)_\delta}}, \quad k_h = \frac{0.89}{\sqrt{M_\delta(R_x)_\delta}}$$

whereas the work of reference 2, which is strictly relevant to this particular condition of the uninclined flat plate at hypersonic speeds, yields

$$c_f = \frac{1.70}{\sqrt{M_\delta(R_x)_\delta}}, \quad k_h = \frac{0.86}{\sqrt{M_\delta(R_x)_\delta}}$$

Such a measure of agreement must be largely fortuitous; however it does imply that, as (1.4) and (1.6) are relevant to plates at finite inclinations, but also give nearly appropriate values for small inclinations, their use for all angles of inclination, provided only that M_1 is large, can be justified.

The values from (1.4) and (1.6) involve a knowledge of the local Reynolds number $(R_x)_\delta$, based on conditions outside the boundary layer. It is more convenient in general to use a Reynolds number based on free stream conditions, or better still

$$\frac{\rho_o a_o x}{\mu_o} = (R_x)_o$$

where suffix zero denotes some datum condition - say I.C.A.N. standard sea-level. Now this parameter can only be introduced if we can relate the change of μ from its value (μ_δ) at the large temperature T_δ to its value at normal temperatures. To do this we assume, as in ref. 1, that there is no change in the molecular weight of the air, - i.e. no dissociation. Then we see that

$$\frac{\rho_\delta q_\delta}{(R_x)_\delta^{\frac{1}{2}}} = \left[\frac{\rho_o a_o}{(R_x)_o^{\frac{1}{2}}} \left(\frac{1 + c_o}{1 + c_\delta}\right) \left(\frac{p_\delta}{p_o}\right) \left(\frac{\gamma_\delta}{\gamma_o}\right)^{\frac{1}{2}} M_\delta\right]^{\frac{1}{2}}$$

or from (1.1) and (1.2)

$$\frac{\rho_\delta q_\delta}{(R_x)_\delta^{\frac{1}{2}}} = 0.978 \left(\frac{p_\delta}{p_o}\right)^{\frac{1}{2}} \left[(1 + c_o) M_2\right]^{\frac{1}{2}} \frac{\rho_o a_o}{(R_x)_o^{\frac{1}{2}}} \qquad (1.7)$$

From (1.7) in (1.4) and (1.6)

$$\left.\begin{aligned} F &= 1.67 \left(\frac{q_2}{q_1}\right) \left(1 + \frac{19.8}{M_2^2}\right)^{-1/4} \frac{\rho_o a_o q_1 (1+c_o)^{\frac{1}{2}}}{(R_x)_o^{\frac{1}{2}}} \left(\frac{p_\delta}{p_o}\right)^{1/2} \\ Q &= 0.87 \left(\frac{q_2}{q_1}\right)^2 \left(1 + \frac{19.8}{M_2}\right)^{-1/4} \left(1 + \frac{7.91}{M_2^2}\right) \frac{\rho_o a_o q_1^2 (1+c_o)^{\frac{1}{2}}}{(R_x)_o^{\frac{1}{2}}} \left(\frac{p_\delta}{p_o}\right)^{\frac{1}{2}} \end{aligned}\right\} \qquad (1.8)$$

Relative to an equal distance behind the leading edge of an uninclined plate, moving at the same speed, but having the same surface pressure

$$\left.\begin{aligned} \frac{F}{F_{\text{flat plate}}} &= \left(\frac{q_2}{q_1}\right)\left(1 + \frac{19.8}{M_2^2}\right)^{-1/4} = F(\delta), \text{ say} \\ \frac{Q}{Q_{\text{flat plate}}} &= \left(\frac{q_2}{q_1}\right)^2\left(1 + \frac{7.91}{M_2}\right)\left(1 + \frac{19.8}{M_2^2}\right)^{-1/4} = Q(\delta), \text{ say} \end{aligned}\right\} \quad (1.9)$$

The variation of $\frac{q_2}{q_1}$ and M_2 is deduced in the Appendix II, and the functions $F(\delta)$ and $Q(\delta)$ are plotted in fig. 6.

APPENDIX II

Conditions behind an Intense Inclined Shockwave in Air

Let us consider the conditions behind an inclined shockwave, taking into account the variation of gas specific heats, and molecular weight. Denoting conditions upstream - downstream of the shock by suffices '1' and '2' respectively, and using other symbols as explained in the list of symbols at the end of this Appendix and in figure 7, we have from continuity,

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} \qquad \text{.............(2.1)}$$

Conservation of momentum across the shock requires that

$$\frac{p_2}{p_1} = 1 + \gamma_1 \left(1 - \frac{u_2}{u_1}\right) \frac{u_1^2}{a_1^2} \qquad \text{.............(2.2)}$$

and normal to it, that

$$v_1 = v_2 \qquad \text{.............(2.3)}$$

Conservation of energy may be expressed as

$$H_2 - H_1 = \tfrac{1}{2}\,(u_1^2 - u_2^2) \qquad \text{.............(2.4)}$$

The perfect gas law states that

$$\frac{p_1}{p_2}\,\frac{\rho_2}{\rho_1} = \frac{m_2}{m_1}\,\frac{T_1}{T_2} \qquad \text{.............(2.5)}$$

and is taken as being obeyed by the air. If we define

$$\bar{c}_p = \frac{H-H_1}{T-T_1}\ , \quad \Gamma = \frac{m\,c_p}{m_1 c_{p_1}}$$

then (2.4) can be written, from (2.5), as

$$\frac{p_2}{p_1}\,\frac{\rho_1}{\rho_2} = \frac{m_1}{m_2} + \frac{\gamma_1 - 1}{2\,\Gamma_2}\,\frac{u_1^2}{a_1^2}\left(1 - \frac{u_2^2}{u_1^2}\right) \text{.....(2.6)}$$

whereas from (2.1) and (2.2)

$$\frac{p_2}{p_1}\ \frac{\rho_1}{\rho_2} = \left[1 + \gamma_1\ \frac{u_1}{a_1}\left(1 - \frac{u_2}{u_1}\right)\right]\frac{u_2}{u_1} \quad \ldots\ldots\ldots(2.7)$$

Equation (2.6) and (2.7) together yield a quadratic for $\frac{u_2}{u_1}$ which can be solved to give

$$\frac{u_2}{u_1} = \frac{1}{2}\left\{\left(\gamma_1 + \frac{a_1^2}{u_1^2}\right) - \left[\left(\gamma_1 - \frac{a_1^2}{u_1^2} - \frac{\gamma_1 - 1}{\Gamma_2}\right)^2 - \frac{4a_1^2}{u_1^2}\left(\frac{m_1}{m_2} - 1\right)\left(\gamma_1 - \frac{\gamma_1 - 1}{2\Gamma}\right)\right]\right\} \Bigg/ \left(\gamma_1 - \frac{\gamma_1 - 1}{2\,\Gamma_2}\right) \quad \ldots\ldots\ldots(2.8)$$

If ψ is the inclination of the shockwave we have

$$u_1 = q_1 \sin\psi\,, \quad v_1 = q_1 \cos\psi$$

where q is the speed; similarly if δ is the anglular deflection of the stream through the shock,

$$\tan(\psi - \delta) = \frac{u_2}{v_2}$$

or using (2.3)

$$\tan\delta = \left(1 - \frac{u_2}{u_1}\right)\tan\psi \Bigg/ \left(1 + \frac{u_2}{u_1}\tan^2\psi\right) \quad (2.9)$$

We note that

$$\frac{u_1^2}{a_1^2} = M_1^2 \sin^2\psi$$

and in the limit as this becomes large (i.e. $M_1 \to \infty$, $\psi \neq 0$), the shockwave becomes intense and

$$\frac{u_2}{u_1} = \frac{\gamma_1 - 1}{(2\Gamma_2 - 1)\gamma_1 + 1}\left[1 + 0\left(\frac{1}{M_1^2\,\psi^2}\right)\right] \quad \ldots\ldots\ldots(2.10)$$

From this equation and (2.3) it follows that

$$\left(\frac{q_2}{q_1}\right) = \left(\frac{u_2}{u_1}\right)\sin^2\psi + \cos^2\psi = \left\{ \left[\frac{\gamma_1 - 1}{(2\Gamma_2-1)\gamma_1+1} \sin\psi \right] + \cos^2\psi \right\} \left\{ 1+0\left(\frac{1}{M_1^2}\right) \right\} \qquad \ldots\ldots\ldots\ldots(2.11)$$

Again, from (2.6) and (2.10)

$$\frac{a_2^2}{a_1^2} = \frac{\gamma_2}{\gamma_1}\,\frac{p_2}{p_1}\,\frac{\rho_1}{\rho_2} = \frac{2\gamma_1(\gamma_1-1)\left[(\Gamma_2-1)\gamma_1+1\right]}{\left[(2\Gamma_2-1)\gamma_1+1\right]^2}\, M_1^2 \sin^2\psi \left[1+0\left(\frac{1}{M_1^2\ \psi^2}\right)\right] \qquad \ldots\ldots\ldots\ldots(2.12)$$

so that

$$a_2^2 = \frac{2\gamma_2(\gamma_1-1)\left[(\Gamma_2-1)\gamma_1+1\right]}{\left[(2\Gamma_2-1)\gamma_1+1\right]^2}\, u_1^2 \left[1+0\left(\frac{1}{M_1^2\ \psi^2}\right)\right] \qquad \ldots\ldots\ldots\ldots(2.13)$$

It also follows from (2.10) that

$$M_2^2 = \frac{(\gamma_1-1)^2 + \left[(2\Gamma_2-1)\gamma_1+1\right]^2 \cot^2\psi}{2\gamma_2(\gamma_1-1)\left[(\Gamma_2-1)\gamma_1+1\right]} \left\{1+0\left(\frac{1}{M_1^2\ \psi^2}\right)\right\} \qquad \ldots\ldots\ldots\ldots(2.14)$$

Also from (2.10) and (2.2)

$$p_2 = 2\,\frac{(\Gamma_2-1)\gamma_1+1}{(2\Gamma_2-1)\gamma_1+1}\,\rho_1 q_1^2 \sin^2\psi \left[1+0\left(\frac{1}{M_1^2\ \psi^2}\right)\right] \qquad \ldots\ldots\ldots\ldots(2.15)$$

The relation (2.9) becomes

$$\tan\delta = \frac{2\left[(\Gamma_2-1)\gamma_1+1\right]\tan\psi}{(2\Gamma_2-1)\gamma_1+1+(\gamma_1-1)\tan^2\psi} \left[1+0\left(\frac{1}{M_1^2\ \psi^2}\right)\right] \qquad \ldots\ldots\ldots\ldots(2.16)$$

The maximum deflection through the shockwave is evidently

$$\delta_{max} = \tan^{-1}\sqrt{\frac{\left[(\Gamma_2-1)\gamma_1+1\right]^2}{(\gamma_1-1)\left[(2\Gamma_2-1)\gamma_1+1\right]}} \left[1+0\left(\frac{1}{M_1^2}\right)\right] \qquad \ldots\ldots\ldots\ldots(2.17)$$

which is achieved when

$$\tan^2\psi = \tan^2\psi_{max} = \frac{(2\Gamma_2-1)\gamma_1+1}{(\gamma_1-1)} \left[1+0\left(\frac{1}{M_1^2}\right)\right] \quad (2.18)$$

For the air upstream of the shockwave we may take

$$\gamma_1 = 7/5$$

Noting from (2.12) that the temperature behind the shockwave is high if $M_1^2 \psi^2$ is large, it is often appropriate to use the values

$$\gamma_2 = \Gamma_2 = 9/7$$

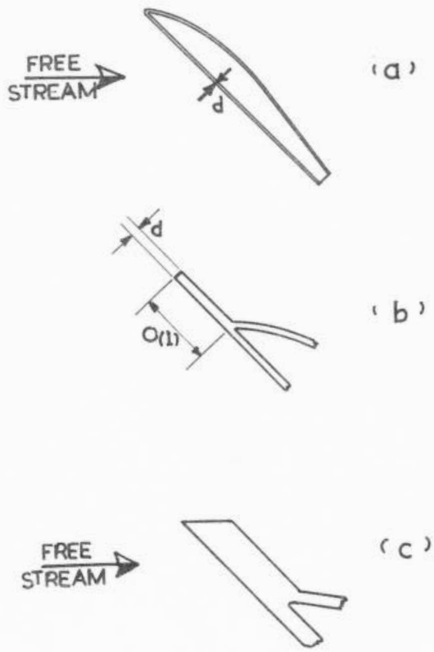
which are appropriate to air at elevated temperatures.

If we define an 'intense shockwave' as being the limiting form given by taking $M_1^2 \psi^2 \to \infty$, and adopting the above numerical values, we find that

$$\left.\begin{aligned}
\left(\frac{q_2}{q_1}\right)^2 &= \cos^2\psi + \frac{1}{64}\sin^2\psi \\
M_2^2 &= \frac{1 + 64\cot^2\psi}{9} \\
p_2 &= \frac{7}{8}\rho_1 q_1^2 \sin^2\psi \\
\tan\delta &= \frac{7\tan\psi}{8+\tan^2\psi} \\
\delta_{max} &= \tan^{-1}\frac{7}{8}\sqrt{2} = 51^\circ \text{ approx.} \\
\text{and} \quad \psi_{max} &= \tan^{-1}\sqrt{8}\,.
\end{aligned}\right\} \quad (2.19)$$

We note that if $\delta = \delta_{max}$, $M_2 = 1$, so that in general $M_2 > 1$.

FIG 1



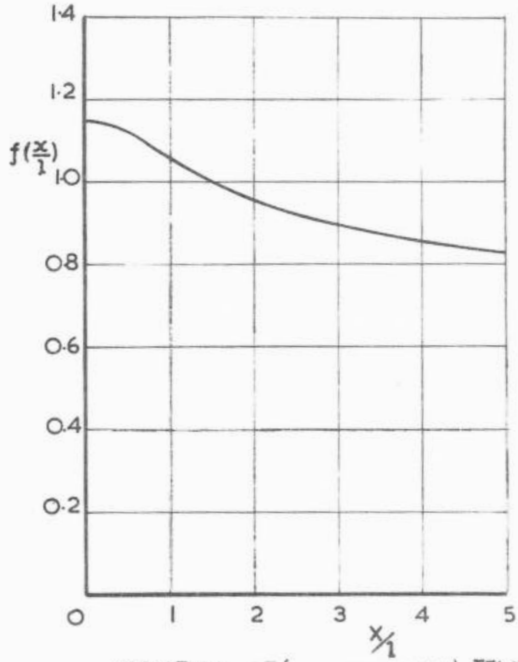


FIG. 2. VARIATION OF (NON-DIMENSIONAL) TEMPERATURE WITH DISTANCE ALONG SURFACE.

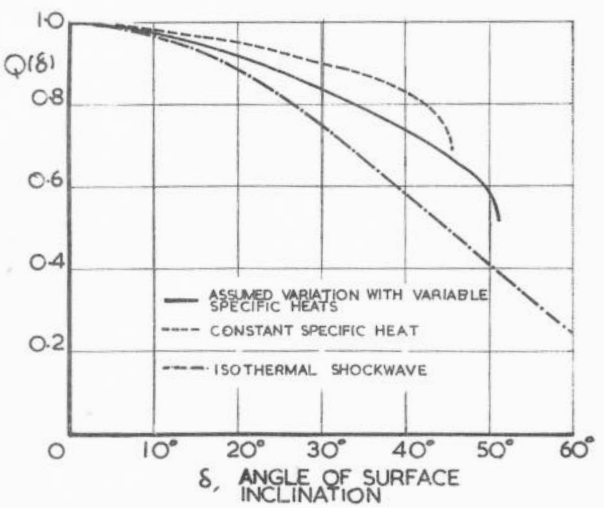


FIG. 3. RELATIVE REDUCTION IN HEAT TRANSFER ON INCLINED SURFACE AS EFFECTED BY ASSUMED STATE OF AIR PROPERTIES.

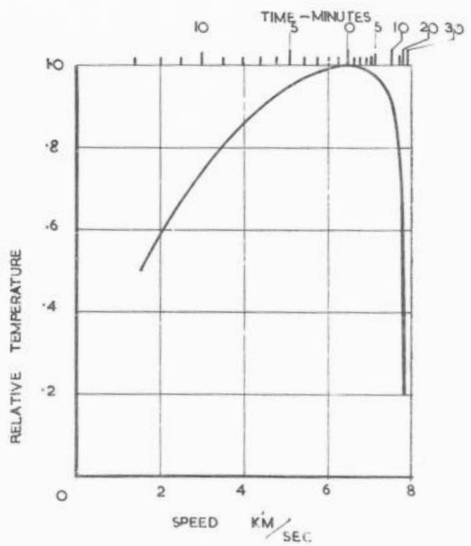


FIG 4 NOSE TEMPERATURE AT VARIOUS SPEEDS RELATIVE TO T_{MAX}. (achieved at 6·5 Km/sec)

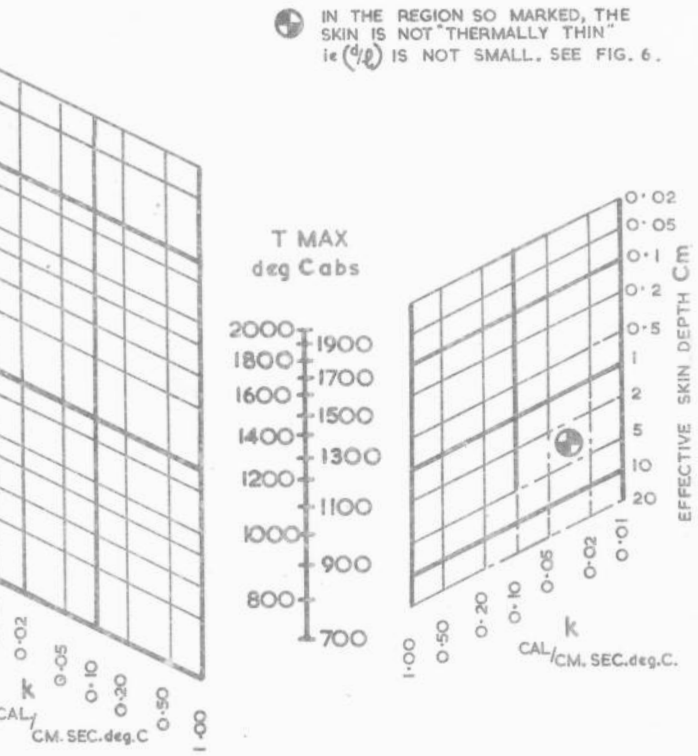


FIG. 5. ALIGNMENT TO DETERMINE T MAX FROM K. L. AND δ

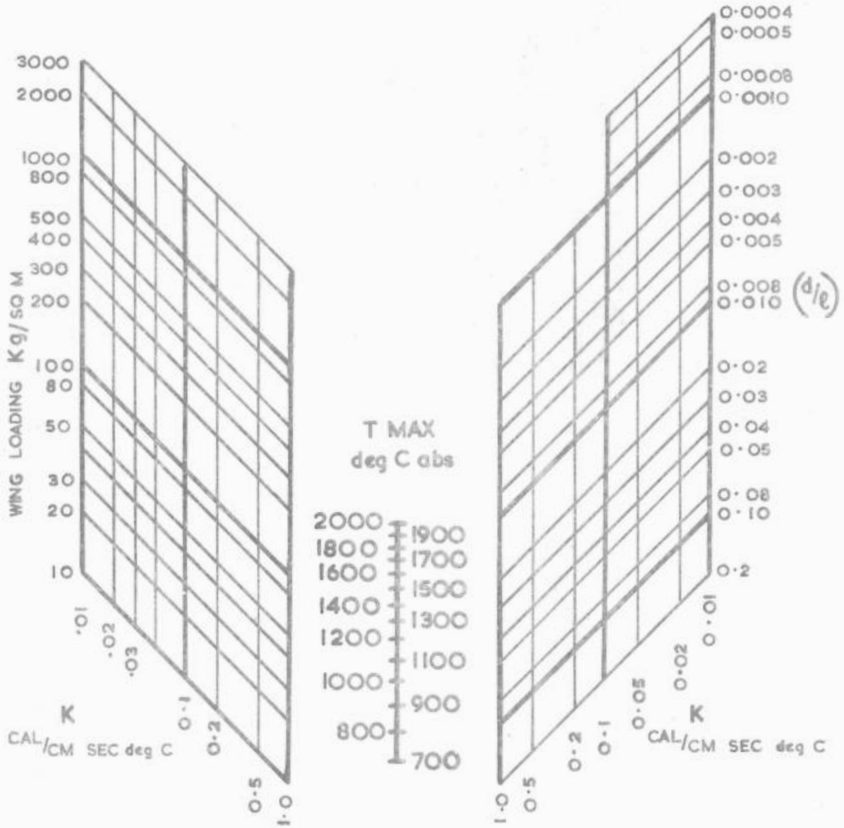


FIG. 6. ALIGNMENT TO DETERMINE (d/ℓ) FROM TMAX K AND L

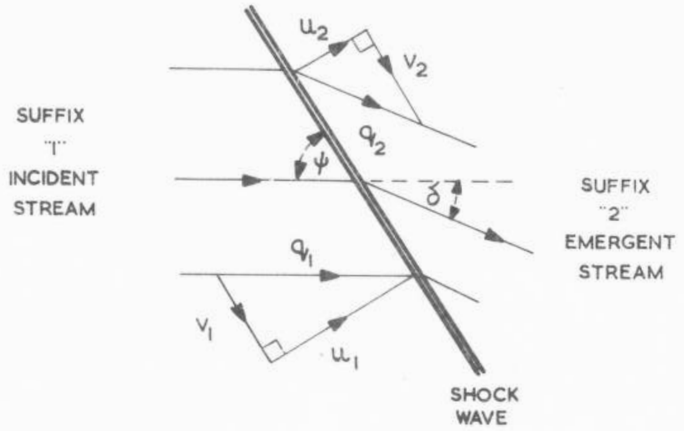


FIG. 7.

NOTATION USED IN APPENDIX II

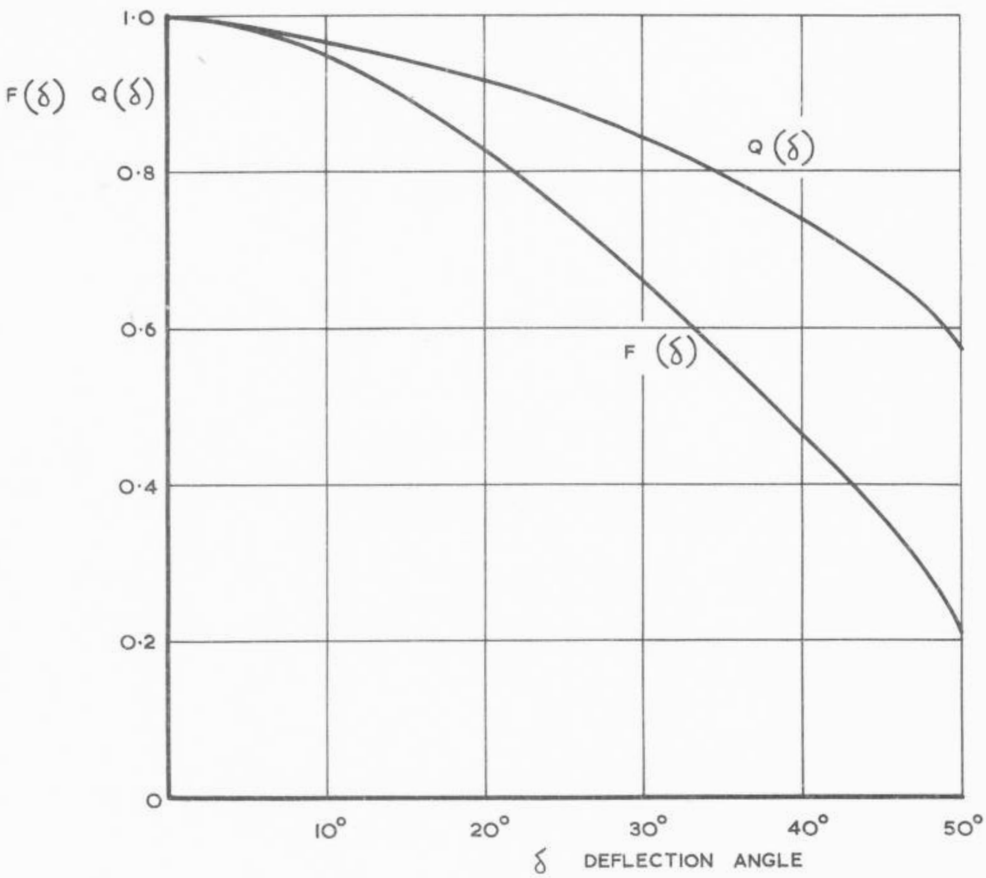


FIG. 8.

RELATIVE REDUCTION IN SKIN FRICTION AND HEAT TRANSFER ON INCLINED WEDGES RELATIVE TO THOSE ON UNINCLINED PLATE MOVING AT SAME SPEED AND WITH SAME SURFACE PRESSURE.