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Estimation of the Effects of Distortion on the Longitudinal Stability of Swept Wing Aircraft at High Speeds (Sub-Critical Mach Numbers)

-by-

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SUMMARY

The effects of distortion on the longitudinal stability of swept wing aircraft at high speeds (sub-critical Mach numbers) are considered on a quasi-static basis. The method employed is based on the theory of Gates and Lyon[1] but involves some extension of this theory.

The treatment of wing distortion is considered in some detail and the effects of built-in twist and camber and wing weight are included using the so-called superposition method[3]. The application of the analysis of Lyon and Ripley[2] for investigating fuselage, tail and control circuit distortion is suggested, but means of modifying and simplifying this procedure where desirable are put forward.

The analysis is illustrated by means of a simple example of a swept wing fighter aircraft for which wing, fuselage and tail distortion effects are considered, and the results are discussed with reference to the individual and combined distortion effects as well as the effect of compressibility.

MEP

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1. Introduction

The longitudinal stability of an aircraft is usually considered in terms of 'static stability' (measured by the static margins), 'manoeuvrability' (measured by the manoeuvre margins) and general dynamic stability. If the stability derivatives are very little modified by frequency effects (i.e. they approximate closely to the quasi-static values), then the static and manoeuvre margins as normally defined can be related to the coefficients E_1 and C_1 in the 'stability quartic' given by the usual small displacement theory. The values of the static and manoeuvre margins then largely determine the characteristics of the phugoid and short period oscillations.

When structural distortion effects are introduced, it is again necessary to consider whether the motion of the aircraft (in the long and short period oscillations) occurs under quasi-static conditions, or whether the dynamics of the separate aircraft components should be considered, introducing additional degrees of freedom corresponding to oscillations of individual components. This question is discussed in ref. 4 where it is pointed out that if the frequency of the short period oscillation is of the same order as the lowest natural frequency of any component (e.g. the wing) the simple quasi-static approach is suspect. Once the quasi-static approach is abandoned, however, the treatment of dynamic stability when distortion effects are included becomes very difficult.[5] The general treatment of the dynamic stability of a flexible aircraft can be similar to that employed in flutter problems, although the difficulties are enhanced by the fact that coupled oscillations of wing, fuselage, tail, etc., are combined with the overall rigid body motion of the aircraft. An attempt to formulate the equations governing the motion of an aircraft with flexible fuselage and wings is made in ref. 5, and the problem is considered briefly in ref. 4.

In this report the 'quasi-static' approach only is considered, as in refs. 1 and 2. By this method the equations of motion for a rigid aircraft are used but the values of the aerodynamic derivatives are modified to include distortion effects.

The basic theory is in essentials that of refs. 1 and 2 with certain modifications and extensions which it is believed will permit of a more logical treatment of the effects of wing distortion in cases where there is built-in twist or camber, and the treatment also permits the ready inclusion of the effects of aircraft weight. The conditions for the balance of aerodynamic, elastic and inertia forces are obtained by the superposition method of ref. 3, which

/it is considered ...

it is considered has many advantages over other methods.

The analysis is illustrated by means of an example of a high speed swept wing fighter, and the results of the analysis are discussed in detail. This example presents a number of features of general interest.

For compressibility effects on a rigid aircraft the important parameter is the Mach number so that the variation of true air speed must be considered. When the aircraft is flexible however, the distortions produced by aerodynamic loading are dependent on the equivalent air speed. In general, therefore, we must consider the two parameters M and $q = \frac{1}{2}\rho V^2$. In this report the suffix M indicates that a derivative is taken at constant Mach number (e.g. $(\partial C_L/\partial \alpha)_M$), and in such cases it is also implied that q is constant.

The corrections for variations of inertia loading due to normal acceleration introduced in para. 4 are 'quasi-static' i.e. it is assumed that the normal accelerations of all parts of the aircraft are the same as that of the C.G. and that the structure is always in equilibrium under the applied aerodynamic and inertia loading. This assumption is similar to the assumption that frequency effects on the aerodynamic derivatives, etc. may be neglected and will similarly become invalid when the short period frequency is high.

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2. Wing distortion

When considering longitudinal stability it is usual to assume that the ailerons will remain in the neutral position when the wing distorts. Distortion of the main wing structure only is then considered. For most swept wings the loss of incidence due to upward bending is greater than the increase due to twist (referred to an ideal straight flexural axis). Thus wing distortion produces a net loss of lift curve slope accompanied by forward movement of the aerodynamic centre (see 2.2).

When the quasi-static approach is used in obtaining stability criteria and derivatives it is assumed that the structure is always in equilibrium under the applied loading. The problem of treating wing distortion is then basically that of solving for the 'aeroelastic equilibrium' of a flexible lifting surface.

2.1. Solution of the aeroelastic equilibrium problem for a flexible lifting surface

The problem of calculating the aerodynamic characteristics of a flexible lifting surface is one of some difficulty due to the fact that distortion under load produces a change of load. Mathematically the problem takes the form of the solution of an integral equation.

It is possible to solve the problem (using 'strip' theory) by successive approximation (see ref. 4) or by the method of semi-rigid representation (refs. 4,8). These methods are well-known and widely used. They suffer from the disadvantage that since 'strip' theory must be used induced aerodynamic effects due to the distortion itself are neglected; also with the semi-rigid method the accuracy is reduced by the need for approximate representation of the distortion mode. More recently the 'superposition' method has been put forward (ref. 3)* and since it was felt that this method is in some respects superior to the above it has been adopted here. The following is a brief description of it. The usual assumption of linearity between loading and incidence is fundamental to the method.

At any point on the span of an elastic wing in equilibrium under aerodynamic load, the final geometric incidence α_F will be given by:

$$\alpha_F = \alpha_I + \alpha_E$$

where α_I = geometric incidence of undistorted wing ('initial' incidence)

α_E = change of incidence due to distortion.

/If α_F ...

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* The superposition method has also been developed independently at Handley Page Ltd., but this work remains unpublished.

If α_F is known, the final loading is known and α_E may be calculated, giving α_I. Thus the problem is easily solved 'backwards'. If a number of arbitrary 'final' incidence distributions α_{F1}, α_{F2}, etc. are chosen and the corresponding 'initial' incidence distributions α_{I1}, α_{I2}, etc. are so obtained, then any given initial incidence distributions may be approximately represented by a linear combination of the arbitrary initial distributions.

Thus $$\alpha_I = A_S\alpha_{I1} + B_S\alpha_{I2} + C_S\alpha_{I3} + \dots$$

In practice three or four such terms may be sufficient.

The coefficients A_S, B_S, C_S etc. are functions of the parameter qA_R, where A_R is the rigid wing lift curve slope allowing for compressibility effects.

But $$A_S\alpha_{I1} = A_S\alpha_{F1} - A_S\alpha_{E1}$$

$$B_S\alpha_{I2} = B_S\alpha_{F2} - B_S\alpha_{E2} \quad \text{etc.}$$

and therefore

$$\alpha_F - \alpha_E = A_S\alpha_{F1} + B_S\alpha_{F2} + C_S\alpha_{F3} + \dots$$
$$- (A_S\alpha_{E1} + B_S\alpha_{E2} + C_S\alpha_{E3} + \dots)$$

so that $$\alpha_F = A_S\alpha_{F1} + B_S\alpha_{F2} + C_S\alpha_{F3} + \dots$$

and hence for the given initial incidence distribution, α_I, the final incidence distribution α_F and hence the final aerodynamic characteristics can be obtained. It will be seen that by means of this method the aerodynamic and structural problems are separated and may be considered independently.

This method is likely to yield more accurate results for many problems than the other methods mentioned previously, due mainly to the fact that induced aerodynamic effects due to distortion are readily included and no sweeping assumptions need be made about the mode of distortion . Once the calculations have been completed for the arbitrary cases chosen, equilibrium conditions for any combination of q and M are quite easily obtained. As with other methods, some difficulty is encountered if the form of the aerodynamic loading varies appreciably with Mach No. (see 5.2).

2.2. Effects of wing distortion on the wing lift and pitching moment contributions

2.2.1. Wing with zero built-in twist and camber; distortions due to wing weight neglected.

If the incidence of the wing (α) is defined as the angle between the chord line of the wing and the direction of flight, measured at the wing root, then we may write

$$C_{Lw} = \text{wing lift coefficient} = \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M \alpha = A\alpha \quad ..(1)$$

where $C_{L\alpha w}$ = wing lift coefficient of flexible wing due to (root) incidence α.

A = wing lift curve slope (following ref. 1) $= \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M$

It is possible to find $\left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M$ using the superposition method as described above (see also ref. 3).

Similarly, taking moments about the rigid wing aerodynamic centre, we may write

$$C_{m_w} = \left(\frac{\partial C_{m\alpha w}}{\partial \alpha}\right)_M \alpha \quad ....(2)$$

where $C_{m\alpha w}$ is the pitching moment coefficient corresponding to $C_{L\alpha w}$ and may also be found by the superposition method. For the 'rigid wing' C_{mw} as defined above is of course zero, but with the flexible wing there is a pitching moment about the rigid wing mean aerodynamic centre (H_{oR}) which is proportional to incidence i.e. there is a movement of the wing aerodynamic centre given by

$$\triangle H_o = -\frac{\partial C_{m\alpha w}}{\partial \alpha} \Big/ \frac{\partial C_{L\alpha w}}{\partial \alpha} \quad ....(3)$$

giving the mean aerodynamic centre of the flexible wing

$$H_o = H_{oR} + \triangle H_o .$$

The wing pitching moment coefficient about the aircraft C.G. is then

$$C_{mw_{C.G.}} = A\alpha\,(h - H_o) \quad ....(4)$$

and the wing pitching moment coefficient about the new mean aerodynamic centre (H_o) is zero

i.e. $\quad (C_{m_o})_{wing} = 0 .$

2.2.2. Wing with built-in twist and camber; distortions due to wing weight neglected.

When the undistorted wing has twist or camber it is

/possible to

possible to consider the effects of incidence, twist and camber separately. We then have, applying the principle of superposition as in ref. 3, that

$$C_{Lw} = \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)\alpha + C_{Lcw} + C_{Ltw}$$

$$= A\alpha + C_{Lwo} \qquad \text{............ (5)}$$

where α = root incidence as defined in 2.2.1.

$C_{L\alpha w}$ = lift coefficient due to root incidence α on wing with zero built-in twist and no built-in camber.

C_{Lcw} = lift coefficient on wing with zero root incidence and zero built-in twist, but with built-in camber.

C_{Ltw} = lift coefficient on wing with zero root incidence and zero built-in camber but with built-in twist.

$$C_{Lwo} = C_{Lcw} + C_{Ltw}$$

$$A = \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M \text{ (as before)}$$

C_{Lcw} and C_{Ltw} may be found using the superposition method and are functions of speed and Mach No.

$$\text{Thus} \left(\frac{\partial C_{Lw}}{\partial \alpha}\right)_M = \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M = A \text{ as in 2.2.1.}$$

Also, the pitching moment coefficient about the rigid wing mean aerodynamic centre is

$$C_{mw} = \left(\frac{\partial C_{m\alpha w}}{\partial \alpha}\right)\alpha + C_{mcw} + C_{mtw}$$

where $C_{m\alpha w}$, C_{mcw}, C_{mtw} are the pitching moment coefficients corresponding to $C_{L\alpha w}$, C_{Lcw}, C_{Ltw} and hence

$$\left(\frac{\partial C_{mw}}{\partial \alpha}\right)_M = \left(\frac{\partial C_{m\alpha w}}{\partial \alpha}\right)_M$$

$$\text{i.e.} \quad \Delta H_o = - \frac{\partial C_{m\alpha w}}{\partial \alpha} \Big/ \frac{\partial C_{L\alpha w}}{\partial \alpha} \text{ as in 2.2.1.}$$

Then

$$C_{mw_{C.G.}} = \frac{\partial C_{m\alpha w}}{\partial \alpha}.\alpha + C_{mwo} + (h - H_{oR})\, C_{Lw}$$

where $C_{mwo} = C_{mcw} + C_{mtw}$,

$$\text{i.e.} \quad C_{mw_{C.G.}} = A\alpha.\Delta H_o + C_{mwo} + (h - H_{oR})(A\alpha + C_{Lwo})$$

$$= (C_{mo})_{wing} + A\alpha(h - H_o), \qquad \text{............ (6)}$$

/where ...

where $(C_{mo})_{wing} = C_{mwo} + (h - H_{oR})\, C_{Lwo}$

and $H_o = H_{oR} + \Delta H_o$.

This expression for $C_{mw_{C.G.}}$ is different from that normally used in that $C_{Lw} \neq A\alpha$. It will be seen, however, that it is convenient to express quantities in the form $X + Y\alpha$ where X and Y are functions of speed and Mach No. but not of incidence.

2.2.3. Wing with built-in twist and camber when distortions due to wing weight are included

Following the method adopted in 2.2.2 we may write

$$C_{Lw} = A^o\alpha + C^o_{Lwo} + n\left(\frac{\partial C_{Lnw}}{\partial n}\right)_M \qquad \ldots\ldots\ldots\ldots(7)$$

where $n-1 = 1/g \times$ normal acceleration of aircraft (positive upwards)

i.e. $n = \dfrac{C_L \frac{1}{2}\rho V^2 S}{W}$, in flight for which the inclination of the flight path to the horizontal is small.

C_{Lnw} = lift coefficient on wing with zero root incidence, zero built in twist and no built-in camber, due to deflection of the wing under its effective weight.

The affix ^{o} denotes the condition $n = 0$, so that the effects of wing weight are entirely contained in the last term.

As with C_{Lwo} etc., $\dfrac{\partial C_{Lnw}}{\partial n}$ may be found using the superposition method.

Similarly, taking moments about the rigid wing aerodynamic centre we have

$$C_{mw} = \left(\frac{\partial C_{m\alpha w}}{\partial \alpha}\right)^o_M .\alpha + C^o_{mwo} + n\left(\frac{\partial C_{mnw}}{\partial n}\right)_M \quad \ldots\ldots\ldots(8)$$

and about the C.G. we have

$$C_{mw_{C.G.}} = \left(\frac{\partial C_{m\alpha w}}{\partial \alpha}\right)^o_M .\alpha + C^o_{mwo} + n\left(\frac{\partial C_{mnw}}{\partial n}\right)_M$$

$$+ (h - H_{oR})(A^o\alpha + C^o_{mwo}) + n\left(\frac{\partial C_{Lnw}}{\partial n}\right)_M (h - H_{oR})$$

$$= (C_{mo})^o_{wing} + (h - H^o_o)\, A^o\alpha + (h - H^n_o)A_n.n \qquad \ldots\ldots\ldots\ldots(9)$$

/where ...

where $H_o^o = H_{oR} + \triangle H^o$, $\triangle H_o^o = -\left(\frac{\partial C_{m\alpha w}}{\partial \alpha} \Big/ \frac{\partial C_{L\alpha w}}{\partial \alpha}\right)^o$

$$A_n = \left(\frac{\partial C_{Lnw}}{\partial n}\right)_M, \quad \triangle H_o^n = -\left(\frac{\partial C_{mnw}}{\partial n} \Big/ \frac{\partial C_{Lnw}}{\partial n}\right)$$

$$H_o^n = H_{oR} + \triangle H_o^n .$$

In practice the affix o could be dropped from A^o, C^o_{Lwo}, C^o_{mwo} and $(C_{mo})^o_{wing}$ without confusion occurring.

The effect on existing longitudinal stability theory of using these more complicated expressions for C_{Lw}, $C_{mw_{C.G.}}$ is considered in §4.

2.3. <u>Effect of wing distortion on the downwash at the tail</u>

The incidence of any point on the rigid tailplane is given by

$$\alpha_{T1} = \alpha + \eta_T - \varepsilon_1$$

where ε_1 = local downwash angle.
Hence, following the method employed in dealing with wing distortion, we could write

$$\triangle C_{LT} = A_{1\alpha}\ (\alpha + \eta_T) - A_{1\varepsilon}\ \varepsilon_R$$

due to tailplane incidence

where $A_{1R} = \frac{\partial C_{LT}}{\partial \alpha_c}$

α_c = root incidence of tailplane with zero wing downwash

$= \alpha + \eta_T$

$A_{1\varepsilon} = -\frac{\partial C_{LT}}{\partial \varepsilon_R}$

ε_R = downwash angle at tailplane root.

Values of $A_{1\alpha}$ and $A_{1\varepsilon}$ might then be found using the superposition method,* provided that the spanwise downwash distribution were known.

We could then write

$$\triangle C_{LT} = A_1\ (\alpha + \eta_T - \bar{\varepsilon}_R)$$

/due to ...

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* This is strictly true only for an all-moving tailplane, see 3.2.

due to incidence

where $\bar{\varepsilon}_R = \varepsilon_R \left(\frac{A_{1\varepsilon}}{A_{1\alpha}} \right)$

and $A_1 = A_{1\alpha}$

In practice, however, the downwash distribution is not usually known with any accuracy even for the rigid wing, and since it will change slightly with speed due to wing distortion it will not be possible to carry out a single aeroelastic equilibrium calculation for the tailplane applying to all speeds. In view of these difficulties it seems acceptable to use a mean downwash angle, the mean being weighted in favour of the regions of greatest tailplane lift.

Since the downwash is produced by wing lift it is logical to use an expression for the mean downwash angle $\bar{\varepsilon}$ having the same form as the expression for C_{Lw}.

Then $$\bar{\varepsilon} = A^o \alpha \bar{\bar{\varepsilon}}_\alpha + \bar{\varepsilon}_o + A_n . n \bar{\bar{\varepsilon}}_n = \bar{\varepsilon}_\alpha + \bar{\varepsilon}_o + \bar{\varepsilon}_n \quad ..(10)$$

where the lift distribution corresponding to $A^o\alpha$ produces a mean downwash angle $\bar{\varepsilon}_\alpha$, the lift distribution corresponding to C_{Lwo} produces a mean downwash angle $\bar{\varepsilon}_o$, and the lift distribution corresponding to nA_n produces a mean downwash angle $\bar{\varepsilon}_n$.

The value of $\bar{\varepsilon}$ will approximate to the value of $\bar{\varepsilon}_R$ defined above, and we write

$$\bar{\varepsilon} \simeq \bar{\varepsilon}_R = \bar{\varepsilon}_{\alpha R} + \bar{\varepsilon}_{oR} + \bar{\varepsilon}_{nR} ,$$

so that the use of $\bar{\varepsilon}$ does not necessarily imply an approximation.

Since the lift distributions $A^o\alpha$ etc. may be obtained by using the superposition method, it is possible to obtain the downwash distributions corresponding to these lift distributions by superposition of the downwash distributions corresponding to the arbitrary lift distributions employed. This can, however, be a somewhat involved process, and in practice it may be acceptable to use estimated values of $\bar{\bar{\varepsilon}}_\alpha$ etc. with semi-empirical corrections for the effects of wing distortion.

The above expression for $\bar{\varepsilon}$ is more complicated than the usual expressions for downwash angle, and the effect of using it in stability theory is considered in §4.

/3. ...

3. Fuselage, control circuit and tail distortion

Distortion of the fuselage, tail and control circuits modifies the tail lift and pitching moment contributions, and also the elevator hinge moment. This has been considered in ref. 2, where however compressibility effects have not been explicitly considered.

Lyon and Ripley[2] write

$$C_{LT} = A_1 \alpha_{To} + A_2 \eta + A_3 \beta$$

$$C_H = B_o + B_1 \alpha_{To} + B_2 \eta + B_3 \beta$$

where $\alpha_{To} = \alpha + \eta_{To} - \bar{\varepsilon}$

η_{To} = tail setting angle at zero windspeed ('built-in' tail setting angle)

η, β are control angles equivalent to movements of the pilot's controls (η_p, β_p of ref. 2) as is η_{To} if a 'variable incidence' tailplane is used.

Alternatively, we may write

$$C_{LT} = A_{1T}\, \alpha_T + A_2 \eta + A_3 \beta$$

$$C_H = B_o + B_{1T} \alpha_T + B_2 \eta + B_3 \beta$$

where $\alpha_T = \alpha + \eta_T - \bar{\varepsilon}$

η_T = true (root) tail-setting angle

$$A_{1T} = \frac{\partial C_{LT}}{\partial \alpha_T}$$

= value of A_1 when fuselage is rigid,✱

and likewise

$$B_{1T} = \frac{\partial C_H}{\partial \alpha_T} .$$

3.1. Effect of fuselage inertia loading

The above expressions may be used to include the effects of all distortion under purely aerodynamic loading, the values of A_1, A_2 etc. being modified to take account of these effects. If

/the effect ...

✱ This applies to tailplanes of 'fixed' incidence - with a 'variable incidence' tailplane A_{1T} is the value of A_1 when both fuselage and incidence control circuit are rigid.

the effect of bending of the fuselage under its own weight is to be included a slight modification of the expressions for C_{LT} and C_H is necessary.

Suppose application of a (total) normal acceleration of ng produces a net change of $n.\partial\alpha_T/\partial n$ in the value of α_T.

Then $$C_{LT} = A_{1T}\alpha_T^o + nA_{1T}\frac{\partial\alpha_T}{\partial n} + A_2\eta + A_3\beta \quad \text{.........}(11)$$

where α_T^o = true root tailplane incidence for $n = 0$.

This may be written

$$C_{LT} = C_{LT}^o + nA_{1T}\frac{\partial\alpha_T}{\partial n} \quad \text{............}(12)$$

and similarly

$$\left.\begin{aligned} C_H &= C_H^o + nB_{1T}\frac{\partial\alpha_T}{\partial n} \\ &= B_o + B_1\alpha_{To} + nB_{1T}\frac{\partial\alpha_T}{\partial n} + B_2\eta + B_3\beta \end{aligned}\right\} \quad \text{........}(13)$$

If application of ng produces a net change of tail setting angle $\Delta\eta_T$, we have

$$\Delta\eta_T = \Delta\eta_{Ti} - \frac{A_{1T}\,\Delta\eta_T\, qS_T}{F_f}$$

where $\Delta\eta_{Ti}$ = change of η_T due to ng at zero windspeed

$q = \frac{1}{2}\rho V^2$

F_f = fuselage bending stiffness = $\dfrac{\text{load on tail}}{\text{change in } \eta_T}$

Then

$$\Delta\eta_T = \frac{\Delta\eta_{Ti}}{1 + \dfrac{A_{1T}q\,S_T}{F_f}}$$

and $$\frac{\partial\alpha_T}{\partial n} = \frac{\Delta\eta_T}{n} = \frac{\partial\eta_{Ti}}{\partial n}\left(\frac{1}{1 + A_{1T}\dfrac{q\,S_T}{F_f}}\right) \quad \text{.........}(14)$$

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial\alpha_T}{\partial n}\right) = 0\,;\quad \frac{\partial}{\partial M}\left(\frac{\partial\alpha_T}{\partial n}\right) = -\frac{\partial\eta_{Ti}}{\partial n}\,\frac{\left(\dfrac{\partial A_{1T}}{\partial M}\,.q + A_{1T}\dfrac{2q}{M}\right)}{\left(1 + A_{1T}\dfrac{qS_T}{F_f}\right)^2}\,.\left(\frac{S_T}{F_f}\right) \quad \text{............}(15)$$

/3.2. ...

3.2. Calculation of A_1, A_2 etc. including distortion effects

The analysis of ref. 2 is adopted here but with certain modifications and the inclusion of compressibility effects.

Fuselage distortion (aerodynamic loading)

Formulae are deduced in ref. 2 for the ratios A_1/a_1, A_2/a_2 allowing for fuselage distortion and these expressions may be applied in general if a_1, a_2 etc. are interpreted as the values of A_1, A_2 etc. when compressibility and all distortion effects except fuselage distortion are included.

Tailplane-elevator distortion

As pointed out in ref. 2, tailplane distortion and elevator distortion cannot be considered separately since the two are completely interdependent. For this reason it is not strictly possible to use the superposition method for the estimation of distortion effects on the tailplane-elevator combination. In ref. 3 a method of treating the 'flap-deflection' case is given which uses the superposition method, but this is based on the assumption that the local flap angle is not changed by distortion of the main surface or flap, so that the form of the spanwise load distribution due to the flap deflection remains the same at all speeds. While this assumption may not lead to serious errors when dealing with the wing-aileron combination (although it seems dubious even in this case), it seems likely that it would lead to appreciable errors if used for a tailplane and elevator. For the treatment of the latter, therefore, the method of 'semi-rigid representation' as given in ref. 2 would seem to be more suitable. If the effects of tab distortion and elevator skin distortion can be neglected (see below), the effects of compressibility may be introduced (by means of linearised theory corrections) to the 'strip' derivatives used. The corrections applied should be those appropriate to the three dimensional tail surface, so that the overall lift and hinge moment coefficients vary in the correct way although changes in the form of the lift distribution due to compressibility are ignored, as with the superposition method. It is not possible to carry out the calculations in terms of a 'compressibility - distortion' parameter such as qA_R (used in the superposition method) since the aerodynamic coefficients a_1, a_2, a_3, b_1, b_2, b_3 are not all modified by compressibility in the same way. It is therefore strictly necessary to carry out calculations for each combination of q and M within the required range.

/Elevator tab ...

Elevator tab and tab circuit distortion

In ref. 2 it is shown that if C_1 and C_2 are small (as they usually are), A_1, A_2, B_1 and B_2 are almost unaffected by tab distortion and A_3, B_3 and C_3 are modified by the factor $(K/K-C_3')$ where C_3' is the value of C_3 when tab distortion is ignored and K is proportional to $(1/q)$ x tab stiffness. On this basis the elevator angle to trim, and hence the stick fixed static margin K_n, is not appreciably changed while the stick free static margin K_n' is slightly changed. Similarly the stick fixed manoeuvre margin H_m is almost unaffected while the stick free manoeuvre margin H_m' is slightly changed. If the tab circuit can be designed so that K is large compared with C_3' for all q, tab distortion effects are then obviously negligible. If this cannot be done, tab distortion effects are easily included by applying the above factor to A_3, B_3, C_3, on the reasonable assumption that the secondary effects of tab distortion on elevator and tailplane distortion are negligible.

If power operated controls are fitted this type of distortion does not, of course, arise.

Elevator skin distortion

The effect of elevator skin distortion is also considered in ref. 2 where it is shown that distortion of the elevator skin, caused by pressure differences between the inside and outside of the control surface, can modify A_1, A_2 and A_3 slightly and B_1, B_2 and B_3 to quite a large extent. The treatment is approximate, since skin distortion of a fabric or metal covered surface is not proportional to load.

The curves in ref. 2 of panel deflection against load show that for an unstiffened metal skin the rate of change of deflection with load is small once the small initial deflection required to take any load has been exceeded. If the skin is applied with initial tension (as is normal practice) this small initial deflection can be minimised. In view of this it seems reasonable to ignore the effects of elevator skin distortion for design purposes on the grounds that it should be possible to design the elevator so as to keep the skin distortion and its effects small enough to be negligible.

The main effect of any skin distortion is to decrease the stick free static and manoeuvre margins, so that if powered controls are used the effects are even less serious than with manually operated controls.

/Elevator control ...

Elevator control circuit distortion

The treatment of elevator control circuit distortion given in ref. 2 can be applied in general if a_1, a_2 etc. are interpreted as the values of A_1, A_2 etc. when compressibility and all distortion effects except control circuit distortion are included.

4. Modifications to existing longitudinal stability theory

When the new expressions of § 2 and 3 for C_{Lw}, C_{mw} etc. are used we have, in general.-

$$C_L = A^o\alpha + C_{Lwo} + nA_n + \frac{S_T}{S}\left\{A_1(\alpha - A^o\dot{\bar{\varepsilon}}_\alpha.\alpha - \bar{\varepsilon}_o - A_n n\,\dot{\bar{\varepsilon}}_n + \eta_{To}) + nA_{1T}\frac{\partial\alpha_T}{\partial n} + A_2\eta + A_3\beta\right\} \qquad (16)$$

$$C_m = C_{mf} + (C_{mo})_{wing} + (h-H^o_o)A^o\alpha + (h-H^n_o)A_n.n - \bar{V}_{C.G.}\left\{A_1(\alpha - A^o\dot{\bar{\varepsilon}}_\alpha\alpha - \bar{\varepsilon}_o - A_n.n\,\dot{\bar{\varepsilon}}_n + \eta_{To}) + nA_{1T}\frac{\partial\alpha_T}{\partial n} + A_2\eta + A_3\beta\right\} \qquad (17)$$

where $C_{mf} = C_m$ contribution from fuselage

$\bar{V}_{C.G.} = S_T l_T/S\bar{c}$

l_T = distance from tailplane mean aerodynamic centre to aircraft C.G.

The other new symbols are defined in §2 and 3.

4.1. Static margin

In ref. 1

$$K_n = -\left(\frac{dC_m}{dC_R}\right)_{\frac{d\eta}{dC_R}=0}$$

$$K'_n = -\left(\frac{dC_m}{dC_R}\right)_{C_H=0}$$

where $C_R\rho V^2$ = constant = $\frac{W}{\frac{1}{2}S}$

i.e. in level flight $n = 1$ = constant.

/Following ...

Following ref. 1 the expression for C_m can be modified by the introduction of a new tail-volume coefficient V_T. Thus, when $n = 1$,

$$C_m = C_{mf} + (C_{mo})_{wing} + (h-H_o^o)(C_L - C_{Lwo} - A_n) + (h-H_o^n)A_n$$

$$- V_T \left\{ A_1 \left[\left(\frac{1}{A^o} - \dot{\varepsilon}_\alpha \right)(C_L - C_{Lwo} - A_n) - \bar{\varepsilon}_o - A_n \dot{\varepsilon}_n + \eta_{To} \right] + A_{1T} \frac{\partial \alpha_T}{\partial n} + A_2 \eta + A_3 \beta \right\} \quad \text{.............(18)}$$

where $$V_T = \frac{\bar{V}_{AC}}{1 + A_1 \frac{S_T}{S}\left(\frac{1}{A^o} - \dot{\varepsilon}_\alpha\right)} = \frac{\bar{V}_{AC}}{1+\bar{F}}$$

and $$\bar{V}_{AC} = \bar{V}_{C.G.} + (h-H_o^o)\frac{S_T}{S}$$

On differentiating equation (18) with respect to C_R and making use of the fact that $\frac{d\eta}{dC_R} = 0$, we finally obtain, as in ref. 1:-

$$K_n = - V_T A_2 \left(\frac{d\eta}{dC_R}\right)_{C_m = 0}$$

Similarly for the stick free case: - $(n = 1)$

$$C_m = C_{mf} + (C_{mo})_{wing} + (h-H_o^o)(C_L - C_{Lwo} - A_n) + (h-H_o^n) A_n$$

$$- \bar{V}_T \left\{ \bar{A}_1 \left[\left(\frac{1}{A^o} - \dot{\varepsilon}_\alpha \right)(C_L - C_{Lwo} - A_n) - \bar{\varepsilon}_o - A_n \dot{\varepsilon}_n + \eta_{To} \right] + \bar{A}_{1T} \frac{\partial \alpha_T}{\partial n} + \bar{A}_3 \beta - \frac{A_2}{B_2} B_o \right\} \quad \text{.............(19)}$$

where $$\bar{V}_T = \frac{\bar{V}_{AC}}{1 + \bar{A}_1 \frac{S_T}{S}\left(\frac{1}{A^o} - \dot{\varepsilon}_\alpha\right)} = \frac{\bar{V}_{AC}}{1+\bar{F}}$$

$$\bar{A}_1 = A_1 - \frac{A_2}{B_2} B_1$$

$$\bar{A}_{1T} = A_{1T} - \frac{A_2}{B_2} B_{1T}$$

$$\bar{A}_3 = A_3 - \frac{A_2}{B_2} B_3 .$$

This yields similarly,

$$K'_n = - \bar{V}_T \bar{A}_3 \left(\frac{d\beta}{dC_R}\right)_{\substack{C_H = 0 \\ C_m = 0}}$$

as in ref. 1.

/4.2. ...

4.2. Manoeuvre Margin

The manoeuvre margin, H_m, is given by[1]

$$H_m = -\left(\frac{\partial C_m}{\partial C_L}\right)_M - \frac{\ell}{\bar{c}}\frac{m_q}{\mu_1}$$

The second term is usually fairly small and decreases with increase of altitude, so that the value of the manoeuvre margin is greatly influenced by the value of $(\partial C_m/\partial C_L)_M$ ✱.

When n is variable we have

$$C_m = C_{mf} + (C_{mo})_{wing} + (h-H_o^o)(C_L - C_{Lwo} - A_n.n) + (h-H_o^n)A_n.n$$
$$- V_T\left\{A_1\left[\left(\frac{1}{A^o}-\dot{\varepsilon}_\alpha\right)(C_L - C_{Lwo} - A_n n) - \bar{\varepsilon}_o - A_n\dot{\varepsilon}_n.n + \eta_{To}\right]\right.$$
$$\left. + A_{1T}\frac{\partial\alpha_T}{\partial n}.n + A_2\eta + A_3\beta\right\} \qquad \ldots\ldots\ldots\ldots(20)$$

Differentiating (20) with respect to C_L, we have with stick fixed (i.e. η and β const.)

$$\left(\frac{\partial C_m}{\partial C_L}\right)_M = \left(\frac{\partial C_{mf}}{\partial C_L}\right)_M + (h-H_o^o)\left(1 - A_n\left(\frac{\partial n}{\partial C_L}\right)_M\right) + (h-H_o^n)A_n\left(\frac{\partial n}{\partial C_L}\right)_M$$
$$- V_T\left\{A_1\left(\frac{1}{A^o}-\dot{\varepsilon}_\alpha\right)\left(1 - A_n\left(\frac{\partial n}{\partial C_L}\right)_M\right) + A_{1T}\frac{\partial\alpha_T}{\partial n}\left(\frac{\partial n}{\partial C_L}\right)_M\right\}$$
$$\ldots\ldots\ldots\ldots(21)$$

If $C_L = \frac{n\,W}{\frac{1}{2}\rho V^2 S}$ and $C_{Lo} = \frac{W}{\frac{1}{2}\rho V^2 S}$

$$C_L = nC_{Lo} \quad\text{and}\quad \left(\frac{\partial n}{\partial C_L}\right)_M = \frac{1}{C_{Lo}} \qquad \ldots\ldots(22)$$

With stick free we have

$$\left(\frac{\partial C_m}{\partial C_L}\right)_M = \left(\frac{\partial C_{mf}}{\partial C_L}\right)_M + (h-H_o^o)\left(1 - A_n\left(\frac{\partial n}{\partial C_L}\right)_M\right) + (h-H_o^n)A_n\left(\frac{\partial n}{\partial C_L}\right)_M$$
$$- \bar{V}_T\left\{\bar{A}_1\left(\frac{1}{A^o}-\dot{\varepsilon}_\alpha\right)\left(1 - A_n\left(\frac{\partial n}{\partial C_L}\right)_M\right) + \bar{A}_{1T}\frac{\partial\alpha_T}{\partial n}\left(\frac{\partial n}{\partial C_L}\right)_M\right\}$$
$$\ldots\ldots\ldots\ldots(23)$$

/The tail ...

✱ Note that ℓ = distance between tail and no-tail aerodynamic centres $\neq \ell_T$.

The tail contribution to $\left(\frac{m_q}{\mu_1} \cdot \frac{\ell}{\bar{c}}\right)$ is, approximately,

$$- \frac{V_T A_1}{2\,\mu_1} \quad \text{stick fixed}$$

and $$- \frac{\bar{V}_T \bar{A}_1}{2\,\mu_1} \quad \text{stick free}\ .$$

The wing contribution to m_q is not necessarily negligible if the wing is swept.

In the manoeuvre margin theory it is implicitly assumed that m_q is not appreciably changed by movement of the C.G. Thus in deriving the above expressions for tail contribution to m_q the tail moment arm used is the distance between the tail and 'no-tail' aerodynamic centres. This is not strictly correct since the angular velocity q is about the C.G.

When the tailplane is swept its centre section and tips may be fairly large distances forward and aft of the mean tail aerodynamic centre, so that more accurately

$$m_{q_{tail}} = - \frac{V_T A_1}{2\mu} + m_{q_T}$$

where m_{q_T} = contribution to m_q due to rotation of tailplane about its mean aerodynamic centre.

In general, however, m_{q_T} will be small.

4.3. Dynamic Stability - quasi-static theory

Dynamic stability and its relation to the static and manoeuvre margins is discussed in ref. 1 where it is shown that the slow divergence associated with $K_n < 0$ is less serious than the rapid divergence or rapid unstable oscillation corresponding to $H_m < 0$. This means that the value of $(\partial C_m/\partial C_L)_M$ is of greater importance than the value of dC_m/dC_L. In this paragraph we shall discuss the effects of distortion on the quasi-static longitudinal stability derivatives.

If distortion effects are included on a quasi-static basis, the usual small-displacement equations of motion can be used for the flexible aircraft, but the derivatives have to be suitably modified.

We have, ignoring the thrust contributions, (see ref. 1)

$$\left.\begin{aligned} x_u &= -C_D - \frac{M}{2}\left(\frac{\partial C_D}{\partial M}\right)_\alpha \; ; \quad z_u = -C_L - \frac{M}{2}\left(\frac{\partial C_L}{\partial M}\right)_\alpha \\ m_u &= \frac{M}{2}\;\frac{\bar{c}}{\ell}\left(\frac{\partial C_m}{\partial M}\right)_\alpha \; ; \quad x_w = \frac{1}{2}\left(C_L - \left(\frac{\partial C_D}{\partial \alpha}\right)_M\right) \\ z_w &= -\tfrac{1}{2}\left[\left(\frac{\partial C_L}{\partial \alpha}\right)_M + C_D\right]; \quad m_w = \tfrac{1}{2}\;\frac{\bar{c}}{\ell}\left(\frac{\partial C_m}{\partial \alpha}\right)_M \end{aligned}\right\} \quad \ldots\ldots\ldots (24)$$

The largest contributions to m_q and $m_{\dot{w}}$ are normally from the tail, and we have*

$$\mu_1 \underset{(tail)}{m_{\dot{w}}} \doteqdot -\frac{A^o}{2(1+F)}\,\frac{S_T}{S}\,A_1\,\dot{\varepsilon}_\alpha\,, \qquad \underset{(tail)}{m_q} \doteqdot -\frac{A_1}{2(1+F)}\,\frac{S_T}{S} \quad \ldots\ldots\ldots (25)$$

These expressions are approximate only and correspond to those given in ref. 1. The wing contribution to $m_{\dot{w}}$ is normally very small at subsonic subcritical speeds, but the wing contribution to m_q may be appreciable for a swept wing. An estimate of m_q(wing) may be obtained by the method that is described in Appendix III.

The derivatives x_q and z_q occur in the stability equations divided by μ_1 and are then normally small enough to be neglected.

If the distortions of the aircraft components are small it seems reasonable to ignore distortion effects on the drag derivatives, so that C_D, $(\partial C_D/\partial\alpha)_M$ and $(\partial C_D/\partial M)_\alpha$ may be estimated for the rigid aircraft. It remains to determine $(\partial C_L/\partial\alpha)_M$, $(\partial C_L/\partial M)_\alpha$, $(\partial C_m/\partial\alpha)_M$ and $(\partial C_m/\partial M)_\alpha$ in terms of A^o, A_1, H_o^o, etc.

Differentiating the expression for C_L given at the beginning of this section (equation 16).-

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M = A^o + A_n\left(\frac{\partial n}{\partial \alpha}\right)_M + \frac{S_T}{S}\left\{A_1\left[1 - A^o\,\dot{\varepsilon}_\alpha - A_n\left(\frac{\partial n}{\partial \alpha}\right)_M \dot{\varepsilon}_n\right] + \left(\frac{\partial n}{\partial \alpha}\right)_M A_{1T}\cdot\frac{\partial \alpha_T}{\partial n} + A_2\left(\frac{\partial \eta}{\partial \alpha}\right)_M\right\}$$

assuming $\left(\dfrac{\partial \eta_{To}}{\partial \alpha}\right)_M = \left(\dfrac{\partial \beta}{\partial \alpha}\right)_M = 0$

$$\left(\frac{\partial n}{\partial \alpha}\right)_M = \left(\frac{\partial n}{\partial C_L}\right)_M\left(\frac{\partial C_L}{\partial \alpha}\right)_M = \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial \alpha}\right)_M \quad \ldots\ldots\ldots\ldots (26)$$

/Therefore ...

---

* For the stick free case, A_1 is replaced by $\bar{A}_1$.

Therefore

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M = \frac{A^o + \frac{S_T}{S}\left\{A_1\left(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha\right) + A_2\left(\frac{\partial \eta}{\partial \alpha}\right)_M\right\}}{1 - \frac{A_n}{C_{Lo}} + \frac{S_T}{S}\,\frac{1}{C_{Lo}}\left\{A_1 A_n\,\dot{\bar{\varepsilon}}_n - A_{1T}\,\frac{\partial \alpha_T}{\partial n}\right\}} \quad \ldots (27)$$

With stick fixed $\left(\frac{\partial \eta}{\partial \alpha}\right)_M = 0$, so that

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M \text{ stick fixed } = \frac{A^o + \frac{S_T}{S}A_1\left(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha\right)}{1 - \frac{A_n}{C_{Lo}} + \frac{S_T}{S}\,\frac{1}{C_{Lo}}\left\{A_1 A_n\,\dot{\bar{\varepsilon}}_n - A_{1T}\,\frac{\partial \alpha_T}{\partial n}\right\}} \qquad (28)$$

With stick free (using 13).-

$$\left(\frac{\partial \eta}{\partial \alpha}\right)_M = -\frac{B_1}{B_2}\left(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha\right) + \left(\frac{\partial n}{\partial \alpha}\right)_M\left(\frac{B_1}{B_2}A_n\,\dot{\bar{\varepsilon}}_n - \frac{B_{1T}}{B_2}\,\frac{\partial \alpha_T}{\partial n}\right)$$

$$= -\frac{B_1}{B_2}\left(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha\right) + \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial \alpha}\right)_M\left(\frac{B_1}{B_2}A_n\,\dot{\bar{\varepsilon}}_n - \frac{B_{1T}}{B_2}\,\frac{\partial \alpha_T}{\partial n}\right)$$

giving

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M \text{ stick free } = \frac{A^o + \frac{S_T}{S}\bar{A}_1\left(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha\right)}{- \frac{A_n}{C_{Lo}} + \frac{S_T}{S}\,\frac{1}{C_{Lo}}\left(\bar{A}_1 A_n\,\dot{\bar{\varepsilon}}_n - \bar{A}_{1T}\,\frac{\partial \alpha_T}{\partial n}\right)} \qquad (29)$$

Similarly

$$\left(\frac{\partial C_L}{\partial M}\right)_\alpha = \alpha\,\frac{\partial A^o}{\partial M} + \frac{\partial C_{Lwo}}{\partial M} + \left(\frac{\partial n}{\partial M}\right)_\alpha A_n + n\,\frac{\partial A_n}{\partial M}$$

$$+ \frac{S_T}{S}\left\{\frac{\partial A_1}{\partial M}\left(\alpha - A^o\,\dot{\bar{\varepsilon}}_\alpha\,\alpha - \bar{\varepsilon}_o - A_n\,n\,\dot{\bar{\varepsilon}}_n + \eta_{To}\right)\right.$$

$$+ A_1\left[-\alpha\left(\frac{\partial A^o}{\partial M}\,\dot{\bar{\varepsilon}}_\alpha + A^o\,\frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M}\right) - \frac{\partial \bar{\varepsilon}_o}{\partial M} - \frac{\partial A_n}{\partial M}\,n\,\dot{\bar{\varepsilon}}_n - A_n\left(\left(\frac{\partial n}{\partial M}\right)_\alpha \dot{\bar{\varepsilon}}_n + n\,\frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M}\right)\right]$$

$$+ A_{1T}\,\frac{\partial \alpha_T}{\partial n}\left(\frac{\partial n}{\partial M}\right)_\alpha + n\,A_{1T}\,\frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + n\,\frac{\partial \alpha_T}{\partial n}\,\frac{\partial A_{1T}}{\partial M} + \frac{\partial A_2}{\partial M}\,\eta$$

$$\left. + A_2\left(\frac{\partial \eta}{\partial M}\right)_\alpha + \frac{\partial A_2}{\partial M}\,\beta\right\}$$

assuming $\left(\frac{\partial \beta}{\partial M}\right)_\alpha = 0$.

Since $n = \frac{C_L}{C_{Lo}}$, $\left(\frac{\partial n}{\partial M}\right)_\alpha = \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial M}\right)_\alpha$ ....(30)

and initially $n = 1$.

/With stick ...

With stick fixed $\left(\frac{\partial \eta}{\partial M}\right)_\alpha = 0$, so that finally

$$\left(\frac{\partial C_L}{\partial M}\right)_\alpha \text{stick fixed} = \frac{\left[\begin{array}{l} \alpha \frac{\partial A^o}{\partial M} + \frac{\partial C_{Lwo}}{\partial M} + \frac{\partial A_n}{\partial M} + \frac{S_T}{S}\left\{ \frac{\partial A_1}{\partial M}(\alpha - A^o \dot{\bar{\varepsilon}}_\alpha \alpha - \bar{\varepsilon}_o - A_n \dot{\bar{\varepsilon}}_n + \eta_{To}) \right. \\ \quad + A_1 \left[-\alpha \left(\frac{\partial A^o}{\partial M} \dot{\bar{\varepsilon}}_\alpha + A^o \frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M} \right) - \frac{\partial \bar{\varepsilon}_o}{\partial M} - \frac{\partial A_n}{\partial M} \dot{\bar{\varepsilon}}_n - A_n \frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M} \right] \\ \quad \left. + \frac{\partial \alpha_T}{\partial n} \frac{\partial A_{1T}}{\partial M} + A_{1T} \frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + \frac{\partial A_2}{\partial M}\eta + \frac{\partial A_3}{\partial M}\beta \right\} \end{array}\right]}{1 - \frac{A_n}{C_{Lo}} + \frac{S_T}{S} \cdot \frac{1}{C_{Lo}}\left(A_1 A_n \dot{\bar{\varepsilon}}_n - A_{1T}\frac{\partial \alpha_T}{\partial n}\right)}$$

$$\ldots\ldots\ldots\ldots (31)$$

With stick free, it can be shown that (using 13).-

$$\left(\frac{\partial \eta}{\partial M}\right)_\alpha = -\frac{1}{B_2}\frac{\partial B_2}{\partial M}\eta - \frac{1}{B_2}\left\{ \frac{\partial B_o}{\partial M} + \frac{\partial B_1}{\partial M}(\alpha + \eta_{To} - A^o \dot{\bar{\varepsilon}}_\alpha \alpha - \bar{\varepsilon}_o - A_n \dot{\bar{\varepsilon}}_n) \right.$$

$$+ B_1\left(-\alpha\left[\frac{\partial A^o}{\partial M}\dot{\bar{\varepsilon}}_\alpha + A^o \frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M}\right] - \frac{\partial \bar{\varepsilon}_o}{\partial M} - A_n \frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M} - \frac{\partial A_n}{\partial M}\dot{\bar{\varepsilon}}_n \right)$$

$$\left. + B_{1T}\frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + \frac{\partial B_{1T}}{\partial M} \cdot \frac{\partial \alpha_T}{\partial n} + \beta \frac{\partial B_3}{\partial M} \right\} + \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial M}\right)_\alpha \left(B_1 \frac{A_n}{B_2}\dot{\bar{\varepsilon}}_n - \frac{B_{1T}}{B_2}\frac{\partial \alpha_T}{\partial n} \right)$$

$$\ldots\ldots\ldots\ldots (32)$$

and hence finally

$$\left(\frac{\partial C_L}{\partial M}\right)_\alpha \text{stick free} = \frac{\left[\begin{array}{l} \alpha \frac{\partial A^o}{\partial M} + \frac{\partial C_{Lwo}}{\partial M} + \frac{\partial A_n}{\partial M} + \frac{S_T}{S}\left\{ \frac{\overline{\partial A}_1}{\partial M}\left(\alpha + \eta_{To} - A^o \dot{\bar{\varepsilon}}_\alpha \alpha - \bar{\varepsilon}_o - A_n \dot{\bar{\varepsilon}}_n\right) \right. \\ \quad + \bar{A}_1 \left(-\alpha \left[\frac{\partial A^o}{\partial M} \dot{\bar{\varepsilon}}_\alpha + A^o \frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M} \right] - \frac{\partial \bar{\varepsilon}_o}{\partial M} - \frac{\partial A_n}{\partial M} \dot{\bar{\varepsilon}}_n - A_n \frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M} \right) \\ \quad \left. + \frac{\partial \alpha_T}{\partial n} \cdot \frac{\overline{\partial A}_{1T}}{\partial M} + \bar{A}_{1T} \frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + \frac{\overline{\partial A}_2}{\partial M}\eta + \frac{\overline{\partial A}_3}{\partial M}\beta - \frac{A_2}{B_2}\frac{\partial B_o}{\partial M} \right\} \end{array}\right]}{1 - \frac{A_n}{C_{Lo}} + \frac{S_T}{S} \cdot \frac{1}{C_{Lo}}\left(\bar{A}_1 A_n \dot{\bar{\varepsilon}}_n + \bar{A}_{1T}\frac{\partial \alpha_T}{\partial n}\right)}$$

$$\ldots\ldots\ldots\ldots (33)$$

where

$$\frac{\overline{\partial A}_1}{\partial M} = \frac{\partial A_1}{\partial M} - \frac{A_2}{B_2}\frac{\partial B_1}{\partial M} \neq \frac{\partial \bar{A}_1}{\partial M}, \quad \frac{\overline{\partial A}_{1T}}{\partial M} = \frac{\partial A_{1T}}{\partial M} - \frac{A_2}{B_2}\frac{\partial B_{1T}}{\partial M} \neq \frac{\partial \bar{A}_{1T}}{\partial M}$$

$$\frac{\overline{\partial A}_2}{\partial M} = \frac{\partial A_2}{\partial M} - \frac{A_2}{B_2}\frac{\partial B_2}{\partial M} \neq \frac{\partial \bar{A}_2}{\partial M}, \quad \frac{\overline{\partial A}_3}{\partial M} = \frac{\partial A_3}{\partial M} - \frac{A_2}{B_2}\frac{\partial B_3}{\partial M} \neq \frac{\partial \bar{A}_3}{\partial M}.$$

Similarly

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_M \text{stick fixed} = \left(\frac{\partial C_{mf}}{\partial \alpha}\right)_M + (h - H_o^o)A^o - \bar{V}_{C.G.} A_1 (1 - A^o \dot{\bar{\varepsilon}}_\alpha)$$

$$+ \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial \alpha}\right)_M \left\{ (h - H_o^n) A_n + \bar{V}_{C.G.}\left[A_1 A_n \dot{\bar{\varepsilon}}_n - A_{1T}\frac{\partial \alpha_T}{\partial n} \right] \right\}$$

$$\ldots\ldots\ldots\ldots (34)$$

/and ...

and

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_M \text{stick free} = \left(\frac{\partial C_{mf}}{\partial \alpha}\right)_M + (h-H_o^o)\,A^o - \bar{V}_{C.G.}\,\bar{A}_1\,(1 - A^o\,\dot{\bar{\varepsilon}}_\alpha)$$
$$+ \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial \alpha}\right)_M \left\{(h-H_o^n)\,A_n + \bar{V}_{C.G.}\left[\bar{A}_1 A_n \dot{\bar{\varepsilon}}_n - \bar{A}_{1T}\,\frac{\partial \alpha_T}{\partial n}\right]\right\} \qquad \text{............(35)}$$

Also

$$\left(\frac{\partial C_m}{\partial M}\right)_\alpha \text{stick fixed} = \left(\frac{\partial C_{mf}}{\partial M}\right)_\alpha + \frac{\partial (C_{mo})_{wing}}{\partial M} + \alpha\left\{(h-H_o^o)\,\frac{\partial A^o}{\partial M} - A^o\,\frac{\partial H_o^o}{\partial M}\right\}$$
$$+ (h-H_o^n)\,\frac{\partial A_n}{\partial M} - A_n\,\frac{\partial H_o^n}{\partial M} - \bar{V}_{C.G.}\left\{\frac{\partial A_1}{\partial M}\,(\alpha - A^o\,\dot{\bar{\varepsilon}}_\alpha \alpha - \bar{\varepsilon}_o - A_n \dot{\bar{\varepsilon}}_n + \eta_{To})\right.$$
$$+ \dot{A}_1\left[-\alpha\left(\frac{\partial A^o}{\partial M}\dot{\bar{\varepsilon}}_\alpha + A^o\,\frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M}\right) - \frac{\partial \bar{\varepsilon}_o}{\partial M} - \frac{\partial A_n}{\partial M}\,\dot{\bar{\varepsilon}}_n - A_n\,\frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M}\right]$$
$$\left. + \frac{\partial A_{1T}}{\partial M}\,\frac{\partial \alpha_T}{\partial n} + A_{1T}\,\frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + \eta\,\frac{\partial A_2}{\partial M} + \beta\,\frac{\partial A_3}{\partial M}\right\}$$
$$+ \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial M}\right)_\alpha \left\{A_n\,(h-H_o^n) + \bar{V}_{C.G.}\left[A_n \dot{\bar{\varepsilon}}_n . A_1 + A_{1T}\,\frac{\partial \alpha_T}{\partial n}\right]\right\} \qquad \text{............(36)}$$

$$\left(\frac{\partial C_m}{\partial M}\right)_\alpha \text{stick free} = \left(\frac{\partial C_{mf}}{\partial M}\right)_\alpha + \frac{\partial (C_{mo})_{wing}}{\partial M} + \alpha\left\{(h-H_o^o)\,\frac{\partial A^o}{\partial M} - A^o\,\frac{\partial H_o^o}{\partial M}\right\} + (h-H_o^n)\frac{\partial A_n}{\partial M}$$
$$- A_n\,\frac{\partial H_o^n}{\partial M} - \bar{V}_{C.G.}\left\{\frac{\partial \bar{A}_1}{\partial M}\,(\alpha - A^o\,\dot{\bar{\varepsilon}}_\alpha \alpha - \bar{\varepsilon}_o - A_n \dot{\bar{\varepsilon}}_n + \eta_{To})\right.$$
$$+ \bar{A}_1\left[-\alpha\left(\frac{\partial A^o}{\partial M}\,\dot{\bar{\varepsilon}}_\alpha + A^o\,\frac{\partial \dot{\bar{\varepsilon}}_\alpha}{\partial M}\right) - \frac{\partial \bar{\varepsilon}_o}{\partial M} - \frac{\partial A_n}{\partial M}\,\dot{\bar{\varepsilon}}_n - A_n\,\frac{\partial \dot{\bar{\varepsilon}}_n}{\partial M}\right]$$
$$\left. + \frac{\partial \bar{A}_{1T}}{\partial M}\,\frac{\partial \alpha_T}{\partial n} + \bar{A}_{1T}\,\frac{\partial}{\partial M}\left(\frac{\partial \alpha_T}{\partial n}\right) + \frac{\partial \bar{A}_2}{\partial M}.\eta + \frac{\partial \bar{A}_3}{\partial M}\,\beta - \frac{A_2}{B_2}.\frac{\partial B_o}{\partial M}\right\}$$
$$+ \frac{1}{C_{Lo}}\left(\frac{\partial C_L}{\partial M}\right)_\alpha \left\{A_n\,(h-H_o^n) + \bar{V}_{C.G.}\left[A_n \dot{\bar{\varepsilon}}_n . \bar{A}_1 + \bar{A}_{1T}\,\frac{\partial \alpha_T}{\partial n}\right]\right\} \qquad \text{............(37)}$$

The value of β (initial value) to be used in the above will be given by equation (19) with $C_m = 0$. The initial value of η may then be found from (18), again with $C_m = 0$. Equation (16) then gives α. In the above it is assumed that $\bar{V}_{C.G.}$ is constant i.e. that ℓ_T is constant. This is not strictly true since in general the mean aerodynamic centre of the tailplane will move, but the error involved is probably small.

/4.4. ...

4.4. Application of the theory

Static margins

Perhaps the simplest way of determining the static margins is to use the formulae

$$K_n = - V_T A_2 \left(\frac{d\eta}{dC_R}\right)_{C_m = 0},$$

$$K'_n = - V_T A_3 \left(\frac{d\beta}{dC_R}\right)_{\substack{C_m = 0 \\ C_H = 0}} .$$

The values of η and β to trim for a given value of $C_R \doteqdot C_L$ are given by equations (18) and (19) with $C_m = 0$. These values can then be plotted against C_R (or M) and values of $(d\eta/dC_R)_{C_m = 0}$ and $(d\beta/dC_R)_{\substack{C_m = 0 \\ C_H = 0}}$ are then obtained by graphical or numerical differentiation. This procedure is suggested in ref. 2, and must involve some loss of accuracy; although since the trim curves can be obtained by calculation, this loss can be minimised by the use of a sufficiently large number of ordinates. When the trim curves are plotted against C_R the high speed end of the range becomes 'compressed' and the low speed end elongated, so that it is probably better to plot control angles against Mach number.

Then $\qquad \dfrac{d\eta}{dC_R} = - \dfrac{M}{2C_R} \dfrac{d\eta}{dM}$, since $C_R M^2$ = constant.

It would be possible to find the value of $\dfrac{dC_m}{dC_R}$ from the relation

$$\frac{dC_m}{dC_R} = \frac{\left(\frac{\partial C_m}{\partial \alpha}\right)_M \left(1 + \frac{M}{2C_R}\left(\frac{\partial C_R}{\partial M}\right)_\alpha\right) - \frac{M}{2C_R}\left(\frac{\partial C_R}{\partial \alpha}\right)_M \left(\frac{\partial C_m}{\partial M}\right)_\alpha}{\left(\frac{\partial C_R}{\partial \alpha}\right)_M}$$

This method, however, suffers from the disadvantage that values of $\dfrac{\partial A_1}{\partial M}$, $\dfrac{\partial A_2}{\partial M}$ etc. must be obtained by graphical differentiation (in general) so that the loss of accuracy is likely to be greater than that involved in finding $\dfrac{d\eta}{dC_R}$.

/In general ...

In general we have

$$\frac{T_N}{\frac{1}{2}\rho V^2 S} + C_L = C_R \operatorname{Cos} \gamma_e$$

$$\frac{T_T}{\frac{1}{2}\rho V^2 S} - C_D = C_R \operatorname{Sin} \gamma_e$$

where T_N = component of thrust force normal to flight direction

T_T = component of thrust force in flight direction.

γ_e = angle between flight path and horizontal.

Thus in level flight $C_R = C_L$ for $T_N = 0$. This result is not, as is suggested in ref. 6, dependent upon the drag being small compared with the lift.

Manoeuvre margins

The manoeuvre margins may be obtained by direct substitution of values for A^o, A_1 etc. in the equations of 4.2. It is thus a simpler matter to obtain the manoeuvre margins than to obtain the static margins due to the absence of derivatives with respect to forward speed in the expressions for the former margins.

5. Miscellaneous refinements

5.1. Inclusion of effects of change of density with altitude on stability

The effect of density variation with altitude on longitudinal stability is considered by Dr. Neumark in ref. 18. Only the level flight condition is there considered, so that $C_L \doteqdot C_R$.

Static and manoeuvre margins

Neumark gives:

$$K_n = -\left(\frac{\partial C_m}{\partial C_L}\right)_M + \frac{M}{2C_L}\left(\frac{\partial C_m}{\partial M}\right)_{C_L},$$

$$K_{n\beta} = -\left(1 + \frac{\gamma M^2}{2N}\right)\left(\frac{\partial C_m}{\partial C_L}\right)_M + \left(1 - \frac{\gamma M^2}{2}\,\frac{N-1}{N}\right)\frac{M}{2C_L}\left(\frac{\partial C_m}{\partial M}\right)_{C_L}$$

where $N = \dfrac{g}{g - RK_\theta}$

and R = gas constant for air, K_θ = lapse rate.

/The static ...

The static margin (stick fixed or free) as defined by Gates and Lyon in ref. 1 is K_n above (for $C_L = C_R$); $K_{n\beta}$ is the 'generalised static margin' (stick fixed or free) which is proportional to E_1 the last term in the stability quintic when density variation with height is included. However, the quantity $K_{n\beta}$ is no longer a measure of the stick movement (or force) to change speed.

From the above formulae given K_n and $\left(\frac{\partial C_m}{\partial C_L}\right)_M$ (which is found when calculating the manoeuvre margin) $K_{n\beta}$ can be evaluated.

There are no effects of density variations with altitude on the manoeuvre margins.

Effect on stability derivatives

Additional stability derivatives x_β, z_β are introduced which are derived in full in ref. 18. These are, however, functions of C_D, $\partial C_D/\partial M$, C_L, $\partial C_L/\partial M$ and of quantities unaffected by distortion so that they are readily determined.

5.2. Inclusion of effects of changes in the form of the wing lift distribution and movement of aerodynamic centre due to compressibility

If distortion effects are to be included it is very difficult to allow correctly for variations in the form of the wing lift distribution or in the aerodynamic centre position of the rigid wing. If they are functions of Mach number it is strictly necessary to perform a complete set of calculations for each combination of M and q. To avoid the heavy labour of such a procedure it is suggested that if the shift of aerodynamic centre is not very large then the aeroelastic equilibrium calculations can be made first ignoring the shift of aerodynamic centre. Then allowance for the shift can be made without correcting for the secondary distortion effects introduced. If the movement of the aerodynamic centre is large, however, it might be advisable to perform calculations with a range of representative positions, an interpolation procedure being subsequently adopted.

If the tailplane aerodynamic centre shift is also appreciable it may be necessary to modify the tail arm.

/6. ...

6. A simple example

As an example the hypothetical aircraft illustrated in Fig. 1 was considered. It has a wing of 45^o sweep and aspect ratio 3.81, and an 'all-moving' tailplane of similar planform to the wing. For simplicity the structural characteristics of the wing and tailplane were assumed to be similar, so that calculations of lift curve slope etc. carried out for the flexible wing apply also to the tailplane. Compressibility effects were allowed for by applying linearised theory corrections to the wing and tail lift curve slopes, and also to an assumed value of C_{mo} introduced into the calculations as an extra 'fuselage' pitching moment contribution,- the distorted wing has zero twist at all sections and is of symmetrical section giving zero contribution to C_{mo}. No other fuselage or thrust contributions to pitching moment or lift were included. Tail lift was included in the total C_L, but wing weight effects were neglected.

Distortion of the wing, fuselage and tailplane was taken into account, but not control circuit distortion, and there is no tab. This is consistent with a system of completely rigid power operated controls, and hence the 'stick fixed' case only was considered.

6.1. Calculation of lift and pitching moment coefficients for the flexible wing and tailplane

Our first problem is to find values of $A = \left(\frac{\partial C_{Lw}}{\partial \alpha}\right)_M = \left(\frac{\partial C_{L\alpha w}}{\partial \alpha}\right)_M$ and ΔH_o for a suitable range of values of $q(= \frac{1}{2}\rho V^2)$ and Mach number. The wing is an example of the case considered in 2.2.1, and hence the superposition method was used.

The method used was exactly as described in ref. 3, the procedure being as follows.-

i) The method of Kuchemann[7] was employed to give (for incompressible flow) the lift distributions corresponding to incidence distributions $\alpha = \eta_1$, $\alpha = \eta_1^2$, $\alpha = \eta_1^3$, α = constant = 1 radian on the rigid wing. $(\eta_1 = y/_{b/2})$. These lift distributions were integrated graphically to give values of C_{LR}/A_R, C_{L1}/A_R, C_{L2}/A_R, C_{L3}/A_R, where

C_{LR} = rigid wing lift coefficient per radian of incidence

C_{L1} = rigid wing C_L corresponding to $\alpha = \eta_1$, with α = 1 radian at the tip

C_{L2} = rigid wing C_L corresponding to $\alpha = \eta_1^2$, with α = 1 radian at the tip

/C_{L3} = ...

C_{L3} = rigid wing C_L corresponding to $\alpha = \eta_1^3$, with $\alpha = 1$ radian at the tip.

A_R = lift curve slope of rigid wing.

It was found that $C_{LR}/A_R = 1.0$, $C_{L1}/A_R = 0.435$, $C_{L2}/A_R = 0.264$, $C_{L3}/A_R = 0.184$.

ii) In stage (i) the locus of aerodynamic centres was obtained and hence the pitching moment distributions corresponding to $\alpha = \eta_1$, $\alpha = \eta_1^2$ etc. were plotted and integrated to give the position of the mean aerodynamic centre of the rigid wing, and the pitching moment coefficients about that point for the various incidence distributions considered.

Then

$$\frac{C_{mR}}{A_R} = 0, \quad \frac{C_{m1}}{A_R} = -0.0775, \quad \frac{C_{m2}}{A_R} = -0.0730, \quad \frac{C_{m3}}{A_R} = -0.0675$$

where C_{mR}, C_{m1} etc. are the pitching moment coefficients about the rigid wing mean aerodynamic centre corresponding to C_{LR}, C_{L1} etc.

iii) A relation between A_R and Mach number was obtained using the method due to Collingbourne.[16]

iv) The lift distributions for the rigid wing were integrated to give shear force and bending moment distributions, it being assumed initially that the wing had a straight flexural axis lying along the 0.45 chord line. Torque distributions about this flexural axis were also obtained.

v) Twist and slope distributions for the four cases were obtained using assumed stiffness distributions, and these distributions were then modified at the root in an attempt to introduce corrections corresponding roughly to the root constraint effects on a swept wing.

vi) The elastic incidence changes of (v) were matched as described in ref. 3 and in 2.1.1. to give the superposition coefficients A_S, B_S, C_S for a range of values of qA_R.

vii) Using (iii) and (vi) a graph of A and ΔH_o against Mach number was produced for the condition $q/\hat{q} = M^2$, where $\hat{q}$ = value of q corresponding to maximum allowable E.A.S. (Fig.2).

In the above it was assumed that compressibility effects modified the two and three-dimensional lift curve slopes without

/appreciably ...

appreciably modifying the form of the lift distribution, or the position of the mean aerodynamic centre, of the rigid wing. The relation referred to in (iii) connects two and three-dimensional lift curve slopes, so that when compressibility corrections are applied to the aspect ratio, sweep angle and two dimensional slope, the required relation between M and A_R is obtained.

In (iv) and (v) it was first assumed that, with a straight flexural axis at 0.45c, the ratio $\frac{\text{bending stiffness}}{\text{torsional stiffness}}$ was constant along the span, and that each stiffness varied as the cube of the local chord. On the basis of information given in R.A.E. Structures Reports 9 and 58 it was decided that a representative value of the above ratio was 4.0, and that a representative root torsional stiffness was given by

$$\text{Twist per unit torque at root} = \frac{S\ell_f^2 \hat{q}}{6.6} \quad \text{where} \quad \ell_f = \text{length of flexural axis.}$$

It is shown in ref. 19 that with a swept wing of moderate aspect ratio and conventional construction, the concept of an effective root may be used. The wing may be considered to behave like an unswept wing outboard of this effective root, but inboard of this the root restraint effects are predominant. The information given in ref. 19 suggests that for a wing of 45° sweep the effective root might be about 0.2 semi-span out from the root. On this basis the twist and slope distributions were modified as shown in Fig. 11. Since this modified the overall values of twist and slope considerably the root stiffness used was decreased from the value previously quoted, giving

$$(GJ)_R = \frac{S\ell_f^2 \hat{q}}{10} .$$

Methods of calculating the rigid wing lift distributions and the elastic distortions of swept wings are discussed in Appendices I and II.

6.2. Introduction of fuselage distortion

Following the treatment in ref. 2, we have

$$A_1 = \frac{A_{1T}}{1+JA_{1T}q}$$

where J = constant, inversely proportional to the fuselage bending stiffness.

Using the A.P.970 fuselage bending stiffness criterion, with $M = 0.8$ and $K = 0.12$

$$J \doteqdot \frac{0.05}{\hat{q}}, \quad \text{where } \hat{q} \text{ corresponds to } V_D,$$

/where ...

where V_D = 'maximum allowable diving speed'

Since the wing and tailplane are similar, $A_{1T} = A$.

6.3. Trim curves

Trim curves for the example aircraft are shown in Figs. 3 and 4. Two sets were plotted, one corresponding to $C_{mo} = 0$ (zero fuselage contribution and zero wing contribution about corrected aerodynamic centre) and the other to $C_{mo} = -\frac{0.015}{\sqrt{1-M^2}}$ (taken as $(C_{mo})_{fuse.}$, $(C_{mo})_{wing}$ remaining zero).

It was assumed for simplicity that $\dot{\varepsilon}_\alpha = 0.1$ = constant, and the relation

$$C_L = 0.0420\left(\frac{\hat{q}}{q}\right) = \frac{0.0420}{M^2}$$

was used, corresponding to $W/S = 50$ lb./ft^2 and $\hat{q} = \frac{1}{2}\rho_o V^2$ where $V = 1000$ f.p.s.

Curves of η_{To} (tail setting angle equivalent to movement of pilot's control) against C_L were produced for the rigid aircraft and for the following cases of distortion.-

(i) Wing distortion only

(ii) Fuselage distortion only

(iii) Tail distortion only

(iv) Wing, fuselage and tail distortion.

6.3.1. Trim curves with $C_{mo} = 0$

The curves are shown in Fig. 3. and it is clear that the distortion and compressibility effects introduced have had little effect on the slopes of the trim curves except at the highest speeds. Let us consider these effects in turn.

The effect of compressibility in the absence of distortion is to displace the trim curve a small amount which is nearly constant for all values of C_L. The curve, which is linear and passes through the origin when no compressibility effects are included, remains very nearly linear down to $C_L \doteq 0.1$. Below this C_L the slope of the curve becomes slightly more positive i.e. a stabilising effect occurs. Ref. 6 predicts that for $C_{mo} = 0$ the increment of elevator angle to trim due to compressibility is very nearly constant over the whole speed range the approximation becoming less exact as Mach number increases. This is in agreement with the present results.

/The distortion ...

The distortion of the wing alone then produces a negligible change in the trim curve. The reason for this appears to be that the effect of the loss of lift curve slope due to distortion (the wing tips bend upwards) is offset by the forward movement of the wing aerodynamic centre. Thus, for a given C_L, wing distortion makes it necessary to fly at a slightly higher incidence so that for a given value of η_T the nose down tail contribution is increased. This effect is almost exactly cancelled by the extra nose-up moment resulting from the forward movement of the wing aerodynamic centre, so that the value of η_T to trim is unchanged.

Distortion of the fuselage alone produces a constant increment of η_T to trim over the whole range, thus leaving the slope unchanged. This is because with $C_{mo} = 0$ and a fixed C.G. position the tailplane load is constant over the whole speed range.

Distortion of the tailplane alone also again produces a constant increment of tail angle to trim and the reason for this is again that the tail load is constant at all speeds. Since the form of the lift distribution due to any given twist distribution of the tailplane is assumed independent of Mach number, the twist due to a given overall load is the same at all speeds and hence the increment of tail-setting angle to trim arising from twist is constant.

When all the distortion effects are combined the total increment of η_{To} to trim is slightly greater than the algebraic sum of the separate increments taken individually. This is because wing distortion makes necessary a slight increase in incidence for a given C_L which, for a given η_{To}, causes an increase in the tail load. When the tailplane and fuselage are rigid this has little effect on η_{To} to trim since it is largely cancelled by forward movement of the wing aerodynamic centre. When the tailplane and fuselage are flexible, however, the increased tail load causes extra tail and fuselage distortion which in turn require a small additional increment of η_{To}.

6.3.2. Trim curve with $C_{mo} = -\dfrac{0.015}{\sqrt{1-M^2}}$

The curves are shown in Fig. 4 and it will be seen that in this case distortion and compressibility effects have modified the form of the trim curves considerably.

The effects of compressibility alone are very marked at

/the higher ...

the higher Mach. numbers considered. As the lift coefficient decreases and the Mach number increases, C_{mo} becomes more and more negative - i.e. the tail setting angle to trim out C_{mo} becomes more and more negative, and at high speeds this contribution to η_{To} is large compared with that required to trim out the other wing and tail pitching moment contributions. At low speeds the reverse is the case, so that as speed increases, and the C_{mo} contribution becomes more dominant, the slope of the trim curve becomes less negative, then zero, and finally positive.

The effect of wing distortion alone, however then makes no further appreciable difference to the trim curve, as in the case when $C_{mo} = 0$. Again, the forward movement of the wing aerodynamic centre is offset by the increased nose down pitching moment contribution due to the higher tail incidence as before.

Distortion of the fuselage alone makes the slope of the trim curve more positive (destabilising), the effect increasing with speed. The change in C_{mo} produces a change in tail load, which becomes more and more negative (tail down) as the Mach number increases. As the speed increases the fuselage distortion due to the increasingly negative tail load produces an increasingly positive incidence of the tail and therefore an increasingly negative increment of η_{To} is required to trim.

Tail distortion alone has a very similar effect to that of fuselage distortion. The all-moving tail behaves like a wing, unlike the usual tailplane-elevator combination. Thus a positive tail load causes the tips of the tailplane to bend upwards, producing a positive increment of η_{To} to trim, and conversely for a negative tail load. The changes in η_{To} to trim are therefore in the same sense as those caused by fuselage bending.

It will be seen that the result of the combined distortion effects at the highest speeds is slightly less than the algebraic sum of the separate effects. This is because, as for $C_{mo} = 0$, wing distortion causes a slight positive change in tail load which in this case at high speeds reduces slightly the magnitude of the tail load, which is negative being largely determined by C_{mo}. Thus the fuselage and tail distortions are slightly reduced and there is a small reduction in the overall (negative) increment of η_{To} due to distortion when the distortion effects are combined.

/6.4. ...

6.4. Stick fixed static margin

For an aircraft fitted with an all-moving tailplane we have

$$K_n = -V_T A_1 \left(\frac{d\eta_{To}}{dC_L}\right)_{C_m = 0} \quad (\text{for } C_L = C_R)$$

Values of $\left(\frac{d\eta_{To}}{dC_L}\right)_{C_m = 0}$ were accordingly obtained from the trim curves for the cases $C_{mo} = 0$, $C_{mo} = -\frac{0.015}{\sqrt{1-M^2}}$.

6.4.1. $C_{mo} = 0$

The resulting curves of K_n against $q/\hat{q}$ for $C_{mo} = 0$ are given in Fig. 5. Consider first the effects of compressibility. It will be seen that the static margin increases slightly with increase of Mach number. This is because $(d\eta_{To}/dC_L)_{C_m = 0}$ is very nearly constant except at the highest speeds where it becomes slightly more negative, whilst A_1 is increased by compressibility.[x] This result is in agreement with that predicted in ref. 6 for the case $C_{mo} = 0$. When K_n is positive in incompressible flow, the restoring tail pitching moment due to a change of speed and corresponding change of incidence ($C_R V^2$ = constant) will exceed the destabilising wing contribution. The difference between these contributions will be increased if both wing and tail lift curve slopes are increased by compressibility in roughly the same ratio, C_{mo} remaining zero.

Considering now the effects of distortion, we see that since wing distortion alone does not appreciably change the slope of the trim curve or the value of V_T or of A_1 for the aircraft considered, K_n is almost completely unaffected by wing distortion. The effect of fuselage and tail distortion however is to decrease the value of A_1 (V_T still remains very nearly constant) so that although for $C_{mo} = 0$ the trim curve slopes are not appreciably modified by these effects, the value of K_n is decreased progressively as $q/\hat{q}$ increases compared with the value for the rigid aircraft.

In this example the loss of static margin due to fuselage distortion is greater than that due to tail distortion, and the combined effects produce a maximum loss of static margin of the

/order of ...

---

x V_T is very nearly constant.

order of 10 - 15 per cent at a Mach number of about 0.8.

6.4.2. $C_{mo} = -\frac{0.015}{\sqrt{1-M^2}}$

Curves of K_n against $q/\hat{q}$ for the case when $C_{mo} = -\frac{0.015}{\sqrt{1-M^2}}$ are given in Fig. 6.

Here it will be seen that compressibility effects alone produce a large loss of static margin at high Mach numbers. The static margin, which at low M is approximately 0.10, becomes zero at $q/\hat{q} \doteqdot 0.5$ ($M \doteqdot 0.7$) and rapidly increases negatively at higher speeds. This loss of K_n with increase of M is predicted in ref. 6, where it is shown that the loss of K_n due to compressibility depends on the value of $(C_{mo}/K_n)_{M=0}$. As shown above, the static margin is increased by compressibility for $C_{mo} = 0$ by an amount dependent on the low speed value of K_n. When $C_{mo} < 0$ however, the value of dC_{mo}/dC_L is positive and increases rapidly as M increases, producing an increasing loss of static margin. These two effects are in opposition, but the latter is dominant in this example. The loss of static margin will depend on the relative size of the two effects - i.e. on the value of $(C_{mo}/K_n)_{M=0}$.

Considering now distortion effects we will see that wing distortion alone causes no appreciable change in K_n as in the case $C_{mo} = 0$ and the reason is the same as in that case. Fuselage and tail distortion however cause a reduction in A_1 and also make the value of $(dm_{T_o}/dC_L)_{C_m = 0}$ more positive (see Fig. 4) so that they produce a large loss of K_n. This reduction in K_n due to fuselage and tail distortion is actually greater than with $C_{mo} = 0$, but since here the reduction due to compressibility is very great, the distortion effects appear less important.

6.5. Stick fixed manoeuvre margin

For this simple example the formula of 4.2 (equation 21) becomes

$$-\left(\frac{\partial C_m}{\partial C_L}\right)_M = -(h-H_o) + V_T A_1 \left(\frac{1}{A} - \frac{\varepsilon}{c}\right)$$

and

$$\left(\frac{m_q}{\mu_1} \cdot \frac{\ell}{c}\right) \doteqdot -\frac{V_T A_1}{2\mu_1} + \frac{m_{qw}}{\mu_1} \frac{\ell}{c}$$

The value of μ_1 was taken as 50. The wing contribution

/to m_q ...

to m_q (m_{qw}) was estimated as described in Appendix III.

The resulting values of H_m as functions of $q/\hat{q}$ are shown in Fig. 7.

It will be seen that compressibility alone produces a loss of H_m which steadily increases as M increases, and this is in agreement with the results of the analysis of ref. 6.

We can see how this arises from the formula

$$H_m = -(h-H_o) + V_T A_1 \left(\frac{1}{A} - \dot{\varepsilon}_\alpha\right) - \frac{m_q}{\mu_1}\frac{\ell}{\bar{c}}$$

The effect of compressibility is to increase A_1, A and m_q. The net result is a reduction in the second term, which is large and an increase in the third term which is small.

As with the static margin, we find that wing distortion alone has no effect on manoeuvre margin for the example considered. Wing distortion reduces the values of H_o and A, so that both the first and second terms in the above expression for H_m are increased, the two changes however almost exactly cancel each other.

Both the fuselage and tail distortion effects on the other hand leave H_o unchanged but reduce A_1 and m_q so that there is a resulting decrease of H_m which, in this example, is greater for fuselage distortion than for tail distortion.

The combined effect of wing, fuselage and tail distortion is very nearly the algebraic sum of the effects taken separately. This suggests that it might be possible to calculate the loss of manoeuvre margin due to each and add the separate contributions, but in fact this would take longer than the single calculation for the combined effects.

To assess the error in H_m due to using $\bar{V}$ instead of V_T (i.e. due to neglecting tail lift in obtaining C_L) values of H_m were calculated using the formula quoted above, but substituting (i) $\bar{V}_{C.G.}$ and (ii) $\bar{V}_{AC}$ for V_T. The results are shown in Fig. 8, where the curve corresponding to the true value of V_T is also shown for comparison. It is interesting to note that the error due to using $\bar{V}_{AC}$ instead of V_T is roughly twice that due to using $\bar{V}_{C.G.}$ and that the error due to using $\bar{V}_{C.G.}$ is roughly equal to the magnitude of the value of m_q/μ_1 (in this example).

/6.6. ...

6.6. Quasi-static stability derivatives

For an aircraft with all moving tail and no tab the equations of 4.3 with stick fixed and taking $\dot{\varepsilon}_\alpha$ as constant simplify to.-

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M = A + \frac{S_T}{S} A_1 \left(1 - A\dot{\varepsilon}_\alpha\right)$$

$$\left(\frac{\partial C_L}{\partial M}\right)_\alpha = \alpha \frac{\partial A}{\partial M} + \frac{S_T}{S} \left\{ \frac{\partial A_1}{\partial M} (\alpha + \eta_{To} - \bar{\varepsilon}) - A\alpha \dot{\varepsilon}_\alpha \frac{\partial A}{\partial M} \right\}$$

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_M = A\,(h-H_o) - \bar{V}_{C.G.} \left\{ A_1 \left(1 - A\dot{\varepsilon}_\alpha\right) \right\} = \left(\frac{\partial C_m}{\partial C_L}\right)_M \left(\frac{\partial C_L}{\partial \alpha}\right)_M$$

$$\left(\frac{\partial C_m}{\partial M}\right)_\alpha = \frac{\partial C_{mo}}{\partial M} + \alpha \frac{\partial A}{\partial M}(h-H_o) - A\alpha \frac{\partial H_o}{\partial M} - \bar{V}_{C.G.} \left\{ \frac{\partial A_1}{\partial M} (\alpha + \eta_{To} - \bar{\varepsilon}) - A_1 \alpha \dot{\varepsilon}_\alpha \frac{\partial A}{\partial M} \right\}$$

Values of α, η_{To}, $\bar{\varepsilon}$ were calculated when determining the static margin.

The longitudinal stability derivatives involve the above partial derivatives (see para. 4.3). If it is assumed, as suggested in para. 4.3, that the drag contributions are unaffected by distortion, then the effects of distortion on the stability derivatives may be demonstrated by evaluating $(\partial C_L/\partial \alpha)_M$, $(\partial C_L/\partial M)_\alpha$, $(\partial C_m/\partial \alpha)_M$, $(\partial C_m/\partial M)_\alpha$ with and without distortion effects. This was done for the aircraft considered using the values of A, A_1, H_o, etc. previously calculated, and these partial derivatives are plotted against $q/\hat{q}$ ($= M^2$ in this case) in Figs. 9 and 10. Only the case $C_{mo} = 0$ is considered, and curves are given for the rigid aircraft and for the aircraft with all distortion effects included (wing, fuselage and tail distortion).

The derivative $\left(\frac{\partial C_L}{\partial \alpha}\right)_M$

As might be expected, the curves describing the variation of $(\partial C_L/\partial \alpha)_M$ with $q/\hat{q}$ are very similar to those of A against $q/\hat{q}$. The tail contribution is of the order of 10 - 15 per cent of the wing contribution, so that it is not negligible.

The derivative $\left(\frac{\partial C_m}{\partial \alpha}\right)_M$

The curves for $(\partial C_m/\partial \alpha)_M$ were obtained from the values of $(\partial C_m/\partial C_L)_M$ calculated when determining the manoeuvre margin and the values of $\left(\frac{\partial C_L}{\partial \alpha}\right)_M$ already derived.

/It will be ...

It will be seen that for the rigid aircraft $\left(\frac{\partial C_m}{\partial \alpha}\right)_M$ is very nearly constant over the whole range. Since $(\partial C_m/\partial C_L)_M$ is decreased by increase of M (see 6.5) and $(\partial C_L/\partial \alpha)_M$ is increased at the same time, the resultant change of $(\partial C_m/\partial \alpha)_M$ is small. Distortion effects reduce both $(\partial C_m/\partial C_L)_M$ and $(\partial C_L/\partial \alpha)_M$ as speed is increased, so that $(\partial C_m/\partial \alpha)_M$ decreases steadily with increase of M for the flexible aircraft.

<u>The derivative</u> $\left(\frac{\partial C_L}{\partial M}\right)_\alpha$

Since $(\partial C_L/\partial M)_\alpha$ is a 'constant incidence' derivative, the expression for it contains terms of the type $\alpha \frac{\partial A}{\partial M}$. The magnitude of $(\partial C_L/\partial M)_\alpha$ therefore depends on the magnitudes of α, α_T and $\frac{\partial A}{\partial M}$, $\partial A_1/\partial M$. For small values of $q/\hat{q}$ $\frac{\partial A}{\partial M}$ is small but α changes rapidly, whilst for high values of $q/\hat{q}$ $\frac{\partial A}{\partial M}$ changes rapidly but α then changes slowly. The net result is $(\partial C_L/\partial M)_\alpha$ has a maximum value for moderate values of $q/\hat{q}$.

Distortion increases α and reduces $\frac{\partial A}{\partial M}$; the latter effect is dominant and there is consequently a reduction in $(\partial C_L/\partial M)_\alpha$ due to distortion which is nearly constant over the whole range.

<u>The derivative</u> $\left(\frac{\partial C_m}{\partial M}\right)_\alpha$

We have $\partial C_{mo}/\partial M = C_{mo} = 0$, and for the rigid aircraft $(h-H_o)$ is constant i.e. $\partial H_o/\partial M = 0$. For the rigid aircraft, therefore, the expression for $(\partial C_m/\partial M)_\alpha$ quoted above is reduced to two terms, viz.

$$\alpha \frac{\partial A}{\partial M}(h-H_o) \quad \text{and} \quad \bar{V}_{C.G.}\left\{\frac{\partial A_1}{\partial M}\alpha_{To} - A_1 \dot{\alpha\varepsilon}_\alpha \frac{\partial A}{\partial M}\right\}.$$

The second term is small compared with the first (i.e. the wing contribution is large compared with that from the tail) so that we have

$$\left(\frac{\partial C_m}{\partial M}\right)_\alpha \doteqdot \alpha \frac{\partial A}{\partial M}(h-H_o)$$

Since $(h-H_o)$ is constant, the form of the curve of $(\partial C_m/\partial M)_\alpha$ for the rigid aircraft is very similar to that of $(\partial C_L/\partial M)_\alpha$, and thus

$$\left(\frac{\partial C_m}{\partial M}\right)_\alpha \doteqdot (h-H_o)\left(\frac{\partial C_L}{\partial M}\right)_\alpha .$$

/When ...

When distortion effects are introduced, $\frac{\partial H_o}{\partial M}$ is no longer zero, and in fact the term $(- A\alpha \ \partial H_o/\partial M)$ is dominant, being large and positive. At low speeds, where α increases rapidly as $q/\bar{q}$ is reduced, this term causes a large increase in $(\partial C_m/\partial M)_\alpha$. At higher speeds the increase is somewhat less although still considerable.

Effects of distortion on stability derivatives

For the aircraft considered, with $C_{mo} = 0$, we see that distortion reduces $(\partial C_L/\partial \alpha)_M$, $(\partial C_m/\partial \alpha)_M$ and $(\partial C_L/\partial M)_\alpha$ so that z_w, m_w and z_u are decreased. The effects on z_w and m_w are appreciable even at moderate speeds, but the effect on z_u will be serious only at high M and low C_L. Distortion increases $(\partial C_m/\partial M)_\alpha$ so that m_u is increased and this effect is most marked at the lowest speeds. Tail contributions to m_q, $m_{\dot{w}}$ will be decreased by distortion since both A and A_1 are reduced. On the assumption that drag derivatives are unchanged by distortion, x_u and x_w will not be affected.

It must be emphasised that the above results apply only to the aircraft considered and for the condition $C_{mo} = 0$, and one must be cautious in attempting to generalise from these results.

7. Concluding Remarks

It is clear that there is no intrinsic difficulty in including distortion effects on longitudinal stability if frequency effects are neglected. The amount of calculation involved may, however, be considerable, especially if wing twist, camber and weight effects have to be included. If, therefore, there is good reason to believe that distortion effects are small (as they may be on a fighter-type aircraft with high stiffnesses) a simple crude assessment may be adequate. On a large bomber or transport aircraft, however, for which load factors and hence stiffnesses are likely to be lower, and which are likely to have higher aspect ratio wings, it may be essential to make a detailed analysis of distortion effects on the lines discussed. This is particularly true of the 'podded engine' layout used in conjunction with thin swept wings.

For the aircraft considered in §6 the results show that if $C_{mo} = 0$ neither compressibility nor distortion effects modify the static margin to any serious extent. When C_{mo} is negative, however, both effects are heavily destabilising as far as the static margin is concerned. It therefore seems advisable to give a high-speed aircraft a layout which is as nearly symmetrical as possible about the plane of the wings.

/List of References ...

List of References

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/List of Symbols ...

List of Symbols

		Defined in para.
A	Wing lift curve slope with compressibility and distortion effects included	2.2.1.
A_R	Rigid wing lift curve slope when compressibility effects are included	6.1.
A^o	Value of A for condition $n = 0$ (i.e. neglecting inertia loading)	2.2.3.
A_n	$= (\partial C_{Lnw}/\partial n)_M$	2.2.3.
A_S, B_S, C_S	Superposition coefficients	2.1.1.
A_1, A_2, A_3 $\bar{A}_1, \bar{A}_2$, etc.	Tail lift coefficient derivatives (Ref.1) when compressibility and all distortion effects are included	Refs. 1,2.
A_{1T}, B_{1T} $\bar{A}_{1T}$	Tail lift and hinge moment derivatives corresponding to true tail-setting angle	3 4.1.
$\frac{\partial \bar{A}_1}{\partial M}$, $\frac{\partial \bar{A}_{1T}}{\partial M}$ etc.		4.3.
B_o, B_1, B_2, B_3	Tail hinge moment derivatives when compressibility and all distortion effects are included	Refs. 1,2.
$\bar{c}$	Standard mean chord	
C_{Lo}	Overall lift coefficient based on wing area in steady level flight $= W/\frac{1}{2}\rho V^2 S$	4.2.
C_{Lw}	Wing lift coefficient	2.2.1,2,3.
$C_{Lcw}, C_{Ltw}, C_{Lnw}$	Wing lift coefficient increments due to built-in camber and twist and wing weight	2.2.2, 2.2.3.
$C_{L\alpha w}$	Wing lift coefficient due to incidence only	2.2.1, 2.2.2.
C_{Lwo}	$= C_{Lcw} + C_{Ltw}$	2.2.2.
C_{mw}	Wing pitching moment coefficient about <u>rigid</u> wing mean aerodynamic centre	2.2.2.
$C_{mcw}, C_{mtw}, C_{mnw}$	Wing pitching moment coefficients corresponding to C_{Lcw}, C_{Ltw}, C_{Lnw}.	2.2.1, 2.2.2. 2.2.3.
$(C_{mo})_{wing}$	Wing pitching moment coefficient about <u>flexible</u> wing mean aerodynamic centre	2.2.2.
C_{mf}	Fuselage pitching moment coefficient (based on wing area)	4.
$C_R = \frac{W}{\frac{1}{2}\rho V^2 S}$	'Resultant vertical force coefficient'	4.1.
F	$A_1 \frac{S_T}{S}\left(\frac{1}{A^o} - \dot{\bar{\varepsilon}}_\alpha\right)$	4.1.

/F ...

		Defined in para.
$\bar{F}$	$\bar{A}_1 \dfrac{S_T}{S}\left(\dfrac{1}{A^o} - \dot{\bar{\varepsilon}}_\alpha\right)$	4.1.
H_m, H'_m	Stick fixed, stick free manoeuvre margins	Refs. 1, 2.
H_o, H_o^o	Value of h corresponding to position of (flexible) wing mean aerodynamic centre of lift due to incidence only - affix o refers to $n = 0$.	2.2.1, 2.2.2, 2.2.3.
H_o^n	'Inertial aerodynamic centre' position	2.2.3.
H_{oR}	Value of H_o for rigid wing	2.2.1.
K_n, K'_n	Stick fixed, free static margins	Ref. 1
ℓ	Distance from tail aerodynamic centre to wing aerodynamic centre (corresponding to H_o^o)	4.2. 4.1. (by implication)
ℓ_T	Distance from tail aerodynamic centre to C.G. of aircraft.	4.
q	$= \frac{1}{2}\rho V^2$; angular velocity in pitch	
$\bar{V}_{AC}$	$= \dfrac{S_T \ell}{S\bar{c}}$ (Tail Volume Coefficients)	4.1.
$\bar{V}_{C.G.}$	$= \dfrac{S_T \ell_T}{S\bar{c}}$ (Tail Volume Coefficients)	4.
V_T	$= \dfrac{\bar{V}_{AC}}{1+F}$ (Tail Volume Coefficients)	4.1.
$\bar{V}_T$	$= \dfrac{\bar{V}_{AC}}{1+\bar{F}}$ (Tail Volume Coefficients)	4.1.
α	Wing incidence measured at root	2.2.1.
α_T, α_{To}	Tailplane root incidences corresponding to η_T, η_{To}	3.
β	Tab angle corresponding to movement of pilots trimmer control	3.
$\bar{\varepsilon}$	Mean downwash angle at tailplane	2.3.
$\bar{\varepsilon}_\alpha$	Mean downwash angle at tailplane due to wing root incidence	2.3.
$\bar{\varepsilon}_o$	Mean downwash angle at tailplane corresponding to C_{Lwo}	2.3.
$\bar{\varepsilon}_n$	Mean downwash angle at tailplane due to distortion of wing under inertia loading	2.3.
$\dot{\bar{\varepsilon}}_\alpha$	$= \bar{\varepsilon}_\alpha / A^o\alpha$	2.3.
$\dot{\bar{\varepsilon}}_n$	$= \bar{\varepsilon}_n / A_n . n.$	2.3.
η	Elevator angle corresponding to movement of control column	3.
η_{To}	Tail setting angle corresponding to setting on ground (if 'fixed') or to movement of pilots control (if 'variable')	3.
η_T	True (root) tail setting angle	3.

APPENDIX I

CALCULATION OF AERODYNAMIC LOADING ON RIGID SWEPT WINGS

Several methods of calculating the lift distribution on swept wings have been produced. Among these are those due to Multhopp[9], Garner,[10] Falkner, Weissinger,[11] De Young,[12] Kuchemann[7] and Diederich[13]. Stanton Jones[14] has derived empirical formulae based on the results of Weissinger which enable lift distribution to be calculated very rapidly for trapezoidal untwisted wings. None of these methods includes thickness, viscosity or compressibility effects.

Of the methods quoted above, those due to Falkner, Multhopp and Garner might be described as 'lifting surface' theories by means of which estimates of span and chordwise lift distributions, lift curve slope and aerodynamic centre locus position may be obtained. All three methods require a considerable amount of computation if the full advantagesof the 'lifting surface' method are to be obtained.

The other methods quoted are mainly modifications of the Prandtl 'lifting line' theory used for unswept wings and suffer from the disadvantage that reliable estimates of the aerodynamic centre locus position cannot be obtained. However, they require less computation than the true lifting surface methods. The method of Kuchemann is different from the others in that it uses assumed chordwise lift distributions for the tip, approximately mid-span, and root sections, with distributions at the intermediate sections based on an empirical interpolating relation. A modified version of the flat plate 'loading law' is used. It is thus possible to obtain an estimate of aerodynamic centre locus position, though the estimate cannot be more accurate than the initial assumptions involved. The method is, however, simple to apply and provides a reasonably accurate estimate of the spanwise lift distribution, and it was used in the example of §6 in this report.

/Appendix II ...

APPENDIX II

CALCULATION OF DISTORTION OF SWEPT WING UNDER GIVEN AERODYNAMIC LOADING

The mode of distortion of a swept wing under load is different from that of an unswept wing, due to the effects of the oblique restraint at the root. Whereas an unswept wing can be treated as a simple cantilever beam in bending and as a tube in torsion, if we accept the concept of a straight flexural axis, a swept wing presents a more difficult problem and the flexural axis concept cannot strictly be used. Near the root the wing tends to bend about the root (streamwise) chord, due to the restraint, but the outer portion of the wing behaves more like a straight wing i.e. it bends about a line roughly normal to the leading and trailing edges. Just outboard of the root there is a 'transition' between these two types of distortion, so that the overall mode of distortion is complicated.

For the purpose of estimating deflections as required for aeroelastic calculations it is possible to use the simple beam theory for a high aspect ratio swept wing. The root restraint effects are confined to a small fraction of the semi-span when the aspect ratio is high. The wing is then treated as a straight wing from the structural point of view, but the slopes and twists are resolved into the line of flight to give the incidence changes. There is some ambiguity about the position of the effective root, and this method can only be approximate. It is, however, quite widely used and it is put forward in refs. 3, 20 and 21.

Experimental evidence (ref. 19) shows that as the aspect ratio is decreased and sweep angle is increased this 'effective root' method ceases to yield useful results, because the region in which root restraint effects are large now occupies a large fraction of the semi-span. The deflections of quite a large portion of the wing cannot then be estimated by this simple approach. It is possible to make a correction for moderate aspect ratios (ref 20) but for low aspect ratios (or where root stress distributions are required) it will be necessary to use a more refined approach. The simple beam theory can be applied together with a 'self equilibrating' stress system (ref. 22), and this method has been used successfully in practice. More exact methods have been put forward (e.g. ref 23) but these are somewhat more complicated. Once the simple beam theory is discarded the simple integrations previously used to obtain deflections can no longer be used, and matrix or some other methods must be employed in conjunction with

/influence ...

influence coefficients. A review of recent methods considered in the U.S.A. is given in ref. 24.

Finally, we must note that with a very low aspect ratio (delta) wing, there may be changes of camber which produce effects of the same order of magnitude as the twist effects.

APPENDIX III

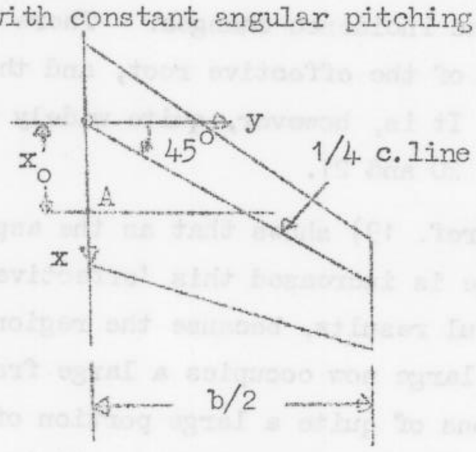
ESTIMATION OF WING CONTRIBUTION TO m_q

With a swept wing the wing contribution to m_q may be appreciable compared with the tail contribution. For the example of §6 m_{qw} was estimated as follows.

Following ref. 1, we obtain a change of manoeuvre margin for an increment of wing pitching moment ΔC_{mw}, arising from the wing contribution to m_q, given by

$$\Delta H_m = - \frac{\Delta C_{mw}}{nC_L} = - \frac{m_{qw}}{\mu} \frac{\ell}{c}$$

ΔC_{mw} is the pitching moment increment due to rotating the wing with constant angular pitching velocity q. Suppose all the wing lift to be generated at the quarter chord line, and that the wing is rotating about the point A, distance x_o from the root quarter chord point taken as the origin.



The incidence change at the point (x,y) on the quarter chord line due to q is then

$$\Delta \alpha = + \frac{q(x-x_o)}{V}$$

and $x = y \tan 45^o = y$

i.e. $\Delta \alpha = \frac{q(y-x_o)}{V}$.

The component $-\frac{qx_o}{V}$ is constant along the span.

For the purpose in hand it is sufficient to neglect

/distortion ...

distortion effects, i.e. we shall determine m_{qw} for the rigid wing.

The constant incidence change $-\frac{qx_o}{V}$ provides zero pitching moment contributions about the rigid wing mean aerodynamic centre. We need therefore consider only the incidence change

$$+\frac{qy}{V} = \frac{q\ell}{V} \cdot \frac{y}{\ell} \ .$$

This is an incidence distribution that increases linearly with spanwise distance from the root.

From the calculations of (ii) in §6.1 for the case $\alpha = \eta_1$, we have $\frac{C_{m1}}{A_R} = -0.0775$ per radian incidence at tip. Hence it follows that the pitching moment coefficient about the rigid wing mean aerodynamic centre due to the rotation at angular velocity q is

$$\Delta C_{mw} = -0.0775\, A_R \cdot \frac{b/2}{\ell} \left(\frac{q\ell}{V}\right)$$

In this case, $\frac{b/2}{\ell} = 0.907$.

$$\text{Also } q = \frac{ng}{V} = \frac{nC_L}{2\mu_1} \cdot \frac{V}{\ell}$$

$$\text{i.e. } nC_L = 2\mu_1 \left(\frac{q\ell}{V}\right).$$

Therefore

$$\Delta H_m = -\frac{\Delta C_{mw}}{nC_L} = +\frac{0.0775\, A_R \times 0.907}{2\mu_1}$$

$$= +\frac{0.070\, A_R}{2\mu_1}$$

The corresponding tail contribution is

$$+\frac{V_T A_1}{2\mu_1}$$

A_1 and A_R are of the same order, and $V_T \doteq 0.3$, so that in this case $m_{qw} \doteq 25\%\ m_{q_{tail}}$.

Note that ΔC_{mw} is the pitching moment about the mean aerodynamic centre and not the centre of gravity. This is, however, consistent with the use of ℓ instead of ℓ_T in the formula: $\Delta \alpha_T = \frac{q\ell}{V}$ and also with the assumption already made in §4.2 that m_{qw} is independent of C.G. position.

distortion effects, i.e. we shall determine m_{qw} for the rigid wing.

The constant incidence change $-\frac{qx_0}{V}$ provides zero pitching moment contributions about the rigid wing mean aerodynamic centre. We need therefore consider only the incidence change

$$+\frac{qy}{V} = \frac{q\ell}{V} \cdot \frac{y}{\ell} .$$

This is an incidence distribution that increases linearly with spanwise distance from the root.

From the calculations of (ii) in §6.1 for the case set 1, we have $\frac{C_{m\ell}}{F} = -0.0775$ per radian incidence at tip. Hence it follows that the pitching moment coefficient about the rigid wing mean aerodynamic centre due to the rotation at angular velocity q is

$$\Delta C_{mw} = -0.0775\, A_R \cdot \frac{b/2}{\ell} \left(\frac{q\ell}{V}\right)$$

In this case, $\frac{b/2}{\ell} = 0.907$.

$$\text{Also } \quad q = \frac{ng}{V} = \frac{nC_L}{2\mu_1} \cdot \frac{V}{\ell}$$

$$\text{i.e. } \quad nC_L = 2\mu_1 \left(\frac{q\ell}{V}\right) .$$

Therefore

$$\Delta H_m = -\frac{\Delta C_{mw}}{nC_L} = +\frac{0.0775\, A_R \times 0.907}{2\mu_1}$$

$$= +\frac{0.070\, A_R}{2\mu_1}$$

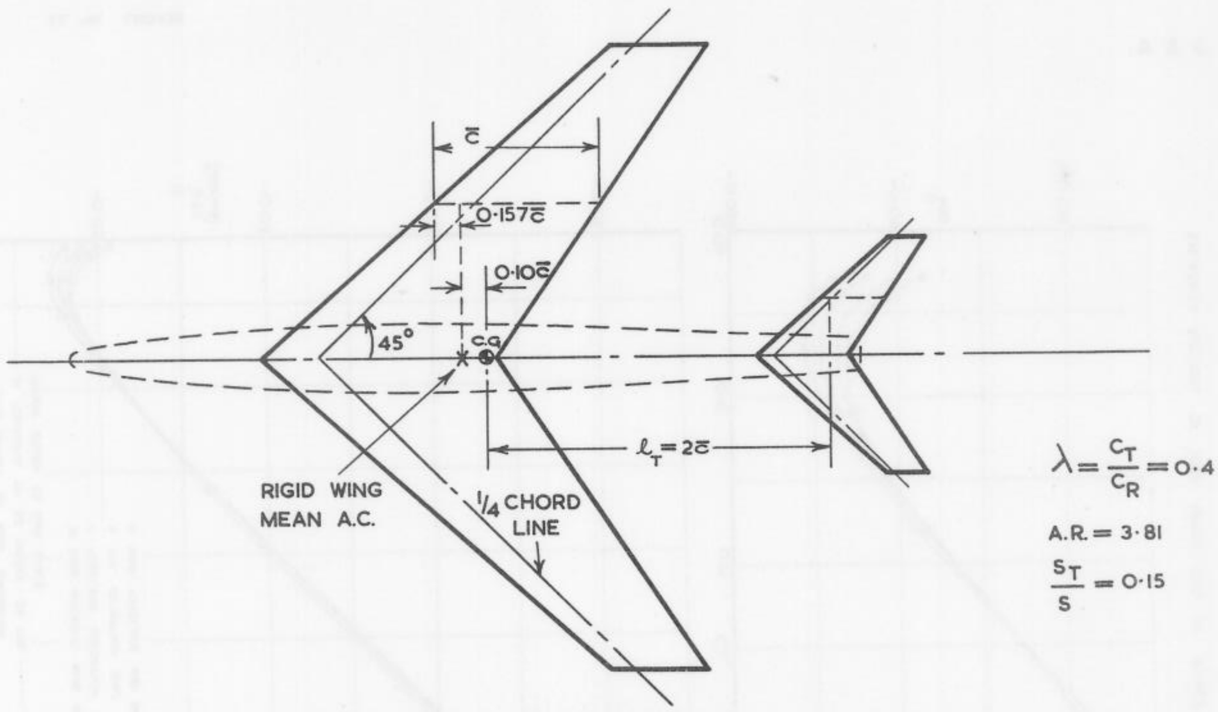
The corresponding tail contribution is

$$+\frac{V_T A_1}{2\mu_1}$$

A_1 and A_R are of the same order, and $V_T \doteqdot 0.3$, so that in this case $m_{qw} \doteqdot 25\%\, m_{q_{tail}}$.

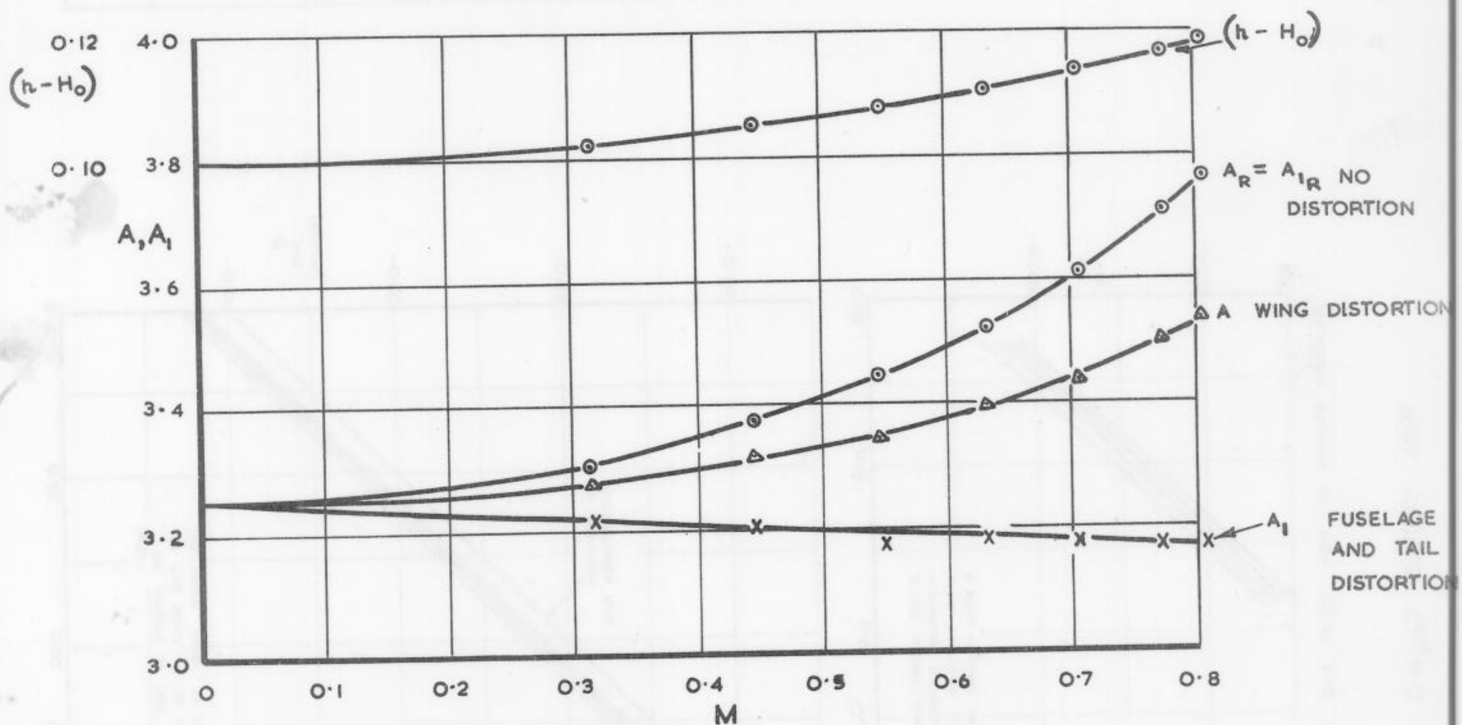
Note that ΔC_{mw} is the pitching moment about the mean aerodynamic centre and not the centre of gravity. This is, however, consistent with the use of ℓ instead of ℓ_T in the formula: $\Delta\alpha_T = \frac{q\ell}{V}$ and also with the assumption already made in §4.2 that m_{qw} is independent of C.G. position.

FIG. 1.

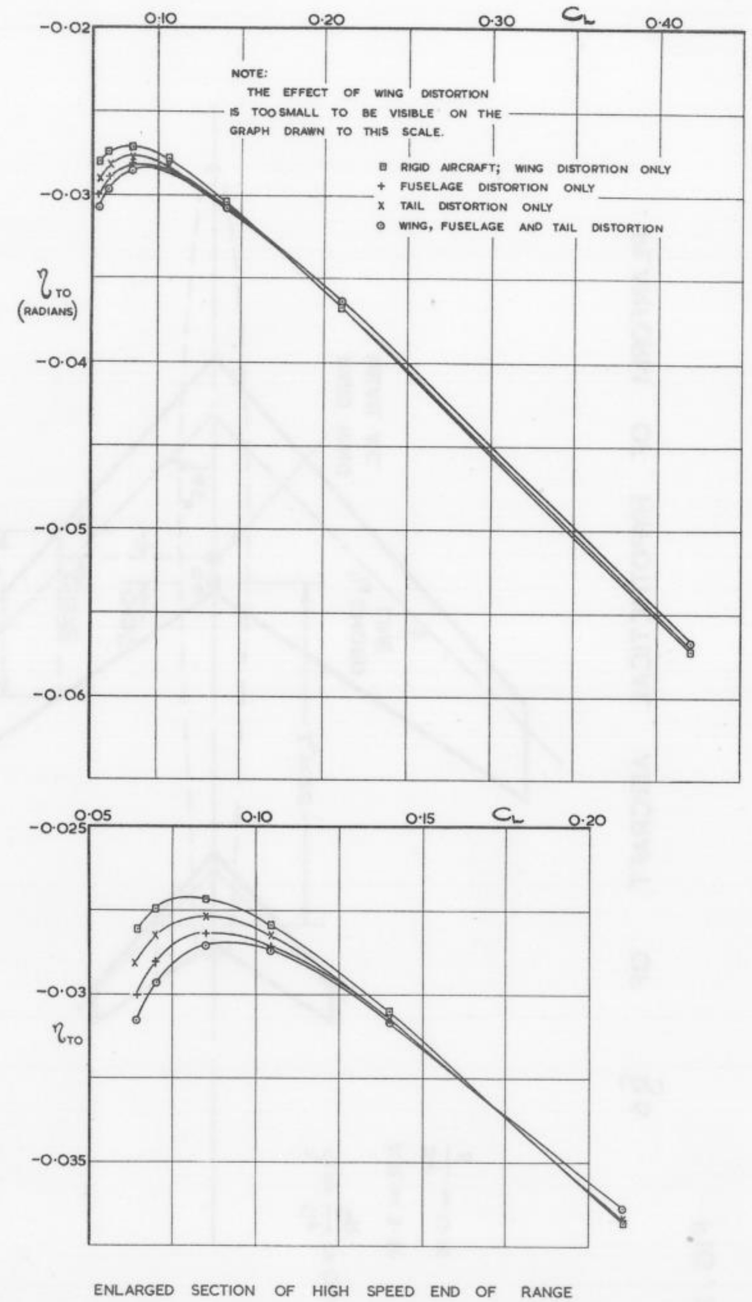


PLANFORM OF HYPOTHETICAL AIRCRAFT OF §6

FIG. 2.

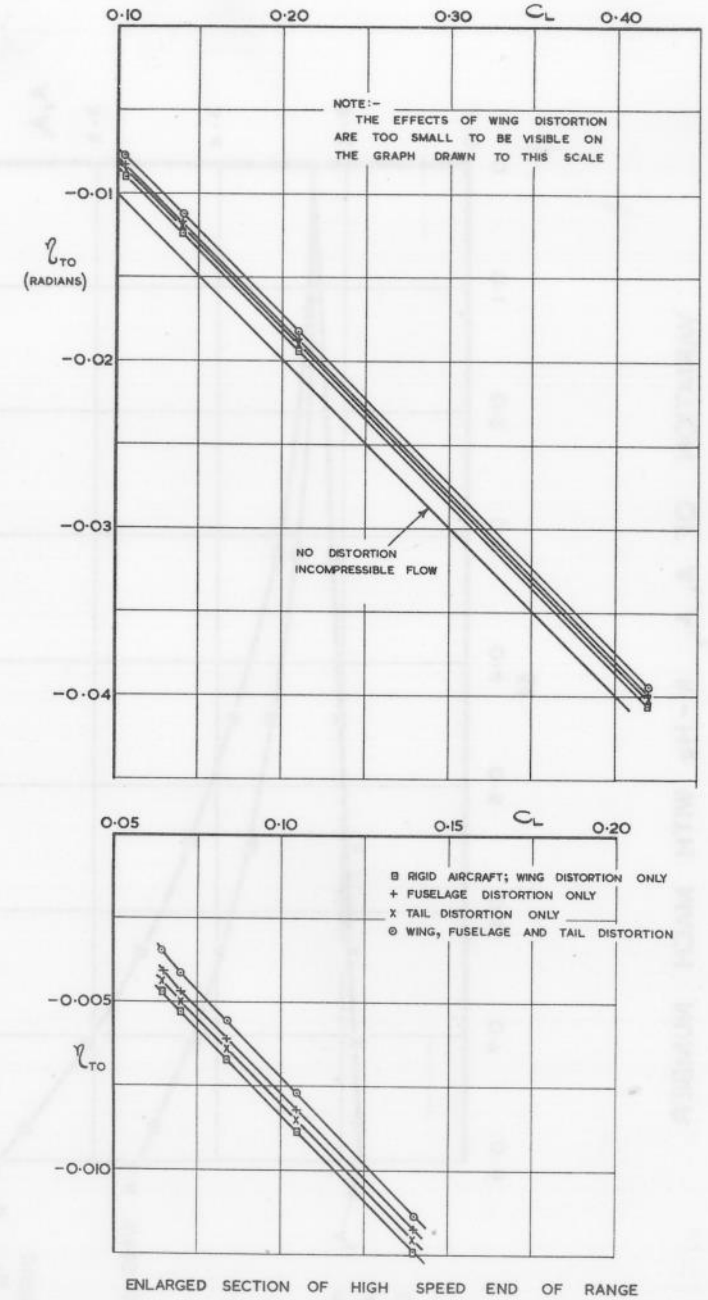


VARIATION OF A, A_1, h - Ho WITH MACH NUMBER
WITH AND WITHOUT DISTORTION



TRIM CURVES $C_{m_0} = \frac{-0\cdot015}{\sqrt{1-M^2}}$

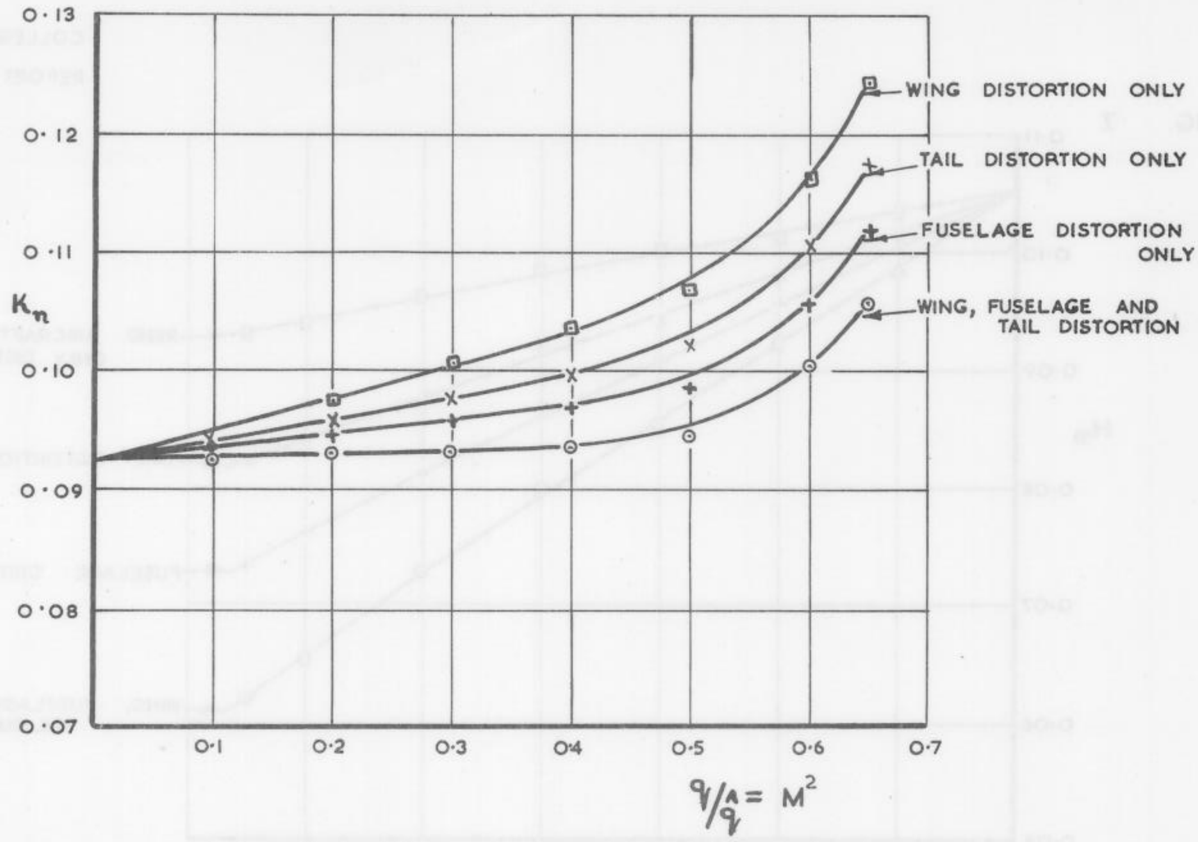
FIG. 4.



TRIM CURVES $C_{m_0} = 0$

FIG. 3.

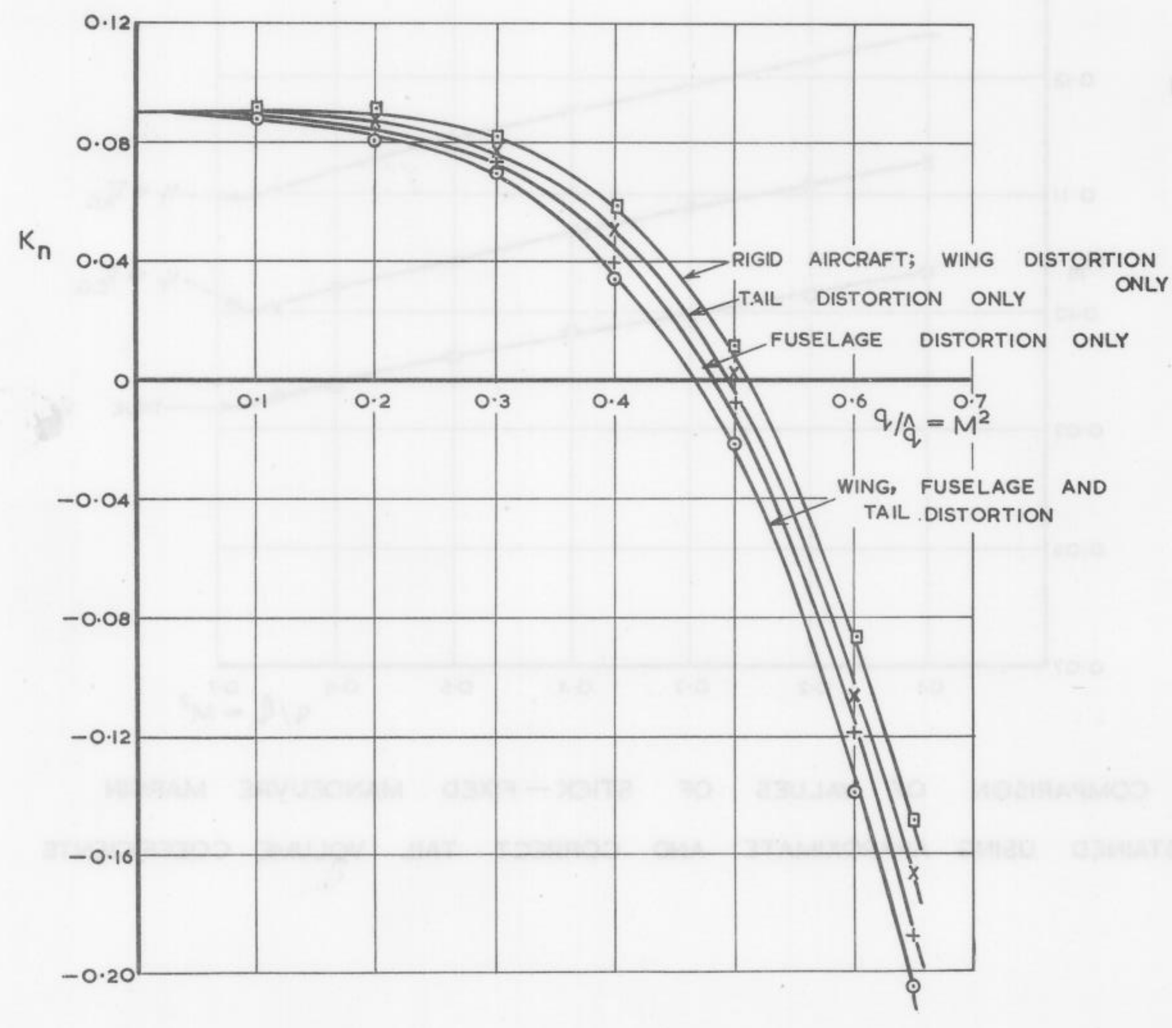
FIG. 5



VARIATION OF STICK – FIXED STATIC MARGIN WITH $q/\hat{q}$

$C_{m_0} = 0$

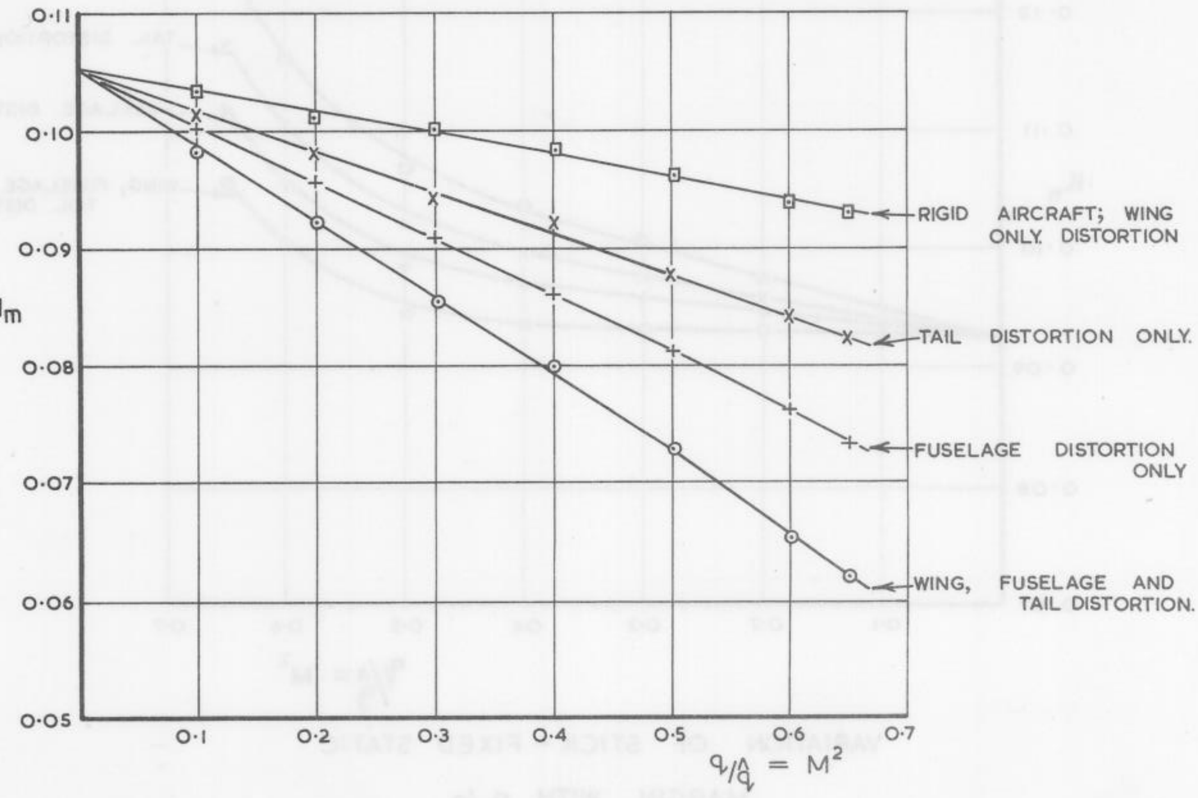
FIG. 6.



VARIATION OF STICK FIXED STATIC MARGIN WITH $q/\hat{q} = M^2$

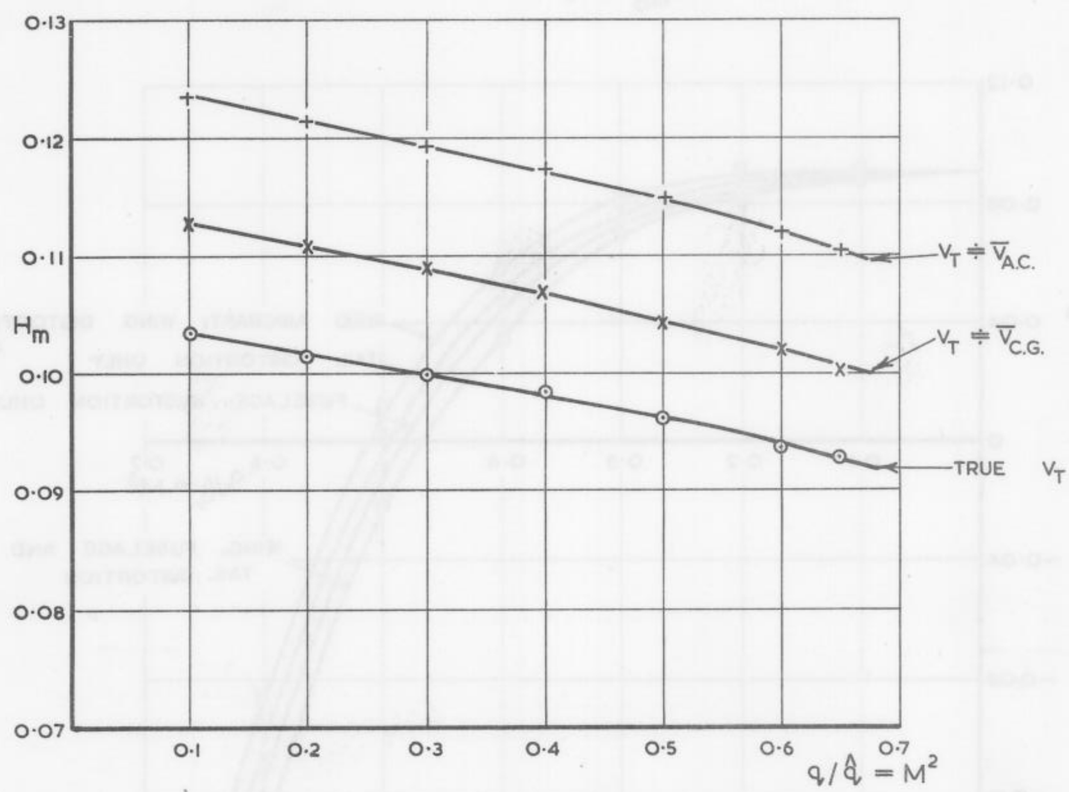
$$C_{m_0} = \frac{-0\cdot015}{\sqrt{1 - M^2}}$$

FIG. 7.



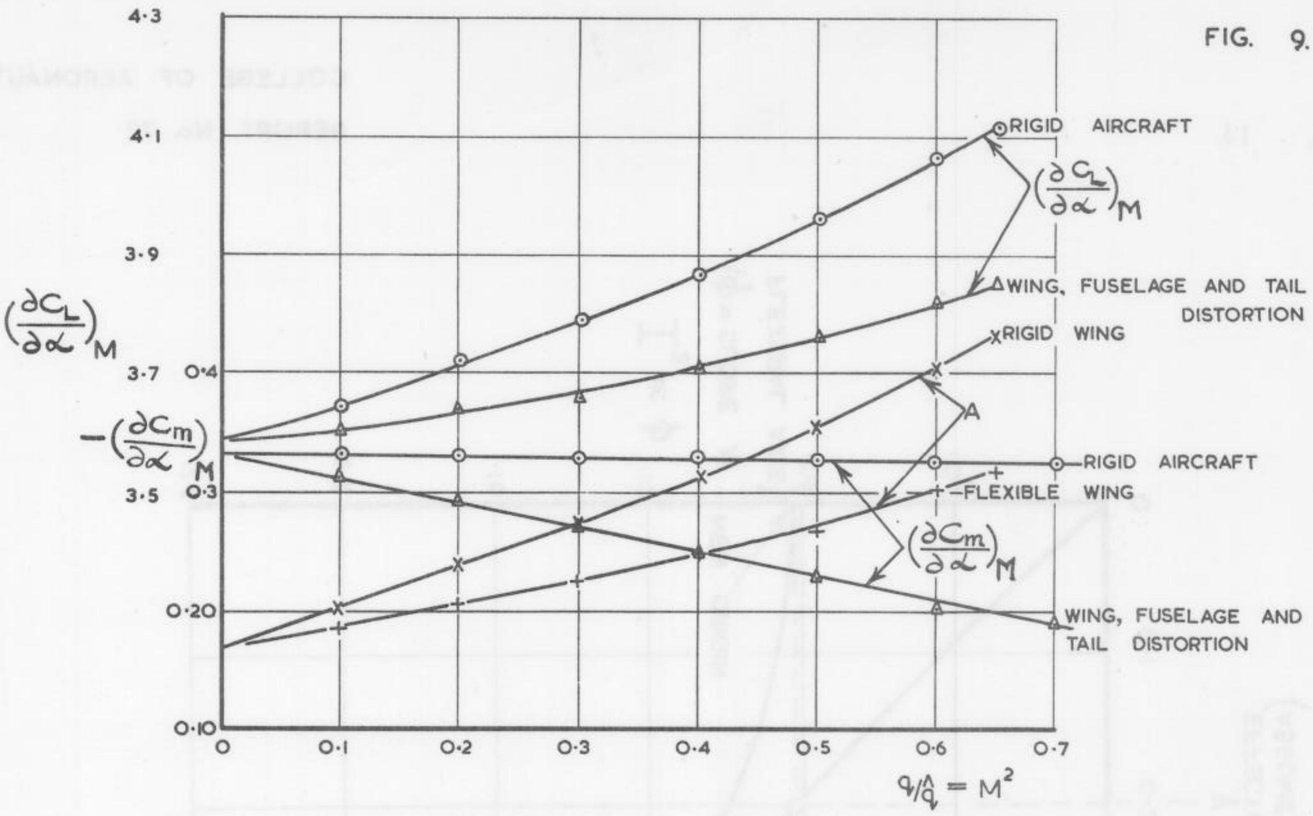
VARIATION OF STICK—FIXED MANOEUVRE MARGIN WITH $q/\hat{q}$

FIG. 8.



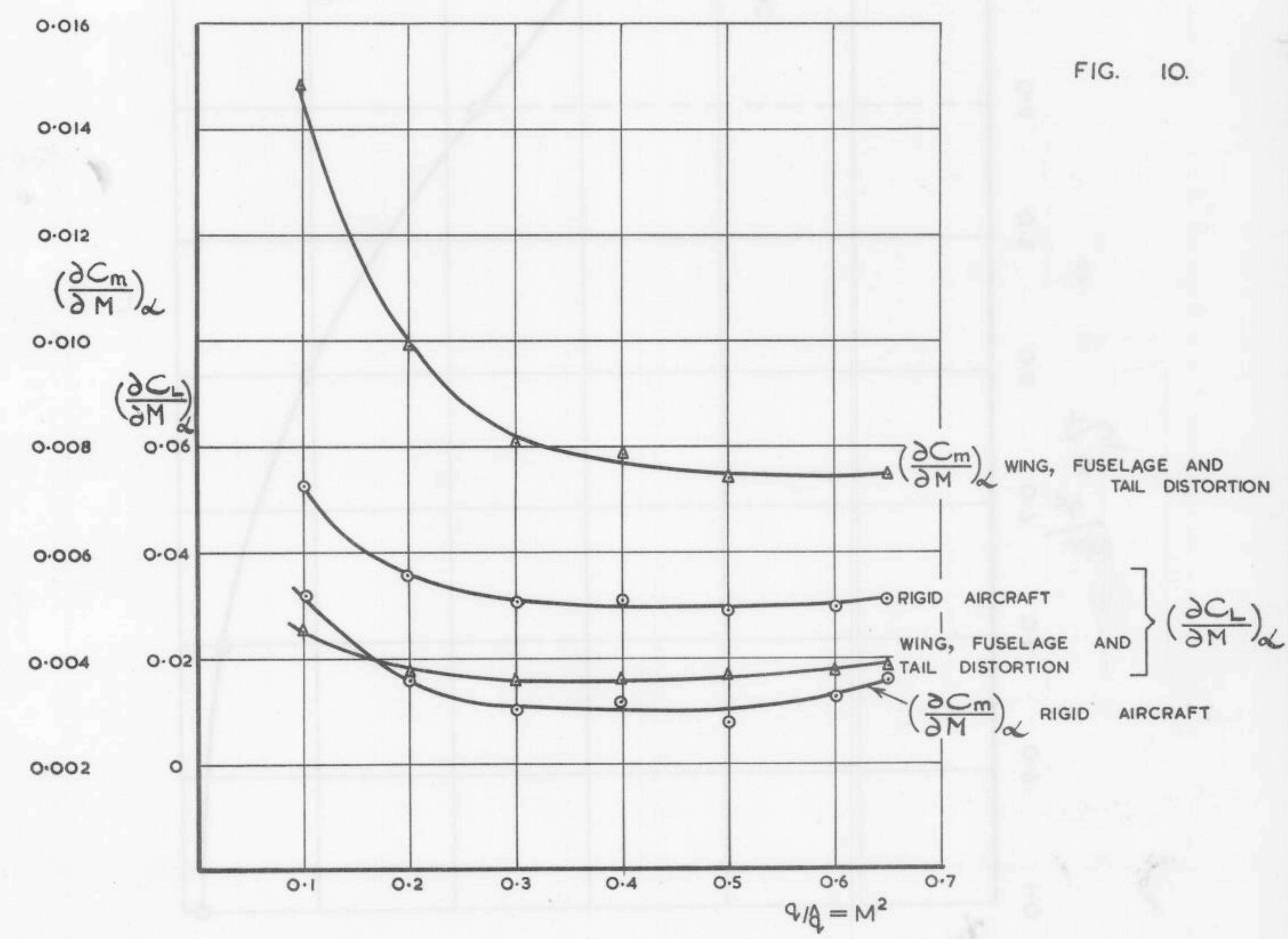
COMPARISON OF VALUES OF STICK — FIXED MANOEUVRE MARGIN OBTAINED USING APPROXIMATE AND CORRECT TAIL VOLUME COEFFICIENTS

FIG. 9.



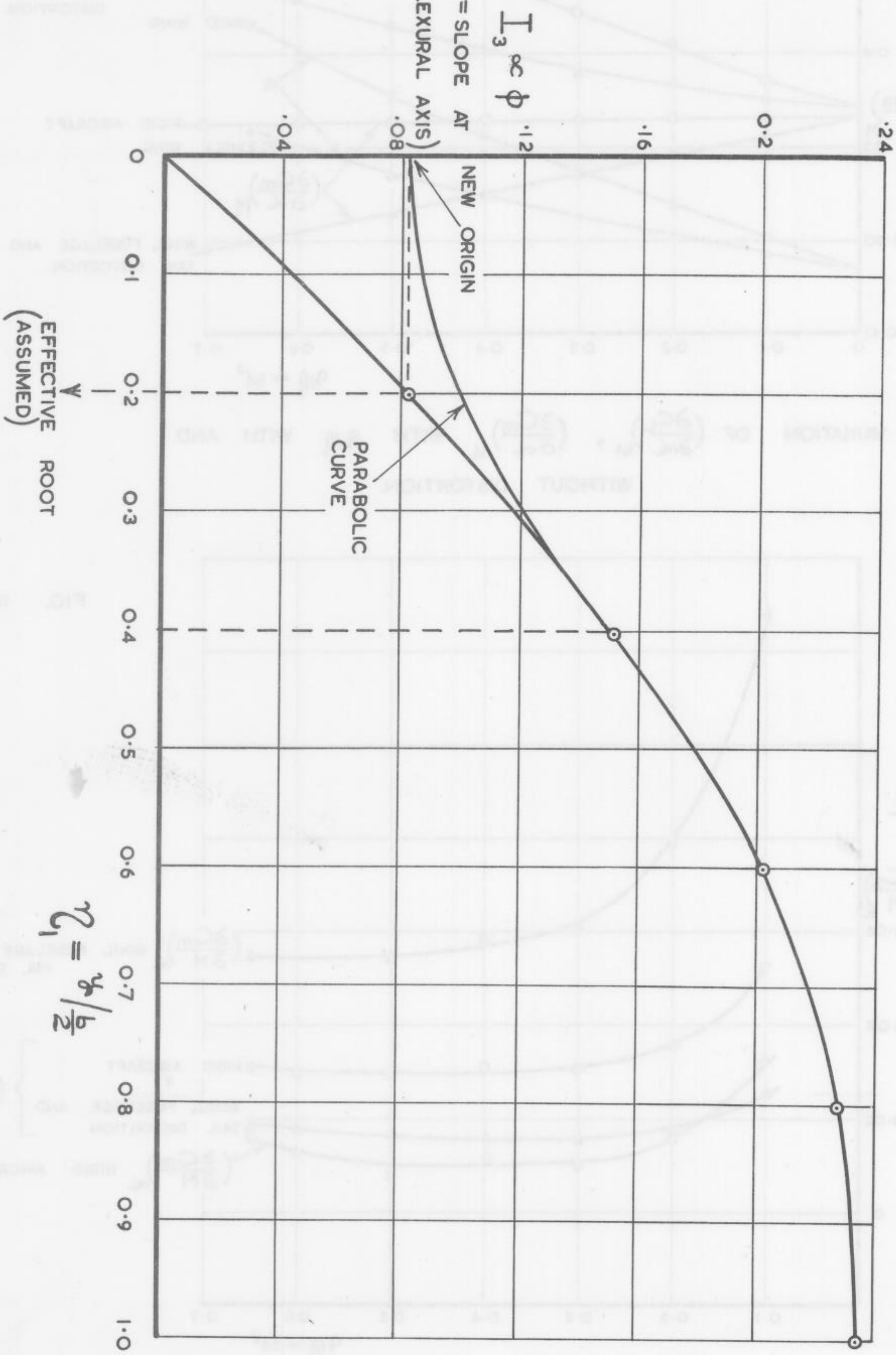
VARIATION OF $\left(\frac{\partial C_L}{\partial \alpha}\right)_M$, $\left(\frac{\partial C_m}{\partial \alpha}\right)_M$ WITH $q/\hat{q}$ WITH AND WITHOUT DISTORTION.

FIG. 10.



VARIATION OF $\left(\frac{\partial C_L}{\partial M}\right)_\alpha$, $\left(\frac{\partial C_m}{\partial M}\right)_\alpha$ WITH $q/\hat{q}$ WITH AND WITHOUT DISTORTION

FIG. 11.



SPANWISE DISTRIBUTION OF SLOPE AT FLEXURAL AXIS
FOR $\alpha = 1$ RADIAN = CONSTANT