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Flutter prediction, suppression and control in aircraft composite wings as a design prerequisite

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Abstract

Emergence of flutter compromises not only the long term durability of the wing structure, but also the operational safety, flight performance and energy efficiency of the aircraft. Effectual means of flutter prevention are, therefore, mandatory in the certification of new flight vehicles. This work intends to address the flutter phenomenon highlighting the above issues, and reviews some of the most recent theoretical and experimental developments in flutter analyses. In the following subchapters, theoretical, computational and experimental flutter for composite structures is pursued. In particular, panel flutter, thrust induced flutter, wing/store type flutter, non-linear flutter, damaged panel flutter, flutter in compressed flow and flutter control *via* neural networks are covered. Effects of fibre/ply orientation on flutter are also briefly covered. The review further looks in to aerothermoelastic behaviour of composite structures buckling problem and hopf bifurcation point determination. Flutter analysed of actively/passively controlled composite structures is critically reviewed due to the emphasized importance in modern structures. It is appreciable that the knowledge gained from the study of flexible structures and unsteady airflows in aircraft can be transitioned back to more traditional flutter studies.

Keywords: Flutter, prediction, suppression, control, composites

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1. Introduction

Flutter denotes a characteristic form of self-excited oscillations that can arise through the interaction of an aerodynamic flow with the elastic modes of a mechanical structure, *e.g.* the bending and torsion modes of an aircraft wing. Emergence of flutter compromises not only the long term durability of the wing structure, but also the operational safety, flight performance and energy efficiency of the aircraft. Effective means of flutter prevention are, therefore, mandatory in the certification of new flight vehicles, and considerable effort, theoretically as well as experimentally, is devoted to the study of methods for active flutter control and of the interaction between structural dynamics and unsteady airflows. If left unchecked, flutter vibrations can completely destroy the structure leading to catastrophe (Figure 1) [1].

Figure 1

Furthermore, knowledge gained from the study of flexible structures and unsteady airflows in aircraft can be transitioned back to more traditional flutter studies. It is also well acceptable that carefully designed and embedded sensors and actuators can lead to actively/passively-controlled structures (Figure 2) with some beneficial contributions as per minimisation of flutter occurrences.

Figure 2

Classically, the flutter properties of a system are the lowest critical speed (U_F) and the associated reduced frequency (ω_F) for which a structure at a given density and temperature will sustain simple harmonic motion. Flight at U_F represents a neutral stability boundary, as all small structural oscillations must be stable below U_F. Above U_F, however, the small oscillations are not damped out and the structure is unstable for a range of speed (or at all speeds) above U_F. The calculation can be broken down into the following steps [2]:

(ii) Determination of the vibration modes of the structure with no aerodynamic forces present.

(iii) Calculation of the aerodynamic forces on the structure due to simple harmonic oscillations of the in vacuo normal modes as functions of speed and reduced frequency.

(iiii) Search for combinations of these parameters for which simple harmonic motion yields equilibrium between the structural inertial forces and the unsteady aerodynamic forces. These combinations are the flutter boundary.

In practice the calculation of (i) assumes that the vibration modes are a superposition of a finite number of preassigned mode shapes, i.e., all the vibrations are linear. Torsion and bending can be coupled but all modes are linear. The aerodynamic force calculations in (ii) generally assume some sort of linearized aerodynamic theory and hence do not capture the effect of flow separation, even though generally vortex flows can precipitate flutter. The methods for flutter analysis are classified according to the characteristics of unsteady aerodynamics, and the governing equations used to calculate the unsteady aerodynamic forces. For subsonic and supersonic flows, where the unsteady aerodynamic forces show strong linearity, DPM (doublet point method) [3], HGM (harmonic gradient method) [4], and KFM (Kernel function method) [5] and frequency-domain flutter analysis method [6] have been used, since these linear methods are effective with small computing time. In the same sense, the lifting surface methods are widely used in the field of aeroelasticity.

More so, understanding the fluid–structure interaction is more fundamental than simply plugging a hole in the existing analysis. Numerical and experimental work so far has centred on rigid wings because the aeroelastic interaction between the wing and surrounding fluid could then be neglected and the overall complexity of the problem is greatly reduced [7]. Given the already knotty fluid mechanics problem at hand, it is reasonable to simplify the problem in order to start the analysis. Computational means and the fluid mechanical analyses have advanced to the point, however, where the aeroelastic interaction can now be included. The design of aircraft wings requires knowledge of how a highly flexible airfoil will deform under aerodynamic loading and the effect of that deformation on airfoil efficiency. The wing shape itself depends upon many physical parameters such as camber, chord and span length, and, most importantly, the mass and stiffness distribution. But dynamic quantities such as the time dependent pressure loading, wing speed, freestream velocity, and local acceleration of the wing surface also directly drive the instantaneous wing deformation. Therefore, it is the dynamic coupling between the wing and surrounding air that decides the final lift and thrust force. With this in mind, it poses the interesting question: can manipulation of the wing's aeroelastic properties lead to improved performance? Clearly, changes in the wing deformation will affect the aerodynamics and so it seems quite possible that changes in the physical properties of the wing could yield better performance. In many respects, this is an inverse problem, where the desired result is known, but not the wing shape needed to achieve it. How this could be accomplished both with passive and active control methods remains an open research question, but one worth future exploration.

In the following subchapters, theoretical, computational and experimental flutter for composite structures is pursued. In particular, panel flutter, thrust induced flutter, wing/store type flutter, non-linear flutter, damaged panel flutter, flutter in compressed flow and flutter control *via* neural networks are covered. Effects of fibre/ply orientation on flutter are also briefly covered. The review further looks in to aerothermoelastic behaviour of composite structures buckling problem and hopf bifurcation point determination. Flutter analysed of actively/passively controlled composite structures is critically reviewed due to the emphasized importance in modern structures.

2. Flutter investigations

2.1 Panel flutter

Panel flutter is a dynamic aeroelastic instability phenomenon resulting from the interactions between motions of an aircraft structural panel and aerodynamic loads exerted on that panel by air flowing past one of the faces. Recently, considerable research efforts have been engaged to address panel flutter. For instance, a high-precision higher-order triangular-plate element that can be used to deal with transverse shear effects based on a simplified higher-order shear deformation plate theory (SDPT) and von Karman large deformation assumption has been developed for the nonlinear flutter analysis of composite laminates [8]. The element presents no shear-locking problem due to the assumption that the total transverse displacement of the plate is expressed as the sum of the displacement due to bending and that due to shear deformation. Quasi-steady aerodynamic theory was employed for the flutter analysis while Newmark numerical time integration method was applied to solve the nonlinear governing equation in time domain. Study results showed that the in-plane force on the plate increased the maximum plate displacement but would not influence the maximum plate motion speed. However, the investigation noted an aerodynamic pressure increases both at the maximum displacement and velocity of the plate. The transverse shear was found to have profound influence on the flutter boundary for a thick plate and under certain conditions changed the plate motion from buckled but dynamically stable to a limit-cycle oscillation. On further developments, the finite-strip method so discussed was later on applied to the flutter analysis of aircraft composite panels [9]. Elsewhere, following adaptive composites modelling and application in panel flutter plate, Suleman [10] concluded that can be inferred that it is possible to achieve an increase in the flutter envelope using piezoceramics, however the application of the electromechanical adaptive composite plate concept is dependent on the mass to stiffness ratio and on the configuration and placement of the actuator patches.

Jinsoo and Younhyuck [11] developed a frequency-domain flutter analysis scheme for wings using an unsteady 3D panel method. The unsteady aerodynamic force calculation was based on the s-plane unsteady nonplanar lifting surface method while a finite element method was used to structurally model the wing. The supersonic flutter analysis was done using the normal mode approach and a U-g method in frequency-domain. The U-g procedure requires the generalized aerodynamic forces for a range of reduced frequencies calculated from the aerodynamic module. The method was validated by comparing the generalized aerodynamic forces and the flutter points with other numerical results and measured data for various types of wings. In a corresponding work, a study on subsonic flutter suppression using self-straining actuators for the Goland wing model with torsion mode flutter has been reported [12]. Regrettably, the work found out that while effective in increasing structure damping prior to flutter, self-straining controllers have little or no effect on the flutter speed.

2.2 Thrust induced flutter

Considering that engine thrust can be represented as a follower force, it is possible that thrust could lead to instability of the wing [13]. Even if the thrust force were not high enough to induce instability on its own, it is quite likely that thrust could interact with other destabilizing mechanisms, for example, aeroelastic flutter. Even for propeller-driven aircraft, thrust could be important although, in the case of prop-whirl flutter, the thrust follows the propeller tip-path plane rather than the nacelle. For stiff propellers, however, it would nearly follow the nacelle. The effect of thrust on the flutter speed may be important, especially in the case of aircraft with very flexible wings. If thrust were to lead to a lowering of the aeroelastic flutter speed, one would certainly want to know about that in order to make appropriate adjustments in the design. Even if thrust were to increase the flutter speed, this could lead to an overly conservative design. In either case, the inclusion of thrust effects in flutter analysis should lead to a more complete analysis.

Kurnik and Przybyowicz [14] have studied on an extended problem of the stability of Leipholz's slender column with rotation effect taken into account. Any flexible rotor with permanent energy supply maintaining a constant rotation speed starts to exhibit orbital motion of its deflected form around the axis placed between the supports. This phenomenon appears at a certain angular velocity, and the necessary condition is, like in the case of non-rotating columns subject to tensile tangential force, the presence of internal damping. According to the study, the two sources of the instability, i.e. follower load and rotation are different in nature, and their interaction presented an interesting dynamic problem. However, although both effects when treated separately were recognised more or less thoroughly, the work failed to study their combination. The follow-up work examined a rotating cantilever column (slender shaft) subject to a tip-concentrated follower load and actively stabilised by piezoelectric elements [15]. The shaft was made of an active laminate - the piezoelectric fibre composite since such systems exhibit flutter-type instability as a result of energy transfer from rotation and to transverse motion of the shaft [16]. A velocity feedback was assumed in the system of active stabilisation. Also, non-linear bifurcation analysis was carried out to predict type of the self-excitation (either soft or hard), near-critical vibration amplitude and jump phenomena. Critical analysis proved that rotation and follower load contradict each other eventually stabilising the system. Unexpectedly, the stability region appeared to be a concave set, primarily, application of active stabilisation yielded desirable effects as the area of safe working enlarged. It was found out that shafts undergoing compression were particularly sensitive to such a stabilisation method; however, in the case on tensile loads the approach became ineffective since the system could be stabilised with respect to *e.g.* follower load but at the cost of angular velocity, the critical threshold of which dropped.

Hodges [17] focused on a uniform beam without bending-torsion coupling. In the model shown in Figure 3, three non-dimensional parameters that govern the dimensionless critical load were taken as the ratio of the cross-sectional mass centre offset from the elastic axis to the beam length (e), the ratio of the cross-sectional mass radius of gyration to the beam length (σ), and the ratio of the fundamental bending and torsional frequencies of an unloaded and uncoupled beam (r). Remarkably, when $e = 0$ the problem ceased to be dependent on σ, and the critical load depended only on r.

Figure 3

When $e \neq 0$, there was a rich dependency of the critical load on both e and σ. The researcher suggested that bending-torsion elastic coupling of the beam, tip mass/inertia, and aeroelastic effects could be included in a more generalized approach. In a later work, the effect of thrust on the flutter of a high-aspect-ratio wing represented by a beam was investigated using a nonlinear mixed finite element method [13]. Aerodynamic forces were calculated using a finite-state, a 2-D unsteady aerodynamic model.

The effect of thrust was modelled as a follower force of prescribed magnitude. Without the thrust force, the wing was shown to become unstable for freestream airspeeds greater than the flutter speed. On the other hand, in the absence of aerodynamic forces, the wing became unstable for values of the thrust in excess of a critical magnitude of the force. When both effects were present, the airspeed at which the instability occurs depended on the thrust magnitude. For validation, an analytical solution for the *in vacuo* case (accounting only for the effect of thrust) was developed and shown to closely match results from the numerical method. Parametric studies showed that the predicted stability boundaries are very sensitive to the ratio of bending stiffness to torsional stiffness (Figure 4).

Figure 4

Indeed, it was proposed that the effect of thrust can be stabilizing or destabilizing, depending on the value of this parameter. An assessment whether or not the magnitude of thrust needed to influence the flutter speed in practice was also conducted for one configuration.

2.3 Wing/store type flutter

Within the context of aeroelastic tailoring, the influence of external stores attached to the wing structure has to be considered during the preliminary aircraft design phases. The mathematical model of aircraft structure for flutter analysis is usually symmetric. When different stiffness or mass properties exist between the left and right external stores, asymmetric configuration would be taken into account. For certain external store configurations, a small amount of structural asymmetry may increase the wing/store type flutter speed significantly. In this case, if feedback control is introduced to adjust automatically the stiffness of the external store on either side, then flutter can be suppressed effectively. Liu *et al.* [18] study has shown feasibility of this flutter suppression scheme - suppressing aircraft flutter by means of disrupting the external stores symmetric state through some actuators driven by the structural-response signals. In principle, the semi-active control method investigated has the merits of less control power required, simpler control strategy and better control effects that may provide an emergency or reserved measure to deal with the flutter encounters.

Investigations on wing/store type flutter have been pursued indeed. Gern and Librescu [19, 20] addressed on structural and aeroelastic tailoring applied to advanced straight and swept aircraft wings' carrying external stores *via* a wing structure modelled as a laminated composite plate exhibiting flexibility in transverse shear and warping restraint effects. The relevant equations of motion as well as the appropriate boundary conditions were obtained *via* Hamilton's variational principle and application of generalized function theory in order to exactly consider the spanwise location and properties of the attached stores. To achieve a realistic representation of the store influence upon static and dynamic aeroelastic behaviour of the system, static weights and dynamic inertias of the attached stores were modelled. For a

comprehensive representation of the stores, their static weights and inertia terms were considered. 3-D modified strip theory aerodynamics was employed, and the obtained eigenvalue/boundary value problems were solved using the extended Galerkin method. The results were found to be in good agreements with other published work highlighting the effects of underwing and tip stores on flutter instability [19].
Hu and Zhao [21] proposed a simple automatic control device that has been adopted to adjust the friction force of an electro-magnetic damper which is installed at the junction between the external store and the wing model (Figure 5).

Figure 5

Since some wing/store configurations application of an impact damper can have significant effects on raising the flutter speed, the damping effects of the friction force was utilized to suppress the wing/store flutter. For the proposed model, when the electrical circuit is established, the armature was pressed to the yoke by the electro-magnetic attraction force proportional to which a friction force iwas introduced between the friction disc and the yoke; the attraction force was proportional to the square of the magnetic induction intensity which itself was proportional to the current intensity fed to the coil of the damper. When the damper was used to attenuate the rotational motion of a single-degree-of-freedom system, then, the frictional torque produced by the damper varied proportionally to the intensity squared of the current fed to the damper.

2.4 Non-linear flutter

The next generation aeronautical and space vehicles are likely to feature increasing structural flexibility, operate in severe environmental conditions and feature greater maneuverability capabilities than the present ones. In order to satisfy such contradictory requirements, an exhaustive exploitation of both the load carrying capacity of their structures and the capabilities that can provide should be used. The non-linear approach of the aeroelastic stability problem enables one to determine the conditions in which, due to the character of the influence of non-linearities that are inherently present in the aeroelastic system, the critical flutter velocity can be exceeded without an immediate failure of the structure as well as conditions in which undamped oscillations may appear at velocities below the critical flutter velocity [22-24]. Matching this behaviour with the post-buckling behaviour, the structures subjected to compressive and/or lateral loads experience some features similar to that of the post-flutter response, in the sense of being benign that is mainly the behaviour of flat panels, or accompanied by a snap-through jump, in which case the failure is imminent (case that is proper for curved panels) [1]. These facts emphasize the considerable importance of at least two issues of including in the aeroelastic analysis the various non-linear effects on the basis of which it is possible to gain a better understanding of their implications upon the character of the flutter boundary for benign or catastrophic effects. And also, the importance of devising powerful methodologies based on both passive and active feedback control algorithms enabling one not only to increase the flutter speed, but also to convert the catastrophic flutter boundary into a benign one. For the later, the desirability of using passive means based upon the use of directionality property of advanced composite materials and of active feedback control methodology appears to be evident.

Structural geometric nonlinearities can be attributed to non-tri*via*l steady-state deformation and/or large motion. The basis of the nonlinearity lies in kinematics, particularly the relations expressing generalized velocity and strain measures in terms of displacement and rotation variables of the wing. Aerodynamic geometrical nonlinearities are manifested in a 3-D aerodynamic model because of the dependence of the pressure at a given point on the disturbances throughout the surface. Using linear aerodynamic theory, the pressure at a point can be linearly related to displacement at another point in terms of an influence coefficient. But since the influence coefficient itself is a function of the wing geometry, the pressure becomes a nonlinear function of the displacements. Chandiramani *et al.* [25] examined the non-linear dynamic behaviour of a uniformly compressed, composite panel subjected to non-linear aerodynamic loading due to a high-supersonic co-planar flow. In the model, the effects of in-plane edge restraints, small initial geometric imperfections, transverse shear deformation, and transverse normal stress were considered in the structural model which satisfied the traction-free condition on the panel faces. The panel flutter equations, derived *via* Galerkin's method, were solved using arclength continuation for the static solution and a predictor-corrector type shooting technique to obtain periodic solutions and their bifurcations. The possibility of hard flutter was demonstrated when considering non-linear aerodynamics.

Furthermore, edge compression yielded multiple buckled states or coexistence of multiple periodic solutions with the stable static solution, that is, the panel could either remain buckled or flutter. Edge restraints normal to the flow appeared to stabilize the panel, whereas those parallel to the flow resulted in a buckled-flutter-buckled transition. For perfect panels, results obtained by the shooting technique and the method of multiple scales were in agreement only within the immediate post-flutter regime indicating that a shear deformation theory was required for moderately thick composite panels. Meanwhile, Librescu *et al.* [1] have recently studied the benign and catastrophic characters of the flutter instability boundary of 2-D lifting surfaces in a supersonic flow field . The work based on the first Liapunov quantity was used to study the bifurcational behaviour of the aeroelastic system in the vicinity of the flutter boundary. It was demonstrated that the increase of hard non-linearities yielded an increase of the 'benign' portions of the flutter instability boundary; the opposite conclusion appeared for 'soft' non-linearities. Further, with the increase of the supersonic flight speed that resulted in an increase of aerodynamic non-linearities, an increase of the catastrophic portions of the flutter instability boundary was experienced (Figure 6 and 7).

Figure 6
Figure 7

The first order shear deformable plate and Timoshenko beam theories have been used for the finite element modelling of a skin panel and stiffeners considering von Karman non-linear strain-displacement relationships, and the nonlinear transient response of fluttering stiffened composite plates subject to thermal loads [26]. A supersonic piston theory was used for modelling aerodynamic loads. In order to find a critical flutter speed, linear flutter analysis of stiffened laminated panels considering large aero-thermal deflections was performed. The flat and stable motion, limit cycle oscillation, and buckled but dynamically stable and chaotic motion of stiffened laminated panels were investigated using the implicit Newmark integration method. The investigations results showed that the increase of the height and number of stiffeners to reduce an aero-thermal deflection could dramatically drop the boundaries of a dynamic stability at a certain point. Also, the non-linear behaviours, such as dynamically stable motions with static aero-thermal deflections, limit cycle oscillations, periodic motions with large amplitude and chaotic oscillations, were observed in the time domain analysis. In a similar development, the thermal postbuckling and aerodynamic-thermal load analysis of cylindrical laminated panels has been performed using the finite element method in a follow-up work [27]. Again, the von Karman nonlinear displacement strain relationships based on layerwise theory was applied to consider large deflections due to thermomechanical loads. The cylindrical arc-length method was used to take account of the snapping phenomena while panel flutter analysis of cylindrical panels subject to thermal stresses was carried out using Hans Krumhaar's supersonic piston theory [27].

On the other hand, much research has also been in progress in the field of passive suppression systems which are believed to be always more simple and robust than active control in practical operation. The feasibility of passively dissipating mechanical energy with electrical shunt circuits has been pursued. Such capabilities are likely to result in a tremendous increase in aircraft efficiency, range, speed, and maneuverability rate as well as in higher payloads characteristics. Hagood and von Flotow [28] have recently formulated the equations of the mechanical and electrical characteristics with piezoelectric material shunted with electrical circuits for the case of a resistor alone and of an inductor-resistor resonant shunt to provide damping for the beam. Hollkamp [29] showed that multiple modes can be suppressed using a single piezoelectric patch connected to a multiple inductor-resistor-capacitor for a beam model. A finite element formulation has been presented for the nonlinear flutter suppression of an isotropic panel under uniform thermal loading by using the modal reduction scheme and LQR linear control [30]. This active control system has the advantages of being adaptable to variable system changes through feedback or feedforward actions and higher performance compared to a passive system. However, application of this active control to practical flutter suppression has a few difficulties because a large amount of power is required to operate actuators, and active system has the spillover problem and is sensitive to system uncertainties. Kim *et al.* [31] investigated a lag mode suppression of hingeless helicopter rotor blades with an L-R shunt circuit. Moon and Kim [32] presented a finite element formulation for a passive suppression scheme of nonlinear panel flutter using piezoelectric inductor-resistor series shunt circuit. However, since this approach is a fixed design, the damping would not be optimal when the system or operating conditions change.

Other studies have developed on active/passive hybrid control system, which integrates PZT actuators with an external voltage source and an inductor-resistor circuit in series. Feasibility studies demonstrate that such an active/passive hybrid control system can suppress vibration effectively with less control effort as compared to a purely active system, if passive parameters such as inductance and resistance were selected correctly. A systematic design/control method to ensure that the passive and active actions are optimally synthesized has been presented [33]. In a recent work, Moon and Kim [34], have proposed a new optimal active/passive hybrid control design (Figure 8) for the suppression of nonlinear panel flutter, with piezoceramic actuators using finite element methods.

Figure 8

The researchers claimed that this approach has the advantages of both active (high performance, feedback action) and passive (stable, low power requirement) systems. Piezoceramic actuators were connected in series with an external voltage source and a passive resonant shunt circuit that consisted of an inductor and resistor. The shunt circuit had to be tuned correctly to suppress the flutter effectively with less control effort as compared to purely active control. To obtain the best effectiveness, active control gains were simultaneously optimized together with the value of the resistor and inductor through a sequential quadratic programming method. The governing equations of the electromechanically coupled composite panel flutter were derived through an extended Hamilton's principle, and a finite element discretization was carried out. The adopted aerodynamic theory was based on the quasi-steady first-order piston theory, and the von Karman nonlinear strain displacement relation was used. Nonlinear modal equations were obtained through a modal reduction technique. Optimal control design was based on linear modal equations of motion, and numerical simulations were based on nonlinear-coupled modal equations. Using the Newmark integration method, suppression results of a hybrid control and a purely active control were presented in the time domain.

Bauchau *et al.* [35] have presented a methodology for the analysis of backlash, freeplay and frictional effects in joints, within the framework of non-linear finite element multibody procedures and their applications, incorporating the effects of friction in joint elements together with effective computational strategy. These non-standard effects were formulated within the framework of finite element based multibody dynamics that allowed the analysis of complex, flexible systems of arbitrary topology. The versatility and generality of the approach were demonstrated by presenting applications to aerospace systems i.e. the flutter analysis of a wing-aileron system with freeplay, the impact of an articulated rotor blade on its loop stop during engagement operation in high wind conditions, and the dynamic response of a space antenna featuring joints with friction. The model was demonstrated by flutter analysis of a wing-aileron system with freeplay, the impact of an articulated rotor blade on its loop stop during engagement operation in high wind conditions, and the dynamic response of a space antenna featuring joints with friction.

A new strategy, based on the nonlinear phenomenon of saturation, has been proposed for controlling the flutter of a wing [36]. The concept was illustrated by means of an example with a rather flexible, high-aspect wing of the type found on such vehicles as high-altitude long-endurance aircraft and sailplanes. The wing was modelled structurally as an Euler-Bernoulli beam with coupled bending and twisting motions. A general unsteady nonlinear vortex-lattice technique was used to model the flow around the wing and provide the aerodynamic loads. The structure, the flowing air, and the controller were considered the elements of a single dynamic system, and all of the coupled equations of motion were simultaneously and interactively integrated numerically in the time domain. The results indicated that the aerodynamic nonlinearities alone can be responsible for limit-cycle oscillations and that the saturation controller can effectively suppress the flutter oscillations of the wing when controller frequency is actively tuned.

In still another development, Patil *et al.* [37] have looked at the effect of structural geometric nonlinearities on the flutter behaviour of high-aspect-ratio wings. A steady-state deflection of the wing was calculated based on constant distributed loading and the changes in structural and aeroelastic characteristics were presented. The results indicated a significant change in the structural frequencies and a significant reduction in the flutter speed. A 2-D aerodynamic model was used and thus the aerodynamic nonlinearities (due to curvature) were not present. Theoretical and experimental investigation of flutter and limit cycle oscillations using a nonlinear beam model and a stall model has been conducted by Tang and Dowell [38]. Hall *et al.* [39] presented the results obtained by using a 3-D geometrically exact (nonplanar) aerodynamic theory coupled with a linear structural analysis. The results that were based on free-wake aerodynamic analysis, also, illustrated that flutter instability speed was drastically reduced with wing curvature

In conclusion, there are many potential sources of non-linearities, which can have a significant effect on an aircraft's aeroelastic response. One essential limitation involving the linearized analysis is that it can only provide information restricted to the flight speed at which the aeroelastic instability occurs. Furthermore, the linearized analyses are restricted to cases where the transient aeroelastic response amplitudes are small. Often this assumption is violated prior to the onset of instability. Thus, to study the behaviour of aeroelastic systems either in the post-instability region or near the point of instability, the structural, physical and aerodynamic non-linearities must be accounted for [1, 40]. Nowadays, there is an increased attention to address the non-linearlity issues as highlighted in this subchapter.

2.5 Flutter of a damaged panel

To meet the increased performance and standards for the existing aircraft as well as new generation aerospace vehicles, engineers are using a combination of metallic and composite structures in their design. These repair structures may be skewed in shape and may have developed cracks due to manufacturing process or fatigue loading during service. It is therefore important to study the flutter behaviour of such panels caused by the complex interaction of aerodynamic, inertia and structural forces. Flutter takes place at a critical air speed and it is important to capture this accurately for arbitrary panel configurations as it might otherwise lead to catastrophic failure. Strganac and Kim [41] have studied the aeroelastic behaviour of composite plates subject to damage growth and suggested that there is a need to develop damage as part of the flutter solution. Thus, Pidaparti and Chang [42] carried out an investigation of skewed and cracked panels under supersonic flow using the general plate/shell finite element based on tensorial mathematics. The work results illustrated the effects of flow angle, boundary conditions and fibre orientation on the flutter bounds and indicate that the fibre angle, flow angle and boundary conditions strongly influence the flutter boundaries for laminated composite cracked panels.

Another work studied the supersonic flutter behaviour of isotropic thin cracked panels using the hybrid finite element method [43]. Recently, Lin *et al.* [44] studied the panel flutter problems of thin plate-like composite panels with patched cracks using a finite element method (FEM). They showed that flutter performance could be improved by isotropic patching. The flutter characteristics of 2-D delaminated composite panels at high supersonic Mach numbers were investigated by Shiau [45] and concluded that the presence of a delamination decreases the flutter boundary. A finite element method was also employed to investigate the free vibration and supersonic flutter analysis of arbitrary damaged composite panels [46]. The FEM employed 48 degrees of freedom (DOF) general plate element and used the classical lamination theory, microstructural continuum damage theory and linearized piston theory. Finite element results were obtained to illustrate the effect of damage on the eigenvalues and flutter boundaries. Supportively, the results obtained indicated that damage had a strong influence on both free vibration and flutter boundaries.

2.6 Flutter in compressed flow

With the advent of active control technology for flutter suppression, and gust load alle*via*tion, and with the increasing need of evaluating the time-dependent subcritical aeroelastic response of lifting surfaces, the time domain representation of unsteady aerodynamic loads becomes a necessary prerequisite toward achieving these goals [47]. This is in contrast to the flutter instability analysis in which context a frequency domain formulation is required. The accurate modeling of the unsteady aerodynamics plays a key role toward approaching the aeroelastic problems. By definition, an indicial function, also called indicial admittance, is the response to a disturbance generated by a step function [40]. The unsteady lift and aerodynamic moment in time and frequency domains in the compressible flight speed range are obtained by using the pertinent aerodynamic indicial functions. As it is well-known, if the indicial function can be determined, then by using Duhamel superposition principle, these can be applied toward determination of the aerodynamic lift and moment in time or frequency domain, for any arbitrary variation in angle of attach α and/or inflow velocity. The representation in the time domain of the unsteady aerodynamic loads is necessary toward determination, in the subcritical flight speed regime, of the dynamic response of aeroelastic systems exposed to arbitrary time-dependent pressure pulses. In addition, when a feedback control system is implemented, this representation of aerodynamic loads is essential toward determination of its closed-loop aeroelastic response [48]. In this context, both the open and closed loop dynamic responses of the aeroelastic system can be analysed. On the other hand, the representation of aerodynamic loads in the frequency domain is necessary toward determination of the flutter instability boundary [47]. Also, since the aircraft design is primarily based on the principle of thin-walled beams, it is desirable to investigate the aeroelastic instability and aeroelastic response directly within the framework of thin-walled beams [41].

Marzocca *et al*. [48] investigated on the subcritical aeroelastic response of 2-D lifting surfaces in an incompressible flow field to gust and explosive type loadings, including also that due to a sonic-boom pressure pulse employing the concept of the indicial functions (Table 1 and 2) to determine the unsteady aerodynamic loads for a 2-D lifting surface in various flight speed regimes that include the compressible subsonic, linearized transonic, supersonic and the hypersonic ones.

Table 1

Table 2

In a later work, an unified approach enabling one to obtain the unsteady lift and aerodynamic moment in the time and frequency domains for 2-D lifting surfaces was developed - the concept of the indicial functions was employed to determine the unsteady aerodynamic loads for a 2-D lifting surface in various flight speed regimes that include the compressible subsonic, linearized transonic, supersonic and the hypersonic ones [49]. In this context, an aeroelastic formulation of 2-D lifting surfaces in various flight speed regimes was presented and the usefulness, in this context, of the aerodynamic indicial functions concept was emphasized. The importances of the 2-D approach stems from the fact that the obtained aeroelastic predictions are more critical than the one obtained *via* a 3-D analysis and *via* experiments, and as a result, are appropriate in the pre-design process. Validations of the aerodynamic model, obtained by comparing the indicial aerodynamic functions with the ones based on other unsteady aerodynamic theories show an excellent agreement. Moreover, the predictions of the subcritical aeroelastic response in subsonic compressible, transonic, supersonic and hypersonic flight speed regimes to external pulses, based on the aerodynamic models developed [49], revealed an excellent agreement with the ones generated *via* the application of other aerodynamic theories. The numerical simulations assessing the versatility of this approach enabled the researchers to treat both the subcritical aeroelastic response and flutter instability.

Plate-beam model has been used for investigating the implications of warping restraint and transverse shear on the static divergence, and flutter of aircraft wings [41]. Lately, the problems of the aeroelastic instability and dynamic aeroelastic response of advanced aircraft wings modelled as anisotropic composite thin-walled beams in compressible subsonic flow and exposed to a sharp-edged gust load have been approached in a unified framework [39]. The aircraft wing was modelled, as an anisotropic composite thin-walled beam featuring circumferentially asymmetric stiffness lay-up, which generates preferred elastic couplings. A number of non-classical effects such as transverse shear, warping restraint, and the 3-D strain effects are incorporated in the structural model. The unsteady aerodynamic loads in subsonic flow are based on 2-D indicial functions in conjunction with aerodynamic strip theory extended to a 3-D wing model. For the wings used in the investigated cases, the non-uniformity of the contour-wise shear stiffness was noted to become immaterial. However, directionality property of composite material plays a significant role on the enhancement of free vibration and aeroelastic behaviour. The work concluded that elastic coupling can be effectively used to suppress the onset of flutter but this may be achieved at the cost of dramatically increasing the response intensity. In addition, warping restraint has a significant influence on the dynamic response, even for large aspect ratio wings. Therefore, warping restraint effect has always to be considered in the structural model of composite aircraft wings. Compared with the warping restraint effect, the influence of transverse shear appearred to be much less significant.

In yet another development, aeroelastic model was developed toward investigating the influence of directionality property of advanced composite materials and non-classical effects such as transverse shear and warping restraint on the aeroelastic instability of composite aircraft wings [50]; the model developed dealt with both divergence and flutter instabilities simultaneously. The aircraft wing was modelled as an anisotropic composite thin-walled beam featuring circumferentially asymmetric stiffness lay-up that generates, for the problem at hand, elastic coupling among plunging, pitching and transverse shear motions. In conclusions, the directionality property of anisotropic composite materials were found to play a complex role on the aeroelastic instability, however, this complex role can be explained by well established aeroelastic concepts such as wash-in, wash-out, twist/bending stiffness and coupling among them. Furthermore the warping restraint effect had a significant influence that considered in the design process on both the flutter and divergence speeds when the aspect ratio was moderate. Finally, transverse shear deformation appeared to have a marginal influence on the aeroelastic instability. However, the results showed that the discard of transverse shear did not always yield conservative predictions.

As addressed earlier on, for the aeroelastic response in the compressible flow it is necessary to express the lift and moment *via* the proper indicial functions expressed in time domain. For the approach of the flutter problem in the subsonic compressible and supersonic flight speed regimes, an analogous procedure based on the generalized counterparts of Theodorsen's function for these flight speed regimes has to be used. Also, for simple harmonic motions and are normally sinterpreted as the forces on the imaginary axis in the complex ω-plane, Vepa's matrix Padé approximants of the aerodynamic forces, have been widely used in aeroelastic analyses. Edwards [51] used the classical theories of Theodorsen and of Garrick and Rubinow for 2-D lifting surfaces, and obtained the corresponding ω-domain solutions for incompressible and supersonic flows. Ueda and Dowell [52] have generalized their doublet point method to the ω-domain for 3-D subsonic lifting surfaces and developed a simple method for calculating the unsteady aerodynamics on harmonically oscillating thin wings in subsonic compressible flow.

2.7 Flutter control *via* neural networks

There exist a great anticipation that artificial neural networks, once properly trained, can be used to significantly speed up the design and analysis process of aerospace systems by allowing rapid trade analysis as well as quick evaluation of potential impacts of design changes – mark you, training time is not included in this assertion [53]. It is envisioned that the training of networks can be done autonomously during off hours, so that they are made available to the designer/analyst when required. However, there are certain drawbacks associated with neural networks that need to be addressed fully to date. Such include the time-consuming nature of the training process, training difficulties, such as optimisation problems, and a lack of a meaningful way to establish network accuracy.

One of the main advancements in neural networks approach is the match-point solution for robust flutter analysis. The computation of robust flutter speeds presents a significant advancement over traditional types of flutter analysis. In particular, μ-method analysis is able to generate robust flutter speeds that represent worst-case flight conditions with respect to potential modelling errors. Robust flutter speeds may be computed using a model formulation that has been previously presented; however, that formulation has limitations in its ability to generate a match-point solution. Of late, a model formulation has been introduced for which μ-method analysis is guaranteed to compute a match-point solution that is immediately realized by analysing a single model thus reducing the computation time and significantly eliminating the normally required iterations [54]. The proposed model was claimed to be able to consider parametric uncertainty in any element. The match-point formulation was derived by properly treating the nonlinear perturbations and uncertainties that affect the equation of motion - the aerostructures test wing was used to demonstrate that the μ-method analysis computes match-point flutter speeds using this new formulation.

3 Aerothemoelastic behaviours

Structures made with composite materials are more sensitive and vulnerable to temperature change than their isotropic counterpart since thermal expansion coefficients of different constituents of the material are usually dramatically different from each other resulting in high stresses due to sudden temperature change. It is also important that the influence of thermal effect may vary from isothermal, dynamic, modulated or flash conditions resolving to different levels of complexity in the composite structure. One of the undesirable and by far most important effects of elevated temperature is the deterioration in mechanical properties of materials of structures that result changes in stiffness due to such changes in material properties. Experimental evidence has revealed a linear relationship between Young's moduli and temperature provides a good correlation for most engineering materials [55, 56]. In such a case, when a steady thermal gradient is considered, the elastic coefficients of the material become functions of the space variables. Consequently, although the considered structure is of uniform thickness, in such circumstances the structure will feature a material-induced variable stiffness.

Unfortunately, the heterogeneity and anisotropy of composite materials make the traditional analysis method used for designing *homo*geneous and isotropic structures obsolete. Nevertheless, there exist an increased interest on thermal-structural aspects of composite structures. For instance, Liu and Huang [57] researched the nonlinear free vibration of composite plates subject to uniform temperature changes using FEM. Zhou *et al.* [58]

investigated the vibration of the thermally buckled composite plate. The initial deflection was considered, and triangular elements were used for the FEM based on the classical plate theory. Lee and Lee [59] studied the vibration of the thermally post-buckled composite plate using the first-order shear deformation plate theory (FSDT). The predictor-corrector approach has been proposed as the most effective technique for obtaining accurate transverse stresses and for thermal loading [55], thermalelastic models derived from 3-D thermoelasticity theory [60-62], a thermoelastic model for analysing laminated composite plates under both mechanical and thermal loadings constructed by the variational asymptotic method [63], and extended 2D method for mechanical loads and thermal loads [64, 65]. Averill and Reddy [66] studied the nonlinear response of laminated composite panels subjected to thermal loads using the refined theory and finite element method. Chen and Chen [67] investigated the thermal post-buckling behaviours of laminated composite plates using the finite element method. Thermal and elastic properties of the material were assumed to be temperature-dependent. The temperature-dependent material properties lowered the critical temperatures, and increased the post-buckling deflections.

Investigations have also been conducted on the effect of the *nonhomogeneity* of the structure induced by a steady thermal gradient and the accompanied degradation of material properties on natural frequencies and mode shapes of a pretwisted rotating beam [68]. The analysis was carried out in the framework of a refined theory of thin-walled anisotropic composite beams encompassing a number of nonclassical features. It was assumed that the beam was subjected to a steady temperature field uniform in the *s*-direction, but featuring a linear distribution in the spanwise direction. This led to a scenarios of temperature variations denoted as $T(z) = {T_0 z}/{L}$ or $T(z) = T_0\left(1 - {z}/{L}\right)$, where T denotes the temperature above a stress-free reference temperature at any point along the beam spanwise, whereas T_0 denotes the temperature excess above the reference temperature at the beam cross-sections $z = L$ or z = 0, respectively. The study results revealed that the deterioration of material properties induced by the temperature rise yield a decay of eigenfrequencies and modification of natural modes, a fact that is likely to have detrimental repercussions upon their dynamic response and flutter instability characteristics, see for example Figure 9.

Figure 9

Moreover, as it was highlighted, the proper use of the tailoring technique can overcome the deleterious effect associated with the thermal degradation of material properties of blade structures.

In one more development, coupled thermal-structural analysis, which includes the interaction between structural deformations and incident heating, was performed for spinning thin-walled composite beam appendages whereas thermal flutter was studied by the coupled thermal-structural analysis [69]. Vitally, the coupled analysis provided data on the stability characteristics, as well as dynamic responses. The beam model was assumed to have transverse shear deformation and rotary inertia, as well as primary and secondary warping effects. For the thermal analysis, the two-dimensional heat transfer in the axial and circumferential directions of a thin-walled beam was considered - the changes of the heating surface involved due to the spinning motion of the beam. And, the spinning speed played an important role in the stability of the spinning beam under solar heat flux. When the spinning speed was equal to the bending frequency of the beam, then the structure was unconditionally unstable. The thermal and the structural analyses were based on the principle of conservation of energy and Hamilton's principle, respectively.

4 Buckling problem and Hopf bifurcation point determination

Elastic stability (or buckling) of beams, plates, and shells is one of the most important criteria in the design of any structure. Often, it is the critical design issue (even more than strength) in sizing certain structural elements. Because of this crucial role of elastic stability, it is extremely useful to have results of buckling analysis expressed in closed form, even if they are approximate, whenever possible for design analysis [70-73]. Although such approximate analyses cannot replace an over-all elastic stability analysis of the entire structure, the ease of implementation and the physical insight that such forms give allows valuable design tradeoffs to be made in the preliminary design phase; and that can lead to significant improvements in cost and performance of the structural design. On this line of interest, active buckling controls of laminated composite plates [74, 75], beams [76-79] and beam columns [80] with surface bonded or embedded piezoelectric sensors that are either continuous or segmented have been presented.

In a recent work, a procedure has been developed to produce and solve algebraic equations for any order aeroelastic systems, with and without frequency-dependent aerodynamics, to predict the Hopf bifurcation point [81]. Apparently, the estimation of the Hopf bifurcation point is an important prerequisite for the non-linear analysis of non-linear instabilities in aircraft using the classical normal form theory. For unsteady transonic aerodynamics, the aeroelastic response was frequency-dependent and therefore a very costly trial-and-error and iterative scheme, frequency matching, was used to determine flutter conditions. Also, the standard algebraic methods have usually been used for systems not bigger than two degrees of freedom and do not appear to have been applied for frequency-dependent aerodynamics. In the proposed approach the computation was performed in a single step using symbolic programming and did not require trial and error and repeated calculations at various speeds required when using classical iterative methods. The method has been evaluated *via* a Hancock two-degrees-of-freedom aeroelastic wing model and a multi-degree-of-freedom cantilever wind model. Hancock experimental data was used for curve fitting the unsteady aerodynamic damping term as a function of frequency. As claimed, a fairly close agreement was obtained between the analytical and simulated aeroelastic solutions with and without frequency-dependent aerodynamics.

5 Flutter analyses of actively/passively controlled composite structures

Smart structures sense external stimuli, process the sensed information, and respond with active control to the stimuli in real or near-real time. A response can consist of deforming or deflecting the structure or communicating the information to another control centre. Smart materials deform or deflect the structure by changing their physical properties when subjected to electric, magnetic or thermal loads. An extension of this is the intelligent, self-healing vehicle whose built-in redundancy and on-board self-inspection detects damage and responds with autonomous adjustments and repair. Further, smart structures may be time-variant and non-linear. It is a challenge to control such structures. Conventional active controller design methods, *e.g.*, eigenstructure assignment and optimal control, require accurate mathematical models. But accurate models are almost impossible for smart structures, because of non-ideal behaviour, simplifications in modelling, manufacture error, parts wear, and environment change.

In this case, adaptive control may be an attractive alternative. There are two radically different adaptive control approaches: adaptive feedback control and adaptive feedforward control. Adaptive feedforward control was originally used for noise control and then extended to vibration control. Up to now, it has been studied in large space structure control, civil structure vibration under seismic or wind excitation, helicopter vibration control, wing flutter suppression, and vibration reduction for automobiles [82, 83]. Its recursive capability makes it very suitable for digital signal processors. In the meantime, the rapid development of digital signal processors also expedites the application of this technology. However, adaptive feedforward control is not available to control the higher harmonic components in responses. This is just because the reference signal does not correlate with the higher harmonic components in these responses. In order to solve this problem, one idea proposed is to introduce a non-linear functional block into the adaptive feedforward control strategy [83]. This non-linear functional block must be able to receives an input reference signal that only contains the fundamental component and its output contains both the fundamental and higher harmonic components, i.e., it can non-linearize the reference signal.

The dynamic response of elastic structures under time-dependent external excitations is a subject of much interest in the design of aeronautical and aerospace vehicles. Such time-dependent external excitations may be induced by atmospheric turbulence, nuclear blast, sonic boom, shock wave, fuel explosion, *etc*. Due to the damaging effects upon the structural integrity and operational life of these vehicles, adequate methods to predict and control their structural dynamic response have to be devised. *E.g*. Civil and military airplane wings are designed to carry heavy external mounted stores along their span. Depending on their magnitude and location, drastic reduction of natural eigenfrequencies and modification of the eigenmodes are experienced. These modifications can result in a deterioration of the dynamic response to time-dependent excitations and can also precipitate the occurrence of the flutter instability. This is imperative, as the next generations of aeronautical and space vehicles are likely to feature increasing structural flexibility and operate in severe environmental conditions.

One of the possible options enabling one to control the dynamic response of these structures under time-dependent external excitations and eliminate their damaging effects without weight penalties consists of the incorporation of adaptive materials technology. In a structure featuring adaptive capabilities, the dynamic response characteristics can be controlled in a known and predictable manner and, as a result, one can avoid the occurrence of the structural resonance and of any dynamic instability and enhance the dynamic response to transient loadings [84, 85]. The adaptive capability can be achieved through the converse piezoelectric effect, which consists of the generation of localized strains in response to an applied voltage. This induced strain field produces, in turn, a change in the dynamic response characteristics of the structure. Employment of a control law relating the applied electric field with one of the kinematical response quantities according to a prescribed functional relationship, results in a dynamic boundary-value problem whose solution yields the closed-loop dynamic response characteristics. Based upon the adaptive materials control technology, a methodology based upon adaptive materials technology has been applied towards the goal of enhancing the dynamic response of cantilevered beams carrying externally mounted stores, and subjected to time-dependent external loads [86]. The results revealed that the control methodology described here can play a noticeable role of damping the oscillations induced in the structure by the action of time dependent external excitations.

It is regrettable that although much work has been carried out to study the fundamental problems in smart structures, there have been few experimental demonstrations of such active solutions, because the control algorithms are complex and require a significant processing power so that the signals can be calculated in real-time. Previous simulation work has indicated that the computing resources required for addressing the main problems are considerable, and hence attempts have been made to make major simplifications. Normally, this means that the modelling orders cannot be too high, and only the dominant modes can be included so that the computing resources are reduced; however, this inevitably leads to degradation in the accuracies that are achievable in practice. In control applications, model reduction is vital in order that real-time performances can be achieved especially in direct digital control situations coupled with good overall accuracies. Published work in this area is rather scarce, since even with the reduced order models effective on-line control is difficult because uni-processor systems cannot achieve real-time performances. But such works are beginning to appear. Most recently Virk and Al-Dmour [87] successively demonstrated a good way of reducing the dimensionality of the control problem by deleting the modes that were the least controllable and observable dependent on system configuration, i.e. the precise locations of the sensors and actuators, Figure 10.

Figure 10

It was shown that the accuracy demanded needs to be balanced with the real-time data throughput that is practically achievable by the computer system available to perform the processing. Clearly, this trade-off need to be resolved for each situation but having a network of processing elements will increase the number of operations that can be performed, and hence lead to better solutions, assuming the parallel solutions are efficient.

5.1 Structures controlled by various adaptive smart materials

5.1.1 Piezoelectric actuators and sensors

Piezoelectric actuators are strain actuators, in that application of a voltage across the piezoelectric material causes a large strain force to be exerted. Vibration control of structural components by means of piezoelectric sensors and actuators is an effective tool in designing active structures capable of damping out excessive vibrations *via* a feedback control mechanism. Saravanos and Heyliger [88] have presented a detailed survey on laminated piezoelectric composite plates, beams and shells. Lin *et al.* [89] demonstrated macroscale use of distributed actuator technology to solve aerodynamic problems. This team of researchers tested distributed piezoelectric actuators bonded onto NACA 66-012 airfoil with 30^0 sweep as a means to suppress flutter and increase the flutter speed. Additionally, strain gauges and accelerometers were bonded to the wing and acted as local sensors. The investigation results, however, showed that the control authority was limited due to saturation of the piezoelectric actuators. The researchers believed that increased coverage would improve the control authority, however, the investigations achieved an increase in flutter dynamic pressure by 12%. Utilizing piezoelectric actuation, Heeg [90] demonstrated a 20% increase of the flutter speed in wind tunnel testing. In this case, a simple single-input single-output feedback controller was used, and the analysis and design was based on rational function approximations for the aerodynamics. Elsewhere, a numerical approach for the solution of structural problems, based on the concept of parameter transfer finite elements, was adopted to study the dynamic behaviour of composite wings [91].

Lee [92] has recently suggested a numerical simulation model for random large amplitude vibration control of composite plate using piezoelectric material. The H_∞ control design was employed to suppress the large amplitude vibrations of composites plates under random loading. The numerical simulation model was based on the finite element method. The finite element governing equation includes fully coupled structural and electrical nodal degrees of freedom, and considered the von Karman large amplitude vibration. The modal reduction method using the structural modes was adopted to reduce the finite element equations into a set of modal equations with fewer DOF. The modal equations were then employed for controller design and time domain simulation. In the simulations without control, the value of the linear mode to the nonlinear deflection was quantified - and the minimum number of linear modes needed for accurate model was obtained. In the simulations with control, it was shown that the truncated modes, which are neglected in the control design, deteriorate the controller performance. The vibration reduction level was not monotonically increasing with the size of the piezoelectric actuator. The optimal piezoelectric actuator size depended on the excitation level. For higher excitation level, optimal actuator size was larger. The H_∞ controller based on the linear finite element formulation gave better vibration reduction for small amplitude vibration, but it still gave reasonable performance for large amplitude vibration provided that the piezoelectric actuator was big and powerful enough.

A new two-pronged approach suitable for the general aeroelastic and aeroservoelastic scaling of any aeroelastic configuration has been presented recently [85]. It was claimed that the method produces aeroelastic-scaling laws for general configurations, and was particularly useful for situations involving active controls and smart-materials-based actuation. This approach was illustrated by applying it to a 2-D wing section in compressible flow, combined with a trailing-edge control surface. Augmented aerodynamic states were reconstructed using a Kalman filter, and linear optimal control was used to design a full-state feedback flutter suppression controller, and also, considerations of constraints on actuator deflection and rate were taken. Further, flutter suppression for a typical cross section with a conventional trailing-edge control surface was compared with that obtained with piezoelectric actuation utilizing bend/twist coupling.

Agneni *et al.* [93] proposed a procedure for a modal-based modelling and analysis of the effectiveness of shunted piezoelectric devices in increasing passive damping of elastic and aeroelastic systems. In the study, linear aeroelastic modelling was reduced to a rational polynomial transfer function allowing extension of the proposed piezo modelling to a general linear aeroelastic system. Dynamical models of a cantilever beam (Figure 11) and a simply supported plate were used to validate the results obtained with an aeroelastic application on a wing and with different numerical formulation [94-96].

Figure 11

The aeroelastic application on a wing of a fully composite glider made of fibre glass skin and two sandwiches spars with carbon fibre and glass fibre skin and spruce core showed the weak capability of these passive devices, also when shunted with resistors and inductors electrical load, in improving the stability margin of an actual aeroelastic configuration (the wing of a remote controlled unmanned glider). These aeroelastic applications showed a weak capability of improving the stability margin, but a significant performance in the reduction of the gust response level in proximity of the critical condition of the system (when the aerodynamic damping becomes less significant) *e.g*. when the flight speed was close to the flutter speed. The results were based on the opportunity and capability to express explicitly the system dependency on the flight speed *via* finite-state description for the aerodynamics and thus to design a semi-active control law for the optimal tuning of *piezo* devices within the flight-speed parameter range.

In recent years, a semi-active control technique has been proposed, in which active and passive control principles are combined, that may find applications in flutter control [97]. It follows that the well known 'decoupler pylon' invented by Reed *et al.* [98] may be considered as a successful application of a semi-active control technique. Morino [99] proposed a tentative idea of 'flutter taming' which means reduction of the limit cycle flutter amplitude by applying feedback control. The work demonstrated this idea by theoretical analysis using a multiscale asymptotic method. When the limit cycle amplitude was not excessively large, instantaneous failure did not occur in the wing structure; instead, only a fatigue problem arose. It was evident that from the viewpoint of fatigue, smaller amplitude results in less accumulation of damage. Hence, in the circumstance that flutter is inevitable for a structural configuration, the aforementioned 'flutter taming' concept would appear to be attractive. In the study of structural non-linear flutter [100], it was found that the non-linear stiffness has direct effect on the limit cycle flutter amplitude. Therefore there was a naturally emergence of a scheme of reducing the flutter amplitude by active control of some stiffness factors. Along the same line, theoretical and experimental investigations have been performed for a scheme for 'flutter taming' - semi-active control of structural non-linear flutter [97]. For a 2-D non-linear flutter system, digital simulation methods were used to verify the principle of semi-active flutter control and to study the response characteristics of the closed loop flutter system. Simulation results showed that by adjusting automatically the non-linear stiffness parameter of the flutter system, the amplitude of the flutter response could be suppressed. In accordance with the theoretical analysis, a wind tunnel test model for semi-active flutter control was designed. A micromotor-slide block system served as the parameter control executive element with the monitoring of response signal and the controlling of micromotor performed by a microcomputer. Wind tunnel tests confirmed that the non-linear flutter could be controlled effectively by this technique.

Substantial research has been carried out on panel flutter suppression by active control using smart materials since the pioneer work by Scott and Weisshaar [101] to perform an active suppression research on linear panel flutter with piezoelectric material using the Ritz method. Active and passive suppression schemes for nonlinear flutter of composite panel have been investigated using lead zirconate titanate (PZT) [102]. The work involved designing the optimal controller based on the linear optimal control theory for flutter suppression of the panel in the active control method. Further, a passive damping technology, using one shunt circuit and two independent shunt circuits, which is believed to be a more robust suppression system in practical operation, was proposed. This approach requires very little or no electrical power. In this passive method, the piezoelectric shunt circuit, which consists of resistor-inductor-capacitor elements in series, was applied. In both methods, a finite element formulation for composite plates with PZT layers was derived using classical laminated plate theory. The adopted aerodynamic theory was based on the quasi-steady piston theory, and the von Karman nonlinear strain-displacement relation is used. A modal reduction technique was used to reduce the number of modes involved and to simplify the nonlinearity of the model. Numerical results, which were based on the reduced nonlinear modal equations of active and passive suppression for nonlinear panel flutter, were presented in the time domain using the Newmark-β method. To achieve the best suppression effect, optimal shape and location of the PZT patches were determined using genetic algorithms. The results clearly demonstrate that the passive damping scheme as well as active control can effectively attenuate the flutter.

In a further development on the suppression of nonlinear panel flutter, Moon *et al* [103] have proposed an optimal active/passive hybrid control design with piezoceramic actuators using finite element methods. This approach has the advantages of both active (high performance, feedback action) and passive (stable, low power requirement) systems. Piezoceramic actuators were connected in series with an external voltage source and a passive resonant shunt circuit, which consisted of an inductor and resistor. Like in Ref. [102], the shunt circuit was tuned correctly to suppress the flutter effectively with less control effort as compared to purely active control. To obtain the best effectiveness, active control gains were simultaneously optimised together with the value of the resistor and inductor through a sequential quadratic programming method. The governing equations of the electromechanically coupled composite panel flutter were derived through an extended Hamilton's principle, and a finite element discretization was carried out. The adopted aerodynamic theory was based on the quasi-steady first-order piston theory, and the von Karman nonlinear strain-displacement relation was used. Nonlinear modal equations are obtained through a modal reduction technique. The optimal control design was based on linear modal equations of motion, and numerical simulations were based on nonlinear-coupled modal equations. Using the Newmark integration method, suppression results of a hybrid control and a purely active control were presented in the time domain.

5.1.2 Shape memory alloy (SMA)

The use of embedded SMA actuators for composite beams deflection and buckling controls provided very promising results in which the composite beams could sustain further external loads without failures by high deflection or buckling [104]. Many studies related to the uses of embedded SMA actuators for noise reduction in rotor blade systems have been found recently. The major achievements of those works were to control the stiffness, angle of blade twist, tip-configuration, natural frequency and damping property of composite rotor blades in order to improve the unstable flutter and noise of the blades due to blade vortex interaction. For instance, coupled conventional constitutive law for shape memory materials, non-linear beam deflection analysis and numerical approach to simulate the deflection of simple SMA composite beams with and without actuations by embedded SMA actuators are beginning to appear [105]. More so, since the SMA materials possess super-elastic property at a temperature greater than austenite finish temperature (A_f), the superelasticity of SMA fibres is able to suppress or damp structural vibration by applying internal forces (distributed and/or localised) to structures in such a way as to dissipate the energy within the structures. Therefore, the superelastic SMA fibres could reduce vibration amplitudes and increase the damping ratio of the structure [106].

Ostachowicz and Kaczmarczyk [107] have demostrated that a SMA-based control system can be designed to reduce the adverse dynamic response of composite structures due to flutter phenomena. Specifically, the natural frequencies were modified *via* a direct SMA heating and this way, the flutter instability regions were controlled. A finite element model to predict the dynamic response of the system with embedded SMA fibres was proposed and the flutter boundaries identified with and without delamination damage, Figure 12.

Figure 12

It was assumed that the plate material properties are a function of temperature expect density, the angle of the graphite fibres and the coefficient of expansion are functions of temperature. It was also shown that for the delamination case the flutter boundaries are exceeded at lower critical aerodynamic loads, as compared with the undamaged case. Since in the FEM formulation there are no restrictions as far as the panel geometry is concerned, the computer model proposed in the study can be applied to design efficient SMA fibre-based control mechanism to avoid flutter instability phenomenon in composite structures with delamination.

5.1.3 Damping layer treatments

Composite materials are widely used for structures due to their strength and stiffness. The durability and low cost of a structural material add to the attraction of using a structural material to enhance damping. By the use of the interfaces and *visco*elasticity provided by appropriate components in a composite material, the damping capacity can be increased with negligible decrease, if any, of the storage modulus. The attaining of a significant damping capacity while maintaining high strength and stiffness is the goal of the structural material tailoring to address undesirable large-amplitude vibrations and radiated noise often impede the effective operation of various types of dynamic civilian and military systems such as rotorcrafts, missiles, land vehicles, and weapon systems. It is therefore prudent to introduce structural damping into a dynamic system to achieve a more satisfactory response.

It is a well-established fact that *visco*elastic nonstructural materials commonly provide damping in composite structures. Due to the large volume of structural materials in a structure, the contribution of a structural material to damping can be substantial. The development of effective and economical structural damping approaches that can suitably adjust mechanical properties to appropriate specifications could be beneficial in designing future systems [108]. New concepts for enhancing the structural damping characteristics have been explored in recent times to the study of adaptive structures. Such active damping techniques, based on combinations of *visco*elastic, magnetic, and/or piezoelectric materials, magnetorheological (MR) fluids, shunted electric circuits, and active non-linear control strategies, integrated methods using smart materials and passive circuits to dissipate energy, have emerged as several likely candidates for improving structural performance and reliability. This development involves tailoring through composite engineering and results in reduction of the need for nonstructural damping materials. Some of the concepts gaining grounds include active constrained layer damping control [109, 110], passive constrained layer damping treatments [111, 112], active–passive hybrid technology [112, 113], micro-cellular foams damping treatments [111, 114, 115], enhanced active constrained layer [116, 117] and state-switchable vibration absorbers [118, 119]. Other concepts such as application of damping properties *via* nanomaterials are still at the very early stage of development and are expected to play a big role in the next generation of aerospace vehicles.

5.2 Fibre and ply orientation

The influences of fibre and ply orientaion have substantial aeroelastic benefits and have been widely studied. For instance, Georghiades and Banerjee [120-122] examined the significance which *wash-out* can have on the flutter characteristics relating to swept and unswept wings composite wings. The coupling effect between the bending and twisting modes of structural deformation in flutter analysis were also investigated. Sarigul-Klijn and Oguz [123] reported on the effect of aspect ratio and ply orientation on aeroelastic response of composite plates and explored the importance of divergence and flutter behaviour in flight vehicles. Another work reported on the nonlinear effects of fibre orientations on the transonic and supersonic flutter characteristics of a composite missile wing model [124]. Free vibration of composite wings has been extensively investigated too [125, 126]. Khot [127] reported on a composite wing structure with enhanced roll maneuver capability at high dynamic pressures using a control system to retwist and recamber the wing. In the first step, minimum weight design satisfying requirements on strength, aileron efficiency and flutter for a specified set of fibre orientations was obtained. The control system was then designed to retwist and recamber the wing to counteract the detrimental twisting moment produced by the aileron. The distribution of control forces was obtained from 'fictious control surfaces' technique, and demonstrated a full recovery of roll rate at all dynamic pressures.

The lay-ups of some structural components may have a significant effect on the structural rigidities but may have little effect on flutter behaviour. In fact, a careful selection of initial lay-up and/or design variables may improve the optimisation efficiency. However, an unconstrained aeroelastic optimisation for maximum flutter speed might reduce the laminate strength of a composite wing structure. The strength set in the initial design can be regained without compromising the maximum flutter speed and the weight by optimising the wing structure in a constrained optimisation. Guo *et al.* [128] showed that a careful choice of initial lay-up and design variables leads to a desirable bending, torsional and coupling rigidities, with the provision of an efficient approach when achieving a maximum flutter speed with a minimum mass of a composite wing. An analytical study on optimisation of a thin walled wing box model made of laminated composite material showed that up to 18% increase in flutter speed and 13 wt.% reductions can be achieved without compromising the strength. The work also reported that in comparison with the bending rigidity, both the torsional and the coupling rigidities have much more significant effects on the flutter speed of a composite wing. The torsional rigidity also played a relatively more dominant role in aeroelastic tailoring although the work failed to determine a clear trend of the coupling rigidity effect on the flutter speed.

5.3 Microelectromechanical systems (MEMS), nanoelectromechanical systems (NEMS) and nanostructrures

Polymer micro- and nano-particles are fundamental to a number of modern technological applications, including polymer blends or alloys, biomaterials for drug delivery systems, electro-optic and luminescent devices, coatings, polymer powder impregnation of inorganic fibres in composites, and are also critical in polymer-supported heterogeneous catalysis. Basic microelectromechanical systems (MEMS) fabrication techniques include bulk micromachining, surface micromachining, and wafer bonding. Also, since the same processes are often used to create both MEMS devices and traditional integrated circuit (IC) circuits, by carefully designing the fabrication process flow it becomes possible to integrate transducers and microelectronics on the same wafer chip. This normally results in both cost savings and better performance. Some examples of MEMS products are micropressure sensors, accelerometers, inkjet printer heads, and digital mirror devices for projection systems, optical switches, and lab-on-a-chip systems for separation, preparation, and detection of DNA or pathogens. MEMS fluidic sensors that are now available include piezoresistive pressure sensors, shear stress sensors, and micromachined hotwires. In aerodynamics, flexible MEMS bubble actuators have been used to affect the rolling moment of a delta wing [129]. Flexible shear stress sensors have also been used to detect the separation line on a rounded leading edge of a delta wing as well as on a cylinder [130]. MEMS actuators are known to be relatively power thrifty and can interact with and manipulate the relevant flow structures to effect global flow property changes from local actuation. This ability is due to the length scale of the actuator (anywhere from hundreds of microns to a few millimeters) being comparable to the flow structure, thus allowing the actuator to directly excite flow instabilities at their origin. A distributed field of such actuators can therefore efficiently achieve large aerodynamic performance improvements. Of equal importance is also the ability to batch fabricate these devices on thin films and distribute them on the aerodynamic surface of interest to form a distributed control system.

Ho *et al*. [7] have perfomed experimental studies on metal wings and paper wings embended with MEMS during their studies on micro-air-vehicles (MAVs). Comparrison of two generation of the titanium–parylene wing with the original paper wing (Figure 13) showed that the lifting ability of the first-generation metal wings closely matches that of the paper wings for large advance ratios.

Figure 13

On Figure 14 the actuator effect is seen from the plots of lift and thrust. The highest lift and thrust peak values (approximately 40 and 20 g, respectively) occurred when the actuators were turned on.

Figure 14

When the actuators are ‘ON’ both lift and thrust differ up to 50% at a given instant in time in comparison to the ‘OFF’ value, which clearly indicates the effect of the valve. The valves locally manipulated the unsteady leading-edge vortex and changed the pressure distribution on the wings. It was also observed that flow unsteadiness rose relatively to lift and thrust due to growth in the size and strength of the vortex, allowing the actuators to be more effective. Nanoparticle-polymer nanocomposites synergistically combine the properties of both the host polymer matrix and the discrete nanoparticles there in. For instance, the polyurethane foam is popular for some of its moderate properties like good moisture resistance, high shear strength, excellent sound dampening, vibrational and oscillating environment tolerance. Recently researchers have investigated nanocomposites containing polymer-nanoparticle to improve their physical, mechanical, and chemical properties [131-134]. Nanoparticles embedded in polymer matrix have attracted increasing interest because of the unique properties displayed by nanoparticles. Due to nanometer size of these particles, their physicochemical characteristics differ significantly from those of molecular and bulk materials. Such nanocomposite materials are expected to have novel electrical and mechanical properties [135].

6 Conclusions

Emergence of flutter compromises not only the long term durability of the wing structure, but also the operational safety, flight performance and energy efficiency of the aircraft. Effectual means of flutter prevention are, therefore, mandatory in the certification of new flight vehicles, and considerable effort, theoretically as well as experimentally, is devoted to the study of methods for active flutter control and of the interaction between structural dynamics and unsteady airflows. It is appreciable that the knowledge gained from the study of flexible structures and unsteady airflows in aircraft can be transitioned back to more traditional flutter studies. With the advent of active/passive control technology for flutter suppression, and gust load alle*via*tion, and with the increasing need of evaluating the time-dependent subcritical aeroelastic response of lifting surfaces, the time domain representation of unsteady aerodynamic loads becomes a necessary prerequisite toward achieving these goals. It is also well acceptable that carefully designed and embedded sensors and actuators can lead to actively/passively-controlled structures with some beneficial contributions as per minimisation of flutter occurrences. It is also of paramount importance to understanding the fluid–structure interaction for successful flutter analysis. In other words, the design of aircraft wings requires knowledge of how a highly flexible airfoil will deform under aerodynamic loading and the effect of that deformation on airfoil efficiency. This work intends to address the flutter phenomenon highlighting the above issues, and reviews some of the most recent theoretical and experimental developments in flutter analyses.

7 References

1. Librescu L, Chiocchia G, Marzocca P. Implications of cubic physical/aerodynamic non-linearities on the character of the flutter instability boundary. *International Journal of Non-Linear Mechanics* 2003; **38**:173-199.
2. Bisplinghoff RL, Ashley H, Halfman RL. *Aeroelasticity*. New York: Dover Publications Inc; 1996.
3. Ueda J, Dowell EH. Doublet-point method for supersonic unsteady lifting surfaces. *AIAA Journal* 1984; **22**:179.
4. Chen PC, Liu DD. A harmonic gradient method for unsteady supersonic flow calculations. *Journal of Aircraft* 1985; **22**:371.
5. Lottati I, Nissim E. Nonplanar, supersonic, three-dimensional, oscillatory, piecewise continuous-kernel function method. *Journal of Aircraft* 1987; **24**:45.
6. Cho J, Williams MH. S-plane aerodynamics of nonplanar lifting surfaces. *Journal of Aircraft* 1993; **30**:433.
7. Ho S, Nassef H, Pornsinsirirak, Nick; Tai Y-C, Ho C-M. Unsteady aerodynamics and flow control for flapping wing flyers. *Progress in Aerospace Sciences* 2003; **39**:635-681.
8. Shiau LC, Wu TY. Nonlinear flutter of laminated plates with in-plane force and transverse shear effects. *Mechanics of Structures & Machines* 2001; **29**:121.
9. Shiau LC, Hwang ST. Application of Finite Strip Method to Composite Panel Flutter Analysis. *Journal of Aircraft* 2000; **37**:740.
10. A. Sulema. Adaptive composites modelling and application in panel flutter and noise suppression. *Computers and Structures* 2000; **76**:365-378.
11. C. Jinsoo, C.Younhyuck. Supersonic flutter analysis of wings using an unsteady 3D panel method. *Computers and Fluids* 2001; **30**:237-256.
12. Balakrishnan AV. Subsonic flutter suppression using self-straining actuators. *Journal of the Franklin Institute* 2001; **338**:149-170.
13. Hodges DH, Patil MJ, Chae S. Effect of thrust on bending-torsion flutter of wings. *Journal of Aircraft* 2002; **39**:371 - 376.

14. Kurnik W, Przybyowicz PM. Stability of rotating cantilever columns under follower loads. *In: Proceedings of the IX International Symposium on Dynamic Problems of Mechanics DINAME. Florianpolis, Santa Catarina, Brazil, 2001.*
15. W. Kurnik.P. M. Przybyowicz. Active stabilisation of a piezoelectric fiber composite shaft subject to follower load. *International Journal of Solids and Structures* 2003; **40**:5063-5079.
16. De Rosa MA, Franciosi C. The influence of an intermediate support on the stability behaviour of cantilever beams subjected to follower forces. *Journal of Sound and Vibration* 1990; **137**:107-115.
17. D. H. Hodges. LATERAL-TORSIONAL FLUTTER OF A DEEP CANTILEVER LOADED BY A LATERAL FOLLOWER FORCE AT THE TIP. *Journal of Sound and Vibration* 2001; **247**:175-183.
18. H. J. Liu, Z.C. Yang, L.C. Zha. Semi-active flutter control by structural asymmetry. *Journal of Sound and Vibration* 2000; **229**:199-205.
19. Gern FH, Librescu L. Effects of externally mounted stores on aeroelasticity of advanced swept cantilevered aircraft wings. *Aerospace Science and Technology*; **2**:321-333.
20. Gern FH, Librescu L. Aeroelastic tailoring of composite wings exhibiting nonclassical effects and carrying external stores. *Journal of Aircraft* 2000; **37**:1097-1104.
21. W.B. Hu, L. C Zhao. Semiactive flutter control of wing:store system with electro!magnetic friction damper. *Journal of Sound and Vibration* 1997; **200**: 625-630.
22. Dowell EH, Ilgamov M. *Studies in Nonlinear Aeroelasticity, (1988), Springer, New York.*
23. Lee BHK, Price SJ, Wong YS. Nonlinear aeroelastic analysis of airfoils: bifurcation and chaos. *Progress in Aerospace Sciences* 1999; **35**:205-334.
24. Librescu L. *Elastostatics and kinetics of anisotropic and heterogeneous shell-type structures, in: Aeroelastic Stability of Anisotropic Multilayered Thin Panels, Noordhoff International Publishers Leiden, The Netherlands, 1975, pp. 106-158.*
25. Chandiramani NK, Plaut RH, Librescu LI. Non-linear flutter of a buckled shear-deformable composite panel in a high-supersonic flow. *International Journal of Non-Linear Mechanics* 1995; **30**:149-167.

26. Oh IK, Lee I, Lee DM. Non-linear transient response of fluttering stiffened composite plates subject to thermal load. *Journal of Sound and Vibration* 2001; **245**:715-736.
27. Lee I, Roh J-H, Oh I-K. Aerothermoelastic phenomena of aerospace and composite structures. *Journal of Thermal Stresses* 2003; **26**:525.
28. Hagood NW, Flotow AV. Damping of structural vibrations with piezoelectric materials and passive electrical networks. *Journal of Sound and Vibration* 1991; **146**:243-268.
29. Hollkamp JJ. Multimodal passive vibration suppression with piezoelectric materials and resonant shunts. *Journal of Intelligent Matererials, Systems and Structures* 1994; **5**:49-57.
30. Zhou RC, Lai Z, Xue DY, Hauang JK, Mei C. Suppression of nonlinear panel flutter with piezoelectric actuators using finite element. *AIAA Journal* 1995; **33**:1098-1105.
31. Kim SJ, Han CH, Yun CY. Improvement of aeroelastic stability of hingeless helicopter rotor blade by passive piezoelectric damping. *Proc. SPIE Smart Struct. Mater. Conf., Volume: 3672, (1999), pp. 1552-1561.*
32. Moona SH, Kim SJ. Active and passive suppressions of nonlinear panel flutter using finite element method. *AIAA Journal* 2001; **11**:2042-2050.
33. Agnes G. Active/passive piezoelectric vibration suppression. *Proc. SPIE Smart Struct. Mater. Conf.* 1994; **2193**:24-34.
34. Moon SH, Kim. SJ. Suppression of nonlinear composite panel flutter with active/passive hybrid piezoelectric networks using finite element method. *Composite Structures* 2003; **59**: 525-533.
35. Bauchau OA, Rodriguez J, Bottasso CL. Modeling of Unilateral Contact Conditions with Application to Aerospace Systems Involving Backlash, Freeplay and Friction. *Mechanics Research Communications* 2001; **28**:571-599.
36. Hall BD, Mook DT, Nayfeh AH, Preidikman S. Novel strategy for suppressing the flutter oscillations of aircraft wings. *AIAA Journal* 2001; **39**:1843-1850.
37. Patil MJ, Hodges DH, Cesnik CES. Characterizing the effects of geometrical nonlinearities on aeroelastic behavior of high-aspect-ratio wings. *In Proceedings of the International Forum on Aeroelasticity and Structural Dynamics, Williamsburg, Virginia.* 1999.
38. Tang DM, Dowell EH. Experimental and theoretical study on aeroelastic response of high-aspect-ratio wings. *AIAA Journal* 2001; **39**:1430-1441.

39. Hall B, Preidikman S, Mook DT. A time-domain simulation for evaluating smart wing concepts for reducing gust loads. *ASME Mechanics and Materials Conference, Blacksburg, Virginia, 1999.*

40. Bisplinghoff RL, Ashley H, Halfman RL. *Aeroelasticity*. New York: Dover Publications; 1996.

41. Karpouzion G, Librescu L. Nonclassical effects on divergence and flutter of anisotropic swept aircraft wings. *AIAA Journal* 1996:786-794.

42. Pidaparti RMV, Chang CC. Finite element supersonic flutter analysis of skewed and cracked composite panels. *Computers and Structures* 1998; **69**:265-270.

43. Wei Z, Yam LH, Cheng L. Detection of internal delamination in multi-layer composites using wavelet packets combined with modal parameter analysis. *Composite Structures* 2004; **64**:377-387.

44. Lin KJ, Lu PJ, Tarn JQ. Flutter analysis of anisotropic panels with patched cracks. *Journal of Aircraft* 1991; **28**:899-904.

45. Shiau LC. Flutter of composite laminated beam plates with delamination. *AIAA Journal* 1992; **30**:2504-11.

46. Pidaparti RMV. Free vibration and flutter of damaged composite panels. *Composite Structures* 1997; **38**:477-481.

47. M. Karpel. Design for active flutter suppression and gust alleviation using state-space aeroelastic modeling. *Journal of Aircraft* 1982; **19**:221-227.

48. Marzocca LL, G. Chiocchia. Aeroelastic response of 2-D lifting surfaces to gust and arbitrary explosive loading signatures. *Int. J. Impact Eng.* 2001; **25**:41-65.

49. Marzocca P, Librescu P, Liviu; Chiocchia G. Aeroelastic response of a 2-D airfoil in a compressible flow field and exposed to blast loading. *Aerospace Science and Technology*; **6**: 259-272.

50. Qin Z, Librescu L. Aeroelastic instability of aircraft wings modelled as anisotropic composite thin-walled beams in incompressible flow. *Journal of Fluids & Structures* 2003; **18**:43-61.

51. Edwards JW, Ashley H, Breakwell JV. Unsteady aerodynamic modeling for arbitrary motions. *AIAA Journal* 1979:365-374.

52. Ueda T, Dowell EH. Flutter analysis using nonlinear aerodynamic forces. *Journal of Aircraft* 1984; **21**:101-109.

53. Dean W. Sparks J, Maghami. PG. NEURAL NETWORKS FOR RAPID DESIGN AND ANALYSIS. *AIAA-98-1779*:1-9.
54. Rick L. Match-point solutions for robust flutter analysis. *Journal of Aircraft* 2002; **39**:91-99.
55. Noor AK, Malik. M. An assessment of five modeling approaches for thermo-mechanical stress analysis of laminated composite panels. *Computational Mechanics.*; **25**:43 - 58.
56. Noor AK, Venneri SL, Paul DB, Hopkins MA. Structures technology for future aerospace systems. *Computers & Structures* 2000; **74**:507-519.
57. Liu CF, Huang CH. Free vibration of composite laminated plates subjected to temperature changes. *Computers and Structures* 1996; **60**: 95-101.
58. Zhou RC, Xue DY, Mei C. Vibration of thermally buckled composite plates with initial deflections using triangular elements. *In: The 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA-93-1321-CP, 1993. p. 226-35.*
59. Lee DM, Lee I. Vibration behaviors of thermally postbuckled anisotropic plates using first-order shear deformable plate theory. *Computers and Structures* 1997; **63**:371-378.
60. Savoia M, Reddy JN. Three-dimensional thermal analysis of laminated composite plates. *International Journal of Solids and Structures* 1995; **32**:593-608.
61. Noor AK, Burton SW. Computational models for high-temperature multi-layered composite plates and shells. *Applied Mechanics Reviews* 1992; **45**:419-446.
62. Noor AK, Malik M. An assessment of five modeling approaches for thermo-mechanical stress analysis of laminated composite panels. *Computational Mechanics* 2000; **25**:43-58.
63. W.Yu, D.H. Hodges. An Asymptotic Approach for Thermoelastic Analysis of Laminated Composite Plates.
64. Rolfes R, Rohwer K. Improved transverse shear stresses in composite finite elements based on first-order shear deformation theory. *Int. J. Numer. Meth. Engng.* 1997: 51-60.
65. Rolfes R, Rohwer K, Ballerstaedt M. Efficient linear transverse normal stress analysis of layered composite plates. *Computers and Structures* 1998:643-652.

66. Averill RC, JN. Thermomechanically postbuckling analysis of laminated composite shells. *in: The 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA-93-1337-CP, 1993. p. 351-60.*

67. Chen LW, Chen LY. Thermal postbuckling behaviors of laminated composite plates with temperature-dependent properties. *Composite Structures* 1991; **19**:261-283.

68. Song O, Librescu L, Oh S-Y. Dynamics of pretwisted rotating thin-walled beams operating in a temperature environment. *Journal of Thermal Stresses* 2001; **24**:5.

69. K. Kyung-Eun, K. Ji-Hwan. Thermally induced vibrations of spinning thin-walled composite beam. *AIAA Journal* 2003; **41**:296-303.

70. D. H. Hodges, Peters DA. Lateral-torsional buckling of cantilevered elastically coupled composite strip- and I-beams. *International Journal of Solids and Structures* 2001; **38**: 1585-1603.

71. Volovoi VV, Hodges DH, Berdichevsky VL, Sutyrin V. Asymptotic theory for static behavior of elastic anisotropic I-beams. *International Journal of Solids and Structures* 1998; **36**:1017-1043.

72. Bauld Jr. NR, Tzeng L-S. Vlasov theory for fiber-reinforced beams with thin-walled open cross sections. *International Journal of Solids and Structures* 1984; **20**:277-297.

73. Pandey M, Kabir M, Sherbourne A. Flexural-torsional stability of thin-walled composite I-section beams. *Composites Engineering* 1995.; **5**:321-342.

74. Chandrashekhara K, Bhatia K. Active buckling control of smart composite plates-finite-element analysis. *Smart Materials and Structure* 1993:31-39.

75. Chase JG, Bhashyam S. Optimal stabilization of plate buckling. *Smart Materials and Structure* 1999: 204-211.

76. Meressi T, Paden B. Buckling control of a flexible beam using piezoelectric actuators. *Journal of Guidance Control and Dynamics* 1993:977-980.

77. Thompson SP, Loughlan J. The active buckling control of some composite column structure strips using piezoceramic actuators. *Composite Structures* 1995; **32**: 59-67.

78. Sloss JM, Bruch JC, Sadek IS, Adali S. Piezo patch sensor/actuator control of the vibrations of a cantilever under axial load. *Composite Structures*; **2003**:423-428.

79. Yang Y, Ju C, Soh CK. Analytical and semi-analytical solutions for vibration control of a cantilevered column using a piezoelectric actuator. *Journal of Smart Materials and Structures* 2003; **12**:193-203.

80. Wang Q. On buckling of column structures with a pair of piezoelectric layers. *Engineering Structures* 2002; **24**:199-205.
81. Sedaghat A, Cooper JE, Leung AYT, Wright JR. Estimation of the hopf bifurcation point for aeroelastic systems. *Journal of Sound and Vibration* 2001; **248**:31-42.
82. Roy ID, Eversman W. Adaptive flutter suppression of an unswept wing. *Journal of Aircraft* 1996; **33**: 775-783.
83. K. Mam. Adaptive non-linear control of a clamped rectangular plate with PZT patches. *Journal of Sound and Vibration* 2003; **264**:835-850.
84. Hodges DH. Lateral-torsional flutter of a deep cantilever loaded by a lateral follower force at the tip. *Journal of Sound and Vibration* 2001; **247**:175-183.
85. Friedmann PP, Presente E. Active control of flutter in compressible flow and its aeroelastic scaling. *Journal of Guidance, Control and Dynamics* 2001; **24**:167-175.
86. Sungsoo NA, Librescu L. Dynamic response of adaptive cantilevers carrying external stores and subjected to blast loading. *Journal of Sound and Vibration* 2000; **231**:1039-1055.
87. G.S. Virk, A.S.Al-Dmour. Real-time vibration suppression in a flexible cantilever rig. *Microprocessors and Microsystems* 1999; **23**:365-384.
88. Saravanos DA, Heyliger PR. Mechanics and computational models for laminated piezoelectric beams, plates, and shells. *Applied Mechanics Reviews* 1999; **52**:305-320.
89. Lin CY, Crawley EF, Heeg J. Open- and closed-loop results of a strain-actuated active aeroelastic wing. *Journal of Aircraft* 1996:987-994.
90. Heeg J. Analytical and experimental investigation of flutter suppression by piezoelectric actuation. *NASA TP 93-3241, NASA.* 1993.
91. Gasbarri P, Mannini A, Barboni R. Dynamic modeling of swept wing with PTFE method. *Computers & Structures* 2002; **80**:1255.
92. Lee YY, Yuen KK, Ng CF, Cheng. GF. Numerical simulation model of vibration responses of rectangular plates embedded with piezoelectric actuators. *Thin-Walled Structures* 2002; **40**:1-28.
93. Agneni FM, Polli GM. Shunted piezoelectric patches in elastic and aeroelastic vibrations. *Computers and Structures* 2003; **81**:91-105.
94. Giesing JP, Kalman TP, Rodden WP. Subsonic unsteady aerodynamics for general con.guration. *AFFDL-TR-71-5, 1971.*

95. Rodden WP, Johnson EH. *MSC.NASTRAN--aeroelastic analysis. User's Guide, ver. 68. The MacNeal-Schwendler Corporation, 1994.*

96. McGowan AMR. An examination of applying shunted piezoelectrics to reduce aeroelastic response. *CEAS/AIAA/ICASE/NASA Langley International Forum on Aeroelasticity and Structural Dynamics, Williamsburg;* 1999:553-71.

97. Yang Z-C, Zhao L-C, Jiang J-SA. semi-active flutter control scheme for a two-dimensional wing. *Journal of Sound and Vibration* 1995; **184**:1-7.

98. W. H. Reed III, Jr. JTF, Runyan HL. Decoupler pylon: a simple, effective wing/store flutter suppressor. *NASA-79-0791.* 1979.

99. Morino L. Flutter taming - a new tool for the aeroelastic designer. *CCAD-TR-84-01* 1984.

100. Z. C.Yang, L. C. Zhao. Analysis of limit cycle flutter of an airfoil in incompressible flow. *Journal of Sound and Vibration* 1988.; **123**:1-13.

101. Scott RC, Weisshaar TA. Controlling panel flutter using adaptive materials. *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics, and Materials Conference (Baltimore, MD), (1991), pp. 2218-2229 AIAA, Washington, DC.*

102. Moon SH, Kim SJ. Active and passive suppressions of nonlinear panel flutter using finite element method. *AIAA Journal* 2001; **39**:2042-2050.

103. Moon S, Hwan KSJ. Supression of nonlinear composite panel flutter with active/passive hybrid piezoelectric networks using finite element method. *Composite Structures* 2003; **59**:525-533.

104. Lee JJ, Choi. S. Thermal buckling and postbuckling analysis of a laminated composite beam with embedded SMA actuators. *Composite Structures* 1999; **47**:695-703.

105. Brinson LC, Huang MS, Boller C, Brand W. Analysis of controlled beam deflection using SMA wire. *Journal of Intelligent Material Systems and Structures* 1997; **8**:12-25.

106. Ju DY, Shimamoto A. Damping property of epoxy matrix composite beams with embedded shape memory alloy fibres. *Journal of Intelligent Material Systems and Structures* 1999; **10**: 514-520.

107. Kaczmarczyk OWM. Vibrations of composite plates with SMA fibres in a gas stream with defects of the type of delamination. *Composite Structures* 2001; **54**:305-311.

108. Garg DP, Anderson GL. Structural vibration suppression via active/passive techniques. *Journal of Sound and Vibration*; **262**:739-751.
109. Baz A. Robust control of active constrained layer damping. *Journal of Sound and Vibration* 1998; **211**:467-480.
110. Trindade MA, Benjeddou A, Ohayon R. Piezoelectric active vibration control of damped sandwich beams. *Journal of Sound and Vibration* 2001; **246**:653-677.
111. Yellin J, Shen I, Reinhall P, Huang P. An analytical and experimental analysis for a one-dimensional passive stand-off layer damping treatment. *Journal of Vibration and Acoustics* 2000; **122**:440-447.
112. M. Arafa, Baz. A. Energy-dissipation characteristics of active piezoelectric damping composites. *Composites Science and Technology* 2000; **60**:2759-2768.
113. Agnes GS, Napolitano K. Active constrained layer viscoelastic damping. *In: Proceedings of the 34th SDM Conference, 1993. p. 3499-506.*
114. Huang P, Reinhall P, Shen I, Yellin J. Thickness deformation of constrained layer damping□an experimental evaluation. *Journal of Vibration and Acoustics* 2001; **123**:213-223.
115. Jeung Y, Shen I. Development of an isoparametric, degenerate constrained layer element for plate and shell structures. *American Institute of Aeronautics and Astronautics Journal* 2001; **23**:280-284.
116. Liu Y, Wang YK. Active-passive hybrid constrained layer for structural damping augmentation. *Journal of Vibration and Acoustics* 2000; **122**:254-262.
117. Lam MJ, Inman DJ, Saunders WR. Variations of Hybrid Damping. *Proceedings of SPIE on Smart Structures and Materials, Vol. 3327, pp. 32-43.* 1998.
118. Cunefare K, De Rosa S, Sadegh N, Larson G. State switched absorber/damper for semi-active structural control. *Journal of Intelligent Material Systems and Structures* 2001; **11**:300-310.
119. Cunefare K. State-switched absorber for vibration control of point-excited beams. *Proceedings of ASME International Mechanical Engineering Conference and Exposition, 2000, Vol. 60, pp. 477-484.*
120. Georghiades GA, Banerjee JR. Significance of Wash-Out on the Flutter Characteristics of Composite Wings. *Journal of Aircraft* 1998; **35**:823.
121. Georghiades GA, Banerjee JR. Role of modal interchange on the flutter of laminated composite wings. *Journal of Aircraft* 1998; **35**:157.

122. Georghiades GA, Guo SJ. Flutter characteristics of laminated composite wings. *Journal of Aircraft,* 1996; **33**:1204.
123. Sarigul-Klijn N, Oguz S. Effect of aspect ratio and ply orientation on aeroelastic response of composite plates. *Journal of Aircraft* 1998; **35**:657.
124. Kim D-HL. Nonlinear Flutter Characteristics of Composite Missile Wing in Transonic/Low-Supersonic Flows. *Journal of Aircraft* 2002; **39**:889.
125. J. Njuguna. Dynamics of composite wings. *School of Engineering and Mathematical Sciences*. MSc. Thesis, vol. London, United Kingdom.: City University, London; 2001.
126. Njuguna J, Su H, Cheung CW, Banerjee J.R. The influence of ply orientation on the free vibration of composite beams. *In UNSW 2002 Conference Proceedings, Sydney, Australia, Eds. Bandyopadhyay, N. Gowripalan, S. Rizkalla, ACUN-4, p.307-314* 2002.
127. Khot NS. Design of composite smart wing to enhance roll maneuver. *Proceedings of - The Iinternational Society for the Optical Engineers (SPIE)* 1996; **3321**:436-446.
128. Guo SJ, Bannerjee JR, Cheung CW. The effect of laminate lay-up on the flutter speed of composite wings. *Proceedings of the Institute of Mechanical Engineers Vol. 217 Part G: J. Aerospace Engineering* 2003:pp.115-122.
129. Huang PH, Folk C, Silva C, Christensen B, Chen Y, Ho CM, Jiang F, Grosjean C, Tai YC, Lee GB, Chen M, Newbern S. Applications of MEMS devices to delta wing aircraft: from concept development to transonic flight test. *AIAA Paper No. 2001-0124* 2001.
130. Fukang J, Lee GB, Tai YC, Ho CM. A flexible micromachine-based shear stress array and its application to separation-point detection. *Sensors Actuators A* 2000:194-203.
131. Njuguna J, K. Pielichowski. Polymer nanocomposites for aerospace applications: properties. *Advanced Engineering Materials* 2003; **5**:769.
132. Njuguna J, K. Pielichowski. Polymer nanocomposites for aerospace applications: Characterisation. *Advanced Engineering Materials* 2004; **6**:204.
133. Njuguna J, K. Pielichowski. Polymer nanocomposites for aerospace applications: Preparation. *Advanced Engineering Materials* 2004; **6**:193.
134. J. Njuguna, K. Pielichowski, Banerjee JR. *In the Proceedings of The First Seminar on Structure and Properties of Advanced Materials for Environmental Applications*

under the auspices of EC Marie Curie Programme (FP 5), Cracow University of Technology, Kraków, December 16-18, 2004, P. 92-96. 2004.

135. Mahfuz H, Rangari VK, Islam MS, Jeelani S. Fabrication, synthesis and mechanical characterization of nanoparticles infused polyurethane foams. *Composites Part A* 2004; **35**:453-460.

2 Caption of Figures and Tables

2. Figures

Figure 1 Generic representation of the catastrophic and benign portions of the flutter critical boundary. The upper half is in the (L − M_{flight}) plane, while the bottom half of the graph is in the (L − M_{flight}) plane [1].

Figure 2 Using distributed strain actuators to affect aerodynamic control [3, 4].

Figure 3 Schematic of wing showing co-ordinate systems and follower force [18].

Figure 4 Critical load versus r for $\sigma = 0.05$ with $e = 0$ (-) and 0.005 (----) [18].

Figure 5 Sketch of wing/store model. 1 wing; 2 store; 3 electromagnetic friction damper [22].

Figure 6 Influence of the aerodynamic non-linearities and of the elastic axis position on the first Liapunov quantity L in the presence of physical non-linearities (B=50) [1].

Figure 7 Benign and catastrophic portions of the flutter critical boundary as a function of the elastic axis position [1].

Figure 8 Schematic diagram of a panel with the hybrid control system in which PZT actuators are connected with an external voltage source and an L–R circuit in series [34].

Figure 9 First coupled £ap-lag natural frequency vs. y for selected values of a and for both temperature gradient scenarios (– temperature profile $T(z) = {T_0 z}/{L}$, - - - temperature profile $T(z) = T_0\left(1 - {z}/{L}\right)$) [68].

Figure 10 The complete beam rig system set-up [87].

Figure 11 Finite element model of the plate with five bonded piezo patches [93].

Figure 12 Delamination zone modelled with variable thickness finite elements [107].

Figure 13 Phased averaged lift and thrust for integrated MEMS check-valve wing. J = 0:48 [7].

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2. Tables

Table 1 Coefficients for approximating the indicial lift and moment at selected subsonic Mach numbers. These quantities identified as (ϕ_{α}^{c}; $\phi_{M\alpha}^{c}$) and (ϕ_{q}^{c}; ϕ_{Mq}^{c}) are the compressible lift and moment indicial functions associated to the pitch angle α and the pitch rate q in subsonic compressible flow, respectively [48].

Table 2 Coefficients for approximating the indicial lift and moment at selected supersonic Mach numbers [48].

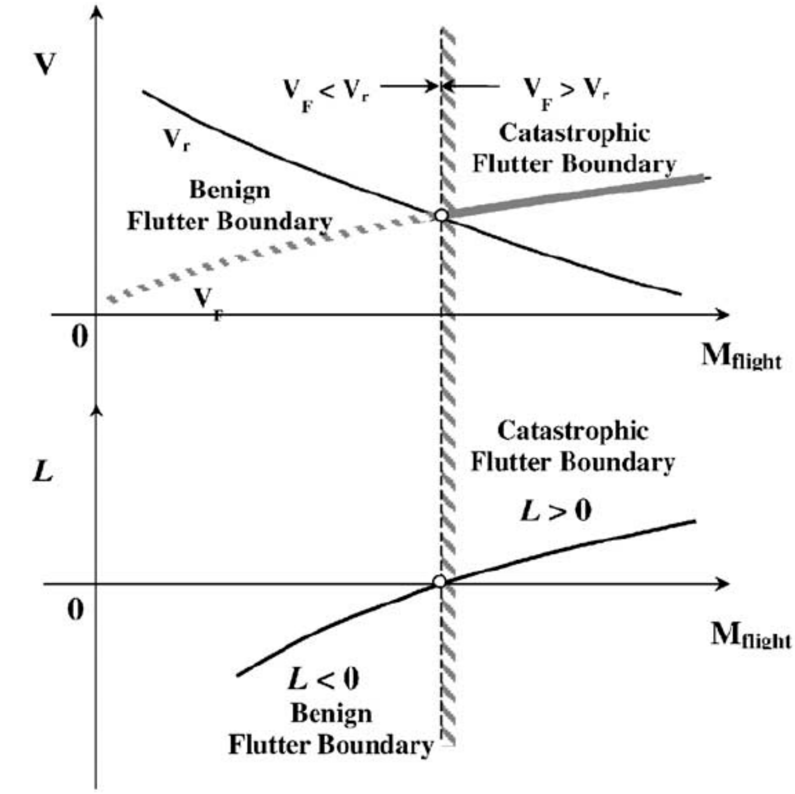


Figure 1 Generic representation of the catastrophic and benign portions of the flutter critical boundary. The upper half is in the (L $-M_{\text{flight}}$) plane, while the bottom half of the graph is in the (L $-M_{\text{flight}}$) plane [1].

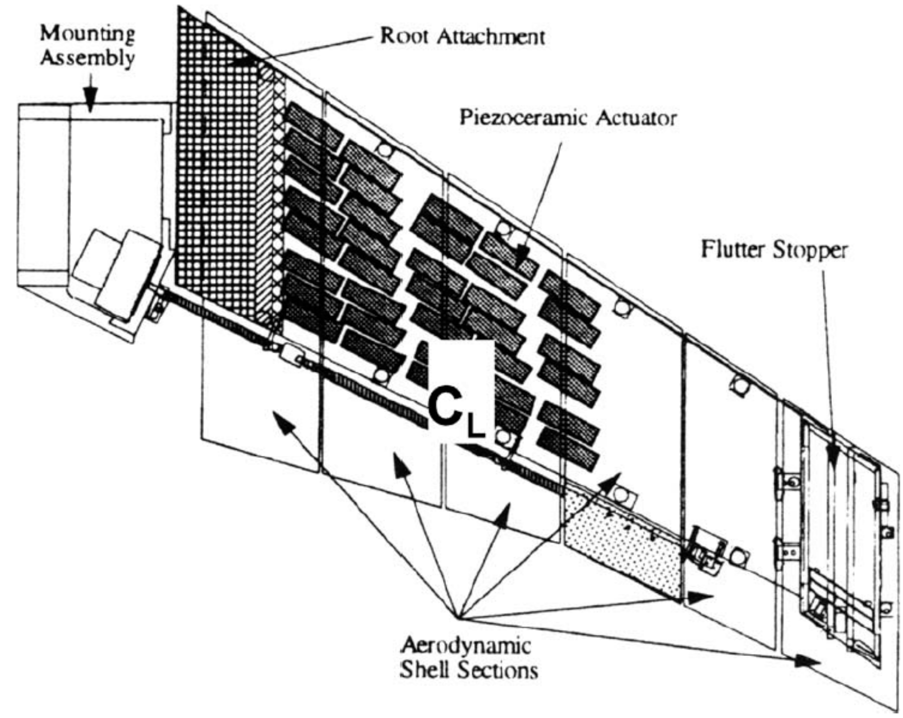


Figure 2 Using distributed strain actuators to affect aerodynamic control [7, 89].

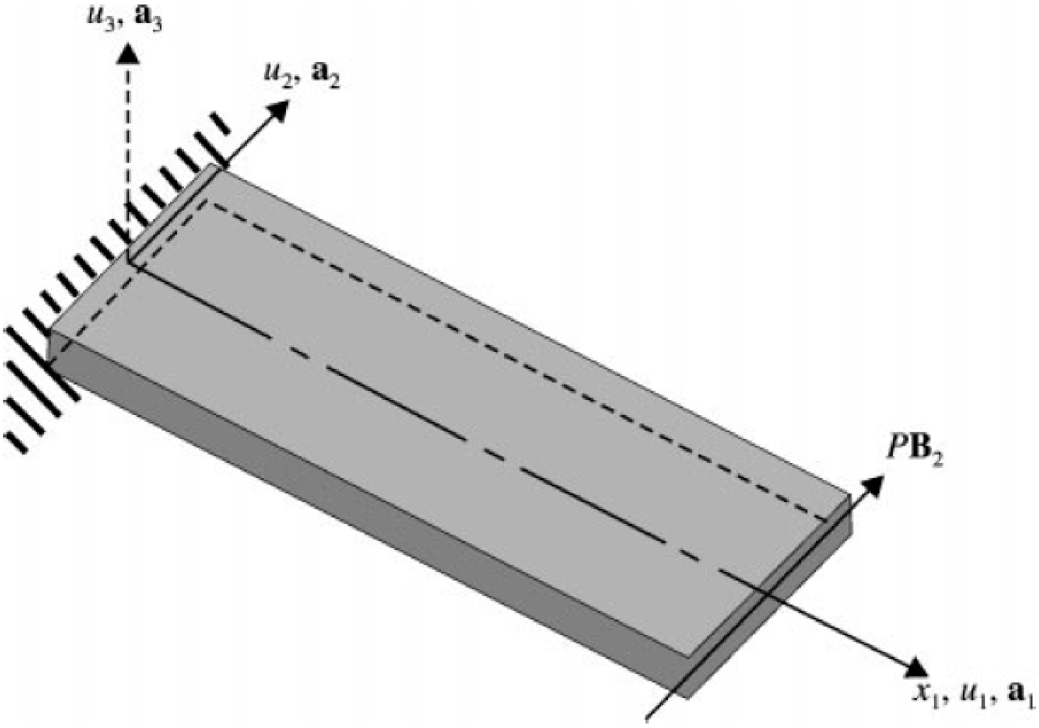


Figure 3 Schematic of wing showing co-ordinate systems and follower force [17].

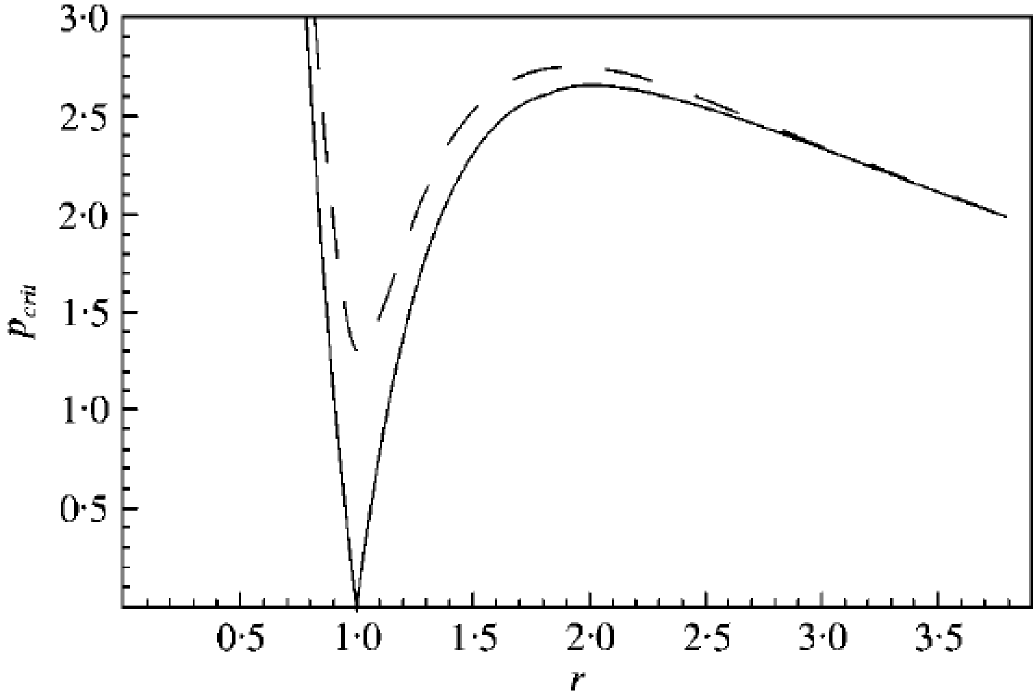


Figure 4 Critical load versus r for $\sigma = 0.05$ with $e = 0$ (-) and 0.005 (----) [17].

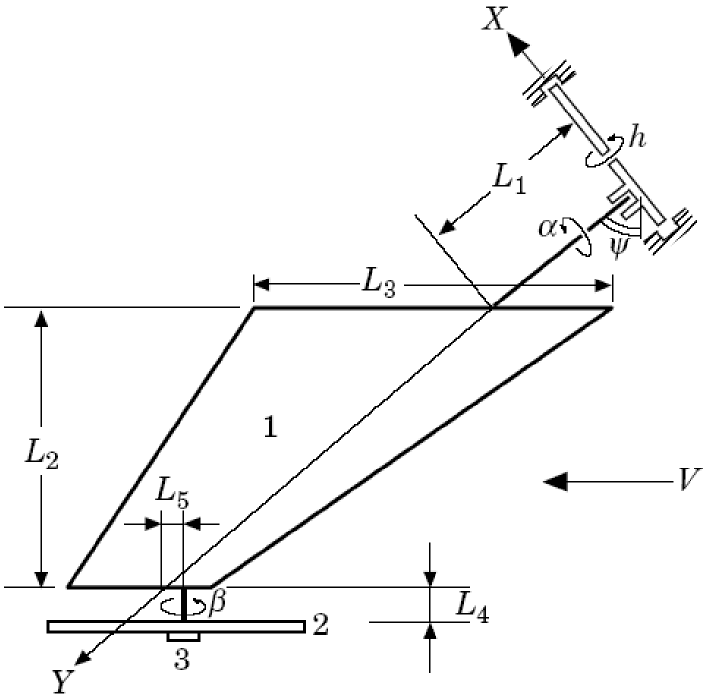
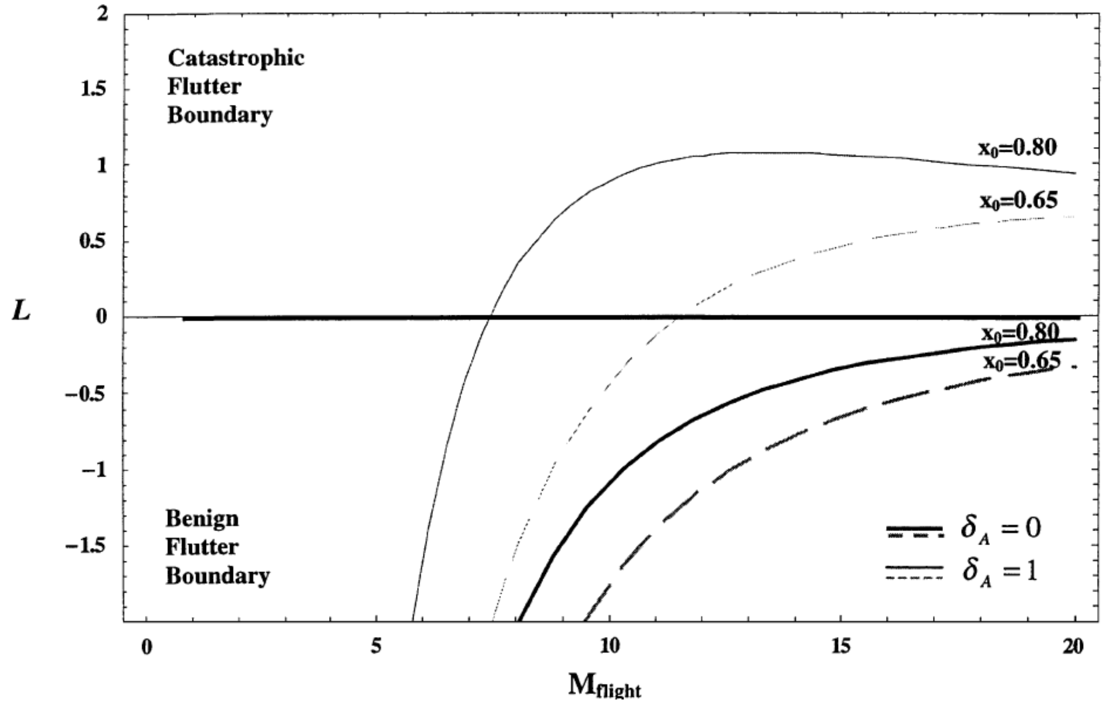


Figure 5 Sketch of wing/store model. 1 wing; 2 store; 3 electromagnetic friction damper [21].



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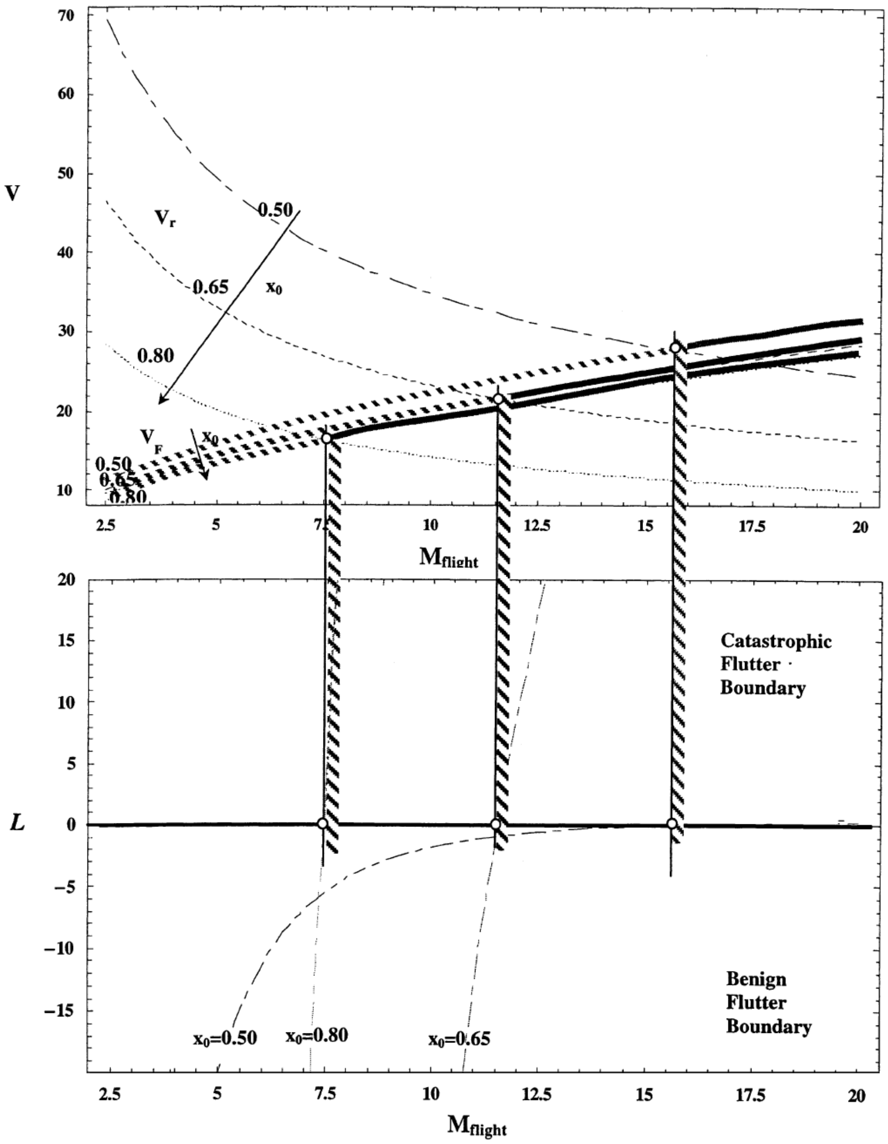

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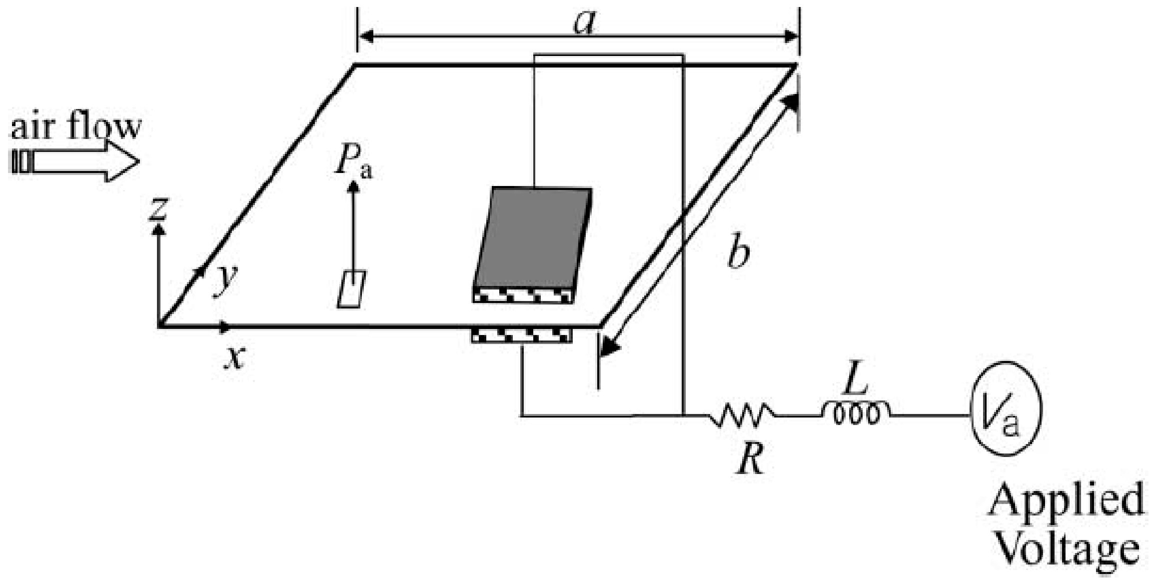

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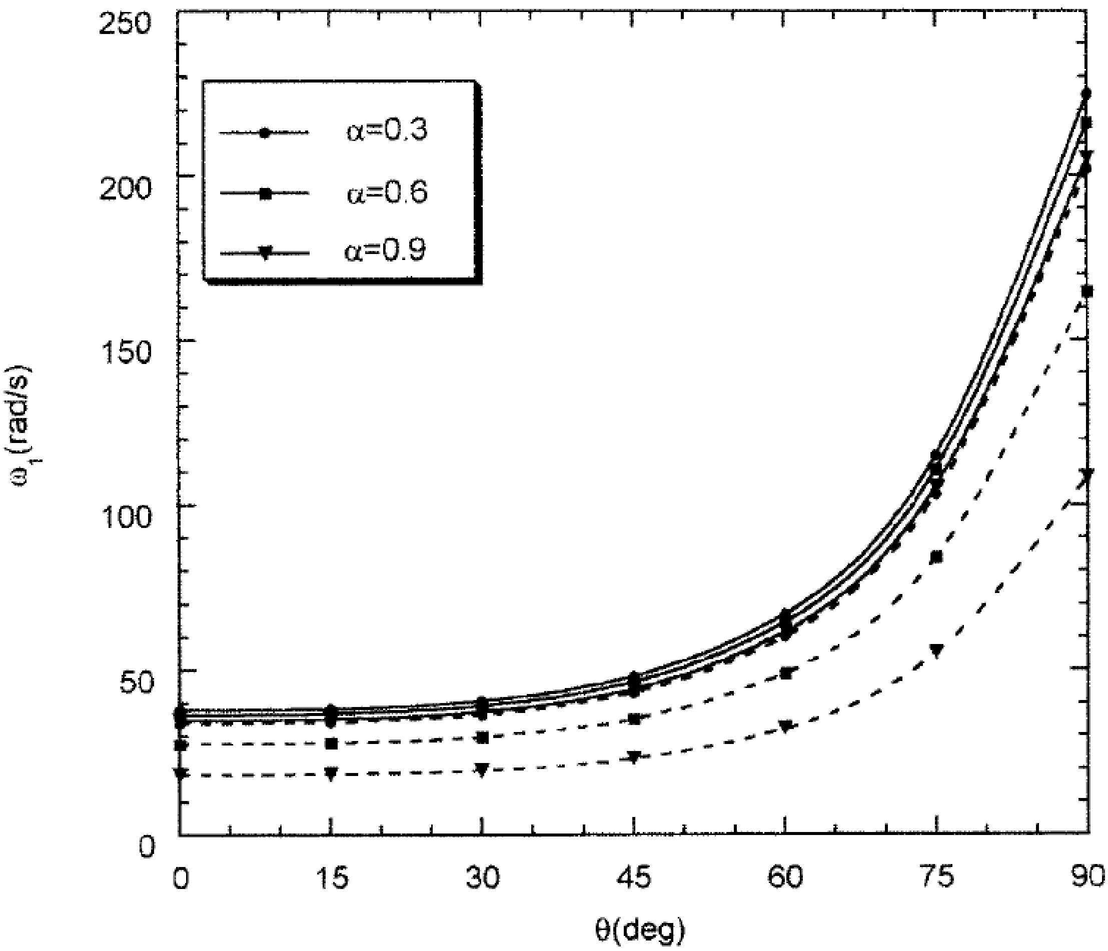

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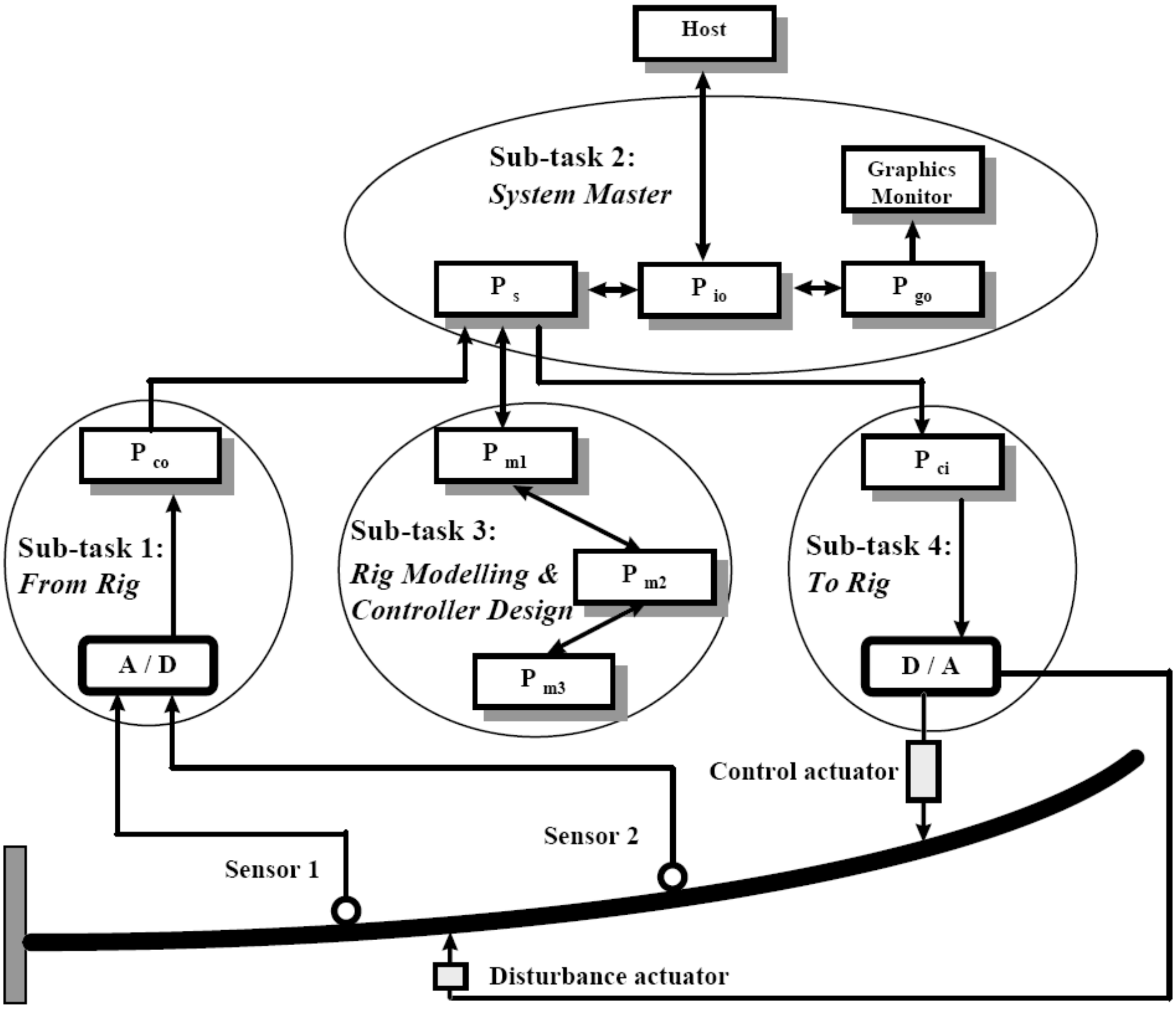

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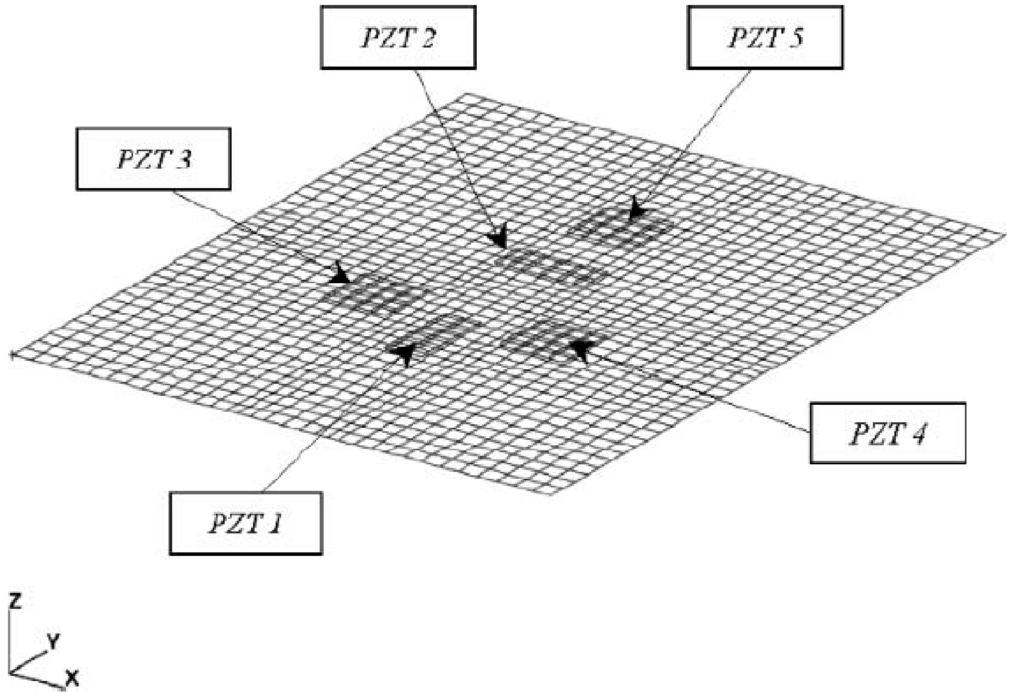

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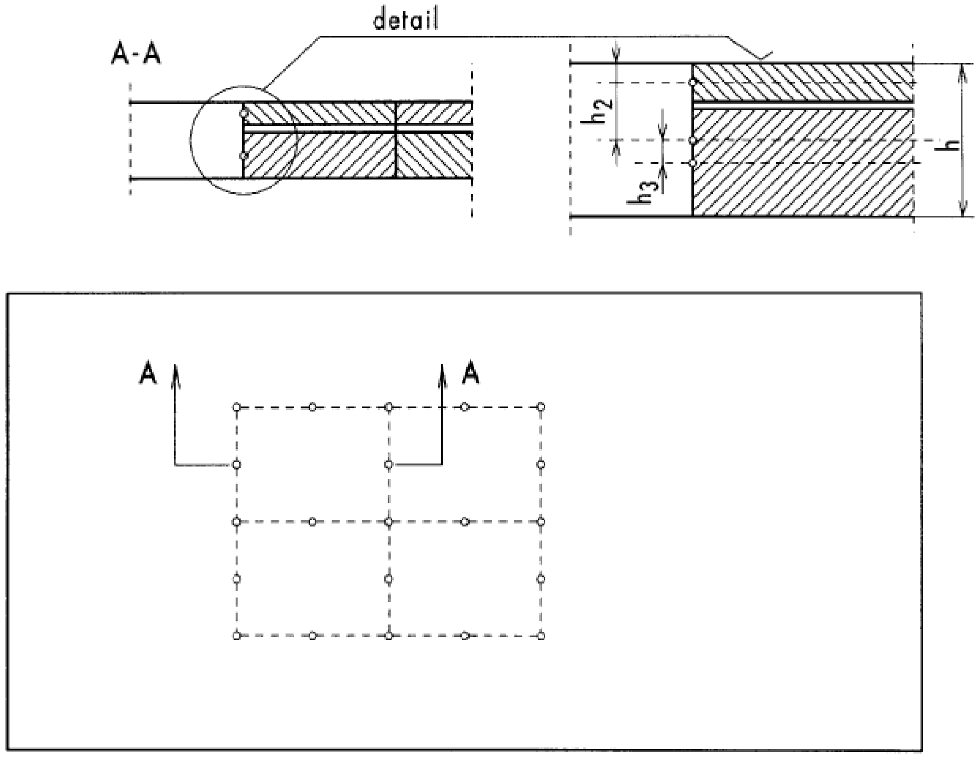

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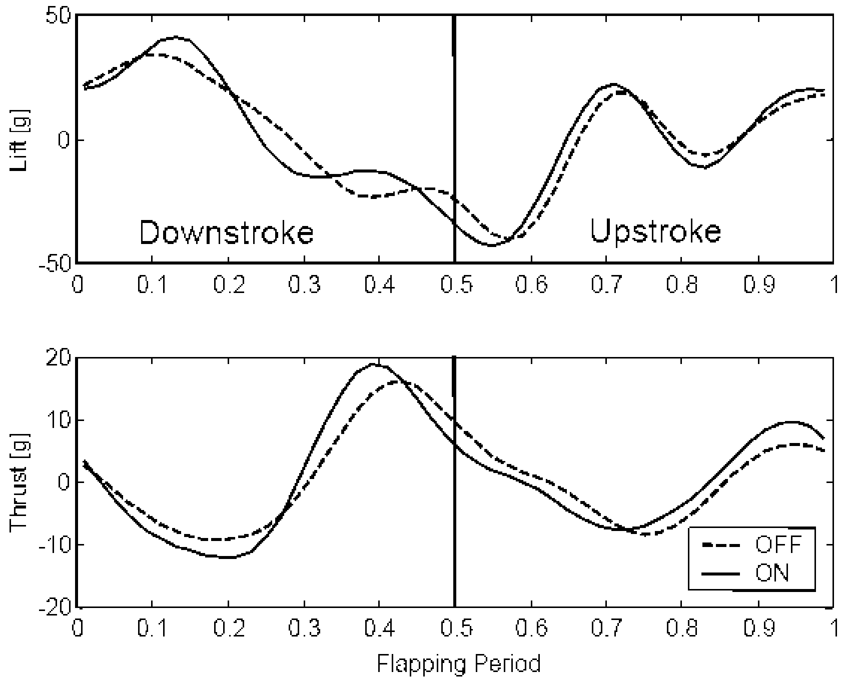

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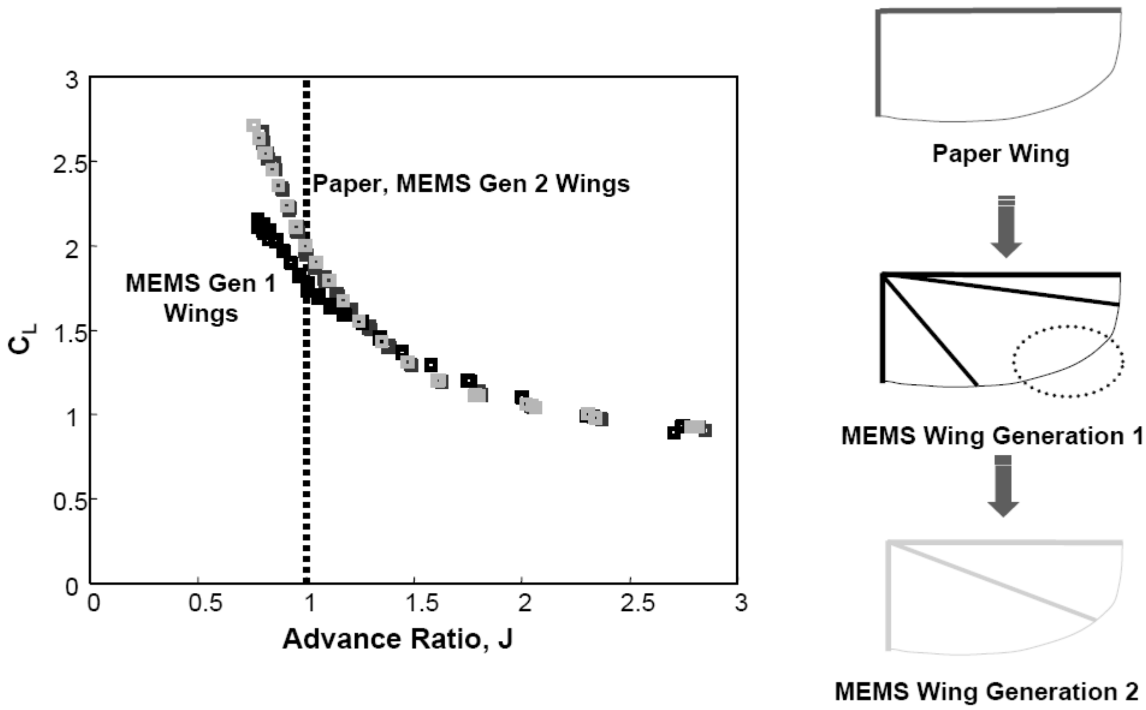

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$$\left\{\phi_{\alpha}^{c}(\tau);\phi_{M\alpha}^{c}(\tau);\phi_{q}^{c}(\tau);\phi_{Mq}^{c}(\tau)\right\}=A_0+A_1\mathrm{e}^{-\beta_1\tau}+A_2\mathrm{e}^{-\beta_2\tau}+A_3\mathrm{e}^{-\beta_3\tau}$$

Indicial function	M	A_0	A_1	A_2	A_3	β_1	β_2	β_3
$\phi_{\alpha}^{c}(\tau)$	0.5	1.155	0.723	1.413	−1.998	1.157	0.139	0.139
	0.6	1.250	0.480	3.154	−3.804	1.126	0.121	0.121
	0.7	1.400	0.328	−1.887	1.089	1.190	0.103	0.103
	0.8	1.667	−6.322	6.538	−1.095	2.111	2.049	0.082
$\phi_{M\alpha}^{c}(\tau)$	0.5	0.289	0.913	0.878	−1.979	0.184	0.184	0.184
	0.6	0.312	1.243	1.451	−2.939	0.181	0.181	0.181
	0.7	0.350	0.863	0.877	−2.049	0.154	0.154	0.154
	0.8	0.417	0.045	−0.309	−0.151	2.138	0.229	0.056
$\phi_{q}^{c}(\tau)$	0.5	0.289	0.794	0.697	−1.791	0.232	0.232	0.232
	0.6	0.312	0.612	0.468	−1.405	0.198	0.198	0.198
	0.7	0.350	0.120	−0.354	−0.115	1.046	0.322	0.075
	0.8	0.417	2.029	4.006	−6.460	0.116	0.116	0.116
$\phi_{Mq}^{c}(\tau)$	0.5	0	−0.097	0.434	−0.442	0.836	0.150	0.145
	0.6	0	0.141	0.146	−0.364	0.337	0.337	0.337
	0.7	0	−0.080	0.041	−0.034	0.397	0.182	0.079
	0.8	0	1.042	−1.086	−0.021	0.583	0.575	0.037

Table 2 Coefficients for approximating the indicial lift and moment at selected supersonic Mach numbers [48].

$$\left\{\phi_\alpha^c(\tau);\phi_{M\alpha}^c(\tau);\phi_q^c(\tau);\phi_{Mq}^c(\tau)\right\} = A_0 + A_1 e^{-\beta_1\tau} + A_2 e^{-\beta_2\tau} + A_3 e^{-\beta_3\tau}$$

Indicial function	M	A_0	A_1	A_2	A_3	β_1	β_2	β_3
$\phi_\alpha^c(\tau)$	1.25	0.849	−0.490	2.372	−2.225	1.226	0.714	0.498
	2	0.367	0.618	−0.959	0.302	0.954	0.733	0.558
	3	0.225	0.485	−0.005	−0.488	1.324	1.284	1.245
	4	0.164	0	0	0	0	0	0
$\phi_{M\alpha}^c(\tau)$	1.25	−0.424	−0.079	−0.344	0.561	1.706	0.641	0.332
	2	−0.184	−0.197	−0.200	0.411	1.205	0.952	0.890
	3	−0.112	−0.200	0.328	−0.148	2.000	1.278	1.020
	4	−0.082	0	0	0	0	0	0
$\phi_q^c(\tau)$	1.25	0.424	0.142	−0.083	−0.223	1.455	0.785	0.436
	2	0.184	0.096	−0.144	0.026	1.529	0.976	0.568
	3	0.112	0.078	−0.109	0.026	1.405	0.895	0.426
	4	0.054	0	0	0	0	0	0
$\phi_{Mq}^c(\tau)$	1.25	−0.283	−2.950	2.983	0.077	0.676	0.646	0.583
	2	−0.122	3.021	−3.036	0.036	0.312	0.310	0.210
	3	−0.075	0.102	−0.097	0.000	0.743	0.742	0.740
	4	−0.055	0	0	0	0	0	0

Flutter prediction, suppression and control in aircraft composite wings as a design prerequisite: a survey

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