

Report No. 64

March, 1953

THE COLLEGE OF AERONAUTICS

CRANFIELD

Analysis of Two-Cell Swept Box with Ribs parallel to the Line of Flight under Loading by Constant Couples ^X

- by -

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SUMMARY

The method of oblique co-ordinates⁽¹⁾ is used to analyse the problem associated with the strength and deformation of a uniform, rectangular, two-cell swept box beam having ribs parallel to the line of flight. The case of loading by constant couples is considered, but no account of root effects is taken.

The ribs are assumed to be continuously distributed, the rib boom area, together with the stringer area, being distributed over the skins. A degree of flexibility is allowed to the rib webs.

Results are presented in the form of cross sectional rotations and stress resultants.

BHF

* This investigation was made during the tenure by the author of a Clayton Fellowship awarded by the Institution of Mechanical Engineers.

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NOTATION

Oxyz	Main system of oblique Cartesian Coordinates			
OXYz	Auxiliary system of oblique coordinates			
Aij' A'ij	(i=1,2,3, j=1,2,3) Matrix inversions for rear and front skins respectively.			
A _i (i=1,2,3	3) Areas of rear, main and frontspar booms respectively.			
$\sum A = A_1 + A_2 + A_3$				
A _R , A [†] _R	Rib boom areas in rear and front cells respectively			
A _s , A's	Stringer areas in """""""			
a _R , a' _R	Rib pitch in """"""			
a _s , a's	Stringer pitch " " " " "			
Ъ	Half depth of box in direction of z axis.			
C _{ij}	Coefficients used in expression of rates of section rotation.			
с	Half width of box in direction of y axis			
°1, °2	Width of rear and front cells respectively			
Е	Young's Modulus of Elasticity			
$\left(e^{xx} = \frac{9x}{90}\right);$	$\left(e_{yy}=\frac{\partial V}{\partial y}\right);$ $\left(e_{xy}=\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\right);$ $\left(e_{xx}^{\dagger}=\frac{\partial U}{\partial x}^{\dagger}\right);$ $\left(e_{yy}=\frac{\partial V}{\partial y}^{\dagger}\right):$			
$\left(e_{xy}^{\dagger} = \frac{\partial V}{\partial x}^{\dagger} + \frac{\partial U}{\partial y}^{\dagger}\right)$; Strain components referred to axes Oxy; in rear and front cells respectively.				
$G = \frac{E}{2(1 + \sigma)}$ Shear Modulus				
K _{ij}	Coefficients used in expression of stress resultants.			
(L ₁ , M ₁)	Oblique components of couple, axes OXY.			
(p, q, r)	Oblique components of rotation about axes Oxyz.			
(p ₁ , q ₁)	Rates of oblique components of rotation			
S, S'	Shear stress resultants in rear and front cells respectively.			
S _i (i=1,2,3) Shear flows in rear, main, and front-spar webs respectively				
Т.1 ° Т'	Direct stress resultants in x direction in the rear and front cells respectively.			
T ₂ , T [†] ₂	Direct stress resultants in y direction in rear and front cells respectively.			

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/ t,

t, t'	Skin thicknesses of rear and front cells respectively
t _i (i=1,2,3)	Thickness of rear, main and front spar webs respectively
U, U'	Displacements of rear and front cell skins in x direction
u _i (i=1,2,3)	Displacements of rear, main and front spar webs in x direction
V, V'	Displacements of rear and front cell skins in y direction
W _i (i=1,2,3)	Displacements of rear, main and front spar webs in z direction
W _{RO}	Displacements of rib web on y axis in z direction
(X, Y, Z)	Components of force, axes Oxyz
a	Angle between Ox and Oy axes
Δ , Δ'	Distortion of section in rear and front cells respectively
Δ_1, Δ_1	Constants in equations for distortion of section
ε)	
×	Terms used in expression of coefficients
μ	C _{ij} , K _{ij}
×)	
σ	Poisson's Ratio
ພ ູ ພ້	Warping of section in rear and front cells respectively
ω_{1}, ω'_{2}	Constants in equations for warping of section

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1. INTRODUCTION

The method used in this analysis is essentially that developed by Hemp in Part 3 of his work⁽¹⁾ on the application of oblique coordinates to swept wing structures. Fig. 1 shows the construction of the box and the notation used.

The sweep back angle is $(\pi/2 - \infty)$, and the box is defined by a set of oblique axes 0xyz. An auxiliary set of oblique axes 0XYz are also used. The upper and lower surfaces are given by $z = \pm$ b respectively, and they are assumed to be reinforced by closely spaced stringers parallel to the x axis, and closely spaced ribs parallel to the y axis. The skin thickness in the rear cell is t, the rib boom area A_R , rib pitch a_R , stringer area A_S , and stringer pitch a_S . The comparable dimensions in the front cell are t', A_R^* , a_R^* , A_S^* and a_S^* respectively. The rear spar web is defined by y = c and has thickness t_1 , the mainspar web by $y = (c - c_1)$ and has thickness t_2 , whilst the front spar is given by y = -c, and has thickness t_3 . The areas of the rear, main, and front spar booms are A_1 , A_2 and A_3 respectively.

Where the spar and rib webs are capable of carrying end loads, their effective area is considered to be included in the appropriate boom area, the webs themselves being assumed to carry only shear loads. All the materials have a Young's Modulus of E, and Poisson Ratio σ . The rib webs are considered to be rigidly connected to the spar webs, but are allowed a limited flexibility in themselves.

The effect of root constraint is not investigated, and the box is considered to be loaded by constant couples.

2. THEORY

Assume a linear variation of the rotation of the box with \mathbf{x} .

Rotation component about x axis $p = p_1 x$ Rotation component about y axis $q = q_1 x$ (1)

/ The warping

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The warping and distortion of a cross section of the box are assumed to be linear in y

Warping:- Rear Cell:-
$$\omega = \omega_1 y + \omega_2$$

Front Cell:- $\omega' = \omega'_1 y + \omega'_2$
(2)

Similarly distortion:-

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$$\Delta = \Delta_1 y + \Delta_2 \qquad \dots \qquad (3)$$
$$\Delta = \Delta_1' y + \Delta_2'$$

Using Eqs (1) to (3), the displacements become:-

For the Skins:-
$$U = q_1 x b \sin \alpha + \omega(y)$$

 $U' = q_1 x b \sin \alpha + \omega'(y)$
... (4)

$$V = -p_1 x b \sin \alpha + \Delta(y)$$

$$V' = -p_1 x b \sin \alpha + \Delta'(y)$$
(5)

For the Spar Webs:- $u_1 = q_1 x \cdot z \sin \alpha + (\omega)_{y=c} \cdot \frac{z}{b}$ $u_2 = q_1 x \cdot z \sin \alpha + (\omega)_{y=(c-c_1)} \cdot \frac{z}{b}$... (6) $u_3 = q_1 x \cdot z \sin \alpha + (\omega')_{y=-c} \cdot \frac{z}{b}$

The displacement in the z direction, of the rib webs, on the centreline of the box is given by Ref. 1 Eqs. (94) and (98) as:-

$$(w_{R})_{y=0} = -e_{xx} \cdot \frac{x^{2}}{2b}$$
Hence:-
$$w_{1} = p_{1}x c \sin \alpha - e_{xx} \cdot \frac{x^{2}}{2b}$$

$$w_{2} = p_{1}x(c-c_{1})\sin \alpha - e_{xx} \cdot \frac{x^{2}}{2b}$$

$$w_{3} = -p_{1}x c \sin \alpha - e_{xx} \cdot \frac{x^{2}}{2b}$$
.... (7)

/ Eqs.

Eqs. (4) to (7) are used to obtain the strains in the skins.

$$e_{xx} = \frac{\partial U}{\partial x} = q_{1}b \sin \alpha$$

$$e'_{xx} = q_{1}b \sin \alpha$$

$$e'_{xx} = q_{1}b \sin \alpha$$

$$e'_{xx} = q_{1}b \sin \alpha$$

$$e'_{yy} = \Delta'_{1}$$

$$e'_{yy} = \Delta'_{1}$$

$$e'_{xy} = -p_{1}b \sin \alpha + \omega_{1}$$

$$e'_{xy} = -p_{1}b \sin \alpha + \omega_{1}$$

$$e'_{xy} = -p_{1}b \sin \alpha + \omega_{1}$$

The box is loaded by constant couples

••• X = Y = Z = 0

The Stress Resultants are restricted.

 T_1 , and T_1' are functions of y only T_2 and T_2' are zero S and S' are constant

• • $e_{xx} = A_{11}T_1 + A_{13}S$ $e_{yy} = A_{21}T_1 + A_{23}S$ $e_{xy} = A_{31}T_1 + A_{33}S$ $e_{xy} = A_{31}T_1 + A_{33}S$ $e_{xy} = A_{31}T_1 + A_{33}S$

Compatibility of warping at the mainspar requires:-

 $\omega_1(c - c_1) + \omega_2 = \omega_1(c - c_1) + \omega_2^{\dagger}$

Using Eqs. (8) and (9):-

$$\begin{array}{c} q_{1}b \ \sin\alpha = A_{11}T_{1} + A_{13}S = A_{11}T_{1} + A_{13}S' \\ A_{21}T_{1} + A_{23}S - A_{1} = A_{21}T_{1} + A_{23}S' - A_{1} = 0 \end{array} \right\} (11)$$

$$-p_{1}b \ \sin\alpha = A_{31}T_{1} + A_{33}S - \omega_{1} = A_{31}T_{1} + A_{33}S' - \omega_{1} \end{array}$$

/ Using

Using the stress-strain relation for the spar webs:-

$$S_{1} = Gt_{1} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$S_{1} = Gt_{1} \left\{ p_{1}c \sin \alpha + \frac{(\omega_{1}c + \omega_{2})}{b} \right\}$$

$$S_{2} = Gt_{2} \left\{ p_{1}(c-c_{1})\sin \alpha + \frac{[\omega_{1}(c-c_{1}) + \omega_{2}]}{b} \right\}$$

$$\cdots (12)$$

$$S_{3} = Gt_{3} \left\{ -p_{1}c \sin \alpha + \frac{(-\omega_{1}c + \omega_{2})}{b} \right\}$$

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Equilibrium of the spar flange joints requires that:-

$$s_1 + s = 0$$

 $s_3 - s' = 0$
 $s - s_2 - s' = 0$
(13)

Eq. (13) implies that: $S_1 + S_2 + S_3 = \frac{Z}{2b} = 0$

For overall equilibrium:-

$$L_1 = 2bc S_1 - 2bc S_3 + 2b(c-c_1)S_2 - 2b Sc_1 + S'c_2$$

and using Eq. (13):-

$$\frac{L_{1}}{4b} = -c_{1}S - c_{2}S'$$
or:- $S' = -S \frac{c_{1}}{c_{2}} - \frac{L_{1}}{4bc_{2}}$

$$S = -S' \frac{c_{2}}{c_{1}} - \frac{L_{1}}{4bc_{1}}$$
.... (14)

 $M_{1} = 2bE(\Sigma A)(q_{1}b \sin a) + 2b(T_{1}c_{1} + T_{1}c_{2}) \qquad \dots \qquad (15)$

where $\Sigma A = A_1 + A_2 + A_3$

/ Substituting

Substituting from Eq. (13) into Eq. (12):-

$$\frac{S'}{Gt_3c} = -p_1 \sin\alpha + \frac{(-\omega_1' c + \omega_2')}{bc}$$

$$\frac{(S - S')}{Gt_2(c-c_1)} = p_1 \sin\alpha + \frac{[\omega_1(c-c_1) + \omega_2]}{b(c-c_1)}$$
... (16)
$$\frac{-S}{Gt_1c} = p_1 \sin\alpha + \frac{(\omega_1c + \omega_2)}{bc}$$

Eqs. (16) and (10) give ω_2 and ω_2'

$$\omega_{2} = \frac{bc(c-c_{1})}{Gc_{1}} \left[\frac{(s-s')}{t_{2}(c-c_{1})} + \frac{s}{t_{1}c} \right]$$

$$\omega_{2}^{*} = \frac{bc(c-c_{1})}{Gc_{2}} \left[\frac{(s-s')}{t_{2}(c-c_{1})} + \frac{s'}{t_{3}c} \right]$$
(17)

Using Eqs. (11) and (16):-

 $\frac{Sb}{Gt_{1}c} = A_{31}T_{1} + A_{33}S - 2\omega_{1} - \frac{\omega_{2}}{c}$ and $\frac{S'b}{Gt_{3}c} = A_{31}T_{1} + A_{33}S' - 2\omega_{1} + \frac{\omega_{2}}{c}$... (18)

Elimination of ω_1 and ω_1' from Eq. (18) by using Eq. (10) yields:-

$$\frac{b}{Gc}\left[\frac{s'}{t_3} - \frac{s}{t_1}\right] = A'_{31}T'_1 + A'_{33}s' - A_{31}T_1 - A_{33}s + \omega_2 \left[\frac{(c-c_1)+2c}{c(c-c_1)}\right] + \omega_2 \left[\frac{(c-c_1)-2c}{c(c-c_1)}\right]$$
(19)

Substitution from the first pair of Eqs. (11) into Eq. (15) and elimination between the resulting two equations gives:-

$$T_{1} = \left[c_{2}(A_{13}^{*}S' - A_{13}^{*}S) + A_{11}^{*}(\frac{M}{2D} - EA_{13}^{*}\Sigma A \cdot S) \right] / A_{11}^{*} \left[EA_{11}^{*}\Sigma A + c_{1}^{*}\right] + c_{2}^{A_{11}} \right]$$

$$T_{1}^{*} = \left[c_{1}(A_{13}^{*}S - A_{13}^{*}S') + A_{11}(\frac{M}{2D} - EA_{13}^{*}\Sigma A \cdot S') \right] / A_{11}^{*} \left[EA_{11}^{*}\Sigma A + c_{1}^{*}\right] + c_{2}^{A_{11}} \right]$$
(20)

/ Using

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Using Eqs. (17) and (19) and rearranging:-

$$A'_{31}T'_{1} - A_{31}T_{1} = S \left[A_{33} + \frac{2bc}{Gt_{2} \cdot c_{1}c_{2}} + \frac{b}{Gt_{1}c_{1}} \right] - S' \left[A'_{33} + \frac{2bc}{Gt_{2}c_{1}c_{2}} + \frac{b}{Gt_{3}c_{2}} \right]$$
.... (21)

$$\left[\frac{A_{31}^{i}A_{31}c_{1} + A_{31}^{2}(c_{2} + A_{11}^{i} \mathbb{E}\Sigma A)}{A_{11}^{i} \mathbb{E}A_{11}\Sigma^{i}A + c_{1}^{i} + A_{11}c_{2}^{i}} - \left[A_{33}^{i} + \frac{2bc}{Gt_{2}c_{1}c_{2}} + \frac{b}{Gt_{1}c_{1}} \right] \right] s$$

$$= \left[\frac{A_{31}^{i}A_{31}c_{2}^{i} + A_{31}^{i}(c_{1} + A_{11}\mathbb{E}\Sigma A)}{A_{11}^{i} \mathbb{E}A_{11}\Sigma^{i}A + c_{1}^{i} + A_{11}c_{2}} \right] - \left\{ A_{33}^{i} + \frac{2bc}{Gt_{2}c_{1}c_{2}} + \frac{b}{Gt_{3}c_{2}} \right] s'$$

$$+ \left\{ \frac{A_{31}A_{11}^{i} - A_{31}^{i}A_{11}}{A_{11}^{i} \mathbb{E}A_{11}\Sigma^{i}A + c_{1}^{i} + A_{11}c_{2}} \right\} = \frac{M_{1}}{2b}$$

$$\cdots (22)$$

Eqs. (22) and (14) enable S, S' to be found and thence T_1 , T_1' from Eq. (20)

 ω_1^* and ω_1 follow from Eqs. (17) and (18):-

From Eqs. (23) and (11):-

$$P_{1} = \frac{dp}{dx} = -\frac{cosec\alpha}{2b} \left[A_{31}T_{1} + S \left[A_{33} + \frac{b}{Gc_{1}} \left[\frac{1}{t_{2}} + \frac{1}{t_{1}} \right] \right] - \frac{bS'}{Gt_{2}c_{1}} \right] \left[q_{1} = \frac{dq}{dx} = \frac{cosec\alpha}{b} \left[A_{11}T_{1} + A_{13}S \right]$$
(24)

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The displacements can be found using Eqs. (24), (23) and (17). The strains follow from Eq. (9)

3. <u>RESULTS</u>

$$T_{1} = K_{11} L_{1} + K_{12} M_{1}$$

$$T_{1} = K_{11}^{\prime} L_{1} + K_{12}^{\prime} M_{1}$$

$$S = K_{21} L_{1} + K_{22} M_{1}$$

$$S_{1} = K_{31} L_{1} + K_{32} M_{1}$$

$$S_{2} = K_{41} L_{1} + K_{42} M_{1}$$

$$S_{3} = K_{51} L_{1} + K_{52} M_{1}$$

$$P_{1} = \frac{dp}{dx} = C_{11}L_{1} + C_{12}M_{1}$$

$$q_{1} = \frac{dq}{dx} = C_{21}L_{1} + C_{22}M_{1}$$

$$(25)$$

Where:-

$$K_{11} = \left[c_{2}(A_{13}\mu - A_{13}^{*}\lambda) + A_{13}A_{11}^{*}\mu = \sum A_{1}^{*}/4b e^{\lambda} + A_{11}^{*} + A_{13}^{*}A_{11}^{*}\mu = \sum A_{13}^{*}/4b e^{\lambda} + A_{13}^{*}A_{11}^{*}\mu = \sum A_{13}^{*}c_{13}^{*} + A_{13}^{*}c_{2}^{*} + A_{13}^$$

$$C_{11} = -\frac{\cos e c \alpha}{8b^{2} e} \left[\frac{A_{31}}{\lambda} \left[c_{2} (A_{13} \mu - A_{13}^{\dagger} \lambda) + A_{13} A_{11}^{\dagger} \mu E \cdot \Sigma A \right] - \mu A_{33} - \frac{b}{Gc_{1}} \left\{ \frac{\mu}{t_{1}} + \frac{\mu - \lambda}{t_{2}} \right\} \right]$$

$$C_{12} = C_{21} = -\frac{\cos e c \alpha}{4b^{2} \lambda} \left[A_{13} A_{11}^{\dagger} - \frac{\lambda \nu^{2}}{e} \right]$$

$$C_{22} = \frac{\cos e c \alpha}{2b^{2} \lambda} \left[A_{11} A_{11}^{\dagger} + \frac{c_{1} \chi^{2}}{\kappa e} \right]$$

$$(27)$$

/ K ==

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$$\begin{aligned} & \mathcal{K} = A_{11}^{i} \left\{ \mathbb{E}A_{11} \Sigma A + c_{1} \right\} + A_{11}c_{2} \\ & \mathcal{K} = A_{31} A_{11}^{i} - A_{31}^{i} A_{11} \\ & \mu = \left[A_{31}A_{31}^{i}c_{2} + A_{31}^{i2}(c_{1} + A_{11}\mathbb{E}\cdot\Sigma A) - A_{33}^{i} - \frac{b(c_{1}+c_{2})}{Gt_{2}c_{1}c_{2}} - \frac{b}{Gt_{3}c_{2}} \right] \\ & \lambda = \left[A_{31}A_{31}^{i}c_{1} + A_{31}^{2}(c_{2} + A_{11}^{i}\mathbb{E}\cdot\Sigma A) - A_{33} - \frac{b(c_{1}+c_{2})}{Gt_{2}c_{1}c_{2}} - \frac{b}{Gt_{1}c_{1}} \right] \\ & \varepsilon = \lambda c_{2} + \mu c_{1} \end{aligned}$$

$$(28)$$

+• SPECIAL CASE

Two Equal Cells

 $t_1 = t_2 = t_3 = t_w$ $c_1 = c_2 = c$ t = t'

(Equal cells with constant web and skin thicknesses)

$$T_{1} = T_{1}' = \left(\frac{A_{13}E \cdot \sum A L_{1}}{4c} + M_{1}\right) / 2b(2c + EA_{11} \cdot \sum A)$$

$$S = S' = -\frac{L_{1}}{8bc}$$

$$S_{1} = -S_{3} = \frac{L_{1}}{8bc}$$

$$S_{2} = 0$$

.... (29)

$$K_{11} = \frac{1}{8bc} \cdot \frac{A_{13}E \cdot 2A}{(2c+A_{11}E \cdot 2A)} \qquad K_{12} = \frac{1}{2b(2c+A_{11}E \cdot 2A)} \qquad K_{21} = -\frac{L_{1}}{8bc} \qquad K_{22} = 0 \qquad K_{21} = -\frac{1}{8bc} \qquad K_{21} = -\frac{1}{8bc} \qquad (30)$$

$$K_{31} = \frac{1}{8bc} \qquad K_{41} = 0 \qquad K_{51} = -\frac{1}{8bc} \qquad (30)$$

$$K_{32} = K_{42} = K_{52} = 0$$

/ C₁₁

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$$C_{11} = \frac{\cos e c \alpha}{8 b c} \left[\frac{(1+\sigma)}{E t_{w} c} + \frac{A_{33}}{2 b} - \frac{A_{13}^{2} E \sum A}{2 b (2 c + A_{11} E \cdot \sum A)} \right]$$

$$C_{12} = C_{21} = -\frac{\cos e c \alpha}{4 b^{2} (2 c + A_{11} E \cdot \sum A)}$$

$$C_{22} = \frac{\cos e c \alpha}{2 b^{2} (2 c + A_{11} E \cdot \sum A)}$$

(31)

 $\lambda = \mu$ $\varepsilon = 2\lambda c = 2\mu c$ $\lambda = 2A_{11}c + A_{11}^2 E \cdot \sum A$

5. DISCUSSION

It can be seen from Eqs (26), that there is a contribution to the oblique shear stress from the "bending couple" M_1 . This contribution is dependent upon the value of γ , and it can be shown that > itself is dependent upon the relative values of A_R , a_R , and t in the two cells.

For the case of constant rib pitch, with the ratio of the rib boom area to skin thickness the same in both cells, i.e. $\frac{A_R}{a_R t} = \frac{A_R'}{a_R' t'}$, the value of x is zero. Under these conditions, there is no oblique shear stress due to M₁.

The results for the special case of two equal cells with constant web and skin thicknesses given in Eqs (29) to (32), are directly comparable to the single cell results, Eqs (78), (83) and (100) of Ref. 1.

Complete analysis of the box beam subjected to constant couples is achieved by using the above results in conjunction with the relevant parts of Ref. 1 $\frac{5}{3}$ 3.2.

/ References ...

... (32)

REFERENCES

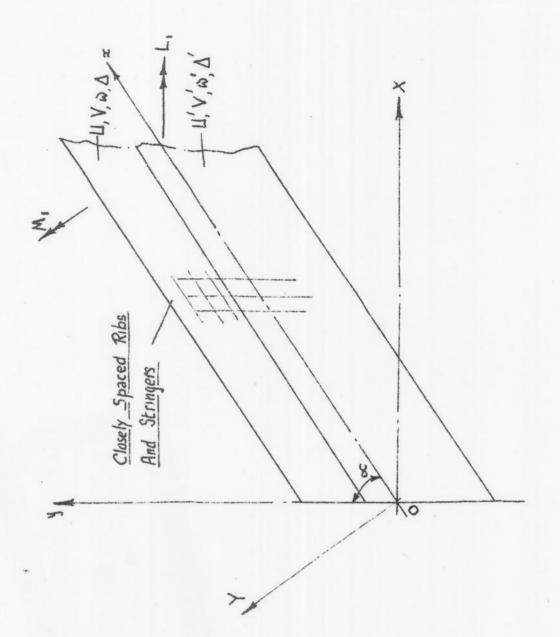
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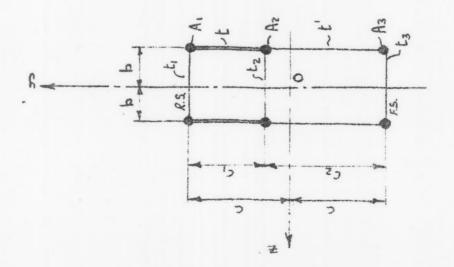
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RIDS Parallel toline OF Flight

F.9.1