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Analysis of Two-Cell Swept Box with Ribs
    parallel to the Line of Flight under
    Loading by Constant Couples \({ }^{\approx}\)
                                    - by -
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## SUMMARY

The method of oblique co-ordinates ${ }^{(1)}$ is used to analyse the problem associated with the strength and deformation of a uniform, rectangular, two-cell swept box beam having ribs parallel to the line of flight. The case of loading by constant couples is considered, but no account of root effects is taken.

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The ribs are assumed to be continuously distributed, the rib boom area, together with the stringer area, being distributed over the skins. A degree of flexibility is allswed to the rib webs.
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Results are presented in the form of cross sectional rotations and stress resultants.

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## NOTATION

Oxyz Main system of oblique Cartesian Coordinates
OXYz Auxiliary system of oblique coordinates
$A_{i j}$ ．$A_{i j}^{\prime} \quad(i=1,2,3, j=1,2,3)$ Matrix inversions for rear and front skins respectively．
$A_{i}(i=1,2,3)$ Areas of rear，main and frontspar booms respectively．
$\sum A=A_{1}+A_{2}+A_{3}$
$A_{R}, A_{R}^{p} \quad$ Rib boom areas in rear and front cells respectively
$A_{s}, A_{s}^{\prime}$ Stringer areas in＂＂＂＂
$a_{R}, a_{R}^{\prime}$ Rib pitch in＂＂＂＂＂
$a_{s}$ ，$a_{s}^{\prime}$ Stringer pitch＂＂＂＂＂
b Half depth of box in direction of $z$ axis．
$C_{i j} \quad$ Coefficients used in expression of rates of section rotation．
c Half width of box in direction of $y$ axis
$c_{1}, c_{2} \quad$ Width of rear and front cells respectively
E Young＇s Modulus of Elasticity
$\left(e_{x x}=\frac{\partial U}{\partial x}\right) ; \quad\left(e_{y y}=\frac{\partial V}{\partial y}\right) ; \quad\left(e_{x y}=\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\right) ; \quad\left(e_{x x}^{\prime}=\frac{\partial U^{\prime}}{\partial x}\right) ; \quad\left(e_{y y}=\frac{\partial V^{\prime}}{\partial y}\right):$
$\left(e_{X y}^{\prime}=\frac{\partial V^{\prime}}{\partial \mathrm{X}}+\frac{\partial U^{\prime}}{\partial y}\right) ;$ Strain components referred to axes oxy；in rear and front cell．s respectively．
$G=\frac{E}{2(1+\sigma)}$ Shear Modulus
$\mathrm{K}_{i j} \quad$ Coefficients used in expression of stress resultants。
（ $L_{1}, M_{1}$ ）Oblique components of couple，axes OXY。
（ $p, q, r$ ）Oblique components of rotation about axes Oxyz．
$\left(p_{1}, q_{1}\right) \quad$ Rates of oblique components of rotation
S，S＇Shear stress resultants in rear and front cells respectively．
$S_{i}(i=1,2,3)$ Shear flows in rear，main，and front－spar webs respectively
$T_{1}$ 。 $T^{\prime} \quad$ Direct stress resultants in x direction in the rear and front cells respectively．
$\mathrm{T}_{2}, \mathrm{~T}_{2}^{\prime} \quad$ Direct stress resultants in y direction in rear and front cells respectively．


## 1. INTRODUCTION

The method used in this analysis is essentially that developed by Hemp in Part 3 of his work (1) on the application of oblique coordinates to swept wing structures. Fig. 1 shows the construction of the box and the notation used.

The sweep back angle is ( $\pi / 2-\alpha$ ), and the box is defined by a set of oblique axes oxyz. An auxiliary set of oblique axes OXYz are also used. The upper and lower surfaces are given by $z= \pm b$ respectively, and they are assumed to be reinforced by closely spaced stringers parallel to the $x$ axis, and closely spaced ribs parallel to the $y$ axis. The skin thickness in the rear cell is $t$, the rib boom area $A_{R}$, rib pitch $a_{R}$, stringer area $A_{S}$, and stringer pitch $a_{S}$. The comparable dimensions in the front cell are t', $A_{R}^{\prime}$, $a_{R}^{\prime}, A_{S}^{\prime}$ and $a_{s}^{\prime}$ respectively. The rear spar web is defined by $y=c$ and has thickness $t_{1}$, the mainspar web by $y=\left(c-c_{1}\right)$ and has thickness $t_{2}$, whilst the front spar is given by $y=-c$, and has thickness $t_{3}$. The areas of the rear, main, and front spar booms are $A_{1}, A_{2}$ and $A_{3}$ respectively.

Where the spar and rib webs are capable of carrying end loads, their effective area is considered to be included in the appropriate boom area, the webs themselves being assumea to carry only shear loads. All the materials have a Young's Modulus of $E$, and Poisson Ratio $\sigma$. The rib webs are considered to be rigidly connected to the spar webs, but are allowed a limited flexibility in themselves.

The effect of root constraint is not investigated, and the box is considered to be loaded by constant couples.

## 2. THEORY

Assume a linear variation of the rotation of the box with $x$.
$\left.\begin{array}{ll}\text { Rotation component about } x \text { axis } & p=p_{1} x \\ \text { Rotation component about } y \text { axis } & q=q_{1} x\end{array}\right\}$

The warping and distortion of a cross section of the box are assumed to be linear in y

Warping:- $\left.\begin{array}{ll}\text { Rear Cell:- } \quad \omega=\omega_{1} y+\omega_{2} \\ & \text { Front Cell:- } \quad \omega^{\prime}=\omega_{1}^{\prime} y+\omega_{2}^{\prime}\end{array}\right\}$

Similarly distortion:-

$$
\left.\begin{array}{rl}
\Delta & =\Delta_{1} y+\Delta_{2}  \tag{3}\\
\Delta & =\Delta_{1} y+\Delta_{2}^{\prime}
\end{array}\right\}
$$

Using Eqs (1) to (3), the displacements become:-

$$
\text { For the Skins:- } \begin{align*}
U & =q_{1} \times b \sin a+\omega(y) \\
U^{\prime} & =q_{1} \times b \sin a+\omega^{\prime}(y)  \tag{4}\\
V & =-p_{1} \times b \sin \alpha+\Delta(y)  \tag{5}\\
V^{\prime} & =-p_{1} \times b \sin \alpha+\Delta^{\prime}(y)
\end{align*}
$$

For the Spar Webs:- $u_{1}=q_{1} x_{0} z \sin \alpha+(\omega)_{y=c} \cdot \frac{z}{b}$

$$
\begin{align*}
& u_{2}=q_{1} x \cdot z \sin a+(\omega)_{y=\left(c-c_{1}\right)^{\prime}} \cdot \frac{z}{b}  \tag{6}\\
& u_{3}=q_{1} x \cdot z \sin a+\left(\omega^{\prime}\right)_{y=-c^{\circ}} \frac{z}{b}
\end{align*}
$$

The displacement in the $z$ direction, of the rib webs, on the centreline of the box is given by Ref. 1 Eqs. (94) and (98) as:-

$$
\left(w_{R}\right)_{y=0}=-e_{x x} \cdot \frac{x^{2}}{2 b}
$$

Hence:- $\quad w_{1}=p_{1} x c \sin a-e_{x x} \cdot \frac{x^{2}}{2 b}$

$$
\left.\begin{array}{l}
w_{2}=p_{1} x\left(c-c_{1}\right) \sin a-e_{x x} \cdot \frac{x^{2}}{2 b}  \tag{7}\\
w_{3}=-p_{1} x c \sin a-e_{x x} \cdot \frac{x^{2}}{2 b}
\end{array}\right\}
$$

Eqs. (4) to (7) are used to obtain the strains in
the skins.

$$
\begin{array}{ll}
e_{x x}=\frac{\partial U}{\partial x}=q_{1} b \sin \alpha & e_{z x}^{\prime}=q_{1} b \sin \alpha \\
e_{y y}=\frac{\partial V}{\partial y}=A_{1} & e_{y y}^{\prime}=\Delta_{1}^{\prime}  \tag{8}\\
e_{x y}=\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}=-p_{1} b \sin \alpha+\omega_{1} & e_{x y}^{\prime}=-p_{1} b \sin a+\omega_{1}^{\prime}
\end{array}
$$

The box is loaded by constant couples

$$
\therefore X=Y=Z=0
$$

The Stress Resultants are restricted.

$$
\begin{aligned}
& T_{1} \text { and } T_{1}^{\prime} \text { are functions of } y \text { only } \\
& T_{2} \text { and } T_{2}^{\prime} \text { are zero } \\
& S \text { and } S^{\prime} \text { are constant }
\end{aligned}
$$

$$
\begin{array}{rlrl}
\therefore e_{x x} & =A_{11} T_{1}+A_{13} S & e_{x x}^{\prime}=A_{11}^{\prime} T_{1}^{\prime}+A_{13}^{\prime} S^{\prime} \\
e_{y y} & =A_{21} T_{1}+A_{23^{S}} S & e_{y y}^{\prime}=A_{21}^{\prime} T_{1}^{\prime}+A_{23^{\prime}}^{\prime} S^{\prime} \\
e_{x y} & =A_{31} T_{1}+A_{33^{S}} S & & e_{x y}^{\prime}=A_{31}^{\prime} T_{1}^{\prime}+A_{33^{\prime}}^{\prime} S^{\prime}
\end{array}
$$

Compatibility of warping at the mainspar requires:-

$$
\omega_{1}\left(c-c_{1}\right)+\omega_{2}=\omega \dot{1}\left(c-c_{1}\right)+\omega_{2}
$$

or rewriting:-

$$
\begin{equation*}
\omega_{1}=\omega_{1}^{\prime}+\frac{\left(\omega_{2}^{1}-\omega_{2}\right)}{\left(c-c_{1}\right)} \tag{10}
\end{equation*}
$$

Using Eqs. (8) and (9):-

$$
\left.\begin{array}{rl}
q_{1} b \sin \alpha= & A_{11} T_{1}+A_{13} S=A_{11}^{\prime} T_{1}^{\prime}+A_{13}^{\prime} S^{\prime} \\
& A_{21} T_{1}+A_{23^{\prime}}-A_{1}=A_{21}^{\prime} T_{1}^{\prime}+A_{23}^{\prime} S^{\prime}-A_{1}^{\prime}=0  \tag{11}\\
-p_{1} b \sin \alpha= & A_{31} T_{1}+A_{33^{S}}-\omega_{1}=A_{31}^{\prime} T_{1}^{\prime}+A_{33^{\prime}} S^{\prime}-\omega i
\end{array}\right\}
$$

Using the stress-strain relation for the spar webs:-

$$
\begin{align*}
& S_{i}=G t_{i}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \\
& S_{1}=G t_{1}\left\{p_{1} c \sin \alpha+\frac{\left(\omega_{1} c+\omega_{2}\right)}{b}\right\} \\
& S_{2}=G t_{2}\left\{p_{1}\left(c-c_{1}\right) \sin \alpha+\frac{\mid \omega_{1}\left(c-c_{1}\right)+\omega_{2}}{b}\right\}  \tag{12}\\
& S_{3}=G t_{3}\left\{-p_{1} c \sin \alpha+\frac{\left(-\omega_{1}^{\prime} c+\omega_{2}^{\prime}\right)}{b}\right\}
\end{align*}
$$

Equilibrium of the spar flange joints requires that:-

$$
\begin{array}{ll}
s_{1}+s & =0  \tag{13}\\
s_{3}-s^{\prime} & =0 \\
s-s_{2}-s^{\prime} & =0
\end{array}
$$



Eq. (13) implies that: $-S_{1}+S_{2}+S_{3}=\frac{Z}{2 b}=0$

For overall equilibrium:-

$$
I_{1}=2 b c s_{1}-2 b c s_{3}+2 b\left(c-c_{1}\right) s_{2}-2 b-s c_{1}+s^{\prime} c_{2}
$$

and using Eq. (13):-

$$
\frac{I_{1}}{4 b}=-c_{1} s-c_{2} s^{\prime}
$$

or:- $S^{\prime}=-S \frac{c_{1}}{c_{2}}-\frac{I_{1}}{4 b c_{2}}$

$$
\begin{equation*}
s=-s^{\prime} \frac{c_{2}}{c_{1}}-\frac{L_{1}}{4 b c_{1}} \tag{14}
\end{equation*}
$$

$M_{1}=2 b E\left(\sum A\right)\left(q_{1} b \sin a\right)+2 b\left(T_{1} c_{1}+T_{1} c_{2}\right)$

$$
\text { where } \Sigma A=A_{1}+A_{2}+A_{3}
$$

Substituting from Eq. (13) into Eq. (12):-

$$
\begin{aligned}
& \frac{S^{\prime}}{G t_{3} c}=-p_{1} \sin \alpha+\frac{\left(-\omega_{1}^{\prime} c+\omega_{2}^{\prime}\right)}{b c} \\
& \frac{\left(S-S^{\prime}\right)}{G t_{2}\left(c-c_{1}\right)}=p_{1} \sin \alpha+\frac{\left[\omega_{1}\left(c-c_{1}\right)+\omega_{2}\right]}{b\left(c-c_{1}\right)} \\
& \frac{-S}{G t_{1} c}=p_{1} \sin \alpha+\frac{\left(\omega_{1} c+\omega_{2}\right)}{b c}
\end{aligned}
$$

Ens. (16) and (10) give $\omega_{2}$ and $\omega_{2}^{\prime}$

$$
\begin{align*}
& \omega_{2}=\frac{b c\left(c-c_{1}\right)}{G c_{1}}\left[\frac{\left(S-S^{\prime}\right)}{t_{2}\left(c-c_{1}\right)}+\frac{S}{t_{1} c}\right.  \tag{17}\\
& \omega_{2}^{!}=\frac{b c\left(c-c_{1}\right)}{G c_{2}}\left[\frac{\left(S-S^{\prime}\right)}{t_{2}\left(c-c_{1}\right)}+\frac{S^{\prime}}{t_{3} c}\right]
\end{align*}
$$

Using Eqs. (11) and (16):-

$$
\begin{align*}
\frac{S b}{G t_{1}^{c}} & =A_{31} T_{1}+A_{33} S-2 \omega_{1}-\frac{\omega_{2}}{c}  \tag{18}\\
\text { and } \frac{S^{\prime} b}{G t_{3}^{c}} & =A_{31}^{\prime} T_{1}^{\prime}+A_{33}^{\prime} S^{\prime}-2 \omega_{1}^{\prime}+\frac{\omega_{2}^{\prime}}{c}
\end{align*}
$$

Elimination of $\omega_{1}$ and $\omega_{1}^{i}$ from Eq. (18) by using Eq. (10) yields:$\frac{b}{G c}\left[\frac{S^{p}}{t_{3}}-\frac{S}{t_{1}}\right]=A_{31}^{1} T 1+A_{33^{\prime}} S^{i}-A_{31} T_{1}-A_{33} S+w_{2}\left[\frac{\left(c-c_{1}\right)+2 c}{c\left(c-c_{1}\right)}\right]+\omega_{2}\left[\frac{\left(c-c_{1}\right)-2 c}{c\left(c-c_{1}\right)}\right]$

Substitution from the first pair of Eqs. (11) into Eq. (15) and elimination between the resulting two equations gives:$T_{1}=\left[c_{2}\left(A_{13}^{i} S^{i}-A_{13} S\right)+A_{11}^{i}\left(\frac{M}{2 b}-E A_{13} \sum A_{0} S\right)| | A_{11}^{i}\left\{E A_{11} Z A+c_{1}\right\}+c_{2} A_{11}\right\}$
$\left.T i=\left[c_{1}\left(A_{13} S-A_{1}^{\prime} S^{\prime}\right)+A_{11}\left(\frac{M}{2 b}-E A_{13}^{\prime} A \cdot S^{\prime}\right) / A_{11}^{\prime} E A_{11} \Sigma A+c_{1}\right\}+c_{2} A_{11}\right)$

Using Eqs. (17) and (19) and rearranging:-
$A_{31}^{\prime} T_{1}^{\prime}-A_{31} T_{1}=s\left|A_{33}+\frac{2 b c}{G t_{2} \cdot c_{1} c_{2}}+\frac{b}{G t_{1} c_{1}}\right|-s^{\prime}\left|A_{33}^{\prime}+\frac{2 b c}{G t_{2} c_{1} c_{2}}+\frac{b}{G t_{3}{ }^{c} 2}\right|$
... (21)
Eqs. (20) and (21) give:-
$\left[\left\{\frac{A_{31}^{1} A_{31} c_{1}+A_{31}^{2}\left(c_{2}+A_{11}^{1} E \Sigma A\right)}{A_{11}^{1}\left\{E A_{11} \sum A+c_{1}\right\}+A_{11} c_{2}}\right]-\left\{A_{33}+\frac{2 b c}{G t_{2} c_{1} c_{2}}+\frac{b}{G t_{1} c_{1}}\right] S\right.$
$=\left[\frac{A_{31}^{\prime} A_{31} c_{2}+A_{31}^{2}\left(c_{1}+A_{11} E \Omega A\right)}{A_{11}^{\prime}\left\{E A_{11} \sum A+c_{1}\right\}+A_{11} c_{2}}\right]-\left\{A_{33}^{\prime}+\frac{2 b c}{G t_{2} c_{1} c_{2}}+\frac{b}{G t_{3} c_{2}}\right\} S^{\prime}$
$+\left\{\frac{A_{31} A_{11}^{\prime}-A_{31}^{\prime} A_{11}}{A_{11}^{\prime}\left[E A_{11} \sum_{1}^{Z A}+c_{13}^{3}+A_{11} c_{2}\right.}\right\} \quad \frac{M_{1}}{2 b}$

Eqs. (22) and (14) enable $S$, $S^{\prime}$ to be found and thence $T_{1}, T_{i}^{\prime}$ from Eq 。 (20)
$\omega_{1}$ and $\omega_{1}$ follow from Eqs. (17) and (18):-
$\omega_{1}^{\prime}=\frac{A_{31}^{\prime} T^{\prime}}{2}+\frac{b S}{2 G t_{2} c_{2}}+\frac{S^{\prime}}{2}\left[A_{33}^{\prime}-\frac{b}{G c_{2}}\left[\frac{1}{t_{3}}+\frac{1}{t_{2}}\right]\right.$
$\left.\omega_{1}=\frac{A_{31} T_{1}}{2}+\frac{b S^{\prime}}{2 G t_{2} c_{1}}+\frac{S^{\prime}}{2}\left[A_{33}-\frac{b}{G c_{1}} \frac{1}{t_{1}}+\frac{1}{t_{2}}\right]\right]$

From Eqs. (23) and (11):-
$p_{1}=\frac{d p}{d x}=-\frac{\operatorname{cosec} a}{2 b}\left[A_{31} T_{1}+s\left[A_{33}+\frac{b}{G c_{1}}\left[\frac{1}{t_{2}}+\frac{1}{t_{1}}\right]-\frac{b S^{\prime}}{G t_{2} c_{1}}\right]\right.$
$q_{1}=\frac{d q}{d x}=\frac{\operatorname{cosec} \alpha}{b}\left[A_{11} T_{1}+A_{13} S\right]$

The displacements can be found using Eqs. (24), (23) and (17).
The strains follow from Eq. (9)

## 3. RESULTS

$$
\begin{array}{ll}
T_{1}=K_{11} I_{1}+K_{12} M_{1} & T_{1}^{\prime}=K_{11}^{\prime} I_{1}+K_{12}^{\prime} M_{1} \\
s=K_{21} I_{1}+K_{22} M_{1} & s^{\prime}=K_{21}^{\prime} I_{1}+K_{22}^{\prime} M_{1} \\
s_{1}=K_{31} I_{1}+K_{32} M_{1} & \\
s_{2}=K_{41} I_{1}+K_{42} M_{1} & \\
S_{3}=K_{51} I_{1}+K_{52} M_{1} &  \tag{25}\\
p_{1}=\frac{d p}{d x}=c_{11} I_{1}+c_{12} M_{1} & q_{1}=\frac{d q}{d x}=c_{21} I_{1}+c_{22^{\prime}} M_{1}
\end{array}
$$

Where:-

$$
c_{11}=-\frac{\operatorname{cosec} a}{8 b^{2} \varepsilon}\left[\frac{A_{31}}{K}\left[c_{2}\left(A_{13} \mu-A_{13}^{\prime}\right)+A_{13} A_{11}^{\prime} \mu E \cdot \sum \cdot A_{j}\right\}\right.
$$

$$
\begin{equation*}
-\mu A_{33}-\frac{b}{G c_{1}}\left[\frac{\mu}{t_{1}}+\frac{\mu-\lambda}{t_{2}}\right] \tag{27}
\end{equation*}
$$

$$
C_{12}=C_{21}=-\frac{\operatorname{cosec} \alpha}{4 b^{2} \gamma}\left[A_{13} A_{11}^{\prime}-\frac{\lambda \gamma_{2}}{\varepsilon}\right]
$$

$$
c_{22}=\frac{\operatorname{cosec} a}{2 b^{2} x}\left[A_{11} A_{11}+\frac{c_{1} x^{2}}{M_{\varepsilon}}\right]
$$

$$
\begin{align*}
& K_{11}=\left[c_{2}\left(A_{13} \mu-A_{13} \lambda\right)+A_{13^{A}} 1_{1} \mu E \cdot E A_{j} / 4 b \varepsilon k\right. \\
& K_{12}=\frac{c_{2} \psi}{2 b \varepsilon k^{2}}\left\{-A_{13}\left(c_{2}+E A_{11}^{\prime} Z_{i}^{\prime} A\right)-A_{13}^{\prime} c_{1}\right\}+\frac{A_{11}^{\prime}}{2 b K} \\
& K_{11}=\left\{c_{1}\left(A_{13}^{\prime} \lambda-A_{13}{ }^{\mu}\right)+A_{13}^{\prime} A_{11} \lambda \cdot E \cdot \sum_{A}\right\} / 4 b \varepsilon k \\
& K_{12}^{\prime}=\frac{c_{1} \times}{2 b \varepsilon \times x^{2}}\left\{A_{13}^{\prime}\left(c_{1}+E A_{11} \Sigma A\right)+A_{13} c_{2}\right\}+\frac{A_{11}}{2 b \times} \\
& K_{21}=-\frac{u}{4 b \varepsilon}  \tag{26}\\
& K_{21}^{\prime}=-\frac{\lambda}{4 b \bar{\varepsilon}} \\
& \mathrm{~K}_{22}=\frac{\mathrm{c}_{2} \%}{2 \mathrm{~b})_{8}} \\
& \mathrm{~K}_{31}=\frac{\mu}{4 b \varepsilon} \\
& K_{41}=\frac{(\lambda-u)}{4 b \varepsilon} \\
& K_{22}^{\prime}=-\frac{c_{1} \nless}{2 b ;} \\
& K_{32}=-\frac{c_{2} \psi}{2 b ; \%} \\
& K_{42}=\frac{\chi\left(c_{1}+c_{2}\right)}{2 b / 8} \\
& K_{51}=-\frac{\lambda}{4 b \varepsilon} \\
& K_{52}=-\frac{c_{1} \phi}{2 b}
\end{align*}
$$

$$
\begin{aligned}
& K=A_{11}^{\prime}\left\{E A_{11} \Sigma A+c_{1}\right\}+A_{11} c_{2} \\
& x=A_{31} A_{11}^{\prime}-A_{31}^{\prime} A_{11} \\
& \mu=A_{31} A_{31}^{\prime} c_{2}+A_{31}^{\prime 2}\left(c_{1}+A_{11} E \cdot \Sigma A\right)-A_{33}^{\prime}-\frac{b\left(c_{1}+c_{2}\right)}{G t_{2} c_{1} c_{2}}-\frac{b}{G t_{3} c_{2}} \\
& \lambda=A_{31} A_{31}^{\prime} c_{1}+A_{31}^{2}\left(c_{2}+A_{11}^{\prime} E \cdot \Sigma A\right)-A_{33}-\frac{b\left(c_{1}+c_{2}\right)}{G t_{2} c_{1} c_{2}}-\frac{b}{G t_{1} c_{1}} \\
& \varepsilon=\lambda c_{2}+\mu c_{1}
\end{aligned}
$$

## SPECIAL CASE

## Two Equal Cells

$$
t_{1}=t_{2}=t_{3}=t_{w} \quad c_{1}=c_{2}=c \quad t=t^{\prime}
$$

(Equal cells with constant web and skin thicknesses)

$$
\begin{align*}
& \left.T_{1}=T_{1}^{\prime}=\left(\frac{A_{13} E \cdot \Sigma A I_{1}}{4 c}+M_{1}\right) \right\rvert\, 2 b\left(2 c+E A_{11} \cdot \Sigma \cdot A\right) \\
& S=S^{\prime}=-\frac{L_{1}}{8 b c} \\
& S_{1}=-S_{3}=\frac{I_{1}}{8 b c}  \tag{29}\\
& S_{2}=0
\end{align*}
$$

$K_{11}=\frac{1}{8 b c} \cdot \frac{A_{13^{E}} \cdot \sum A}{\left(2 c+A_{11} E \sum A\right)}$
$K_{21}=-\frac{L_{1}}{8 b c}$

$$
\mathrm{K}_{12}=\frac{1}{2 \mathrm{~b}\left(2 \mathrm{c}+\mathrm{A}_{11} \mathrm{E} \mathrm{\Sigma} \mathrm{~A}\right)}
$$

$$
\begin{equation*}
\mathrm{K}_{22}=0 \tag{30}
\end{equation*}
$$

$K_{31}=\frac{1}{8 b c}$
$K_{41}=0$
$K_{51}=-\frac{1}{8 \mathrm{bc}}$
$K_{32}=K_{42}=K_{52}=0$

$$
\begin{align*}
C_{11} & =\frac{\operatorname{cosec} a}{8 b c}\left[\frac{(1+\sigma)}{E t}+\frac{A_{w 3}}{2 b}-\frac{A_{13}^{2} E A}{2 b\left(2 c+A_{11} E \cdot \delta A\right)}\right] \\
C_{12} & =c_{21}=-\frac{\operatorname{cosec} \alpha A_{13}}{4 b^{2}\left(2 c+A_{11} E \cdot \Sigma A\right)}  \tag{31}\\
C_{22} & =\frac{\operatorname{cosec} a A_{11}}{2 b^{2}\left(2 c+A_{11} E \cdot \Sigma A\right)} \\
X & =0 \\
\lambda & =\mu  \tag{32}\\
\varepsilon & =2 \lambda c=2 \mu c \\
K & =2 A_{11} c+A_{11}^{2} E \cdot \Sigma A
\end{align*}
$$

## 5．DISCUSSION

It can be seen from Eqs（26），that there is a contribution to the oblique shear stress from the＂bending couple＂$M_{1}$ 。 This contribution is dependent upon the value of $x$ ，and it can be shown that $\gamma$ itself is dependent upon the relative values of $A_{R}, a_{R}$ ，and $t$ in the two cells．

For the case of constant rib pitch，with the ratio of the rib boom area to skin thickness the same in both cells， i．e．$\frac{A_{R}}{a_{R} t}=\frac{A_{R}^{\prime}}{a_{R}^{!} t^{t}}$ ，the value of $x$ is zero．Under these conditions，there is no oblique shear stress due to $M_{1}$ 。

The results for the special case of two equal cells with constant web and skin thicknesses given in Eqs（29）to （32），are directly comparable to the single cell results， Eqs（78），（83）and（100）of Ref． 1.

Complete analysis of the box beam subjected to constant couples is achieved by using the above results in conjunction with the relevant parts of Ref． 1 § 3.2 。

## REFERENCES

1) Hemp, Wos. On the Application of Oblique Co-ordinates to Problems of Plane Elasticity and Swept Back Wing structures.

College of Aeronautics Report No. 31 Jan. 1950.


Geometry Of Swept Wing



[^0]:    * This investigation was made during the tenure by the author of a Clayton Fellowship awarded by the Institution of Mechanical Engineers.

