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DESIGN CHARTS FOR CARBON FIBRE COMPOSITES

by

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SUMMARY

This report contains charts of elastic properties and buckling coefficients of a simply supported compression panel, based on theory of multi layer plates of orthotropic material, for a typical carbon fibre composite. In addition the optimum orientation of plies, of a three ply system, is considered for a corrugated compression panel together with the modifications necessary for other panel shapes. The computer programmes used are contained in the Appendix.

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NOTATION

\( D_{1,2} \) flexural rigidity (eq.4a)

E elastic modulus

F efficiency factor (eq.5n)

G shear modulus

K buckling coefficient

\( K' \) buckling coefficient of composite plate (eq.4d)

L strut length

M panel geometry parameter (eq.5l)

\( N = \frac{a/b}{(D_2/D_1)^{1/4}} \)

R angle ply thickness/total thickness

S measured shear strength of single ply.

\( V_f \) filament volume fraction

X measured longitudinal strength of single ply

Y measured transverse strength of single ply

Z material parameter (eq.5l)

\( a_{11}, a_{22} \) flexibility coefficients of single layer (eq.2c)

\( b_{11}, b_{22} \) stiffness coefficients of single layer (eq.2b)

a plate length

b plate width

\( r_{11}, r_{22} \) flexibility coefficients of total composite (eq.3b)

t plate thickness

\( \bar{t} \) effective thickness (eq.5d)

w end load/in.

\( \varepsilon \) direct strain

\( \gamma \) shear strain

\( \Theta \) ply angle (fig.2)

\( \phi \) corrugation angle (fig.9)

\( \mu \) Poisson's ratio

\( \sigma \) direct stress

\( \tau \) shear stress
Notation ctd.

Subscripts

A  applied
C  compression
T  tension
CR buckling
LI local instability
LW long wave
f  flank
c  crest
x, y, z applied stress axes
α, β, γ filamentary axes
o longitudinal axis of single ply.
1. Introduction

The increasing availability of high strength and stiffness, lightweight filaments and in particular carbon fibre has led to a study of the potential structural weight saving with this new material. In the course of this work the need for methods of selecting the most suitable orientation of multi-layer composites, for various types of loading, became apparent. Techniques similar to those used for predicting the behaviour of plywood and glass reinforced plastics have been used.

The results are presented as follows,

i) graphs of elastic and strength properties together with a computer programme with which it is possible to select layups suitable for combined loading conditions

ii) a chart of buckling coefficients for a simply supported panel, together with the modification necessary for other edge conditions

iii) a design chart for a corrugated compression panel.

The material chosen for the study was a high modulus carbon fibre composite with a 60 per cent filament volume. However the results can be modified for other materials.
2.0 Elastic Properties

Methods of predicting the elastic properties of a single uni-directional laminate from the properties of its constituents, are available, but all rely on a knowledge of the void content, straightness of filaments, etc. Data regarding these properties is practically non-existent for carbon composites hence the measured elastic properties of a single sheet of composite have been used. The fibre content chosen was 60 per cent by volume as this appears to be approaching a working maximum, further increases resulting in a marked reduction in transverse strength and stiffness.

The properties used were as follows:-

Material - Fibre: Type I High Modulus, treated
Resin: Epoxy

$V_f = 0.60$
$E_\alpha = 30 \times 10^6$ lb/in$^2$
$E_\beta = 1.1 \times 10^6$ lb/in$^2$
$G_{\alpha\beta} = 0.7 \times 10^6$ lb/in$^2$
$\mu_{\alpha\beta} = 0.3$

Methods of determining the elastic properties of multi-directional laminates are well established (ref.1) and have been used with success for plywoods and glass laminates.

2.1 General relationship for orthotropic laminates

Each layer has three moduli of elasticity in the direction of the three axes (see fig.1) one of which is parallel to the filaments, three moduli of rigidity $G_{\alpha\beta}$, $G_{\beta\gamma}$, $G_{\gamma\alpha}$ and six Poisson's ratios, two associated with each axis, $\mu_{\alpha\beta}$, etc. The latter are not independent but are associated as follows

$$\frac{\mu_{\alpha\beta}}{E_\alpha} = \frac{\mu_{\beta\alpha}}{E_\beta}$$

$$\frac{\mu_{\alpha\gamma}}{E_\alpha} = \frac{\mu_{\gamma\alpha}}{E_\gamma}$$

$$\frac{\mu_{\beta\gamma}}{E_\beta} = \frac{\mu_{\gamma\beta}}{E_\gamma}$$

... (2a)

It thus requires nine independent properties to completely define the elastic behaviour of an anisotropic material. For sheet materials it is normal to assume (ref.1) that
\[ E_\gamma = E_\beta, \quad \gamma_\beta \gamma = \gamma_\alpha \gamma = \gamma_\alpha \beta, \quad \mu_\beta \gamma = \mu_\gamma = \mu_\alpha \gamma \]
this reduces the required number of elastic constants to
\[ E_\alpha, \quad E_\beta, \quad \gamma_\alpha \beta, \quad \mu_\alpha \beta. \]

2.2 Stress-strain relationship

Consider now the effect of rotating the filamentary axes, \( \alpha, \beta \) through an angle \( \theta \) with respect to the axes of applied stress, \( x, y \) (see fig. 2).

It can be shown that (ref. 1)
\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = 
\begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} \quad \quad \quad \quad \quad \quad \ldots \ (2b)
\]

where
\[ b_{11} = \frac{1}{\lambda} \left[ E_\alpha \cos^4 \theta + E_\beta \sin^4 \theta + \sin^2 \theta \cos^2 \theta (2E_\alpha \mu_\beta \alpha + 4\lambda \gamma_\alpha \beta) \right] \]
\[ b_{22} = \frac{1}{\lambda} \left[ E_\beta \cos^4 \theta + E_\alpha \sin^4 \theta + \sin^2 \theta \cos^2 \theta (2E_\alpha \mu_\beta \alpha + 4\lambda \gamma_\alpha \beta) \right] \]
\[ b_{33} = \frac{1}{\lambda} \left[ \sin^2 \theta \cos ^2 \theta (E_\alpha + E_\beta - 2E_\alpha \mu_\beta \alpha) + \lambda \gamma_\alpha \beta (\cos^2 \theta - \sin^2 \theta)^2 \right] \]
\[ b_{21} = b_{12} = \frac{1}{\lambda} \left[ \sin^2 \theta \cos ^2 \theta (E_\alpha + E_\beta - 4\lambda \gamma_\alpha \beta) + E_\alpha \mu_\beta \alpha (\cos^4 \theta + \sin^4 \theta) \right] \]
\[ b_{31} = b_{13} = \frac{1}{\lambda} \left[ \sin^3 \theta \cos \theta (E_\beta - E_\alpha \mu_\beta \alpha - 2\lambda \gamma_\alpha \beta) + \sin \theta \cos^3 \theta (E_\alpha - E_\alpha \mu_\beta \alpha - 2\lambda \gamma_\alpha \beta) \right] \]
\[ b_{32} = b_{23} = \frac{1}{\lambda} \left[ \sin^3 \theta \cos \theta (E_\beta - E_\alpha \mu_\beta \alpha - 2\lambda \gamma_\alpha \beta) - \sin^3 \theta \cos \theta (E_\alpha - E_\alpha \mu_\beta \alpha - 2\lambda \gamma_\alpha \beta) \right] \]

where \( \lambda = 1 - \mu_\beta \alpha \mu_\alpha \beta \)
and also by inversion
\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = 
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = [b_{11}]^{-1} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} \quad \ldots \ (2c)
\]
where \( \mu_{xy} = -\frac{a_{12}}{a_{11}} \); \( \mu_{yx} = -\frac{a_{12}}{a_{22}} \)

\( E_x = \frac{1}{a_{11}} \); \( E_y = \frac{1}{a_{22}} \); \( \gamma_{xy} = \frac{1}{a_{33}} \)

2.3 Laminated composites

Only 'balanced' laminates are considered, that is plates that have a symmetrical distribution of plies about the median plane. The plates do not therefore twist when subjected to in-plane forces or bending moments. (See fig. 2)

a) Plate Stretching

\[
b_{11} \text{ COMP} = \frac{1}{t} \sum_{i=1}^{n} b_{111} t_{i} \quad \ldots (2d)
\]

e etc.

b) Plate Bending

\[
b_{11} \text{ COMP} = \frac{1}{t} \sum_{i=1}^{n} b_{111} I_{i} \quad \ldots (2e)
\]

e etc.

Inversion of the \( b_{i} \) matrix for the composite will yield the composite elastic moduli.

For this study the number of layers has been assumed to be large, as thin prepregs (\(.001\)) are available and the effect of asymmetry could be minimised. Consequently eq.(2d) has been used to calculate the laminate properties. This was achieved with the use of an ICL 1905 computer together with a simple program in JEAN language. The results are tabulated for various values of cross ply angle \( (\theta) \) and proportion of cross plies \( (R) \), (App.1). The results are also plotted in figs. 3, 4 and 5. It will be noted that with \( R = 1.0 \) (i.e. all cross plies) at small angles (say 10\(^\circ\)) \( \mu_{xy} > 1.0 \). This is verified by Cox (ref. 2) and although it is calculated would be accompanied by an appropriate Poisson's ratio \( \mu_{xz} \) through thickness of the laminate in order that the volume of the plate does not change. This is also borne out by Rothwell (ref.3) for a similar layup.
3.0 Strength of Multi Layer Composites

Methods of predicting the strength of uni-directional sheets based on the constituent properties have been suggested (ref.4). However for practical purposes the values obtained are dubious in that they rely on a knowledge of the void content, arrangement of fibres, etc. Consequently measured unidirectional sheet strength properties have been used in this analysis. Very little experimental data is available on the behaviour of carbon composites loaded at axes other than the filamentary axes.

The data used is based on the same material as that used in Appendix 1 and the measured strength values used were as follows

\[ X_T = 150,000 \text{ lb/in}^2 \text{ tension} \]
\[ X_C = 120,000 \text{ lb/in}^2 \text{ compression} \]
\[ Y = 6,000 \text{ lb/in}^2 \text{ tension and compression} \]
\[ S = 8,000 \text{ lb/in}^2. \]

3.1 Theoretical analysis

Failure criteria are discussed in detail by TSAI (ref.4) and by Chamis (ref.5). The most effective method appears to be that proposed by Hill (ref.6) for an anisotropic material.

The criterion used is

\[ K = \frac{X}{\sqrt{\sigma_x^2 - \sigma_x \sigma_y + \frac{X^2}{Y^2} \cdot \sigma_y^2 + \frac{X^2}{S^2} \cdot \tau_{xy}^2}} \quad \ldots (3a) \]

where \( X, Y, \) and \( S \) are the measured longitudinal transverse and shear strengths respectively.

Consider now the stresses applied to a multi layer composite.

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
= \begin{bmatrix} a_{ij} \end{bmatrix}_{COMP}^{-1}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix} a_{ij} \end{bmatrix}_{COMP}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix} r_{ij} \end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

(3b)
Then using the strains calculated above stresses in a layer of the composite can be calculated

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_{\text{LAYER}} = \begin{bmatrix}
b_{ijLAYER}
\end{bmatrix}_{ij} \begin{bmatrix}
r_{ij}
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_{\text{COMP}} \ldots (3c)
\]

In order that these may be applied to the failure criterion then these stresses must be transferred to the filamentary axes of the layer, hence:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_{\text{LAYER}} = \begin{bmatrix}
mn
\end{bmatrix} \begin{bmatrix}
b_{ijLAYER}
\end{bmatrix}_{ij} \begin{bmatrix}
r_{ij}
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}_{\text{COMP}} \ldots (3d)
\]

where \[
\begin{bmatrix}
mn
\end{bmatrix} = \begin{bmatrix}
\cos^2\theta, \sin^2\theta, -2\sin\theta\cos\theta \\
\sin^2\theta, \cos^2\theta, 2\sin\theta\cos\theta \\
\sin\theta\cos\theta, -\sin\theta\cos\theta, (\cos^2\theta - \sin^2\theta)
\end{bmatrix}
\]

These stresses can then be substituted into the chosen strength criterion and the strength factor \( K \) determined. Failure will obviously occur in one of the layers and if required further analysis could be carried out to determine final strength of the laminate without the failed layer. This latter exercise has not been attempted as it was felt that for components subjected to a complex stress system almost invariably the remaining layers would be incapable of supporting the stresses imposed and total failure would occur. For simple tension however it might be possible to use the initial failure as a proof load but it is very unlikely to be of practical value.

The analysis has been carried out for various ratios of applied stresses and the results are shown in figs. 6 and 7 for simple tension and compression.

The analysis was achieved by use of an ICL 1905 and a Fortran IV computer program. This is listed in App. 2. As the data and programs are on cards, substitution of new data and deletion of WRITE statements where not required is simple. Using this program and by substitution of appropriate values of \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \) (i.e. applied stress system) it is possible to determine the best layup, based on maximum strength, for a 3-fibre system. The program is being modified to include other types of layup (e.g. a 4-fibre system) and to allow for possible asymmetry.
4.0 Buckling of Plates

The theoretical analysis of the buckling of anisotropic plates in compression and shear is well established (ref.7) and in the case of plywood and C.R.P. plates has been verified experimentally (ref. 8, 9). Experimental testing has been carried out at the College as part of student theses (ref.10, 11) and results indicate the theory is applicable to C.R.P. composites.

4.1 Buckling in compression of simply supported orthotropic plates

An expression for the buckling stress of an orthotropic plate is derived in ref.12 as shown below.

\[ \sigma_{CR} = \frac{2 \pi^2}{b^2 t} \left[ \left( \frac{D_1 D_2}{D_3} \right)^{1/2} + D_3 \right] \]  \hspace{1cm} \ldots (4a)

where

\[ D_1 = \text{flexural rigidity corresponding to bending moment } M_x \]
\[ = \frac{(EI)_x}{\lambda} \]

\[ D_2 = \text{flexural rigidity corresponding to bending moment } M_y \]
\[ = \frac{(EI)_y}{\lambda} \]

\[ D_3 = \frac{1}{2}(\mu_{xy} D_2 + \mu_{yx} D_1) + 2(GI)_{xy} \]

where \((GI)_{xy}\) is the average torsional rigidity of the plate.

\[ \lambda = 1 - \mu_{xy} \cdot \mu_{yx} \]

For a thin plate, thickness \(t\), these stiffnesses may be expressed as (ref.12)

\[ D_1 = E_x \cdot t^3 / 12 \lambda \]
\[ D_2 = E_y \cdot t^3 / 12 \lambda \]
\[ D_3 = t^3 (\mu_{xy} \cdot E_y + \mu_{yx} \cdot E_x) / 24 \lambda + t^3 G_{xy} / 6 \]  \hspace{1cm} \ldots (4b)

Then substituting (4b) into (4a) gives

\[ \sigma_{CR} = \frac{\pi^2}{6 \lambda} \left[ \left( \frac{E_x \cdot E_y}{2} \right)^{1/2} + \mu_{xy} \cdot E_y / 2 + \mu_{yx} \cdot E_x / 2 + 2G_{xy} \right] \]
\[ \times \left[ \frac{t^2}{B} \right] \]  \hspace{1cm} \ldots (4c)

or

\[ \sigma_{CR} = E_{x0} \left( K' \right) \left( \frac{t}{B} \right)^2 \]  \hspace{1cm} \ldots (4d)
where $E_{xo}$ = longitudinal modulus of a single unidirectional sheet.

$$K' = \frac{1}{E_{xo}} \cdot \frac{\pi^2}{6h} \left[ (E_x \cdot E_y)^{1/2} + \mu_{xy} E_y/2 + \mu_{yx} E_x/2 + 2\lambda G_{xy} \right]$$

The expression $K'$ has been evaluated in the JEAN programme used in App.1 for plates with a balanced layup and the results are plotted in fig. 8. It can be seen that the optimum layup for plate buckling occurs when $R = 1.0$ (all angle ply), $\theta = 45^\circ$. However the permissible compressive stress for this layup is only 15,000 psi when compared with a maximum possible of 120,000 psi for unidirectional material.

It can be shown that the optimum stress level,

$$\sigma_{OPT} = \sqrt[3]{E_{xo} \cdot K' \left( \frac{P}{b^2} \right)^{2/3}} \quad \ldots \quad (4e)$$

where $P$ = applied compressive load

$$\frac{P}{b^2} = \text{structural index.}$$

As the structural index is increased it will be necessary to introduce an increasing number of axial plies or angle plies at a small angle, $\theta$, in order to increase the optimum stress level. By superimposing lines of constant stress from fig. 9 on the chart for buckling coefficient, $K'$ (fig. 9) a 'path to follow' from the optimum layup (i.e. $R = 1.0, \theta = 45^\circ$) may be determined. This is shown as a dotted line on fig. 8.

Fig. 10 shows optimum stress level vs. structural index for carbon fibre composite compared with steel and aluminium alloy.

The buckling chart, fig. 8 'may also be used to determine the buckling coefficient for plates with different edge conditions and aspect ratios. Wittrick (ref. 13) has suggested that the buckling curves in ref. 14 which give buckling coefficient $K$ versus $a/b$ (panel aspect ratio) for various edge conditions may be used by substituting a value

$$N = a \cdot \left( \frac{D_2}{D_1} \right)^{1/4}$$

for aspect ratio $a/b$.

Then buckling stress,

$$\sigma_{CR} = \frac{K \cdot R \cdot A \cdot E \cdot S}{3.52} \cdot K' \left( \frac{2}{b} \right)^{2/3} \quad \ldots \quad (4f)$$
5.0 Design of Wide Compression Panels

Optimum design procedures for wide panels subjected to compressive loading almost invariably establish a design criteria that all modes of buckling occur simultaneously. The procedure adopted in this section is similar in that local and long wave instability modes are assumed to be coincident. The types of panels considered do not buckle in the torsional mode and in fact types of construction prone to this mode of buckling appear to be unattractive in filamentary composites for most purposes.

5.1 Corrugated compression panels

Consider the buckling of a corrugated compression panel (see fig. 11), the modes of buckling to be anticipated are

i) local buckling of plate elements
ii) long wave buckling of whole panel.

For i) the plate is assumed to be simply supported and infinitely long, although if necessary the interaction effect with the adjacent plates could be included and for ii) ends of the panel are assumed to be simply supported.

In this initial analysis the column is assumed to be made from a constant thickness sheet where

\[ t_c = t_f = t, \quad b_c = b_f = b \]  \quad ... (5a)

Then local buckling stress of plate,

\[ \sigma_{LI} = E_{xo}(K')(\frac{t}{b})^2 \]  \quad ... (5b)

where \( E_{xo} \) and \( (K') \) are defined in 4.1.

Long wave buckling stress of panel,

\[ \sigma_{LW} = \frac{\pi^2 E_x I}{L^2 t} \]  \quad ... (5c)

where

\[ t = \frac{2t}{(1 + \cos \phi)} \]  \quad ... (5d)

\[ I = \left[ 2bt(\frac{h}{2})^2 + \frac{2t}{\sin \phi} \cdot \frac{h^3}{12} \right] \cdot \frac{1}{2b(1 + \cos \phi)} \]  \quad ... (5e)

for \( 0 < \phi < 120^\circ \)

\[ h = b \cdot \sin \phi \]  \quad ... (5f)

Substituting (5f) into (5e) we obtain

\[ I = \frac{1}{2} \frac{b^2 t \sin^2 \phi}{(1 + \cos \phi)} \]  \quad ... (5g)
and by substituting (5g) and (5d) into (5e) we obtain
\[ \sigma_{LW} = \frac{\pi^2 E_x \sin^2 \phi}{6} \cdot \frac{b^2}{L^2} \quad \ldots (5h) \]

now applied stress,
\[ \sigma_A = \frac{\omega}{t} = \frac{\omega (h \cos\phi)}{2t} \quad \ldots (5i) \]

As all buckling modes occur simultaneously
\[ \sigma_{LI} = \sigma_{LW} = \sigma_A = \sigma \quad \ldots (5j) \]

and by examining equations (5b), (5c) and (5h) we can show
\[ \sigma^4 = \sigma_{LI} \cdot \sigma_{LW} \cdot \sigma_A^2 = \frac{\pi^2 E_x \sin^2 \phi}{6} \cdot \frac{b^2}{L^2} \cdot E_xo(K') \left( \frac{t}{b} \right)^2 \]
\[ \times \frac{\omega^2}{4t^2(1 + \cos\phi)^2} \]

Rearranging this becomes
\[ \sigma^4 = \frac{\pi^2}{24} \left[ E_x \cdot E_xo(K') \right] \left[ \sin^2 \phi (1 + \cos\phi)^2 \right] \left[ \frac{\omega}{L} \right]^2 \quad \ldots (5k) \]
or
\[ \sigma^4 = \frac{\pi^2}{24} E_x^2 \cdot E_xo^2 \cdot Z \cdot M \cdot \left[ \frac{\omega}{L} \right]^2 \quad \ldots (5l) \]

where \( Z = \frac{E_x \cdot E_xo \cdot K'}{E_xo^2} \)

\[ M = \sin^2 \phi (1 + \cos\phi)^2 \]

By examining eq.(5l) it can be seen that the parameters for material properties, \( Z \) and cross sectional geometry, \( M \) are independent and for a given loading index, \( \omega/L \) may be maximised separately in order to achieve the maximum stress, \( \sigma \) and hence the lightest panel. The maximum value of \( M \) occurs when \( \phi = 60^\circ \) and substituting this value into eq.(5l) yields
\[ \sigma = 0.914 \, \sqrt{\frac{E^2}{E_{xo}}} \cdot Z \cdot \sqrt{\frac{\omega}{L}} \quad \ldots \quad (5m) \]

The material property parameter, \( Z \), has been evaluated in the JEAN programme in App.1 and the results are shown in fig.12. It can be seen that the maximum value of \( Z \) occurs when \( R = 0.33, \theta = 55 \). This 'optimum' orientation will only apply up to the stress level that can be achieved by this particular configuration. In order to increase the stress level (i.e. at a higher loading index, \( \omega/L \)) then the layu must be altered and the 'path to follow' from the optimum is obtained as in 4.0 by superposition of lines of constant allowable stress and the path is shown on fig.12. By following this path a graph of optimum stress level, \( \sigma_{opt} \) versus loading index, \( \omega/L \) may be constructed and is shown in fig.13 in comparison with aluminium alloy.

When the material considered is isotropic i.e. \( E_x = E_{xo} = E \) etc. eq.\( (5m) \) becomes

\[ \sigma = F \sqrt{E} \sqrt{\frac{\omega}{L}} \quad \ldots \quad (5n) \]

where \( F = \text{Efficiency Factor} (=1.26 \text{ for corrugated panel}) \) and this agrees with the expression obtained by Emero and Spunt (Ref.15)

For other cross sectional shapes it is suggested that eq.\( (5m) \) can be modified as follows

\[ \sigma = 0.914 \, \sqrt{\frac{E_{xo}^2}{E_{xo}}} \cdot Z \cdot \sqrt{\frac{\omega}{L}} \cdot \frac{F'}{1.26} \quad \ldots \quad (5p) \]

where \( F' = \text{Efficiency Factor of panel (see table 1)} \)

However, this will only apply to panels prone to similar modes of failure (i.e. no torsional instability)
6.0 Conclusions

Before attempting to use these highly anisotropic materials careful consideration must be given not only to the type, magnitude and variations in the applied loadings but also to the design specifications. Merely attempting to substitute carbon fibre for metal and expecting exactly similar performance will not realise the full potential of the material.

The methods presented should be useful as a guide for designers faced with selecting fibre orientations for the chosen material. They will also serve as a guideline for similar highly anisotropic materials as the buckling charts are presented in terms of the longitudinal elastic modulus of a single ply.

It is proposed to extend the work and to present a family of charts to include materials with less marked anisotropy eg. glass fibre. If required the methods and computer programme are also suitable with little modification for use with composites composed of layers of different materials.

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    Moment Loads.
APPENDIX I - Evaluation of Elastic Properties and Buckling Coefficients

THIS IS A JIEN PROGRAMME FOR EVALUATING THE HJJ MATRIX

1.0 TYPE "311"
1.1 SET Z=E*COS(A)+4*N+SIN(A)+4
1.2 SET Y=(Z+SIN(A)+2*COS(A)+12*(2*M+J+4*S*R))/S
1.3 TYPE Y
2.0 TYPE "3P2"
2.1 SET X=N+COS(A)+4*M+SIN(A)+4
2.2 SET Y=(X+SIN(A)+2*COS(A)+12*(2*M+J+4*S*R))/S
2.3 TYPE X
3.0 TYPE "333"
3.1 SET Y=SIN(A)+2*COS(A-12+M+N-2*S*R)
3.2 SET J=(V+S+2*(COS(A)+4+2*SIN(A)+12+COS(A)+12+SIN(A)+12))/S
3.3 TYPE J
4.0 TYPE "312"
4.1 SET C=SIN(A)+2*COS(A)+12*M+N-4*S*R)
4.2 SET D=(C+M+SIN(A)+12+SIN(A)+12))/S
4.3 TYPE D
5.0 TYPE "313"
5.1 SET E=SIN(A)+3*COS(A)+4*(N-M+J-2*S*R)
5.2 SET F=(E-SIN(A)+COS(A)+3*(M+J-2+S*R))/S
5.3 TYPE F
6.0 TYPE "332"
6.1 SET G=SIN(A)+COS(A)+3*(N-M+J-2+S*R)
6.2 SET H=(G+SIN(A)+3+COS(A)*(M+J-2+S*R))/S
6.3 TYPE H
### Results of JEAN Programme I

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<tr>
<th>B_{11}</th>
<th>B_{22}</th>
<th>B_{33}</th>
<th>B_{12}</th>
<th>B_{13}</th>
<th>B_{23}</th>
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<td>3.310926056</td>
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<td>0</td>
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---

- SET M = 30.0  \( E_\alpha \)
- SET N = 1.1  \( E_\beta \)
- SET G = 0.011  \( \mu_{\beta\alpha} \)
- SET R = 0.7  \( G_{\alpha\beta} \)
JEAN Programme 2 - Buckling Coefficients

2.00 SET M=A+(E-A)*R
2.01 SET N=B+(F-B)*R
2.02 SET O=C+(G-C)*R
2.03 SET P=D+(H-D)*R
2.04 SET Q=M*N+P+2*Q
2.05 SET S=N*Q/Q
2.06 SET T=M*Q/Q
2.07 SET U=(M*N-P+P)/Q
2.08 SET V=P*Q/Q
2.09 TYPE R IN FORM 1
2.10 SET W=1/S
2.11 SET X=1/T
2.12 SET Y=1/U
2.13 SET I=V/S
2.14 SET J=V/T
2.15 TYPE W,X,Y,I,J IN FORM 2
2.16 SET K=(W*10*5*X*10*5+I*X/2+J*W/2+2*Y*(1-I*J))
2.17 SET K=K*1.6499/(1-I*J)
2.19 SET Q=K/30  Q = K^1
2.20 TYPE Q IN FORM 3
2.21 SET Z=W*K/900
2.22 TYPE Z IN FORM 4

A = B11  B = B22  C = B33  D = B12  E = B11  F = B22  G = B33  H = B12
R = Cross Ply Ratio
Set of Typical Results for Jean Programme 2

| SET E = 28.414 |
| SET F = 1.1667 |
| SET G = 1.5113 |
| SET H = 1.1424 |
| θ = 10° |

**DO PART 2 FOR R = 0.0110**

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<td>0.2495</td>
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<tr>
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<td>1.0113</td>
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<td>0.2418</td>
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APPENDIX 2 - FORTRAN Programme For Composite Strength

MASSIVE COMPOSITE STRENGTH

WRITE (2, 7)
1 FORMAT (2AH: TELL YOUR COMPOSITE STRENGTH)
READ (1, 2) EA, EB, G, PB, BL, B11, B22, B33, B12, B13, B23
C EA IS ALPHA, EB IS BETA, G IS SHEAR MOD. PB IS MUHA, BL IS LAMBDA
C SUFFIX 0 IN N TERMS REFERS TO ANGLE THETA
2 FORMAT (9F0.0)
READ (1, 3) SX, SY, TXY
5 FORMAT (3F0.0)
DIMENSION R (10)
C R IS TRANSPLANTED DIMENSION 1:10
C I IS ANGLE THETA IN RADIANS
READ (1, 4) R
4 FORMAT (10F0.0)
READ (1, 5)
C SX, SY, TXY ARE APPLIED STRESSES USE 1000 AS MAX
5 FORMAT (3F0.0)
WRITE (2, 7) SX, SY, TXY
7 FORMAT (3F0.0)
DU 01 = 1, 1
DU 02 = N, 1
B11 = EA * COS(T(I))**4 + EB * SIN(T(I))**4
B11 = (B11 + 4. * EA * PB + 4. * BL * G) / BL
B22 = (B22 + 4. * EA * PB + 4. * BL * G) / BL
B33 = (B33 + 4. * EA * PB + 4. * BL * G) / BL
C SUFFIX 0, 0 TOTAL STRESS
B11S = (1. - R(I)) * B11 + R(N) * B11T
B22S = (1. - R(I)) * B22 + R(N) * B22T
B33S = (1. - R(I)) * B33 + R(N) * B33T
B12S = (1. - R(I)) * B12 + R(N) * B12T
B13S = (1. - R(I)) * B13 + R(N) * B13T
B23S = (1. - R(I)) * B23 + R(N) * B23T
S = B11S * B22S + B33S - B12S**2 * B33S
K11 = B11S / B33S
K22 = B22S / B33S
K33 = (B11S + B22S - B12S**2) / B33S
K12 = (B12S - B13S**2) / B33S
K13 = B13S / B33S
K23 = B23S / B33S
C SX, SY, TXY ARE STRESSES IN THE 0 LAYER
SXU = B11U * SX + B12U * SY + B13U * TXY
SYU = B22U * SX + B23U * SY + B23U * TXY
TXY = B33U * SX + B33U * SY + B33U * TXY
49 FORMAT (5F0.0)
WRITE (2, 50)
50 FORMAT (5F0.0)
C F10.1 IS STRENGTH FACTOR FOR FIT FIT SIMILAR
50 E=150. / S=CT(SX**2-SX*SY+0.025).*SY**2+352.*TXY*2)
 WRITE(2,51) 101
51 FORMAT(SH E=,3PF10.0)
 GO TO 54
52 EUC=120. / S=CT(SX**2-SX*SY+400).*SY**2+225.*TXY*2)
 WRITE(2,53) 10C
53 FORMAT(SH EUC=,3PF10.0)
54 SXI=971.1*EX+9121.*LY+9131.*EY
 SYI=971.1*EX+9211.*LY+9231.*EY
 TXY=9131.*EX+9231.*EY+9331.*EY
 C SXI, SYI, TXY ARE STRESS IN CROSS PLY LAYER - Positive.
 WRITE(2,67) SXI,SYI,TXY
47 FORMAT(SH)
 SA=CT(S(I(J))+2*SXI+SIN(T(I))**2*SYT-2*SINT(I)*COS(T(I))*TXY
 SB=CT(S(I(J))+2*SXI+COS(T(I))**2*SYT+2*SINT(I)*COS(T(I))*TXY
 TAB=CT(S(T(I)))*COS(T(I))*SX1-SINT(I)*COS(T(I))*SY1
 TAB=TAB+(CO(T(I))**2-SINT(I)**2)*TXY
 C SA,SB,Tab ARE STRESS IN CROSS PLY REFERED TO FILAMENT AXES.
 WRITE(2,68) A,SB,Tab
48 FORMAT(SH)
 IF (SA>57.5,55)
55 E=150. / S=CT(SA**2-SA*SB+0.02).*SB**2+352.*TAB**2)
 WRITE(2,56) 10
56 FORMAT(SH E=,3PF10.0)
 GO TO 59
57 EUC=120. / S=CT(SA**2-SA*SB+400).*SB**2+225.*TAB**2)
 WRITE(2,58) 10C
58 FORMAT(SH EUC=,3PF10.0)
 WRITE(2,60) (I), (T)
60 FORMAT(I6,3,F8.4)
62 CONTINUE
61 CONTINUE
 STOP
END

END OF SEGMENT LENGTH 989. NAME COMPOSITE STRENGTH
### Typical Set of Results

For $\sigma_x = 1000$.

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<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
<th>$\theta$</th>
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### Data Table

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<th>$F_{in}$</th>
<th>$F_{out}$</th>
<th>$F_{in}$</th>
<th>$F_{out}$</th>
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<td>21.</td>
<td>945.</td>
<td>21.</td>
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</table>
TABLE 1: Efficiency Factors for Wide Columns  
(Derived from ref.15, Emerso and Spunt)

\[ \sigma_{opt} = F \sqrt{E} \sqrt{\frac{d}{L}} \]

<table>
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</tr>
<tr>
<td>Trapezoidal corrugated, semi sandwich</td>
<td>0.83</td>
</tr>
<tr>
<td>Truss core, semi sandwich</td>
<td>0.83</td>
</tr>
<tr>
<td>Semi trap. corrugated semi sandwich</td>
<td>0.85</td>
</tr>
<tr>
<td>Top hat stiffened</td>
<td>0.96</td>
</tr>
<tr>
<td>Truss core corrugation</td>
<td>1.07</td>
</tr>
<tr>
<td>Semicircle corrugation</td>
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</tr>
<tr>
<td>Truss core sandwich</td>
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FIG. 1. SINGLE UNIDIRECTIONAL PLY
FIG. 2. SIGN CONVENTION
FIG. 3. LONGITUDINAL ELASTIC MODULUS, $E_x$. 
FIG. 8. SIMPLY SUPPORTED COMPRESSION PANEL, BUCKLING COEFFICIENTS
FIG. 10. OPTIMUM STRESS LEVEL FOR S.S. PLATE IN COMPRESSION

STEEL DTD 5052

ALL AXIAL

CARBON FIBRE COMPOSITE

STEEL (ELASTIC)

INCREASING NO. OF AXIAL PLYS

AL. ALLOY. L 73

OPTIMUM STRESS, \( \sigma_{opt} \) (MPa)

\((P/L)^{2/3}\)

0

100

200

300

400

500

600

700

800

100,000

120,000

140,000

160,000
FIG. 11. CORRUGATED COMPRESSION PANEL
FIG 12, CORRUGATED COMPRESSION PANEL, BUCKLING CHART