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Disposal orbits for GEO spacecraft: a method for evaluating the orbit height distributions resulting from implementing IADC guidelines

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Abstract

Geostationary orbit (GEO) is the most commercially valuable Earth orbit. The Inter-Agency Space Debris Coordination Committee (IADC) has produced guidelines to help protect this region from space debris. The guidelines propose moving a satellite at the end of its operational life to a disposal orbit, which is designed so that satellites left there will not infringe the operational GEO region within a period of at least 100 yr.

Standards are being developed through the International Organisation for Standardization to translate the IADC guidelines into engineering practice. This article presents an analytical method for calculating the distribution of final orbits assuming the IADC guidelines in GEO are implemented, as a function of distributions of satellite parameters (mass per unit area, solar radiation pressure reaction coefficient), the fuel measurement uncertainty, and the desired reliability of the disposal manoeuvre.

Results show that typically the fuel measurement uncertainty dominates the distribution of perigee heights rather than the scatter in satellite properties or desired manoeuvre reliability. The method is simple to implement and allows the effects of changes in system parameters to be evaluated quickly.

Key words: GEO, IADC, disposal orbit, ISO, space debris

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1 Introduction

Space debris is a serious and growing problem. From the launch of Sputnik-1 in 1957, most satellites at the end of their mission have been left for the orbit to decay naturally. More than two decades ago this was recognised as a potential problem, and space debris is now an active area of study. Papers presented at the Fifth European Conference on Space Debris (Klinkrad, 2009) provide a good overview of current work, especially in related engineering and technology. Current activities include implementing debris mitigation measures in advance of eventual debris remediation, and are focussed on low Earth orbit where the problem is most urgent; geosynchronous orbit (GEO) is also important because of its commercial significance. However, space debris is an issue which goes far beyond the science and engineering of satellites and their orbits: legal, security and financial issues are also involved. Taylor (2006) provides a helpful overview of space debris including these wider issues.

Several studies of long-term orbit behaviour relevant to space debris mitigation in GEO have been published. Using orbits propagated over 100 yr, Anselmo & Pardini (2008) identify the importance of orbit eccentricity management and passivation as mitigation precautions, as well as presenting results for orbit perturbations which lead to highly eccentric orbits for some light objects (e.g. collision fragments). Chao & Gick (2004) show results for navigation satellite orbits and show how orbit eccentricity at some inclinations can grow as large as 0.7. Such eccentricity growth would dramatically reduce orbit lifetime (the satellites experience much higher atmospheric density at perigee) and also increases collision risk between active and defunct satellites (as defunct satellites leave disposal orbits and cross regions containing active spacecraft): both are significant issues for debris mitigation. Other useful studies of long-term orbit behaviour include Lewis et al. (2004), Westerkamp et al. (1997) and Wytrzyszczak & Breiter (2001). These all illustrate that over long periods of time, subtle perturbations can accumulate to have significant effects on orbit evolution. Studies such as these provide the scientific basis for practical debris mitigation measures now coming into use.

Since the 1990's efforts to understand space debris and to develop mitigation methods have been underway internationally and are coordinated by the Inter-Agency Space Debris Coordination Committee (IADC). Guidelines have been published by IADC (2002) and are being adopted almost universally by space-faring nations. In the last decade an increasing number of missions included deliberate action to remove a satellite from useful orbit regions at the end of the operational phase (e.g. SPOT 1 (Alby, 2005), Inmarsat F3 (Hope, 2007)). More recently, the International Organisation for Standardization (ISO) is supporting the development of international standards which translate the IADC guidelines into engineering practice though its sub-

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committee TC20/SC14 (Aircraft and Space Vehicles / Space Systems and Operations). Several of these standards will shortly be published, and in particular ISO 24113 (Space systems - Space debris mitigation) and ISO 26872 (Space systems - Disposal of satellites operating at geosynchronous altitude) relate to satellite disposal from GEO.

A key requirement for GEO missions drawn from the IADC guidelines is the need for the disposal orbit perigee to be a minimum height (h / km) above the geostationary altitude as expressed in Equation 1.

$$h = 235 + 1000\,C_R A/m \tag{1}$$

C_R is the spacecraft's solar radiation pressure (SRP) reaction coefficient (or SRP coefficient, dimensionless, with a value from 0 (a hypothetical transparent spacecraft) to 1 (totally absorbing - matt black) to 2 (reflecting perfectly back towards the Sun)), and A/m is the spacecraft's area to mass ratio ($\mathrm{m^2\ kg^{-1}}$): these parameters refer to the whole satellite including appendages, averaged over all relevant viewing aspects. Both parameters depend on the reference area used for normalisation: the same area should be used in both cases. The combined parameter $C_R A/m$, sometimes referred to as effective A/m ratio, can be estimated by orbit tracking; example values are 0.02–0.04 $\mathrm{m^2\ kg^{-1}}$ (Hope, 2007).

To enable practical use, ISO 24113 and ISO 26872 require 90% reliability of the disposal manoeuvre (rather than the ideal, but unachieveable, 100%). Combining this with the inherent uncertainty of satellite fuel measurement, designers and operators must be conservative in their fuel budgets so that at least 90% of disposal manoeuvres achieve the IADC perigee increase. The actual perigee heights achieved are thus generally higher than the nominal heights required because of this safety margin. A further effect which can be modelled simply is the spread in perigee height due to gravity and SRP perturbations.

The questions this study aims to answer are (1) what disposal orbits are likely to be achieved once the ISO interpretation of the IADC guidelines is applied, and (2) what are the key factors which determine these orbits? Answers to these questions allow us to evaluate the expected effectiveness of the standards in mitigating space debris, and can be used to quantify satellite collision risks in the GEO region.

The next section derives the analytical relationships describing the disposal orbits expected. Following that, results illustrating the effects of typical parameter values are presented and then discussed. Finally, some conclusions drawn from the research are presented.

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## 2 Method of calculating orbit height distributions

The method presented describes orbits by a probability density distribution of their (perigee) heights above the ideal geostationary height (i.e. probability per unit height interval; the distribution in inclination is assumed to be independent of height). There are three steps:

(1) Calculate the distribution of nominal perigee heights to satisfy the IADC guideline as a function of the satellite properties (C_R, A/m).
(2) Calculate the actual perigee heights achieved allowing for propellant measurement uncertainty and the required disposal manoeuvre reliability.
(3) Allow for the effect of orbit perturbations on orbit height to quantify the long-term distribution of orbit height.

In the following derivations, the symbols p, ρ and P are used for a (a) probability, (b) probability density (e.g. probability per unit height interval in km), and (c) cumulative probability, respectively.

2.1 Distribution of nominal perigee heights

To calculate the distribution of nominal perigee heights using Equation 1 the distribution of the product of satellite parameters $C_R A/m$ is required. This can be evaluated directly if data for a representative satellite population are available, or derived from separate probability distributions for each parameter. For the results presented here, separate distributions for C_R and m/A are combined (m/A is used rather than A/m since it is easily related to the familiar ballistic coefficient $B = m/(C_D A)$). Figure 1 shows the distributions assumed to represent hypothetical and not actual populations of satellites with low or high reflectivity (Figure 1(a)) and low or high mass per unit area (Figure 1(b)); for simplicity the distributions are assumed to be independent. The probability densities are defined conventionally, i.e. as probability per unit x-axis interval, and the integrated area under each curve is exactly 1.0 (dimensionless). The distributions are chosen to bound likely values of the parameters; the distributions are not meant to directly represent populations of existing satellites. Thus real satellite parameters should fall within the range of parameter values assumed (typical values of $C_R = 1.2$ to 1.5 (IADC, 2004) have a tighter distribution than shown in Figure 1(a), so the "dark" and "bright" cases successfully bound actual values; similarly the distribution of A/m values presented by Lewis et al. (2004, Fig. 6) from ESA's DISCOS database falls between the "light" and "heavy" distributions of Figure 1(b)). (Note that the probability densities for A/m and m/A are related by $\rho(A/m) = \rho(m/A)(m/A)^2$.)

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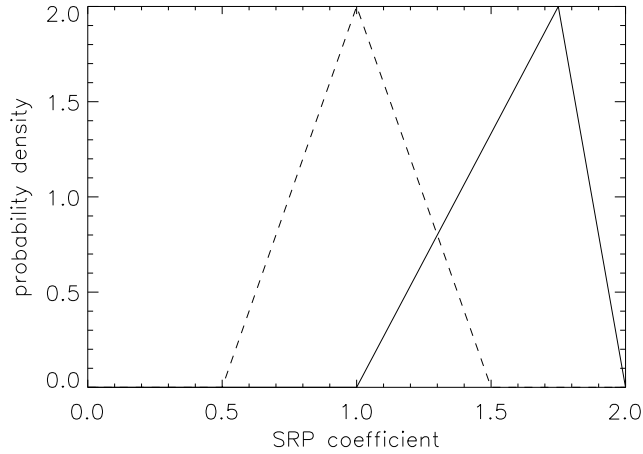
(a) SRP coefficient distribution for the "bright" (solid line) and "dark" (dashed line) cases
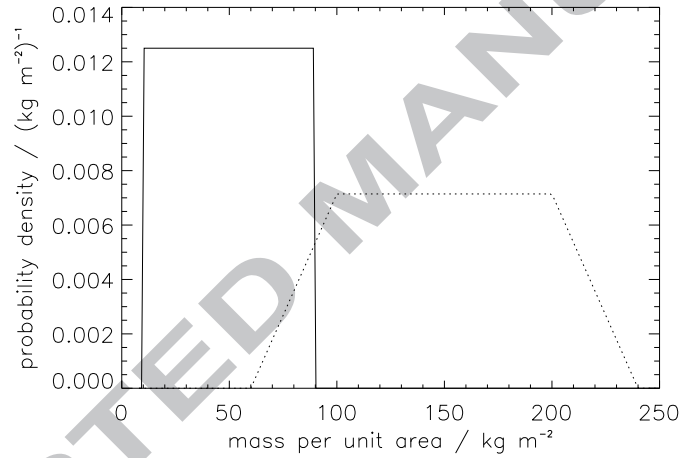
(b) Mass per unit area distributions for the "light" (solid line) and "heavy" (dotted line) cases

Fig. 1. Assumed distributions of parameters for hypothetical satellite populations bounding the values of those likely to be found for current and future satellites.

Figure 2(a) illustrates how the probability for the product of the parameters C_R and A/m is calculated. The integral of the joint probability density over the shaded region gives the cumulative probability that the product is less than or equal to a given value; the differential of the cumulative probability is the probability density.

Consider small intervals in area to mass ratio $(a, a + \delta a)$ and SRP coefficient $(C_R, C_R + \delta C_R)$. The probability that the spacecraft properties are in both these intervals (assuming independent distributions) is
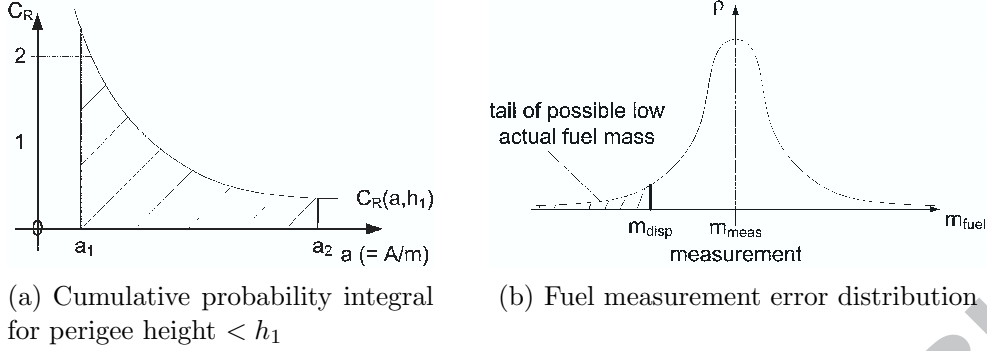
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(a) Cumulative probability integral for perigee height $< h_1$

(b) Fuel measurement error distribution

Fig. 2. Perigee cumulative probability integral and fuel measurement error.

$$\delta p = \rho(a)\,\rho(C_R)\,\delta a\,\delta C_R = \left(\frac{m}{A}\right)^2 \rho(m/A)\,\rho(C_R)\,\delta(m/A)\,\delta C_R \tag{2}$$

Examples of these distributions are shown in Figure 1. The cumulative probability (Equation 3) that the spacecraft properties are such that the re-orbit perigee height is less than or equal to h_1 can be evaluated numerically in two steps:

(1) For a given value of area to mass ratio a, the probability (Equation 2) is integrated from $C_R = 0$ to $C_R(h_1)$ (Equation 4), $C_R(h_1)$ is the smaller of (a) the SRP coefficient which gives a re-orbit height equal to h_1 for that value of $a = A/m$ and (b) 2 (the maximum value of C_R).
(2) The one dimensional integrals of the above step are integrated over all possible values of a, i.e. from a_1 to a_2, to calculate the total cumulative probability.

Thus the cumulative probability that perigee height h (in km) is less than a given value (h_1) is $P(h < h_1)$:

$$P(h < h_1) = \int_{a_1}^{a_2} \int_{0}^{C_R(h_1)} \rho(a)\rho(C_R)\,da\,dC_R \tag{3}$$

$$\text{where } C_R(h) = \min\left(2, \frac{h - 235}{1000\,a}\right) \tag{4}$$

$$\text{and } a = A/m \text{ (m}^2\text{ kg}^{-1})$$

$$\text{Then } \rho(h_1) = \frac{dP(h < h_1)}{dh_1} \tag{5}$$

We write $\rho_{min}(h)$ for this probability density distribution for the minimum required perigee height to satisfy the IADC GEO disposal guideline.

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2.2 Distribution of achieved perigee heights

Figure 2(b) represents the fuel measurement error distribution. No published data documenting actual fuel measurement uncertainty distributions near end-of-life have been found and so the curve represents a general case. Due to the measurement uncertainty, when a given measurement is made (e.g. m_{meas}), the operator is uncertain about the true amount of remaining fuel. To ensure that the actual amount available exceeds the amount necessary for the minimum disposal manoeuvre (m_{disp}) the operator must act conservatively. If the distribution is known, then the margin ($m_{\text{meas}} - m_{\text{disp}}$) can be calculated for any desired reliability. There is still a finite probability (represented by the shaded area in Figure 2(b)) that not enough fuel is available, but this can be kept small enough to satisfy operational requirements by being sufficiently conservative.

For small changes in orbit height (assuming manoeuvres of the same type), the ΔV required is proportional to the height change, and fuel used is proportional to ΔV. Thus the fuel mass required is proportional to the height change, and so the uncertainty in fuel mass is proportional to the uncertainty in achieved orbit height change. It is convenient to express the fuel measurement uncertainty as a fraction of the fuel required to achieve the nominal perigee height: let σ_0 be the ratio between fuel mass measurement standard deviation (σ_{m}) and the fuel mass required to achieve the minimum perigee height for disposal (m_{disp}), i.e. the fractional fuel measurement uncertainty. Then the standard deviation in achieved perigee height change (σ_{h}) is the same fraction of the minimum perigee height (h_{min}).

$$\sigma_0 = \frac{\sigma_{\text{m}}}{m_{\text{disp}}} = \frac{\sigma_{\text{h}}}{h_{\text{min}}} \tag{6}$$

$$\sigma_{\text{h}} = \sigma_0 \, h_{\text{min}} \tag{7}$$

Note that it is assumed that σ_0 is independent of h_{min}. This assumption simplifies the model and without more comprehensive data on fuel measurement uncertainty a more sophisticated model is not justified.

If the distribution of fuel measurement errors is known (e.g. here it is assumed to be Gaussian) then the fuel margin required to ensure a given manoeuvre reliability can be calculated directly. Let z be the normalised deviate for the distribution of fuel measurement errors ($z = \delta m / \sigma_{\text{m}}$ where δm is the difference from the mean of the distribution (the mean is the measured fuel amount), and σ_{m} is the measurement standard deviation), and $P(z')$ be the cumulative probability that $z < z'$. The inverse cumulative distribution gives
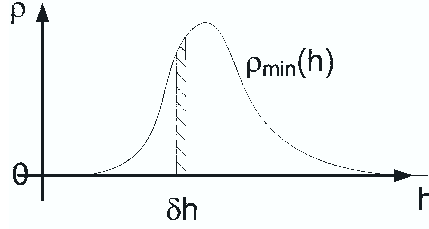
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Fig. 3. Incremental probability $\delta p = \rho \, \delta h$ corresponding to height increment δh for the probability distribution of nominal re-orbit height.

normalised deviate as a function of probability: $z = P^{-1}(p)$, e.g. for a Gaussian distribution $P^{-1}(0.1) = -1.282$ and to ensure $>90\%$ probability that $m > m_{\mathrm{disp}}$ requires $m_{\mathrm{meas}} - m_{\mathrm{disp}} > 1.282\sigma_{\mathrm{m}}$. Assuming that operators use the minimum fuel to achieve the manoeuvre with the required reliability, since fuel mass and orbit height change are proportional, the mean of the distribution of achieved perigee heights with probability p is

$$\overline{h} = h_{\mathrm{min}} - P^{-1}(1 - p)\,\sigma_{\mathrm{h}} \tag{8}$$

$$= h_{\mathrm{min}}\left[1 - P^{-1}(1 - p)\,\sigma_0\right] \tag{9}$$

Thus when the measured fuel is just sufficient to re-orbit to height $\overline{h}$, then the operator should command the manoeuvre since after this time the fuel margin relative to the measurement uncertainty will be too small to achieve the required disposal reliability.

The distribution of achieved perigee heights after the re-orbit is calculated by summing the spreads in height achieved for each nominal perigee height (h_{min}) weighted by the probability of that re-orbit height being required to meet the IADC guideline. This gives the expected distribution of achieved perigee heights for the population of satellites corresponding to the distribution of h_{min}. Figure 3 shows the probability increment corresponding to a small height range for the distribution of nominal re-orbit heights; the corresponding probability increment is $\delta p = \rho_{min}(h)\delta h$. For this value of h_{min}, Equations 9 and 7 can be used to calculate $\overline{h}$ and σ_{h} for the distribution of achieved heights.

This probability increment thus leads to a distribution of achieved heights $\rho(h; \overline{h}, \sigma_{\mathrm{h}})$, which contributes to the total distribution of achieved heights with weighting δp. Because of the proportionality between fuel use and orbit height change, $\rho(h; \overline{h}, \sigma_{\mathrm{h}})$ is just a scaled version of the distribution of fuel measurement errors.

$$\delta\rho_a = \rho(h; \overline{h}, \sigma_{\mathrm{h}})\,\delta p \tag{10}$$

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The complete distribution of actual perigee heights achieved is obtained by summing these contributions over all values of $h_{\min}$.

$$\rho_a(h) = \int\limits_{h_1=h_{min,1}}^{h_{min,2}} \rho(h; \overline{h}, \sigma_{\mathrm{h}}) \, \rho_{min}(h_1) \, dh_1 \tag{11}$$

where $\overline{h}$ and σ_{h} are functions of the required manoeuvre reliability (p), the fractional fuel measurement uncertainty (σ_0), and of $h_{\min}$.

$\rho_a(h)$ is the probability density for the achieved perigee h given the required re-orbit reliability p, the fractional fuel measurement uncertainty σ_0, and the function $h_{\min}(C_R A/m)$ which determines the recommended minimum re-orbit perigee height. As noted above, the probability density $\rho(h; \overline{h}, \sigma_{\mathrm{h}})$ for the height spread at a specific $h_{\min}$ is scaled from the distribution of fuel measurement errors. For a Gaussian error distribution this is given by

$$\rho(h; \overline{h}, \sigma_{\mathrm{h}}) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{h}}} \exp\left(-\frac{(h-\overline{h})^2}{2\sigma_{\mathrm{h}}^2}\right) \tag{12}$$

where $\overline{h}$ and σ_{h} are functions of $h_{\min}$, p and σ_0 (Equations 9 and 7). Data are not currently available to justify a more sophisticated model.

2.3   Distribution including long-term orbit perturbations

Perturbations act on a satellite's orbit in the GEO region so that eccentricity changes cyclically leading to further spreading of orbit height. This results in a long-term distribution of orbit heights which is slightly broader than the distribution of initially achieved heights. Equation 1 includes factors for two aspects of this process. 35 km of the constant value of 235 km are to allow for periodic gravity perturbations which change the orbit eccentricity and lower the perigee over periods of several decades. Solar radiation pressure also changes orbit eccentricity: the term $1000\,C_R A/m$ approximately quantifies this effect (in km). The orbit is thus "smeared" in height by intervals of these sizes over periods of decades and longer; as a first approximation the orbits are assumed to spread uniformly over the corresponding height bands (± 35 km and $\pm 1000\,C_R A/m$).

2.4 Validation and Practical Implementation

To validate a practical implementation of this algorithm it is useful to check the normalisation of the probability density distribution at each stage. The integral of the probability density over all reasonable values of the independent variable should be exactly 1.0 if normalisation has been retained. In practice there will be a small deviation from this due to numerical approximations, but the discrepancy need never exceed 0.01 if satisfactory digitisation resolution is used (i.e. small enough increments in perigee height and C_R, etc.). In addition to this, straightforward tests using plots of intermediate and final results give confidence that equations have been correctly implemented.

3 Results

The algorithm of section 2 has been used to evaluate several specific cases:

- Various satellite properties (low and high SRP coefficient, low and high area per unit mass)
- Alternative fuel measurement uncertainties
- Several levels of disposal manoeuvre reliability

The reference values for disposal manoeuvre reliability and fuel measurement uncertainty are based on the proposed ISO implementation ($p = 0.9$) and on good current propellant monitoring, respectively. Yendler & Jew (2008) indicate that with care the amount of remaining propellant can be estimated with uncertainty equivalent to one month's fuel use. In GEO the typical ΔV allowance for station-keeping is 52 m s^{-1} per year; one month's fuel is therefore equivalent to 4.3 m s^{-1}. A typical ΔV requirement for the nominal IADC disposal manoeuvre is 11 m s^{-1}, so the fractional fuel measurement uncertainty (assumed to be one standard deviation) representative of current best practice is $\sigma_0 = 4.3/11 = 0.4$. Independent estimates of σ_0 for current GEO communication satellites range from 0.2 to 1.35 (for large and small satellites respectively, pers. comm.) suggesting that a typical value of 0.4 is reasonable.

Comparing results for the different cases with various levels of resolution (e.g. 500 or 1 000 point discretization of the probability distributions) indicates that the distribution mean and standard deviation are generally accurate to 1–2 km.
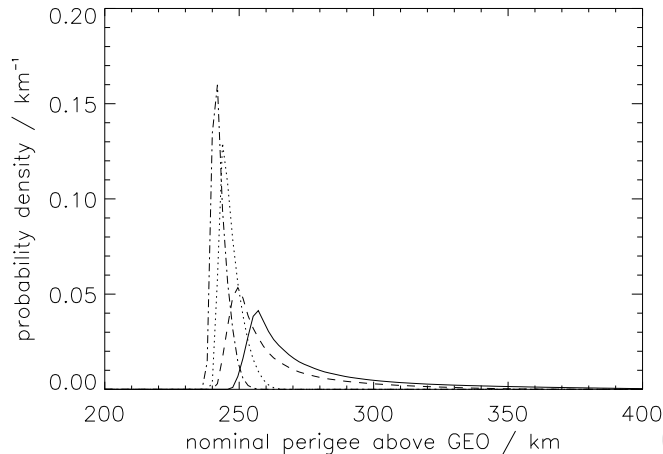
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Fig. 4. Distributions of nominal perigee heights above geostationary derived for the satellite parameter distributions of Figure 1 (and disposal reliability $p = 0.9$, fractional fuel measurement uncertainty $\sigma_0 = 0.4$). The parameter distribution cases are "bright & light" (solid line), "bright & heavy" (dashed line), "dark & light" (dotted line), "dark & heavy" (dash-dot line).

3.1 Orbit height vs satellite properties

Figure 1 shows the satellite properties assumed, i.e. two cases each for the distributions of C_R (bright or dark) and m/A (light or heavy) giving four combinations ("bright & light", "bright & heavy", "dark & light", "dark & heavy"). Figures 4 and 5 and Table 1 show the resulting nominal, achieved, and long-term orbit height distributions for these four cases and the reference parameter values of $p = 0.9$ and $\sigma_0 = 0.4$.

### 3.2 Orbit height vs fuel measurement uncertainty

The two extreme parameter combinations from Figure 1 were used with fractional fuel measurement uncertainties of 0.4 (standard value) and 0.1 (hypothetical more accurate fuel measurement) to investigate the effect of improved fuel measurement accuracy on the orbit distributions (Figure 6(a), Table 2).

3.3 Orbit height vs disposal manoeuvre reliability

Figure 6(b) and Table 3 show the final long-term orbit height distributions for four different disposal manoeuvre reliabilities and fractional fuel measure-

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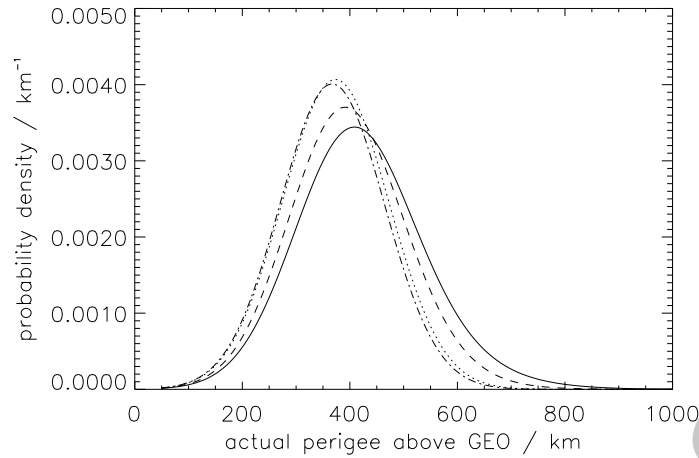
(a) Expected initially achieved disposal orbit perigee distributions allowing for the disposal manoeuvre (reliability and fuel measurement uncertainty)
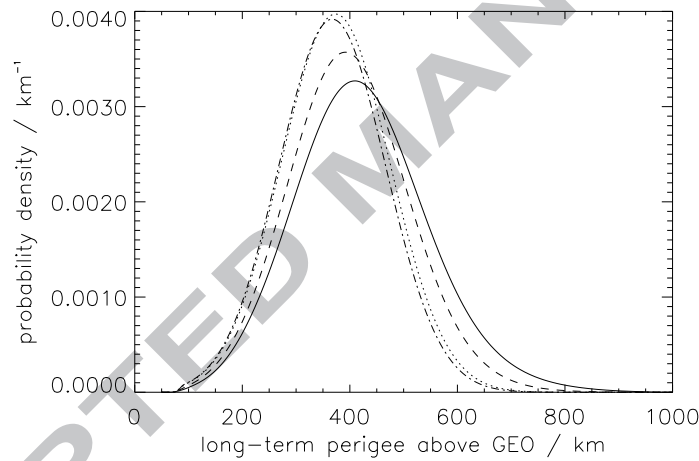

(b) Expected long-term perigee distributions allowing for disposal manoeuvre and orbit perturbations

Fig. 5. Distributions of initially achieved and long-term perigee heights ($p = 0.9$, $\sigma_0 = 0.4$, satellite parameter cases: "bright & light" (solid line), "bright & heavy" (dashed line), "dark & light" (dotted line), "dark & heavy" (dash-dot line)). In this case the initially achieved and long-term distributions are similar but not identical (see Table 1).

ment uncertainty $\sigma_0 = 0.4$.
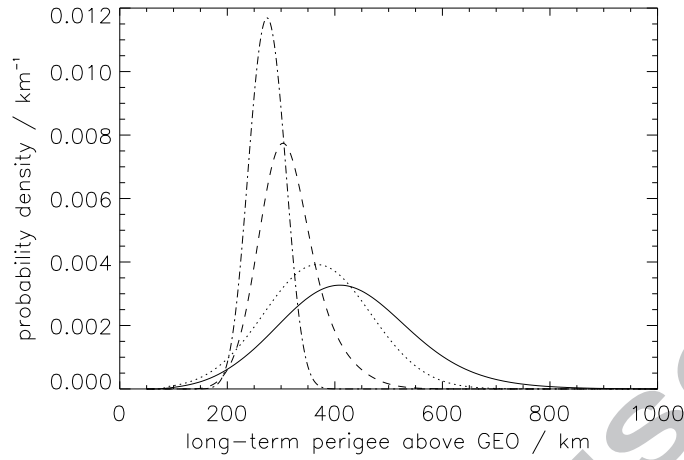
(a) Long term orbit height distributions as a function of fuel measurement uncertainty for the two extreme satellite parameter cases ($\sigma_0 = 0.4$: "bright & light" (solid line), "dark & heavy" (dotted line), and $\sigma_0 = 0.1$: "bright & light" (dashed line), "dark & heavy" (dash-dot line); disposal reliability $p = 0.9$).
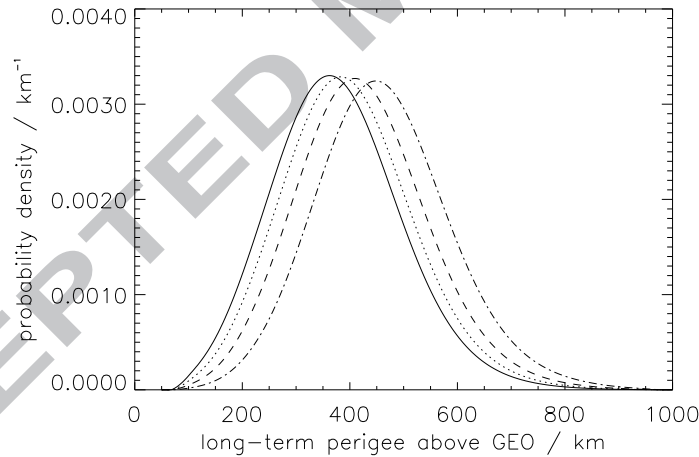
(b) Long term orbit height distributions as a function of manoeuvre reliability ($p = 0.80, 0.85, 0.90, 0.95$ - solid, dot, dash and dash-dot lines respectively; "bright & light" & $\sigma_0 = 0.4$ case)

Fig. 6. Distributions of perigee heights above geostationary derived for two cases of fuel measurement uncertainty and for various disposal reliabilities.

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Table 1

Mean and standard deviation of the orbit height distributions for the four possible satellite parameter distribution combinations from Figure 1 (see Figures 4 and 5, disposal reliability $p = 0.9$ and fractional fuel measurement uncertainty $\sigma_0 = 0.4$)

Case	Nominal IADC		Achieved		Long-term	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
	/ km	/ km	/ km	/ km	/ km	/ km
Bright & light	281	41	422	122	420	124
Bright & heavy	262	70	382	107	384	108
Dark & light	264	28	397	110	396	112
Dark & heavy	248	38	371	100	371	102

Table 2

Mean and standard deviation of the long-term orbit height distribution for two fractional fuel measurement uncertainties (σ_0) and the bright & light and dark & heavy satellite distributions (disposal manoeuvre reliability $p = 0.9$).

Case	$\sigma_0 = 0.1$		$\sigma_0 = 0.4$	
	mean	st. dev.	mean	st. dev.
	km	km	km	km
Bright & light	314	59	420	124
Dark & heavy	279	46	371	101

Table 3

Statistics of the long-term orbit height distribution for disposal manoeuvre reliability p in the range 0.80 to 0.95 (for fractional fuel measurement error $\sigma_0 = 0.4$ and the satellite parameter case, "bright & light", most affected by solar radiation)

p	mean / km	st. dev. / km
0.80	371	120
0.85	393	122
0.90	420	124
0.95	460	126

4 Discussion

Before discussing specific results a few comments on this approach to evaluating disposal orbits are appropriate. The method presented here has both strengths and weaknesses. Its main strengths are that it quantifies the importance of key parameters directly and uses an algorithm which is computationally simple and which requires minimal computing resources. The method is

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relatively transparent, so that results can easily be validated and assumptions are explicit. Its weaknesses include that it does not propagate orbits directly but instead has to rely on parameterizations (e.g. for orbit perturbations) calculated independently. Thus it also ignores effects such as gravitational harmonics which might cause clustering around certain longitudes for orbits very close to the ideal geostationary orbit, and is not time-dependent and so cannot easily capture the process as orbit perturbations disperse the initial disposal orbits. However, as one tool among several, this method is useful for understanding the disposal orbits likely to be achieved by GEO satellites at end-of-life.

Three aspects of the final disposal orbit height distributions are investigated using the results presented. The first is the relative importance of satellite parameters $(C_R, A/m)$ and fuel measurement uncertainty on the spread in long-term orbit heights. Figure 5 shows clearly that for a typical fractional fuel measurement uncertainty of 0.4, it is the fuel measurement uncertainty which predominantly determines the spread of orbit heights. Although the distribution of nominal orbit heights to satisfy the IADC guidelines may vary significantly between different types of spacecraft (Figure 4), once an operator has made appropriate allowances for fuel measurement uncertainty and the required disposal reliability, most of the orbit height spread is determined by the fuel measurement uncertainty and not the satellites' other characteristics. The additional orbit height changes due to long-term perturbations only add a small amount to the width of the orbit height distributions.

The second aspect studied is to quantify the effect of improving the fuel measurement accuracy. Figure 6(a) shows the influence of reducing the fractional fuel measurement uncertainty to 0.1, i.e. about 1 m s^{-1} of ΔV. At this level the height distribution narrows significantly (to about half the width for $\sigma_0 = 0.4$) and satellite parameters become relatively more influential on the final orbit. This fuel measurement accuracy corresponds to about one week of normal station-keeping, and it is unlikely that there are benefits to justify an operator making great efforts to achieve or exceed this. Conversations with operators indicate that σ_0 can vary significantly between spacecraft. If the trend for larger GEO satellites with more accurate fuel measurement continues, small rather than large (> 0.5) values for σ_0 seem appropriate.

The third issue investigated is the influence of the disposal manoeuvre reliability on the final orbits. Figure 6(b) shows the final orbits for the bright & light case (the case for which satellite properties are most influential) for four different levels of reliability (the ISO implementation of the IADC guideline requires a reliability $p \geq 0.9$). The distribution mean increases as p increases, by about 100 km as p increases from 0.80 to 0.95, and its standard deviation increases slightly (from 120 km to 126 km). The safety margin an operator needs so that disposal reliability is above 50% represents a penalty in fuel

which, on average, will be "wasted" since it takes the satellite above the IADC minimum altitude. The term $-P^{-1}(1 - p)\sigma_0$ of Equation 9 quantifies this penalty: for a Gaussian distribution and the reliabilities of 0.80, 0.85, 0.90, 0.95, these penalties are $0.842\sigma_0$, $1.036\sigma_0$, $1.282\sigma_0$ and $1.645\sigma_0$ respectively. Thus if an operator chooses to plan using a disposal reliability of 0.95 instead of 0.90, the extra height (proportional to extra fuel required) is equivalent to $0.363\sigma_0$ or around 1.6 m s^{-1} of ΔV (for $\sigma_0 = 0.4$). This suggests there is a relatively small penalty for an operator to plan conservatively and to design the disposal manoeuvre with a reliability better than 90%.

5   Conclusions

The method presented here efficiently quantifies the influence of the main parameters on the distribution of GEO disposal orbit heights. It explicitly evaluates the probability density distribution of orbit heights, and is designed especially to quantify the effect of the ISO implementation of the IADC guidelines. It includes parameters describing the satellites (SRP coefficient C_R, area to mass ratio, fuel measurement uncertainty), the desired disposal orbit height (calculated using the IADC guideline), and the required probability of success for the disposal manoeuvre.

The main finding from this research is the practical importance of the fuel measurement accuracy. Using representative values for satellite properties, the main factor determining the spread of orbit heights is the fuel measurement uncertainty rather than variation in the satellite's SRP coefficient or area to mass ratio. With a fuel measurement uncertainty corresponding to about 4 m s^{-1} of ΔV (typical current best practice), the width of the disposal orbit height distribution will be approximately 200 km. Increasing the manoeuvre reliability above the ISO requirement of 0.90 has only a small impact on the propellant required.

Future applications of this method include evaluation of alternative GEO disposal manoeuvre requirements and using the orbit height distributions to estimate collision probabilities. It is straightforward to couple this model with different satellite population growth scenarios to quantify the impact of different policy scenarios, for example. The model provides a useful addition to the tools available to evaluate space debris mitigation options.

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6   Acknowledgements

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