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THERMAL DESIGN OF A COMPACT RECUPERATIVE
HEAT EXCHANGER FOR A STIRLING ENGINE

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Variable print quality
TO MY WIFE HEIYAM FOR HER PATIENCE AND REAL HELP.
I wish to express my gratitude and thanks to K. B. United Stirling of Malmö, Sweden for their encouragement and generous support of this project.

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SUMMARY

This thesis is concerned with factors affecting the thermal design of a compact recuperative type crossflowing heat exchanger for the primary heater of a Stirling engine. The exchanger is constructed of small diameter metal tubes (in the range of 3.0 mm to 6.0 mm) and close spacings are maintained between the tubes (i.e. in the range of 0.30 mm to 1.80 mm). These small slender tubes are usually arranged in single or double rows and are mounted around the combustion chamber. The exchanger is used to heat hydrogen or helium which act as a working fluid for the Stirling cycle.

A survey of the published literature indicated that the available data does not include results for the tube geometries of interest in this study. Consequently the heat transfers and hydraulic resistances were measured experimentally for a single row of small diameter, closely-spaced tubes situated in a crossflowing fluid stream. The Reynolds numbers (based on the mainstream fluid velocity and the tube diameter) ranged between 300 and 6500. Where possible the accuracy of the experimental procedure was checked by comparing the present results with those obtained by previous workers. Several arrangements of both bare tubes and tubes fitted with extended surfaces were studied. The results were analysed and discussed, and where appropriate compared with those published in the open technical literature. In most comparative cases excellent agreement was experienced and any departure, could be explained.

For the bare tube arrangements the influences of flow blockage ratio, mainstream turbulence intensity and surface roughness on the average heat transfer performance were investigated. A comparison of the heat transfers and pressure drops characteristics of the different tube arrangements led to proposals for an optimal exchanger geometry. The validity of the empirical corrections suggested by previous workers to account for the influence of flow blockage on average heat transfers was examined. An alternative modified empirical expression was then proposed for the particularly high flow blockage situations (D/H > 0.85).

It was found that at these high flow blockages the overall average heat transfers were independent of the mainstream turbulence intensity. However preliminary tests suggested that an increase in the tube surface roughness increases the average tube heat transfer.

Since the proposed heat exchanger operates at the highest possible mean metal temperatures, it is likely that 'hot spots' occurring due to variations in local heat transfers can lead to premature failure. Consequently a detailed study of the local heat transfer distributions is presented for various geometrical conditions. The influences of blockage ratio and mainstream Reynolds numbers are examined and the results are analysed, and discussed, and where possible compared with other published data. The accuracy of experimental procedure employed in these tests was checked by comparing the results for a single cylinder case with those reported by other investigators.

The influence of fitting a single longitudinal fin to the rear of tubes on both the heat transfer and pumping power was studied. The tube dia-
meter in these tests was kept constant, at 6.0 mm, but the angle of
inclination of this longitudinal fin was varied incrementally so that an
optimal angle for maximum performance is recommended. In a similar
manner, transverse finned tubes with two different fin spacings were
also investigated. The heat transfers and pressure losses obtained for
the different finned tube arrangements were compared with each other and
with those obtained for the bare tube geometries so that an optimal tube
configuration was proposed.

The data presented in this thesis were generalized, where possible, so
that the results should be useful for future work. They should thus
contribute to an understanding of the basic phenomena associated with
modern compact heat exchangers. Recommendations for further work are
also presented.
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NOMENCLATURE

A  constant

\( A_c \)  minimum free flow area, \( m^2 \)

\( A_r, A_t \)  fin side and tube side heat transfer area respectively, \( m^2 \)

\( A_m \)  mean value of the inside and outside surface area, \( m^2 \)

\( A_i, A_o \)  inside and outside surface area of the tube fitted with extended surfaces, \( m^2 \)

\( A_s \)  total heat transfer area, \( m^2 \)

\( a \)  overheating ratio, \( a = (R - R_o) / R \)

\( B \)  constant

\( B \)  volume expansion coefficient; applied to 'Gr'

\( b \)  fin thickness, \( m \)

\( b' \)  average mass transfer coefficient, \( m/sec. \)

\( b'_p \)  local mass transfer coefficient, \( m/sec. \)

\( c \)  constant

\( \dot{c} \)  basic orifice discharge coefficient, dimensionless

\( \bar{c} \)  concentration of Ferrocyanide in \( K mol / m^3 \)

\( C_p, C_v \)  specific heat of mainstream fluid at constant pressure and volume respectively, \( J/ Kg \cdot K \)

\( D \)  tube diameter, \( m \)

\( D_s \)  diffusivity, \( m^2/sec. \)

\( D_v \)  volumetric hydraulic diameter, \( m \)

\( d_{or}, d_d \)  orifice diameter and duct diameter respectively, \( m \)

\( E \)  velocity of approach factor

\( E_1, E_2 \)  empirical curves represent function of \( Re, X \cdot Tu \) and function of \( \lambda_x / D \) respectively

\( e \)  emissivity

\( \bar{e} \)  dimensionless expansibility factor

\( 1/F_0 \)  fouling factor, \( m^2 \cdot X^0 / W \)

\( F, f_1 \)  denotes function

\( f \)  Fanning friction factor
4 X Fanning friction factor

H transverse pitch of a single row of tubes in crossflow; in the case of single cylinder it is equal to wind tunnel width, m

h heat transfer coefficient, W/m²·K

D/H blockage ratio

hₕ, hₜ fin side and tube side heat transfer coefficient, W/m²·K

I electrical current, Ampere

Iₙ local limiting current, Ampere

J₂ dimensionless heat transfer coefficient

J₄ dimensionless mass transfer coefficient

k/x thermal conductance of fin material, W/m²·K

K_film thermal conductivity of fluid at mean film temperature, W/m·K

K_s thermal conductivity of stainless steel, W/m·K

L tube length, m

lₒ segment length, m

M distance between immediate holes in a perforated plate, m

m exponential constant

mₒ dimensionless area ratio

m Naphthalene weigh loss, kg

n exponential constant

n valence change of the appropriate ion

P pressure, N/m²

P_atm atmospheric pressure, N/m²

ΔP pressure difference, N/m²

P_w Naphthalene vapour pressure at T_w, N/m²

R instantaneous value of probe resistance, ohm

R₀ probe resistance at ambient temperature, ohm

R_c cable resistance, ohm
$R_s$ resistance of bridge arm in series with probe arm, ohm

$R_T$ probe resistance at temperature $T$, ohm

$R_v$ gas constant ($J/kg\cdot K$)

$r$ mean radius of connecting bends at ends of tube

$Q$ total input power in watt

$q_{r}, q_r$ volume rate of flow at initial and reference conditions respectively, $m^3/sec$.

$q_{f}, q_b$ heat flux of the fin tip and fin base in watt/m$^2$

$q$ total heat flux, $w/m^2$

$q_c$ net convective local heat flux, $w/m^2$

$q_p$ circumferential conduction, $w/m^2$

$q_y$ axial conduction, $w/m^2$

$q_r$ radiation dissipation, $w/m^2$

$u_o$ mainstream velocity prior to test section, $m/sec.

$u_m$ integrated mean velocity, $m/sec$.

$u_r$ reference velocity, $m/sec$.

$S$ surface area, $m^2$

$S_d$ standard deviation of error, $S_d = \frac{1}{n} \sum_{j=1}^{n} (x_j - \bar{x})^2$

$S_1$ transverse pitch of a multi-row tube bank, $m$

$S_2$ longitudinal pitch of a multi-row tube bank, $m$

$T_a$ temperature of the mainstream air, $^\circ C$

$T$ temperature, $^\circ C$

$T_s$ segment temperature, $^\circ C$

$T_w$ surface temperature of the Naphthalene, $^\circ C$

$T_1, T_2$ true cylinder surface temperature on either side of the measurement segment as indicated by a curve fitted through the uninsulated thermocouple readings, $^\circ C$

$T_3$ temperature at the end of the measurement segment, $^\circ C$

$T_t, T_f$ temperature of the tube side and fin side, $^\circ C$
mass transfer test duration, sec.
y appropriate longitudinal distance between thermocouples, m
$V_o$ actual value of bridge voltage at ambient temperature but with zero fluid velocity, volt
$V_B$ voltage at bridge top (output voltage), volt
$V_{r.m.s}$ root mean square value of output $AC$ voltage, volt
$v_1, v_2$ initial and exit specific volume of fluid, J/kg.k
$V_{iT}, V_{oT}$ bridge voltages at zero fluid velocity and at an arbitrary velocity respectively, both at temperature $T$, volt

Greek Symbols

$\lambda$ overall heat transfer coefficient for fluid inside finned tube modified to include the outside surface area, $\text{w/m}^2/\text{k}$
$\lambda_x$ turbulence scale, m
$\theta$ temperature difference between the fin base and surrounding, k
$\lambda/D$ surface roughness parameter, dimensionless
$\delta$ insulated slot width, m
$\mu$ dynamic viscosity, kg.m/sec.
$\bar{x}$ mean distribution of experimental data, $\bar{x} = \frac{1}{n} \sum_{j=1}^{n} x_j$
$\alpha$ longitudinal fin position relative to the mainstream direction degrees
$\psi$ angle of fluid attack relative to the test section, degrees
$\phi$ positional angle of the local heat transfer measurement from the front stagnation point, degrees
$\gamma$ fin efficiency
$\gamma^0$ weighed value of the surface efficiency
$\rho$ density of fluid at mean film temperature, kg/m$^3$

Dimensionless Numbers

$Bi$ Biot number, $Bi = h.b / 2k$
$N_R$ removal number, $N_R = q_b / h.b.\theta$
$Gr$ Grashof number, $Gr=\frac{g.\beta.\rho.\theta^2.D^3(T_w-T_o)}{\mu^2}$
\( N_G \) generation number, \( N_G = q_f \cdot b / 2 h_\theta \)

\( \text{Nu} \) Nusselt number, \( \text{Nu} = h \cdot D / k_{\text{film}} \)

\( \text{Nu}_B \) an increase in Nusselt number due to an increase in blockage ratio

\( \text{Nu}_T \) an increase in Nusselt number due to an increase in turbulence intensity of the mainstream fluid

\( \text{Nu}_i \) Nusselt number of the inside tube surface

\( \Delta \text{Nu} / \text{Nu} \) total increase in Nusselt number due to the combined effect of turbulence intensity and blockage ratio

\( \text{Pr} \) Prandtl number, \( \text{Pr} = C_p \cdot \mu / k \)

\( \text{Re} \) Reynolds number; \( \text{Re} = \rho \cdot u \cdot D / \mu \)

\( \text{Re}_h \) Reynolds number based on the hydraulic diameter

\( \text{Sc} \) Schmidt number, \( \text{Sc} = \mu / \rho \cdot D_s \)

\( \text{Sh} \) Sherwood number, \( \text{Sh} = b \cdot D / D_s \)

Subscripts

\( b \) base

\( f \) fin

\( i \) inner

\( o \) bulk conditions

\( \text{eff} \) when applied to 'h' this denotes effective heat transfer coefficient based on surface area of bare tube alone

\( \text{act} \) denotes heat transfer coefficient based on actual area (i.e. surface area of fin and tube)

\( r \) reference

\( w \) wall

\( \text{max} \) maximum

\( \text{atm} \) atmospheric

\( m \) integrated mean
CHAPTER 1. GENERAL INTRODUCTION AND DETAILS OF THE PRESENT INVESTIGATION

1.1 Introduction

A knowledge of the convective heat transfers and hydraulic resistances associated with tubular heat exchangers is of considerable importance since these devices are employed in applications such as steam power plant, chemical and process equipment and mobile power plants for both automotive and marine use. Many new requirements have arisen in recent years particularly in the aerospace and nuclear fields. Design information for the types of exchanger commonly employed is provided extensively in a wide range of papers (see Afgan and Schlunder, Ref. 1) as well as in many text-books such as those edited by Kays and London (Ref. 2), Frass and Ozisik (Ref. 3), Hynizak (Ref. 4) and Kern and Kraus (Ref. 5).

The relative heat transfer surface area required for a particular duty can vary significantly depending on the design and type of equipment used. The importance of effective heat exchanger design and specification is indicated by the estimate that in many process and energy conversion systems, at least 30% to 40% of total metal consumption is employed for heat exchanger construction and manufacture (Ref. 6). Thus heat exchangers should be as compact as possible particularly in applications where space and weight considerations are significant e.g. in mobile power plant. This requirement has thus motivated continuous research and development into the production of heat exchangers with intensified heat transfer characteristics and simultaneous low frictional losses.

These units are often classified into three basic types namely:

(i) Direct transfer type e.g. an air heater in which air is passed directly over electric resistance elements or a cooling tower.

(ii) The recuperative design in which heat is exchanged between two fluids separated by a solid partition.

(iii) The regenerative or periodic flow type in which the heat exchange fluids flow alternatively through a storage matrix.

Recuperators are most commonly employed since a regenerative device with the equivalent heat transfer area is less effective. Nevertheless this latter type finds use in certain applications such as:

(i) In processes using gases which are corrosive or at high temperatures and pressures, recuperators are impractical due to the material property limitations of metals. The ceramic matrices employed in regenerators are thus preferable.

(ii) In fields where particularly low exchanger pressure drops are desirable (e.g. in small gas turbine exhausts), the fluid velocities and hence the convective heat transfer coefficients are limited. Consequently to obtain a worthwhile effectiveness from the exchanger, the heat transfer area must be correspondingly high. A large surface/volume ratio can most conveniently obtained with the small passages which can be incorporated in a regenerative storage matrix.
Recuperative exchangers are often constructed of tubes arranged in single or multipass tube banks whereby the fluids are kept separate as they pass through the exchanger. In many cases, the film heat transfer coefficient on the inside of the tube wall is very much higher than that on the outside so that the overall heat transfer coefficient between the two fluids is mainly controlled by the outer film resistance (Ref. 2). Consequently, if information regarding the heat transfer coefficients on the outside of the tube is available thermal design calculations can be carried out. Alternatively inner wall heat transfers can be readily predicted.

The convective heat transfer associated with the outer wall of a tube in a cross-flowing fluid is dictated by the flow structure which exists on the surface. For a tube in a bank flow structure is dependent upon the Reynolds number and turbulence characteristics of the mainstream fluid as well as the geometrical arrangement and surface characteristics of the tubes (see detailed description in chapter 2). Thus any mathematical analysis of the boundary layers is complicated so that a general approach is not possible, except for relatively simple situations (Refs. 8, 9, 10). These theoretical techniques are confined to cases of laminar flow where the governing phenomena are fairly well understood but unfortunately the practical application is limited (Refs. 11, 12). It is, therefore, not surprising that extensive use is made of purely empirical heat transfer correlations based on published experimental results obtained by many investigators over a wide range of test conditions. Comprehensive reviews of the pertinent studies on smooth tubes in crossflowing fluids are provided by Morgan (Ref. 11) and Zukauskas (Refs. 13, 14). These reviewers, however, point out that there are still conditions for which the published data has only limited use and the validity of the generalized correlations is questionable, (Refs. 12, 15, 17). Typical situations in which these uncertainties prevail include, tube banks with very high flow blockages, tubes with high surface roughnesses and highly turbulent flows.

In the case of tubes with extended surfaces, more complicated phenomena than that associated with bare tubes is involved so that generalized empirical formulae are not applicable. Consequently, the available experimental correlations are only useful over a limited range (the system geometries must be similar but not necessarily identical for the similar mainstream fluid flow conditions). This perhaps partially justifies the lack of a comprehensive review of the published studies on heat transfers from tubes with extended surfaces. Nevertheless, a large body of experimental data exists in the technical literature and this is partially surveyed by Kay and London (Ref. 2) and also in other textbooks such as (Refs. 3, 5).

This present thesis is concerned with the basic design of a compact crossflow type of recuperative heat exchanger. The heat transfers and hydraulic resistances of several arrangements of small-diameter, closely-spaced tubes both with and without extended surfaces, were examined. Capital and operating costs are not directly considered in this study but where appropriate consideration is given to constructional features. Thus complicated shapes of secondary surface were not investigated and similar high cost tubes such as those with louvered, elliptic or turbulence generating cross-sections were not studied.
Whilst there is a scope for more work on the subject, an optimal tube arrangement is suggested for the present tube geometric and test flow conditions.

1.2 The Cross Flow Recuperative Heat Exchanger

Tubular cross-flow heat exchangers are employed in the Stirling engines, at present under development, to heat hydrogen which acts as the working fluid for the cycle. A typical prototype unit produced by K. B. Utd. St. at an advanced stage of development is shown in plate 1. A schematic representation of the primary heating system used in this prototype is presented in Fig. 1. A light distillate liquid fuel is burnt in air and the hot combustion gases are then passed through the high temperature heat exchanger. Hydrogen passes through the inside of tube matrix in this high temperature heat exchanger so that it is heated by the combustion gases via the tube wall. This tube matrix consists of one or two rows of slender tubes of diameter 3-8 mm. comparatively close spacings (in the range of 0.5 - 3 mm) are maintained between the tubes.

The effective design of this high temperature exchanger is of paramount importance in the development of a high specific output Stirling engine. This exchanger must be constructed to provide the highest possible cycle temperature i.e. the temperature difference between the working fluid and the combustion gases must be minimized. On the other hand there is a strong incentive to minimize the overall weight, volume, and the pressure drop. Thus with these conflicting demands, an optimum design will only be possible after extensive investigations of the following areas:

(i) The influence of the system geometry (i.e. the tube spacing and diameter) on the overall heat transfer coefficient and pressure drop characteristics at various flow conditions.

(ii) The exchanger tube matrix operates at the highest possible mean temperature, so that 'hot spot' formation due to variations in local heat transfers could lead to premature failure. Consequently, extensive study into the local heat transfer distribution is essential for various test conditions.

(iii) It is likely that the performance of the recuperative exchanger will be significantly improved by the addition of secondary extended surfaces. Thus the influence of various extended surfaces should be studies.

1.3 Need and Justification for the Present Study

The ultimate object of the present thesis is to obtain accurate experimental data on the heat transfer and pressure drop characteristics associated with a single row of small diameter tubes mounted with close spacings. Both bare tubes and those with extended surfaces will be studied so that direct performance comparisons will be made.

As mentioned previously a considerable amount of forced convective heat transfer and pressure data has been published for banks of tubes in crossflowing fluids. However, a survey of this available literature, (see Chapter 2 of this thesis), indicates that the data does not
include results for the tube geometry of interest in this study.

First, it appears difficult to develop dimensionless heat transfer correlations at high blockage ratios. Furthermore, the effects of turbulence intensity, and surface roughness, are not readily apparent for these highly blocked geometries. The available data has generally been determined with large diameter tubes. However, the growth of the boundary layer on the front of the tube, is proportional to a fractional power of the diameter so that these closely spaced small diameter tubes may inhibit the growth of the boundary layer. Surface roughness effects may also be more significant with small diameter tubes.

Secondly, there is a comparative paucity of published information concerned with local heat transfers for cylinders at high flow blockages. The published data are concerned only with a few Reynolds numbers which are considerably higher than those of interest in the present investigation (where \( \text{Re}_0 \) ranges from 300 to 6500).

Thirdly, extended surfaces require further study since generalised empirical correlations are not available in these cases due to the large number of variables which require examination.

The results should thus assist in the design of the primary heat exchanger in the Stirling engine. An effective and economic design for this exchanger will remove one of the principal difficulties in producing a commercial Stirling engine for automotive purposes (Ref. 16). The future use of these engines appears attractive as an alternative to the diesel engine since they are likely to be more efficient, have less noise and vibration problems and in principle are capable of burning a range of fuels with minimal pollutant formation.

The data produced in this present thesis should also contribute to understanding the basic phenomena associated with modern compact recuperative heat exchangers.

1.4 Details of the Present Investigation

As discussed in the previous section this thesis examines the heat transfers and hydraulic resistances associated with a single row of small diameter (3.0 - 6.0 mm), closely spaced (0.3 - 1.8 mm) tubes in a crossflowing air stream. A range of Reynolds numbers between 300 and 6500 (based on the mainstream velocity and tube diameter) is studied. The experiments involved both

(a) Direct heat transfer measurements. These were employed with smooth tubes and those fitted with extended surfaces.

and (b) A heat-mass transfer analogy (sublimation of naphthalene) which was used to determine average heat transfers for smooth cylinders.

Where possible the accuracy of experimental procedure was checked by comparing the present results with those reported previously in the literature. In chapter 3 the average heat transfers associated with a single row of tubes in crossflow were investigated. The influence of
flow blockage ratio, surface roughness, and turbulence intensity on these average heat transfer characteristics is examined. A comparison of the heat transfer and pressure drop characteristics of the different tube configurations led to proposals for an optimal tube arrangement. The validity of the empirical corrections suggested by previous workers to account for the influence of flow blockage was examined, and an alternative modified empirical correction proposed for the higher blockages (> 0.85).

The distribution of local heat transfer coefficient around the perimeter of these tubes was then investigated and is reported in Chapter 4. The influences of tube blockage ratio and mainstream Reynolds number are examined. The results are analysed, and discussed, and where possible, compared with other published data despite the differing test conditions. The heat transfer technique was initially tested using a single cylinder and comparisons with the results of previous investigations were used to establish the suitability of the method.

The influence of fitting a single longitudinal fin to the rear of tubes is examined in Chapter 5. The effect on both heat transfer and pumping power was studied. The angle of inclination of this longitudinal fin was varied incrementally so that an optimal angle is recommended. Transverse finned tubes (with two different fin spacings) were also investigated in this chapter. The heat transfers and pressure drops are compared with those obtained for the smooth tube and longitudinal finned tube arrangements.
FIG. 1. SCHEMATIC REPRESENTATION OF 'STIRLING CYCLE'
CHAPTER 2  GENERAL SURVEY OF THE RELEVANT LITERATURE.
CHAPTER 2 A GENERAL SURVEY OF THE RELEVANT LITERATURE

2.1 Introduction

This review is concerned with the behaviour of single cylinders, banks of smooth tubes, and arrangements of tubes with extended surfaces in crossflowing fluids. The flow characteristics as well as the heat transfer processes are discussed since the latter are considerably influenced by the flow pattern (Ref. 13). Separate sections are devoted to the influences of Reynolds number, turbulence intensity and the tube blockage ratio. The effects of the geometrical configuration, the number of rows and the angle of fluid attack are also discussed. The variation of the hydraulic resistance with parameters such as the Reynolds number and the geometry is then considered. The ratio of heat transfer to the pumping power required to overcome the flow resistance is used to define the optimal arrangement. Most of these aspects have been extensively investigated, but are available only in widely scattered publications. However, because of the wide range of available data, emphasis has been placed on recent publications so that this review is presented in a compact and practical form. Furthermore the review is generally restricted to the subcritical regime of Reynolds number (i.e. for the range 300 to $2 \times 10^5$ based on mainstream velocity and tube diameter). Alternative extensive reviews are available in references (11, 13) for smooth tubes, and references (2, 5) for tubes fitted with extended surfaces. The various experimental arrangements employed by previous investigators are not discussed in this chapter, since these are included where appropriate later in the thesis.

2.2 Single Cylinder Behaviour

2.2.1 Flow Characteristics.

When a fluid flows over a circular tube a laminar boundary layer begins to form at the front stagnation point. The thickness of this layer increases with downstream distance. The flow Reynold's number $Re_\theta$, which expresses the ratio between inertial and viscous forces, is used to distinguish between the several different types of flow which occur. For single tubes $Re_\theta$ is based on mainstream fluid velocity and the tube diameter.

As the Reynolds number is increased, the relative influence of the inertial forces also increases and the laminar boundary layer separates prior to reaching the rear stagnation point. A symmetrical pair of vortices appears at the rear of the tube, forming a circulation zone confirmed by the laminar flow lines. These vortices become extended downstream with further increases of Reynolds's number until at $Re_\theta < 40$ they are periodically shed from the rear of the tube, and the stability of the circulation region is lost. This boundary layer separation phenomena, is associated with the pressure and velocity distribution of the flow around the tube. Energy is dissipated in overcoming the internal friction of the boundary layer, and as the velocity decreases and hence the pressure rises near the rear of the tube, the remaining energy is insufficient to overcome this adverse pressure increase. Consequently, slow moving particles in the boundary layer are brought to rest or eventually their direction of motion is reversed so that
vortices are formed and shed from the rear of the tube. At still higher Reynolds numbers, these vortices tend to be washed away alternately from opposite side of the cylinder. This alternate shedding is a typical example of the so-called 'von Karman vortex streets'. At Reynolds numbers below $2 \times 10^3$ the vortices are decelerated, and the flow quickly reverts to the initially laminar or viscous system. The downstream distance required for the re-attainment of this flow increases with an increase in Reynolds number. Re larger than $2 \times 10^3$, the initially formed re-circulations break up to form a field of smaller vortices which persist for considerable distances downstream. The turbulence associated with this vortex shedding greatly affects the heat transfer. (Refs. 11, 13, 27).

The flow pattern around a cylinder in crossflow is also dependent however on the wind tunnel characteristics, the upstream flow history, the degree of turbulence and the surface parameters (e.g. roughness, etc.). In view of the complexity of the phenomena it is not surprising that further research is still required for full understanding. Moreover this complexity also precludes rigorous theoretical or numerical analysis (Ref. 12). Nevertheless, many measurements of local pressure distributions, skin friction and form drag have contributed to an understanding of the flow characteristics. The separation points, for example, have been clearly identified and in most cases delineated by $\beta = 80^\circ$ for flows up to a critical Reynolds number of $2 \times 10^5$, see (Ref. 14).

2.2.2 Heat Transfers.

The heat transfers depend on the flow patterns. Consequently due to the large number of parameters which affect these flow measurements of the heat transfers can exhibit considerable scatter even at nominally similar Reynolds numbers, see for example McAdams (Ref. 7). However, most previous researchers (e.g. Refs. 7, 20, 21, 23), have shown that the Nusselt number can be predicted by an expression of the form

$$\text{Nu} = (A + B \text{Re}^n) \text{Pr}^m$$

The calculations of both the Nusselt and Reynolds numbers should be based on mean film temperature.

Morgan (Ref. 11) reviewed the heat and mass transfer results of some 75 previous investigators and examined the dependence of the heat transfer on Reynolds number. The effect of yaw angle, combined natural and forced convection, and the temperature loading are also evaluated. Values of the constants $A$, $B$, $n$ and $m$ are presented in a tabulated form for these previous investigators. Morgan shows that the percentage coefficient of variation for the Nusselt number at a given Reynolds number is from 10% to 29%, depending on the flow velocity. However, the variation between the various correlations is from 10% to 46% so that these correlations do not help to reduce the uncertainty in the heat transfer relationship. One possible explanation for the wide scatter is that the blockage ratios encountered in these studies ranged between $7 \times 10^5$ and 0.60. The influence of blockage ratio and turbulence intensity on the average heat transfers from circular cylinders has also been examined by Morgan. He has referred to several published suggestions for correcting for blockage (both solid and wake), but apparently none of these methods is applicable over a wider range.
For Reynolds numbers ranging from $10^2$ to $10^5$, Richardson (Ref. 22) suggested that the average heat transfer may be represented by a two term relationship:

$$\text{Nu} = A \text{Re}^{0.5} + B \text{Re}^{0.67}$$  (3)

The first term is concerned with the laminar flow region up to the separation point (delineated by $\phi = 90^\circ$ for the regime of interest in this thesis). This region is most affected by the bulk steam turbulence and least by blockage ratio effects. The second term is concerned with the separated flow region and thus is more affected by tunnel blockage than by bulkstream turbulence. It was suggested that (a) varied from 0.37 to 0.55, depending on the turbulence level whereas (b) ranged from 0.057 to 0.094 depending on blockage ratio.

It is apparent from the foregoing that the heat transfers are particularly dependent on the Reynolds number, tube blockage ratios and mainstream turbulence so that the effect of these will be considered separately.

**A The Influence of Reynolds Number**

At the low Reynolds numbers (<40) associated with a stable vortex pattern, heat transfers over the upstream half of the cylinder are greater than those associated with the rear half. Increase of the Reynolds number alters this ratio; the heat transfers at the rear increasing at a faster rate than those at the front.

However, Eckert and Soehngen (Ref. 21) studied the heat transfer at low Reynolds numbers (20 - 500) using a Mach-Zehnder interferometer. Local values of heat transfer were obtained from the interferograms. The percentage contribution of the rear half to the overall heat transfer was found to be approximately 15% at the lower Reynolds numbers. Even at the higher Reynolds number studied, the heat transferred from the front half was still larger than that from the rear, see Fig. 2. This is to be expected since the frequency of vortex shedding is still comparatively low. Another study by Krall and Eckert (Ref. 31) at Reynolds numbers between 10 and $4.64 \times 10^3$, has confirmed these conclusions. Similar data is provided by van Meel (Ref. 28) and Dyban et al (Ref. 42) at Reynolds number between $4.5 \times 10^3$ and $45 \times 10^3$, see Fig. 2. Owen (Ref. 32) investigated higher flow ranges (i.e. Re varying from 3.8 to $8.1 \times 10^4$) and confirmed that the rate of increase of heat transfer with Reynolds number is greater over the rear half. However, he found that the mean Nusselt number over this rear section was still lower than that associated with the front. Similar observations have also been reported by Schmidt and Wenner (Ref. 33) at Reynolds numbers up to $3.9 \times 10^4$ and by Giedt (Ref. 41) for flow Reynolds numbers up to $1.01 \times 10^5$, see Fig. 4. These results conflict somewhat with those of Lohrisch (Ref. 36 and Zapp (Ref. 37) for Reynolds numbers up to $3.9 \times 10^4$. These authors predict that the heat transfer at the rear is higher than that at the front. Furthermore, Winding and Cheney (Ref. 38), Small (Ref. 39) at a Reynolds number equal to $3.9 \times 10^4$; and Lohrisch (see p. 406 Ref.40).
at a Reynolds number of \(5 \times 10^4\), have found approximately equal heat transfers over the two halves of their test cylinders. However, at higher Reynolds numbers (1.4 to \(2.19 \times 10^5\)) Giedt's results (Ref. 41) exhibit a rear to front heat transfer ratio of greater than unity at a still higher range of Reynolds this ratio increases as demonstrated recently by Achenbach (Ref. 42).

To summarize, however, it appears that the rates of increase of heat transfer over the two halves are both Reynolds number dependent although this dependence is not similar. The recovery in heat transfer at the rear portion is particularly marked at \(\text{Re}_o > 3.9 \times 10^4\). This is associated with the comparatively high energy vortex shedding.

**B. The Influence of Turbulence Intensity and Scale**

It has been known for sometime, that the experimental results on convective heat transfers reported by different observers, often show discrepancies which exceed the expected experimental errors. These may usually be explained by variations of the free stream turbulence, see Fig. 5. The effects of both turbulence intensity and scale have thus received considerable attention over the last two decades. The intensity
of the turbulence is a measurement of the amplitude of the fluctuating components of velocity whereas the scale is related to the relative size of the turbulent eddies present in the flow. Both markedly affect local and average heat transfers (Ref. 43). In early studies, Ciedt (Ref. 44) concluded that the effect of a grid (used to generate the main stream turbulence) was to produce heat transfers which were normally characteristic of velocities higher than those actually occurring. Kestin and Maeder (Ref. 45) provide convincing proof that a change in the turbulence intensity markedly affects the local rates of heat transfer. Van der Hegge Zijnen (Ref. 46) suggested that an optimum value of turbulence to tube diameter exists. This ratio corresponds to a condition of resonance where the effective frequency of the turbulence coincides with the frequency of eddy shedding. Other studies indicate that the stagnation region is most affected by turbulence and this effect is subsequently partially damped out (Ref. 47). Kestin et al (Ref. 48) have also reviewed turbulence effects. Recently Lowery and Vachon (Ref. 49) have demonstrated that in the laminar boundary region characterized by $0 < \phi < 40^\circ$, the effect of turbulence intensity is always to increase the heat transfer distribution by a constant dependent on the turbulence intensity. This factor was independent for the range of Reynolds number studies.

Most of these investigations (except Ref. 46) were carried out at comparatively high Reynolds number, i.e. in the range equal to $10^5$ to $6 \times 10^5$. It has been observed that the effect of turbulence is different for the heat transfer over the front of the cylinder, than over the back. For example in a low turbulence air stream, the ratio of front to back heat transfer is approximately 0.85, whereas with the more turbulent flows, this increases to around 1.1 (Ref. 45). The change was due both to an increase of the heat transfer on the front, and a decrease on the back half of the cylinder. Turbulence intensities of approximately 1% and 4% were employed during these tests. Petrie and Simpson (Ref. 27) in their study of free stream turbulence, concentrated mainly on the wake region ($\phi > 140^\circ$). They verified similar work by Zijnen (Ref. 46). Between Reynolds numbers of $5 \times 10^3$ and $3.5 \times 10^4$ they found that the heat transfer from the cylinder was extremely sensitive to free stream fluctuations. They attributed the discrepancies in past data to the fact that many investigators did not measure or state the free stream turbulence level, see Fig. 6. They also showed that fluid in all regions around the cylinder is sensitive to free stream fluctuations. The overall values of heat transfer were found to be in close agreement with Richardson (Ref. 22). Unlike studies with much higher Reynolds numbers, Petrie and Simpson (Ref. 27) showed that increases of up to 10% in the free stream turbulence level were likely to give increases of approximately 100% in the heat transfer in the wake. However, it should be emphasized that this relative increase in the wake region varies with Reynolds number, and more importantly, the overall effect on the average heat transfers may not be as significant. This conclusion was confirmed by the extensive investigations of Dyban et al. (Ref. 42) and Oka et al. (Ref. 52), see also Fig. 7.
C. The Influence of Blockage Ratio

In practice, circular tubes are often positioned so that the flow is restricted or blocked by adjacent walls. In this case the blockage ratio may be expressed by the ratio of tube diameter to channel width, D/H. Similar effects are observed with a row of tubes as the blockage ratio increases the average velocity around the cylinder, outside the boundary layer, increases and the pressure and velocity distribution are changed accordingly. In the original separation region particles of fluid near the surface reverse their direction of low (i.e. they continue in the flow direction of mainstream), this causes the minimum pressure point to be displaced from 70° to 90° relative to the front stagnation point. The separation point is moved downstream to 100°, see Zukauskas (Ref. 13). Typical results obtained by Akilbayev (Ref. 50) are shown in Fig. 8. In this figure the flow pattern appears to be significantly affected by increasing the flow blockage and this becomes more pronounced at higher blockage ratios. Accordingly, the distribution of the local heat transfer coefficient will also be markedly altered.

For a single cylinder in crossflow, Akilbayev (Ref. 50) calculated the local heat transfer from the front portion of a cylinder using Merk's method (Ref. 51) and a potential velocity distribution, see Fig. 9. As can be seen, the heat transfer on the front portion increases with an increase in blockage ratio. More important, however, with blockages greater than 0.71 maximum heat transfer no longer ensues at the front stagnation point. Akilbayev's calculations moreover are in good agreement with his experimental data; see also Zukauskas (Ref. 13). However, these reported results are in conflict somewhat with the experimental results of Oka et al. (Ref. 52), see Fig. 9. In both cases the influence of the blockage ratio is clearly evident. In general very few studies have been carried out with high flow blockages, (D/H > 0.65) and the available data is published in Refs. 13, 42, 50. This can be partially explained since most of the heat transfer studies reported on tube banks were mainly concerned with inner rows (see for example Refs. 38, 53). The local convective heat transfer distributions over the rear of the test cylinders are considerably at variance with those for the low blockage situation, see Ref. 52. As the blockage ratio is increased to D/H > 0.8, the influence over the rear portion of the tube is relatively greater as reported in Refs. 13, 52 and see Fig. 10. These changes in the local heat transfer patterns are, of course, reflected in the average heat transfers and consequently, in the dimensionless correlation for average Nusselt number. Corrections are, therefore, necessary. Several methods for correcting the Reynolds number to account for solid and wake blockages have been suggested (Refs. 50, 55, 56, 57). Zukauskas (Ref. 13) suggested that these corrections are only required if D/H > 0.29. Some of these proposed modifications are compared in Fig. 11 and it can be seen, that there are appreciable differences in the correction proposals. For more detailed comparison of these corrections over a wide range of flow blockages, the reader is referred to Morgan (Ref. 11). Perkins and Leppert (Ref. 30) studied several reference velocities and concluded that an empirical formula derived by Pope (Ref. 57) was preferable. Their investigations covered a range of Reynolds number between 40 and 10⁵. The blockage ratios involved, however, did not exceed 0.30.
Akilbayev (Ref. 50) studies flow blockages up to 0.83 and suggested an empirical correction for the reference velocity of the form

\[ U_r = U_o \left(1 + 1.18 \left(\frac{D}{H}\right)^3\right)^2 \]

It should be emphasised, however, that there is very little available information at these geometries.

2.3 Tubes in a Bank

Circular cylinders are often arranged in bundles or banks to obtain a desired heat transfer in a relatively small space so that there is considerable interaction between adjacent tubes. The most common arrangements are either inline, or staggered banks of tubes. However, other arrangements such as crossed-in-line or inclined systems are sometimes used, see Fig. 12.

2.3.1 Flow Characteristics

The flow of a fluid over banks of tubes is similar to that associated with single tubes at moderate or high blockages. The contractions in area cause large pressure gradients and hence corresponding changes in velocity distributions in the boundary layer and also over the rear portions. The flow pattern is thus governed by the geometrical arrangement and size of the tubes. In multiple banks, the effect of the downstream rows on the previous rows are negligible so that the performance of a single row of tubes is close to that of the first row of a multiple arrangement.

Ishigai et al. (Ref. 58) undertook flow studies using Schlieren technique, see plate 2, to examine the regions of vortex formation. They confirmed the similarity in the flow structure for single cylinders and tubes in a single row with low to moderate blockages. However, significant changes ensued at higher flow blockages. Flow in a staggered bank system, is similar to that of a curved channel of sequentially diverging-converging section. In the case of inline banks there are two extremes. If the longitudinal pitch is equal to unity, the flow is similar to that of a straight channel. In the second extreme, the longitudinal pitch approaches infinity and the flow is similar to that over a single transverse row with the velocity profile of the mainstream straightened between the adjacent rows. Most practical situations lie between these extremes, so that rows are situated in circulation regions where the velocity distribution is non-uniform.

At Re 1000 the flow is predominantly laminar; vortices are formed in the separated region, but their effects on the boundary layer of the subsequent row are eliminated by viscous forces and the negative pressure gradient. With higher Reynolds numbers the flow between tubes becomes vortical with a high degree of turbulence. Although this vortical flow will influence the flow over a subsequent tube row, a laminar boundary layer will still persist on the front section of this latter row. The flow over these subsequent tubes is thus mixed, i.e. a laminar boundary layer influenced by the turbulent flow, together with an intensive vortical flow in the rear of the tubes. The intensity of the turbulence is dependent upon the Reynolds's number and the geometry of the bank.
Bresler (Ref. 59) showed that the velocity distribution around a tube in an inner row of an in-line arrangement is substantially different than that for a single tube. The pressure in front of the separation points is higher. The maximum pressure is at $\phi = 40^\circ$ where impact from the main stream occurs. Zukauskas (Ref. 5) suggested that the impact point is moved from close to the front stagnation point at low Reynold's numbers to $\phi = 55^\circ$ as $Re$ increased to $4 \times 10^3$ and then moved back towards the front again at higher Reynold's numbers. Separation occurs at approximately $\phi = 145^\circ$, and as in the single tube situation at high Reynold's number a laminar to turbulent transition can ensue in the boundary layer.

The flow over a tube in a staggered bank is in some ways more closely related to that over a single cylinder. The flow divides at the front stagnation point and a laminar boundary layer is set up. The position of the separation point differs from that of a single tube due to laminar to turbulent transition of the boundary layers. Separation ensues at $\phi = 150^\circ$. However, at higher Reynold's numbers i.e. $Re < 10^6$ this point is brought forward, see (Ref. 6).

The flow characteristics of crossed geometries are discussed by Smith and Goome, (Ref. 60) and Smith (Ref. 61) and Brauer (Ref. 118).

### 2.3.2 Heat Transfer

The variation of heat transfer around a tube in a bank is determined by the flow pattern which is governed by the system geometry. A tube in one of the inner rows is thus influenced by a highly turbulent flow and the boundary layer near the point of impact is solely laminar only at low Reynold's number.

Local heat transfers around the perimeters of tubes have been studied by Winding and Cheney (Ref. 38) Bortoli et al. (Ref. 53), Robinson and Han (Ref. 62), Zukauskas (Refs. 13, 14) and Kostic and Oka (Ref. 63) amongst others. A comparison of the heat transfer variations on inner rows of both staggered and in-line geometries with that for a single tube is presented in Fig. 13. In both banks the higher flow turbulence results in an increase in heat transfer over the tube. For the in-line case 'shading' by the preceding upstream cylinders results in comparatively low heat transfer at the front of the cylinder with the maximum value occurring at $\phi = 50^\circ$ (approximately) i.e. the impact point. The physical properties of the fluid have little effect so that the heat transfer distributions are similar for liquids and gases.

A knowledge of the local variations in heat transfer are particularly useful in applications where overheating of tubes is possible. However, for most practical design purposes mean heat transfer data are sufficient. Generally, the mean heat transfers for rows of tubes are determined by their position in the arrangement. In most cases the first row heat transfer is considerably lower than that associated with inner rows. An exception to this general statement occurs with an in-line arrangement at low flows ($Re < 500$) where 'shading' effects outweigh the flow disturbance mechanism. Furthermore in the special case of the so-called 'tangential' tube banks (i.e. an in-line arrangement with a longitudinal pitch $s_2/D = 1$) the highest convective heat transfers ensue at the initial tube
row. Generally, the individual row heat transfer rate increases up to the third or fourth row of the bank and then remains approximately constant, see Jones and Monroe (Ref. 64). The magnitude of this effect depends on the geometry of the bank and is discussed also by Kays and London (Ref. 2), Pierson (Ref. 65), Cram et al (Ref. 66), Welch and Fairchild (Ref. 67). A typical result for the variation of heat transfer with the number of rows are shown in Fig. 14 for both staggered and in-line arrangements. In effect, upstream rows of a bank act as a turbulence grid so that as would be expected a decrease of longitudinal pitch increases the heat transfer at the inner rows.

The overall heat transfer rates to banks of tubes have been widely studied for various geometries, see (Refs. 69, 70, 71). Grimison (Ref. 72) correlated the results of references (65, 71) in the form:

$$\text{Nu} = C \text{Re}^m \mathcal{F}(\frac{S_1}{D}, \frac{S_2}{D})$$

Banks composed of 10 rows were studied with the Reynolds number based on the tube diameter and the maximum fluid velocity in the bank. The fluid properties were evaluated at the mean film temperature. Grimison tabulated C and m for various arrangements of staggered and in-line banks. The experimental data indicate that staggered geometries yield substantially higher Nusselt numbers for Re 20,000 particularly if the transverse pitch is greater than the longitudinal. Similar data based on 15 rows of tubes are available in Kays and London, (Ref. 68). Reynolds number in this case is based on maximum fluid velocity and the hydraulic diameter, $D_h$, where

$$D_h = \frac{4(S_1S_2 - D^2/4)}{D}$$

Zukaukas (Ref. 5) presents average heat transfer rates for a wide range of tube banks in cross flow. Empirical expressions are presented for the Nusselt number at a tube in one of the inner banks of the form

$$\text{Nu} = C \text{Re}^m \text{Pr}^{0.36} \left(\frac{\text{Pr/Pr}_w}{\text{Pr}}\right)^{0.25}$$

Values of C and m are tabulated in Table 2.1 where Re is based on the maximum velocity and tube diameter. These empirical correlations predict higher heat transfers with staggered arrangements, particularly at wide transverse spacings and low Reynolds number.

The effect of the number of tube rows on the overall heat transfer may be allowed for by applying the correction factors published in Pierson (Ref. 65) or Zukauskas (Ref. 5) amongst others.
TABLE 2.1

VALUES OF CONSTANTS C and m

<table>
<thead>
<tr>
<th>Re</th>
<th>Tube Arrangement</th>
<th>C</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 - 100</td>
<td>In-line</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Staggered</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>10^3 - 2 x 10^5</td>
<td>*In-line,</td>
<td>0.27</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>+Staggered</td>
<td>0.35</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>Staggered</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>2 x 10^5</td>
<td>In-line</td>
<td>0.021</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>Staggered</td>
<td>0.022</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Footnote to table:
* For $D_{in}/h > 0.7$ i.e. close transverse spacings, lower Nusselt numbers than those predicted by the general correlation are obtained.
+ For these staggered arrangements, the predicted Nusselt number should be multiplied by $(S_1/S_2)^{0.2}$.

Crossed in-line arrangements have been studied by Hämmeke et al. (Ref. 73) for Reynold's number ranging from $5 \times 10^3$ to $2 \times 10^5$. The transverse and longitudinal pitches were 2.06 and 1.38 respectively in all their tests. The measured heat transfer rates for this geometry were higher than those obtained with conventional staggered or in-line arrangements.

The discussion in this section has been solely concerned with flows perpendicular to the tube axes. However, in some applications the system geometry may be restricted so that the flow of the fluid will be inclined to the axis of the tube banks. The dependence of heat transfer on the angle of fluid attack is shown in Fig. 15.

2.4 Tubes Fitted with Extended Surfaces

The use of extended surfaces introduces additional variables into the heat transfer and fluid flow relations so that it is not possible to obtain generalized empirical correlations for the wide range of extended
(or secondary) surface geometries (Ref. 2). Extended surfaces are often employed where the heat transfer coefficients associate with one of the fluid streams are significantly different than those for the other, for example in gas-to-liquid heat exchangers. The use of extended surfaces is particularly advantageous when applied to tube banks in crossflow. The principal pressure losses in such a system arise due to the turbulence generated by the rapid changes in the cross-sectional area of the flow passages. The presence of continuous plate fins, circular or helical fins has comparatively little effect on these turbulence patterns in the tube banks so that the pressure loss is relatively unaffected, (Ref. 3). Thus the heat transfer area can be greatly increased (say five to ten fold) by the addition of fins with a proportionately much lower effect on pressure loss. The effect of fin addition on the average heat transfer coefficients depends on fin geometry but in all cases they are likely to be reduced by more than a factor of about two, (for general comparisons, see Kays and London, Ref. 2).

The most commonly used extended surface tubing are the plate finned tubes shown in Fig. 16. A wider range of extended surfaces may also be used for special requirements.

2.4.1 Flow Characteristics

The differences in the flow characteristics of bare tubes and those fitted with extended surfaces are governed mainly by the shape, size and type of the fin (i.e. the fin geometry) as well as by the geometrical arrangement of the tubes. The flow patterns are thus rather complicated and difficult to analyse particularly for the more unconventional extended surfaces. Nevertheless, extensive studies have been made on the more conventional types such as circular and helical transverse fins and continuous plate fins, (Refs. 116, 117, 118, 119).

Lymer and Ridal (Ref. 116) employed water as the visualization fluid together with the techniques developed by Deterding (Ref. 120) to study the flow patterns associated with a staggered bank of circular finned tubes in crossflow (see plate 3). K. B. United Stirling (Ref. 117) have also employed a water model to investigate the flow patterns for single rows composed of plate finned and helical finned tubes (see plates 4, 5). Other finned tube arrangements have been studied in (Ref. 110) and comparisons drawn with plain tube arrangements of similar geometry. The flow patterns for staggered banks of circular finned tubes in crossflow were also studied by Neal and Hitchcock (Ref. 119), see Fig. 17.

Analyses of these flow patterns suggest that the mainstream diverges over the surface of the fins thus introducing a flow component along the axis of the tube between fins. Thus a vortex is generated within the boundary layer on the fin surface near the leading edge. Due to a positive pressure gradient this vortex is shed in the upstream direction and then is subsequently carried away downstream. This shedding phenomenon in the case of finned tubes, however, is not the same as the formation of Karman vortex streets downstream of banks of bare tubes (Ref. 121).
For circular finned tubes in a staggered bank, the flows tends to be steady over the first two rows (see for example Ref. 119). However, in subsequent rows considerable turbulence is generated, both in the mainstream and between the secondary surfaces. The flow conditions become confused so that interpretation is very difficult. The main flow features which are common to most tube arrangements are reproduced in Fig. 17. It can be seen that separation from the tube surfaces occurs shortly after a point halfway around the circumference. A converging wake trails behind the tube. This wake is decelerated by the fins and a reverse flow can ensue along the fin surfaces in a similar manner to the flow on the upstream side. After partially flowing around the tube this reverse flow is then drawn downstream by the main flow. A high level of turbulence forms within this wake and subsequent rapid diffusion into the mainstream affects the entire flow by about the fourth tube row. This has been corroborated by observations of large scale perturbations in the flow after the first few rows (Ref. 119). Local re-circulation regions are also usually formed near the front of the tube at the root of the fin.

The exact nature, however, of the flow over the extended surfaces depends also on the mainstream conditions (e.g. the turbulence characteristics) and particularly on the detailed geometry. It is difficult to summarize the flow of the fins characteristics in this complicated situation but nevertheless Ref. 122 has drawn the following conclusions for plate finned arrangements:

(i) The rear half of the tube is always a region of separated flow.

(ii) The fin surface in the wake of the tube is in an area of high turbulence.

(iii) Separation occurs near all sharp corners e.g. the leading edge of the fin and the front junction of the fin and tube.

(iv) The flow over the remainder of the flat fin surface is smooth and unseparated.

With closely-spaced tubes and continuous plate fins (Ref. 117) there is a blockage effect and the velocity over the fin surfaces can be markedly different from the mainstream value (see plate 4).

2.4.2 Heat Transfers

Because of the interest in the use of extended surfaces to increase heat flows a large body of heat transfer data exists for a wide range of geometrical and flow conditions. The bulk of these data are concerned with average heat transfer performance. Local heat transfers are, however, reported for circular finned tubes in crossflowing fluids (Refs. 114, 116, 119, 127) whilst plate finned tube arrangements, are considered in (Refs. 121, 129, 142, 143). These investigations have revealed that the distribution of local heat transfer over the fin surface area is far from uniform, as is assumed in the earlier mathematical analysis (Refs. 130, 131, 132, 133, 134). This is to be expected due to the complicated
nature of the flows, however, it has been shown that a number of factors control the heat transfer pattern on the outside of a finned tube in crossflow. These are the boundary layer thickness (both thermal and velocity); the nature of the boundary layer including the local intensity and scale of turbulence; the local velocity and fluid temperature, etc., (see Neal and Hitchcock, Ref. 119). No quantitative relationship was drawn between these factors and the local heat transfers. Obviously such a task would be formidable in view of the general lack of understanding of the detailed flow processes. In particular the effects of the overall flow blockage (often expressed in terms of the hydraulic diameter) and the extent of fin area relative to the tube area (i.e. high or low fin) are uncertain.

However, the total finned area can be divided, with regard to heat transfer, into an active section, which is thoroughly swept by the crossflow, and a passive section, which is situated in the wake, (Ref. 118). Observations of a variety of finned tubes have shown that this passive finned area increases as the overall fin area increases. If the relative increase in surface area due to the extended surfaces is small, (for example, where short or widely spaced fins are employed) then this passive area remains small. This can also be the case when high blockage ratios are used (see Fukui and Sakomoto, Ref. 121).

Local heat transfer data for the cases of both circular finned and plate finned tubes in banks, have shown markedly higher heat transfers over the upstream portion of the fins, followed by a steady deterioration towards the rear as the flow progresses downstream. This feature is associated with the flow of the fluid through the fin channels. Initial boundary layer development ensues over the front of the fins followed by separation of the flow (for circular fins only) at or near the $\theta = 90^\circ$ position (Ref. 119, 127). The wake region (in both cases) is a zone of relatively slow moving fluid with considerable eddying which contributes little to the average heat transfers. As can be seen in Figs. 18 and 19, that relatively high heat transfers occurred at a short distance downstream from the leading edges of the fins. These peaks are usually associated with the sudden flow contraction which takes place due to the presence of the transverse square-edged fins. A re-circulation wake is thus formed near the leading edge and results in an associated drop in heat transfer (see for example Wong (Ref. 127) for circular finned tube and Saboya and Sparrow (Ref. 129) who considered one row of plate fins). The subsequent expansion in the flow brings turbulent-less stagnant air into contact with the fin surfaces, with consequent high heat transfers. These phenomena were also observed in (Ref. 114) for the case of circular finned tubes. Weiner, Gross and Paschkis (Ref. 114) reported different heat transfer characteristics for the front region of round-edged fins. Thus it appears that a sharp edge is necessary to 'trip' the re-circulation wake.

The region of high heat transfer near the fin root at the front of the tubes is mainly attributed to surface scrubbing by the fluid and the high turbulence generated due to the complicated flows in this region. On the other hand, the boundary layer thickens in the direction of flow so that the heat transfers are relatively low at the fin sides. Moreover, as the flow progresses round the tube, the heat transfers become
generally lower near the base of the fin. The low heat transfers in the wake region are the result of re-circulation and low fluid velocities (Ref. 119).

For a single row of tubes fitted with plate fins in crossflow, Shepherd (Ref. 128), in an early study reported on the variation of average heat transfer with Reynolds number (based on the hydraulic diameter and the maximum velocity in the minimum free flow area). This study revealed a constant relationship between the two variables although the results were evaluated without account being taken of the role of fin efficiency. The Reynolds number was varied between 175 and $1.2 \times 10^3$ and the hydraulic diameter was expressed as:

$$D_h = 4 \frac{A_o L}{A_s}$$

where $A_o$ is the minimum free flow area, $L$ is the stream wise length, and $A_s$ is the heat transfer surface area.

More accurate measurements were reported by Saboya and Sparrow (Ref. 129) for a similar plate fin and tube geometry and Reynolds numbers range. The two sets of data are compared in Fig. 20. Other average heat transfer results for the plate finned tube has been published by Gebhart (Ref. 122) who employed different geometry than those of (Ref. 128 and 129). Furthermore, the plate fin configurations in Gebhart's case was complicated due to the presence of slots, holes and tabs.

Fukui and Sakamoto (Ref. 121) have studied the variation of average heat transfer coefficients with tube spacing and tube diameter. They reported that the average heat transfer coefficients are increased as the ratio of tube spacing to tube diameter decreased.

Saboya and Sparrow (Refs. 129, 142, 143) employed the same technique as Fukui and Sakamoto and reported measurements of both the average and local heat transfer coefficients for one, two and three rows of tubes fitted with plate fins and positioned in a staggered arrangement. They observed relatively higher average heat transfers over the second and the third rows in comparison with those over the first row, see Fig. 21. It should be noted, however, that these results refer only to the fin surface area and no account is taken of tube heat transfers. Nevertheless, these errors were considered to be small because of the large total fin surface area in relation to that of the tube. Thus, for most finned tube arrangements, it is generally accepted that the first row experiences the lowest heat transfer coefficients. For the third and subsequent rows these coefficients tend to be more or less constant (see also Ref. 118).

For staggered banks of circular finned tubes, Neal and Hitchcock (Ref. 119) reported that the average heat transfer is increased by decreasing the transverse pitch or increasing the longitudinal spacing. They noticed that, with the closer longitudinal pitches, the air flow was concentrated towards the top and bottom of the fins. However, the longitudinal spacings studied by these investigators were such that each downstream row lay in the wake of its predecessor. Consequently after the initial row, lower heat transfers were found over the fronts of the tubes. At this juncture, however, it should be noted that whilst a decrease in longitudinal pitches (at constant transverse spacing)
results in a lower average heat transfer coefficient, the overall heat transfers (per unit volume of heat exchanger) is actually increased owing to the presence of a larger number of finned tubes. Nevertheless (Refs. 119 and 124) suggest that these close longitudinal configurations may be undesirable in practice due to the non-uniformity of the heat transfers.

It is dangerous, however, to draw general conclusions from the results of Neal and Hitchcock, since contradictory information has been published. For example, Zozula, Khavin and Kalinin (Ref. 146) employed aluminium finned tubes in a staggered bank and found that the average heat transfer increased with increasing transverse pitches or (to a lesser extent) decreasing longitudinal spacings. For helically finned tubes in a staggered configuration, Mikrovic (Ref. 147) reported similar observations to those of Zozula et al. Moreover, this study suggests that with constant transverse and longitudinal pitches, the average heat transfers increased as the tube diameter was increased. In an earlier study Jameson (Ref. 150) found that the average heat transfers for staggered helically finned tubes were substantially independent of both longitudinal and transverse spacings. Schmidt (Ref. 151) varied the longitudinal pitch and also concluded that the tube spacing did not affect the heat transfers for finned tubes. Similar conclusions to Jameson were arrived at by Briggs and Young (Ref. 155). Ward and Young (Ref. 154) showed that for a triangular arrangement of finned tubes, the average heat transfers were dependent on the fin root diameter but were independent of the spacing between adjacent fins.

These differences in the reported data can be attributed to differences in the test conditions since the geometrical and flow parameters are complicated and can have differing influences over the flow ranges studied. For example, significant effects have been observed with change in the relative depth of the inter-fin space (Ref. 148); the fin thickness (Ref. 149); the finning factor (i.e., the ratio of the surface area of fin relative to that of tube) (Refs. 2 and 127); and the flow turbulence level (Refs. 124 and 146). Consequently if valid comparisons are to be made all these factors should be considered. Other additional factors may also be involved due to differences in the test facilities employed in the experiments. Furthermore the bond resistance at the root of the fin, the shape and orientation of the leading edges of the fins, and the fin efficiency may also affect the measurements.

Thus it is difficult to draw general conclusions regarding the effect of the presence of secondary surfaces on the average heat transfer coefficients.
A- Effect of Tube Inclination

For arrangements of circular finned tubes, inclined relative to the mainstream flow the effect depends upon the orientation of the fins. For example, if the fins are arranged perpendicularly to the mainflow, the heat transfer is increased by between 20 to 40% in comparison with that for the un-inclined system. This is due to the additional turbulence of the airstream induced by the transverse positioning of fins (Ref. 111). However, if the inclined finned tubes are arranged such that the fins are parallel to the airstream, the heat transfer remains virtually unchanged. This follows since the airflow paths, through the inclined and un-inclined elements, differ only slightly (due to changes in the effective spacing between the tubes).

For a wide variety of fin arrangements, inclination to the mainstream flow may also effect the flows in the entrance and exit regions. The average heat transfer will then be affected. The extent of any change, however, depends upon the particular geometrical arrangement, and the angle of inclination; unfortunately, little detailed information on this subject was found except that provided by Preece, Lis and Hitchcock (Ref. 111).

B- Effect of Fouling Factors

In heat exchangers the use of fouling factors is somewhat indefinite. The effects can often be transient in nature yet a nominal fixed value is added indiscriminately to the steady state heat transfer resistance (p. 458, Ref. 5). The method by which 'fouling' accumulates is little understood and there is considerable scope for further work on this neglected subject (Ref. 115a). In practice, heat exchangers may undergo a decline in thermal performance (especially those used as air pre-heaters in combustion chambers or exhaust stacks). This is due to accumulation of heat insulating substances on the surfaces. However, the main effect of 'fouling' is that an intolerable proportion of the available temperature difference must be used to overcome this resistance. Consequently the exchanger must be over-designed initially. Tables of fouling factors for various conditions are published by the Tubular Heat Exchanger Manufacturers Association (see Appendix-D). However, these tables are intended only as a crude guide and indicate the steady-state resistances arising due to cumulative fouling. Furthermore the operating lives of exchangers are not discussed.

An alternative approach in which a time-dependent fouling factor is introduced has been proposed by Kern and Seaton in 1959 (as quoted in Ref. 5). For fluids which are particularly prone to cause fouling (e.g. those which result in fouling resistances > 3.5 x 10^-4), considerable decline in performance may ensue. Thus the design should be orientated towards the physical suppression of fouling, see Kern (p. 459, Ref. 5).

2.5 Hydraulic Resistance

The evaluation of the performance of any heat exchanger depends on the hydraulic resistance as well as on the heat transfer. This resistance is characterized by the pressure drop across the bank of tubes. It is
a function of the geometrical arrangement, the flow velocity, and the fluid properties. Hence in dimensionless form, the hydraulic resistance may be expressed in terms of the Fanning friction factor \( f \) as:

\[
f = \frac{\Delta P}{\rho U_{\text{max}}^2} = \frac{1}{Y} \left( \text{Re, } \frac{s_1/D}{s /D}, N \right)
\]

where \( U_{\text{max}} \) is the maximum flow velocity in the minimum free flow area between the tubes, \( \Delta P \) is the total pressure drop and \( N \) is the number of tube rows. In the case of finned tube arrangements, the hydraulic resistance can also depend on the fin height, thickness and the number of fins per unit length, etc.

An alternative friction factor is also employed in some publications and this is defined as (see Refs. 2, 68):

\[
f = \frac{2g \Delta P}{\rho U_{\text{max}}^2} = 4 f
\]

In other studies (Refs. 13, 147, 166) the Euler number has been preferred. However, the hydraulic resistance is often defined using an equivalent shear force per unit of frictional area. Whether this equivalent shear force is a true viscous shear, or primarily results from pressure variations (as in the case of a tube bank) is of no consequence. For most flow surfaces it is a combination of skin friction and form drag, although there is no practical advantage in separating these effects. Kays and London (Ref. 2) present the following expression for the total pressure drop associated with tube banks in crossflow:

\[
\Delta P_{\text{tot}} = \bar{f} \frac{A_t}{A_c} \frac{V_m}{V_1} + \bar{a}
\]

where \( A_t \) is the total surface area, \( A_c \) is the minimum free flow area; \( V_m \) and \( V_1 \) are the mean and initial specific volumes respectively of the crossflowing fluid; and \( \bar{a} \) is the difference in static pressure which accompanies any acceleration or deceleration of the fluid. Thus:

\[
\bar{a} = \frac{\rho U_{\text{max}}}{2g} \left( 1 + \sigma^2 \right) \left( V_2 - V_1 \right)
\]

where \( \sigma \) is the ratio of free flow to frontal area and \( V_2 \) is the exit specific volume.

For flows over banks of tubes, (both bare and finned) each tube row offers a contraction and an expansion. The frictional characteristics of the first and last tube rows are thus not materially different from those of the interior rows. Thus the entrance and exit losses are readily taken into account and there is no need to correct the total pressure drop for entrance and exit effects (Ref. 2, 68). Nevertheless an influence of the number of tube rows on the hydraulic resistance can be distinguished, see: Fig.22.

As with the heat transfers, the hydraulic resistances of both bare and finned tube banks have been widely studied. Pierson (Ref. 65) conducted
an early study into the effects of Reynolds number and tube arrangement on the friction factor. In general it was found that this factor decreased, or at most remained constant, with increasing Reynolds number. He also found that the tube arrangement had a more marked effect on the friction factor than on the Nusselt number. Similar observations have been reported in Refs. 72, 150.

Gunter and Shaw (Ref. 68) examined the experimental data of several workers and proposed that the following empirical equation for pressure drop could be applied to both bare and finned tubes:

\[ f/2 = \frac{\Delta P \cdot \xi}{U_{\text{max}}^2} \cdot \frac{D_v}{L} \left( \frac{U}{\mu_w} \right)^{0.14} \left( \frac{D_v}{S_1} \right)^{-0.4} \left( \frac{S_2}{S_1} \right)^{-0.6} = F(Re) \]

where \( D_v \) is the volumetric hydraulic diameter and is equal to \((4 \times \text{Net Free Volume}) / (\text{Exposed Surface Area})\).

Jones and Monroe (Ref. 64); Gram, Mackey and Monroe (Ref. 66); Fairchild and Welch (Ref. 70) and Kays (Ref. 125) have all reported pressure drop results for in-line unfinned tube banks and studied the influence of tube spacing. In all cases decreasing the transverse pitch or increasing the longitudinal pitch, caused a marked increase in the hydraulic resistance. A different affect can be observed with staggered tube banks. For these arrangements the hydraulic resistance is mainly influenced by the transverse pitch and as would be expected the pressure drop increases markedly as the transverse spacing is reduced. If the minimum gas flow area is in the transverse direction an increase in longitudinal pitch can reduce the hydraulic resistance, see (Ref. 2, 13). However, a reverse affect is obtained if the minimum flow area is associated with the diagonal spacing.

These observations are equally true for both bare and finned tubes (Ref. 155). Again, however, the quantitative changes depend on the detailed geometries of the fins (see for example, Mikrovic, Ref. 147). For staggered arrangements of helically finned tubes, Jameson (Ref. 135); Rounthwait and Nicholson (Ref. 137) and Mikrovic (Ref. 147) have reported that a decrease in the longitudinal spacing markedly increased the hydraulic resistance. On the other hand, however, an increase in the transverse pitch had comparatively little effect on the results reported in Ref. 135. This is not corroborated by the data of Refs. 137 and 147. In these latter references, however, increasing the transverse pitch also increased the hydraulic resistance, this is also the affect observed with similar arrangements of bare tubes. The reason for this departure was not explained.

Ward and Young (Ref. 154) and Briggs and Young (Ref. 155) showed that for a triangular pitch arrangement of helically finned tubes, the hydraulic resistance decreased with an increase in the tube spacing. For constant transverse and longitudinal pitches, the hydraulic resistance was found to increase with increasing tube diameter (see for example Ref. 147).

In-line circular finned tube arrangements generally exhibit the same trends as those of the staggered (see Brauer Ref. 118). Increasing the number of fins per unit length, the fin height or the fin thickness increases
the hydraulic resistance for a given Reynolds number.

In general, there is little published data concerned with the pressure drops associated with tube geometries which differ from the conventional staggered and in-line systems. However, Hameke et al (Ref. 73) have reported higher pressure losses for crossed in-line banks of unfinned tubes. For finned tubes arranged either in crossed in-line or crossed staggered configurations rather different characteristics have been reported by Brauer (Ref. 118). He observed that the hydraulic resistance for these crossed arrangements were higher than those associated with conventional in-line finned tubes with similar fin dimensions but less than those for conventional staggered finned banks.

Inclining a banks of bare tubes to the direction of mainstream flow results in a reduction in the pressure loss (see Ref. 13). However, quantitative data on this subject does not appear to be available. Nevertheless, Preece, Lis and Hitchcock (Ref. 111) reported that for arrangements of circular finned tubes the plane of the fin should be parallel to the flow so that a minimum projected area results.

2.6 Optimal Tube Arrangement

The heat exchanger designer is usually concerned with the assessment of the performance characteristics of various tube bank arrangements. Thus much effort has been expended in comparing the relative heat transfer and pressure drop performances. The performance data for exchangers are customarily presented as the heat transfer factor 'j' and the pressure drop factor 'f'. These are a function of Reynolds number (see chap. 10 in Ref. 2). The curves for these factors vary widely both in magnitude and shape so that it is often difficult to draw suitable conclusions (Refs. 78, 111). A worthwhile criterion for comparison purposes is the ratio of the heat transferred per cubic metre of the system to the pumping power needed to overcome the flow resistance. See Fig. 23.

In many cases it has been possible to compare the relative performance of several finned tube banks by directly plotting the heat transfer coefficients against the pressure losses (see Table 1 and Fig. 5 in Ref. 111). This simple method of performance test is particularly advantageous when directly measured values are available. If such values are not available, then, the previously discussed method of plotting j and f factors should be employed. It should be mentioned, however, that the maximum ratio of heat transfer to pumping power can often occur at low flows i.e. at low heat transfer, so that the maximisation of this criterion is not always attainable. Furthermore a comprehensive exchanger design should also consider, for example, the following factors:

(i) The maximum allowable pressure loss.

(ii) The optimal design of the extended surfaces and their effectiveness.

(iii) Fouling, excessive thermal stresses, and constructional (space and weight) problems.
After detailed consideration of these factors, the capital and operating costs can then be identified for a given situation. Further information on exchanger design is provided in textbooks (Refs. 1 - 5) as well as in the references cited in Chapter 5 of this thesis.
FIG. 3. DISTRIBUTION OF LOCAL NUSSELT NUMBERS ON THE OUTER SURFACE OF A SINGLE CYLINDER
(FROM van MEEL, Ref. 28)
FIG. 4. LOCAL SINGLE CYLINDER HEAT TRANSFERS OBTAINED AT Tu=0.3% (FROM DYBAN ET AL. Ref. 42).
FIG. 5. LOCAL NUSSELT NUMBERS FOR A SINGLE CYLINDER IN CROSSFLOW AT HIGH REYNOLDS NUMBERS (FROM GIEDT, Ref. 41).
FIG. 6. COMPARISON OF PREVIOUS EXPERIMENTAL DATA ON THE DISTRIBUTION OF LOCAL HEAT TRANSFERS AROUND A SINGLE CYLINDER

1. $Re_o = 2.69 \times 10^4$, (Ref. 38); 2. $Re_o = 1.05 \times 10^4$, (Ref. 28);
3. $Re_o = 1.95 \times 10^4$, (Ref. 33); 4. $Re_o = 1.5 \times 10^4$, $Tu = 5\%$, (Refs 42, 43);
5. $Re_o = 1.5 \times 10^4$, $Tu = 5.78\%$, (Ref. 27); 6. $Re_o = 1.3 \times 10^4$,
   $Tu = 0.70\%$, (Ref. 27); 7. $Re_o = 8.29 \times 10^3$, (Ref. 33).
FIG. 7. VARIATION OF HEAT TRANSFER AROUND A SINGLE CYLINDER AT DIFFERENT LEVELS OF TURBULENCE (FROM Ref. 52)
FIG. 8. VELOCITY DISTRIBUTION AROUND A TUBE AT DIFFERENT BLOCKAGE RATIOS (FROM Refs. 50 and 54).
FIG. 9. CALCULATED HEAT TRANSFER DATA FROM AKILBAYEV (Ref. 50) IN COMPARISON WITH EXPERIMENTAL DATA FROM OTHER INVESTIGATORS
FIG. 10. DISTRIBUTION OF THE LOCAL HEAT TRANSFERS AT DIFFERENT BLOCKAGE RATIOS (FROM OKA ET AL. Ref. 52)

\( \text{Re}_0 = 3 \times 10^4 \)
FIG. 11. COMPARISON BETWEEN VARIOUS METHODS FOR CORRECTING FOR BLOCKAGE RATIO.
FIG. 12.  TUBE BANK ARRANGEMENTS

(a) STAGGERED

(b) IN-LINE

(c) CROSSED IN-LINE

(d) CROSS INCLINE
CURVE 1—SINGLE TUBE; CURVE 2—IN AN IN-LINE BANK
CURVE 3—IN A STAGGERED BANK (AFTER Ref. 13)

FIG. 13. VARIATION OF LOCAL HEAT TRANSFER OF A SINGLE TUBE AND A TUBE IN A BANK
FIG. 14. CORRECTION FOR THE NUMBER OF ROWS IN THE HEAT TRANSFER CALCULATION. 1 - IN-LINE 2 - STAGGERED (FROM ZUKAUSKAS Ref. 13)

\[ \varepsilon_\psi = \frac{N_u_\psi}{N_u_\psi=90} = (\sin \psi)^{0.6} \]

FIG. 15. THE FACTOR \( \varepsilon_\psi \) TO ACCOUNT FOR TUBE INCLINATION. STAGGERED AND IN-LINE TUBE BANKS (FROM Refs. 13 and 74).
Fig. 16. Most common types of crossflow finned surfaces.

(a) Plate finned tube

(b) Helical finned tube

(c) Annular finned tube
FIG. 17. GENERAL PATTERN OF FLOW AROUND A CIRCULAR FINNED TUBE IN A 2nd. ROW OF STAGGERED ARRANGEMENT (FROM NEAL AND HITCHCOCK Ref. 119).

FIG. 18. DISTRIBUTION OF THE LOCAL MASS TRANSFER ON THE OUTSIDE SURFACE OF A CIRCULAR FIN (FROM WONG Ref. 127).
FIG. 19. LOCAL SHERWOOD NUMBERS OBTAINED FROM A ONE ROW OF THE TEST PLATE FINNED TUBE ARRANGEMENT SHOWN ABOVE (FROM SABOYA AND SPARROW Ref., 129).
FIG. 20. COMPARISON BETWEEN AVERAGE MASS TRANSFER RESULTS OF SABOYA AND SPARROW FOR A SINGLE ROW OF PLATE FINNED TUBES AND DIRECT HEAT TRANSFER DATA

FOR COMPARISON PURPOSES \[ Sh = Nu \left( \frac{Sc}{Pr} \right)^{\frac{1}{3}} \]
TEST GEOMETRIES FOR PLATE FINNED TUBE BANK COMPOSED OF THREE ROWS INVESTIGATED BY SABOYA AND SPARROW (Ref. 129). $h=16.5\text{ mm}$, $D=8.53\text{ mm}$, $S=21.5\text{ mm}$ AND $L=18.5\text{ mm}$

![Diagram showing test geometries for plate finned tube bank](image)

FIG. 21. VARIATION OF AVERAGE MASS TRANSFERS WITH NUMBER OF TUBE ROWS AND $Re_h$ (Ref. 129)
NO IC

6 DIGITS DENOTES THE TOTAL NUMBER OF ROWS.

Reo

ΔP/ρU₀²

10⁻¹

2

4

6

8

10⁻⁴

2

4

6

8

10⁻²

2

4

6

8

10⁻⁰

2

4

6

8

10⁰

2

4

6

8

10²

2

4

6

8

10⁴

2

4

6

8

10⁶

2

4

6

8

10⁸

2

4

6

8

10¹⁰

2

4

6

8

10¹²

2

4

6

8

10¹⁴

2

4

6

8

10¹⁶

Reo

FIG. 22. HYDRAULIC RESISTANCE RESULTS FOR A TYPICAL STAGGERED TUBE BANK, BARE TUBES WITH TRANSVERSE PITCH = 1.5 AND LONGITUDINAL PITCH = 1.04 (FROM ZUKAUSKAS Ref. 14).
FIG. 23. A TYPICAL PLOT OF THE HEAT TRANSFER COEFFICIENT FOR AN ARBITRARY SET OF CONDITIONS TAKEN AS STANDARD AS A FUNCTION OF THE PUMPING POWER. (FROM KAYS AND LONDON, Ref. 4.)
PLATE 2  SCHLIEREN PHOTOGRAPHS OF VORTEX FORMATION REGION FOR SINGLE ROW TUBE BANK AT DIFFERENT FLOW BLOCKAGES (AFTER ISHIKAI, ET AL. Ref. 58).
PLATE 4
FLOW PATTERN THROUGH A SINGLE ROW OF TRANSVERSE PLATE FINNED TUBES (AFTER K.B. UNITED STIRLING, Ref. 11).
CHAPTER 3  AVERAGE HEAT TRANSFERS AND HYDRAULIC RESISTANCES ASSOCIATED WITH A SINGLE ROW OF CLOSELY SPACED TUBES IN CROSSFLOW.
CHAPTER 3

AVERAGE HEAT TRANSFERS AND HYDRAULIC RESISTANCES ASSOCIATED WITH A SINGLE ROW OF CLOSELY SPACED TUBES IN CROSSFLOW.

3.1 Introduction

It was desired to measure the average convective heat transfer coefficients and the hydraulic resistances associated with small diameter (3 - 6 mm) and closely spaced (0.3 - 1.8 mm) circular cylinders in crossflow. These tubes were arranged in a single row so that the blockage ratio \( \frac{D}{H} \) ranged from 0.67 to 0.92. The range of Reynolds numbers based on the mainstream velocity prior to the test section was varied between 300 and 8000. Although extensive information is available in the literature for tube banks in crossflowing fluids (as was explained in the previous chapter), these tube geometries are not included.

For this purpose, two separate measurement techniques were employed in this present study so that the data produced can be compared. These techniques were:

(i) A mass transfer technique (using the sublimation of Naphthalene).

(ii) A direct heat transfer measurement method.

Air was used as the test fluid in the average heat transfer measurements so that both these techniques were readily applicable. The hydraulic resistances were determined by measurements of the pressure drop across the tube arrangements. These experiments yielded new correlations for the average heat transfers together with additional data for the hydraulic resistances. Comparisons are drawn between the present results and those previously reported in the technical literature. The influence of tube diameter and spacing was investigated and an optimal tube arrangement is suggested. Additional data on the influence of surface roughness and turbulence intensity are obtained for a single tube diameter and three different spacings. These results are discussed in terms of the system parameters involved and where possible compared with other data obtained in the previous studies.

3.2 Previous Experimental Arrangements

Previous investigators have generally used the following measurement techniques for investigation of the average convective heat transfer coefficients associated with tube banks.

(a) A heat-mass transfer analogy (together with the sublimation of naphthalene or an electrochemical method).

(b) Direct heat transfer measurement.

(c) Mach-Zender Interferometry (for low flow velocities).

However, in this chapter, it is not intended to present a detailed review of the general criteria and specific details employed by the previous investigators. A comprehensive review of the basic considerations and background are provided by Eckert and Goldstein (Ref. 87) as well as in a wide range of papers. For example a detailed explanation of
mass transfer methods, their range of accuracy, sources of error and practical limitations can be found in (Refs. 18, 35, 36, 38, 84). The electrochemical method is reviewed comprehensively by Mizushina (Ref. 102); the direct heat transfer methods in (Refs. 19, 65, 70, 104) and Mach-Zehnder Interferometry in (Refs. 21 and 87).

3.3 General Test Facility

The experimental test rig consisted basically of a controlled supply of air, a test duct in which the test sections were mounted, and the necessary instrumentation and measuring systems, see Fig. 24. The air supply and test duct were common to all the tests, so that these features are described in this section whilst the instrumentation is discussed under the heading of the appropriate test method.

For the range of tube sizes encountered in the present tests, the required air flow varied from 0.008 to 0.4 m\(^3\)/sec. This flow was supplied by means of a Keith Blackman, 20 type 812 stage centrifugal fan with a maximum delivery gauge pressure of approximately 3.0 m of water. In the mass transfer tests and the pressure drop measurements, it is desirable to maintain the air temperature in the test section at approximately the ambient value. This was achieved by means of a water cooled crossflow air cooler and a 'Secomack' electrical air heater mounted in series in the air delivery, see Fig. 24.

To achieve a uniform mass or heat transfer along the axis of the test cylinder, a uniform velocity profile was required in the test section. Consequently a large diameter settling chamber (Fig. 25) was fitted prior to the test section. The air velocity was reduced in this chamber and settled with the use of baffles. A relatively uniform velocity profile with a narrow boundary layer was achieved by incorporating a sudden contraction at the chamber outlet. The first baffle, manufactured from 16 S.W.G. perforated sheet steel, was welded at a distance of 1.5 D\(_i\) from the chamber inlet. To the centre of this baffle was fixed a blank disc of diameter 1.5 D\(_i\). The effect of this was to spread the in-coming air jet and thus obtain the maximum effect from the baffle. The second baffle, constructed of 40 gauge wire mesh, was fitted a further 1.5 D\(_i\) downstream from the inlet, see Fig. 25. The effect of these baffles was to eliminate excessive eddies and turbulence caused by the fan and delivery pipe bends. The overall length of the chamber was 1 m and its diameter was 0.40 m.

The test duct, manufactured from 18 S.W.G. mild steel, had a rectangular cross-section of internal dimensions 0.10 x 0.08 m., and an overall length of 0.60 m. The sudden contraction inlet, manufactured from laminated wood, had a rectangular bell-mouth section.

A total of fifteen test sections each composed of a single row of tubes were required to cover the various combinations of diameters and transverse spacings. The sections were fabricated from strips of "TUFNOL" Fig.26. The construction of these sections enables new exchanger matrices to be inserted into the test section position without having to dismantle the test duct. The internal dimension of these sections were similar to those of the duct, see Fig. 26. The 'dummy' tubes in the test section were constructed of stainless steel and the single measure-
ment tube was positioned in the centre of the tube matrix. In the mass transfer experiments this tube consisted of a steel former coated with naphthalene whereas electrically heated copper tubes were used for the direct heat transfer measurements. To avoid errors due to variations in the air velocity over the test section, the 'effective length' of the test cylinder was approximately 90 mm.

3.4 General Instrumentation

This section is concerned with the measurement systems which were common to all the tests. The particular instrumentation associated with either the mass transfer or direct heat transfer studies is described later in the sections dealing with these studies.

The flow rate was measured using a sharp edge orifice plate with d and d/2 tappings in accordance with B.S. 1042. The orifice plate and associated pipework, was installed between the air heater and the settling chamber. The pressure drop across the orifice plate, and the gauge pressure at the inlet to this orifice were determined using 1.50 m tall water manometers. The atmospheric pressure was measured using a mercury barometer situated in the laboratory. The air temperature prior to the orifice station was obtained using a mercury-in-glass thermometer graduated in steps of 0.1°C over a range of 0 to 35°C. Three orifice plates (of internal diameters 22.9, 38.1 and 56.1 mm) were necessary to cover the complete range of flow Reynolds numbers. The ducting diameter was 90 mm. Similar instrumentation was used to obtain the pressure and temperature at the test section and the air flow was corrected accordingly. This air flow was controlled by means of a butterfly type valve mounted prior to the entrance to the settling chamber.

3.5 Classification of the Flow in the Test Duct

a) Velocity Profile Measurements

The main object of these measurements was to estimate the velocity characteristics of the airflow prior to the test section, so that a suitable measurement length of the test cylinder may be used. This will avoid errors due to non-uniform air flows (e.g. near the duct walls) over the measurement section.

The general procedure and corrections employed in these measurements followed closely the recommendations given in (Refs. 57, 76, 77). A pitot tube was traversed across the duct to measure the velocity profile at both a high and a low Reynolds number. To avoid error due to the yaw angle (i.e. due to misalignment), the probe was positioned so that the pressure difference between the two tappings on opposite sides was zero. The pressures were displayed on a calibrated electronic micromanometer. For each test a period of approximately ten minutes was allowed for the attainment of steady-state conditions. To reduce the possibility of inherent errors, the zero adjustment of the micromanometer scale was repeated at the conclusion of the test. Typical velocity profiles obtained at two different Reynolds number are presented in Fig. 27 and it can be seen that essentially uniform profiles were obtained.
b) Turbulence Intensity Measurements

The complete description of the upstream flow characteristics also requires measurements of the magnitude and frequency of the irregular velocity fluctuations which can be regarded as being superimposed on the mean flow. Further, it is well known that such irregular velocity fluctuations (turbulence) can significantly affect both average and local heat transfers from a cylinder in crossflowing fluids. Consequently the intensity of the turbulence of the mainstream fluid was measured. A "D.I.S.A." constant temperature anemometer type 55 D 01 together with a 5 μm diameter, tungsten hot wire probe was employed as shown schematically in Fig. 28. The probe support was mounted in the guide tube of a special traversing mechanism which was employed to control the position of the hot wire sensor. A "shorting probe" was used to short-circuit the probe support and cable thereby cancelling out their resistances during alignment of the anemometer system. The dynamic characteristics and the upper frequency limits of the bridge arm were adjusted indirectly by conducting a square-wave test. A sudden decrease in the electrical power supplied to the probe was perceived by the system as a sudden velocity decrease and the resultant signal was observed on the screen of an oscilloscope. This part of procedure played an important role since ideal bridge balance is a condition for using this method correctly. The adjustments of gain, bridge balance, etc., were made in accordance with the standard procedure of reference 81 with a view of achieving a smooth test signal pattern without superimposed damped oscillations.

Since the temperature of the upstream air was maintained constant during all the measurements, corrections due to temperature fluctuations were unnecessary. The standard procedure and major precautions taken in these measurements closely paralleled those of references 79 and 81. The results were interpreted by direct calibration and are accurate within ± 10%, see Appendix A. These results are presented in Fig. 29 as the turbulence intensities against the flow Reynolds numbers, and it can be seen that the turbulence intensity increased as the Reynolds number decreased. However, at the higher Reynolds numbers (i.e. Re > 2 x 10^3) the turbulence intensity tends to be constant. The individual turbulence intensity profile at the plane of the test section was measured at two different Reynolds numbers and are shown in Fig. 30. It can be seen that largely uniform profiles were obtained.

The turbulence scale was not measured since in these present tests, grids were not inserted prior to the test sections (except those in the settling chamber) so that the ratio of turbulence scale to the cylinder diameter was expected to be small (Ref. 80).

3.6 Experimental Details

3.6.1 Mass Transfer Test

In these experiments the mass transfer rate was found by measuring the rate of sublimation from a naphthalene cylinder. The Chilton-Colburn analogy was then employed to estimate the corresponding heat transfer coefficients. The calculation procedure and the heat-mass transfer analogy are discussed in detail in Appendix B, whereas the experimental
details are described in the present section.

a) Test Cylinder Preparation

Fresh naphthalene was melted in a small pyrex beaker placed in boiling water. During this process ingress of water into the naphthalene container was avoided. Cylindrical steel formers were preheated in the steam and then dried prior to coating with the sublimate. This decelerated the solidification process during coating and resulted in a more consistent layer. Coating was achieved by painting the molten naphthalene onto the former to a diameter in excess of that required. Air pockets, if formed, were removed by a heated wire. The correct specimen dimensions were obtained by 'turning down' the diameter on a precision lathe before final finishing with a fine aluminium oxide cloth.

After each test, the old coating of naphthalene was removed and a fresh one applied. This proved beneficial since the old naphthalene tended to crack and fracture during the machining process.

b) Test Procedure and Calculations.

The air-flow and temperature in the test section were adjusted to the required values with a stainless steel 'dummy' tube mounted in place of the naphthalene cylinder. Upon attainment of these conditions the fan was switched off and this 'dummy' was then replaced by the test cylinder which had been previously weighed. This naphthalene tube was positioned by two spigots inserted through holes in the surrounds of the test section, see Fig. 31. The fan was then started and the air flow and temperatures monitored throughout the test. The duration of each test was measured using an accurate stop-clock.

The maximum allowable weight loss was limited in order that the geometry of the naphthalene cylinder was not excessively disturbed. This sublimation loss depended upon the size and hence initial weight of the test cylinder, but in all cases did not exceed 0.07 grams. Consequently, the duration of each test was less than 15 mins. The exact duration depended upon the air flow and temperature in the test duct and experience gained during the experiments proved helpful in selecting an appropriate value. The test cylinder was rapidly removed at the end of a test run by extracting the spigots. This cylinder was immediately re-weighed to minimise sublimation losses in the atmosphere. Accurate weighing was necessary because of the small weight losses discussed previously.

Consequently, a balance capable of weighing to 0.0001 grams was employed.

The mass transfer coefficient, \( b \), may then be evaluated using

\[
b = \frac{\dot{m}}{t \cdot S \cdot \frac{R_v \cdot T_w}{P_w}}
\]

where

- \( \dot{m} \) is the naphthalene weight loss \( (kg) \)
- \( t \) is the test duration \( (sec.) \)
- \( S \) is the surface area of the test cylinder \( (m^2) \)
- \( R_v \) is the gas constant \( (J/kg \cdot \degree K) \)
- \( T_w \) is surface temperature of the naphthalene \( (^\circ K) \)
- \( P_w \) is the naphthalene vapour pressure at \( T_w \) \( (N/m^2) \)
Consequently, a knowledge of the naphthalene surface temperature is required. It is usual to assume that it is equal to the temperature of the air in the test section. Any error in this assumption may be minimised by maintaining this air temperature as close as possible to the ambient value, i.e. the initial temperature of the test cylinder. This precaution is particularly important in the present series of experiments due to their short duration.

The Chilton-Colburn analogy (Ref. 82) may then be employed to evaluate the heat transfer coefficient, $h$, so that

$$h = \frac{\nu}{\rho C_p} \left( \frac{Re}{Pr} \right)^{2/3}$$

A sample calculation is presented in Appendix B. Test runs were carried out to cover the combinations of tube diameters and spacings given in table 3.1.

### 3.6.2 Direct Heat Transfer Test

Direct heat transfer measurements were also employed in the present study in addition to the mass transfer experiments. For this purpose, use was made of a steady-state electrical method involving the 'indirect' heating of a copper cylinder. Previous investigators have employed both steady-state and transient techniques. Direct or indirect electrical heating is generally utilised but on occasions steam or heat transfer fluids have been employed. However, for the present tests a steady state method appeared most suitable. This requires a knowledge of the heat dissipation from the cylinder and also the mean temperature of the tube and air.

#### a) Design of Test Cylinders

The original design consisted of a glass tube coated with a copper film of approximately 0.003 mm thickness. The mean surface temperature of the tube may then be found by measurement of the electrical resistance of this film. The small thickness of the copper ensured a large resistance change as the temperature varies so that comparatively crude measure of resistance change sufficed. However, problems were encountered due to the poor adhesion between the glass and the copper; furthermore it was difficult to connect the necessary copper leads to the film. Hence, it was decided to use copper tubes of the appropriate dimensions as the test cylinders. The design of these cylinders is shown schematically in Fig. 32. Each tube was heated internally by a nichrome resistance element insulated electrically from the copper by a thin ceramic sheath which was a 'push fit' in the tube. The heated length was 90 mm so that the velocity distribution was uniform over the test section. The remainder of the tube was filled with low conductivity compacted insulation and the connecting leads were led out through this material.

Generally to minimise axial heat losses, the length to diameter ratio ($L/D$) of the test section should be high (Ref. 11). However, in the present tests the range of ($L/D$) was equal to (16.7 - 25.3). Nevertheless, due to the high standard of insulation employed and the low heat transfer coefficient at the cylinder ends it was considered that axial
losses would be small.

To measure the mean resistance of the copper cylinder two copper leads were connected at each end of the effective test length to the inner surface of the cylinder. These leads were then led out internally through the insulation.

b) Test Instrumentation and Equipment

The necessary instrumentation and associated apparatus is shown diagrammatically in Fig. 33. The nichrome heating element was supplied from a 0-80 volt d.c. stabilised power source rated at 0.6 amps. A 30 ohms variable resistance was mounted in series with the heater to provide further adjustment of the current as required. To estimate the heater power dissipation to the air flow the current in the circuit was measured by a standard ammeter (of 1% accuracy) and a voltmeter of similar accuracy was used to determine the voltage drops across the heater element.

The electrical resistance of the test length of the copper tube was found by employing a 'four-way' bridge d.c. circuit. A known current was passed through the copper tube. The resistance was determined by measurement of the voltage drop over this test length using a digital voltmeter capable of reading 1 μV. This current was maintained constant at 0.45 amps throughout the tests and was determined by measurement of the voltage drop across a standard 10 ohms resistance. The heating effect of this current was negligible due to the low resistance (approximately 1.9 to 5.5 x 10^-4 ohms) of the copper tubes.

c) Calibration of Test Cylinders

To determine the mean temperature of the test cylinder the relationship between this parameter and the electrical resistance was required over the range 0 to 100°C. This was achieved by immersing the copper test cylinders in a thermostatically-controlled, electrically-heated, well-stirred oil bath. Steady-state conditions were attained and the oil bath temperature was measured by mercury-in-glass thermometers graduated in divisions of 0.1°C. A linear calibration was obtained and a typical relationship is presented in Fig. 34. The results for all the cylinders were in good agreement when compared on a fractional resistance change basis. Any slight variations may be accounted for by differences in the material properties or internal stresses in the tubes.

Although copper has the most linear relationship between resistance and temperature of known metals (Ref. 83) it is less reproducible than platinum since it is subject to oxidation at moderate temperatures. Consequently, the cylinders were re-calibrated at the conclusion of each test series and no important variations were observed.

d) Test Procedure

The test cylinder was inserted in the centre of the tube matrix and the air supply and cylinder heater were switched on. A period of up to 40 mins was required to attain the necessary steady-state conditions.
Upon achievement of these conditions the heat dissipation from the cylinder and its electrical resistance were measured as described in the previous sections. The air temperature prior to the tube bank was measured by a mercury-in-glass thermometer. Any rise in temperature of the air in passing over the tube bank was negligible.

The circumferential non-uniformity of the heat transfer coefficients around the tube, under the test conditions, causes variations in the surface temperature of the copper test cylinder. Consequently, slightly differing temperatures occurred at the positions at which the leads, used for resistance measurement, were soldered to the tube. A small potential difference was thus generated and affected the measurement of voltage drop over the effective length of this tube, and hence the estimation of resistance. This error was estimated and allowed for by switching off the current used for resistance measurement whilst maintaining the heater at its steady conditions. The residual voltage generated due to the 'thermocouple effect' was then measured by the digital voltmeter and the necessary correction applied.

The mean test cylinder temperature was found from the appropriate calibration curve so that the heat transfer coefficient can be calculated.

The construction of test cylinders of diameter 3 mm proved to be difficult so that the direct heat transfer measurements were confined to tubes of larger diameter. Consequently, test runs were carried out to cover the combinations given in table 3.2.

**TABLE 3.1**

**RANGE OF MASS TRANSFER TEST GEOMETRIES**

<table>
<thead>
<tr>
<th>TUBE DIAMETER, mm</th>
<th>TUBE SPACING, mm</th>
<th>BLOCKAGE RATIO PER CENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.35</td>
<td>89.6</td>
</tr>
<tr>
<td>3.0</td>
<td>0.88</td>
<td>77.3</td>
</tr>
<tr>
<td>3.0</td>
<td>1.40</td>
<td>68.2</td>
</tr>
<tr>
<td>4.0</td>
<td>0.60</td>
<td>87.0</td>
</tr>
<tr>
<td>4.0</td>
<td>1.13</td>
<td>78.0</td>
</tr>
<tr>
<td>4.0</td>
<td>1.43</td>
<td>73.7</td>
</tr>
<tr>
<td>5.0</td>
<td>0.55</td>
<td>90.1</td>
</tr>
<tr>
<td>5.0</td>
<td>1.05</td>
<td>82.6</td>
</tr>
<tr>
<td>5.0</td>
<td>1.68</td>
<td>74.9</td>
</tr>
<tr>
<td>6.0</td>
<td>0.30</td>
<td>95.2</td>
</tr>
<tr>
<td>6.0</td>
<td>0.85</td>
<td>87.6</td>
</tr>
<tr>
<td>6.0</td>
<td>1.40</td>
<td>81.1</td>
</tr>
</tbody>
</table>
### TABLE 3.2
RANGE OF DIRECT HEAT TRANSFER TEST GEOMETRIES

<table>
<thead>
<tr>
<th>TUBE DIAMETER, mm</th>
<th>TUBE SPACING, mm</th>
<th>BLOCKAGE RATIO PER CENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>0.62</td>
<td>86.6</td>
</tr>
<tr>
<td>4.00</td>
<td>1.15</td>
<td>77.7</td>
</tr>
<tr>
<td>4.00</td>
<td>1.45</td>
<td>73.4</td>
</tr>
<tr>
<td>4.78</td>
<td>0.66</td>
<td>87.9</td>
</tr>
<tr>
<td>4.28</td>
<td>1.04</td>
<td>82.1</td>
</tr>
<tr>
<td>4.78</td>
<td>1.78</td>
<td>72.9</td>
</tr>
<tr>
<td>6.00</td>
<td>0.30</td>
<td>95.2</td>
</tr>
<tr>
<td>6.00</td>
<td>0.85</td>
<td>87.6</td>
</tr>
<tr>
<td>6.00</td>
<td>1.40</td>
<td>81.1</td>
</tr>
<tr>
<td>6.00</td>
<td>0.50</td>
<td>92.3</td>
</tr>
<tr>
<td>6.00</td>
<td>1.00</td>
<td>85.7</td>
</tr>
<tr>
<td>6.00</td>
<td>1.50</td>
<td>80.0</td>
</tr>
</tbody>
</table>

### TABLE 3.3
RANGE OF HYDRAULIC RESISTANCE TEST GEOMETRIES

<table>
<thead>
<tr>
<th>TUBE DIAMETER, mm</th>
<th>TUBE SPACING, mm</th>
<th>BLOCKAGE RATIO PER CENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.63</td>
<td>82.6</td>
</tr>
<tr>
<td>3.0</td>
<td>0.98</td>
<td>75.4</td>
</tr>
<tr>
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<td>1.45</td>
<td>67.4</td>
</tr>
<tr>
<td>4.0</td>
<td>0.57</td>
<td>87.5</td>
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<td>1.05</td>
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<tr>
<td>4.0</td>
<td>1.45</td>
<td>73.4</td>
</tr>
<tr>
<td>4.5</td>
<td>0.88</td>
<td>83.6</td>
</tr>
<tr>
<td>4.5</td>
<td>1.72</td>
<td>72.3</td>
</tr>
<tr>
<td>5.0</td>
<td>0.47</td>
<td>91.4</td>
</tr>
<tr>
<td>5.0</td>
<td>0.92</td>
<td>84.5</td>
</tr>
<tr>
<td>5.0</td>
<td>1.31</td>
<td>79.2</td>
</tr>
<tr>
<td>6.0</td>
<td>0.47</td>
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<tr>
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<td>0.92</td>
<td>86.7</td>
</tr>
<tr>
<td>6.0</td>
<td>1.43</td>
<td>80.8</td>
</tr>
<tr>
<td>6.0</td>
<td>0.50</td>
<td>92.3</td>
</tr>
<tr>
<td>6.0</td>
<td>1.00</td>
<td>85.7</td>
</tr>
<tr>
<td>6.0</td>
<td>1.50</td>
<td>80.0</td>
</tr>
</tbody>
</table>
3.6.3 Error Analysis

A total of 120 heat transfer tests were required to cover the desired range of test conditions and to assess the repeatability of the measurements. However, at two configurations (namely 6mm x 0.3 mm spacing and 4.78 mm x 0.66 mm spacings) a continuous high-pitched note was emitted particularly at the higher flows. This phenomenon was attributed to vibration of the tubes and appeared to affect the heat transfer measurements. Consequently the measurements at these geometries were ignored and were not included in the final results.

In both the heat and mass transfer measurements, there were several possible sources of experimental errors and these may broadly classified as systematic, individual, independent and random errors, see Appendix C . Attempts were made, however, throughout the tests to reduce or minimise these errors. Wherever possible emphasis was placed on comparing the present results with previously published data. These provided an estimate of the overall accuracy of the measurements. Further evidence of their accuracy was also obtained from an error analysis.

The main sources of experimental error will now be discussed in detail.

(a) Direct Heat Transfer Tests

(i) Axial Heat Losses - this error was minimized by firstly maintaining a comparatively small temperature difference between the test cylinder and the ambient surroundings, secondly by insulating the ends of the cylinder; and thirdly by avoiding using cylinders with high aspect ratios (in the present tests L/D was always greater than 15).

(ii) Radiation Effects - these effects were calculated employing emissivities supplied by McAdams (Ref. 7 ). However, because of the relatively low cylinder surface temperatures the radiative contributions were always less than 2% so that these effects were neglected.

(iii) Average Temperature Measurement - errors in this measurement were reduced by the calibrating of resistance against the tube temperature over the range of -17°C to 100°C. The resistance was measured by digital voltmeters capable of reading to an accuracy of within ± 1% of full scale deflection. Thus the temperature was measured to an accuracy of ± 0.2°C so that the uncertainty in the values of heat transfer coefficient due to this error lies between ± 2% to ± 4%.

(iv) Non-Uniformity in the Tube Geometry - the tube diameters were measured at several positions along the effective lengths. The percentage change in the tube diameter was always less than ± 1.0%. However, this could change the tube spacings by up to ± 12%. The effects on the average heat transfers, however, will be somewhat less serious see McDonald and Eng. (Ref. 89)

(v) Natural Convection - the procedure followed in evaluating natural-convection effects was in accordance with that recommended by McAdams (Ref. 7) and Sarma and Sukhatme (Ref. 88). The tube Grashof numbers (based on the average surface temperature) were always such that Gr < 0.22 Re^2 even at low Reynolds numbers. This criterion indicates
the limit at which natural convection effects become significant and applies to horizontal cylinders. In the present tests the cylinders were mounted vertically so that natural convection was lower for these cases. Thus the effect of natural convection was neglected.

(B) Mass Transfer Tests

(i) Variations in Surface Roughness - These variations occur during the final machining process prior to each test run. Attempts were made to produce a more consistent, smoother surface by casting the test cylinders in a stainless steel mould. Unfortunately due to the very small diameters employed in the tests this procedure was unsuccessful.

The variations in surface roughness could affect the mass transfer due to the close spacings employed. Errors due to this are difficult to estimate quantitatively. However, after conducting some initial heat transfer tests using cylinders of varying surface roughnesses (see section 3.7.6.) it is suggested that the uncertainties in the mass transfers did not exceed ± 10 to ± 15%.

(ii) Extraneous Sublimation Losses - These can occur prior to and at the conclusion of each test. These losses were considered to be very small due to the precautions described earlier.

(C) Errors Arising in Flow and Pressure Drop Measurements

The measurement accuracy for the various parameters were estimated to be:

- Air temperature ± 0.1°C
- Velocity of the air prior to the test section ± 1%
- Pressure drop across the test sections ± 1.0 mm water

3.7 Results and Discussion

3.7.1 Heat and Mass Transfers

The results for both heat and mass transfers are presented in a generalised form as Nusselt numbers plotted against Reynolds number based on the maximum velocity (i.e. the velocity in the minimum free flow area between the tubes) see Fig. 35. This method of calculating Reynolds number is widely used for flows through banks of tubes. Previous results provided by Keys and London (Ref. 2); Zukauskas (Ref. 13) and Pierson (Ref. 65) are also presented. These were obtained from tests on multi-row tube banks, so are corrected to account for the influence of the number of rows (using the correction factors suggested in their references). Inspection of Fig. 35 suggests that for Reynolds number, $Re < 5 \times 10^5$, the present results are in good agreement with those obtained in the previous studies. At higher Reynolds numbers, however, the present results appear to be considerably lower. The small spacings encountered in these present tests, result in wide variations of velocity over the tubes. Consequently the maximum velocity is less representative of the flow over the tubes than in cases studied previously which have greater spacings between adjacent cylinders. Thus using this maximum velocity tends to overestimate the effective Reynolds' number.
which should be employed when correlating very closely spaced tubes (see Refs. 13, 14).

Many alternative reference velocities to that of the maximum velocity have been suggested by previous investigators, (see Chapter 2) to account for the influence of the flow blockage. Perhaps the most common correction is to employ the mean velocity over the cylinder surface so that

$$u_m = u_o (1 - \frac{\pi}{4} \frac{D}{H})^{-1}$$

where \(H\) is the transverse pitch and thus \((D/H)\) is the blockage ratio.

The correct choice of reference velocity enables tube bank data to be compared with those for single cylinders. Thus a comparison between the present results and those of McAdams (Ref. 7) is made in Fig. 36 and Fig. 37. Inspection of these figures reveals the following:

(a) The present heat transfer data obtained from tubes at \(D/H < 0.85\) (i.e. the results represented by the solid symbols) is in excellent agreement with McAdams' correlation.

(b) However, at higher flow blockages, i.e. \(D/H \geq 0.85\) the heat transfers are considerably higher than McAdams' results. Thus the use of a mean Reynolds number can lead to serious underestimations in predicting average heat transfers. For example, at \(R_e_m = 4.3 \times 10^5\) the average Nusselt number predicted by McAdams' relationship is 36, whereas the measured value in the present tests is 52, (i.e. about 30% difference).

(c) The heat transfers inferred from present mass transfer results are higher than those predicted by McAdams, at virtually all blockage ratios.

Consequently, it appears better to deal separately with the present heat transfer results from those obtained by the mass transfer tests. The direct heat transfer measurements (at all blockage ratios) may be correlated as:

$$Nu = 0.77 R_e_m^{0.47}$$

using an integrated mean velocity as the basis for Reynolds number. The mass transfer results yield

$$Nu = 0.152 R_e^{0.67}$$

Statistical analysis (Ref. 90) yields a Standard Error of 9.96% in the estimate for the direct heat transfer results and 8.86% for the mass transfer case. Some of the mass transfer results particularly at \(Re < 10^5\) lie outside the 99% confidence limits for the heat transfer data and vice-versa. Consequently it appears that there is a significant difference in the two sets of results. For this reason the heat and mass transfer results are discussed separately in the remainder of this section with greater emphasis placed on the direct heat transfer results.
As discussed earlier in this section, the mean velocity is not a suitable basis for correlation of the heat transfers at high blockages \((D/H > 0.85)\) so that recourse was made to an empirical reference velocity. A modified form of the correction proposed by Akilbayev (Ref. 50) was employed and resulted in reasonable agreement with McAdams' single cylinder relationship, see Fig. 37 b. Thus for the present heat transfer data for small diameter tubes with close spacings \((0.85 < D/H < 0.92)\) the most suitable reference velocity is expressed by

\[ U_r = U_0 \left[ 1 + 1.82 \left( \frac{D}{H} \right)^3 \right]^2 \]

Thus it is recommended that an empirical velocity of this form should be used to predict the convective heat transfers associated with tubes at high blockage ratios.

### 3.7.2 The Influence of Tube Diameter and Spacing on Heat Transfers

The effect of tube spacing on the heat transfer coefficients derived from the mass transfer tests may be seen from Figs. 38-40 where Nusselt numbers are plotted against the Reynolds numbers (based on approach velocity) for each geometry. A family of curves of the form

\[ Nu = C \text{Re}^m \]

For these curves \(C\) varied from 0.419 to 0.635 and the exponent \(m\) ranged from 0.55 to 0.67. In general, for each size of tubes the heat transfer increased as the tube spacing decreased.

Similar relationships were obtained for the direct heat transfer measurements, see Figs. 41-44. For these experiments \(C\) varied from 0.419 to 3.33 and the exponent changed from 0.39 to 0.621.

However, the designer of a Stirling engine requires the heat exchanger arrangement which will produce the maximum heat transfer rate for a particular fuel and combustion air input rate (i.e. in the present study, a particular air flow rate over the tubes). Consequently the results are replotted as \(\text{Nu}/D\) (which is proportional to the heat transfer coefficient) against \(\text{Re}/D\) (which is proportional to the air flow rate prior to the test section). The mass transfer results for two air flow rates, see Fig. 45, indicate that the heat transfer coefficient increased as the tube spacing decreased. With the exception of the 3 mm sized cylinders at the higher flow rate, the tube diameter had little effect on the rate of heat transfer. A similar presentation of the direct heat transfer data, see Fig. 46, corroborates the conclusion concerning the benefit of employing close spacings. However, the diameter appears to have some effect particularly at the higher flow rate. Nevertheless no consistent relationship with diameter can be observed (the lowest heat transfer occurred with the 5 mm diameter tube in this case and the 3 mm diameter tube in the mass transfer experiments). Thus the rate of heat transfer appears to be relatively insensitive to changes in the tube diameter whereas variations in the spacing can have significant effects.
3.7.3 **Hydraulic Resistances (Pressure Drops)**

In these experiments the temperature of the air was maintained constant at the ambient value throughout all the tests to eliminate varying density effects. The pressure was measured by water manometers and the tappings were positioned 36 mm upstream and 265 mm downstream of the test section. The selection of this latter position eliminated any low pressure regions associated with vortices or eddies created by the tubes.

The mean gap between adjacent tubes over the whole matrix was found by employing feeler gauges and this mean value was employed in the calculations of Reynold's number. The range of geometries is given in Table 3.3.

Hydraulic resistance data is often presented in dimensionless form as the Fanning friction factor plotted against the Reynolds number (based on the maximum velocity at the minimum flow area between the tubes), see Fig. 47 - 48. However, recent studies (see for example Ref. 78) have shown that this method of presentation offers little useful information when the performances of several tube banks in crossflow are examined. In fact, the Fanning friction factor is much more useful in the case of tube bundles or flows inside tubes. In these cases a quantitative relationship exists between the pressure drops and the heat transfers (see Ref. 7). Thus in this present study the hydraulic resistances are expressed as $\Delta P/\rho$ and these results are presented in Figs. 49 - 52 for each tube bank arrangement. The Reynolds number is based on the mainstream velocity. A family of curves of the form

$$\frac{\Delta P}{\rho} = C_1 \text{Re}^m_0$$

were obtained where $C_1$ varied from $9.9 \times 10^{-6}$ to $89.5 \times 10^{-6}$ and the exponent $m$ ranged from 1.82 to 2.08.

The variation of the pressure drop at constant air flow rate in the duct prior to the test section (i.e. at a particular fuel and combustion air in an actual Stirling engine) is presented in Fig. 53. The pressure drop increased markedly as the spacing between the tubes is reduced. Unlike the heat transfer case, however, the diameter had a marked effect and the pressure drop decreased significantly as the tube diameter decreased for both mass flow rates examined. This is not unexpected since decreasing the tube diameter at a fixed tube spacing increases the net free flow area so that the hydraulic resistance is reduced. Thus the pressure drop with 3 mm diameter tubes at 0.5 mm spacings is approximately the same as with 6 mm tubes at 1.0 mm spacings. Consequently there are significant benefits so far as hydraulic resistance is concerned in employing the 3 mm tubes, i.e. the smallest tubes tested at present.

3.7.4 **Optimal Tube Arrangement**

The optimal arrangement of tube bank will have the largest possible value for the ratio of heat transfer to hydraulic resistance. Thus the present measurements indicate that for an exchanger constructed of a single row of tubes, the smallest diameter (i.e. 3 mm) should be employed since the heat transfer is relatively unaffected by diameter and the hydraulic resistance decreases as the tube size decreases.
The heat transfer and the pressure drop both increase as the spacing is decreased so that at a given fuel and air input to an engine the spacing between tubes is dictated by the maximum allowable pressure drop. Thus the minimum spacing allowed by this constraint should be adopted to maximise the heat transfer.

3.7.5 The Influence of Mainstream Turbulence Intensity

As discussed earlier in this thesis there is a lack of information concerned with the influence of mainstream turbulence intensity on the heat transfers associated with tubes in restricted flows. Consequently the present study of average heat transfers was extended to include the influence of this parameter. For this purpose only the 6.0 mm diameter tube was employed with three different spacings (0.5 mm, 1.0 mm and 1.5 mm) so that high blockage ratios (0.8 to 0.92) were studied. The experiments were carried out in conjunction with a MSc. project at the Cranfield Institute of Technology so that full details of the experimental techniques and the hot wire anemometry instrumentation are presented by Al-Kanani (Ref. 75). The turbulence intensities were measured as was described earlier in section 3.5, and the method for measuring the heat transfers was virtually identical to that discussed in section 3.6.2.

Perforated plates or grids, mounted in the duct upstream of the test section were used to generate a range of turbulence intensities. These grids were designed and constructed so that the turbulence intensity and macro-scale of the flow can be readily calculated using the generalised correlation of Ballal (Ref. 80). To minimize the effects of turbulence scale on the heat transfers, the ratio of this variable to the tube diameter was maintained constant at a value of 1.5 throughout the tests. A turbulence macro-scale of this level has comparatively little effect, see Zijnen (Ref. 46). This desired calculated turbulence scale was achieved by varying the distances between the turbulence generating grids and the test sections. Details of the calculations are provided in Appendix A and the appropriate distances which should be maintained as shown in table 3.4.

Experimental measurements of the turbulence intensities at the test sections were in good agreement with the predicted results (see table A.1, Appendix A). However, the measured turbulence intensities varied somewhat with changes in the Reynolds number particularly for $Re < 3000$ (Fig. A.2 of Appendix A). Consequently, higher turbulence intensities ($Tu > 15\%$) occurred at the lower Reynolds numbers, thus the average heat transfer measurements were obtained at $Re > 900$. Typical turbulence intensity profiles downstream of the turbulence-generating grids are shown in Figs. A.3, A.4, A.5 in Appendix A.

The results obtained in the present tests are presented in Figs. 54, 55, 56 as the average Nusselt number plotted against the mainstream turbulence intensity. These results are compared with previous data on single cylinders observed by Dyban et al (Ref. 42) and Lowery and Vachon (Ref. 49). Unfortunately for comparison purposes published data are not available concerning the effect of turbulence in tube banks. Thus an appropriate reference velocity must be used when
comparing the present results with previous single cylinder data, to account for the differences in the blockage ratios. The integrated mean velocity correction was employed in the case of the 6 mm x 1.5 mm tube row (i.e. D/H < 0.85) and the modified empirical correction (see section 3.7.1) in the case of the 6 mm x 1 mm and 6 mm x 0.5 mm configurations (i.e. D/H > 0.85).

Dyban et al (Ref. 42) suggested the following equation for the prediction of heat transfers from single cylinders in turbulent crossflows:

\[ \text{Nu} = C \text{Re}_0^n \]

where \( C \) and \( n \) are empirical constants whose values depend on the turbulence intensity as follows:

<table>
<thead>
<tr>
<th>( \text{Tu} %, \text{Tu} , % )</th>
<th>0.3</th>
<th>2.6</th>
<th>6.5</th>
<th>12</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>0.262</td>
<td>0.262</td>
<td>0.262</td>
<td>0.262</td>
<td>0.25</td>
</tr>
<tr>
<td>( n )</td>
<td>0.60</td>
<td>0.61</td>
<td>0.62</td>
<td>0.63</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Lowery and Vachon (Ref. 49) propose a correlation of the form

\[ \text{Nu} = 0.686 \text{Re}_0^{0.5} + 0.043 \text{Tu} \cdot \text{Re}_0/100 \]

Inspection of Figs. 54, 55 reveals that the results for the two lower blockages are in good agreement with those of Dyban et al although there is less similarity with those of Lowery and Vachon. However, these latter workers employed a considerably higher Reynolds Number than those maintained in the present study. At the highest flow blockage (i.e. the 6 mm x 0.5 mm geometry) there are considerable discrepancies between the present results and previous data particularly at \( \text{Re}_0 < 2.3 \times 10^4 \), see Fig. 56. For this range of Reynolds number (based on the proposed reference velocity) the average Nusselt numbers are unaffected by the turbulence intensity. However, at the higher Reynolds numbers it is evident that the average Nusselt number increased with increasing turbulence intensity. The heat transfers associated with flow blockage \( \text{D/H} = 0.8 \) increased as the turbulence intensity was increased. The same characteristic was generally observed with the remaining geometries although the rate of increase was less marked for \( \text{D/H} = 0.857 \) and even less marked for the highest blockage configuration i.e. \( \text{D/H} = 0.923 \)

### 3.7.6 The Combined Influence of Turbulence Intensity and Blockage Ratio

One of the most extensive reviews of the heat transfers associated with cylinders in cross flows is that by Morgan (Ref. 11) who undertook the formidable task of re-examining previous published studies. He discussed the combined effect of mainstream turbulence intensity (for a range from zero to 16%) and blockage ratio (from 0.01 to 0.36). Morgan calculated the fractional change in the average Nusselt number for particular conditions using the value of zero turbulence intensity and blockage as a base. Two terms, one of which refers to the blockage affect and the other which allows for turbulence, were employed.
Thus:

\[ \Delta \text{Nu} / \text{Nu} = \Delta \text{Nu}_B + \Delta \text{Nu}_T \]

The reference velocity proposed by Vincenti and Graham (Ref. 55) for closed wind tunnels was employed to allow for the increase in the mainstream velocity due to blockage effects. This reference velocity has the form:

\[ U_r = U_0 \left[ 1 + 0.321 \ C_d \ \frac{D}{H} + 1.356 \ \left( \frac{D}{H} \right)^2 \right] \]

where \( C_d \) is a drag coefficient which depends on Reynolds number, blockage ratio, turbulence intensity and aspect ratio \((Z/D)\). For \( 10^2 \leq Re \leq 10^5 \), \( Tu < 1\% \), \( D/H < 1 \), and \( Z/D > 10 \), the value of \( C_d \) was assumed to be 1.2 so that

\[ \Delta \text{Nu}_B = \left[ 1 + 0.385 \ \frac{D}{H} + 1.356 \ \left( \frac{D}{H} \right)^2 \right]^n - 1 \]

where \( n \) is the exponent for \( Re \) in the appropriate heat transfer correlation.

This correction is applied to experimental data obtained for single cylinders at negligible blockage ratios e.g. Hilbert (quoted in Ref. 11).

The correction for the influence of turbulence intensity was derived from the experimental data of van der Hegge Zijnen (Ref. 46) for Reynolds numbers of approximately \( 10^4 \) and \( 0.03 \leq Tu \leq 0.12 \).

Thus:

\[ \Delta \text{Nu}_T = 2.42 \ (Tu)^{2/3} \]

These corrections are applied to determine the combined effect of blockage ratio and turbulence intensity, see Fig. 57. The predictions were in reasonable agreement with previous experimental data, especially at low flow blockage. However, inspection of the figure suggests that at blockage ratios, \( D/H > 0.2 \) significant departures occur between the predicted and experimental data obtained, by investigators such as Perkins and Leppert (Ref. 54). One would expect this departure, however, to be more serious at even higher blockage ratios such as those used in the present study since the reference velocity employed by Morgan is not suitable for these geometries.

Thus the blockage effect can be estimated by using either the integrated mean velocity as the correction or the empirical correction suggested in section 3.7.1 as appropriate. Consequently the total fractional change may be expressed:

\[ \frac{\Delta \text{Nu}}{\text{Nu}} = \left\{ \left( \frac{1}{1 - \frac{n}{4} \ \frac{D}{H}} \right)^n - 1 \right\} + 2.42 \ (Tu)^{2/3} \]

for \( D/H < 0.85 \).
and

$$\frac{\Delta \text{Nu}}{\text{Nu}} = \left[ \left( 1 + 1.82 \left( \frac{D}{H} \right)^{1.3} \right)^{2n} - 1 \right] + 2.42 (\text{Tu})^{2/3}$$

for $D/H > 0.85$

For $35 \leq \text{Re}_o \leq 5 \times 10^3$ \hspace{2cm} $n = 0.471$

and $5 \times 10^3 \leq \text{Re}_o \leq 5 \times 10^4$ \hspace{2cm} $n = 0.633$

It should be noted that these exponents are similar to those derived from McAdams' data (Ref. 7). Use of these alternative values, however, lead to only slight differences in the predictions (4 - 5%).

In Fig. 58 calculated results using these high blockage corrections are compared with the experimental data of this present study. Previous experimental data due to Dyban et al (Ref. 42) are also included together with results obtained by Al-Kanani (Ref. 75).

Inspection of Fig. 58 suggests that as the blockage ratio increases (at a given Reynolds number) the fractional rate of increase in average Nusselt number decreases. The influence of turbulence intensity decreases as the blockage ratio is increased. At very high blockages ($D/H > 0.85$) the heat transfers are virtually unaffected by changes in the level of turbulence.

**TABLE 3.4**

**TEST SPACINGS FOR THE TURBULENCE PRODUCING GRIDS**

<table>
<thead>
<tr>
<th>GRID NO.</th>
<th>M mm</th>
<th>b mm</th>
<th>SPACING mm</th>
<th>Tu %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.8</td>
<td>2.0</td>
<td>63.5</td>
<td>5.4</td>
</tr>
<tr>
<td>2</td>
<td>2.9</td>
<td>0.95</td>
<td>686</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>3.15</td>
<td>1.5</td>
<td>152</td>
<td>12.0</td>
</tr>
</tbody>
</table>

* Experimental results at $\text{Re}_o = 1585$
3.7.7 The Influence of Surface Roughness

All the experiments undertaken to determine the influence of surface roughness employed a 6 mm diameter copper test cylinder. The outside surface of this cylinder was roughened by 'Sand Blasting', (using a grade 2 grit). Thus the surface finish consisted of small irregularities, the mean height of which was determined by using a Taylor-Hobson 'Talysurf' machine. For convenience, this mean height was designated by the 'Centre Line Average' (C.L.A.). For comparison purposes heat transfer measurements were also undertaken on the relatively smooth test cylinder prior to roughening. This smooth cylinder had a C.L.A. value for the surface finish of 0.5 x 10^{-3} mm, whereas the C.L.A. value for the roughened test cylinder was 0.005 mm. These values were then converted into (λ/D) for convenience of comparison of the present results with the previously published data. Thus,

\[
\begin{align*}
\lambda/D \quad \text{(Smooth Test Cylinder)} & = 20.7 \times 10^{-5} \quad \text{Present Study} \\
\lambda/D \quad \text{(Rough Test Cylinder)} & = 20.7\times 10^{-4} \quad \text{Present Study} \\
\lambda/D \quad \text{(Rough Test Cylinder)} & = 30 \times 10^{-4} \quad \text{Achenbach (Ref. 17)}
\end{align*}
\]

Average heat transfer measurements were performed for the single cylinder case (blockage ratio = 0.07) and for both the 6mm x 1.5 mm (0.80 blockage) and 6mm x 0.5 mm (0.92 blockage) tube row configurations. The general test conditions and experimental procedure were virtually identical with those described in section 3.6.2.

The present results were plotted as average Nusselt number versus the Reynolds number based on the mainstream velocity, see Figs. 59 and 60. The first figure shows the present single cylinder results for the smooth and rough tubes. As can be seen in this diagram, the surface roughness did not affect the average heat transfers within the specified range of Reynolds number. This partially corroborates Achenbach's (Ref. 17) conclusion who found that surface roughnesses up to a value of λ/D = 9 x 10^{-3} had no effect on single cylinder heat transfers for Reynolds numbers less than 6 x 10^{4}. McAdams' data for single cylinders is also included in Fig. 59 to illustrate the validity of the measurements.

In Fig. 60 the heat transfers for the tube bank configurations are presented. These exhibit a completely different character in that the average heat transfers were significantly increased due to an increase in the surface roughness. However, the percentage increase in the average heat transfers were not identical for the two rows. For the tube bank geometry of 6 mm x 1.5 mm geometry the percentage increase was 38% to 41% whereas for the 6 mm x 0.5 mm a 22% improvement was obtained. The reasons for this behaviour are obscure at present in view of the almost complete lack of information available on roughened tubes in a bank.

It should be mentioned that a C.L.A. of 0.005 mm implies that some of protuberances on the cylinder surface may well be four to five times this height. They are thus significant when compared to the spacing between adjacent tubes in the test section so that it is, perhaps, not unexpected that surface roughnesses of this magnitude affect the heat transfers. Within the present experimental errors, these differences in heat transfer were surprisingly repeatable. However, a detailed
consideration of the local heat transfer is probably required to shed further light on the phenomena particularly since the greatest improvement occurred at the less blocked arrangement. It is thus suggested that further work should be undertaken to investigate this practically significant finding.

3.8 Concluding Remarks

The average heat transfers associated with a single row of closely-spaced (0.5 mm - 1.5 mm), small diameter (3.0 mm - 6.0 mm) tubes in crossflowing air were measured employing both direct heat transfer and mass transfer techniques. The hydraulic resistances associated with these tube assemblies were also examined. The flow Reynolds numbers (based on the mainstream velocity prior to the tube row) ranged from 300 to $8 \times 10^3$ and the following conclusions can be drawn:

1. For blockage ratios $D/H < 0.85$, the average forced convective heat transfers were in good agreement with those reported in the technical literature for single cylinders provided that an integrated mean velocity was used as the basis for Reynolds number. However, at greater blockages, the heat transfers were significantly higher. In these cases an empirical reference velocity of the form

$$U_r = U_o \left[1 + 1.82 \left(D/H\right)^{2.2}\right]$$

should be employed to correlate with single cylinder data.

2. For a constant air flow rate (i.e. a constant fuel and air input to an actual engine) the average heat transfer coefficient increased with decreasing tube spacing but was relatively insensitive to changes in tube diameter. On the other hand, the pressure drop (hydraulic resistance) increased as the tube spacing decreased and as the tube diameter increased. This is not unexpected since decreasing the tube diameter at a fixed tube spacing increases the net free flow area so that the hydraulic resistance is reduced. Thus, for the range of conditions employed in this study it is recommended that the smaller diameter tubes should be employed together with the closest tube spacing. However, this latter parameter is dictated by the maximum allowable pressure drop for a particular fuel and air input to the engine.

3. For tube rows with low flow blockages the influence of turbulence intensity was to increase significantly the heat transfers. However, the percentage increase in the average heat transfers decreased as the flow blockage was increased. Consequently at high flow blockages the average heat transfers are virtually independent of the mainstream turbulence level.

4. The average heat transfers associated with these closely spaced tubes ($D/H > 0.8$) were affected by the surface roughnesses of the tubes. Roughening the tubes increased the heat transfers.
FIG. 24. RIG COMPONENTS — OVERALL LAYOUT

- ORIFICE PLATES AND DUCT TO BS 1042 Pt. 2.
- SETTLING CHAMBER
- REGULATOR
- FLEXIBLE DUCTING
- WATER COOLED AIR COOLER
- KEITH BLACKMAN 20 TYPE 8 CENTRIFUGAL FAN
- TEST DUCT
- TEST SECTION
- SECOMAC AIR HEATER
FIG. 27. VELOCITY PROFILES AT THE PLANE OF TEST SECTION.
**Fig. 28.** Representative diagram showing the associated equipments employed to measure the turbulence intensity.
FIG. 29. TYPICAL RESULTS OF THE TURBULENCE INTENSITY AT THE PLANE OF THE TEST SECTION.
FIG. 30. TURBULENCE INTENSITY PROFILES AT VARIOUS REYNOLDS NUMBERS, Re.

KEY

\( \bullet \) Re_0 = 500
\( \circ \) Re_0 = 1620
\( \triangle \) Re_0 = 3890

\( \frac{1}{k} \)

\( T_u \)
FIG. 31. SECTIONAL VIEW OF NAPHTHALENE CYLINDER

- TUFNOL END PIECES
- NAPHTHALENE COATING
- STEEL FORMER
- PERSPEX SPIGGOT
- DUMMY TUBES
FIG. 33. SCHEMATIC REPRESENTATION OF EXTERNAL SERVICES AND INSTRUMENTATION USED IN THE DIRECT HEAT TRANSFER TEST
FIG. 34. TYPICAL RESULTS FOR RESISTANCE–TEMPERATURE RELATIONSHIP.
FIG. 35. MEAN NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON THE MAXIMUM VELOCITY (PRESENT STUDY). GENERALISED EMPIRICAL FORMULA FOR A TUBE BANK ARRANGED IN A MULTI-ROW/CROSSFLOW SYSTEM (Ref. 5) CORRECTED TO A SINGLE ROW (1). EMPIRICAL FORMULA FROM (Ref. 72) CORRECTED TO A FIRST ROW (1) EXPERIMENTAL DATA FROM (Ref. 68) ON BARE TUBE BUNDLE. DIA. = 6·35 mm. CORRECTED TO A FIRST ROW (3).
FIG. 36. MEAN NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON THE INTEGRATED MEAN VELOCITY BETWEEN ADJACENT TUBES.
FIG. 37. a. COMPARISON BETWEEN THE PRESENT DIRECT HEAT TRANSFER RESULTS (FOR ALL TUBE BLOCKAGE RATIOS) AND McADAMS BEST FITTED LINE.
FIG. 37.b. COMPARISON BETWEEN THE PRESENT DIRECT HEAT TRANSFER RESULTS FOR $D/H > 0.85$ AND McADAMS BEST FITTED LINE.
FIG. 38. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. (DIRECT HEAT TRANSFER MEASUREMENTS)
FIG. 39. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. (DIRECT HEAT TRANSFER MEASUREMENTS)
FIG. 40. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. (DIRECT HEAT TRANSFER MEASUREMENTS)
FIG. 41. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. (RESULTS OF MASS TRANSFER TECHNIQUE).
FIG. 42. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. RESULTS OF MASS TRANSFER TECHNIQUE.
FIG. 43. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH.
RESULTS OF MASS TRANSFER TECHNIQUE.
FIG. 44. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER BASED ON VELOCITY OF APPROACH. RESULTS OF MASS TRANSFER TECHNIQUE.
FIG. 45. EFFECT OF TUBE ROW GEOMETRY ON HEAT TRANSFERS AT CONSTANT AIR FLOW RATE. (MASS TRANSFER RESULTS)
FIG. 46. EFFECT OF TUBE ROW GEOMETRY ON HEAT TRANSFERS AT CONSTANT AIR FLOW RATE. (DIRECT HEAT TRANSFER RESULTS)
FIG. 47a. VARIATION OF THE FANNING FRICTION FACTOR WITH $Re_{\text{max}}$.

FIG. 47b. VARIATION OF THE FANNING FRICTION FACTOR WITH $Re_{\text{max}}$. 
FIG. 48. a. VARIATION OF THE FANNING FRICTION FACTOR WITH $Re_{max}$

FIG. 48. b. VARIATION OF THE FANNING FRICTION FACTOR WITH $Re_{max}$
FIG. 49. PRESSURE DROP RESULTS.
FIG. 50. PRESSURE DROP RESULTS.
FIG. 51. PRESSURE DROP RESULTS.
FIG. 52. PRESSURE DROP RESULTS.
FIG. 53: EFFECT OF TUBE ROW GEOMETRY ON THE HYDRAULIC RESISTANCE AT CONSTANT AIR FLOW RATE.
FIG. 55. EFFECT OF MAINSTREAM TURBULENCE INTENSITY ON HEAT TRANSFER TO A TUBE ROW OF 6.0 mm. × 1.0 mm. SPACING.
**FIG. 56. EFFECT OF MAINSTREAM TURBULENCE INTENSITY ON HEAT TRANSFER TO A TUBE ROW OF 6.0 mm DIA AND 0.50 mm SPACING.**
FIG. 57. COMBINED EFFECTS OF BLOCKAGE RATIO AND TURBULENCE INTENSITY ON HEAT TRANSFERS FROM TUBES AT LOWER BLOCKAGE RATIOS (REPRODUCED FROM MORGANS, Ref. 11). SOLID SYMBOLS REFER TO PREVIOUS EXPERIMENTAL DATA CITED BY MORGANS.
FIG. 58. COMBINED EFFECTS OF BLOCKAGE RATIO AND TURBULENCE INTENSITY ON HEAT TRANSFERS FROM TUBES IN CROSSFLOWING AIR. THE EXPERIMENTAL POINTS ARE AS ABOVE.
FIG. 59. EFFECT OF SURFACE ROUGHNESS ON HEAT TRANSFERS FROM SINGLE CYLINDERS

<table>
<thead>
<tr>
<th>KEY</th>
<th>D/H</th>
<th>$\lambda/D \times 10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>▲</td>
<td>0.075</td>
<td>20.7</td>
</tr>
<tr>
<td>▲</td>
<td>0.075</td>
<td>207.0</td>
</tr>
</tbody>
</table>
FIG. 60. EFFECT OF SURFACE ROUGHNESS ON HEAT TRANSFERS FROM TUBES AT HIGHER BLOCKAGES
PLATE 6

General view of the experimental rig for average heat transfer tests. 1-D.C power source for the heater. 2-A 30 Ohm variable resistance. 3-Avometer. 4-Test cylinder. 5-D.C power source for resistance measurement. 6-Standard resistance. 7,8-Varios D.V.M's.
CHAPTER 4

LOCAL HEAT TRANSFERS
ASSOCIATED WITH TUBES AT
HIGH FLOW BLOCKAGES.
CHAPTER 4
LOCAL HEAT TRANSFERS ASSOCIATED WITH TUBES AT
HIGH FLOW BLOCKAGES

4.1 Introduction

This chapter is devoted to the measurement of the local heat transfers associated with small diameter (5 or 6 mm) closely spaced tubes (0.5 - 1.5 mm). The blockage ratios thus varied between 0.77 and 0.92. The experimental results were obtained by employing two different test methods and the two sets of data and those previously obtained by other investigators are compared.

The reason for undertaking these measurements is, (as discussed in detail earlier in the thesis) the virtual lack of information regarding the local heat transfers around tubes at high blockages. The only relevant data available are those of Akilbayev (Ref. 50) at Re₀ = 5.0 x 10⁴ and those of Oka et al (Ref. 52) at Re₀ = 3 x 10⁴. Both data differ substantially between each other for approximately similar blockage ratios, see Fig. 9 Chap. 2. The Reynolds numbers based on a suitable reference velocity ranged from 8.07 x 10⁴ to 1.54 x 10⁵. Within this range however, previous studies on single cylinders have shown that considerable changes in the local heat transfer patterns can occur (see chapter two). Apart from the studies of Refs. 50 and 52 there is no available information on local transfers. In the Stirling engine heat exchanger the high temperature of the combustion gases could result in the formation of 'hot spots', thus it is essential to investigate the distribution of local heat transfers so that the magnitudes and positions of maximum heat transfer may be identified. A knowledge of these local values will also assist in assessing the usefulness of fins and extended surfaces. The influence of Reynolds number, in the range (300 < Re₀ < 6500) on these local patterns is also required.

However, as discussed in Chapter 2, it is considered that the experiments should be carried out with the small diameter tubes of interest rather than with larger sized cylinders. This eliminates any influence of tube diameter on the local heat transfers at these very high flow blockages. However, this presented several difficulties in the construction and instrumentation of the test cylinders. It thus appears relevant at this stage to examine previous experimental arrangements and discuss the special criteria and techniques required for the present investigation. Despite the difficulties and the need for elaborate equipment and procedure in the present tests, the direct measurements gave satisfactory results and agreed reasonably well with those reported in the previous studies. In particular to evaluate the accuracy of the present technique tests were carried out on single cylinders. For the tube rows, the results are presented as the local Nusselt number plotted against the angular position, for each blockage and Reynolds number.

4.2 Experimental Techniques of Previous Investigators

Circular cylinders in crossflows have received considerable attention and it is a time-consuming task to compare the details of the various experimental techniques. A comparison of the techniques employed in investigations up to 1948 has been provided by Winding and Cheney (Ref. 28) although they omitted the classic work of Schmidt and Wenner (Ref.
Ten years later, Knudsen and Katz (Ref. 56) provided a similar comparative review. Various other reviews have appeared and perhaps the most extensive is that produced by Eckert and Goldstein (Ref. 87). However, none of these reviews is specifically concerned with local heat transfer measurements on small diameter tubes. In this present section, some of the previous experimental techniques used for the measurements of local heat transfers will be discussed. Emphasis is placed on details of the dimensions of the test cylinders employed (e.g., tube diameter and range and details of the local test elements, etc.). Possible sources of error and particular limitations or difficulties are examined. Previous experimental techniques may be broadly classified as:

(i) Methods employing direct heat transfer measurements.

(ii) Methods based on the analogy between mass and heat transfer.

In the first category the cylinder boundary conditions can vary between the extremes of constant heat flux and constant temperature, whereas in the second group the heat transfers apply to the isothermal case.

A - Direct Heat Transfer Measurements

These methods generally involve more complicated instrumentation and are more time consuming in comparison with those based on the analogy between heat and mass transfer. The local heat transfer coefficients are directly evaluated by measuring the surface temperature and the corresponding heat flux at various positions around the cylinder.

In their experiments, Schmidt and Wenner (Ref. 33) used an electrically heated copper strip as a narrow segment of the test cylinder wall. The remainder of this wall was kept at the same temperature as the strip by means of steam condensing inside the cylinder. They thus maintained isothermal boundary conditions. The diameter of the test-cylinder varied between 50 and 250 mm at blockage ratios ranging between 16.7 to 41.7%. Reynolds number varied between $8.29 \times 10^3$ and $4.26 \times 10^5$. Since corrections for internal conduction are unnecessary the technique has many advantages since the calculations of local heat transfers are simple. However, the electrically heated segment must be small in comparison with the circumference of the tube to provide local data so that the method is restricted to reasonably large diameter cylinders. This technique is not suitable for the present investigations on small diameter tubes. Achenbach (Ref. 17) employed a closely similar method.

A less simple technique was used by Giedt (Ref. 41) with a 102 mm diameter test cylinder. The central 165.1 mm of this cylinder constituted the test section. Five separate heaters wound with nichrome were fitted around this heating ribbon section. The central heater acted as the measurement section and the adjacent heaters acted as guards. The local heat transfers were estimated by measurement of the current input to the ribbons and the temperature variation around the circumference of the test cylinder. The tests thus corresponded to an approximately constant heat flux boundary condition. Giedt (Ref. 41) conducted his experiments with this boundary condition since most previous experimental work had been confined to the isothermal condition.
The calculation of his local results was more complicated and involved solution of heat balance equations at each position. These often quoted measurements of Giedt together with subsequent results obtained by many other investigators using similar methods, suggest that there are considerable difference between the local heat transfers for the constant temperature and constant heat flux boundaries.

Recently Sarma and Sukhatme (Ref. 88) have employed an even more complicated design of test cylinder. This consisted of thirty identical brass pieces (each with separate thermocouple and heat arrangements) placed in grooves in a fibre rod. The assembly was machined to form a smooth and almost continuous brass surface 76.2 mm in diameter and 151 mm long. The heater leads and thermocouple connections were led out internally. The brass surface was polished, to minimise the heat loss due to radiation. The material separating the brass pieces reduced the circumferential heat conduction. Two guard heater assemblies (whose heat input can be adjusted in a circumferential direction) were employed to reduce axial heat losses. The Reynolds' number in the tests ranged from 500 to 700 and the blockage ratio was 0.1. Despite the complicated fabrication and instrumentation, their results indicate that the circumferential conduction was still significant and amounted to about 15% of the total heat input. Moreover, the radiation component amounted to about 25% of this total input. Both these corrections were estimated to within ±15%. Thus it appears that this complicated construction and procedure were not justified. In addition, it is only suitable for large diameter tubes and offers comparatively little advantage in comparison with Schmidt and Wenner's method.

Boulos and Pei (Ref. 29) and Van Meel (Ref. 89) have along with many others, employed platinum films as the measuring elements and these techniques again require elaborate equipment and calibration. Petrie and Simpson (Ref. 27) Dyban et al (Ref. 42) and Perkins and Leppert (Ref. 54) have used simpler methods based on that of Giedt (Ref. 41). However, the major difference was that in some cases direct resistive heating (i.e. passage of a current through the cylinder) was used to generate uniform heat flux. The bore of the cylinder was unfilled so that corrections for heat conduction through the filling were unnecessary, see reference (91). Although the experimental procedures involved are simple enough the method of calculation is complicated since allowance has to be made for internal heat conduction within the cylinder. In addition various heat losses must also be estimated. Generally these corrections etc. can only be estimated to within ±25% and consequently the experimental results are inaccurate if the corrections are large when compared with the heat flux. To overcome this serious defect previous investigators have used either thin walled test cylinders or thin film test segments mounted flush with the surface. Alternatively relatively low conductivity materials have been employed for the test cylinders.

A somewhat different technique was used by Kostic and Oka (Ref. 63). They used 100 mm diameter test cylinders, placed in a 500 x 500 mm open wind tunnel. The Reynolds number was varied between 1.3 and 2.7 x 10^4. These test cylinders were internally heated using an uniformly wound Kanthal wire heater. The cylinder surface temperatures were measured by thermocouples embedded in a narrow segment.
10 mm wide and 200 mm long (this corresponds to an arc of 11°). This segment was separated from the remainder of the cylinder by two grooves filled with epoxy resin. In this manner a constant heat flux was ensured and circumferential heat conduction was reduced and thus neglected. To avoid errors caused by end effects, the voltage drop associated with the power input was measured only along the length of the test segment. Later measurements of the surface temperature have confirmed the validity of using these slots.

B - Methods based on the Analogy between Heat and Mass Transfer

The heat-mass transfer analogy results from the similarity of the equations which relate the transfer of heat, mass or momentum through a boundary layer in terms of the potential gradient and the molecular and eddy diffusion properties of the fluid. For similar fluid flows the transfer of heat and mass from a surface will be related and will be dependent on the relative diffusion properties of the fluid, and the associated thermal and concentration gradients. Consequently, local mass transfer coefficients determined either from the sublimation rate of a solid (e.g. Naphthalene) or by diffusion (e.g. in the electrochemical method) may be used to infer the corresponding local heat transfers. However, if either the thermal or concentration gradients are distorted due to incorrect experimental setting or procedure, the analogy will not hold as has been observed in references (42, 87).

Proessling (Ref. 93) who studied the sublimation of small spheres initiated the techniques using naphthalene to measure local mass transfers. Subsequently, it has been widely used for the estimation of convective heat transfers. Winding and Cheney (Ref. 38) investigated the local mass transfer from a single Naphthalene cylinder in a crossflowing air stream. The diameter of their test cylinder was 31.8 mm and the Reynolds number was maintained at 32,800. They determined the change in profile of the cylinder by means of feeler gauges which measured the gaps between the model and the original mould in which it had been cast. Considering the crudeness of the technique, the angular variation of the heat transfer coefficient compared very favourably with the results determined by more conventional methods, (see, Ref. 7). More accurate methods for determining local mass transfers from Naphthalene cylinders, have been used by Christian (Ref. 94) in axisymmetrical flow and by Sogin et al. (Ref. 84) in crossflowing air. In both studies profile measurements were obtained by means of micrometer dial indicators mounted on standard lathes. In Sogin et al's investigation, the Naphthalene cylinder was 106.7 mm in diameter and the blockage ratio was 13.1% and Reynolds number ranged from 2.18 to 3.42 x 10^5. The profilometer readings were corrected to allow for deformations due to the contact pressure. In the laminar boundary layer region the results were in good agreement with Merk's solution (Ref. 51).

However, for the successful use of these profilometric methods for the estimation of local convective heat transfers the test cylinder diameters should be large so that the many practical difficulties associated with the use of profilometers on small diameter tubes are reduced. These difficulties may be avoided by the use of completely different mass transfer methods (e.g. Electrochemical techniques). In these methods, the local mass transfers are estimated by measuring the limiting
electrical currents resulting from ionic diffusion in a liquid electrolyte. The local mass transfer probe acts as one of the electrodes in the circuit. The movement of the ions towards the electrodes is dictated by diffusion, by convection in the electrolyte and by ion movement in the electric field (see Ref. 95).

The electrochemical method was introduced initially for measurement of the average mass transfers and was employed for this purpose by Wagner (Ref. 96), Wilkie (Ref. 97), Tobias (Ref. 98) and Eisenberg (Ref. 99). Grassman, et al (Ref. 100) extended the technique to the measurement of local mass transfers and Hanratty et al (Ref. 101) have used it for turbulence measurements. An extensive review of the electrochemical technique has been provided by Mizushina (Ref. 102), who discusses transient as well as steady-state measurements, see also (ref. 95, 103). This electrochemical method is more suitable for work with relatively small diameter tubes since a single measuring probe may be rotated incrementally. Thus the test cylinder is relatively easy to fabricate. In addition errors due to conduction effects are eliminated. Nevertheless, there are many limitations in employing the technique, mainly due to the fact that it is confined to liquids. Thus the experimental data are only strictly applicable to fluids with high Schmidt numbers. Furthermore, it is limited to flows with low or moderate velocities since at higher flows a 'true' limiting current does not occur (Ref. 102). Consequently if high Reynolds numbers are to be investigated, suitably large diameter tubes must be used.

4.3 Choice of a Suitable Technique for the Present Study

It is apparent, from the previous discussion that practical difficulties can hinder the measurement of the local heat or mass transfers associated with small diameter tubes. These problems may be minimized in the case of direct heat transfer methods, by suitably designing the test cylinder so that complicated instrumentations and guard heater systems are avoided, see Kostic and Oka (Ref. 63). However, with small diameter test cylinders it is more convenient to employ direct resistive heating (i.e. passage of a current through the cylinder).

However, in the present investigation circumferential conduction can still be significant (despite the presence of inhibiting slots) - this correction was determined by means of auxiliary temperature measurement as described in detail later in the chapter.

Moreover, it was thought to be beneficial in the present study also to employ a suitable mass transfer technique to provide a further check on the results. For these experiments the electrochemical mass transfer technique is more suitable, as discussed in the preceding section.

4.4 Present Experimental Details

4.4.1 The Electrochemical Mass Transfer Technique

This technique was used in conjunction with an M.Sc. project undertaken at Cranfield Institute of Technology, so that full details of the method are described by Mahmood (Ref. 105). Thus only a comparatively brief description of the present test rig and the experimental procedure are
presented. The test cylinder was fabricated and instrumented so that it could be used for both single cylinder and multiple tube geometries. As reported in Ref. 102, such measurements are extremely sensitive to the approaching flow fluctuations so that it was desirable to measure the turbulence intensity at the plane of the test section. Thus a brief description of these measurements are presented (in so far as the procedure differs from that discussed in chapter 3). The mass transfer results are discussed in section 4.5.

A - Description of the Test Rig

The experimental rig which was used for the present local mass transfer tests is shown schematically in Fig. 61. This test rig was originally based on a design proposed by Mizushina et al (Ref. 103) who investigated the effect of free-stream turbulence on mass transfers. It consists of a P.V.C. square cross-section channel (of side length 57 mm). Five copper meshes (mounted in flanges) were placed periodically along the length of this channel to reduce the boundary layer build up and to provide a relatively uniform velocity profile at the plane of the test section. The test section was positioned downstream of these meshes and also mounted between flanges. The fluid employed as the electrolyte consisted of a mixture of both potassium Ferro and Ferri-cyanide contained in sodium hydroxide. A Stuart No. 26 Centrifugal pump was used to circulate this electrolyte. This pump had a maximum discharge of 151.5 litres per minute at a head of 1.5 m. Thus the maximum 'approach' velocity in the test section was approximately 0.7 m/sec. The electrolyte was stored in a 110 litre capacity P.V.C. tank connected to the section of the pump by a galvanized steel pipe. The storage tank was fitted with a lid to minimise deterioration of the electrolyte in the presence of sunlight. The fluid flow rate was measured by means of a 50 mm diameter Fisher-Porter variable orifice type flowmeter. These flows were controlled in the main circuit at entrance to the flowmeter and at outlet from the tank respectively; and a third valve was positioned in a bypass circuit, see Fig. 61. The tube holders were constructed of Tufnol or Ebonite and an appropriate number of dummy tubes were employed to produce the required tube row geometry.

B - Design of the Test Cylinder

A 4.8 mm diameter Copper tube was used and its design is shown schematically in Fig. 62. A local probe was obtained by employing a 0.71 mm diameter insulated copper wire as the cathode. This was mounted inside a stainless steel sheath and the assembly was embedded and mounted flush with the outside surface. Considerable care was taken to maintain a clean surface over this cathode and to isolate it by epoxy-resin from the remainder of the test cylinder. The connections were led cut internally through the bore. Unfortunately, a smaller cathode could not be constructed, owing to the very small limiting current which occurred. Thus the local mass transfer measurements were associated with an area which subtended an angle of 16.300. The test cylinder was rotated incrementally during the experiments to obtain the distribution of local mass transfer coefficients.

To ensure that the limiting current occurs at the cathode its area should be much smaller than that of the anode. This was ensured by connecting together three of the copper meshes in the upstream section of the duct to
act as the anode. Thus the current density at the anode was always less than that at the cathode. The limiting current which is related to the mass transfer rate can fluctuate during the test due to turbulence in the flow. Thus an approximately time averaged value was obtained as described later in this chapter.

C - Test Instrumentation

A block diagram of the associated measurement equipment is shown schematically in Fig. 63. The voltage applied between the electrodes was supplied from a 6 volt d.c power source and was varied by means of a potentiometer of 2900 ohm maximum resistance. This voltage was measured by a DM2 digital multimeter to an accuracy of ± 2%. The current was measured by another digital multimeter capable of measurements in the range of 0 to 200 μA, with a minimum resolution of 0.1 μA.

D - Test Procedure

The electrolyte solution was prepared by adding the appropriate amount of Sodium Hydroxide to the distilled water and pumping the mixture around the circuit. This was continued until the desired test temperature was obtained. The temperature was obtained initially by allowing the pump to circulate the electrolyte for a warming-up period of about 20 minutes. Alternatively this temperature was controlled by means of an electric heater and a separate water cooling coil immersed in the storage tank. The desired flow rate was set by adjusting the by-pass and mainflow valves.

During the test, the applied voltage was varied incrementally and the corresponding currents was observed. This procedure was followed until the limiting current plateau was obtained. A typical current-voltage relationship is shown in Fig. 64. As the negative potential of the cathode is increased, the Ferricyanide ions in the solution which have accumulated at the surface of the cathode are removed. The current thus increases exponentially. Eventually a stage is reached at which the voltage is sufficient to remove all the ions which migrate to the cathode so that any further increase in potential does not effect the current. Since ion migration in this case is mainly due to the diffusion this limiting current is a measure of the convective mass transfer rate. Nevertheless, some residual current due to ionic attraction can be present even after the addition of an indifferent electrolyte (NaOH). In this case correction can be made, see(Ref. 102). Eventually a further increase in voltage removes the hydrogen ions in the solution and a further increase in current ensues.

By rotating the test cylinder in incremental angular steps of 10°, the local values of the limiting current (which corresponds to the local mass transfers) were obtained.

This procedure was repeated for three different flow rates in the Reynolds number range 300 to 1200 for each of three test sections. Care was taken to clean and wash the tank filter, duct and the cylinder surface before employing a new test section. Nitrogen was bubbled through the test solution before each set of readings to reduce the rate of Ferricyanide deterioration.
E - Data Reduction and Method of Calculation

The local values of limiting current fluctuated due to turbulent flow effects so that both maximum and minimum values, at any position, were recorded. A mean of these two readings was then employed. The local mass transfer coefficient $b_\phi$ may be evaluated from

$$b_\phi = \frac{I_\phi}{\overline{n} F_d A C}$$

where $I_\phi$ is the average value of local limiting current; $\overline{n}$ is the valence change of the appropriate ion (in these tests $n = 1$); $F_d$ is equal to $9649 \times 10^4$ Faraday, and $C$ is the concentration of Ferrocyanide in Kmol/m$^3$.

The Schmidt number was evaluated from the appropriate values of Kinematic viscosity $\nu$ and diffusivity $D_s$ since

$$Sc = \frac{\nu}{D_s}$$

These parameters can be found in Ref. 105. The local Sherwood number was calculated from

$$Sh = b_\phi \cdot \frac{D}{D_s}$$

where $D$ is the test cylinder diameter. A suitable heat-mass transfer analogy (e.g. that due to Chilton and Colburn) may then be used to obtain the appropriate heat transfers. The local mass transfers for both single and multiple tube arrangements can be significantly affected by mainstream flow fluctuation (Refs. 95, 100, 101, 102, 103). Thus it was necessary to measure the turbulence intensity of the circulating fluid as described in the following section.

F - Measurement of Fluid Turbulence Intensity

Mizushina (Ref. 102) has suggested that an electrochemical method can be employed to measure the turbulence intensity in a liquid. He preferred this technique since the dimensions of a suitable hot wire probe can be comparable to the scale of flow fluctuations and cause its own disturbance. Other investigators have also used this method, see Grassman et al (Ref. 100) and Shaw and Hanratty (Ref. 106). However, the hot film anemometers employing suitable designed probes are still the most common means of measuring fluctuations in liquids, see for example (Ref. 79). These techniques were used in the present tests. A constant temperature D.I.S.A. type 55 D01 universal anemometer was used together with a heavy coated fibre film probe (type 55 R 13). The sensor film was mounted parallel to the probe axis. This type of probe has been widely employed for measurements in water (Ref. 107). The general procedure adopted in the turbulence intensity measurements is similar to that described in section 3.5. Also the calibration technique for the sensor was similar to that previously discussed and allowed for the requirements mentioned in Refs. 109, 110. The detailed operating procedure employed in the setting; square wave testing; calibration and measurement was in accordance with Ref. 111. Typical results for the turbulence intensity measurements are shown in Fig. 65. Particular precautions which are necessary in the present tests with liquid include:

...
1. The hot film probe body was inserted in the centre of test flange and care was taken to prevent fluid leakage.

2. For the turbulence measurements water was used as the circulating fluid so that any chemical reactions between the electrolyte and the film sensor was avoided. The use of water as an alternative fluid does not significantly affect the turbulence measurements since the difference in fluid densities was only marginal.

3. The water temperature was maintained constant at 30°C throughout the measurement period to avoid the disturbing effects of varying fluid temperatures on the anemometer signal. Further errors due to the variation of the thermal properties of the liquid with temperature were also thus eliminated.

4. Grounding of the anemometer measuring system is necessary to prevent 'hum loops' that can lead to erroneous evaluation of the test signal. This also protects the sensor against destruction. This grounding was achieved by connecting a copper wire between the ground lead of the anemometer and the nearest mesh screen.

5. At the very low velocities 0.06 - 0.7 m/sec employed in the present study, the heat transfer from the sensor is affected by dirt deposition and contamination. In certain cases this may even increase the rate of heat loss if the effect of the increased surface area is larger than the insulating effect of the coating. Consequently substantial changes in the sensitivity and frequency response of the sensor can ensue. To avoid this effect an extra fine filter was fitted at the pump suction.

6. Short circuiting of the probe support inside the test duct was prevented by using a proprietary potting compound (N.I.C.C. for Pyrotex) to seal the connections.

7. In liquids, dissolved gases can form bubbles on the sensor surface and reduce the heat transfer. This causes a downward drift in the anemometer calibration. Local supersaturation caused by the heat given off by the sensor can result in bubbling at zero flow conditions particularly if microscopic bubbles are already present (Ref. 103). Bubbling was observed initially during the present operating procedure. This was eliminated by reducing the overheating ratio, i.e. the temperature of the sensor.

G - Further Remarks on the Use of the Electrochemical Technique in the Present Study

The main advantage of using the electrochemical method is its speed and simplicity. Thus a comparatively large amount of experimental data can be obtained in a short time. Furthermore it has the attraction that the local heat transfers are unaffected by conduction or radiation which can lead to errors in the direct determinations. However, to obtain accurate results careful and precise experimentation is required. The quality of the cathode material and the electrolyte solution must be maintained throughout the test. Measurement of the mean limiting current values at any position can also require elaborate instrumentation, see Refs. 110,
In this present study the relative variation of mass transfer around the circumference of the cylinder is required rather than precise absolute values. These local values can then be compared with previously measured averages. Because of the relatively small diameter (4.8 mm) tubes investigated in the tests, the ratio of overall cathode diameter (including sheath) to that of test cylinder was approximately 0.2. Thus the cathode and cathode holder (the stainless steel sheath) subtended a 22° peripheral angle. The cathode area, moreover, was nearly flat and did not conform to the curvature of the cylinder surface. This arises due to machining and constructional difficulties. These additional features also throw doubt on the absolute value of the measurements but are unlikely to affect the relative variation.

### 4.4.2 Direct Heat Transfer Technique

Direct heat transfer techniques are often employed for the measurements of local heat transfers around circular cylinders (Refs. 13, 41, 42, 52). However, the procedures for evaluating these local heat transfers are by no means simple (as discussed earlier in section 4.1). Thus, a modified version of Kostic and Oka's technique is employed in the present study. This method appeared particularly suited for the small diameter, closely spaced tube geometries. The test apparatus and experimental procedure are described in this section. The test rig was similar to that used for the average heat transfer tests, so that a detailed description is available in the previous chapter.

#### A Description of the Test-Section

It was decided to employ new tube arrays to replace the ones used in the average heat transfer tests. These assemblies were designed to:

1. provide easy access to the test cylinders,
2. allow adjustment of the gaps between the test cylinder and the adjacent tubes.

These tube assemblies are shown in plate 7. The dummy tubes were mounted in brass holders to form tube matrices which were in turn fitted between 'tufnol' surrounds. The test cylinder was positioned between two appropriately sized matrices and the required spacing adjusted using feeler gauges. The tufnol surround pieces were then tightly bolted together so that the tubes were clamped in position. This complete assembly was then mounted in the test duct by means of flanged connections. Asbestos gaskets were used to prevent air leakage.

Local heat transfer variations were determined by incrementally rotating the test cylinder. A means of measuring the angular position was thus required. A suitable indication was constructed using a stainless steel pointer and a graduated protractor.

#### B Design of the Test Cylinder

A schematic representation of the test cylinder is shown in Fig. 66. It consisted of a thin walled type 347 stainless steel tube 6 mm in diameter and about 200 mm long, mounted directly in the centre of the test section. This cylinder was heated by passing a direct current through the stainless steel. The surface temperature at the 'local measurement position'
was obtained by means of an embedded copper constantan thermocouple. The thermojunction was electrically insulated from the remainder of the cylinder. To reduce circumferential heat conduction effects, two longitudinal slots were cut through the wall of the tube. These slots were positioned 1 mm apart on opposite sides of the thermocouple. The slots were 0.5 mm wide and 50 mm long and were filled with epoxy resin. This 'guarded measurement section' subtended an angle of 19° at the centre of the test cylinder.

To estimate any circumferential conduction effect the temperature representative of the remainder of the tube was found by means of a copper constantan thermojunction which was directly soldered to the tube at a position 180° (i.e. directly opposite) from the first thermocouple. This second thermocouple was, as pointed out in (Refs. 112, 113) directly soldered to the cylinder body in order to assess accurately the surface temperature under conditions of dominant circumferential conduction. The thermal resistance offered by any electrical insulation was thus eliminated.

The potential drop over the test length was measured by means of two copper wires soldered to the tube wall. Both these insulated connections and the thermocouple leads were led cut internally through the tube bore. These arrangements should avoid end effects, see (Ref. 63). Nevertheless in order to check these effects and to assess axial heat conduction, a thermocouple was embedded in the tube wall at one end of the measurement segment.

The dimensions of the slotted segment were checked after fabrication and instrumentation since an accurate knowledge of these parameters is important in assessing circumferential conduction effects. These dimensions were measured using both a 'Quantimet Image Analyser' and a standard 'projection microscope'. The measured dimensions on a horizontal plane were then converted to the appropriate distances around the periphery of the tube, see Fig. D.2 in Appendix-D. Incorporation of the thermojunctions and the insulated slots can also modify the surface profile of the tube with consequent disturbance of the boundary layer flow. The surface topography of the test cylinder was, therefore, measured using a 'Talysurf' machine. Typical surface profiles are shown in Fig. 67. An acceptable surface finish was eventually obtained by resoldering as necessary and by careful filing down the protuberances.

The position of the main measurement thermocouple (i.e. the 0° position) was indicated by a milled slot at one end of the tube. A stainless steel needle was then inserted radially through the tube at this point and soldered in position to act as a pointer.

C Calibration of the Test Cylinder

The test cylinder was immersed in a well stirred, and thermostatically controlled oil bath. All the thermocouple readings were then compared with a standard mercury-in-glass thermometer graduated to 0.1°C. Melting ice and boiling water at room temperature were used to check the 0°C and 100°C indications. The estimated maximum variation in the thermocouple readings was approximately 0.003 mv and this had a negligible effect on the temperature measurements.
D. Test Instrumentation

A diagram of the test instrumentation and services is presented in Fig. 68. The heating current was supplied from a 200 Amperes d.c. welding generator modified so that the output could be a wide range. A suitable shunt-resistance of .001 ohms was connected across the power lead to measure the heating current to an accuracy of ± 1%. This resistance gave an associated voltage drop of 1 mv/ampere and the voltage was measured by a calibrated digital voltmeter type DM 200. An electrically powered solenoid valve was incorporated in the supply to act as a circuit breaker. The voltage drop over the effective test length of the cylinder was measured by another DM 200 digital voltmeter.

For the temperature measurements, each thermocouple was connected via a rotary switch to a data-logger system. The thermocouple readings were recorded alternatively and the data printed out directly. The uninsulated thermojunction was corrected for the voltage drop across the junction as described in (Ref. 113). Thus the heating current was switched off after the test and the variation of this thermocouple reading with time was obtained by means of an ultraviolet type recorder.

E. Test Procedure

The test cylinder was inserted in the centre of test section, and adjusted until the pointer coincided with the zero indication on the fixed protractor. The gaps between the test cylinder and the adjacent dummy tubes were also adjusted using suitable feeler gauges. The whole assembly was then mounted in the test duct. The electrical power leads, voltage drop leads and thermocouple wires were then connected as shown in Fig. 68.

The required air flow was obtained by means of the control valves and the air temperatures monitored. The welding generator was then switched on and the appropriate heating current selected. A period of approximately 30 - 45 minutes was required for attainment of steady-state conditions (these were indicated by negligible changes of tube and air temperatures over a period of ten minutes). Upon achievement of these conditions, the following data were measured:

(i) the temperature of the insulated segment of the test cylinder.

(ii) the temperatures at the end of the insulated segment (this was employed to check axial conduction).

(iii) the temperature of the remainder of the tube.

(iv) the air temperature prior to the test section.

(v) the heating current and the associated voltage drop over the test length of the tube.

The reading of the directly soldered thermocouple which measures the bulk temperature of the tube was affected by the passage of the heating current. This effect was determined by switching off the current and extrapolating the resultant variation of temperature with time to zero time (see the following section). The output from this thermocouple was thus monitored using a fast response ultra-violet type recorder.
The cylinder was rotated incrementally and the measurements were recorded for each angular position.

F. Estimation of Circumferential Conduction

Despite the employment of two epoxy-resin filled 'isolating' slots in the test cylinder, the small size and hence low thermal mass of this cylinder suggests that circumferential conduction can still affect the heat transfer measurements. This effect can be estimated by measuring the essentially uniform temperature of the bulk of the tube by means of a second uninsulated thermocouple (as described earlier in the section dealing with the design of the test cylinder). This thermocouple was situated diametrically opposite the 'measurement segment' so that the relative influence of the two slots was minimised.

As the heating current was passed directly through the test cylinder wall the associated potential drop over the thermojunction distorted the temperature indication. A technique for estimating this distortion has been described by Davenport et al (Ref. 113) and was employed in this present study. Accordingly the thermocouple output and the heating current were monitored continuously by a fast response recorder. The flow of heating current was then suddenly interrupted by opening the solenoid circuit breaker (the thermocouple e.m.f. then no longer includes the additional 'pick-up component'). Fig. 69 presents a typical recorder trace. Extrapolation of the temperature curve back to zero time i.e. the instant when the circuit breaker was opened, indicated the 'true undistorted temperature' of the bulk of the tube. Subsequent readings may then be corrected by multiplying them by the ratio of the undistorted to the distorted indication.

Circumferential conduction can now be estimated from a knowledge of these temperatures by performing a heat balance on the 'isolated' measurement segment. The local heat transfer coefficient can be defined as:

\[ h = \frac{q}{(T_w - T_o)} \]

where \( q \) is the net convective local heat flux. An energy balance on the cylindrical element yields

\[ q = q_c + q_\phi + q_y + q_r \]

i.e. the total heat flux dissipated in the segment due to passage of the heating current is composed of the sum of the convective heat flux, the circumferential and axial heat conduction and the radiative dissipation. The conduction contributions are proportional to the temperature gradients i.e. \( \frac{\partial T}{\partial \phi} \) and \( \frac{\partial T}{\partial y} \). The circumferential contribution was approximated by assuming a linear gradient across the slots so that for a unit length of the cylinder:

\[ q_\phi = \frac{K}{L_2} \left( \frac{D_o - D_i}{2} \right) (T_1 - T_2) + \frac{K}{L_2} \left( \frac{D_o - D_i}{2} \right) (T_2 - T_3) \]

where \( T_1 \) and \( T_2 \) are 'true cylinder surface temperatures' on either side of the measurement segment as indicated by a curve fitted through the uninsulated thermocouple readings. \( D_i \) and \( D_o \) are the inner and outer diameters of the test cylinder \( K \) is the thermal conductivity of the
epoxy resin contained in the slots and \( T_s \) is the surface temperature of the measurement segment, \( \delta_1 \) and \( \delta_2 \) are the widths of the slots, see Fig. D.2. The circumferential conduction can be as high as 16% of the total heat flux for the low range of flow velocities. The axial conduction can be similarly approximated by:

\[
q_y = \frac{K_s}{y} (D_o - D_i) (T_3 - T_6)
\]

where \( K_s \) is the thermal conductivity of stainless steel, \( T_3 \) is the temperature at the end of the measurement segment and \( y \) is the distance (appropriate longitudinal) between the thermocouples. The constant allows for the geometry of the slotted segment. This axial conduction contribution was found to be always less than ± 3%, and hence it was neglected.

The radiative component was evaluated using a value of 0.08 for the emissivity of test cylinder, see (Ref. 7). This radiative heat flux was within 1-4% of the total, because the temperature difference between the wall and that of the air was relatively small and this contribution was also neglected.

The local Nusselt number was then evaluated from a knowledge of the heat transfer coefficient since

\[
Nu = \frac{h \cdot d}{K_f}
\]

4.5 Results and Discussion

4.5.1 Results

The mass transfer obtained by the electrochemical method are presented in Figs. 70 to 73. For comparison purposes, the results obtained for a single cylinder geometry are represented in Fig. 74 in the form of \( Sh/ (Re_0^{0.5} \cdot Sc^{0.33}) \) plotted against the angular position, \( \phi \), as measured from the front stagnation point. These single cylinder data are compared with previous mass transfer results obtained for similar test conditions such as those of Dimopoulos and Hanratty (Ref. 95) and Grassman et al (Ref. 100). The present results for the tube rows are also compared with other published results where appropriate.

The direct heat transfer results obtained for a single cylinder are presented in the form of \( Nu/Re_0^{0.5} \) and compared with previous data in the open published literature, see Fig. 75. The present results for the tube rows are then presented in Figs. 76 to 79 as local Nusselt number, \( Nu \), plotted against the angular position, \( \phi' \). For comparison purposes these tube row results are replotted in the form of \( Nu/Re_0^{0.5} \) and compared with similar previous data where applicable, see Figs. 80 and 81.

4.5.2 Discussion of Single Cylinder Results

Comparisons of the experimental data obtained from tests on a single cylinder using the present direct heat transfer technique and results calculated according to the theory of Frössling (Ref. 93) are presented in Fig. 75. The previous experimental results of Dyban et al (Ref. 42)
are also shown in this figure. The general behaviour of these data is quite similar, i.e. they exhibit a maximum in the heat transfer curve at the front stagnation point, and this parameter decreases toward the rear of the cylinder. The close agreement between the present data and that previously reported indicates that the present direct heat transfer technique is suitable for the measurement of local heat transfers for small diameter tubes.

The experimental data obtained, by the electrochemical method, for a single cylinder are presented in the form of the local Sherwood number, $Sh$, plotted against the angular position, $\theta$, see Fig. 70. The blockage ratio in these single cylinder mass transfer tests was $D/H = 0.39$, so that the results can not be compared directly with those determined by the present heat transfer technique at a lower blockage, i.e. $D/H = 0.07$. Moreover much of the previously available mass transfer data obtained by the electrochemical method apply to considerably lower blockages. Thus initially, to eliminate blockage ratio effect, the present mass transfer results are compared with those determined experimentally by Perkins and Leppert (Ref. 54) at blockage ratios of 0.31 and 0.42. In addition, the calculated results of Akilbayev (Ref. 50) for blockage of 0.39 and 0.52 are also presented for the front portion of a tube. Unfortunately, these comparisons exhibit comparatively little agreement. This may be at least partially explained since the mass transfer data correspond to constant cylinder temperature boundary conditions whereas the reported heat transfers are for approximately constant heat flux conditions. These effects are discussed in greater detail in section 4.5.4, but it is worth noting here that previous investigators (see Refs. 13, 29, 31, 92) reported that the constant heat flux condition tends to 'flatten out' the circumferential variation in local heat transfer. Thus comparisons with data obtained at similar boundary conditions are desirable, even if the blockage ratio is different. The present results are compared with other single cylinder mass transfer data provided by Dimopoulos and Hanratty (Ref. 95) and by Grassman et al (Ref. 100), see Fig. 74. Good agreement was obtained over the separation region for the same Reynolds numbers. However, there is a measure of disagreement over the laminar boundary layer region at the front of the cylinder. The present results do not exhibit a maximum at the front stagnation point and this feature is difficult to fully explain, although Mizushina (Ref. 102) has found that this affect can occur at certain levels of mainstream turbulence, see Fig. 82. Similar affects have been reported by Grassman et al (Ref. 100) for local mass transfer results in the range of Reynolds numbers from $3.87 \times 10^3$ to $1.028 \times 10^4$ and a turbulence level of 2.3%. However, the turbulence levels of 1.9% to 3.5% encountered in the present study cannot fully explain the phenomenon. Nevertheless, as was pointed out in the preceding sections, the boundary layer concentration fields may have been disturbed to a certain extent due to the distortion in the surface profile of the cylinder at the nearly flat cathode. The mass transfers may also be affected by the comparatively large size of the cathode and the surrounding holder and insulation.

4.5.3 Discussion of Tube Row Results

A The Influence of Flow Blockage

As can be seen from Figs. 76 to 79, the distribution of local heat transfers were significantly affected by the flow blockage. In all the cases
studied in the investigation, the maximum heat transfer associated with
the front portion of the cylinder did not ensue at the front stagnation.
This can also be observed with the mass transfer data shown in Figs. 71 -
72. Both sets of data corroborate the results of previous investigators
(Ref. 50, 52) at higher Reynolds numbers.

The increase in heat transfer at high flow blockage appears to be associ-
ated with the tendency of a fluid to move out of the way of a downstream
obstacle together with the subsequent acceleration in the flow around the
front of the cylinder. Thus boundary layer growth was inhibited over
the front portion of the tube, and the laminar boundary layer, over this
portion of the cylinder, decreased in the downstream direction. As ex-
pected, the increase in Nusselt number over the front portion of the tube
was greatest for the highest blockage situation i.e. for D/H = 0.92.

The direct heat transfer results at a blockage ratio of 0.80 are compared
with the published measurements of Oka et al (Ref. 52) at a higher free
stream Reynolds numbers, see Fig. 80. Good agreement was obtained over
the laminar boundary layer region but significant differences were
apparent over the rear of the cylinder in the separated flow region. The
distribution of heat transfer over the rear portion of single cylinder is
Reynolds number dependant, see also section 2.2.2 and (Ref. 13, 14, 41,
89) with the heat transfers increasing rapidly with increasing Reynolds
number. One would expect a similar affect with tube rows so that Oka et
al's data (which was obtained at Reo = 3 x 10^4) would be considerably at
variance with the present results in this separated region.

The present heat transfer measurements for the highest blockage ratio,
D/H = 0.92, are also compared with the experimental data of Oka et al.
and calculated data from Aiklbayev (Ref. 50), see Fig. 81. As can be
clearly seen, the general character of the heat transfers in the front
portion was remarkably similar in all cases. Moreover, the discrepan-
cies over the rear portion are similar to those at the slightly low
blockage case, i.e. they are explainable in terms of the differing main-
stream or reference Reynolds numbers. For the same mainstream Reynolds
number, the greatest increase in heat transfer was observed over the rear
of the highest blockage tube row investigated, i.e. the rear heat trans-
fers increased with increasing blockage ratio or increasing maximum velo-
city in the minimum spacings between adjacent tubes.

B. The Influence of Flow Reynolds Number

The Reynolds number based on the maximum velocity appeared to influence
the rate of increase of Nusselt number over the separated flow region.
At low flow rates the heat transfers remained virtually constant around
the rear of the cylinder at a value only slightly greater than the minimum
heat transfer. As the Reynolds number was increased, the heat transfers
over the rear portion also increased for all the tube assemblies tested.
However, the rate of increase of heat transfers around the rear of the
tubes depended on the blockage ratio, see Fig. 83. Thus at the higher
Reynolds, the heat transfers associated with the rear half of the test
cylinder were higher than those over the front, at a blockage ratio of
0.92. For less blocked flows, the ratios of heat transfers around the
rear half to those over the front were less than unity. Moreover, the
ratio of rear to front heat transfers was lowest at the lowest blockage
The influence of Reynolds number on this ratio is more pronounced at the higher blockages. At $D/H = 0.80$, the ratio is unaffected by the flow rate. However, at the higher blockages the rear half heat transfers increase at a faster rate than those over the front half as the flow rate is increased.

### 4.5.4 Comparison Between Heat and Mass Transfer Results

Fig. 84 shows a typical comparison between the local heat transfers associated with a flow blockage of 0.92 and the mass transfers obtained with a blockage of 0.88. Both sets of data exhibit similar character, over the front portion of the cylinder in so far as the maximum heat or mass transfer occurred in the region which approximately corresponded to the minimum flow area between adjacent tubes. Good agreement can also be observed over the rear portion. However, the rate of increase of heat transfer downstream of the front stagnation point is much larger for the electrochemical data.

This may be partially explained at least by the difference in the boundary conditions in the two sets, i.e. the mass transfer measurements correspond to constant surface temperatures whereas the heat transfer data were obtained at approximately constant wall flux. Although previous studies on the influence of boundary conditions have been conducted at lower flow blockages, the general features are nevertheless probably applicable to the present study. It has been shown (Ref. 13) that when the test flow conditions are similar, the local heat transfer patterns for cylinders are much flatter with constant heat flux wall conditions than those obtained at constant temperature. The effect of boundary conditions can be seen in Fig. 85 which presents the recently published results of Boulos and Pei (Ref. 29) at a Reynolds number of $5.4 \times 10^3$. Similar information has been published by Krall and Eckert (Ref. 31) at a lower Reynolds number of 200 (see also Fig. 22 of Ref. 13). Thus one would expect the variations in heat transfer around the cylinder to be greater in the electrochemical results than those found by the direct heat transfer technique.

### 4.6 Concluding Remarks

The distribution of local convective heat transfer coefficient around tubes at high blockage ratios in the range $0.77 \leq D/H \leq 0.92$ have been determined for a range of Reynolds number (based on the mainflow velocity) of 300 to $7 \times 10^3$. Both a direct heat transfer method and an electrochemical method were used for these measurements, and the following conclusions can be drawn:

1. The single cylinder results obtained using the direct heat transfer technique were in excellent agreement with previously reported experimental and theoretical information. Less close agreement was obtained for the electrochemical data although blockage ratio and mainstream turbulence intensity effects made direct comparisons difficult. It was thus considered that the direct heat transfer measurements provided the more reliable data. Nevertheless, the mass transfer results can shed light on the overall heat transfer characteristics.
2. The differences between the present heat and mass transfer results can be largely attributed to the different cylinder wall boundary conditions associated with the test method.

3. The distribution of local heat transfers for the tube rows were affected by both the blockage ratio and the flow Reynolds number. The local heat transfer always increased over the front half of the cylinder, attaining a maximum in the region which approximately corresponded to the minimum flow area between adjacent tubes. This rate of increase was considerably greater at the highest blockage ratio tested (i.e. with the closest spacing between adjacent tubes).

4. For the lower range of Reynolds number investigated in the study (i.e. the range of particular interest to the designer of a Stirling engine), the local Nusselt numbers remained approximately constant over the rear portion of the test cylinder. However, at the higher flow rates these local heat transfers attained minimum values at the separation point and these minima were followed by subsequent increases downstream over the rear of the cylinders.

5. The ratio of heat transfer over the rear portion of the tube to that over the front increases with either a decreasing spacing between the adjacent tubes or an increasing Reynolds number (based on the mainflow velocity).
FIG. 61. SCHEMATIC LAYOUT OF THE ELECTROCHEMICAL TEST RIG.
FIG. 62. CONSTRUCTIONAL REPRESENTATIVE DIAGRAM FOR THE ELECTROCHEMICAL TEST CYLINDER.

FIG. 63. CIRCUIT DIAGRAM OF THE ELECTRICAL CIRCUIT.
FIG. 64. TYPICAL LIMITING CURRENT PLATEAUS AT TWO DIFFERENT FLOW RATES FOR A SINGLE CYLINDER.
FIG. 65. RESULTS OF MAINSTREAM TURBULENCE INTENSITY
AT THE PLANE OF THE TEST-SECTION.

\[ \left[ \frac{V_{B}^2}{V_o^2} - 1 \right] \]

\[ V_o = 0.825 \ V_o \]

WATER TEMP. = 16° C

PROBE RESISTANCE = 9.6 Ω
FIG. 66. SCHEMATIC DIAGRAM OF THE TEST CYLINDER EMPLOYED IN THE DIRECT HEAT TRANSFER TEST.
FIG. 68. REPRESENTATIVE SET-UP OF THE DIRECT HEAT TRANSFER TEST INSTRUMENTATION.
FIG. 70. LOCAL MASS TRANSFERS FROM SINGLE CYLINDER.
FIG. 71. LOCAL MASS TRANSFERS FROM CLOSELY SPACED TUBES
FIG. 72. LOCAL MASS TRANSFERS FROM CLOSELY SPACED TUBES

4.76 mm DIA. TUBES
BLOCKAGE RATIO = 0.79

$\frac{Sh}{Sc}^{1/3}$

$\phi$ (DEGREES)

<table>
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<tr>
<th>KEY</th>
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<tr>
<td>1</td>
<td>565</td>
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<tr>
<td>2</td>
<td>471</td>
</tr>
<tr>
<td>3</td>
<td>376</td>
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FIG. 73. LOCAL MASS TRANSFERS FROM CLOSELY SPACED TUBES

4.76 mm DIA. TUBE
BLOCKAGE RATIO = 0.88

$Sh/Sc^{1/3}$

$\phi$ (DEGREES)

KEY  $Re_\theta$
1  752
2  565
3  376
FIG. 74. COMPARISON BETWEEN THE PRESENT LOCAL MASS TRANSFER RESULTS AND THOSE OF PREVIOUS RESULTS OBTAINED BY THE ELECTROCHEMICAL METHOD. SINGLE CYLINDER RESULTS.
FIG. 75. COMPARISON BETWEEN THE PRESENT SINGLE CYLINDER HEAT TRANSFER RESULT AND THOSE OF PREVIOUS STUDIES.
FIG. 76. LOCAL HEAT TRANSFER RESULTS FOR 6.0 mm x 1.5 mm
 BLOCKAGE RATIO = 0.80

\[ \phi \text{ (DEGREES)} \]

- \( \text{Re}_0 = 6410 \)
- \( \text{Re}_0 = 4130 \)
- \( \text{Re}_0 = 1220 \)
FIG. 77. LOCAL HEAT TRANSFER RESULTS FOR 6.0mm x 1.0mm TUBE ROW AT DIFFERENT REYNOLDS NUMBERS
FIG. 78. LOCAL HEAT TRANSFER RESULT FOR 6.0 mm, × 0.50 mm TUBE ROW.
FIG. 79. LOCAL HEAT TRANSFER RESULTS FOR ① 6.0mm x 1.0mm, ② 6.0mm x 1.50mm, ③ AND ④ FOR 6.0mm x 0.50mm, AT LOWER REYNOLDS NUMBERS.
FIG. 80. COMPARISON BETWEEN THE PRESENT LOCAL HEAT TRANSFER DATA FOR 6·0 mm. × 1·50 mm. TUBE ROW AND THOSE REPORTED BY OKA ET AL. (Ref. 9).

FLOW BLOCKAGE = 0·80 IN ALL CASES

\[
\frac{Nu}{\sqrt{Re_0}}
\]

\(\phi \) (DEGREES)

KEY

1. OKA ET AL (9) \(Re_0 = 30,000\)
2. PRESENT \(Re_0 = 6410\)
3. STUDY \(Re_0 = 4130\)
### Table

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<td>2</td>
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### Diagram

**Fig. 81.** Comparison between the present local heat transfer data for tube row of 6.0 mm x 0.50 mm and those previous investigators.
**FIG. 82a.** TYPICAL TURBULENCE EFFECTS ON LOCAL MASS TRANSFERS PUBLISHED BY KESTIN AND WOOD (Ref. 47). THESE RESULTS ARE SIMILAR TO THOSE PUBLISHED BY KESTIN, MAEDER AND SOGIN (Ref. 48).

**FIG. 82b.** TYPICAL TURBULENCE EFFECTS ON LOCAL MASS TRANSFERS OBTAINED BY THE ELECTROCHEMICAL MASS TRANSFER METHOD. FROM MIZUSHINA (Ref. 102).
FIG. 83. THE RATIO OF HEAT TRANSFERS AT THE REAR HALF OF TUBES TO THAT AT THE FRONT AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER.
FIG. 84. COMPARISONS BETWEEN THE PRESENT LOCAL HEAT TRANSFER RESULTS UNDER DIFFERING TEST CONDITIONS.

<table>
<thead>
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<th>KEY</th>
<th>D/H</th>
<th>Re_α</th>
<th>TEST CONDITIONS</th>
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<tr>
<td>△△</td>
<td>0.92</td>
<td>689</td>
<td>CONSTANT FLUX</td>
</tr>
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<td>○○</td>
<td>0.88</td>
<td>376</td>
<td>CONSTANT TEMP.</td>
</tr>
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<td>●●</td>
<td>0.92</td>
<td>300</td>
<td>CONSTANT FLUX</td>
</tr>
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<td></td>
<td>DIRECT HEAT TRANSFER RESULTS</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>ELECTROCHEMICAL RESULTS</td>
</tr>
</tbody>
</table>

\[
\frac{Nu}{Pr^{1/3}}, \frac{Sh}{Sc^{1/3}}
\]

\[
\phi \text{ (DEGREES)}
\]
FIG. 85. A COMPARISON BETWEEN SINGLE CYLINDER RESULTS AT DIFFERENT TEST CONDITIONS. THESE DATA ARE PUBLISHED BY BOULOS AND PEI (Ref. 29).

<table>
<thead>
<tr>
<th>KEY</th>
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<th>TEST CONDITIONS</th>
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<tr>
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<td>5400</td>
<td>CONSTANT HEAT FLUX</td>
</tr>
<tr>
<td>2</td>
<td>5300</td>
<td>CONSTANT TEMP.</td>
</tr>
</tbody>
</table>

BOTH TEST CYLINDERS HAD THE SAME Tu % AND BLOCKAGE RATIO.
CHAPTER 5

COMPARATIVE PERFORMANCE CHARACTERISTICS OF SOME EXTENDED SURFACES FOR THE SMALL DIAMETER AND CLOSELY SPACED TUBE ROWS.
CHAPTER 5

COMPARATIVE PERFORMANCE CHARACTERISTICS OF SOME
EXTENDED SURFACES FOR THE SMALL DIAMETER AND
CLOSELY SPACED TUBE ROWS

5.1 Introduction

This chapter is concerned with the performance of single rows of longitudinal and transverse finned tubes. The tube diameter was 6 mm. For the longitudinal fins, three different tube spacings were examined (0.5, 1.0 and 1.5 mm) and the angle of inclination of the single longitudinal fin attached to the rear portion of the tubes was maintained at 45°, 60° or 90° relative to the plane of the test section. Thus nine different arrangements were studied. The geometry of the fin was based on previous optimization studies published in the open literature and also on the local heat transfer results, see Chapter 4 of the present thesis. Two different transverse plate finned tube arrangements were tested. The geometry of the individual fins was the same in each case with the number of fins per unit length being 240 fins/meter and 470 fins/meter. The associated heat transfers and pressure losses were measured under steady state conditions and the flow range in the tests was 250 ≤ Re₀ ≤ 4.2 x 10^3. The results were first generalized in dimensionless form and compared with previous data from the literature where possible. The present results were then compared graphically so that the relative performance of a finned tube arrangement is discussed. It should be noted, however, that only the heat transfers and hydraulic resistances were considered. Other factors which enter into the selection of an arrangement include the cost of the tubing, the maximum allowable pressure, the extended surface effectiveness and the likelihood of fouling.

5.2 Previous Experimental Arrangements

Because of the complicated nature of the parameters involved in the investigation of heat transfer rates associated with the finned tubes, conventional experimental techniques cannot always be applied. Consequently it is beneficial, first of all to examine some of the widely employed previous experimental arrangements together with their limitations and sources of errors. Attention is also paid to methods of data reduction. This is followed by a summary of previous studies of fin effectiveness and its role in the heat transfer measurements. This is done in the hope that many of the experimental errors inherent in the previous techniques could be avoided. However, an effort is made throughout this discussion to avoid repetition of the details of the conventional methods described in the previous chapters.

Most of the previous experimental data on banks of finned tubes were obtained by circulating a fluid (e.g. water or steam) through a finned test tube situated in the centre of each row in the tube bank assembly. Measurements of the heat flux and the mean temperature difference between the circulating and outer test fluids enables the overall heat transfer coefficient to be calculated. A knowledge of the thermal resistance of the tube wall and the heat transfer coefficient at its inner surface is then required to evaluate the heat transfers between the test fluid and the tube. The thermal resistance of the wall may be readily calculated.
except in cases where there is appreciable thermal contact resistance between the tube and the fins, see for example, Gardner and Carnavos (Ref. 126) and Kraus (Ref. 138).

The inside film resistance is in many cases difficult to measure accurately. This is mainly due to the fact that allowances must be made for the effect of bends, boundary layer development and the radial temperature distributions inside the tube (Ref. 111). Nevertheless, the general equations correlating these flows are often employed, e.g. Hausen's relationship for flow inside tube (Ref. 7). Alternative correlations are also available which take into account the aspect ratio of the tube and viscosity variations in the fluid, see (Ref. 129). Accordingly, the average inner heat transfer coefficient may be calculated from an equation of the form:

$$Nu_i (Pr_i)^{\alpha_1} \left( \frac{\mu_o}{\mu_i} \right)^{\beta_1} = \frac{C Re_i^{\alpha_2}}{R_{01}^{\beta_2}} \left( \frac{L_i}{d_i} \right)^{\gamma_2} \left( \frac{2 \pi}{d_i} \right)^{\delta_3}$$

where $i$ refers to the inner wall of the test element, $o$ refers to the bulk conditions, $r$ is the mean radius of any connecting bends at the ends of the tube and $C, m, n$ are constants.

Nevertheless, in many cases previous investigators have employed alternative approaches for the prediction of inner heat transfer coefficients, for example, a modified 'Wilson Plot' technique has been employed (Refs. 135, 136, 137, 147). However, this latter method (although simple to perform) is susceptible to error so that the use of empirical formulas is preferable.

Upon estimation of the heat transfer coefficient on the inner surface of the finned tube ($h_i$) and the overall heat transfer coefficient ($\lambda_t$) based on the total outer surface area ($A_{tot}$), the average heat transfer coefficient on the outside tube surface ($A_{tot}$) can be obtained by:

$$h = \gamma_o^{-1} \left[ \frac{1}{\lambda_t} - A_{tot} \left( \frac{1}{h_i A_i} + \frac{x}{A_m K} \right) \right]^{-1}$$

where $\gamma_o$ is the weighted efficiency of the heat transfer surface, i.e. $\gamma_o A_{tot} = \gamma_i A_i + A_t$. The suffices ($i, m, t, f$) refer to the inner surface, mean tube, and fin areas respectively, and $(K/x)$ is the thermal conductance of the fin material.

In cases in which the fin efficiency is low (e.g. with long thin fins or ones constructed of a relatively low conductivity material), it is important to investigate the variation of local heat transfer coefficient on the fin side, see (Ref. 122). Weiner, Gross and Paschkis (Ref. 114) used a water cooled circular fin for this purpose. This fin was 4.76 mm thick and 101.6 mm in diameter. The base cylinder was 48.3 mm in diameter and spacing between fins was 9.5 mm. Two dimensional heat transfers were promoted in the fin by concentrating the heat flows into a 1.27 mm copper strip backed by thermal insulation. The local heat transfer coefficients were then evaluated by means of 21 thermocouples spaced radially at 1.27 mm intervals. The whole assembly was then rotated incrementally.
probable error in the experimental values of the local heat transfers was estimated to be ± 20%. The average heat transfer over the fin surface was estimated by fitting the radial temperature distributions to the theoretical Bessel type relations for circular fins.

A less complicated, but more effective method has been used by Lymer and Ridal (Ref. 116). They measured the local heat transfers directly by inserting a small insulated probe heater (of 6.35 mm in diameter) into a fin so that the copper faces of the probe were flush with the upper and lower surfaces of the fin. The local heat transfers were evaluated from a knowledge of the electrical power input to the heater and the surface temperature of the probe. Neal and Hitchcock (Refs. 119, 124) used a similar technique but they employed a four times scale model of the finned tube. The local heat transfers were determined by some 36 local fluxmeters. The test fins were insulated from the tube by a ring of Sindanyo let into the fin root, and the fin assembly was heated indirectly by an electrical radiant heater. The whole test tube assembly could be rotated incrementally.

Average and local heat transfers for both circular finned and plate finned tubes have been obtained by employing the heat-mass transfer analogy together with the sublimation of Naphthalene. Lewis (Ref. 18), Owen (Ref. 32) and Wong (Ref. 127) have all used this method on circular finned tubes, whereas Fukui and Sakamoto (Ref. 121) and Saboya and Sparrow (Refs. 129, 142, 143) have used it on plate finned configurations. The general techniques applied by these investigators were virtually identical with those described in detail in the previous chapters and so do not merit further discussion.

However, in many practical applications, finned tubes are designed so that fin efficiencies are high (e.g. short compact fins or those constructed of material of high thermal conductivity). In these cases, average heat transfer data are adequate.

5.3 Fin Effectiveness

The fin effectiveness may be defined as the ratio of the actual heat transfer from the extended surface to that which would ensue if all the fin was at the base temperature. Over the last 40 years or so a great deal of attention (both experimentally and theoretically) has been given to fin effectiveness. One of the earliest contributions was that of Harper and Brown (Ref. 130), who presented analytical solutions for fins of both a rectangular and a trapezoidal profile. They concluded that for most practical cases, a one dimensional heat flow model is sufficiently accurate method.

Murray (Ref. 131) considered the problem of annular fins of uniform thickness and employed a two dimensional conduction model which allowed for temperature variations in the radial and circumferential directions. In a similar fashion Avrami and Little (Ref. 132) derived solutions for straight fins of rectangular profile. The problem of annular fins with tapered profiles has been examined by Carrier and Anderson (Ref. 133).

Gardner (Ref. 134) presented a one-dimensional generalized solution for both straight pins and annular fins whose cross-sectional area varied in
power law fashion with the distance from the fin base. However, in all these cases, the effectiveness was analysed by employing the following simplifying assumptions.

1. Steady state conditions apply and heat is not generated internally within the fin.

2. The fin material is homogenous and its thermal properties are invariant.

3. Heat is transferred from the fin by forced convection and both the heat transfer coefficient over the fin surface and the temperature of the surrounding medium are constant.

4. The thickness of the fin is small compared with its height so that temperature gradients across the fin thickness may be neglected.

5. The temperature at the base of the fin is uniform and there is zero contact resistance at the interface.

6. The heat transferred from the fin tip is neglected.

However, the validity of these simplified theoretical analyses particularly that of Gardner has been supported experimentally by Weiner et al (Ref. 114). They suggested that a constant value of the heat transfer coefficient may often be used in the design of a fin without significant error in the overall estimate of its performance. They found that the fin temperature distribution has little effect on the heat transfer coefficients.

However, in other cases the variations in the local heat transfers across the surface of the fin due to the fluid flow structure are severe. They thus concluded that insufficient data were available to decide when the use of average heat transfers is acceptable.

Some of the simplifying assumptions were relaxed in subsequent theoretical studies by Meles (Ref. 142), Keller and Somers (Ref. 145); Han and Lefkowitz (Ref. 152); Bert (Ref. 153) and Cumo, Pinchera and Urbani (Ref. 156). In these studies allowances were made for variations in heat transfer coefficient. This coefficient was assumed to vary in a power law form with the distance from the fin base (e.g. linear and hyperbolic distributions were used). However, the various authors did not attempt to justify the practical significance of these variations. In fact these assumed variations are not confirmed by the experimental data for fins of various geometries in crossflows (as discussed earlier in Chapter 2 of this thesis). These experimental investigations of fin characteristics throw doubt upon the validity of the simplifying assumptions (Refs. 116, 157). This is also true for cases where heat is generated within the fin.

Minkler and Rouleau (Ref. 158) were the first to consider the effect of internal heat generation. They studied longitudinal fins of both rectangular and triangular profile but assumed constant heat transfer coefficients and no fin tip losses. They employed a one-dimensional model and discussed the conditions under which fins are worthwhile. It was concluded that a constant temperature gradient results in a fin which is designed
to have minimum thermal mass. This conclusion was, however, not accepted by Wilkins (Ref. 159) who reported that for finite internal heat generation the temperature gradient varies in minimum mass fins.

Males and Wilkins (Ref. 160) analysed longitudinal fins of various profiles and also allowed variations in the heat transfer coefficients, the rate of heat generation and the thermal conductivity. Rectangular, triangular, trapezoidal profiles and fins of optimal shape were considered. They pointed out that the efficiency of a fin with internal heat generation is always smaller than that of a conventional case. This follows since the temperature gradient in the fin is affected (see Appendix F).

Cumo, Pinchera and Urbani (Ref. 156) conducted a two-dimensional analysis of straight fins and allowed for both variable heat transfer coefficients and internal heat generation. These investigators employed a numerical finite difference technique and concluded that one-dimensional simplified analyses can be misleading.

In a comprehensive piece of work, Kern and Kraus (Ref. 5) also employed a finite difference method. In their study the heat transfer coefficient was allowed to vary in either a linear, parabolic or exponential form. Contact resistance between the fin and base surface was also taken into account. The thermal properties of both the fin material and the fluid could vary. Both radiative and forced convective heat exchange with the surroundings was assumed and fin tip losses were also included. This study was extremely comprehensive but yet again, no attempt was made to compare the chosen variation for the heat transfer coefficients with the distribution over an actual fin.

The experimental data reported by various investigators (Refs. 18, 32, 114, 116, 119, 121, 124, 127, 129, 142, 143) do not show any definite relationship between the local heat transfers and the distance from the base of the fin. Moreover, the presence of a tube (e.g. in a plate fin arrangement in crossflow) can change the distribution of local heat transfers significantly. Consequently, the assumptions made to allow for the variations in heat transfer coefficient must be questioned. Thus in any practical situation where an appreciable temperature gradient is expected along the fin surface, reliance must be placed upon directly measured values of the fin efficiency. It should be emphasized, however, that with the higher conductivity fin materials, the simplified expressions for fin efficiency (such as those of Gardner (Ref. 134), Lymer and Ridal (Ref. 116) and Pshenisov and Luzhnov (Ref. 157) should suffice. Thus in the present study, which mainly deals with copper fins, the fin efficiencies were calculated where necessary using average heat transfers and the simplified one-dimensional analyses of Kern and Kraus (Ref. 5) and Gardner (Ref. 134).

5.4 Present Experimental Details

5.4.1 Choice and Design of the Extended Surfaces

As discussed in the preceding sections, a large variety of extended surfaces and vortex flow turbulence promoters have been developed to augment the heat transfer associated with tubes in crossflow. The capital cost of the heat exchanger is largely dependent on the shape, form and size of
the finned tube. However, there are no generally accepted performance criteria to permit rapid selection of the optimal configuration for a specific application. Thus it was decided to examine in this present study the performances of two groups of finned tubes, namely

(i) Tubes fitted with a longitudinal fin.
(ii) Tubes fitted with transverse fins.

The use of longitudinal fins has received comparatively little attention since (depending on the position and orientation of the fin) the flow can bypass the fin surface. This is similar to the case of axial flow over cylinders fitted with circular fins in which the flows tend to concentrate in the gaps between adjacent finned tubes.

In the present tube row geometries the rear of the tube experiences low heat transfers particularly at the lower Reynolds numbers. This is probably due to the comparatively low energy re-circulatory flows over this portion of the tube. Thus it appeared advantageous to use an angled longitudinal fin, as in Fig. 86, to divert the fast moving fluid stream, exiting from the small gap between adjacent tubes. This diverted flow should then 'wash' the rear of the tube and improve the heat transfers. In addition, the impingement of the comparatively high velocity fluid on the longitudinal fin should result in high heat transfer on the upstream side of this fin. In practice fin buckling problems may arise due to the differences in temperature between the extended surface and the tube. However, these can probably be overcome by periodically slotting the fin along its length. Thus incorporation of a longitudinal fin appeared promising as a cheap and simple method of improving the heat transfer rates.

The determination of the optimal geometry for such a fin will involve an investigation of the effects of fin length and inclination. This involves a study of too many variables in the time available at present so that the length of the fin was fixed by examining the results of Geiger and Collucio (Ref. 161). They studied a tube fitted with a longitudinal fin mounted at the rear stagnation point and parallel to the main flow direction. As can be seen in Fig. 88 Geiger and Collucio examined five different fin heights and the optimal height appeared to be approximately equal to 2/3 of the tube diameter. However, it should be noted that these data were observed with a single cylinder arrangement so that the overall blockage ratio did not exceed 0.12. Moreover, the results apply to considerably higher Reynolds numbers than those encountered in the present work. However, in the absence of any further information it was decided to use a fin length of 0.67 x tube diameter in the present study at high flow blockage.

The other variable, the fin inclination will affect the flow structure over the rear portion of the tubes and hence the heat transfers. However, altering the inclination will affect the form drag due to variations in the projected area of the finned tubes. Thus three angles of inclination were studied in this present chapter.

No information was available concerning the optimal geometry of the transverse fin arrangement. The absence of such information is understandable in view of the large number of variables which affect the heat transfer
performance of transverse plate fins. The time available for the present investigation precluded a comprehensive study to identify the optimal fin dimensions. However, it was decided to investigate a particular fin geometry at two different spacings. The length of the transverse fin was the same as in the longitudinal case so that the fin effectiveness were approximately similar. Furthermore, two different spacings between the transverse fins were employed so that the influence of the number of fins per unit length on the finned tube performance was examined, at least in a preliminary fashion.

Moreover to reduce the number of tests the present experiments were carried out with the tube diameter fixed at 6.0 mm. For the longitudinal fin cases the transverse spacings between adjacent tubes were 0.5 mm, 1.0 mm and 1.5 mm. A constant tube diameters (6.0 mm) and spacings (1.0 mm) were maintained in the transverse finned configurations. The detailed dimensions of these finned arrangements are given in Figs. 86 and 87.

5.4.2 General Test Details

The test rig and instrumentation used in the present tests were virtually the same as those used for the average heat transfer and flow resistance tests performed on bare tubes. They are thus similar to the apparatus described in detail in Chapter 3 of this thesis. However, a different method of heating the test cylinder was employed in the present tests. The copper test cylinder was heated by passing an A.C. current through the cylinder wall so that a uniform heat flux was maintained throughout the test-element body. The electrical current was varied between 130 and 450 Amperes and the voltage was measured only across the effective length of the test tube to avoid end effects. An alternating current supply was used since a direct current generator of the required high current output was not available. Copper/Constantan thermocouples were embedded in both the tube wall and the fin to measure the surface temperatures. These thermocouples were electrically insulated and calibrated against standard Mercury-in-glass thermometers as described in the previous chapters. Standard A.C. digital voltmeters (type Solarton IM1604) were employed to measure the voltages and currents. The phase angle between the current and voltage was measured using a suitable oscilloscope. The instrumentation and associated apparatus are shown in Fig. 89 and plate 10. Three different test assemblies were constructed to cover the range of geometries, see plate 11. The tubes and fins were normally constructed of copper to maintain high fin efficiencies. Moreover, the fins were directly soldered to the tube walls so that the contact resistances at the interface were negligible. In the case of the longitudinal finned tubes, some experiments were carried out with an insulated plate fin made of Perspex material. These provided a check on the measurements made with copper tubes and fins as discussed in section 5.4.5.

5.4.3 Experimental Procedure

a) Tests on the Longitudinal Finned Tube Arrangements

Experiments were conducted at three different angles of inclination (45°, 60° and 90°) of the longitudinal fin relative to the plane of the test section. The dummy finned tubes were assembled between the two halves
of the test-section, as described in Chapter 4 and the test cylinder was then mounted in the centre of the tube matrix. The spacings between this test cylinder and the adjacent tubes were checked and adjusted as necessary using feeler gauges. The correct angle of inclination of the longitudinal fins was obtained using a suitable combination set as shown in plate 12. After completion of this setting up procedure the electrical insulation of the test cylinder from the remainder of the dummy tubes was checked and any necessary adjustments carried out. This ensured that the measured electrical dissipation represented the heat flux transferred from the test tube. The exit section of the duct was then connected and the appropriate air flow switched on. The temperature of this air supply was controlled as described in Chapter 3 of this thesis. The required electrical power was applied and time was allowed for the attainment of steady state conditions. These were achieved within 25 to 45 minutes depending on the heat flux and the average air velocity. Since an A.C. electrical technique was used it was necessary to measure the phase angle between the voltage and the current in each test run so that the heat flux could be corrected accordingly. This was achieved by directly comparing the sinusoidal-wave forms for the voltage and current on an oscilloscope. This phase angle varied from 14° to 19.5° depending on the geometry of the test element and its average temperature.

b) Tests on the Transverse Finned Tube Arrangements

The experimental procedure employed in these tests was similar to that described for the longitudinal configurations. However, care was needed to ensure that the alignment of the test cylinder was such that a virtually continuous plate fin arrangement was obtained, see plate 13 in Appendix-G. The electrical insulation of the test cylinder was also more difficult in these cases. This was achieved by coating the edges of the fins with a thin film of 'shellac'. Any excess coating was then removed and the insulation was checked before and after each test.

c) Hydraulic Resistance Tests

The pressure drops due to the flow over the test section were measured as described in Chapter 3 so that this description is not repeated.

5.4.4 Data Reduction and Error Analysis

Most of the precautions taken during the experiments in order to reduce or avoid errors in the measurements were discussed in previous sections. In each test, steady state conditions were obtained and the following data recorded:

(i) the pressure drop across the orifice plate.
(ii) the temperature of the air prior to the settling chamber.
(iii) the total pressure and temperature of the air prior to the test section.
(iv) the phase angle between the voltage and current.
(v) the electrical current dissipated in the test section.
(vi) the voltage drop across the effective length of the test cylinder.
(vii) the readings from three different thermocouples positioned to measure the average tube temperature.

and (viii) the temperature of the longitudinal fin by means of a thermocouple mounted half-way between the base and the tip.

The Reynolds numbers and the average heat transfer coefficients were evaluated at the mean film temperature (as in all the tests presented in this thesis). This is the arithmetic mean of the surface temperature and the surrounding air temperature.

It is estimated that the following accuracies applied in this series of tests:

(i) individual temperatures ± 0.1°C
(ii) power input to the test element ± 3%
(iii) velocity of flow ± 1%
(iv) phase angle ± 2.0°
(v) Reynolds number ± 1%
(vi) angle of inclination of the longitudinal fin relative to the plane of the test section ± 2.0°
(vii) surface area of the fin ± 1%
(viii) total pressure drop across the tube bank ± 5%

The average heat transfers measured in the present tests appeared to be repeatable within ± 6%. The correction for radiative losses was usually approximately 1% of the total heat input and this loss was estimated to within ± 20%. Thus this correction was negligible and was not included in the calculations. All the measurements were checked for repeatability and two to three tests were carried out at approximately similar Reynolds numbers, thus 126 test runs were needed. For the pressure drop tests a total of 70 test runs were conducted. These measurements also included tests on rows of bare tubes.

The heat transfer coefficients were evaluated from a knowledge of the heat flux and the tube and fin temperatures. Unfortunately for low efficiency fins there is a considerable variation in temperature along the length so that measurement at just one station is not sufficient. Thus it was necessary to estimate the fin efficiencies in the present tests to check the validity of allotting a single uniform temperature to the fin.

These fin efficiencies were thus evaluated for

(1) the longitudinal fin arrangement in which heat is generated within the fin (see Appendix-F and for more details see Kern and Kraus, Ref.5).

(2) the transverse fin geometry in which internal heat generation was negligible since the geometry of such a fin inhibits current flow within the extended surfaces (Ref.168). The appropriate expressions for the fin efficiencies in this case are presented in Appendix-G.

In the absence of alternative values, the heat transfer coefficients.
associated with the bare tubes were employed to calculate the fin efficiencies. In most cases due to the high thermal conductivity of the copper the efficiencies were greater than 0.90 so that errors due to the use of a mean fin temperature were comparatively small.

5.4.5 Separation of Tube and Fin Heat Transfers

In the longitudinal finned tube arrangements one would expect considerable variations in heat transfer coefficient between the tube and fin surfaces. The fin can be mounted so that the comparatively high-velocity fluid (exiting from the small gap between adjacent tubes) impinges on the finned surface. This is likely to lead to high heat transfers over these surfaces. Furthermore, it was intended that the fluid flow should be diverted around the rear of the tube by the longitudinal fin. Thus to assist in understanding the complicated flow and heat transfer behaviour of the system it was desirable to obtain separate heat transfer coefficients for the fin and tube surfaces as well as measurements of the overall value.

Separation of these coefficients was achieved by employing a copper test cylinder fitted with a 'perspex' longitudinal fin. The low thermal conductivity of this fin resulted in fin efficiencies always less than 0.40 so that the contribution of the extended surface to the overall heat transfer was comparatively small. Nevertheless the following iterative procedure was employed to reduce any errors due to heat dissipation from the fin.

The total dissipation from the fin-tube combination may be written as:

\[ Q = h_t A_t (T_t - T_a) + \gamma_f h_f A_f (T_t - T_a) \]  \hspace{1cm} (A)

where \( Q \) = total input power in Watts;
\( h_t, h_f \) = tube heat transfer coefficient and fin heat transfer coefficient respectively, W/m\( \cdot \)K\(^0\);
\( A_t, A_f \) = tube and fin surface area respectively, m\(^2\);
\( T_t \) = average tube temperature, \(^\circ\)C;
\( T_a \) = temperature of the mainstream air, \(^\circ\)C;
and \( \gamma_f \) = fin efficiency

The overall heat transfer coefficient determined previously using a copper finned arrangement was used as an initial estimate for the fin heat transfer. This was employed in Gardner's expression (Ref. 134) for the efficiency of a rectangular fin to determine an initial estimate of \( \gamma_f \).

Upon determination of an initial estimate of the tube heat transfer coefficient can then be obtained from equation (A) and the measured heat dissipation. This tube coefficient was then employed together with the previously measured overall coefficient to obtain an improved estimate of the fin-side coefficients. The procedure was repeated until successive estimates for the tube and fin heat transfers exhibited satisfactory convergence. Generally three to four iterative cycles were sufficient. This calculation procedure was relatively insensitive to changes in the estimated value of the fin efficiency so that the use of Gardner's expression should not lead to significant errors.
In the case of the transverse fins it is a reasonable assumption that the tube heat transfer coefficients were not appreciably affected by the presence of the fins. Accordingly the fin heat transfer coefficients may then be evaluated from a knowledge of the overall coefficient, the bare tube coefficient and the weighted fin efficiency.

5.5 Results and Discussion

5.5.1 Longitudinal Finned Arrangements

The results for the tests on the tube rows fitted with longitudinal fin are presented in Tables 5.3 to 5.5. The actual convective heat transfer coefficients (i.e. those referred to the total fin and tube surface areas) were plotted against the Reynolds number (based on the mainstream velocity) in Figs. 90, 91 and 92 for all the longitudinal finned cases. In these figures, the corresponding results from the tests on rows of smooth bare tubes are also included so that direct comparisons can be drawn. It is apparent from these plots that the performances of the various finned tube arrangements differed appreciably. For the fins angled at 90° and 60° to the plane of the tube row the heat transfer coefficients can be slightly less than those for the bare tubes, particularly at the closest tube spacings i.e. the 6 mm x 9.5 mm section. These reductions occurred at the lower Reynolds numbers i.e. 250 < Re_o < 2 x 10^3. The percentage decrease in the heat transfer increased at the lower Reynolds numbers and reached a maximum decrease of approximately 23% at Re_o = 500 when the longitudinal fins were positioned at <\alpha> = 90° and about 11% when <\alpha> = 60°. However, for the fins angled at 45° to the plane of the tube row the heat transfers were always increased (at all the Reynolds numbers studied). This was true for all tube spacings and thus the heat transfer performance was considerably improved when longitudinal fins were attached to the rear part of tubes at <\alpha> = 45°. This enhanced performance could, no doubt, be attributed, at least in part, to the impingement of the fluid exiting from the spacing between the tubes. Jet impingement heat transfers are often correlated in terms of Re_o^0.8 whereas the heat transfers over the rear of a tube are proportional to Re_o^0.67. Thus impingement may explain the comparatively greater enhancement of heat transfer as the Reynolds number increased.

Any conclusion concerning the effectiveness of adding fins to the tubes should allow for the effect of increasing the surface area of the tube. Thus the present results are replotted in Figs. 93, 94 and 95 as the effective heat transfer coefficients (i.e. based on tube area alone) h_eff against the Reynolds number Re_o. It is apparent from these plots that the increased heat transfers due to an increased surface area depend upon the angle of inclination in the range of Reynolds number investigated, with the greatest enhancement occurring at inclinations of 45° to the plane of the test section. Large increases in heat transfer (two to three fold) can be obtained.

To study the influence of the longitudinal fins on the heat transfers the fin and tube heat transfer coefficients were plotted separately against the mainstream flow Reynolds number, see Figs. 96 to 101. It is apparent from these plots that the fin heat transfer coefficients were less than those for the tubes for the case of the fin at <\alpha> = 90 at the lower Reynolds numbers (e.g. at Re_o < 1.4 x 10^3 for the 6.0 mm x 1.5 mm tube row dimensions; at Re_o < 900 for the 6.0 mm
When the longitudinal fin was inclined at $\alpha = 60^\circ$, the fin heat transfers increased and even at the lower Reynolds numbers were approximately equal to those on the tube side. Further increases in the fin-side coefficient were observed at $\alpha = 45^\circ$. The influence of flow Reynolds number, $Re_0$, on the fin heat transfers is also clearly evident in the figures. In all cases, the fin heat transfers increased rapidly with fluid velocity. The greatest increase was associated with the closest spaced arrangements.

The ratio of fin heat transfer coefficients to tube heat transfers ($h_f/h_t$) are presented in Figs. 102, 103 and 104. Inspection of these figures reveals:

(i) the ratio $h_f/h_t$ increases as the flow Reynolds number increases. This is probably due to the fact that impingement heat transfers on the fin increase at a faster rate than the tube coefficients.

(ii) At a fixed angle and flow the largest values of $h_f/h_t$ are associated with the closest spacings. This is not unexpected since the velocity of the fluid exuding from the gap between the tubes is greater at the closest spacing.

(iii) For a given spacing the ratio $h_f/h_t$ generally increases as the inclination of the fin to the mainstream flow is increased. This affect is particularly marked at the closer spacings. Once again this is probably related to the greater degree of impingement of the comparatively high velocity fluid in the inclined situations.

The tube side heat transfers for the longitudinal finned tube arrangements were compared with those for the bare tubes. The results exhibited considerable scatter and it appeared that the spacing between the tubes was the most significant variable in determining the ratio $h_t$ (finned)/$h_t$ (bare), see Fig. 105. The results indicate that for the most widely spaced tubes (1.50 mm gap) the presence of the longitudinal fin increased the tube-side heat transfers.

The angle of inclination of the fin can also influence the flow resistance and hence the required pumping power, see Fig. 106 which presents the results of the pressure drop measurements. It may be seen from this figure that the tube rows with longitudinal fins positioned at $\alpha = 45^\circ$ exhibit the highest pressure losses. This was not unexpected, however, due to the relatively smaller flow passages in these cases and the larger projected areas offered to the flow (thus creating relatively higher form drag). For $\alpha = 90^\circ$, it was expected that the flow resistance would be less than that associated with smooth tubes. This is due to the reduction of the total drag as indicated in previous studies (Ref. 161). However, in the present tests no such indication was observed for both the 6 mm x 1.0 mm and (6 mm x 0.5 mm) tube banks. Such a reduction did occur, however, in the least closely-spaced arrangement (6 mm x 1.5 mm).

The heat transfer data obtained for the longitudinal fins are presented in general form using a suitable reference Reynolds number to account for blockage effects in Fig. 107.

The velocity used in these presentations was thus either the integrated
mean velocity, or the empirical reference velocity proposed in Chapter 3, as appropriate for the flow blockage.

It is apparent that the heat transfer data for a particular fin inclination may be grouped into a single relationship. This suggests that the use of an appropriate reference Reynolds number can account for the effect of tube spacing. Thus the angle of inclination of the longitudinal fin dictates the heat transfers. This is not unexpected since 'impingement' effects should be more important as the tube inclination to the mainstream flow is increased.

The slope of curve 3 (i.e. for $\alpha = 90^\circ$) is approximately $\frac{2}{3}$ which is in accord with previous data for separated flow regions. The slight deviations at the higher and lower Reynolds numbers can be mainly attributed to experimental error. Curves 1 and 2 of Fig. 107 refer to the results obtained from longitudinal fins positioned at $\alpha = 45^\circ$ and $\alpha = 60^\circ$ respectively. The greater deviations from a slope of $\frac{2}{3}$ of these curves particularly at $Re > 4 \times 10^3$ are probably due to the 'impingement' of the fluid on the angled fins.

5.5.2 Transverse Pinned Tube Arrangements

The actual and effective overall heat transfer coefficients for the tubes fitted with transverse fins are presented in Figs. 108 and 109 respectively. Two different fin spacings are presented and the results for the previously measured heat transfers associated with bare tubes of similar diameter and spacing are also included. It can be seen from Fig. 108 that the actual heat transfers (i.e. those based on the total finned tube area) for the finned arrangements were higher than those for the bare tubes.

The heat transfer coefficients were highest in case of the closest fin spacing and this probably reflects the slightly greater flow blockage in this case. However, any difference in heat transfers between the two finned arrangements was comparatively small and may be accounted for, at least in part, by experimental errors.

The increase in actual heat transfers due to the fins also depended on the Reynolds number. At $Re_0 = 500$ the actual heat transfers increased by 46% with 240 fins/metre and by 67% with 470 fins/metre. At a higher Reynolds number, $Re_0 = 2.0 \times 10^5$, these percentage increases were 73% and 95% respectively.

Fig. 109 presents the improvements in overall heat transfer performance due to the increases in both heat transfer coefficients and surface areas. It is apparent that significant improvements are possible. Thus at $Re_0 = 500$ the effective heat transfer coefficients increased by 150% and 330% when compared with the bare tube data. Moreover, at $Re_0 = 2000$ these increases were 178% and 340% for 240 fins/metre and 470 fins/metre respectively. Thus transverse fin arrangements should be employed to maximise the heat transfer rates.

However, the hydraulic resistance of the tube row is also an important factor. The pressure losses, associated with the transverse fin systems were appreciably higher, see Fig. 110. For example, the percentage increase in pressure loss in comparison with a similar arrangement of
bare tube at $Re_0 = 500$ was 225% and at $Re_0 = 2000$ this increase was 124%. The comparatively less steep increase in hydraulic resistance at the higher Reynolds numbers may be explained by the higher hydraulic drags associated with the finned tubes (for general reference, see Gunter and Shaw, Ref. 68). It is interesting to note that the hydraulic resistances for the two fin spacings were not significantly different. This probably follows since the projected area normal to the direction of flow was only 1.8% greater for the closer spaced surfaces. The pressure drops for the transverse cases were also significantly greater than those for the worst longitudinal fin configuration i.e. $\alpha = 45^\circ$.

Assuming that the tube heat transfer coefficients were unaffected by the presence of the fins, the fin heat transfer coefficients may be calculated and corrected for the fin efficiency. These results are presented in Table 5.6 and also in Fig. 111 as the average Nusselt number (based on the hydraulic diameter plotted against the flow Reynolds number (based on the minimum free flow area and the hydraulic diameter). As discussed earlier in this thesis, this method of presentation is widely used in the literature for the transverse fin configurations. Although direct comparisons between various finned geometries can be misleading the present data are compared with the mass transfer data published by Saboya and Sparrow (Ref. 129) for a single-row plate-finned exchanger. These latter data are for conditions closer to the present geometry than any other available cases in the literature. However, the blockage ratio $D/H$ is much higher in the present tests. These previous data were first converted to heat transfers by employing the Chilton-Colburn heat-mass transfer analogy. Examination of Fig. 111 confirms the difficulty of generalising heat transfer data for the transverse finned geometries. It should be emphasised that it is dangerous to generalise the results for the transverse finned tube arrangements, and that the measurements are only valid for the geometries employed in the present experiments.

5.5.3 Comparison of the Finned Tube Rows

a) Heat Transfers

In Fig. 112 the actual heat transfers associated with the two different transverse finned arrangements are compared with those associated with the longitudinal finned tubes (for similar tube diameters and spacings). As can be seen from this figure, tube rows fitted with transverse fins generally exhibit higher heat transfers than those associated with the longitudinal fins. However, at $Re_0 > 4 \times 10^3$ the longitudinal finned tubes with $\alpha = 45^\circ$ have a better heat transfer performance. A similar behaviour can also be observed at $Re_0 < 250$.

However, consideration of the previously presented pressure drop characteristics, indicate that the transverse finned arrangements exhibit the highest pressure losses.

b) Optimal Fin Arrangement

As was pointed out earlier in this thesis several, sometimes mutually conflicting factors enter into the ultimate selection of the optimal tube configuration. These factors can include the increase in thermal
performance, the added weight, the initial cost and the pumping power or operating cost. Previous studies (Refs. 78, 118, 164, 166) have shown that some of these factors may be difficult to quantify and a generally acceptable selection criterion does not exist. Ultimately, the Stirling engine heater designer must consider all the particular system requirements. However, a useful criterion which should be maximised is the ratio of heat transfer coefficient to the hydraulic resistance of the tube row. This ratio is a crude type of measure of the energy extracted from the fluid to the pumping energy required to maintain the flow. This ratio (based on the effective heat transfer) is plotted against the mainstream Reynolds number, see Figs. 113 to 115, for the longitudinal fin systems.

Similar presentations are available in Fig. 116 for the transverse finned tubes. Since the tube diameter was invariant at 6 mm this mainstream Reynolds number is proportional to the fuel and combustion air input to a practical engine. Since high ratios generally occur at conditions of low heat transfer it should be emphasised that this criterion is constrained by the necessity to attain a required heat transfer. For completeness the results are represented in Figs. 117 to 120 using the actual coefficients as the basis for the heat transfers.

It may be seen from Figs. 113 to 115, that at the less blocked arrangements (i.e. with spacings of 1.0 mm and 1.5 mm) the best overall performance was obtained with the fin parallel to the main flow direction, i.e. \( \alpha = 90^\circ \). An exception to this behaviour occurred at the higher Reynolds numbers with the 1.0 mm spacing. Nevertheless it may be preferable to use a fin inclined at \( \alpha = 45^\circ \) to make use of the overall higher heat transfer. At a spacing of 0.5 mm, see Fig. 115, however, the ratio of effective heat transfer to the pressure drop is greatest with the fin inclined at 45°. Generally except at very low Re the finned configurations were more effective than the rows constructed of smooth tubes. Thus if a small gap is maintained between tubes (as recommended in Chapter 3) it appears preferable to employ an inclined longitudinal fin.

In Fig. 116, the performance of the transverse finned tubes with 240 fins/metre is considerably lower than that associated with the bare tubes for \( \text{Re}_0 < 10^3 \). Increasing the Reynolds number results in the finned tubes having the better performance. For the higher number of fins/metre (i.e. 470) the relative performance of the finned tubes is considerably better than that of the bare tubes. The difference, however, decreases with decreasing Reynolds number. Thus if the extra cost of employing transverse fins can be tolerated it appears advantageous to use an arrangement with closely spaced fins instead of the bare smooth tubes.

It is interesting to compare the relative performances of the longitudinal and transverse finned tubes at similar tube diameter and spacings (6 mm x 1.0 mm). As can be seen in Fig. 121, the highest performance is achieved with the longitudinal finned tubes with \( \alpha = 90^\circ \) at \( \text{Re}_0 < 500 \). However, at higher Reynolds numbers the transverse finned tubes (470 fins/metre) were superior.

It should be emphasised once again that the relative order of merit discussed in the foregoing performance assessment is not necessarily the
sole criterion. Furthermore, this assessment ignores the relative cost of the finned tubes since these were not available. However, the initial cost per unit size for the transverse finned tubes will be higher than the cost of the longitudinal finned tubes, which in turn will be significantly higher than that of the bare tubes. Once this basic cost information is available, then, an economic performance criterion can be employed to investigate the optimal configuration for a specified cost (typical examples of these calculations are available in Refs. 137 and 167). Consideration should also be given to the extended surface effectiveness, the maximum allowable pressure losses, and the possibility of fouling, see Refs. 164, 169, 171, 172, 173 and 174.

5.6 Conclusions

Experimental measurements of the heat transfers and pressure losses have been undertaken for nine different arrangements of longitudinal finned tubes. Two different arrangements of the transverse plate finned tubes have also been studied. The performances of these finned tube arrangements were compared with each other and with data obtained for rows of bare tubes. The following conclusions can be drawn:

1. Both the heat transfer and the pressure drop increase (at constant tube spacing) with an increase in the angle of inclination of the longitudinal fin relative to the mainstream flow direction.

2. At a fixed angle of inclination, both the heat transfer and the pressure drop increase as the tube spacing is reduced.

3. For all longitudinal finned tube arrangements, the rate of increase of the effective heat transfers (based on the tube surface area) increased as the Reynolds number is increased.

4. For the longitudinal finned system, inclinations at $\alpha = 45^\circ$ gave the best heat transfer performance although the measured pressure drop were simultaneously higher.

5. For the transverse finned geometries the actual heat transfer coefficients appear to be relatively insensitive to the fin spacing provided that the flow blockage is taken into account in assessing the correlating Reynolds number. Thus the greatly increased area of tubes with close fin spacings is responsible for these tubes have a higher overall effective heat transfer coefficients.

6. At fixed tube diameter and spacing, the transverse finned tubes with minimum spacing between the fins exhibit the highest ratio of effective heat transfers to pumping power in the range of $Re_\theta>500$. However, at lower $Re_\theta$ the tubes with a longitudinal fin parallel to the mainstream flow is the most effective. Nevertheless, it may be preferable to incline the fin to $\alpha = 45^\circ$ to obtain higher heat transfers provided that the correspondingly higher pressure drops can be tolerated.

7. However, from a thermal viewpoint it appears preferable to employ tubes fitted with transverse fins provided the extra cost can be justified. However, tubes fitted with a slotted longitudinal fin could well be a
cheaper alternative with very little loss in performance. Indeed at low Re, the performance may well be superior although the overall heat transfers are lower. If close spacings are maintained between the tubes, fins inclined at $\approx 45^\circ$ to the mainstream direction are preferable.
TABLE 5.1

TYPICAL DIRECT TEST AND REPEATABILITY TEST RESULTS FOR THE LONGITUDINAL FINNED TUBE ROW OF 6.0 mm DIA. X 0.5 mm SPACING.

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TABLE 5.2
TYPICAL DIRECT TEST AND REPEATABILITY TEST RESULTS FOR THE LONGITUDINAL FINNED TUBE ROW OF 6.0 mm DIA X 1.00 mm SPACING

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TABLE 5.3

TYPICAL DIRECT TEST AND REPEATABILITY TEST RESULTS FOR THE LONGITUDINAL FINNED TUBE ROW OF 6.0 mm DIA X 1.50 mm SPACING

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<td>374</td>
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<td>12060</td>
<td>422</td>
<td></td>
</tr>
<tr>
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<tr>
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<td>4942</td>
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<td>11811</td>
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<tr>
<td>4386</td>
<td>11800</td>
<td>465</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 5.4

**TYPICAL EXPERIMENTAL RESULTS FOR THE LONGITUDINAL FINNED TUBE**

**ARRANGEMENT OF 6.0 mm DIA.. X 0.50 mm SPACING**

<table>
<thead>
<tr>
<th>DEGREES</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(Re_0)</th>
<th>(Re_r)</th>
<th>(h_{act.} \ oF/ W/m². K)</th>
<th>(h_{tubg side} \ W/m . °K)</th>
<th>(h_{fin side} \ W/m . °K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>249</td>
<td>1472</td>
<td>77</td>
<td>90</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>486</td>
<td>2873</td>
<td>110</td>
<td>118</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>1914</td>
<td>11316</td>
<td>312</td>
<td>273</td>
<td>379</td>
</tr>
<tr>
<td></td>
<td>4014</td>
<td>23731</td>
<td>547</td>
<td>375</td>
<td>841</td>
</tr>
<tr>
<td>60</td>
<td>248</td>
<td>1466</td>
<td>84</td>
<td>84</td>
<td>84</td>
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<td></td>
<td>479</td>
<td>2222</td>
<td>115</td>
<td>107</td>
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<tr>
<td></td>
<td>1684</td>
<td>11138</td>
<td>367</td>
<td>250</td>
<td>549</td>
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<tr>
<td></td>
<td>3850</td>
<td>22761</td>
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<td>515</td>
<td>953</td>
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<tr>
<td>45</td>
<td>244</td>
<td>1443</td>
<td>111</td>
<td>98</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>479</td>
<td>2332</td>
<td>161</td>
<td>132</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>1676</td>
<td>11091</td>
<td>462</td>
<td>261</td>
<td>805</td>
</tr>
<tr>
<td></td>
<td>3900</td>
<td>23097</td>
<td>840</td>
<td>495</td>
<td>1427</td>
</tr>
</tbody>
</table>
# Table 5.5

Typical experimental results for the longitudinal finned tube arrangement of 6.0 mm DIA. x 1.00 mm spacing

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Re_0</th>
<th>Re_r</th>
<th>h_{act.} \ W/m^2\cdot^\circ K</th>
<th>h_{tube side} \ W/m^2\cdot^\circ K</th>
<th>h_{fin side} \ W/m^2\cdot^\circ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>244</td>
<td>1124</td>
<td>72</td>
<td>82</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>468</td>
<td>2248</td>
<td>101</td>
<td>95</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>1928</td>
<td>8443</td>
<td>211</td>
<td>203</td>
<td>226</td>
</tr>
<tr>
<td></td>
<td>4595</td>
<td>21164</td>
<td>401</td>
<td>372</td>
<td>451</td>
</tr>
<tr>
<td>60</td>
<td>244</td>
<td>1124</td>
<td>73</td>
<td>77</td>
<td>67</td>
</tr>
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<td></td>
<td>487</td>
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<td>101</td>
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</tr>
<tr>
<td></td>
<td>1843</td>
<td>8489</td>
<td>250</td>
<td>186</td>
<td>358</td>
</tr>
<tr>
<td></td>
<td>4436</td>
<td>20432</td>
<td>616</td>
<td>373</td>
<td>1047</td>
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<td>45</td>
<td>240</td>
<td>1105</td>
<td>85</td>
<td>78</td>
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</tr>
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</tr>
<tr>
<td></td>
<td>1843</td>
<td>8443</td>
<td>298</td>
<td>198</td>
<td>470</td>
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<td></td>
<td>4360</td>
<td>20081</td>
<td>739</td>
<td>379</td>
<td>1352</td>
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## TABLE 5.6

TYPICAL EXPERIMENTAL RESULTS FOR THE LONGITUDINAL FINNED TUBE
ARRANGEMENTS OF 6.00 mm DIA. X 1.50 mm SPACING

<table>
<thead>
<tr>
<th>DEGREES</th>
<th>$Re_0$</th>
<th>$Re_m$</th>
<th>$h_{act.}$ W/m² °K</th>
<th>$h_{tube side}$ W/m² °K</th>
<th>$h_{fin side}$ W/m² °K</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>248</td>
<td>667</td>
<td>67</td>
<td>69</td>
<td>62</td>
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<tr>
<td></td>
<td>492</td>
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<td>89</td>
<td>90</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>1864</td>
<td>5013</td>
<td>177</td>
<td>180</td>
<td>171</td>
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<tr>
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<td>11818</td>
<td>365</td>
<td>285</td>
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<td>60</td>
<td>253</td>
<td>680</td>
<td>70</td>
<td>69</td>
<td>71</td>
</tr>
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<td>422</td>
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<td>616</td>
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<td>491</td>
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<td>4390</td>
<td>11811</td>
<td>480</td>
<td>375</td>
<td>659</td>
</tr>
<tr>
<td>Fins/Metre</td>
<td>$D_h$ m</td>
<td>Flow Blockage %</td>
<td>$Re_o$</td>
<td>$Re_h$</td>
<td>$h_{tot}$ W/m²°C</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
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<td>-------------------</td>
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<tr>
<td>237</td>
<td>5.6</td>
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<td>27000</td>
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<td>470</td>
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<td>120</td>
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<td>5541</td>
<td>500</td>
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<td></td>
<td></td>
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<td>12143</td>
<td>774</td>
</tr>
</tbody>
</table>

**TABLE 5.7**

**EXPERIMENTAL RESULTS FOR THE TRANSVERSE FINNED TUBE ARRANGEMENTS**

**TUBE ROW OF 6.0 mm DIA. X 1.00 mm SPACING**
Fig. 86. Longitudinal Finned Tube Arrangements.

Range of Test Geometries

<table>
<thead>
<tr>
<th>$H/mm$</th>
<th>$D/H$</th>
<th>$C/mm$</th>
<th>$\theta/^{\circ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>0.923</td>
<td>6.5</td>
<td>0.923</td>
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<td>0.857</td>
</tr>
<tr>
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<td>0.857</td>
<td>7.0</td>
<td>0.857</td>
</tr>
<tr>
<td>7.5</td>
<td>0.800</td>
<td>7.5</td>
<td>0.800</td>
</tr>
<tr>
<td>7.5</td>
<td>0.800</td>
<td>7.5</td>
<td>0.800</td>
</tr>
</tbody>
</table>

$A_{tot} = 274.6 \text{ mm}^2$

$A_f/A_t = 0.587$

$A_f/a_t = 0.392$
### FIG. 87.
TRANSVERSE FINNED TUBE ARRANGEMENTS

<table>
<thead>
<tr>
<th>FINS/METRE</th>
<th>$A_f/A_t$</th>
<th>$Z$ mm</th>
<th>$D_s/S$</th>
<th>$A_{tot}$ mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>0.90</td>
<td>3.68</td>
<td>0.857</td>
<td>3167.0</td>
</tr>
<tr>
<td>470</td>
<td>2.01</td>
<td>1.65</td>
<td>0.857</td>
<td>4395.0</td>
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</tbody>
</table>
FIG. 88. THE OPTIMUM DIMENSIONLESS FIN HEIGHT FOR A SINGLE LONGITUDINAL FIN ATTACHED TO THE REAR OF A SINGLE CYLINDER (Ref. 161).

<table>
<thead>
<tr>
<th>No.</th>
<th>Re x 10^-4</th>
<th>x = b/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.9</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8.2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>19.2</td>
<td>5</td>
</tr>
</tbody>
</table>
FIG. 89. BLOCK DIAGRAM OF THE CIRCUIT USED FOR HEAT TRANSFER MEASUREMENTS UNDER CONSTANT FLUX CONDITIONS.
FIG. 90. HEAT TRANSFER RESULTS FOR 6.0 mm x 1.50 mm LONGITUDINAL FINNED TUBE ARRANGEMENTS AT DIFFERENT ANGLES OF INCLINATION.
FIG. 91. HEAT TRANSFER RESULTS FOR 6.0 mm. x 1.0 mm. LONGITUDINAL FINNED TUBE ARRANGEMENTS AT DIFFERENT ANGLES OF INCLINATION.
FIG. 92. HEAT TRANSFER RESULTS FOR 6.0 mm x 0.50 mm LONGITUDINAL FINNED TUBE ARRANGEMENTS AT DIFFERENT ANGLES OF INCLINATION.
FIG. 93. EFFECTIVE HEAT TRANSFER COEFFICIENTS BASED ON TUBE SURFACE AREA AS A FUNCTION OF REYNOLDS NUMBERS $Re_o$ FOR LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. x 1.5 mm.

**KEY**
- x $45^\circ$
- △ $60^\circ$
- ○ $90^\circ$

FINNED TUBES

BARE TUBES

$Re_o$

$h_{eff}$ (W/m$^2$.K)

TUBE DIA. 6.0 mm.
SPACING 1.5 mm
FIG. 94. EFFECTIVE HEAT TRANSFER COEFFICIENTS BASED ON TUBE SURFACE AREAS AS A FUNCTION OF REYNOLDS NUMBERS $Re_0$. LONGITUDINAL, FINNED TUBE ROW OF $6.0 \text{ mm} \times 1.0 \text{ mm}$.
FIG. 95. EFFECTIVE HEAT TRANSFER COEFFICIENTS BASED ON TUBE SURFACE AREA AS A FUNCTION OF REYNOLDS NUMBERS $Re_o$ LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. $\times$ 0.50 mm.
**FIG. 96a.** ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT.
TUBE ROW OF 6.0 mm DIA $\times$ 1.50 mm SPACING WITH $\alpha = 90^\circ$

**FIG. 96b.** ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT.
TUBE ROW OF 60 mm DIA $\times$ 1.50 mm SPACING WITH $\alpha = 60^\circ$
FIG. 97. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA. x 1.50 mm SPACING WITH $\alpha = 45^\circ$.
FIG. 98.a. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA x 1.0 mm SPACING WITH $\alpha = 90^\circ$.

FIG. 98.b. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA x 1.0 mm SPACING WITH $\alpha = 60^\circ$. 

KEY
- FIN
- TUBE
FIG. 99. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA × 1.0 mm SPACING WITH $\alpha = 45^\circ$
FIG. 100. a. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA x 0.50 mm SPACING WITH $\alpha = 90^\circ$

FIG. 100. b. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA x 0.5 mm SPACING WITH $\alpha = 60^\circ$
FIG. 101. ACTUAL HEAT TRANSFER FOR THE FIN SIDE AND TUBE SIDE OF A LONGITUDINAL FINNED TUBE ARRANGEMENT. TUBE ROW OF 6.0 mm DIA x 0.50 mm SPACING WITH $\alpha = 45^\circ$. 

KEY
- FIN
- TUBE

$\Re_0$

$h, \text{ W/m}^2\text{k}$

$6.0 \text{ mm DIA}$

$0.50 \text{ mm SPACING}$

$\alpha = 45^\circ$
Fig. 102. Variations of the ratio $h_f / h_t$ with $Re_0$ at constant angle of 90°.

Key:
1. 6.0 mm Dia. x 0.50 mm Spacing
2. 6.0 mm Dia. x 1.50 mm Spacing
3. 6.0 mm Dia. x 1.00 mm Spacing

$\alpha = 90$ DEGREES

---

$h_{fin} / h_{tube}$
FIG. 103. VARIATION OF THE RATIO $h_f / h_t$ WITH $Re_o$ AT CONSTANT ANGLE OF 60°
FIG. 104. VARIATIONS OF THE RATIO $h_f/h_t$ WITH $Re_0$ AT CONSTANT ANGLE OF 45°
FIG. 105. VARIATION OF $h_t$ (FINNED TUBE) / $h_t$ (BARE TUBE) WITH $Re_o$. 
FIG. 106. PRESSURE DROP RESULTS FOR THE LONGITUDINAL FINNED TUBE ARRANGEMENTS.
Fig. 107. Generalisation of the heat transfer data obtained for the longitudinal fins.
FIG. 108. HEAT TRANSFER COEFFICIENTS BASED ON TOTAL HEAT TRANSFER AREA AS A FUNCTION OF Re₀. TRANSVERSE FINNED TUBE ROW OF 6.0 mm × 1.0 mm.
FIG. 109. EFFECTIVE HEAT TRANSFER COEFFICIENTS BASED ON TUBE SURFACE AREA AS A FUNCTION OF $Re_o$

TRAVVERSE FINNED TUBE BANK OF 6.0 mm. x 1.0 mm.
FIG. 110. COMPARISON BETWEEN THE PRESSURE DROP ASSOCIATED WITH TUBE ROW OF 6.0 mm x 1.0 mm FOR VARIOUS ARRANGEMENTS
FIG. 111. COMPARISON BETWEEN THE PRESENT HEAT TRANSFER DATA OBTAINED FOR THE TRANSVERSE FIN GEOMETRIES AND THE MASS TRANSFER DATA OBTAINED BY SABOYA AND SPARROW (Ref. 129)
FIG. 112. COMPARISON BETWEEN THE HEAT TRANSFERS ASSOCIATED WITH THE TRANSVERSE FINNED TUBES AND THOSE ASSOCIATED WITH THE LONGITUDINAL ARRANGEMENTS. RESULTS OF TUBE ROW OF 6.0mm × 1.0mm
FIG. 113. THE EFFECTIVE HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER, LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. × 1.50 mm.

TUBE DIA. 6.0 mm.
SPACING 1.5 mm.

KEY
1 90°
2 60°
3 45°
4 BARE TUBES

$\frac{h_{eff}}{\Delta P}$, W. N$^{-1}$.K$^{-1}$

$Re_o$

4 6 8 10$^3$ 2 4 6
FIG. 114. THE EFFECTIVE HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER, LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. x 1.0 mm.

KEY

- 1) 90°
- 2) 60°
- 3) 45°
- 4) BARE TUBES

TUBE DIA. 6.0 mm.
SPACING 1.0 mm.
FIG. 115. THE EFFECTIVE HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER, LONGITUDINAL FINNED TUBE-ROW OF 6.0 mm. × 0.50 mm.
FIG. 116. THE EFFECTIVE HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER, TRANSVERSE FINNED TUBE ROW OF 6.0 mm. x 1.0 mm.
FIG. 117. ACTUAL HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER, RESULTS FOR LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. x 1.50 mm.
FIG. 118. ACTUAL HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER. LONGITUDINAL FINNED TUBE ROW OF 6.0 mm. x 0.50 mm.
**Fig. 119.** Actual heat transfer per unit pumping power as a function of mainstream Reynolds number. Longitudinal finned tube row of 6.0 mm. x 1.0 mm.
FIG. 120. ACTUAL HEAT TRANSFER PER UNIT PUMPING POWER AS A FUNCTION OF MAINSTREAM REYNOLDS NUMBER TRANSVERSE FINNED TUBE ROW OF 6·0 mm. × 1·0 mm.
FIG. 121. COMPARISON BETWEEN THE PERFORMANCES OF TRANSVERSE FINNED TUBE ARRANGEMENTS AND THOSE OF LONGITUDINAL GEOMETRY.
PLATE. 10 GENERAL VIEW OF THE EXPERIMENTAL RIG FOR HEAT TRANSFER TESTS ON TUBES FITTED WITH EXTENDED SURFACES. 1- A.C. POWER SOURCE. 2- POWER SHUNT. 3- TEST SECTION. 4- STANDARD D.V.M. 5- OSCILLOSCOPE UNIT. 6- DATA LOGA UNIT.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Conclusions

This thesis reports on an investigation of some of the factors affecting the thermal design of a compact recuperative heat exchanger for use in the primary heater of a Stirling engine. The exchanger consisted of an arrangement of small diameter metallic tubes, and in a practical engine, the working fluid (hydrogen) flows inside these tubes. The heat transfer coefficient on the tube inner surface is thus comparatively high so that it is important to maximize the rate of heat transfers from the combustion gases flowing over the outside of the tubes. This present investigation has thus studied the effect of recuperator geometry (e.g., tube diameter and spacing, type of fins) on the heat transfer coefficient to the outside of the tube. The pressure drop due to the gases passing over the tubes has also been studied. The tubes varied in diameter from 3.0 mm to 6.0 mm and the spacings between adjacent tubes ranged from 0.30 mm to 1.60 mm. The tubes were mounted in a single row. The main conclusions which can be drawn are as follows:

1. Normally, for tubes mounted in a tube bank or in a restricted channel, single cylinder correlations may be used to predict the heat transfer provided an integrated mean velocity for the flow between the tubes is used as a reference velocity in the correlation. This approach was found to be adequate for some of the geometries investigated in this present study. However, for the more highly blocked arrangement (that is D/H > 0.85) such a procedure was found to be unsatisfactory and could underestimate the actual heat transfers by up to 50%. For these very highly blocked flows, it was found possible to use a single cylinder correlation, provided an empirical reference velocity of the form

\[ U_r = U_0 \left( 1 + 1.82 \left( \frac{D}{H} \right)^3 \right)^2 \]

was used for the Reynolds number. Thus in a single row of tubes mounted so that the blockage ratio is high, extrapolation of existing data can lead to underestimation of the heat transfers unless the appropriate reference velocity is applied.

2. For a constant Reynolds number mainstream flow in the duct prior to the test section (that is a constant fuel and combustion air input to an actual engine) the heat transfers were relatively insensitive to the tube diameter at a fixed tube spacing. However, the pressure drops were found to be very dependent on the diameter and decreased as the tube diameter was decreased. This is not unexpected since decreasing the tube diameter at a fixed tube spacing increases the net free flow area so that the hydraulic resistance is reduced. Thus it is recommended the smaller diameter tube should be used for the exchanger. The heat transfers also increased as the spacing was decreased so that it is recommended that the lowest spacing commensurate with the allowable pressure drop should be employed.

3. It was found that the turbulence intensity of the mainstream fluid prior to the test section had little effect on the heat transfers. This is contrary to the observations of previous investigators on single cylin-
ders. However, this is probably not unexpected with these highly blocked arrangements in which the flow blockage would ammend the turbulence structure.

4. The surface roughness of the tubes appears to affect the heat transfers with these very close spacings (0.30 mm to 1.80 mm) between the tubes. However, the results are of a preliminary nature and a systematic investigation is needed to quantify the effect of surface roughness. It appears nevertheless from these present results that an increase in surface roughness increased the overall heat transfer coefficient on the outside of the tubes.

5. The local heat transfers associated with these tube arrangements were also studied. It was found that the heat transfer coefficients increased downstream from the front stagnation point. This appears to be associated with the acceleration of the fluid as it passes through the gap between the tubes. The heat transfer coefficient reached a maximum at the position delineated by $\theta = 90^\circ$, that is near the region of minimum gap. Subsequent to this maxima, the heat transfer coefficients decrease rapidly and in the case of very low Reynolds number (i.e. $Re < 10^3$) the heat transfer coefficient remain low around the rear of the tube. This is probably due to the fact that the vortices formed in this separated region have comparatively low energy. However, at the higher Reynolds numbers, there is some increase in heat transfer coefficient around the rear of the tube. The results indicate, however, that in general there is a considerable variation in heat transfer coefficient around the circumference of a tube in a highly blocked flow situation.

6. The ratio of heat transfer over the rear portion of the tube to that over the front increases with either a decreasing spacing between the adjacent tubes or an increasing Reynolds number (based on the mainstream velocity).

7. It was found that adding extended surfaces to the tubes increased the effective heat transfer coefficient (i.e. the coefficient based on the actual bare tube area). Tests were carried with both longitudinal and transverse fins. The longitudinal fins were of fixed length but were inclined at three different angles of inclination to the axis of the tube row, (i.e. 45°, 60° and 90°). The transverse fins were also of fixed size but two fin spacings (namely 240 fins/metre and 470 fins/metre) were tested. The tube diameter was fixed at 6.00 mm.

8. It was found, in the case of the longitudinal fins, that with a fixed spacing between the tubes, and with a constant $Re_o$ (i.e. a constant mainstream Reynolds number in the duct prior to the test section) the highest effective heat transfer coefficient occurred with the fin inclined at 45° to the axis of the test section. Furthermore, when the angle of inclination of this fin was fixed, the heat transfer coefficients increased as the spacing between the tubes was decreased. Thus with a spacing of 0.5 mm and the fin inclined at $\alpha = 45^\circ$, the effective heat transfer coefficient was 200% to 300% better than that for the equivalent bare tube row. However, the pressure drops were also greater for the inclined fin cases.

The performance of the longitudinal fins were studied by estimating the ratio of the effective heat transfer coefficient to the pressure drop.
It was desired to maximize this ratio. Generally except at the closest spacing, it was found that the longitudinal fin positioned at $\alpha = 90^\circ$ gave the highest ratio. At the closest spacing of 0.5 mm employed in this study, it was found that the highest ratio was obtained with the longitudinal fin inclined at $\alpha = 45^\circ$ to the axis of the test section. Thus if close spacings are to be maintained between the tubes (as recommended previously) it is suggested that a longitudinal fin inclined at $\alpha = 45^\circ$ is a reasonably cheap means of increasing the heat transfer rates to the tubes. At the less close spacing, it is suggested that perhaps, the fin inclined at this angle should also be used in order to make use of the higher heat transfer coefficients.

The ratio of the fin side to the tube side heat transfers were also determined. This ratio was found to be highest at the closest tube spacing. It also increased markedly as the mainstream Reynolds number was increased and as the angle of inclination to the mainstream direction was increased. These effects are considered to be due to the impingement of the comparatively fast moving fluid exiting from the gap between the tubes upon the fin surface. These impingement heat transfers will increase at a faster rate with Reynolds number than the heat transfers around the rear of the tubes, thus explaining the marked increase as the mainstream flow is increased. The increase as the spacing between adjacent tubes is decreased is probably due to the higher velocity of the fluid exiting from the gap between the tubes; furthermore, a greater degree of impingement occurs with the inclined fins, thus explaining the higher ratio of fin to tube heat transfers in these cases.

9. With the transverse fins it was found that the actual heat transfer coefficient was virtually independent of fin spacing providing allowance was made for the extra blockage effect. However, the greater heat transfer area which occurs with a close fin spacing means that the effective heat transfer coefficient is much higher in this case. The pressure drops for these two cases were found to be virtually independent of fin spacing and this is probably due to the only slightly greater blockage which ensues in the more closely spaced case. Thus it is recommended that if transverse fins are to be fitted that a comparatively close fin spacing should be maintained. The transverse fins at the close spacing 470 fins/metre gave the highest ratio of effective transfer coefficient to pressure drop in the present tests. However, it is likely that these fins will be more costly than the simple longitudinal fin configuration. Nevertheless, at $Re_0 < 500$ the longitudinal fin fitted parallel to the mainstream flow direction gave the highest ratio of effective heat transfer coefficient to pressure drop, although this configuration may not be practical due to the overall lower heat transfer.

10. It may thus be concluded that if the extra cost of transverse fins can be justified, these fins should be used since they give the highest overall heat transfer and also at $Re_0 > 500$ the greatest ratio of effective heat transfer coefficient to pressure drop. However, a cheaper alternative appears to be longitudinal fin (slotted to avoid thermal distortion due to expansion of the fin at higher temperatures). In this case if close spacings are maintained between the tubes, a fin inclined at 45° to the axis of the mainstream flow should be employed.
6.2 Recommendations for Future Work

1. It appears from an examination of the overall heat transfer results for the bare tubes that the results at the highest blockage ratio were higher than those for any other blockage ratio even when an appropriate reference velocity was used. This suggests that the constant in the reference velocity may well depend upon the blockage ratio. A further series of tests should thus be carried out at higher blockage ratios to determine the most suitable form of this constant and to establish if it is blockage dependent.

2. It was observed that the effect of high blockage ratio was to increase the average heat transfer coefficient for the bare tube compared with the values estimated from correlations for single cylinders. However, it is considered that there may well be a diameter effect at these higher blockage ratios since for a fixed blockage ratio the spacing between the tubes is proportional to the diameter whereas the thickness of the boundary layer will be proportional to some fractional power of the diameter. Thus the ratio of boundary layer thickness to spacing will be dependent upon some power of the diameter so that the inhibition of the boundary layer growth is likely to be less at larger diameters. It is proposed that tests be conducted with a wide range of tube diameters to investigate this possible diameter effect.

3. The preliminary tests discussed in this thesis suggest that an increase in the tube surface roughness with these small diameters and closely spaced arrangements can increase the overall heat transfer coefficients associated with the bare tubes. It is recommended that further tests be conducted to examine this surface roughness effect in detail. Again tests should be carried out with tubes of varying diameter to assess whether the roughness effect is proportional to blockage ratio or to the absolute value of the spacing.

4. The measurements made in this present investigation have all been made with the main flow direction perpendicular to the axis of the test section. In a practical Stirling engine heater this is unlikely to occur and the flow will impinge upon the test section at an angle. Thus it is recommended that tests be undertaken to investigate the effect of maldistributions in the initial mainstream flow on the overall coefficients.

5. The local heat transfer coefficients reported are somewhat sparse. Therefore, it is suggested that further studies be carried out on the local heat transfers at high flow blockages. In particular the apparent discrepancy between the electrochemical (mass transfer) results and the direct heat transfer results should receive further study.

6. Further work should be carried out to optimize the size and shape of the extended surfaces:
   a) The length and angle of inclination of the longitudinal fins should be varied over a wider range in order to optimize the geometry of such a fin.
   b) In the case of the transverse fins, again, their length, width, shape
and the number of fins per unit length should all be varied comprehensively in order to optimize their geometry. In particular, due to the high temperature of the combustion gases prior to this heat exchanger it may be necessary to restrict these transverse fins to the rear of the tube and this effect should receive attention.

7. For both fin configurations, the optimization process would be assisted by a knowledge of local coefficients over the fin surface. It is thus suggested that measurements of these local coefficients be obtained if possible and the electrochemical (mass transfer) technique appears to be the most suitable technique.

8. Tests should be made with hot combustion gases (i.e. on a hot system) to assess the effect of fin distortion due to expansion in practical situation. Furthermore such an actual hot system should also be used to study the effect of extended surface design on fouling of the exchanger particularly if fuels other than light distillate oils are to be burned. Corrosion problems in the exchanger could also be studied with such a hot system.
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<td>Sills, E.</td>
<td>Personal Communications, Instrumentation Section, Cranfield Institute of Technology.</td>
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APPENDIX-A PRODUCTION AND MEASUREMENT OF TURBULENCE INTENSITY.
APPENDIX - A

PRODUCTION AND MEASUREMENT
OF TURBULENCE INTENSITY

A.1 Introduction

In chapter 2 of the present thesis reference was made to the heat transfer performance of heat exchangers comprising several tube rows. It was shown that higher heat transfers are associated with the second row and subsequent rows than the first row, and this was connected with the influence of the wake shedding from the first row. Upon subsequent flow patterns it is apparent that the first row generates considerable turbulence (i.e. irregular high frequency velocity fluctuations). The 'lumps' of downstream fluid consist of macroscopic eddies of various sizes and velocities. The size of these fluid eddies which continually agglomerate and disintegrate determine the scale of the turbulence. This size is determined by the external conditions associated with the flow, e.g. by the geometry of the tube row, or the screen (grid) through which the fluid stream has passed. The departure of the instantaneous velocity of these eddies from the mean velocity determines the turbulence intensity. However, the values of both the turbulence intensity and scale generated by a tube row or mesh depend upon the downstream distance and the Reynolds number (based on the mesh size). Considerable work both theoretical and experimental has been conducted to determine turbulence decay i.e. the relationship between the downstream distance and the turbulence characteristics (see Refs. 79a, 80).

In the present investigation perforated plates were placed in the flow to generate turbulence. This Appendix describes the method of calculating the decay of turbulence intensity and scale downstream of the four different perforated plates using an empirical formula provided in (Ref. 80). This decay relationship was used to establish the spacing between the plates and test section so that a constant turbulence scale and different turbulence intensities were obtained at the plane of the test section. A suitable value of turbulence scale was chosen from the work of Van der Zegelinen (Ref. 46), on the combined effect of turbulence intensity and scale on heat transfer. The turbulence intensities were checked by measurement in the plane of test sections using a hot wire anemometer.

The calibration procedure for the hot wire anemometer is also described.

A.2 Turbulence Decay

If a grid or a perforated plate is placed normal to the direction of flow of a fluid, then two distinct effects result at a given Reynolds number (based on the grid geometry). The first result is the distortion of any macroscale eddies in the approaching flow and the formation of new eddies. The second is the creation of high intensity fluctuations of small scale eddies in the wake of each obstruction, thus generating additional turbulent energy. In an isotropic turbulence field there is a continuous transfer of inertial energy of large scale eddies to the small scale ones. As the downstream distance from the plate or grid increases, it becomes difficult to distinguish between the size of the largest and smallest eddies, and after a long period, there is virtually no energy transfer be-
tween the size of the largest and smallest eddies, and after a long period, there is virtually no energy transfer between the different sized eddies. There is also a reduction in energy of all the eddies due to viscous effects. This interaction process at different energy levels is governed by the Reynolds number. However, in practice a perfect isotropic or homogenous turbulence can never be achieved even at large downstream distances from the grid because of the unidirectional characteristics of the macroscale permanent eddies. It has been pointed out in (Ref. 80) that to achieve an approximately isotropic turbulence, a distance of at least 50 bar sizes downstream of the grid is required. A reduction in Reynolds number appears to make matters worse, because it decreases the energy content of the microscale eddies and consequently the macroscale eddies prevail. When the downstream distance exceeds 80 bar sizes, isotropy is more nearly achieved because of the better diffusive properties of the turbulence field.

The turbulence parameters downstream of grids or perforated plates have been measured by several investigators who have proposed various equations to correlate the data. The following expressions are widely accepted.

For low turbulence intensities (i.e. 2 to 10%)

$$T_u^2 = \left( \frac{u}{u'} \right)^2 = A \left( \frac{x}{M} \right)^n,$$ and

For high turbulence intensities (8 to 20%)

$$\frac{u'}{u} = B \left( \frac{x}{b} \right)^{-m}$$

whereas the turbulence microscale may be predicted by

$$\lambda_x = \left( \frac{10 \cdot \nu}{n \cdot u}(x) \right)^{\frac{1}{2}}$$
and the turbulence macroscale by

\[
\left( \frac{L}{b} \right) = 0.3 \left( \frac{X}{b} \right)^{0.5}
\]

The values of the constants \( A \) and \( n \) are see (Ref. 179)

<table>
<thead>
<tr>
<th>( \text{Re}_b )</th>
<th>( A )</th>
<th>( n )</th>
<th>Area Blockage Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 10^4 )</td>
<td>70</td>
<td>1.13</td>
<td>&lt; 0.30</td>
</tr>
<tr>
<td>( 2.5 \times 10^4 )</td>
<td>8.5</td>
<td>1.40</td>
<td>0.30 &lt; ABR &lt; 0.50</td>
</tr>
</tbody>
</table>

Ballal (Ref. 80) recommended that \( A \) and \( n \) should be taken equal to 60 and 1.17 respectively for \( M/b > 4 \) at the low range of turbulence, and \( B \) and \( m \) should be taken equal to 1.12 and 1.14 respectively for \( M/b < 4 \) at the higher range of turbulence intensity.

Four perforated plates having different bar sizes were chosen, see Table A.1, and the predictions of the decay of turbulence intensity and scales are shown in Fig. A.1

### A.3 Selection of Grid to Test Section Spacings

From the foregoing it is clear that both turbulence intensity and scale varies with the downstream distance from the generation point. However, both turbulence parameters have an influence on the heat transfers associated with tubes, and thus in order to investigate solely the effect of turbulence intensity it was necessary to keep the turbulence scale constant throughout the present tests. Moreover, the value of turbulence scale at the plane of test section should be such that its influence on the heat transfers is a minimum. Van der Hegge Zijnen (Ref. 46) seems to be the only investigator who studied the combined influence of the turbulence scale \( \lambda_X \), and intensity of turbulence, \( T_u \). His extensive measurements were so arranged that the scale of turbulence was either very large or comparable with the diameter of the cylinder. The test cylinders ranged from very thin wires (1.0 mm dia. minimum) to small tubes (41.9 mm dia. maximum). The range of Reynolds numbers covered was from \( 2 \times 10^3 \) to \( 2.5 \times 10^4 \) and the turbulence intensity was up to 14%. Zijnen established that the ratio

\[ \frac{\text{Nu}^*}{\text{Nu}_{\text{lam}}} = \frac{\text{N}_{\text{turb}}}{\text{Nu}_{\text{lam}}} \]

systematically increased with turbulence intensity, \( T_u \), and decreased or increased with the scale \( \lambda_X \). He presented his results in the form of the correlation equation

\[ \frac{\text{Nu}^*}{\text{Nu}_{\text{lam}}} = 1 + E_1 \left( \text{Re} \times T_u \right) \times E_2 \left( \lambda_X / D \right) \]

where \( D \) is the diameter of test cylinder, and \( E_1 \) and \( E_2 \) are two empirical curves reproduced in Fig. A.
FIG. A.1. VARIATION OF TURBULENCE INTENSITY AND SCALE WITH DOWNSTREAM DISTANCE
Consequently in the present tests the value of $\lambda_x / D$ was chosen from Fig. A.5 (b) to be equal to 0.25 so as to have little effect on the heat transfer. This value of scale together with the turbulence curves for the various perforated plates shown in Fig. A.1 was used to establish downstream positions of the test section relative to the perforated plates and the results are given in Table A.1. However, as a check these predicted values of turbulence intensities were also measured using a D.I.S.A hot wire anemometer. These measurements were also
used to obtain the turbulence intensity profiles at the test section. This was achieved as described in the text of the thesis and the results are given in Figs. A.3 - A.5. It can be seen that generally flat turbulence profiles were obtained for most perforated plates except at the highest turbulence intensity. This later profile was, however, accepted on the basis that the influence of turbulence scale to test cylinder diameter ratio \( \lambda / D \) was considered more important than the turbulence isotropy, at least for the purpose of the present experiments (for more detail see Ref. 75). A comparison between the predicted and directly measured values of turbulence intensity is made in table A.2 and this reveals a satisfactory level of agreement between the two sets of data. However, the intensity of turbulence was also measured at different flow velocities and the results are shown in Fig. A.6.

It can be seen, the turbulence intensity decreased with increasing Reynolds numbers (based on the test cylinder diameter) in the range of 900 to \( 2 \times 10^3 \). At the higher values of Reynolds numbers the turbulence intensity tends to be constant.

### A.4 Calculation of the Turbulence Intensity

All the measurements of turbulence intensity with the constant temperature hot wire anemometer were evaluated on a basis of the experimentally plotted calibration curve \( V_1^2 = f (u \lambda) \) for the anemometer and the probe, where \( V_1 \) denotes the measured output voltage and \( u \) is the velocity of fluid. This method of plotting provides a straight-line curve (in accordance with King's law) for the investigated range of fluid velocity from 0.8 to 25 m/sec., providing a double logarithmic representation of the function \( (V_1 / V_0) - 1 \) vs \( U \) is used. A proper choice of \( V_0 \) is also required (where \( V_0 \) is the output voltage at zero flow velocity, and \( V_0 = 0.925 V_0 \) for air and \( V_0 = 0.86 V_0 \) for water). Since turbulence measurements are based on the slope of the calibration curve, this type of representation offers an advantage. However, in many measurements especially where the probe was solidly mounted, the determination of \( V_0 \) is somewhat difficult as this required that the fluid flow be brought to rest while maintaining the same cold resistance \( R/R_0 \sim 1 \) (i.e. a constant temperature). This difficulty was overcome by maintaining the fluid (the air and/or the water) temperature at constant value and thus avoiding the complicated procedure of readjusting the overheating ratio or the alternative computation method to substitute for the changes in the physical data of the air. These changes can affect the constant \( A \) in the King's law:

\[
Q = I^2_R = (A + BU^2) \Delta T .
\]

If the overheating ratio \( a = (R - R_i)/R \) is kept constant then \( \Delta T / R \sim (R - R_o)/R = a / (1 + a) \) will be constant, and consequently \( I^2 - I^2_o \) will be independent of temperature at a given flow velocity. If, for example, bridge voltages \( V_{OT} \) and \( V_{IT} \) were measured at a fluid temperature of \( T \), for the same value of \( a \) as during calibration, then:

\[
\frac{V_{IT}^2 - V_{OT}^2}{(R_T + R_s + R_o)^2} = \frac{V_1^2 - V_o^2}{(R + R_s + R_o)^2}
\]

which by rearranging gives:
\[ \frac{V_1^2 - V_0^2}{V_0^2} = \frac{V_{IT}^2 - V_{OT}^2}{V_0^2} \frac{R + R_a + R_c}{R_T + R_s + R_c} \]

where \( R_s = 50 \text{ ohm (standard)} \) and \( R_c \) is the cable resistance and equals to 0.3 ohm for a 5 - metre cable.

During velocity changes, the output velocity \( V_1 \) will generally follow the typical calibration curve, see Fig. A.11, up to the upper frequency limit. This limit is determined by the probe type, the excess temperature on the probe sensor, the anemometer gain, bandwidth, and the bridge configuration, and by the nature and velocity of the medium under measurement (Ref. 81). Once the slope \( n = \frac{dy}{dx} \) of the calibration curve was determined then the values of turbulence intensity were calculated using the following formula provided in Ref. 81.

\[ Tu = \frac{V_1 R_s R_a}{V_1} \times 2 \frac{V_1^2}{n \frac{V_1^2}{n} - \frac{V_0^2}{n}} \]
### TABLE A. 1

**TEST GEOMETRIES OF THE TURBULENCE PRODUCING GRIDS**

<table>
<thead>
<tr>
<th>Grid No.</th>
<th>M (mm)</th>
<th>b (mm)</th>
<th>T (mm)</th>
<th>Tu% Predicted</th>
<th>( \frac{L_x}{b} ) % Predicted</th>
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<tbody>
<tr>
<td>1</td>
<td>4.8</td>
<td>2.0</td>
<td>63.5</td>
<td>6.5</td>
<td>0.74</td>
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<td>2</td>
<td>2.9</td>
<td>0.95</td>
<td>686</td>
<td>1.49</td>
<td>1.58</td>
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<tr>
<td>3</td>
<td>3.15</td>
<td>1.50</td>
<td>152.</td>
<td>4.50</td>
<td>1.0</td>
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### TABLE A. 2

**COMPARISON BETWEEN THE PREDICTED VALUES OF TURBULENCE INTENSITY AND THOSE OBTAINED BY DIRECT MEASUREMENTS**

<table>
<thead>
<tr>
<th>Grid No.</th>
<th>( Re_0 )</th>
<th>Exp. Tu%</th>
<th>Pred. Tu%</th>
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<tr>
<td>1</td>
<td>910</td>
<td>5.8</td>
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<td></td>
<td>1598</td>
<td>5.4</td>
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</tr>
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<td></td>
<td>2500</td>
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<td></td>
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<td>11</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>3889</td>
<td>10.4</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
FIG. A.3. TYPICAL TURBULENCE INTENSITY PROFILES FROM GRIDS No. 1 AND No. 2
FIG. A.4. A TYPICAL TURBULENCE INTENSITY PROFILE ASSOCIATED WITH GRID NO. 3.

GRID No. 3
Re₀ = 3889

FIG. A.5. A TYPICAL CALIBRATION CURVE FOR THE CONSTANT TEMPERATURE HOT WIRE ANEMOMETER.

V₀ = 2.73 VOLT
\dot{V}_0 = 0.925 V₀
Rₛ = 3.77 \Omega
FIG. A.6: RESULTS OF TURBULENCE INTENSITY MEASUREMENTS AT FIXED TURBULENCE SCALE
APPENDIX-B  HEAT-MASS TRANSFER ANALOGY.
CHAPTER 6  GENERAL CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK.
APPENDIX - B

HEAT - MASS TRANSFER ANALOGY

B.1 Introduction

Convective heat transfer coefficients in cross flow tube heat exchangers, may be determined by near-ambient isothermal model techniques, using the analogy between heat transfer and mass transfer under conditions of geometric, dynamic and kinematic similarity. In the naphthalene sublimation technique, the surfaces of the model corresponding to the heat transfer surfaces of the exchanger are coated with naphthalene, and air is passed through the model to simulate the flow of hot gases from the combustion chamber. The resulting weight losses from the naphthalene surfaces, are used to obtain the mass transfer coefficients for the model, and hence the convective heat transfer coefficients in the exchanger.

Chilton and Colburn (Ref. 82) in 1934, established a correlation between the rate of mass transfer and the skin friction for various geometric shapes by using the similarity between the mechanics of heat transfer and mass transfer. Thus measurements of mass transfer rates in isothermal models, may be used to estimate convective heat transfer coefficients. Under the conditions of geometric, dynamic and kinematic similarity, the dimensionless heat and mass transfer coefficients are equal according to the Chilton-Colburn analogy.

For heat transfer, the dimensionless coefficient is:

$$j_H = \frac{h}{\sqrt{\rho U C_p}} (Pr)^{\frac{3}{8}}$$

and for mass transfer:

$$j_M = \frac{b}{U} (Sc)^{\frac{3}{8}}$$

These dimensionless coefficients are found experimentally to be approximately equal to $\frac{1}{2} f$ for certain simple streamline shapes, but to be much less than $\frac{1}{2} f$ for flow past bluff bodies, including spheres and cylinders. This heat-mass transfer analogy has been employed by many investigators for determination of convective heat transfer coefficients. The range of problems studied include flow over cylinders and tube banks, impingement of jets, rotating discs, and tangentially fired heating furnaces.

B.2 The Advantages of Naphthalene as a Subliming Solid

In the past work on the Heat Mass transfer analogy, naphthalene has been a predominant choice as the sublimate. Other solid subliming substances can, however, be used. These include solid hydrocarbons such as camphor, urethane, hexachloroethane and aceto-naphthalene. These however, have unsuitable sublimation rates (Ref. 176); some would diffuse from the cylinder surface within a few seconds, whilst others would diffuse far too slowly. Within the temperature range of this work, (160°C to 200°C) naphthalene was the obvious choice; having a suitable vapour pressure
(Fig. B.1) to allow an exposure time of between 10 and 20 mins. Naphthalene also allows for the measurement of both local and average mass transfer rates, and hence heat transfer coefficients.

The relatively hard finish of a naphthalene cylinder facilitates machining, and hence an accurate model can be obtained for the test.

B.3 Evaluation of Average Heat Transfer Coefficients from Mass Transfer Measurements

The mean mass transfer coefficient \( \dot{b} \) is defined by

\[
\frac{\dot{m}}{\dot{t}} = \dot{b}(c_w - c_0)
\]

where subscript 'w' refers to the mass transfer surface and subscript 'o' refers to the bulk stream.

Since the partial pressure of the naphthalene vapour is small compared with ambient pressure, the vapour can be treated as a perfect gas having a temperature \( T_w \) at the mass transfer surface. Thus

\[
c_w = \frac{P_w}{R_t T_w}
\]

The vapour concentration of the naphthalene in the free stream is insignificant. Therefore \( c_o = 0 \), and

\[
\dot{b} = \frac{\dot{m}}{\dot{t}S} \cdot \frac{R_t T_w}{P_w}
\]

Although sublimation depressed the surface temperature of the naphthalene below that of the free stream this difference has been found to be negligible (Ref. 127). Consequently, the saturation vapour pressure \( P_w \) is evaluated at the mean free stream absolute temperature \( T_0 \) (\( T_w \)) and is given by (Ref. 177).

\[\log_{10} P_w = 11.55 - \frac{3765}{T_0}\]

where \( P_w \) is measured in torr.

From the Chilton-Colburn analogy (Ref. 54) \( J_H = J_m \) so that

\[
h = \frac{\dot{b} \rho \dot{c}}{\dot{P} \dot{c}} \left( \frac{S_c}{\dot{P} \dot{c}} \right)^{2/3}
\]

It has been estimated by Sherwood and Truss (Ref. 181) that \( S_c = 7.00 \frac{T_o}{T_0} \), for \( 100K < T_o < 500K \).
Thus the average convective heat transfer coefficient may be evaluated from mass transfer data.

B.4 Properties of Naphthalene

(a) General Properties:

Chemical Formula: \( \text{C}_8 \text{H}_{10} \)

Description: White, crystaline volatile flakes with aromatic odour.

Physical Properties:

- Density: 1145 kg/m\(^3\) @ 20°C
- Molecular Weight: 128.16
- Melting Point: 80.1°C
- Boiling Point: 217.9°C
- Flash Point: 86°C
- Vapour Pressure: imm. Hg. @ 52.6°C
- Vapour Density: 4.42
- Gas Constant, \( R_v \): 64.7 J/kg. deg. K.
- Lower Explosive Limit in Air: 0.9% by volume
- Upper Explosive Limit in Air: 5.9% by volume
- Auto-ignition Temperature: 526°C
- Latent heat of Sublimation: 559 kJ/kg
- Coefficient of Volumetric Expansion: 0.00028/deg. C

(b) Hazardous Properties:

- Fire Hazards: Moderate, when exposed to heat or flame, reacts with oxidising materials.
- Spontaneous Heating: No
- Explosion Hazard: Moderate, in the form of dust, when exposed to heat or flame.

B.5 Sample Calculations

Measured Data.

- Cylinder Weight Loss = 0.449 gms.
- Air Temperature at Test Section = 2.88.2K
- Surface Area of Cylinder = \( 1.6 \times 10^{-3} \) m\(^2\)
- Duration of Test = 1213 seconds.
ALL MISSING PAGES ARE BLANK

IN

ORIGINAL
FIG. B.1. VARIATION OF VAPOUR PRESSURE WITH TEMPERATURE FOR NAPHTHALENE AND CAMPHOR
APPENDIX-C

GENERAL ERROR ANALYSIS.
GENERAL ERROR ANALYSIS

C.1 Introduction

In the calculation of the convective heat transfer coefficients from the experimental data, the measurement of the various quantities is involved. The subsequent large amount of data was reduced to forms suitable for presentation as shown in the text of the thesis. However, measurement of an experimental variable is usually subject to error and the accuracy and validity of any conclusions is dependent upon these errors. Such errors may be broadly classified (Refs. 178, 179) as systematic, individual, independent and random errors.

The systematic errors are of a constant or similar form and often result from improper procedures or conditions that are consistent in their effect. They are primarily due to imperfections in the experimental techniques or in the use of measuring apparatus. They may be reduced by careful instrumentation and their presence may be detected by cross checking the measurement technique.

Individual errors refer to the errors introduced by each potential source of error. These errors may be 'systematic' errors, or alternatively 'random'. The random errors are distinguishable by their lack of consistency but their magnitudes are usually predictable within limits. Random errors may be due to inherent inaccuracy of the instrumentation, random behaviour of the measuring system, etc. The statistical basis for error analysis is reviewed in this appendix and an attempt made to outline the relevant mathematical arguments.

C.2 Independence of Errors

The output of a measuring system depends not only upon the quantity to be measured but also upon other undesirable inputs to which the system is also sensitive. Hence it is assumed that the instantaneous output of a measuring system is the linear or algebraic sum of the instantaneous values corresponding to the measured quantity and to each of the possible system errors. However, this does not imply that the probable sum of the errors is as large as the sum of the probable errors. This assumption is normally valid if the system errors (both random and systematic) are small with respect to its output range. This is equivalent to assuming that the sensitivity of the system to a given error input is not significantly altered by the values of the other error inputs or by the quantity to be measured. For example in the case of the average heat transfer test cylinder, errors are introduced approximately in proportion to the deviation of the temperature and heat flux from their nominal values. The assumption is then that the heat flux sensitivity of the instrument is not appreciably altered by the temperature, and that the temperature sensitivity is not appreciably altered by the heat flux. However, the question of independence of various parameters is presumably resolved by consideration of the physical mechanisms for a given situation. The extent to which sensitivities to other inputs constitute errors depends on the manner in which the measurement system is used as well as the system sensitivities. For example the turbulence intensity of the mainstream fluid which is also sensitive to temperature to the extent that
the temperature and temperature sensitivity are known. The temperature response to the system may be subtracted from the measurement system output as a calibration correction and will not be a measurement error. The temperature response is properly considered an error only if the system sensitivity is not precisely known and/or it is not feasible to measure and correct for the system temperature. In this sense, an error is a quantity whose value is not completely known. If the value of an error was known, it would simply be a correction. Hence the question arises of how to measure or describe quantities whose values are not known. At this juncture the statistical approaches becomes of value.

If a number of measurements are made, they may be described in terms of their average and variation. For example, if the heat transfer coefficient of a single cylinder were measured many times at virtually the same Reynolds number, the average measured value might be 210 W/m²·K and the spread indicated for example by noting that 90% of the measurements fell within ±6% of this value. In the case of single measurement taken at a single Reynolds number, the concept of probability takes on slightly different meaning. If for a single measurement, the estimated heat transfer coefficients were within the range 197.4 to 222.6 it may mean that there is a 90% chance of it being correct. However, it may be argued that there is no such thing as a probability with a single event, i.e. the estimation is either correct or incorrect. A more practical view would be to excuse this on the basis that if the estimating process was repeated a large number of times, then correct results were obtained within this range 90% of the time.

C.3 Summation of Errors (Ref. 83)

If the output of a measurement system is accepted as a sum of values corresponding to the measured quantities and the various errors, the next step is to sum the errors in some manner so as to obtain an estimate of the system error. The simplest approach would be to add the absolute values of the probable or maximum values of each individual error, and assume that the probable or maximum system error is somewhat less. This approach would ordinarily yield an upper limit which may be so large as to be of no practical value. As an example, the measurement of limiting current is the measurement of the average motion of individual ions in the electrolyte and the measurement of gas pressure is a measurement of an average of momentum reactions from individual gas molecules. A sum of absolute values of errors which may be introduced by each ion or each gas molecule would far exceed the magnitude of any practical limiting current or pressure measurement errors which would be made. In such cases, statistical averaging in which errors are assumed to partially cancel one another is not only a reasonable approach but is necessary, if a meaningful estimate is to be made. Consequently it is convenient to describe error distributions in terms of their mean and variance. The mean is the average value of the error, and the variance is the average value of the square of the difference between the error value and the mean. Another parameter frequently used to describe error distributions is the standard deviation, $\sigma_d$. The standard deviation is the square root of the variance, and is expressed in the same units as the individual sample values and the mean. The variance and standard deviations are thus measures of the spread or probable range of error value. For discrete distributions, the mean $\mu$ and variance $\sigma_d^2$ may be expressed as:
Thus it is convenient to consider error distributions as having a mean
of zero since a non-zero mean could be absorbed as part of the calibra-
tion of the system. However, the extent to which individual errors are
identified will depend partly upon the system characteristics and partly
upon convenience. As an example, if the measurement system consists of
nine elements each sensitive to vibration, the system error due to vibra-
tion may be considered as the sum of the nine individual vibration errors
or a single vibration error for the entire system. For valid statistical
averaging of a number of errors to be some degree of independence
must be assumed with respect to the individual error sources. Such inden-
dependence would not be a characteristic of the measuring system under
evaluation but it would rather be a characteristic of the environment
and the manner in which the system is to be used. For example, consider
a humidity measuring instrument which is subject to temperature and
pressure error. If the instrument is used to measure humidity at a
ground station over an extended period of time, the temperature and baro-
metric pressure will fluctuate essentially independently and the errors
due to temperature and pressure will be correspondingly independent.
However, if the same instrument were used to measure the characteristics
of the atmosphere as a function of altitude over a limited period of time,
the temperature and barometric pressure would change together and an
assumption of their independence would lead to erroneous results.

If errors due to two or more independent sources are added the mean and
variance of the sum of the errors will be the sums of the means and vari-
ances of the individual error distribution. If error from two or more
sources which were not independent are added the variance of the resulting distribution will be more or less than the sum of the individual variances depending whether the relationship between the errors is such that they tend to cancel or not. If a small number of errors are not independent it may be convenient to consider them as a single error source independent with respect to the remaining error sources. However, in spite of this the question of independence of the individual error sources is not usually a major problem. Error dependence seldom arises except through physical relationships which may be realised through an understanding of the particular application.

The 'Gaussian distribution' is an important part of statistics and probability series. In one form it may be written as:

\[ B(x) = \frac{1}{\sqrt{2\pi} s_d} e^{-\frac{(x - \mu)^2}{2 s^2}} \]

where \( B(x) \) is the probability of occurrence. The probability the sample value falling between two limits \( m_1 \) and \( m_2 \) is

\[ \int_{m_1}^{m_2} B(x) \, dx \]

The importance of the Gaussian distribution was recognised early in the development of statistical theory because of the limiting distribution of the means of samples from populations with a variety of distribution functions. Recently, the central limit theorem which is frequently regarded as one of the most important theorems in statistics has shown that the Gaussian distribution is the limiting distribution for the sample from means of populations with any distribution function limited only by the requirement that they have finite variances.

Perhaps the simplest case is when the total system error is a sum of a number of individual error distributions which are known to be independent and Gaussian. The total system error will then have a Gaussian distribution whose mean is equal to the sum of the means. The variance is equal to the sum of the individual variances and the total standard deviation is the square root of this total variance.

In many cases a condition exists where the individual error distribution functions are not well known, and the assumption that they are Gaussian in nature is questionable. In these cases, the effect of the central limit theorem is to justify the practical use of a Gaussian distribution to the extent that no error is introduced in many instances where such an assumption is not strictly valid. So long as the total system error may be considered as a sum of many independent individual errors, and it is not dominated by a small number of error distributions, the total system error may be estimated by assuming a Gaussian distribution whose mean is a sum of the individual error distribution means, and whose variance is the sum of the individual variances. However, in a physical measurement system the error may be viewed as a sum of a number of individual error contributions. For statistical purposes, a single value which is the sum of \( n \) individual independent errors is closely related to
the mean of a sample of n elements. Thus the application of the central limit theorem to error analysis is a sum of a number of individual error contributions which are independent and of finite mean and variance. The total system error distribution will then approach a Gaussian distribution with variance equal to the sum of individual variances and mean equals to the sum of individual means, as the value of the former becomes large with respect to the largest of the individual variances. An important aspect of the theorem is that nothing is required of the distribution functions beyond a finite mean and variance. The condition that the variance be finite is a minor restriction as far as error analysis is concerned because infinite values are not encountered in physical measurements.

If in some practical cases the errors are known to be independent but are not Gaussian so that none of the aforementioned methods are applicable, it is still possible to make some meaningful estimates of the limits of the system error. From the Tchebysheff inequality the probability of a system error departing from its mean by a value greater than j times the standard deviation of the system error will not exceed $1/j^2$. The only requirement upon the individual distributions is that they be of finite mean and variance and are independent. The Tchebysheff inequality states that the probability of the mean of a sample of size n from a distribution with finite mean and variance differing from the population mean by a value greater than a is less than the ratio of the variance to the square of the deviation interval, divided by sample size $n$ i.e.

$$ \frac{1}{n} \leq \frac{(x - \mu)^2}{n} $$

It is seen that the standard deviations may be summed by the root sum square method without regard to their distribution functions, provided that the individual errors are independent. Thus the standard deviation or probably value of the total system error will be the square root of the sum of the squares of the individual standard deviations or probable error values.
APPENDIX - D

SAMPLE CALCULATIONS.
APPENDIX - D

SAMPLE CALCULATION

D.1 Reynolds Number Calculations

Experimental Data a 6.0 mm x 0.50 mm Single Tube Row

General

Atmospheric pressure, \( P_{\text{atm}} \) 29.602 in.Hg
Orific plate diameter, \( d_{\text{or}} \) 50.8 mm
Duct size, \( d_d \) 82.5 mm
Air temperature prior to orific plate, \( T_1 \) 14.5°C
Pressure difference across the orific plate, \( \Delta P \) 307.0 mm water
Static pressure prior to orific plate, \( P_1 \) 1046 mm water
Air temperature prior to test section, \( T_2 \) 19.5°C
Static pressure prior to test section, \( P_2 \) 732.0 mm water
Average test cylinder temperature, \( T_w \) 35.5°C

Now:

\[
\begin{align*}
\bar{P}_1 &= P_{\text{atm}} \times 0.491 + P \times 0.0361 \\
&= 29.602 \times 0.491 + 41.2 \times 0.0361 \\
&= 16.02 \text{ lb/in} \\
\bar{P}_a &= 14.53 + 28.8 \times 0.0361 \\
&= 15.57 \text{ lb/in} \\
T_L_1 &= 14.5 \times \frac{9}{5} + 32 + 459.67 \\
&= 517.8 \text{ °R} \\
T_L_2 &= 19.5 \times \frac{9}{5} + 32 + 459.67 \\
&= 526.8 \text{ °R} \\
\rho_1 &= 2.7 \frac{\bar{P}_a}{T_L} \\
&= 0.0836 \text{ lb/ft}^3 \\
\rho_2 &= 2.7 \frac{\bar{P}_a}{T_L} \\
&= 0.0809 \text{ lb/ft}^3 \\
\bar{Q} &= 359.2 \cdot C \cdot Z \cdot E \cdot d_{\text{or}}^2 \sqrt{\left( \frac{\Delta P}{\rho} \right)}
\end{align*}
\]
where \( C \) is the basic orifice discharge coefficient

\[
Z = Z_R \times Z_d
\]

\( Z_R \) and \( Z_d \) are correction factors due to the Reynolds number range and duct size

\( \gamma \) is the dimensionless expansibility factor

\[
E = \frac{1}{(1 - m^2)^2}, \text{ and is known as the velocity of approach factor}
\]

\[ \dot{m} = \frac{d_{or}}{d_d}, \text{ dimensionless, area ratio} \]

These correction factors and constants are obtained from B.S. 1042 and hence,

\[
\frac{d_{or}}{d_d} = 0.6154 \quad E = 1.0804
\]

\[
\dot{m} = 0.3787 \quad C = 0.6075
\]

\[
\frac{\Delta P_a}{P_{ia}} = \frac{12.07}{16.02} = 0.753 \quad Z = 1.006
\]

\[
\gamma = 1.400 \quad \text{Air at}
\]

\[
\zeta_v = 1.0044 \quad T = 14.5^\circ C
\]

\[
\zeta_r = 0.7173 \quad \gamma = 1.400
\]

\[
\overline{Q} = 359.2 \times 0.6075 \times 1.006 \times 1.80 \times 2^2 \times \frac{12.07}{0.0836} = 190.0 \text{ ft}^3 / \text{min.}
\]

\[
= 5.38 \text{ m}^3 / \text{min.}
\]

To convert this into reference conditions at the test section:

\[
\overline{Q_r} = \overline{Q} \left( \frac{\overline{P} - \overline{P}_r}{K \overline{T}} \right) \left( \frac{\overline{P}_r - \overline{P}_v}{R \overline{T}} \right)
\]

where the suffix \( r \) denotes the reference conditions

\[
\overline{Q_r} = \frac{190.0}{16.02} \times \frac{16.02}{517.8} \times \frac{525.8}{15.57} = 198.9 \text{ ft}^3 / \text{min.}
\]

\[
= 5.63 \text{ m}^3 / \text{min.}
\]
\[ U_0 = \frac{Q}{A_0} \] velocity of approach

where \( A_0 \) is the area of test section

\[ U_0 = \frac{198.9}{0.08 \times 0.10 \times 60 \times 35.31} = 11.73 \text{ m/s} \]

\[ T_{\text{film}} = \frac{T_w + T}{2} \]

\( T_w = 35.5 \degree C \), average surface temperature of the test cylinder.

\[ T_{\text{film}} = 27.9 \text{ thus } \mu_f = 1.906 \times 10^5 \text{ kg/m/sec.} \]

i.e. the dynamic viscosity of air at film temperature.

\[ \frac{\rho_f}{\mu_f} = 0.0808 \times \frac{35.31}{2.205} = 1.294 \text{ kg/m}^3 \]

then:

\[ Re = \text{Reynolds number} \]

\[ = \frac{\rho u D}{\mu} = \frac{1.294 \times 11.73 \times 0.006}{1.906 \times 10^{-5}} \]

\[ = 4778 \]

D.2 Average Nusselt Number Calculation

If \( T_w = 35.0 \degree C \)

\( T_2 = 23.0 \degree C \)

\( T_{\text{film}} = 29.0 \degree C \)

\( k_f = \text{thermal conductivity} = 2.64 \times 10^{-2} \text{ w/m K} \)

\( Q = \text{heat dissipation} = 4.74 \text{ W} \)

\( S = \text{test cylinder surface area} = 1.7 \times 10^{-3} \text{ m} \)

then:

\[ h = S \left( \frac{q}{T_w - T} \right) = \frac{4.74}{1.7 \times 10^{-3} \times 12} \]

\[ = 232.4 \text{ w/m}^2 \text{ K} \]

\[ Nu_{\text{av.}} = \frac{hD}{k_f} = \frac{232.4 \times 0.006}{2.64 \times 10^{-2}} \]

\[ = 52.8 \]
D.3 Local Nusselt Number Calculation

Data from 6.0 mm x 0.50 mm tube row at Re = 689

The temperature of the (T_2) air prior to test section was maintained approximately constant over the range 18.2 to 18.5°C.

The heat dissipation (d.c, Ohmic heating), q = 6.17 to 6.3 watt

The surface temperatures and other related or derived data are given in table D.1. However, these data do not present the temperature correction due to the influence of the potential drop over the uninsulated thermojunction, for details see Chapter 4. In this sample run the correction factor is 0.562. Both $T_{w1}$ and $T_{w2}$ were plotted against the angular position so that their integrated mean values are approximately similar. The unshielded values were corrected, however, by comparison of these integrated mean readings. In this manner any errors due to the calculation of the correction factor from the ultraviolet recorder traces was avoided.

A typical plot of temperature distribution is shown in Fig. D.1.
FIG. D.1. TYPICAL TEMPERATURE DATA AT Re₀ = 689 FROM A TUBE ROW OF 6.0 mm DIA. x 0.50 mm SPACING.
## TABLE D.1

TYPICAL EXPERIMENTAL DATA FOR A SINGLE ROW 6.0 mm DIA. x 0.50 mm SPACING

<table>
<thead>
<tr>
<th>( \phi ) (DEG.)</th>
<th>( T_0 ) °C</th>
<th>( T_m ) °C</th>
<th>( T_m ) (Corr.), °C</th>
<th>I.V.</th>
<th>C/C W</th>
<th>h W/m² °K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18.0</td>
<td>50.2</td>
<td>49.3</td>
<td>5.76</td>
<td>-0.230</td>
<td>165</td>
</tr>
<tr>
<td>+20</td>
<td>18.0</td>
<td>50.1</td>
<td>49.5</td>
<td>5.72</td>
<td>-0.263</td>
<td>163</td>
</tr>
<tr>
<td>+40</td>
<td>18.0</td>
<td>50.0</td>
<td>49.0</td>
<td>5.74</td>
<td>-2.253</td>
<td>165</td>
</tr>
<tr>
<td>+60</td>
<td>18.1</td>
<td>49.0</td>
<td>48.9</td>
<td>5.78</td>
<td>+0.126</td>
<td>183</td>
</tr>
<tr>
<td>+80</td>
<td>18.0</td>
<td>47.0</td>
<td>49.9</td>
<td>5.68</td>
<td>+0.631</td>
<td>209</td>
</tr>
<tr>
<td>+90</td>
<td>18.1</td>
<td>47.3</td>
<td>49.8</td>
<td>5.65</td>
<td>+0.669</td>
<td>212</td>
</tr>
<tr>
<td>+100</td>
<td>18.1</td>
<td>49.2</td>
<td>49.5</td>
<td>5.68</td>
<td>+0.163</td>
<td>181</td>
</tr>
<tr>
<td>+110</td>
<td>18.1</td>
<td>50.2</td>
<td>49.4</td>
<td>5.64</td>
<td>+0.146</td>
<td>176</td>
</tr>
<tr>
<td>+120</td>
<td>18.2</td>
<td>50.5</td>
<td>49.7</td>
<td>5.64</td>
<td>-0.290</td>
<td>159</td>
</tr>
<tr>
<td>+140</td>
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<td>50.4</td>
<td>49.5</td>
<td>5.64</td>
<td>-0.300</td>
<td>159</td>
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<td>+160</td>
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<td>50.6</td>
<td>49.3</td>
<td>5.77</td>
<td>-0.375</td>
<td>155</td>
</tr>
<tr>
<td>+180</td>
<td>18.2</td>
<td>50.6</td>
<td>49.1</td>
<td>5.76</td>
<td>-0.388</td>
<td>177</td>
</tr>
<tr>
<td>0</td>
<td>18.5</td>
<td>50.7</td>
<td>49.3</td>
<td>5.63</td>
<td>-0.233</td>
<td>161</td>
</tr>
<tr>
<td>-20</td>
<td>18.5</td>
<td>50.5</td>
<td>49.5</td>
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<td>-0.263</td>
<td>162</td>
</tr>
<tr>
<td>-40</td>
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<td>49.8</td>
<td>49.1</td>
<td>5.66</td>
<td>-0.252</td>
<td>165</td>
</tr>
<tr>
<td>-60</td>
<td>18.4</td>
<td>48.4</td>
<td>48.9</td>
<td>5.66</td>
<td>-0.128</td>
<td>186</td>
</tr>
<tr>
<td>-80</td>
<td>18.4</td>
<td>46.3</td>
<td>49.9</td>
<td>5.65</td>
<td>+0.630</td>
<td>216</td>
</tr>
<tr>
<td>-90</td>
<td>18.5</td>
<td>46.7</td>
<td>49.7</td>
<td>5.73</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>-100</td>
<td>18.4</td>
<td>47.8</td>
<td>49.6</td>
<td>5.71</td>
<td>+0.163</td>
<td>192</td>
</tr>
<tr>
<td>-110</td>
<td>18.4</td>
<td>48.6</td>
<td>49.5</td>
<td>5.63</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>-120</td>
<td>18.4</td>
<td>49.4</td>
<td>49.7</td>
<td>5.62</td>
<td>-0.291</td>
<td>165</td>
</tr>
<tr>
<td>-140</td>
<td>18.4</td>
<td>49.5</td>
<td>49.5</td>
<td>5.66</td>
<td>-0.306</td>
<td>165</td>
</tr>
<tr>
<td>-160</td>
<td>18.3</td>
<td>49.7</td>
<td>49.3</td>
<td>5.73</td>
<td>-0.375</td>
<td>163</td>
</tr>
<tr>
<td>-180</td>
<td>18.2</td>
<td>49.7</td>
<td>49.2</td>
<td>5.76</td>
<td>+0.388</td>
<td>188</td>
</tr>
</tbody>
</table>
A radiative test cylinder dimensions.

\[
\sin \theta = \frac{\bar{x}_r}{2r_0}
\]

\[
x_r = 2r_0 \theta
\]

\[
\bar{x}_0 = 0.59 \text{ mm}
\]

\[
\bar{x}_t = 0.56 \text{ mm}
\]

\[
\bar{x}_r = 1.01 \text{ mm}
\]

\[
\theta = 9.72^\circ
\]

\[
\theta_1 = 21.50^\circ
\]

\[
\theta_2 = 20.80^\circ
\]

\[
x_r = 1.02 \text{ mm}
\]

\[
x_t = 0.62 \text{ mm}
\]

\[
x_2 = 0.59 \text{ mm}
\]

**FIG. D.2 TEST CYLINDER DIMENSIONS.**
APPENDIX-E

NORMAL FOULING FACTORS FOR HEAT TRANSFER EQUIPMENT.
The Tubular Exchanger Manufacturers Association have established standards to define design practices not covered by the ASME Code for Unfired Pressure Vessels. Since the ASME Code is primarily concerned with safe pressure containment and the means of inspection during construction, the contribution of TEMA to the sound mechanical construction has been substantial. In addition, TEMA published a table of fouling factors to assist the designer in preventing the premature fouling of a single item in a process including several items of heat transfer equipment. These fouling factors were intended as a crude guide toward the equalization of cumulative fouling in various fouling streams.

Fouling factors are time-independent according to the TEMA definition. They are not present when the apparatus is placed on stream; yet at some indefinite time in the future, when the apparatus has lost some of its heat transfer performance, the fouling factor is deemed present. The inbetween fouling process was not defined, however, and the fouling factor has shed little knowledge on the nature of fouling. It is significant that an item of equipment which fails to comply with the TEMA notion after a desired period of continuing operation can still contain fouling problems. Typical long term fouling factors proposed by the Association are presented in Table E.1.

**TABLE E.1**

*TEMA* FOULING FACTORS* FOR SOME INDUSTRIAL FLUIDS

<table>
<thead>
<tr>
<th>Fluid Type</th>
<th>Fouling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organic Vapours</td>
<td>2.84 x 10^-3</td>
</tr>
<tr>
<td>Steam (non-oil bearing)</td>
<td>2.84 x 10^-3</td>
</tr>
<tr>
<td>Alcohol vapours</td>
<td>2.84 x 10^-3</td>
</tr>
<tr>
<td>Steam, exhaust (oil-bearing from reciprocating engines)</td>
<td>5.68 x 10^-3</td>
</tr>
<tr>
<td>Refrigerating vapours (condensing from reciprocating compressors)</td>
<td>1.14 x 10^-2</td>
</tr>
<tr>
<td>Air</td>
<td>1.14 x 10^-2</td>
</tr>
<tr>
<td>Coke oven gas and other manufactured gas Diesel</td>
<td>5.68 x 10^-2</td>
</tr>
<tr>
<td>Diesel engine-exhaust gas</td>
<td>5.68 x 10^-2</td>
</tr>
<tr>
<td>Fouling factors - industrial liquids</td>
<td>5.68 x 10^-3</td>
</tr>
<tr>
<td>Refrigerating liquids, heating, cooling or evaporating</td>
<td>5.68 x 10^-3</td>
</tr>
<tr>
<td>Brine (cooling)</td>
<td>5.68 x 10^-3</td>
</tr>
</tbody>
</table>

* Fouling factor = \( \frac{1}{F_0}, \frac{m^2 \cdot K}{V} \)
APPENDIX-F  THE INFLUENCE OF INTERNAL HEAT GENERATION ON THE FIN EFFECTIVENESS.
APPENDIX - F

THE INFLUENCE OF INTERNAL HEAT GENERATION
ON THE FIN EFFECTIVENESS (REF. 5)

Consider a longitudinal fin of arbitrary profile and let the length coordinate originate at the fin base. The fin profile is confined by two symmetrical curves \( y = f_1(x) \) and \( y = f_2(x) \). The fin cross section for a unit fin depth is \( a = f_2(x) - 2f_1(x) \) and the temperature excess over the surrounding fluid at any point on the fin surface is \( T_f - T_d \), where \( T_f \) is the temperature on the fin and \( T_d \) is the ambient temperature. The differential equation for the fin temperature profile is formulated from a steady-state heat balance over a differential element of height \( dx \). The difference between heat entering the element by conduction at \( x \) and that leaving the element by conduction at \( x + dx \) is

\[
d = 2 \frac{d}{dx} \left[ K f_1(x) \frac{dT}{dx} \right] dx ;
\]

the convective heat loss is

\[
2h(T - T_d) dx ;
\]

and the heat generated within the element is

\[
2 q_f f_1(x) dx
\]

where \( q_f \) is the rate of internal heat generated.

This leads to the generalized differential equation

\[
\frac{d}{dx} \left[ K f_1(x) \frac{d\theta}{dx} \right] - h \theta + q_f f_1(x) = 0
\]  

(1)

where \( d\theta = dT \).

For the rectangular profile, \( f_1(x) = b/2 \) and with constant thermal conductivity Eq. (1) becomes

\[
\frac{d^2 \theta}{dx^2} - m^2 \theta = - \frac{q_f}{K}
\]

(2)

where \( m = \sqrt{2h/K} \).

This first order, nonhomogeneous equation has a complementary function

\[
\theta_1 = B_1 e^{mx} + B_2 e^{-mx}
\]

and a particular integral

\[
\theta_2 = \frac{q_f b}{2h}
\]

The general solution of Equation (2) is

\[
\theta = \theta_1 + \theta_2 = B_1 e^{mx} + B_2 e^{-mx} + \frac{q_f b}{2h}
\]

(3)
where the arbitrary constants $B_1$ and $B_2$ are evaluated from the boundary conditions at the fin base and the fin tip where heat losses are negligible. Thus at

\[ x = 0 \quad \Rightarrow \quad -Kb \frac{d\theta}{dx} \bigg|_{x=0} = \frac{q}{b} \]

\[ x = l \quad \Rightarrow \quad \frac{d\theta}{dx} \bigg|_{x=1} = 0 \]

If these boundary conditions are used for the solution of Eq. (2) then:

\[ \theta = \frac{q}{2hKb} \left( \frac{\cosh mx}{\tanh mx} - \frac{q_f b}{2h} \right) \quad (4) \]

It is convenient to express this equation in terms of three dimensionless parameters which pertain to the longitudinal fin of rectangular profile.

The ratio of heat passing through the fin base by conduction to the heat lost from the surface if no fin were present sometimes used and is called the Removal number and thus can be expressed per unit length as:

\[ N_R = \frac{q}{h b \theta_b} \quad (5) \]

The value of $N_R$ must exceed unity for the fin to be worthwhile. On the other hand if $N_R < 1$ the fin acts as an insulator.

The generation number is the ratio of the heat generated to the heat dissipated by the fin if all the fin is at the base temperature. For two fin faces and considering unit fin length we have:

\[ N_G = \frac{q_f b L}{2h \theta_b} = \frac{q_f b}{2h \theta} \quad (6) \]

The generation number is related to the fin efficiency. Multiplying the numerator and denominator of Eq. (6) by $2h \int_0^L \theta dx$ yields

\[ N_G = \frac{q_f b L}{2h_0 \int_0^L \theta dx} \cdot \frac{2h_0 \int_0^L \theta dx}{2h_0 \int_0^L \theta dx} \quad (7) \]

The first term in Eq. 7 is the ratio of the heat generated to the heat actually dissipated by the fin. The second term is the fin efficiency when no heat generation is present. Thus Eq. 7 can be written as

\[ N_G = \frac{q_f b L}{2h_0 \int_0^L \theta dx} \quad (8) \]

It can be seen that the generation number is a measure of the fin 'inefficiency' due to heat generation. When $N_G = 0$ there is no heat generation. When $N_G = 1$ the fin is generating heat precisely as fast as it can be removed by convection. When $N_G > 1$ heat flows into the fin base and, as given in Eq. 6 $N_R$ is negative.
The Biot number $Bi$ measures the ratio of the surface conductance to the internal conductance in the fin. It is usually written employing the fin half thickness $b/2$ thus

$$Bi = \frac{hb}{2K}$$

and this is sometimes used as a useful criterion. If the term preceding the parentheses is adjusted we have

$$\frac{q}{b} \left( \frac{b}{2} \right) = \frac{q_b \theta_0 (h b)^{\frac{1}{2}}}{h b \theta_0 (2K)^{\frac{1}{2}}} = N_R (Bi)^{\frac{1}{2}} \theta_0$$

and the last term becomes $NG \theta$. Thus

$$\theta = \theta_0 N_R (bi)^{\frac{1}{2}} \left( \frac{\cosh \frac{mx}{\tanh mx}}{\sinh mx} \right) + \theta_0 NG$$

and at $x = 0$, where $\theta = \theta_0$. Eq. 10 reduces to

$$N_R Bi^{\frac{1}{2}} = (1 - NG) \tanh \frac{mb}{2}$$

Fig. F.1 is a plot of Eq. 11. It can be seen, as $mb$ increases, the value of $N_R Bi^{\frac{1}{2}}$ becomes asymptotic. At $NG = 0$ the value of $N_R Bi^{\frac{1}{2}}$ approaches unity, and for $NG = 1$ the value of $N_R Bi^{\frac{1}{2}}$ is always zero. The latter can be deduced from Eq. 11 and represents the case where the heat generated by the fin just equals the heat convected through its surface.

Fig. F.2 is a typical plot of the temperature ratio $\theta/\theta_0$ for a copper fin 50.8 mm height and 1.6 mm thickness, with $h = 234$ w/m °K. This figure is based on Eq. 10 and shows that the edge temperature increases significantly as the heat generation increases.
FIG. F.1. PLOT OF $N_R (B_i)^{1/2}$ FOR VARIOUS VALUES OF GENERATION NUMBER $N_g$.

(from Kern and Kraus, Ref. 5)
**FIG. F.2.** TEMPERATURE DISTRIBUTION IN LONGITUDINAL FIN OF RECTANGULAR PROFILE SHOWING EFFECT OF HEAT GENERATION. COPPER FIN IS 1.6mm THICKNESS, 50.8mm HEIGHT, WITH $h = 284 \text{ W/m}^2\text{°C}$. 

*(FROM KERN AND KRAUS (Ref. 5))"
APPENDIX - G

FIN EFFICIENCY.
Heat is transferred from the base to a finned surface by conduction through the fins and subsequent convection to the surrounding fluid. There is thus a temperature gradient along the fins. The fin efficiency may be defined as the ratio of the actual heat transferred to that which would be transferred with an infinitely conducting fin, i.e. one whose temperature is at the base temperature.

\[ \gamma = \frac{\int_0^s h \theta \, ds}{(T - T_a) \, h \, dA} \]

If \( h \) is constant, then

\[ \gamma = \frac{h \int_0^s \theta \, ds}{h (T_s - T_a)} = \frac{\theta_m}{T_s - T_a} \]

where,

- \( \theta_m \) is the mean temperature difference between the finned surface and the fluid, °C
- \( \theta \) is the local temperature difference between the gas and the finned surface, °C
- \( T_s \) is the mean outside tube wall temperature, °C
- \( T_a \) is the bulk fluid temperature °C
- \( s \) is the side volume per unit length, m²

For a uniform heat transfer coefficient, the rate of heat transfer between a gas and a unit length of finned tube may be expressed by:

\[ q_t = h_t S \, (T_s - T_a) \quad (3) \]

The area of the surface can be calculated and the temperature difference measure but \( h \) and \( \theta \) cannot be found by any simple means. To refer the mean heat transfer coefficient to some fluid property other than that of the experiment, it is necessary to evaluate the fin efficiency. The usual Gardner (Ref. 134) expression for calculating fin efficiency relies upon a mathematical solution of the heat flow problem associated with the fin. Thus for the simple rectangular fin of thickness \( b \) and of finite length \( l \),

\[ \gamma = \frac{\tanh ml}{ml} \quad (4) \]

where

\[ m = \sqrt{\frac{2 \, h}{k \, b}} \quad (5) \]

\( h \) = film heat transfer coefficient \( W/m² \, °C \)
\( k \) = thermal conductivity of the fin material, \( W/°C \, m \)

(see Table G.1)
Gardner has generalized the relationships for extended-surface heat exchanger systems to include not only the simple rectangular case but also several other shapes. These results are useful in design and are thus presented in Figs. G.1 to G.4. Most of this information is available in more detail in heat transfer textbooks. However, it is worth noting that several attempts were made experimentally to verify the validity of Gardner's expression (as already discussed in Chapter 5 of this present thesis) for the case of crossflowing finned tubes. Of these attempts, Lymer and Ridal (Ref. 116) assume that for the longitudinal fins of rectangular profile, the following simplified expression is valid:

$$\gamma = \frac{1}{1 + \frac{\pi^2}{3} - \frac{N_4}{45} + \frac{2N_5}{945}}$$  \hspace{1cm} (6)$$

where \(N = m\) thus as a first approximation

$$\gamma = \frac{1}{1 + \frac{2D^2h}{3bK}}$$  \hspace{1cm} (7)$$

and similarly for an

$$\gamma = \frac{1}{1 + B \frac{2D^2h}{3bK}}$$  \hspace{1cm} (8)$$

where the factor \(B\) requires experimental determination, then

$$h \gamma = \frac{\frac{h}{1 + B \frac{2D^2h}{3bK}}}{\frac{Ka}{h} \gamma} = \frac{ka}{hd} + B \cdot \frac{Ka}{K} \frac{2D^2}{3bd}$$ \hspace{1cm} (9)$$

and,

$$\frac{Ka}{hd} \gamma = \frac{ka}{hd} + B \cdot \frac{Ka}{K} \frac{2D^2}{3bd}$$ \hspace{1cm} (10)$$

where \(d\) is the finned tube diameter and \(Ka\) is the gas thermal conductivity. If \(Ka/hd\) is plotted against \(Ka/K \cdot 2D^2/3bd\), the slope of the line is \(B\), and the intercept on the ordinate is \(K_a/3bd\) hd. Solution of this equation for a specific tube geometry and Reynolds number enables the prediction of the heat transfer rate for any gas and metal conductivity. Heat transfers from similar finned tubes of different thermal conductivity were examined and the results are compared with the Gardner fin efficiency as shown in Fig. G.5. \(K_a\) and \(h\) were evaluated at the arithmetic mean of the tube wall and air temperature. It can be seen in the figure that there is a close agreement for the higher conductivity materials whereas a considerable difference is clearly evident at lower values of thermal conductivity. It is thus preferable to carry out experimental tests with high conductivity materials (such as copper) so that the extrapolation to a material of infinite conductivity is relatively easy.
**Fig. G.3 Efficiency of Annular Fins of Constant Thickness**

\[ \eta = \frac{2}{\frac{1}{h}(k_{in} - \frac{d}{2}k_{out})} \]

- \( h = \frac{u}{l} \)
- \( k_{in} = \frac{\sqrt{h_{in}}}{u_{in}} \)
- \( u_{in} = \frac{\sqrt{h_{in}}}{l} \)
- \( \omega = u_{in}(r_{i}/r_{o}) \)
- \( r_{b} = b/2 \)

**Fig. G.4 Efficiency of Annular Fins With Constant Metal Area for Heat Flow**

\[ \eta = \frac{4}{3\left[\frac{1}{h_{in}} + \frac{d}{2}k_{out}\right]} \]

- \( h_{in} = \frac{\sqrt{h_{in}}}{u_{in}} \)
- \( u_{in} = \frac{\sqrt{h_{in}}}{l} \)
- \( \omega = u_{in}(r_{i}/r_{o}) \)
- \( r_{b} = b/2 \)
TABLE G.1

COMPARISON OF FIN MATERIALS

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity K, W/m. °K</th>
<th>Density, kg/m³</th>
<th>/K, kg K/W m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>383</td>
<td>8970</td>
<td>2.34</td>
</tr>
<tr>
<td>Aluminum, pure</td>
<td>225</td>
<td>2722</td>
<td>1.21</td>
</tr>
<tr>
<td>Alloy</td>
<td>156</td>
<td>2670</td>
<td>1.71</td>
</tr>
<tr>
<td>Magnesium, pure</td>
<td>173</td>
<td>1761</td>
<td>1.02</td>
</tr>
<tr>
<td>Steel</td>
<td>55</td>
<td>7850</td>
<td>14.26</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>14</td>
<td>7850</td>
<td>56.0</td>
</tr>
</tbody>
</table>

FIG. G.5. COMPARISON OF GARDNER FIN EFFICIENCY WITH Experimentally Derived Value.