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Optimal Design of Planar Parallel Manipulators 3RRR Through Lower Energy Consumption

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Abstract

In most existing studies, the solutions of planar parallel manipulators are restricted to a feasible region of solution. This research provides an optimal solution in link dimensions of planar parallel manipulators to a defined trajectory and structure of the links, minimizing the mechanical energy of the manipulator. An algorithm will be obtained that allows adequate dimensioning of the manipulator for a specific task, by means of a passive reconfiguration. With this method most of the energy is used by the manipulator to execute a task, not for the manipulator's movement. The process is illustrated with an example.

Keywords:

Parallel manipulator, Parallel and serial singularities, Energy, Optimization

1 INTRODUCTION

In contrast to serial manipulators, the parallel manipulators present a higher complexity of analysis, since they have more complex closed kinematic chains, correlated to give a solution to the system. Not only serial singularities (*q*) appear, in which the geometry does not allow to reach certain points when the movement is blocked once the links are aligned [1], but also parallel singularities (θ). Those refer to points in which rigidity of the system is lost, as well as control of movement, since the number of degrees of freedom of the system changes [2, 3, 4]. Each of these singularities (*q* or θ) can cause the loss of control over the desired movement, be it separately or when both appear simultaneously. The feasible region is not only defined by the task area of the manipulator, but the orientations and singularities within the task area have also to be taken into account, to determine if the point is a part of the above mentioned region [5]. For different models of serial manipulators conditions have been defined to determine the feasible regions [4].

Some studies related to parallel manipulators focus on defining the feasible regions according to the Jacobian matrix of the system [6,7]. Serial and parallel matrix are generated which, when reduced in range, are at a singular point. One way to plan trajectories with this type of models is using redundant systems, leaving a free variable as the orientation of the end effector link, to be able to avoid the singularity points for a defined task trajectory [7].

The parallel planar manipulator 3RRR consists of a triangular mobile plate connected by three arms to a base. Every arm has three rotational joints, with parallel axes among them and among the arms. This defines the planar movement of the triangular plate with change in the task point (center of the plate) and its orientation. The motorized joints are those that are connected to an inertial base, one in every arm. The triangular plate in movement as well as the base are equilateral triangles (Figure 1).

The optimum design depends on what is considered to be optimum. If the price is the most important requirement, the optimum will be minor cost. Or minor infrastructure for construction or a better weight-strength relation is the important requirement. The suitable solution depends on what is needed for a specific problem [4, 8].

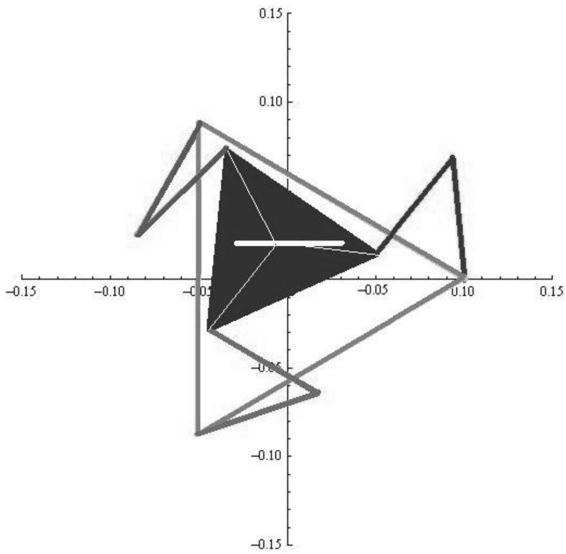

Figure 1. Parallel manipulator 3RRR

In this paper, the lengths of the links are obtained for a parallel manipulator, and the one that uses the least energy for its task is defined as optimum. This method uses an algorithm which uses the lengths of the manipulator to determine the objective function. This solution not only must comply with the criterion of less energy, it also must be within the feasible region, restricted by the singularities of the system.

2 THEORETICAL MODEL

In the optimization process the mechanical energy of the model is the objective function to consider. This function depends on different variables, like the dimensions of the links, speed, masses and task at hand. A pre-established task is to be executed, with position and orientation of the movable plate predetermined. This work trajectory is established and cannot be changed, be it because it complies with a specific task or because it is a trajectory optimized by other methods. Examples would be specific painting or welding processes, o in microchip assembling, where there is an initial and final position to be complied with. In this paper the dimensions of the manipulator's links are given as variables to be optimized, leaving the whole system based on them.

For the optimization process and the computing of the examples described Mathematica 6 software will be employed, which allows the symbolic use of the equations, as well as the implementation of the search method of Hooke and Jeeves (Simplex Method).

The following steps will have to be taken for this method:

1. Define the energy model of the manipulator.
2. Define the dynamic characteristics of the manipulator.
3. Analyze the task trajectory.
4. Define the energy function with the design variables.
5. Define the criteria of the feasible region for the model.
6. Optimize the energy function of the manipulator with the restrictions of the model.

2.1 Energy model

The model is based on the concepts of classic thermodynamics. For this case, the moving links of the manipulator are defined as the system. It is considered to be a closed system with boundaries around the movable links and the energy flows between the system and its surroundings will be defined [9].

The energy depends on the current conditions of the system, such as speed and position of the center of mass of each of the links that are taken into account. The following equation includes each and every task and energies of the system:

$$\delta Q + \delta W^{ext} + \delta W^{int} = dE^{ext} + dE^{int} \tag{1}$$

Of the interactions between the system and its surroundings, only those of mechanic origin are to be considered. The actuators are considered idealized, that is, heat flows (δQ) or internal energy changes of the link are not considered, since it is modeled as rigid body, reason why the expansion or deformation tasks are eliminated as well (δW^{int}). By way of simplifying, the following equation is obtained:

$$\delta W^{ext} = dEP_{Grav} + dEP_{Electr} + dEP_{Magnet} + dEC_{Lin} + dEC_{Rot} \tag{2}$$

Where only mechanical interactions are considered, reason why the magnetic and electric interactions are also eliminated from the equation. The energy transformations that may exist within the actuators are not to be taken into consideration.

From this model the following flows are obtained:

1. Energy of the inertial base to the manipulator,
2. Energy of the manipulator to the task to be performed,
3. Energy dissipated in the actuators in form of heat,
4. Energy used for the movement of the manipulator.

Of those flows, the one to be optimized is the energy required for the movement of the manipulator, so the energy introduced into the system is used in the execution of the main task. To simplify, in this case ideal actuators (that do not dissipate energy) are used as a first approximation to reality. Thereby the following schematic results (Figure 2).

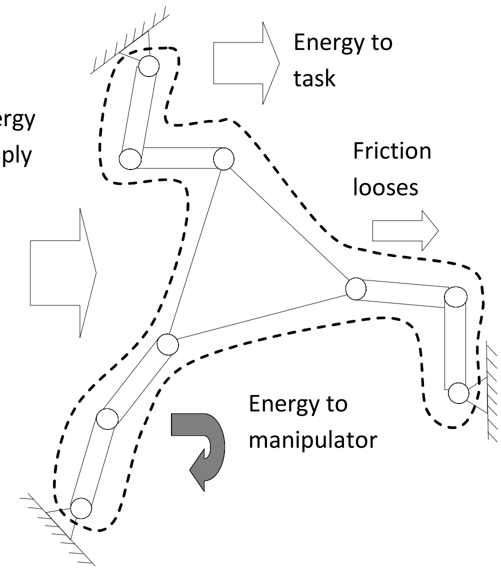

Figure 2. Energy flow

It is important to point out the consideration taken into account for the analysis of the system. The links are rigid bodies, reason why the deformation and expansion effects will not be considered. The actuators involved are ideal, without friction losses. With these simplifications the model only has one input, the energy provided to the manipulator, and two outputs, the energy provided for the task and the movement of the manipulator. This last one would be the one we look to optimize.

2.1 Dynamics of the manipulator

The equations of the mechanical energy of the proposed model are developed. The arms of the manipulator are defined as constant cross-section links (Figure 3), whereas the movable plate is considered of constant thickness, both of materials of uniform density (ρ). This density is given as data, in the case of the bar as density by length unit, and in the case of the triangular plate as density by surface unit.

$$m = \rho * Li$$

$$I = \frac{\rho * Li}{12}$$

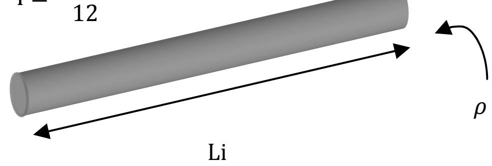

Figure 3. Mass and inertia of link i

Knowing the masses and inertias of the links in movement of the manipulator, the energy associated to each one of them can be defined as follows (Eqs. 3).

$$ECLineal_{esl\,i} = \frac{m_i {v_i}^2}{2} \tag{3a}$$

$$\text{ECAngular}_{\text{esl i}} = \frac{I_i \omega_i^2}{2} \quad (3b)$$

$$\text{EP}_{\text{esl i}} = m_i g h_i \quad (3c)$$

$$E_{\text{esl i}} = \text{ECLineal}_{\text{esl i}} + \text{ECAngular}_{\text{esl i}} + \text{EP}_{\text{esl i}} \quad (3d)$$

For each of the binary links (arms) and for the ternary link (plate) similar equations are generated, with the speeds being those of its mass centers and the angular velocities of each link.

In order to determine the speeds of the mass centers, the inverse kinematics of the model is used. For each orientation of the movable plate, 8 possible solutions or operation modes are generated (Figure 4), depending on the way the plate is moving [6]. The equation generated involves the dimensions of design in the speed equations. This way, as much the mechanical properties of mass and inertia of each link as well as the speeds of each link element is function of the dimensions of design.

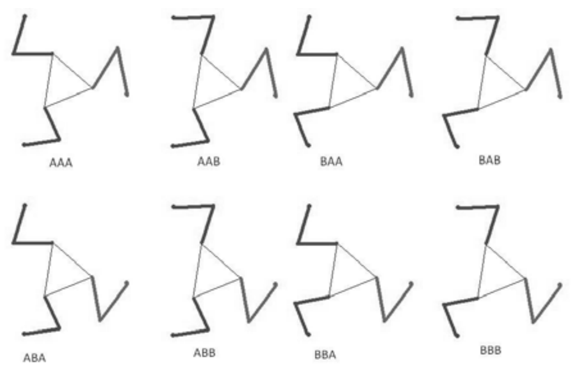

Figure 4. 3RRR modes of operation

2.3 Task trajectory

The task trajectory involves not only the position, but also the orientation of the movable platform. For the analysis, a defined trajectory, which can not be changed, is used to optimize the elements of the manipulator. The variables of the trajectory are $(Xt, Yt, \varphi t)$. By means of inverse kinematics equations are established the angles of each link, not only based on the trajectory, but also on the lengths of the links to be optimized.

The task trajectory must be within the feasible region, that is to say, where the manipulator is without singularities. Within this feasible region, depending on the combination of angles and lengths of the link, the orbits leading to singularities may be found [2, 3]. These singularities may be of Type 1 or serial, of Type 2 or parallel, or Type 3 when both singularities appear simultaneously. The model used to analyze the type of singularity is based on the Jacobian matrix associated with the manipulator (Eqs. 4), [1].

$$J_\theta t = J_q \dot{\theta} \quad (4a)$$

$$t = [\dot{X}t, \dot{Y}t, \dot{\varphi}t] \quad (4b)$$

$$\dot{\theta} = [\dot{\theta}_{11}, \dot{\theta}_{12}, \dot{\theta}_{13}] \quad (4c)$$

Where $\dot{X}t, \dot{Y}t, \dot{\varphi}t$ are the reasons for the change of the trajectory with respect to time, and $\dot{\theta}_{11}, \dot{\theta}_{12}, \dot{\theta}_{13}$ are the angular velocities of the active links of the arms of the manipulator. The relation between those would be the one that determines the conditions of singularity. When the range of the parallel and serial Jacobian matrix (J_θ, J_q) is zero, the system is in singularity.

Therefore, in order to evaluate if the point analyzed is able to generate a solution with less energy, it is also necessary to verify that it is within the feasible region, by means of the behavior of the Jacobian matrix of the model, as well as for the whole trajectory.

2.4 Objective function, restrictions and optimization

The objective function is the energy associated to the links of the manipulator, based on the lengths of links and the trajectory of the task (Eqs. 5). The restrictions that the model presents are introduced through the determinant of the Jacobian matrix, which are also based on the lengths of design and the trajectory of the task. With the previous information the optimization begins.

$$\text{Eesl} = \sum_{i=1}^{n} E_{\text{esl i}} \quad (5a)$$

$$\text{Eesl} = f[\text{Xt}, \text{Yt}, \text{L1}, \text{L2}, \dots, \text{Li}] \quad (5b)$$

$$\text{Etot} = \sum_{t0}^{tf} \text{Eesl} \quad (5c)$$

$$\text{Etot} = f[\text{Xt(t)}, \text{Yt(t)}, \varphi(t), \text{L1}, \text{L2}, \text{L3}] \quad (5d)$$

As data we have the trajectory and direction of the central point of the movable triangular plate, the lengths of the links being free variables. For this type of manipulators the symmetry has great advantages for the description of different trajectories, choosing the primary links with the same length in each one of the arms, same as for the secondary links. For the movable triangular plate, a triangle circumscribed to a circumference is assumed whose radio would be the design dimension of the plate. As optimization method the Simplex method is used, through the following steps:

1. For each set of lengths of the manipulator, a point whose coordinates are $(L1, L2, L3)$ is considered, and the energy of the manipulator is obtained through the whole trajectory. For this, $n + 1$ initial points are chosen, being n the number of dimensions to optimize, in this case, $n = 3$.
2. For each point it has to be evaluated if the criterion of non-reduction of the margin of the associated Jacobian determinants is fulfilled the whole time. If a specific point does not comply with this requirement, another one is chosen.
3. The point with the highest value of energy (objective function) is eliminated, and the following point is generated, through the reflection of the point with greater value with respect to the origin of the subspace generated by other points (Figure 5)
4. Another iteration is done, until the criteria of convergence necessary for the method are met.

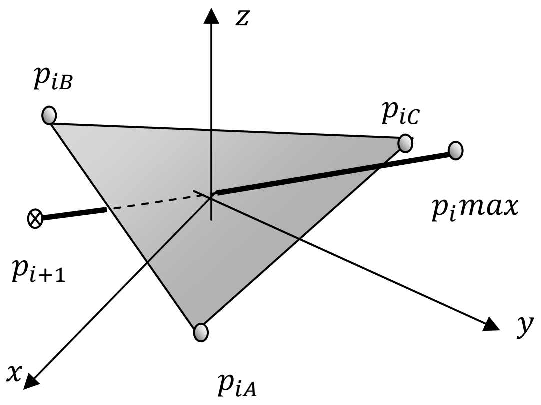

Figure 5. 3D simplex method

This analysis can be complex, depending on the type of proposed trajectory or for other types of manipulators and the complexity of the inverse kinematics of the manipulator. In this case, having 8 options of operation, it will be necessary to review each one, since it is possible that some do not fulfill the criteria of the feasible region but others do. The search for the optimal point will be as much in the energy as in the non-singularity of each of the work modes of the system.

3 EXAMPLE

In this case a defined trajectory will be considered, initiating with proposed dimensions and optimizing the model, according to the behavior of the solutions that are generated by the method up to the optimal point. The mentioned simplifications are taken from ideal actuators, without losses through friction or heat. The trajectory and direction proposed of the movable plate are given by the equations (Eqs. 6) that generate a no simple desired task and direction graphs (Figure 6):

$$\mathrm{xt} = 0.3\ \mathrm{Sin}(2\mathrm{t}) * \mathrm{Sin}(4\mathrm{t}) \tag{6a}$$

$$\mathrm{yt} = 0.3\ \mathrm{Sin}(2\mathrm{t}) * \mathrm{Sin}(4\mathrm{t}) \tag{6b}$$

$$\varphi\mathrm{t} = \mathrm{Sin}\ (4\pi\mathrm{t}) \tag{6c}$$

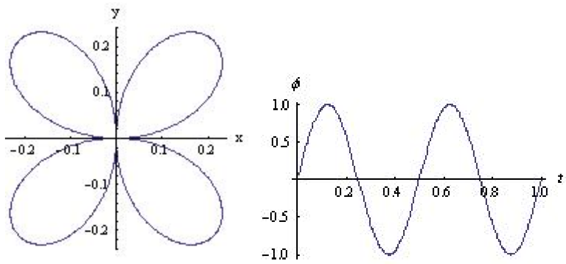

Figure 6. Task trajectory and orientation

The movement of the links is made on planes perpendicular to the field of gravitation, in order to eliminate the effects of potential energy.

With the initial dimensions for a point in the optimization process, which will be denominated original, the following results for this model are achieved, with the corresponding graphs of energy and singularities (Figure 7).

$\mathrm{l1} = 1.0\ [\mathrm{m}]$

$\mathrm{l2} = 1.0\ [\mathrm{m}]$

$\mathrm{l3} = 0.4\ [\mathrm{m}]$

$\mathrm{ETot_{BAA}} = 6855.77\ [\mathrm{J}]$

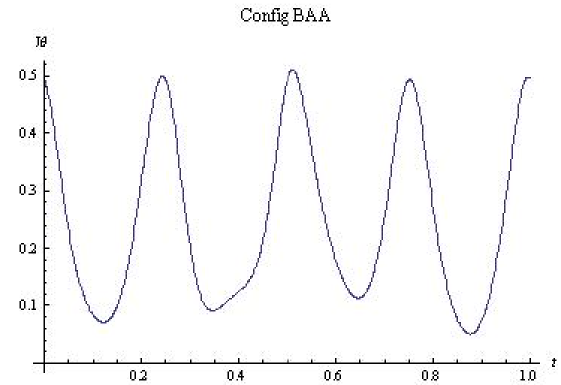

Figure 7a. Original serial singularity

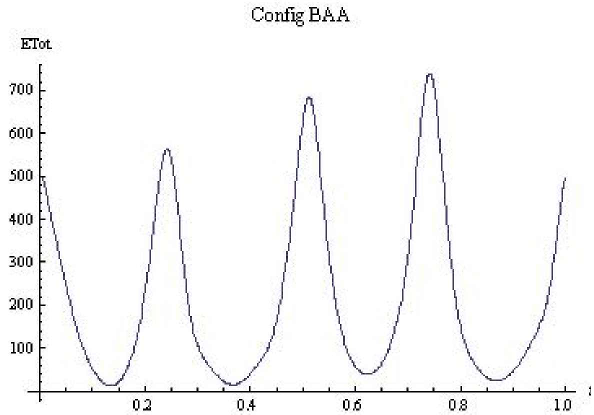

Figure 7b. Original energy

For this case, the best configuration is BAA, which has the lower energy and neither serial nor parallel singularities apply. The amount of energy against which to compare the results of the following generated points solution is noted.

Based on the first iterations, models of smaller dimensions are generated, but not all comply with the criteria of singularities, with parallel singularities (Figure 8) and serial singularities (Figure 9), where the behavior of the dimensions and the objective function during the search are shown. For the case of serial singularity, the following dimensions apply in ABB mode:

$\mathrm{l1} = 0.519\ [\mathrm{m}]$

$\mathrm{l2} = 0.489\ [\mathrm{m}]$

$\mathrm{l3} = 0.197\ [\mathrm{m}]$

$\mathrm{ETot_{ABB}} = 2249.23\ [\mathrm{J}]$

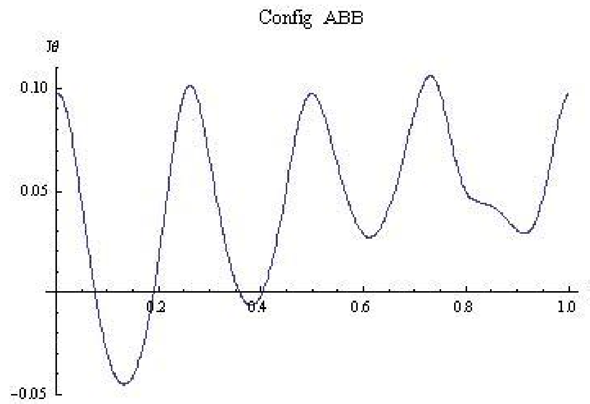

Figure 8a. Parallel Jacobian singularity

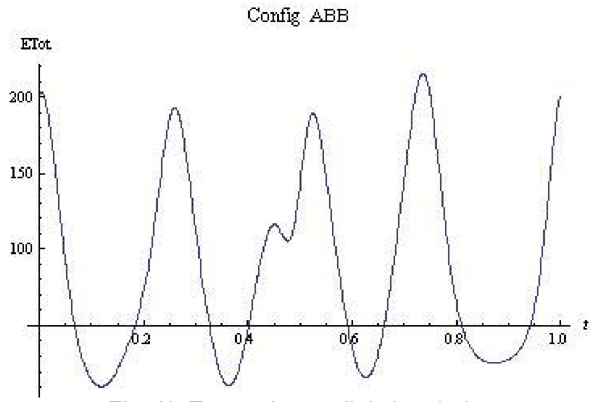

Fig. 8b Energy in parallel singularity

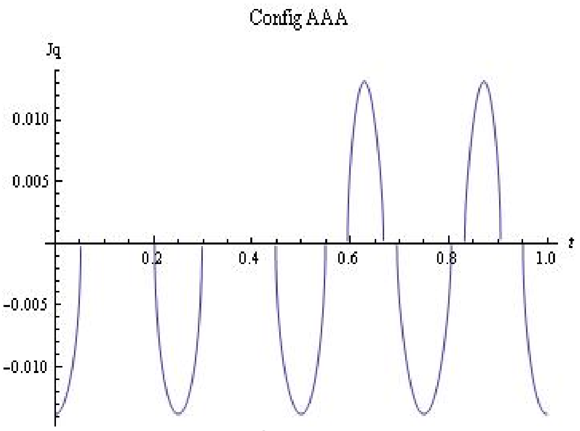

Figure 9a. Serial singularity

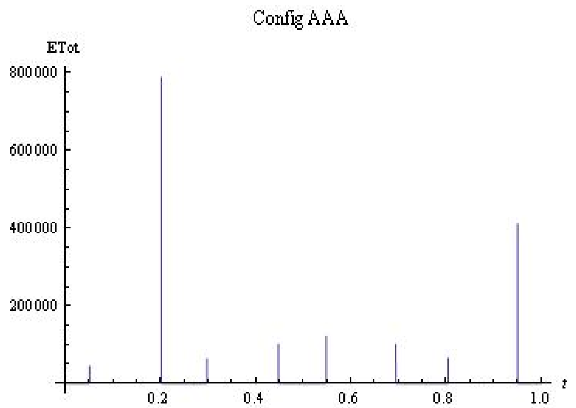

Figure 9b. Energy in serial singularity

When serial singularities exist, the points are eliminated when the generate infinite energies, as shown in the case of the following combination of dimensions:

$l1 = 0.565\ [\mathrm{m}]$

$l2 = 0.547\ [\mathrm{m}]$

$l3 = 0.2\ [\mathrm{m}]$

For iteration 14, the system presents the necessary criteria of convergence, generating the following solution, in which the diminution of energy can be appreciated (Figure 10), as well as the nonexistence of singularities in the development of its trajectory (the graphics of the Jacobian determinant is never zero for any point):

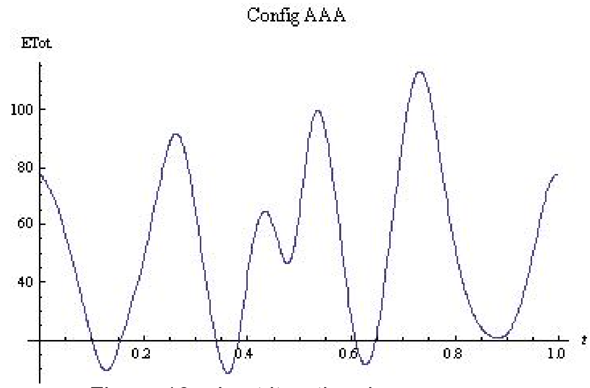

Figure 10a. Last iteration, lower energy

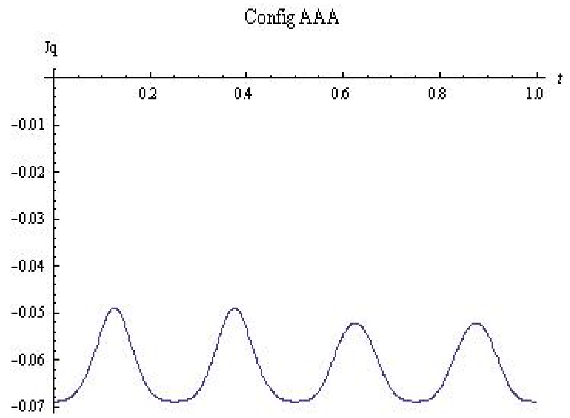

Figure 10b. Serial Jacobian in last iteration

$l1 = 0.652\ [\mathrm{m}]$

$l2 = 0.653\ [\mathrm{m}]$

$l3 = 0.2\ [\mathrm{m}]$

$\mathrm{Etot}_{\mathrm{AAA}} = 1256.2\ [\mathrm{J}]$

In this case, the energy required by the initial model is 6855,77 [J], optimized at a value of 1256,2 [J], which represent 81,68% savings.

4 CONCLUSIONS

When applying this method to parallel manipulators:

1. The model with smaller consumption of energy according to the mentioned criteria is obtained.
2. The model can be reconfigured in order to use the least energy possible for the required process.
3. The solution, in spite of being complex equations, does not require a lot of computer time.

A disadvantage would be that at the moment of computation of the solutions, where the behavior of each of the operation modes will have to be compared, since some of them may fall into singularity, whereas others may be a solution. The decisive factor in models that display the same amount of energy (the speeds and the masses of the links are similar) is then the one that does not fall within a singular configuration, but this may improve when analyzing each one of the operation modes separately, and comparing the final result with the other operation modes, that could have different dimensions. With regard to serial singularities, they will be eliminated as they would need infinite energy values when the system is blocked, however, for the analysis of parallel singularities, the use of the Jacobian matrix is indispensable.

The advantage of this radical method is that it leads quickly to a solution. In the example of a manipulator 3RRR, it takes only 5 minutes with an Intel in 1.4 GHz processor. As the complexity of the system increases, it will be increasingly harder to find a solution, since then it would depend on the capacity of the used optimization method, as well as on the complexity of the equations of the proposed inverse kinematics, as is the case with space parallel manipulators. However, saving energy in a task of multiple repetitions, as could be the assembly of a circuit or executing laser cuts, is an even greater advantage. The benefits arise when the least amount of energy is consumed for a certain process.

5 ACKNOWLEDGMENTS

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