Covariance Structural Models of the Relationship between the Design and Customer Domains

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This paper addresses the problem of modelling and mapping of difficult to quantify customer needs to technical requirements and subsequently to design parameters. Proposed is a covariance structural equation model, which incorporates a confirmatory and a structural component. The former is used for the decomposition of the qualitative customer needs, modelled as latent variables, onto a generally larger number of measurable technical requirements. The structural component maps the technical requirements to design parameters. The concept is illustrated by an example. The model is confined to the linear dependence between the variables, but in general the approach can handle a number of non-linear relations through variable transformation. The conclusion is that the proposed synthetic procedure, named SEMDES (Structural Equation Models for the Design of Engineering Systems) represents a sufficiently rich and generic structure capable of bridging the gap between the customer and the design domains.

Keywords: Covariance Structural Equation Models; Axiomatic Design; Quality Engineering; Requirements Engineering

1. Introduction

Established standards for the engineering of systems such as ANSI/EIA 632 and ISO/IEC 15288 support a seamless process of converting customer needs into systems/technical requirements, which are subsequently transformed into logical representations and finally into physical solution representations. The process is applied recursively to subsequent, lower levels of the product decomposition. Integration, test and verification follow from the lower to the higher levels of granularity. Axiomatic Design (Suh 1990 and 2001) follows to an extent this philosophy through a process called zigzagging. However, neither Axiomatic Design (AD) nor the standards prescribe a process for customer needs identification – traditionally this has been the territory of market analysis. Consistency and communication between the customer and the design domains is therefore essential in the definition of adequate requirements and the identification of possible variability of the already defined requirements. Variability of design characteristics can contribute to the complexity of the design (El-Haik and Yang, 1999) which, in turn, can affect technical risk and costs. It is proposed in this work that in addition to variability, any statistical correlations of requirements and design parameters have to be taken into consideration when exploring a particular design in order to identify performance sensitivities.

The broader aim of the paper is to demonstrate that a proposed combination of covariance structure models and latent variable models, called SEMDES, represents an underlying generic structure which bridges the gap between the customer and the design domains. Consequently SEMDES (Structure Equation Models for the Design of Engineering Systems) should be able to aid the definition of customer needs, their subsequent

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mapping to functional requirements, and then mapping the latter to design parameters. SEMDES is also intended to provide a richer model of the design domain by accounting for the effects of variability and correlation of the functional requirements (FRs), possible (historic) correlation between FRs and Design Parameters (DPs), and also for any correlation or causal relations between the DPs.

The following two sections (2 and 3) introduce the basic postulates, definitions and notation of Axiomatic Design and the Covariance Structure Models, both of which are extensively referred to in the main body of the paper. These two sections are based exclusively on Suh (1990, 2001) and Long (1983- a, b), respectively. SEMDES is introduced in Section 4 with the help of an example derived from an Axiomatic Design case study. A discussion on the scope, limitations and future work on SEMDES is presented in section 5. Finally conclusions are drawn in section 6.

2. Axiomatic design

The underlying hypothesis of the Axiomatic Design (AD) theory (Suh, 1990 and 2001) is that there exist fundamental principles that govern good design practice. The main distinguishable components of AD are domains, hierarchies, and design axioms. The foundation axioms are:

Axiom 1. Maintain the independence of the functional requirements

Axiom 2. Minimise the information content of the design.

According to the AD theory, the design process takes place in four domains (figure1): Customer, Functional, Physical and Process. Through a series of iterations, the design process converts customer’s needs (CNs) into Functional Requirements (FRs) and constraints (Cs), which in turn are mapped to Design Parameters (DPs). DPs determine (but also can be affected by) the manufacturing or Process Variables (PVs). The decomposition process starts with the decomposition of the overall functional requirement. Before decomposing a FR at a particular hierarchical level in the functional domain, the corresponding DP must be determined for the same hierarchical level in the physical domain. This iterative process is called zigzagging (See also Tate, 1999, for a more thorough description of the decomposition problem).

Zigzagging also involves the other domains since manufacturing considerations may constrain design decisions, while too “tight” requirements could virtually prohibit the discovery of feasible design solutions.

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* It appears that systems engineers realised this independently, while learning from some bad designs in the automotive industry (see Stevens, Myers, and Constantine, 1974, p.139).
At each level of the design hierarchy, the relations between the FRs and the DPs can be represented in an equation of the form:

\[ \text{FR} = [A]\text{DP} \]  

where each element of the design matrix \([A]\) can be expressed as:

\[ A_{ij} = \frac{\partial \text{FR}_i}{\partial \text{DP}_j}, \ (i = 1, \ldots, m \text{ and } j = 1, \ldots, n) \]  

Equation (1) is called the design equation and can be interpreted as “choosing the right set of DPs to satisfy given FRs”. Each element \(A_{ij}\) is represented as a partial derivative to indicate dependency of a FR \(i\) on a DP \(j\). For simplicity the value of an element \(A_{ij}\) can be expressed as 0 (i.e. the functional requirement does not depend on the particular design parameter), or otherwise X. Depending on the type of the resulting design matrix \([A]\), three types of designs exist: uncoupled, decoupled and coupled (figure 2).

Figure 2. Examples of design types. Matrix entries marked with X mean \(A_{ij} \neq 0\)

Uncoupled design occurs when each FR is satisfied by exactly one DP. The resulting matrix is diagonal and the design equation has an exact solution, i.e., Axiom 1 is satisfied. When the design matrix contains non-zero entries on the main diagonal and some elements below it, the resulting design is decoupled. This means that a sequence exists, where by adjusting DPs in a certain order, the FRs can be satisfied. The design matrix of a coupled design contains mostly non-zero elements and thus the FRs cannot be satisfied independently. A coupled design can be decoupled, for example, by adding components to carry out specific functions, however, this comes at a price.

The complexity of the problem increases tremendously when the manufacturing process factors are being considered simultaneously with the design ones. By analogy to equation (1), the design parameters can be considered as requirements of the manufacturing process. Thus the design equation of the manufacturing process is:

\[ \text{DP} = [B]\text{PV} \]
The two matrix equations can be combined into a single relation by substituting equation (3) into equation (1), thus linking the requirements with the manufacturing process:

\[
\mathbf{FR} = [A][B]\mathbf{PV} = [C]\mathbf{PV}
\]

(4)

The multiplication order reflects the chronological order of the design and manufacturing processes. In theory, if the resulting matrix \([C]\) is diagonal, then the design is uncoupled and all the design and manufacturing parameters satisfy the functional requirements. This does not happen very often in practice and either \([A]\), i.e. the design, or \([B]\), i.e., the manufacturing process has to be modified during the product development process.

3. Covariance structural models

*Covariance Structural Models* is arguably the broadest term encompassing a number of methods which aim to explain the relationships among a set of observed variables in terms of a generally smaller number of unobserved and/or other observed variables. Its origins can be traced back to the early years of research in genetics (see for example Wright (1920), quoted in Loehlin (1998). In this section two specific models are outlined, the Confirmatory Model and the Structural Equation Model. The combination of these models forms the SEMDES method which is proposed in Section 4. The notation and the description of the Confirmatory and the Structure Equation Models in this section follow exclusively that of Long (1983-a, b) for consistency. Loehlin (1998) provides a very good introduction to the subject.

3.2 The confirmatory model

The confirmatory model is used predominantly in the behavioural sciences. In this model some variables of interest cannot be directly observed, for example, psychological disorders. These unobserved variables are referred to as *latent variables* or *common factors*. While latent variables cannot be directly observed, information about them can be obtained indirectly by noting their effect on observed variables (e.g. psychophysiological symptoms). In this way one takes a hypothesised structure and tries to find out how well it accounts for the observed relationships in the data. Figure 3 illustrates a Confirmatory model.

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**Notation:**
- \(x\) - observed variable
- \(\xi\) - common factor (latent variable)
- \(\lambda\) - loading which indicates how change in a common factor affects an observed variable
- \(\delta\) - unique factor (residual)
- \(\phi\) - covariance (correlation) between two common factors
- \(\theta\) - covariance (correlation) between two unique factors (residuals)
The latent variables $\xi$ are represented by circles while the observed variables $x$ are indicated by squares. Each latent variable in this particular model is measured by two observed variables. In the terminology of factor analysis it is said that $x_1$ and $x_2$ load on $\xi_1$, and $x_3$ and $x_4$ load on $\xi_2$. The loadings are indicated by the straight arrows connecting the latent and the observed variables. The relationships among the observed and latent variables can be expressed in a matrix from:

$$x = \Lambda \xi + \delta$$

(5)

where $x$ is $(q \times 1)$ vector of observed variables; $\xi$ is a $(s \times 1)$ vector of latent variables; $\Lambda$ is a $(q \times s)$ matrix of factor loadings relating the observed to the latent variables; and $\delta$ is a $(q \times 1)$ vector of residual or unique factors. The above equation, called also the factor equation, can be thought of as the regression with zero intercept of observed variables on unobserved (latent) variables. The loadings $\lambda$ correspond to slope coefficients, that is, a unit change in the latent variable results in an expected change of $\lambda$ units in the observed variable. For example (see also figure 3), the factor equation, $x_2 = \lambda_{21} \xi_1 + \delta_2$, indicates that a unit increase of $\xi_1$ results in an expected increase of $\lambda_{21}$ units in $x_2$. In this equation, $\delta_2$ can be thought of as an error term indicating that $\xi_1$ does not perfectly predict $x_2$. Both the observed and latent variables are assumed to be measured as deviations from their means. Thus, the expected value of each vector is a vector containing zeroes: $E(x) = 0$; $E(\xi) = 0$; and $E(\delta) = 0$. Since this assumption involves only a change in origin of the distribution, it does not affect the covariances among the variables. A practical advantage of assuming zero means is that the covariances are equivalent to expectations of the product of variables with zero means. This allows the population covariance matrix $\Sigma$ of the observed variables to be obtained by multiplying the matrix of the observed variables $x$ by its transpose $x'$ and taking expectation. This is accomplished by multiplying equation (5) by its transpose and taking expectations:

$$\Sigma = E(xx') = E[(\Lambda \xi + \delta)(\Lambda \xi + \delta)']$$

Since the transpose of a sum of matrices is equal to the sum of the transpose of the matrices, and the transpose of a product of matrices is the product of the transposes in reverse order, it follows that:

$$\Sigma = E[(\Lambda \xi + \delta)(\xi'A' + \delta')]$$

Using the distributive property of matrices and taking expectations:

$$\Sigma = E[\Lambda \xi \xi'A'] + E[\Lambda \xi \delta'] + E[\delta \xi'A'] + E[\delta \delta']$$

The parameter matrix $\Lambda$ does not contain random variables, since the population values of the parameters are constant (even if unknown). This allows us to write:

$$\Sigma = \Lambda E[\xi \xi'A'] + \Lambda E[\xi \delta'] + E[\delta \xi'A'] + E[\delta \delta']$$

Finally, since $E[\xi \xi']$ is defined as $\Phi$, $E[\delta \delta']$ is defined as $\Theta$, and $\delta$ and $\xi$ are assumed to be uncorrelated, the above equation can be simplified to:

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† It should be noted, however, that in the regression analysis both the dependent and the independent variables are observed.
\[ \Sigma = \Lambda \Phi \Lambda' + \Theta \] (6)

Equation (6) is referred to as the covariance equation.

The \((i,j)\)th element of \(\Sigma\), \(\sigma_{ij}\), is the population value of the covariance between \(x_i\) and \(x_j\). \(\Sigma\) is a \((q \times q)\) symmetric matrix, since \(\text{cov}(x_i, x_j) = \text{cov}(x_j, x_i)\). The main diagonal contains variances, since \(\text{cov}(x_i, x_i) = \text{var}(x_i)\). If the observed variables were standardised to have a variance of one, \(E(x_i, x_j)\) would be the correlation between \(x_i\) and \(x_j\), and \(\Sigma\) would be the population correlation matrix.

Similarly the covariances (or correlations) among the latent variables are contained in \(\Phi\), an \((s \times s)\) symmetric matrix. The covariances (correlations) among the residuals are contained in the population matrix, \(\Theta\), a \((q \times q)\) symmetric matrix. The above assumptions and results are summarised in table 1.

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Dimension</th>
<th>Mean</th>
<th>Covariance</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi)</td>
<td>((s \times 1))</td>
<td>0</td>
<td>(\Phi = E(\xi \xi'))</td>
<td>((s \times s))</td>
<td>common factors</td>
</tr>
<tr>
<td>(x)</td>
<td>((q \times 1))</td>
<td>0</td>
<td>(\Sigma = E(xx'))</td>
<td>((q \times q))</td>
<td>observed variables</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>((q \times s))</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>loading of (x) on (\xi)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>((q \times 1))</td>
<td>0</td>
<td>(\Theta = E(\delta \delta'))</td>
<td>((q \times q))</td>
<td>unique factors</td>
</tr>
</tbody>
</table>

Factor Equation: \(x = \Lambda \xi + \delta\) (5)
Covariance Equation: \(\Sigma = \Lambda \Phi \Lambda' + \Theta\) (6)

Assumptions:

a. Variables are measured from their means: \(E(\xi) = 0\); \(E(x) = E(\delta) = 0\).

b. The number of observed variables is greater than the number of common factors; i.e., \(q > s\).

c. Common factors and unique factors are uncorrelated: \(E(\xi \delta') = 0\) or \(E(\delta \xi') = 0\).

Two important issues are associated with solving the covariance equation, identification and estimation. Identification precedes estimation and is concerned with whether the covariance equation has unique solution, that is, whether the parameters of the model are uniquely determined. The left side of the covariance equation contains \(q(q + 1)/2\) distinct variances and covariances among the observed variables\(^\ddagger\). The right side of the equation contains \(q\times s\) possible loadings from \(\Lambda\), \(s(s + 1)/2\) independent variances and covariances among the \(\xi\)’s; and \(q(q + 1)/2\) independent variances and covariances among the \(\delta\)’s. Thus, the covariance equation decomposes the \(q(q + 1)/2\) distinct elements of \(\Sigma\) into \([qs + s(s + 1)/2 + q(q + 1)/2]\) unknown independent parameters from the matrices \(\Lambda\), \(\Phi\), and \(\Theta\). It follows that a confirmatory model is unidentified unless at least \([qs + s(s + 1)/2]\) constraints are imposed. Hence a necessary (but not sufficient) condition for identification is that the number of independent, unconstraint parameters in the model must be less than or equal to \(q(q + 1)/2\).

\(^\ddagger\) Recall that \(\Sigma\) is a \((q \times q)\) symmetric matrix. Of the total of \(q^2\) elements in \(\Sigma\), the \(q\) diagonal elements are variances (or unities if we are dealing with correlations, i.e. with standardised data). Half of the remaining \(q^2 - q\) elements are redundant since the covariance (correlation) matrix \(\Sigma\) is symmetric. Thus there are \(q + (q^2 - q)/2 = q(q + 1)/2\) unique elements in \(\Sigma\).
Estimation assumes that the model is identified. It uses sample data to construct the sample matrix of covariances, $S$, to estimate the parameters in $\Lambda$, $\Phi$, and $\Theta$.

This is done by obtaining an estimate of the covariance matrix through the covariance equation $\hat{\Sigma} = \hat{\Lambda} \Phi \Lambda' + \hat{\Theta}$, where the $\hat{}$ indicates that the matrices contain estimates of the parameters. These estimates must satisfy the constraints imposed on the model. Thus estimation involves finding values of $\hat{\Lambda}$, $\hat{\Phi}$, and $\hat{\Theta}$ that generate an estimated covariance matrix $\hat{\Sigma}$ that is as close as possible to the sample covariance matrix $S$.

A function which measures how close is a given $\hat{\Sigma}$ to the sample covariance matrix $S$ is called a fitting function. Three fitting functions are commonly used in confirmatory factor analysis, which correspond to the methods of Unweighted Least Squares (ULS), Generalised Least Squares (GLS), and Maximum Likelihood (ML).

The methods and criteria for identification and estimation are of great importance, but fall beyond the scope of this paper. There are several commercial model fitting programs which can help the researcher or the practitioner with identification and estimation (see for example Loehlin, 1998). One of these programs, LISREL, is used in the example presented in Section 4.

### 3.2 The structural equation model

Structural Equation Modelling is a fundamental tool used extensively in econometrics. A Structural Equation Model (SEM) specifies the causal relationship among a set of variables. SEM is illustrated in figure 4 and is summarised in table 2. Unlike confirmatory factor analysis, all variables in SEM are observed and measurable. Those variables that are to be explained by the model are called *endogenous* variables. Endogenous variables are causally dependant on other endogenous variables and/or what are called *exogenous* variables. Exogenous variables are determined outside the model.

![Figure 4. A structural equation model.](image)

<table>
<thead>
<tr>
<th>Notation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$ - exogenous variable</td>
</tr>
<tr>
<td>$\eta$ - endogenous variable</td>
</tr>
<tr>
<td>$\gamma$ - coefficient relating an exogenous variable with an endogenous variable</td>
</tr>
<tr>
<td>$\phi$ - covariance (correlation) between two exogenous variables</td>
</tr>
<tr>
<td>$\beta$ - coefficient relating two endogenous variables to one another</td>
</tr>
<tr>
<td>$\zeta$ - error in measurement of $\eta$</td>
</tr>
</tbody>
</table>

Let $\eta$ be a $(r \times 1)$ vector of endogenous variables and let $\xi$ be a $(s \times 1)$ vector of exogenous variables (see Figure 4). In SEM it is assumed that the variables are related by a system of linear structural equations:

$$\eta = B\eta + \Gamma \xi + \zeta$$  \hspace{1cm} (7)
where $B$ is a $(r \times r)$ matrix of coefficients relating the endogenous variables to one another; $\Gamma$ is $(s \times s)$ matrix of coefficients relating the exogenous variables to the endogenous variables; and is $\zeta$ a $(r \times 1)$ vector of errors in equations, indicating that the endogenous variables are not perfectly predicted by the structural equations. Restricting the elements of $\Gamma$ and $B$ to equal zero indicates the absence of a causal relationship between the appropriate variables. For example fixing $\gamma_{ij} = 0$ implies that the exogenous variable $\xi_j$ does not have a causal effect on the endogenous variable $\eta_i$. Similarly fixing $\beta_{ij} = 0$, the endogenous variable $\eta_i$ is assumed to be unaffected by $\eta_j$. The diagonal elements of $B$ are assumed equal to zero, indicating that an endogenous variable does not cause itself.

Subtracting $\tilde{B}\eta$ from each side of equation (7) (resulting in $\eta - \tilde{B}\eta = \Gamma\xi + \zeta$) and defining $\tilde{B}$ as $(I - B)$, were $I$ is the identity matrix, results in:

$$\tilde{B}\eta = \Gamma\xi + \zeta,$$

which is the form more commonly found in the econometric literature (Long 1983-b). Equation (8) in turn can be presented in what is known as the reduced from of the structural equation:

$$\eta = \tilde{B}^{-1}\Gamma\xi + \tilde{B}^{-1}\zeta$$

In this form the endogenous variables are represented as functions only of the exogenous variables and the errors in equations.

As in the confirmatory model all variables in SEM are assumed to be measured as deviations from their means: $E(\eta) = E(\zeta) = 0$ and $E(\xi) = 0$. The errors in equations are measured from zero (i.e., $E(\zeta_i) = 0$) and are assumed to be uncorrelated with the exogenous variables, that is, $E(\xi\zeta'_i) = 0$ and $E(\zeta\xi'_i) = 0$.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>$(r \times 1)$</td>
<td>0</td>
<td>$COV(\eta) = E(\eta\eta')$</td>
<td>$(r \times r)$</td>
<td>endogenous variables</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$(s \times 1)$</td>
<td>0</td>
<td>$\Phi = E(\xi\xi')$</td>
<td>$(s \times s)$</td>
<td>exogenous variables</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>$(r \times 1)$</td>
<td>0</td>
<td>$\Psi = E(\zeta\zeta')$</td>
<td>$(r \times r)$</td>
<td>errors in equations</td>
</tr>
<tr>
<td>$B$</td>
<td>$(r \times r)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>direct effects of $\eta$ on $\eta$</td>
</tr>
<tr>
<td>$\tilde{B}$</td>
<td>$(r \times r)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>defined as $(I - B)$</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>$(r \times s)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>direct effects of $\xi$ on $\eta$</td>
</tr>
</tbody>
</table>

Structural Equations:

$$\eta = B\eta + \Gamma\xi + \zeta$$

$$\tilde{B}\eta = \Gamma\xi + \zeta$$

Reduced Form Equation:

$$\eta = \tilde{B}^{-1}\Gamma\xi + \tilde{B}^{-1}\zeta$$

Covariance Equation:

$$\Sigma = \begin{bmatrix} \tilde{B}^{-1}(\Gamma\Phi\Gamma' + \Psi)\tilde{B}^{-1} & \tilde{B}^{-1}\Gamma\Phi \\ \Phi\Gamma'\tilde{B}^{-1} & \Phi \end{bmatrix}$$

Assumptions:

a. Variables are measured from their means: $E(\eta) = E(\zeta) = 0; E(\xi) = 0$. 

Table 2. Summary of the Structural Component of the Covariance Structure Model
With the above assumptions, the following covariance matrices can be defined (see also table 2): $\Phi$, a $(s \times s)$ symmetric matrix, contains the covariances among the exogenous variables; the covariances among the errors in equations are contained in the symmetric matrix $\Psi$ of dimension $(r \times r)$. The values of $\Psi$ are generally unknown, although off-diagonal elements can be restricted to zero to indicate that errors in equations are uncorrelated across two equations.

As in the confirmatory factor model discussed in the previous subsection, a covariance equation needs to be constructed in order to link the covariances of the observable variables with the unknown structural parameters and covariances. In this case the structural parameters are contained in $B$ and $\Gamma$, and the unknown covariances in $\Phi$ and $\Psi$.

The covariances among the observable variables can be stacked in a $(r+s \times r+s)$ covariance matrix:

$$
\Sigma = \begin{bmatrix}
\text{cov}(\eta) & \text{cov}(\eta, \xi) \\
\text{cov}(\xi, \eta) & \text{cov}(\xi)
\end{bmatrix}
$$

(10)

The covariance equation can be derived from equation (10) by substituting $\text{cov}(\eta)$ with $E(\eta \eta')$, where $\eta$ is substituted with the right-hand side of the structural equation (9):

$$
\Sigma = \begin{bmatrix}
\bar{B}^{-1}(\Gamma \Phi \Gamma' + \Psi)\bar{B}^{-1} & \bar{B}^{-1}\Gamma \Phi \\
\Phi \Gamma \bar{B}^{-1} & \Phi
\end{bmatrix}
$$

(11)

In practice $\Sigma$ is unknown, therefore sample data from field studies must be obtained in order to construct the sample matrix $S$. The process of estimation involves finding values for $B$, $\Gamma$, $\Phi$, and $\Psi$ that produce a covariance matrix according to the covariance Equation (11) which is as close as possible to the observed (sample) covariance matrix $S$. The criteria for model identification as well as the estimation methods are beyond the scope of this brief introduction. For complex models identification and estimation are performed with the help of computer programs such as LISREL (see for example Long 1983-b or Loehlin 1998).

4. SEMDES

The proposed combined application of the Confirmatory Factor Model and the structural component of the Covariance Structure Model form a procedure named, SEMDES - Structure Equation Models for the Design of Engineering Systems. SEMDES is introduced by way of an example which has been adapted from a case study by Suh (1990). The choice of the case study has been influenced by a number of considerations such as the requirement to be self contained and simple enough so that at least the matrix calculations are reproducible by hand. This would make SEMDES easier to understand and compare with the AD approach. The case study was augmented for the purposes of this research with an imaginary extension to illustrate modelling and simulation of vague, non functional customer needs. The applicability of the example to other industries is addressed below.

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$^9$ The detailed derivation is presented in Long (1983-b). It is conceptually similar to the derivation of equation (6).
4.1 Case Study

A shoe manufacturer decided to invest in injection moulding machines that can make foamed plastic shoe soles. The first task facing the company was to determine the needs of the potential customers. The marketing department interviewed a number of persons from a particular gender, age range, and income bracket. The interviews revealed that the potential customers in that particular market segment were less fashion conscious and the majority preferred quality shoe wear **. The next problem was how to translate the customer need (CN) *quality shoe (sole)* into functional requirements (FR). The market researchers managed to extract a few characteristics of a ‘quality shoe (sole)’ from their initial interviews. It was decided to use a Confirmatory Factor model to check the hypothesis that characteristics such as *flexibility*, *weight*, and *durability* are the FRs which will satisfy the CN in this particular market segment.

![Figure 5. Confirmatory factor model suggested for the injection moulding machine example.](image)

At this point a second interview with the customers was conducted to test the confirmatory model shown in Figure 5. It should be noted here that while desirable, the interviewees do need to be the same persons for the two interviews.

The collected data was standardised and the sample correlation matrix $S$ constructed. In the suggested model, the matrix of the latent variables $\mathbf{\xi}$ contains only one element, $\xi_{11}$ which is the single customer need, quality shoe (sole). For this reason the correlation matrix of the latent variables, $\Phi$, also contains only one element, **In this example only the sole is considered.**
\[ \phi_{11} = 1. \] The matrix of the observed variables \( \mathbf{x} \) contains three elements, the three FRs. Similarly the matrix \( \mathbf{\Lambda} \) containing the loadings of \( \mathbf{x} \) on \( \mathbf{\xi} \) has only three elements. It is assumed that the errors (the residual factors) are uncorrelated. It follows then that the matrix containing the covariances of the residual factors, \( \mathbf{\Theta} \) is diagonal - the diagonal elements being the variances of the individual residual factors. With these assumptions it is easy to see that the model is identified. The covariance equation (6) of the model in this example can be represented as:

\[
\begin{bmatrix}
1 & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & 1 & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & 1
\end{bmatrix}
\begin{bmatrix}
\lambda_{11} \\
\lambda_{21} \\
\lambda_{31}
\end{bmatrix}
+ 
\begin{bmatrix}
\theta_{11} & 0 & 0 \\
0 & \theta_{22} & 0 \\
0 & 0 & \theta_{33}
\end{bmatrix}
\]

Note that in the above equation \( \sigma_{ij} = \sigma_{ji} \) since \( \Sigma \) is a symmetrical matrix.

In order to estimate the model, the above equation is solved by giving values to the parameters on the right hand side of the equation that would generate the predicted matrix, \( \hat{\Sigma} \), on the left hand side. \( \hat{\Sigma} \) should be as close as possible to the observed correlation matrix \( \mathbf{S} \) in figure 5. This process was performed with the help of LISREL (Joreskog and Sorbom, 2001) which handles a large class of covariance structure models. LISREL checked whether the model is identified, and after confirmation, indicated a nearly perfect fit. This was not a surprise, given such a straightforward model. The estimated values of the loadings \( \lambda_{ij} \) and the errors in variance \( \delta_j \) are shown in figure 5. (The estimations of LISREL can be reproduced by manually performing the calculations at the right hand side of the above equation, assuming that \( \theta_{11} = 0.37, \theta_{22} = 0.22, \) and \( \theta_{33} = 0.68. \) Note also that \( \lambda_{ij}^2 + \delta_j = 1. \))

The conclusion of the model fitting procedure is that, indeed, flexibility, weight, and durability are predicted by the latent variable ‘quality shoe (sole)’ since the model is identified, estimated and the loadings \( \mathbf{\Lambda} \) turned out to be significant, that is, greater than 0.3. These findings justified that the observed variables be accepted as functional requirements of the new product:

- FR1 = Flexibility.
- FR2 = (Light) Weight.
- FR3 = Durability.

In general there may be more than one latent variable representing qualitative customer needs. In such cases the structural part of the model specifies the relationships between the latent variables while the measurement (confirmatory) part specifies the relationships of the latent to the observed variables. Thus one latent variable can be linked to one or more latent variables and to one or more observed variables. For example, it may be assumed in a model that latent variables such as quality, comfort and appeal are related, but nevertheless distinct factors. In practice, the usual procedure is to solve the measurement and structural models simultaneously, because in doing so, one brings to bear all information available (Loehlin, 1998, p. 92).

The next task of the engineering process was to find design parameters (DPs) which will satisfy the FRs. The design team decided that the core of the sole (figure 6) was to be made of polyvinylchloride (PVC) of uniform density, \( \rho \), for durability, light weight, and flexibility.

\[\text{†† Standardised values above 0.3 are considered to be significant (see for example Child, 1990).}\]
The outer skin was to be a solid PVC layer for good wear resistance (i.e. durability). A team of designers, manufacturing and production engineers decided that the machine which will produce the sole will be based on the injection moulding process, shown at the bottom of figure 6.

Thus for the DPs it was decided that:

- DP1 = Thickness of the foamed core, $a$.
- DP2 = Density of the foamed plastic core, $\rho$.
- DP3 = Thickness of the solid plastic layer, $b$.

Suh (1990) concluded in the original case study that such design is uncoupled, that is, the design matrix is diagonal:

$$
\begin{bmatrix}
FR1 \\
FR2 \\
FR3
\end{bmatrix} =
\begin{bmatrix}
X & 0 & 0 \\
0 & X & 0 \\
0 & 0 & X
\end{bmatrix}
\begin{bmatrix}
DP1 \\
DP2 \\
DP3
\end{bmatrix}
$$

However, it is demonstrated in the next section, that this result may not hold if a possible correlation between the FRs is accounted for.

The next step of the product development process was to search for process variables (PVs) which could yield diagonal design matrix. In order to obtain uniform density, the incompressible plastic material needed to be uniformly distributed throughout the mould before being let to foam under uniform pressure. For this reason a construction of the mould was chosen which incorporated a moving half (figure 6). This would allow controlling the expansion of the volume at nearly constant pressure. On the other hand, the thickness, $b$, of the skin layer at the surface could be controlled by either cooling the mould surface or by varying the elapsed...
time before the mould expansion, since plastic could not expand at low temperatures even when blowing agents were present (Suh, 1990). Thus the following PVs were chosen:

PV1 = Expansion rate of the mould, $\varepsilon$.
PV2 = Injection velocity, $V$.
PV3 = Temperature of the mould surface, $T$.

The DP-PV design equation is:

$$
\begin{bmatrix}
a \\
\rho \\
b
\end{bmatrix} =
\begin{bmatrix}
X & 0 & 0 \\
X & X & 0 \\
0 & X & X
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
V \\
T
\end{bmatrix}
$$

The design matrix suggests a decoupled design of the manufacturing process. The matrix reveals that the expansion rate, $\varepsilon$, affects both the foamed core thickness, $a$, and its density, $\rho$. Similarly, the injection velocity, $V$, controls for the core density, $\rho$, and the thickness of the solid skin layer, $b$.

The Structure Equation Model (SEM) corresponding to the structure of the FR-DP design equation is shown in figure 7. The FRs are considered to be the exogenous variables, $\xi$, since they were determined outside this model. The DPs are the endogenous variables, $\eta$. A causal link between an FR and a DP is represented by a solid arrow. The double headed curved arrows represent error correlations. These were fitted by the software on the basis of the correlations among the FRs from the previous stage - the confirmatory model. The FR correlations are preserved (as constraints) in the sample correlation matrix - the highlighted region of the table in figure 7. The model fitting program LISREL was used again to identify and estimate the model. It is seen from figure 7 that each loading $\lambda$ is significant at 0.5.
The model fits reasonably well for sample sizes between 20 and 100. For example, for a sample size of 50 and 9 degrees of freedom, the $\chi^2$ (chi-square) is 44.23 and the P-value is 0.0000. The P-value is the probability of obtaining a $\chi^2$ value larger than the value obtained, given that the model is correct. It is worth pointing out, however, that the $\chi^2$ is sensitive to sample size and to departure from multivariate normality of the observed variables. Thus for a larger sample, the difference between the chi-square and the degrees of freedom would be large, as in this example, suggesting that further inspection is needed of fitted residuals, the standardised residuals and modification indices. Often these quantities will suggest ways to relax the model by introducing more parameters (Joreskog and Sorbom, 2001). Indeed, LISREL suggested that in order to reduce the Chi-square, four paths are introduced between the exogenous and endogenous variables. The software estimated the following values (weights):

- Weight to Foam thickness - 0.33;
- Weight to Thickness of solid plastic layer - 0.69
- Flexibility to Density of foamed plastic core - 0.33;
- Durability to Density of foamed plastic core - 0.69;

<table>
<thead>
<tr>
<th>Sample Correlation Matrix $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Flexibility</td>
</tr>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Durability</td>
</tr>
<tr>
<td>Foam_a</td>
</tr>
<tr>
<td>Density_(\rho)</td>
</tr>
<tr>
<td>Solid_b</td>
</tr>
</tbody>
</table>

Figure 7. Structural Equation Model constructed for the injection moulding machine example.
From axiomatic design point of view this result is significant as it indicates that, given a certain correlation between the functional requirements, a model with a better fit may not necessarily be the envisaged uncoupled design.

4.2 The SEMDES procedure

The approach described above could be adapted to other sectors such as the defence and aerospace where certain customer needs such as ‘intensity’, ‘impact’, etc., are subjective and difficult to quantify. The same applies to the transport, construction as well as the consumer industries. Taking the automotive industry as an example (see also Sobek II et al 1998 and Nevins and Winner 1999), comfort may mean different things to different age, gender and income groups. It can be mapped to technical requirements such as vibrations and deflections transmitted to the passengers. These, in turn can be mapped to characteristics such as the constants of the seat springs, the stiffness of the suspension, the acceleration limits and so forth. While QFD tools can be used for this type of mapping, it is the application of the latent variable model which should give the designer reasonable confidence that he/she has chosen the right mapping.

Following the examples above, the SEMDES procedure is summarised in figure 8 and generalised as follows:

**Step 1:** Perform customer elicitation studies, for example, structured interviews. Use these as inputs to exploratory factor analysis and conjoint analysis to extract and prioritise the customer needs (CN).

/* Comment: CNs can be quite vague and difficult to quantify and therefore are modelled as latent variables. */

**Step 2:** Formulate a hypothesis on the potential functional requirements (FR) which satisfy the CNs – build the Confirmatory model part of SEMDES. Ensure that the model is identified.

**Step 3:** Perform a second set of interviews with (a representative sample of) customers in order to obtain the covariance matrix of the observable variables - the FRs.

**Step 4:** Evaluate the hypothesis by performing (goodness of fit) estimation of the model.

**Step 5:** If the model fits, then:
- accept the FRs as representative of the customer need, store the correlation coefficients between the FRs;
- else: go to Step 2.

**Step 6:** Map the FRs as identified in Step 5 to design parameters (DP).

**Step 7:** Translate the FR-DP mapping into the Structure Equation Model part of SEMDES. Keep the correlation coefficients from Step 5 as constraints and also consider possible causal relationships between the DPs, which may need to be incorporated into the SEM.

**Step 8:** Find the best model fit in order to obtain the values of the design matrix entries ($\gamma$) and the casual links ($\beta$), if any, between the DPs.

**Step 9:** The best fit model may differ from the one devised by the application of the design equation in Step 6. Then it will be up to the designer to decide if the model represents a good compromise in terms of functionality (Axiom 1), weight and other cost-benefit factors (see for example Guenov 2002). If the designer is not convinced, then the process should be repeated from Step 6 if only DPs are modified or from Step 2 if FRs are added and/or replaced.
5. Discussion

In this section the advantages, scope and limitations of SEMDES are considered and avenues for further research are outlined.
5.1 Causality and Correlation

The possible correlation between DPs, and between FRs and DPs is considered in this subsection. Such correlations usually reflect practical experience gained from development of similar products over a long period of time. For example, empirical curves or equations may link aspect ratio, wing loading, etc., with empty weight fractions in aircraft design (Raymer, 1992).

![Diagram of Structural Equation model](image)

In the model presented in figure 7 all FRs and DPs are uncorrelated (the zero entries in S). The question is whether and how the FR-DP structure will change as a result of such correlation combined with the FRs correlation. It should be emphasised at this point that causality and correlation are two distinct issues, which if overlooked can lead to a significant misinterpretation of the model. Thus if one assumes that if a FR “causes” a DP they should be correlated, otherwise they should be uncorrelated, may lead to wrong conclusions. Due to the fact that the exogenous variables can be correlated, then controlling for one such variable, a strong (causal) relationship elsewhere in the model can vanish and a zero relationship can become strong‡‡. The problem is illustrated in figure 9, where it is assumed that certain FRs are correlated with certain DPs, but are

‡‡ This is also known as specification error (Kenny, 1979).
not causally linked. The correlation coefficients are highlighted in the correlation matrix and are also represented by double headed dashed-line arrows in the path model. After running the model it turns out that $\lambda_{11} \approx 0$. (This means that varying the foam thickness within the tolerance will not have a strong effect on the flexibility of the sole.) While this phenomenon has been known to the SEM theorists (Kenny 1979, pp. 62-63) it will require further research to understand the interplay of causality and correlation in engineering design. This extends also to the modelling of constraints involving the DPs, which may require the introduction of causal links ($\beta_{ij}$ in figure 4; see also next section and figure 10) between the endogenous variables (the DPs). For example, the power of one mechanism may affect the performance of another one, due to, say electromagnetic interference even though the design may be uncoupled. Such effects have been modelled as supplemental constraints in other systems design tools such as SLATE (Talbott et al 1994).

5.2 SEMDES and Axiomatic Design

Another question that arises from the above examples is how covariance structure models, SEM in particular, relate to Axiomatic Design (AD). In the case of SEM, the loadings ($\lambda_{ij}$) can be considered partial regression coefficients (known also as beta weights) because of the condition that the errors are uncorrelated with the exogenous variables, that is, $E(\xi \zeta') = 0$ and $E(\zeta \xi') = 0$ (Kenny 1979). This suggests that in the linear case the design equation matrix can be expressed by a SEM. The latter has potentially a stronger descriptive power compared to the design equation since it allows also for representing causal links between the endogenous variables (the DPs in Axiomatic Design). The latter are illustrated by the model fragment shown in figure 10. Wing dihedral is the angle of the wing with respect to the horizontal. The dihedral tends to roll the aircraft level whenever it is banked. It is also known that wing sweep, which is used primarily to reduce the adverse effects (shocks) of transonic and supersonic flow, causes an effective dihedral which adds to any geometric dihedral. As a rough approximation, 10 deg of sweep provides about 1 deg of effective dihedral (see for example, Rymer (1992)). This assumption is modelled as a constraint ($\beta_2 = 0.1$) in the model in figure 9. In this example the effective dihedral is a derived design parameter which adds to the difficulty in selecting the geometric dihedral.

The example also shows that the number of Functional Requirements (FR) and Design Parameters (DPs) may differ while Axiomatic Design assumes a one-to-one mapping between FRs and DPs, i.e., square design matrixes. While desirable, this is not always possible (see also Guenov and Barker 2005).

The linearity assumption in SEMDES is founded on previous research. For example, Fray et all (2000) argue that: “…assumption of nearly linear behaviour within the design range can safely be made for most products, because the design range is typically very small given the tight tolerances on performance of modern systems§§”. The authors quote a survey of literature on parametric error models in machine tools and also a case study on electronics packaging in support of their argument.

§§ A prior knowledge of the range is important. For example, a relatively small change of speed, from subsonic to transonic, can cause a significant increase of drag due to the formation of shocks.
Nonlinearity in SEM can be handled through variable transformation. For example single bend transformations (i.e. functions whose curves have a single bend) such as logarithm, square root and reciprocal are more appropriate for variables that have a lower limit of zero and no upper limit. Two bend transformations such as arcsin are useful for variables which have lower and upper limit (Kenny, 1979). Current versions of tools such as LISREL can handle a number of multilevel, non-linear models. Exploration of non-linearity forms a part of the future work on SEMDES.

5.3 SEMDES and quality engineering

As part of the covariance structure methods SEMDES shares a conceptual similarity with the Taguchi methods. The Taguchi methods use design of experiments which is based on the analysis of variance, ANOVA. The latter has been recognised as a special case of multiple regression (Kenny 1979). Kenny also outlined the advantages of multiple regression over ANOVA. As it was noted above, SEM can be classed as a multiple regression model, given the $E(\xi\xi') = 0$ and $E(\zeta\zeta') = 0$ conditions.
It is perhaps easier to demonstrate the richness of SEMDES and its apparent similarity to QFD in the pictorial representation shown in figure 11. Some of the constituent matrices such as COV(\eta) and A, appear, albeit less formally in the House of Quality as the relationships matrix and the technical correlations matrix, respectively (Houser and Clausing 1988 and Cohen 1995), while others (Γ) represent the Axiomatic Design matrix. Matrix B is also known as the Design Structure Matrix (DSM) or Design Dependency Matrix. The term DSM is used here in relation only to design parameters. An extensive review of the classification and capabilities of DSM is given by Browning (2001).

Unlike QFD, SEMDES does not deal with planning and technical targets. For example, metrics in the QFD Planning Matrix, such as the Importance to the Customer and its multiplier, the Improvement Ratio can serve as a guide to which parameters should be varied and by how much in the SEMDES model. In this respect SEMDES is complementary to QFD and for this reason would be applicable to evolutionary designs. However, in contrast to the general description of QFD (including Cohen’s one) this work has shown that both the technical requirements, or Substitute Quality Characteristics as they are known in QFD terminology and the design parameters can co-vary. The latter also can have direct (causal) effect on each other.

5.4 SEMDES and marketing

The confirmatory factor analysis part of SEMDES can either confirm or reject a customer need hypothesis, but it does not uniquely determine the number of variables which describe the functional requirements mapping. In the shoe example, a hypothesis was confirmed that Flexibility, Weight and Durability describe
well the customer need (Quality). In fact, a model including characteristics such as Water Resistance, Softness, etc., as additional requirements, could have represented even a better mapping of the customer need. In this respect fitting a particular latent variable model does not mean that it is a unique description of the customer need. The determination of the number and importance of the customer needs is an issue which involves subjectivity (Step 1 of the SEMDES Procedure). Experience, intuition and qualitative research are needed to develop the list of key product attributes. There are a number of marketing techniques which utilise a combination of qualitative and quantitative methods. These can be traced back to methods such as the Q-methodology (Stephenson 1953; see also McKeown and Thomas 1988), and (exploratory) factor analysis (Thurstone 1947; see also Child 1990). These methods have evolved in recent years to combine conversation fragments, writings, pictures and (video) images to extract common factors. For example, the ZMET method (Zaltzman and Higie, 1993) uses images and metaphors to build a consensus map- a graph diagram for representing and understanding the voice of the customer. A less formal method is Concept Engineering (1998) associated with the Language Processing Method (LPM 1997) which shares conceptual similarity with the Original KJ method (Kawakita 1975), better known for its affinity maps.

After the attributes (customer needs) have been identified a decision has to be made on the number and importance of these attributes and their combinations. One of the relatively mature techniques currently applied in marketing is Conjoint Analysis (see for example Louviere 1988; Wittink and Bergestuen 2001). Depending on the type of conjoint survey conducted, different statistical methods are used to translate respondents’ answers into importance values or utilities. These reveal the underlying value which the customer consciously or subconsciously place on each attribute and on combinations of attributes. One of those techniques is similar to what engineers may recognise as factorial experiments (see for example Montgomery, 2001). Regardless of the method used, it is critical to have a carefully thought out list of attributes. Too many attributes can greatly increase the burden on respondents while too few attributes can severely reduce the predictive power of a model because key pieces of information would be missing. One limitation of the conjoint models is the assumption that all respondents (customers) are equally well informed about the products and product attributes, which may not be the case in practice, due to advertising, marketing, distribution and other reasons. In such cases adjustments have to be made to account for bias.

5.5 Summary of SEMDES limitations

As a qualitative process SEMDES can help the practicing designers understand the decomposition and mapping of customer needs onto technical requirements and subsequently mapping of the latter onto design parameters. However, as an analytical procedure it can be applicable mainly to evolutionary design. This is because the need for models and computational processes existing prior to the application of SEMDES. For example, the early conceptual design phase of complex engineering systems such as aircraft requires the application of hundreds of low fidelity empirical models and thousands of variables representing design parameters and performance constraints associated with the technical requirements. The models are coupled through shared variables. Obtaining values for the outputs given input variables specified by the designer requires the employment of iterative and optimisation methods in order to dynamically assemble a computational process (see for example Guenov et al. 2006). Such a computational process is a prerequisite for obtaining the sample matrix S in the structural equation part of SEMDES.

Another limitation is that the sampling of the input parameters needs to be performed in a limited range in order to ensure the linearity of the covariance structural model.

Last, but not least, some caution must be exercised when applying covariance structure models. If the (latent) variables are not standardised, their scale must be established. This is due to the indeterminacy between the variance of a latent variable and the loadings of the observed variables on that factor (Long...
This makes it impossible to distinguish between the case of a latent variable with a large variance and small loadings on it, and the case of small variance and large loadings from the observed variables on the latent variable. It is also very important to keep in mind that a complex design model may not be identified. This means that an infinite number of models may fit the covariance equation. In relation to engineering design this may not be fatal, since any feasible design, that is, a design which satisfies the FRs and constraints is acceptable. However, unidentified models are not acceptable when determining functional requirements from customer needs (i.e. the confirmatory factor analysis).

6. Conclusions

As a tool for design and analysis, SEMDES is intended to help with the understanding and specification of the relationships between customer needs, technical requirements and design parameters. It is based on a combination of a confirmatory factor model and a structure equation model. This work has demonstrated for the first time that these models can be applied together to engineering design.

SEMDES bridges an important gap between engineering design and marketing, as it incorporates the definition and the seamless decomposition of customer needs to functional requirements and to design parameters. It will be particularly useful in practice for simulation of variability of customer needs and requirements in order to see their effect on the evolving design.

It was also shown that SEMDES is a richer model compared to the (axiomatic) design equation and complementary to the House of Quality (QFD) in that it allows for the introduction of correlation between the functional requirements (FRs), between the FRs and the design parameters (DPs), as well as correlation and causal relations between the DPs. However, more research is needed to achieve a better understanding of the interplay between correlation and causality in engineering systems design, as well as any non-linear effects of the latent and measured variables. Extensive practical trials will prove the usefulness of the approach in a broader context. Part of these future efforts is the intended application of the structural component of SEMDES as a local sensitivity analysis tool within a multidisciplinary design optimisation and analysis research framework.

References


