Accurate, fast and stable solver for electromagnetic scattering of absorbing layer materials

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Introduction

- The Boundary Element Method (BEM) is a powerful tool to model the scattering of electromagnetic waves by conducting and penetrable objects. It finds applications in forward and inverse problems such as radar footprint determination, stealth technology and imaging for diagnostics and security.
- The Impedance Boundary Condition (IBC) model specifies a relationship on the surface of the scatterer between the magnetic and electric currents $\boldsymbol{m} = -z\hat{\boldsymbol{n}} \times \boldsymbol{j}$. IBC is especially well suited to simulate metals coated by a dielectric or absorbing layer which is the base of current stealth technologies.
- Classic IBC formulations suffer from low frequency and dense discretisation breakdowns. In other words, the accuracy of the solution deteriorates and the computation time increases when the frequency is low or when the number of unknowns in the problem N is high because more iterations are required to solve the linear system. The formulation presented here solves these problems using a multiplicative preconditionner that makes it stable and accurate at arbitrary low frequency or dense mesh.

Figure: F-117 Nighthawk

Figure: Stealth frigate

Formulation

- The classic Electric Field Integral Equation (EFIE) reads $\eta T\boldsymbol{j} - \left(\frac{1}{2} + K\right)\boldsymbol{m} = \boldsymbol{e}^i \times \hat{\boldsymbol{n}}$

$$T(\boldsymbol{j}) = -ikT_s(\boldsymbol{j}) + \frac{1}{ik}T_h(\boldsymbol{j}) = -ik\hat{\boldsymbol{n}} \times \int_\Gamma \frac{e^{-ikR}}{4\pi R}\boldsymbol{j}(\boldsymbol{r}')d\boldsymbol{r}' + \frac{1}{ik}\hat{\boldsymbol{n}} \times \nabla \int_\Gamma \frac{e^{-ikR}}{4\pi R}\nabla' \cdot \boldsymbol{j}(\boldsymbol{r}')d\boldsymbol{r}'$$

$$K(\boldsymbol{j}) = \hat{\boldsymbol{n}} \times \text{p.v.} \int_\Gamma \nabla \frac{e^{-ikR}}{4\pi R} \times \boldsymbol{j}(\boldsymbol{r}')d\boldsymbol{r}'$$

- The unknown currents $\boldsymbol{j}$ and $\boldsymbol{m}$ are respectively discretized with normalized Rao-Wilton-Glisson (RWG) basis functions ($\boldsymbol{f}_i$) and Buffa-Christiansen (BC) basis functions ($\boldsymbol{g}_j$) which results in the classic IBC-EFIE formulation:

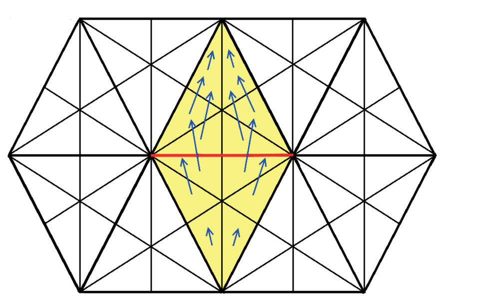

Figure: RWG basis function

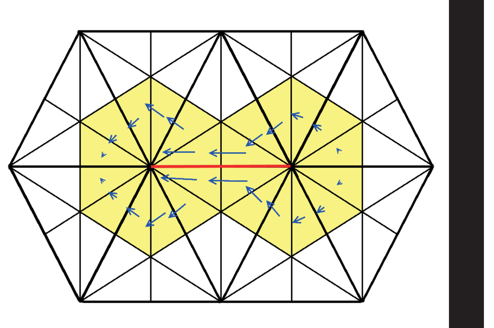

Figure: BC basis function

$$\mathrm{m} = -z\mathbf{G}_{mix}^{-1}\mathbf{G}\mathrm{j} \qquad \mathbf{S}\mathrm{j} = \left(\eta\mathbf{T} - z\left(\mathbf{K} + \frac{1}{2}\mathbf{G}_{mix}\right)\mathbf{G}_{mix}^{-1}\mathbf{G}\right)\mathrm{j} = \mathrm{V}$$

Where $(\mathbf{G}_{mix})_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, \boldsymbol{g}_j\rangle$, $(\mathbf{G})_{ij} = \langle \boldsymbol{f}_i, \boldsymbol{f}_j\rangle$, $(\mathbf{T})_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, T(\boldsymbol{f}_j)\rangle$, $(\mathbf{K})_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, \boldsymbol{g}_j\rangle$ and $(\mathrm{V})_i = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, \boldsymbol{e}^i \times \hat{\boldsymbol{n}}\rangle$.
The condition number of $\mathbf{S}$ or equivalently the number of iteration required to solve the system grows proportionnally to the number of unknowns and the inverse of the frequency.

- Consider the quasi-Helmholtz projectors: $\mathbf{P}^\Sigma = \mathbf{\Sigma}\left(\mathbf{\Sigma}^T\mathbf{\Sigma}\right)^+\mathbf{\Sigma}^T$ and $\mathbf{P}^{\Lambda H} = \mathbf{I} - \mathbf{P}^\Sigma$ where $\mathbf{\Sigma}$ is the star to RWG connectivity matrix. We define $\mathbf{M}_1$ to rescale the incident plane wave in frequency and $\mathbf{M}_2$ to rescale the currents:

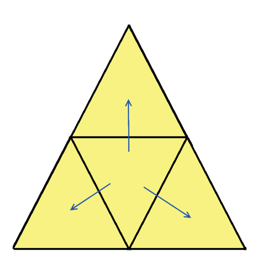

Figure: Star built with RWG

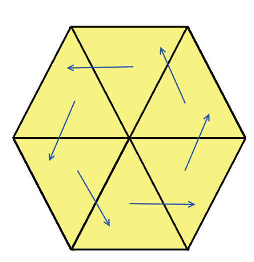

Figure: Loop built with RWG

$$\mathbf{M}_1 = \mathbf{P}^\Sigma + \frac{1}{ika}\mathbf{P}^{\Lambda H} \qquad \mathbf{M}_2 = ika\mathbf{P}^\Sigma + \frac{i\eta ka}{i\eta ka + z}\mathbf{P}^{\Lambda H}$$

The following formulation is immune from the low frequency breakdown:

$$\mathbf{M}_1\mathbf{S}\mathbf{M}_2\mathrm{Y} = \mathbf{M}_1\mathrm{V} \qquad \mathrm{j} = \mathbf{M}_2\mathrm{Y}$$

- Consider the dual projectors $\mathbf{P}^\Lambda = \mathbf{\Lambda}\left(\mathbf{\Lambda}^T\mathbf{\Lambda}\right)^+\mathbf{\Lambda}^T$ and $\mathbf{P}^{\Sigma H} = \mathbf{I} - \mathbf{P}^\Lambda$ where $\mathbf{\Lambda}$ is the loop to RWG connectivity matrix. The operator $\mathbf{M}_3$ uses a Calderon type preconditionning to solve the dense discretization breakdown:

$$\mathbf{M}_3 = \frac{1}{a}\mathbf{P}^{\Sigma H}\mathbb{T}_s\mathbf{P}^{\Sigma H} + \mathbb{T}_h$$

where $(\mathbb{T}_s)_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{g}_i, T_s(\boldsymbol{g}_j)\rangle$ and $(\mathbb{T}_h)_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{g}_i, T_h(\boldsymbol{g}_j)\rangle$

- To regularize correctly the system we replace $(\mathbf{G})_{ij} = \langle \boldsymbol{f}_i, \boldsymbol{f}_j\rangle$ by $(\mathbf{T}_\delta)_{ij} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, T_\delta \boldsymbol{f}_j\rangle$:

$$T_\delta = \frac{1}{\delta}\hat{\boldsymbol{n}} \times \int_\Gamma \frac{e^{-\frac{R}{\delta}}}{2\pi R}\boldsymbol{j}(\boldsymbol{r}')d\boldsymbol{r}'$$

T_δ tends to $\hat{\boldsymbol{n}} \times I$ when $\delta \to 0$ and T_δ inherits the properties of T_s as $T_\delta = \frac{2}{\delta}T_s(k = -\frac{i}{\delta})$. In particular the singular values of T_δ scales as $\frac{1}{N}$ so T_δ can be regularized by T_h those singular values scales as N. The following system is immune from the dense discretization breakdown in addition to the low frequency breakdown.

$$\mathbf{M}_3\mathbf{G}_{mix}^{-1}\mathbf{M}_1\mathbf{S}\mathbf{M}_2\mathrm{Y} = \mathbf{M}_3\mathbf{G}_{mix}^{-1}\mathbf{M}_1\mathrm{V}$$

Results

- Our formulation for IBC remains well conditioned when the discretization increases, contrary to other well established ones. In addition, relevant physical models for the impedance (e.g. the Drude model) have an impedance that is function of the frequency. In these cases, our formulation remains stable and accurate. The following figures show the condition number of sereral formulations for a unit sphere.

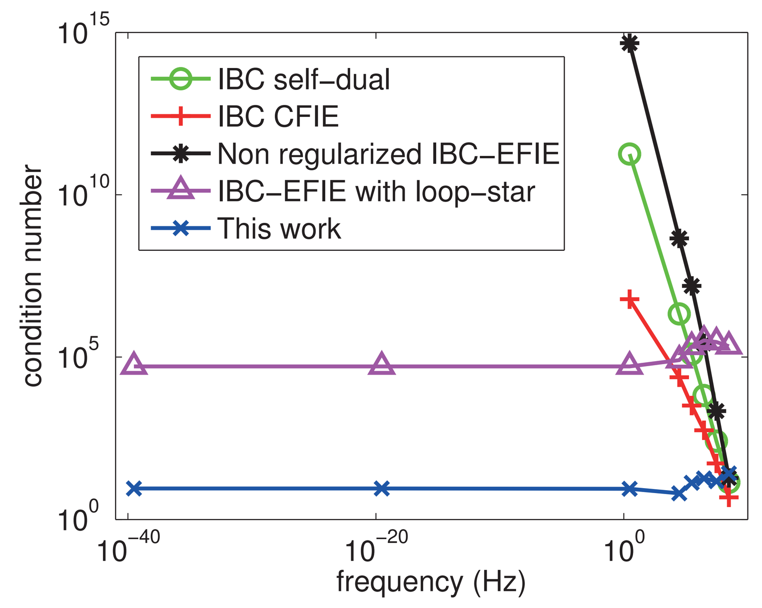

Figure: Condition number as a function of f for the copper ($z = (1-i)\sqrt{\frac{\mu\pi f}{\sigma}}$, $h = 0.15$m)

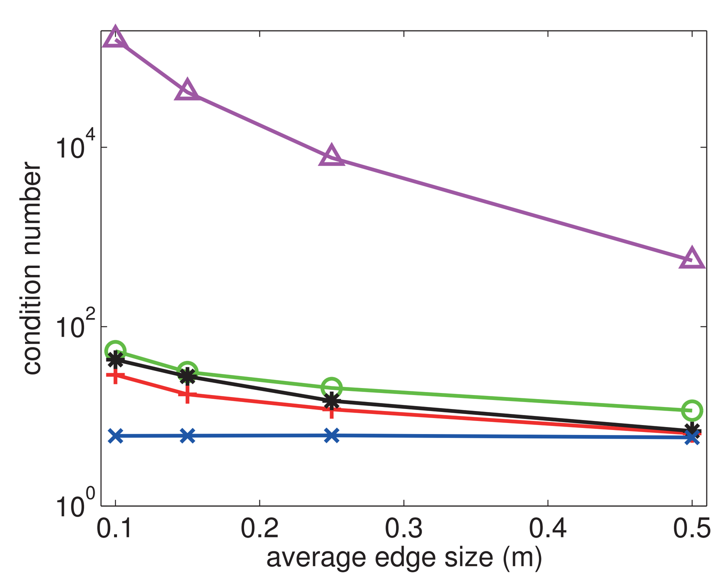

Figure: Condition number as a function of h ($z = 0.7 + 0.6i$, $f = 60$MHz)

- Another use for IBC is the scattering of a metal coated by an absorbing layer, this is particularly relevant to simulate stealth planes.
- The magnitude of the electric current on the stealth airplane surface induced by an incident plane wave is represented on this figure.

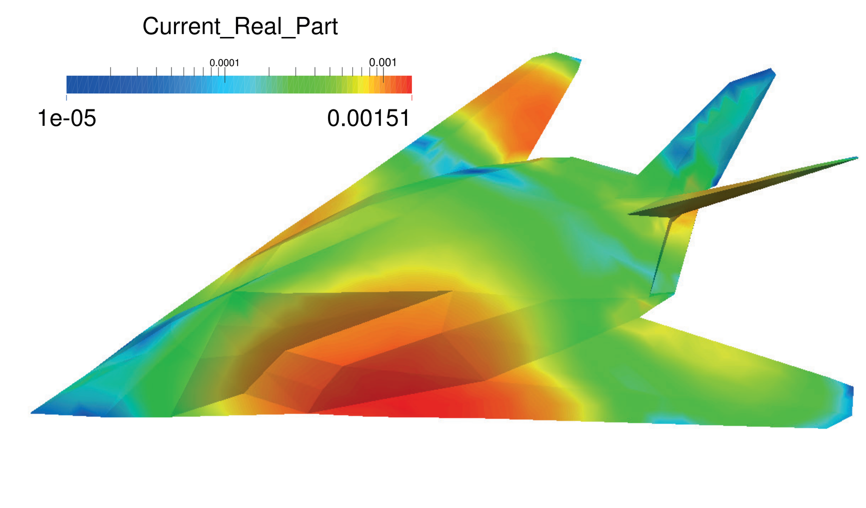

Figure: Distribution of the electric current on an airplane surface

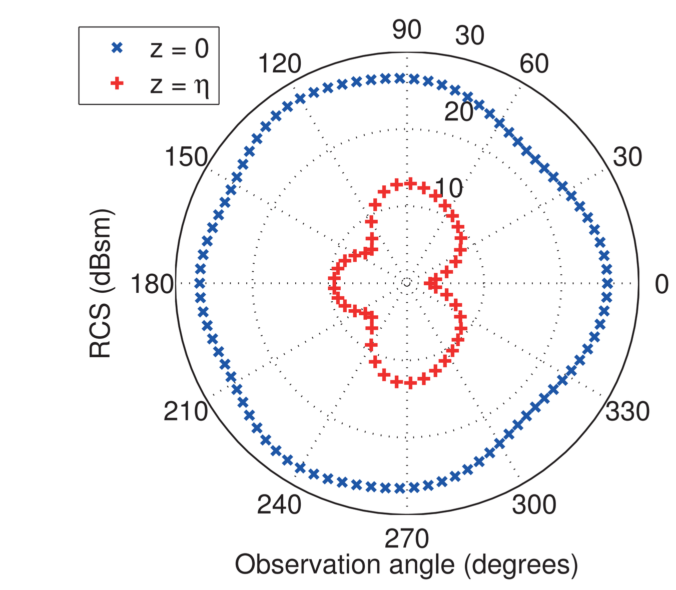

Figure: RCS of the airplane for different incidence angles ($f = 10$MHz)

- Our formulation shines especially when there are several right hand side of the system that need to be computed. This is the case when the radar cross section has to be simulated from each incident direction.
- On this figure, the effect of the absorbing layer that coats the airplane on the radar footprint is evident.

Conclusion

- The impedance boundary condition has a large range of applications and it is particularly well suited to simulate absorbing layer coated materials.
- This new formulation for the IBC-EFIE significantly improve the performance to solve the IBC problem: the linear system is solved with a number of iterations independent of the frequency and the number of unknowns. This property holds even when the impedance z tends to 0 (Perfect Electric Conductor) or when it is a function of the frequency as in physical models for the impedance.
- It enables solving the IBC-EFIE in a O(N.log(N)) time complexity using a fast multipole method.

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