AIAA SCITECH 2024 Forum, 8-12 January 2024, Orlando, USA. Paper number AIAA 2024-0569
DOI:10.2514/6.2024-0569

Non-linear Control of a Quadrotor with Actuator Delay

Muhammed R. Kartal* , Dr. Dmitry Ignatyev†, and Prof. Argyrios Zolotas‡
School of Aerospace, Transport and Manufacturing, Cranfield University, Bedford, MK43 0AL, UK

During the last decade, Unmanned Aerial Vehicles (UAVs) gained significant interest for use in various application domains such as monitoring/surveillance, freight/cargo shipping, and agriculture spraying. Developing such vehicle platforms requires the utilization of robust control approaches to maintain stable and appropriate maneuvering capabilities, as well as to address system uncertainty such as payload variation or dynamic variations (e.g., spraying drones are affected by such uncertainty). Moreover, due to the rotor-based structure, rotorcraft UAVs are quite vulnerable to UAV systems control input signal delay. In terms of maintaining a robust approach for a rotorcraft UAV, it is essential to provide stability against UAV systems control input signal delay. This paper makes an analysis throughout to improve control efficiency against UAV systems control input signal delay. Through investigation and improved fault rejection, three controlling algorithms were designed and applied. The analysis focused on three different scenarios. Insights are discussed within the remit of command tracking performance with UAV systems control input signal delay.

I. Nomenclature

m	=	mass of the UAV
I_x	=	inertia of the UAV at X direction
I_y	=	inertia of the UAV at Y direction
I_z	=	inertia of the UAV at Z direction
$6DoF$	=	6 Degree of Freedom

II. Introduction

In recent decades, through the development of technology, Unmanned Aerial Vehicles (UAVs) have gained significant usage in various areas, including monitoring, cargo shipping, and agricultural spraying. In terms of reliability and sustainability, the use of UAVs and the expectations placed on them should be clearly defined[1]. A recent study conducted by the European Union Aviation Safety Agency (EASA) identifies safety, security, noise, and wildlife impact as key concerns regarding the urban use of drones[2]. Rotorcraft UAVs are suitable for diverse operations due to their capability to operate in confined spaces, agile maneuverability, and flexible design. Given the wide range of operating conditions, the utilization of robust controller algorithms is necessary. However, as the application areas expand to include cargo carrying and agricultural spraying, achieving robustness becomes more challenging. Overcoming this challenge necessitates considering specific scenarios during the controller design process.

Since the flight dynamics of rotorcraft UAVs primarily depend on actuators, any actuator malfunction has the potential to cause chaotic output. Moreover, due to their small structure, UAVs are highly susceptible to external disturbances and model uncertainties [3]. Particularly, multitasking systems require robust algorithms to mitigate modeling errors. On the other hand, the system dynamics model is approximated using physical interpretations. Given the highly nonlinear flight conditions, it is common for the system to deviate from the desired actuator dynamics [4].. Numerous approaches have been developed in the literature to ensure stability. Some researchers focus on refining the system model, while others aim to achieve control methodologies that are less reliant on a predefined system model. However, model-based methodologies lose effectiveness as uncertainty increases. To address this issue and attain robust behavior, sensor-based algorithms show promising results [5][6]. This approach ensures that changes in the model do not significantly impact the system.

*PhD Candidate, Autonomous Systems and Control Centre for Autonomous and Cyberphysical Systems / m.r.kartal@cranfield.ac.uk
†Senior Research Fellow in Autonomous Systems and Control Centre for Autonomous and Cyberphysical Systems / D.Ignatyev@cranfield.ac.uk
‡Professor of Autonomous Systems Control Centre for Autonomous and Cyberphysical Systems / A.Zolotas@cranfield.ac.uk

One potential sensor-based technique is the utilization of incremental dynamics (ID) to create controllers that are less dependent on the system model [7] [8]. The incremental nonlinear dynamics inversion (INDI) and incremental backstepping (IBKS) are two control techniques developed using this approach [9] [10]. To capture information about unmodeled dynamics, both INDI and IBKS employ acceleration feedback (sensor measurements) to generate incremental instructions. This eliminates the need for time-consuming and expensive dynamics-related model data that is solely dependent on the system's states in the design of these sensor-based nonlinear controllers.

Building upon the aforementioned discussion, this study presents a comprehensive analysis of actuator faults in Unmanned Aerial Vehicles (UAVs). The analysis includes a comparison of PID, Integrator Backstepping, and Incremental Backstepping control designs. Additionally, a 6DoF nonlinear quadrotor model is implemented throughout the study. Two distinct scenarios addressing actuator faults are defined, designed, and simulated. The comparison is conducted by evaluating command tracking performance under normal and signal delay conditions. The paper follows the following structure: Section 2 discusses the vehicle platform model employed in this study. Proposed control designs are presented in Section 3, followed by simulations and a discussion of the results in Section 4. The paper concludes with Section 5.

III. Model

The UAV model utilized in this study comprises two aspects. Firstly, a linear model is prepared to determine controller gains. Secondly, a nonlinear model is developed for simulations. The research focuses on a quadcopter-style rotorcraft as the proposed UAV structure. By applying Newton's second law, the dynamic model of the UAV is formulated for 6DoF. Consequently, the dynamic model of the quadrotor in the inertial frame can be expressed as follows:

$$\ddot{x} = -\frac{f_t}{m}[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] \tag{1}$$

$$\ddot{y} = -\frac{f_t}{m}[\cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi] \tag{2}$$

$$\ddot{z} = g - \frac{f_t}{m}[\cos\phi\cos\theta] \tag{3}$$

$$\ddot{\phi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} + \frac{u_2}{I_x} \tag{4}$$

$$\ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} + \frac{u_3}{I_y} \tag{5}$$

$$\ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\theta}\dot{\phi} + \frac{u_4}{I_z} \tag{6}$$

Where $[x, y, z]^T$ presents coordinate positions, and $[\phi, \theta, \psi]^T$ presents angular states. Also moments of inertia are represented as $[I_x, I_y, I_z]^T$. The quadrotor system is an under-actuated system with 4 inputs and 6 dimensions for the control. Input signals represented in the equations as $[f_t, u_2, u_3, u_4]^T$. More precisely, f_t presents total force, which affects all the position states. However, u_2, u_3 and u_4 targets moment values of the UAV.In further steps of the control algorithm, these calculated values modified with control allocation matrix, but this action is not in the scope of this study.

Simulation conditions are chosen as the continuous-time with nonlinear airframe modelling to catch the closest results as real-life conditions. Closed-loop dynamics stability of the designed systems validated by the Lyapunov theory [11]. In the following section, proposed control algorithms will be presented.

IV. Control

In this study, three distinct control methodologies were proposed. One of the controllers employs a PID approach, while the other two controllers are based on the backstepping technique, specifically Integrator Backstepping and Incremental Backstepping. This enables the measurement of performance and behavior across different approaches, such as PID-Backstepping and Incremental-Integrator. The suggested algorithms were validated using Lyapunov Theory to ensure finite-time convergence and stability.

All controllers are implemented within a cascade structure, which works in conjunction with the PID position controller. Given the highly agile dynamics of rotorcraft UAVs, a cascade system design proves to be an effective methodology for this underactuated system. The cascade design comprises an outer controller responsible for X, Y, and Z position control, and an inner controller managing the angular states.

The outer controller corresponds to the attitude control loop, which computes the desired angular setpoints. Conversely, the inner loop represents the angular control loop that governs the motors of the UAV. A visual representation of the cascade system design is depicted in Figure 1.

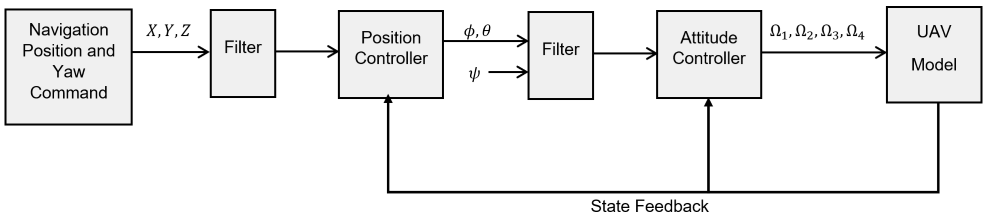

Fig. 1 Proposed cascade control system diagram

In the following discussion of the topic, proposed controlling algorithms will be explained briefly.

A. Proportional–Integral–Derivative

For all three controllers, the Proportional-Integral-Derivative (PID) controller has been employed as the position controller. It can be incorporated into the cascade design as either the outer or higher controller, placed in the last layer of the structure. Furthermore, one of the controllers also utilizes a PID algorithm for the angular states. Therefore, the controller structures are designed as PID-PID, PID-IBS, and PID-IBKS. Figure 2 depicts a parallel PID controller structure.

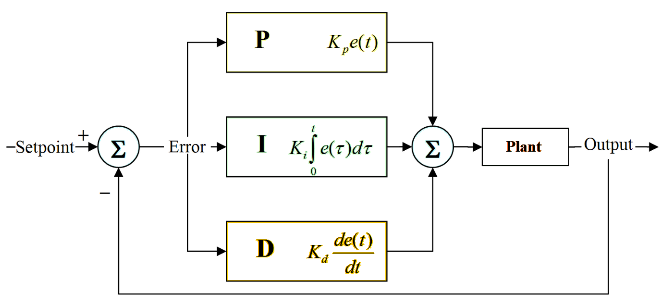

Fig. 2 Proportional–Integral–Derivative(PID) controller diagram

The PID control algorithm is a widely recognized controller that combines Proportional, Integral, and Derivative components. In our system, the PID controller is utilized in both PID and PD forms. The Proportional-Derivative (PD) form was chosen for the X and Y positions, while the Proportional-Proportional-Integral-Derivative (P-PID) form was preferred for the Z position. Equations (7) and (8) present the general PID logic.

$$e_x = x_d - x \tag{7}$$

$$u(t) = K_{p_x} e_x + K_{i_x} \int e_x + K_{d_x} \dot{e}_x \tag{8}$$

The backstepping based controller algorithms are briefly explained below.

B. Integrator Backstepping(IBS)

The Backstepping control methodology is a widely employed nonlinear flight control strategy. Linear controllers, in general, aim to mitigate some of the nonlinear dynamics of the system and determine controller gains. However, this cancellation of dynamics can have drawbacks on the performance of the controller. Nonetheless, linear controllers have been successfully applied in various operations. On the other hand, nonlinear controllers do not suffer from this limitation. However, the conventional Backstepping algorithm heavily relies on an accurate system model, making it less feasible in the presence of modeling uncertainties [12].

In this study, the Integrator Backstepping and Incremental Backstepping methods were employed as nonlinear controllers for comparison. The stability of both controllers has been proven using Lyapunov functions [13].

To ensure global asymptotic stability (GAS), the backstepping algorithm can be enhanced by incorporating an integrator. This allows for the design of a virtual input signal (ϵ) based on its desired value α(x). To achieve this, an error signal (z) needs to be designed, resulting in an error-neutralizing approach. The proposed modification can be illustrated as follows:

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\xi \\
\dot{\xi} &= u \\
\xi &= \alpha(x) \\
\dot{\alpha}(x) &= \left(\frac{\partial \alpha}{\partial x}\right)\dot{x} = \left(\frac{\partial \alpha}{\partial x}\right)[f(x) + g(x)\xi]
\end{aligned}
$$

Where V is proposed control Lyapunov function(CLF) and V_a is a proposed positivity definite function;

$$V(x) = \frac{1}{2}x^2 \tag{9}$$

$$\dot{V}(x) = \left(\frac{\partial V}{\partial x}\right)(f(x) + g(x)\alpha(x)) \leq -V_a(x) \tag{10}$$

The block diagram of the Integrator Backstepping controller is shown in Fig. 3.

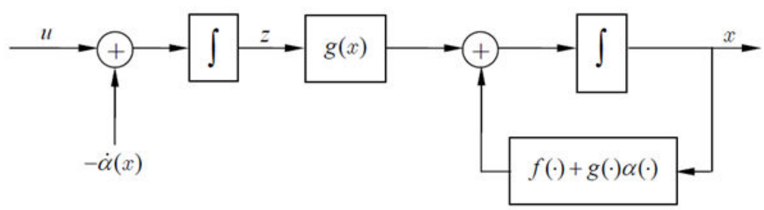

Fig. 3 Integrator Backstepping Algorithm diagram

C. Incremental Backstepping(IBKS)

A promising sensor-based technique known as Incremental dynamics has been developed to address unmodeled system dynamics by utilizing acceleration feedback information. Within this framework, two distinct control algorithms have been developed: Incremental Dynamic Inversion (INDI) and Incremental Backstepping (IBKS) algorithms [14].

However, it is important to note that the IBKS algorithm, despite its benefits, lies in a middle ground between model-based and sensor-based methodologies. As a result, it is more susceptible to disturbances originating from sensors, such as noise, bias, and delays within the closed-loop system. [15]. The block diagram illustrating the Incremental Backstepping approach is depicted in Fig. 4.

As presented in the diagrams, feedback from the system gains quite a lot of importance. In the following part of the section, a mathematical explanation of the incremental backstepping algorithm has been proposed.

$$\dot{x} = f(x) + g(x)u \tag{11}$$

We can linearize the equation around$[x_0, u_0]$

$$\dot{x} = \dot{x_0} + g(x_0)\Delta u \tag{12}$$

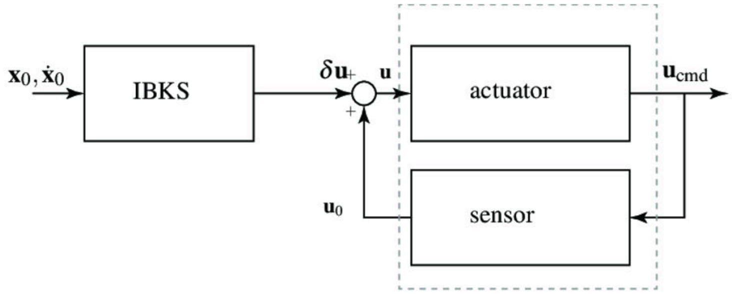

Fig. 4 Incremental Backstepping methodology variations diagram

Where

$$\Delta u = u - u_0 \tag{13}$$

If we design a backstepping controller with control Lyapunov function(CLF) such as;

$$V(x) = \frac{1}{2}x^2 \tag{14}$$
$$\dot{V} = x\dot{x} \tag{15}$$
$$= x(\dot{x_0} + g(x_0)\Delta u) \tag{16}$$

And a control equation such as;

$$V_a = kx^2 \tag{17}$$
$$\dot{V} \leq -V_a \tag{18}$$
$$x\dot{x} = x(\dot{x_0} + g(x_0)\Delta u) \leq -kx^2 \tag{19}$$

This gives us the incremental controller

$$\Delta u = -g(x_0)^{-1}(\dot{x_0} + kx) \tag{20}$$

In this case, the assumption that [equation] and [equation] becomes applicable for a real system, as the control input directly influences the angular accelerations, while the angular rates are modified through the integration of these angular accelerations.

The dynamics equation (20) states that the incremental dynamics of the system is produced by the control input increment. For the implementation of such a concept, it is assumed that the sampling time is small. In this case, the assumption that and becomes possible for a real system because the control input directly affect the angular accelerations, whereas the angular rates are only changed by integrating these angular accelerations.

Moreover, it is assumed that the actuators are highly responsive, enabling them to achieve the desired input increment within the small sampling time. Additionally, the sensors are assumed to be ideal, providing state derivatives without errors.[6]

D. Filter

To avoid infeasible commands provided by the controller, a command filter (CF) is added to the controller[16]. filter includes bandwidth, magnitude and rate limiter as seen in the Fig. 5.

Commands filters are low pass filters which shape the command inputs to match the aircraft dynamics. This technique has been used in backstepping strategies, constraining the pseudo-control in each step. Therefore, differentiation variables need to be estimated as a function of the measurements. To estimate signal derivatives, the proposed filter also

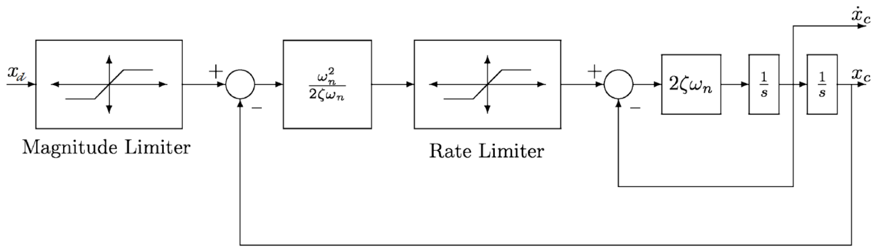

Fig. 5 Anti windup command filter diagram

contains a differentiator.

$$\begin{bmatrix} x_c \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} x_c \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} q_1 \\ 2\xi\omega_n \left(S_R\{\frac{{\omega_n}^2}{2\xi\omega_n}[S_M(x_d) - q_1]\} - q_2 \right) \end{bmatrix} \tag{22}$$

Where S_M is magnitude and S_R rate constraints.

V. Simulation

Performance and robustness are two essential aspects of a controlling algorithm. A well-designed controller should effectively track the desired signal and reject disturbances. In order to evaluate these characteristics, two distinct scenarios were designed, and three different controllers were investigated within these scenarios.

The base rotorcraft model for simulations is Parrot Mambo quadrotor drone. Parrot Mambois a research drone which widely used in many research studies. Parrot Mambo drone has 63 gram mass with PF070235 motors and 66 mm propellers. According to MATLAB database, inertia values J_X, J_y and J_z are as given respectively; $5.82857e-05$, $7.16914e-05$, $10e-05$. The proposed simulation pathway and the corresponding controller outputs are presented in Fig. 6.

Throughout the study, two different operation scenario defined:
- Command tracking performance
- Input delay rejection

Firstly, a flight mission involving a circular route was defined to assess the controllers' capability in handling angular attributes.

The performance of each controller is depicted in the graphs, where the desired command is represented by a black line, PID by a green dotted line, IBS by a red line, and IBKS by a blue line.

A. Performance comparison

As a starting point of the study, it is crucial to verify the command tracking performances of the controllers under normal conditions. For this purpose, a circular route was defined to assess the angular controller's performance.

In Fig. 7, the outputs of the X, Y, and Z coordinates are compared. The command tracking performances were analyzed by comparing the controller position outputs.

Upon a general review, it can be observed that all three controllers are capable of providing satisfactory tracking performance. Specifically, in the X position, the PID controller performs slightly better, while in the Y position, the order of performance is reversed, with the differences becoming more noticeable. Regarding altitude control, the command to maintain the same level was successfully executed by all controllers. However, Fig. 8 displays the angular states during the simulation.

Similar behavior to the position states can be observed in the angular aspects. Based on the obtained results, it can be concluded that the proposed controllers are capable of effectively manipulating the system according to the given commands. This paves the way for further analysis in subsequent steps.

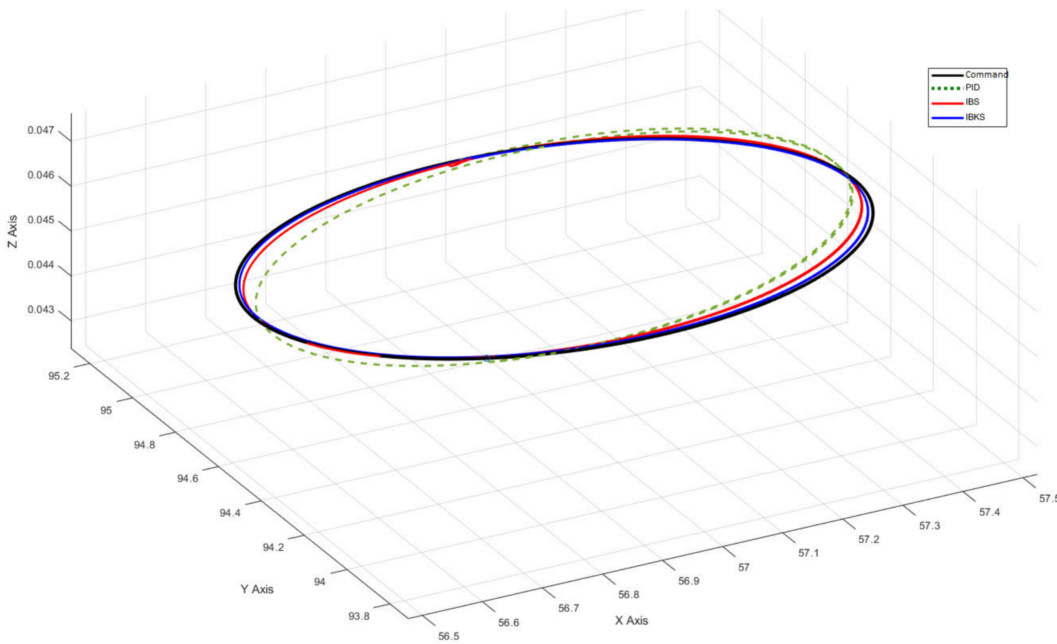

Fig. 6 Base mission 3D representation

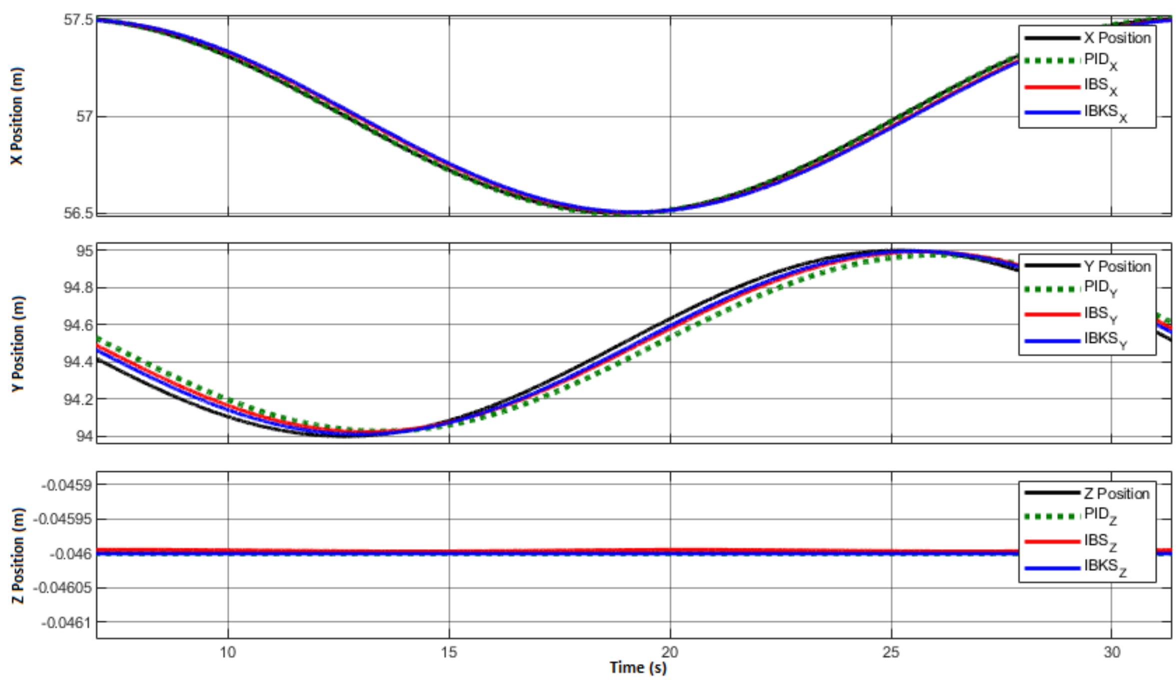

Fig. 7 Command tracking performance position graph

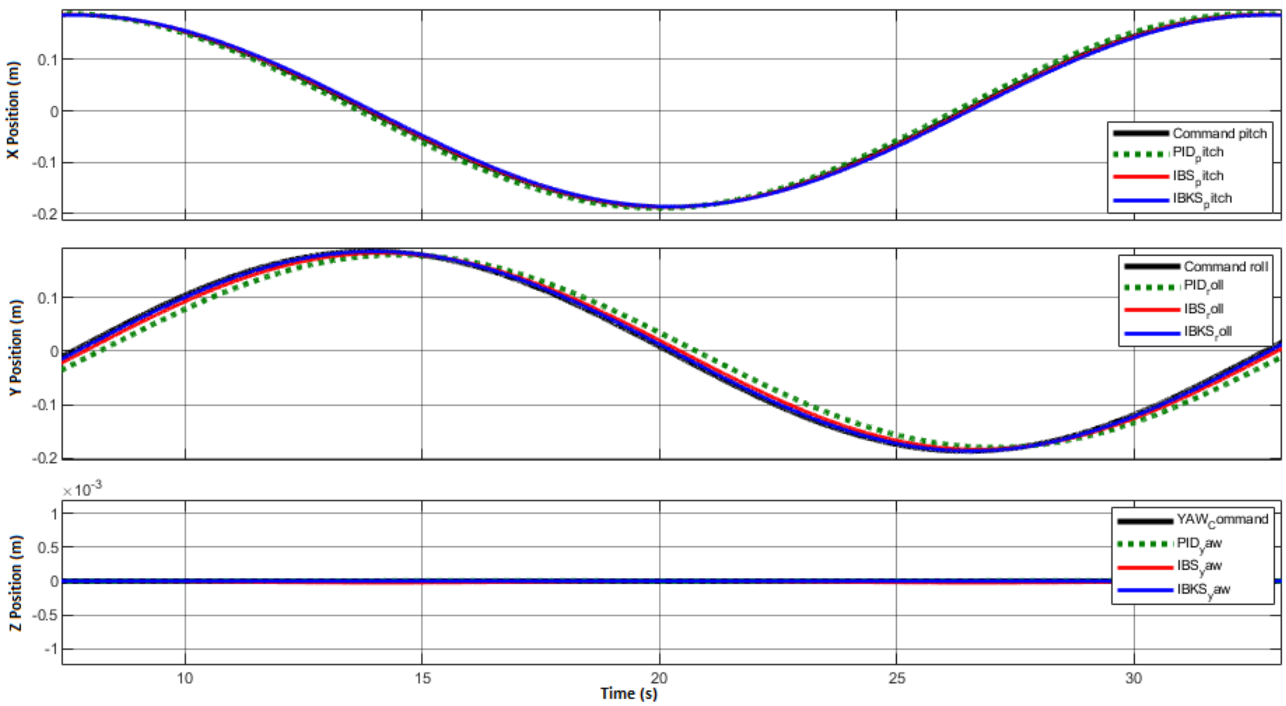

Fig. 8 Command tracking performance angular state graph

B. Input Delay

Input delay poses a significant challenge for physical UAV systems, as it can arise due to factors such as flight computer computing time, signal transmission time, and the time required for the motors to achieve the desired mechanical response. To incorporate cumulative delays into the system, a delay function is applied to the input signals. The application of the delay function is explained with a diagram representation in Fig. 9.

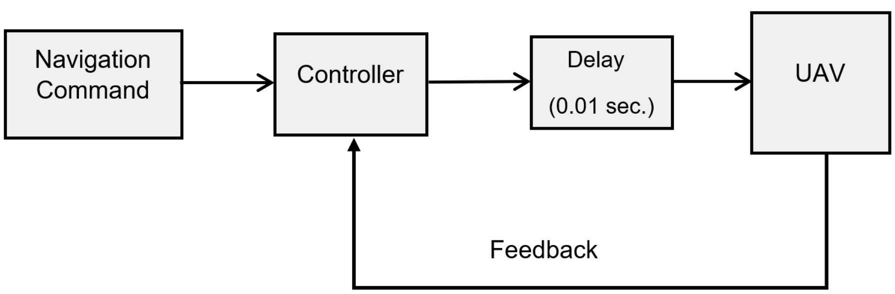

Fig. 9 Input delay diagram

In Fig. 10, the proposed delay of 0.01 sec is implemented in the system to estimate the lag effect between the controller command and the actual state of the motors. The lag time value is chosen empirically, considering the maximum acceptable value for all controllers. Since the target issue is the lag caused by hardware components, it is treated as a continuous constant delay.

The model-based algorithms, PID and IBS, failed to effectively reject the proposed disturbance. However, both controllers did not yield chaotic results, except for the altitude state. Fig. 11 illustrates that IBS exhibited undershoot behavior, while PID showed overshoot behavior. Fig. 12 presents the angular position outcomes solely influenced by the angular controllers, indicating that the position controllers handled the situation appropriately. Similar to the performance simulation, instability in the yaw angle led to altitude loss.

In this phase of the study, IBKS warrants a closer examination. Among all three controllers, only IBKS was able to compensate for the input signal delay and produce acceptable outputs. The sensor-based algorithm IBKS successfully

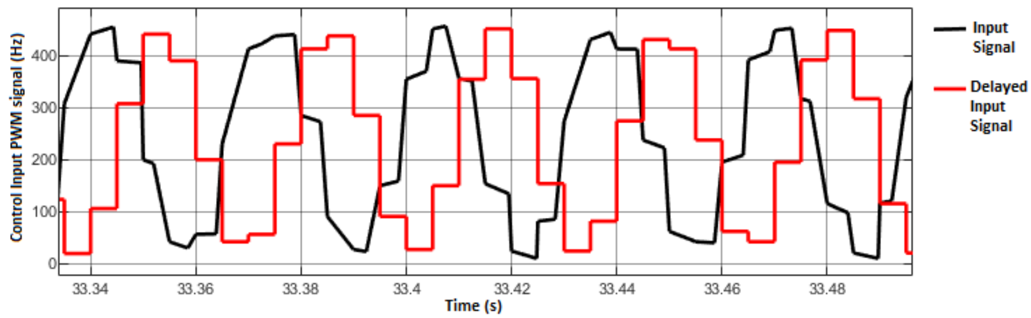

Fig. 10 Input PWM signal and delayed input PWM signal graph

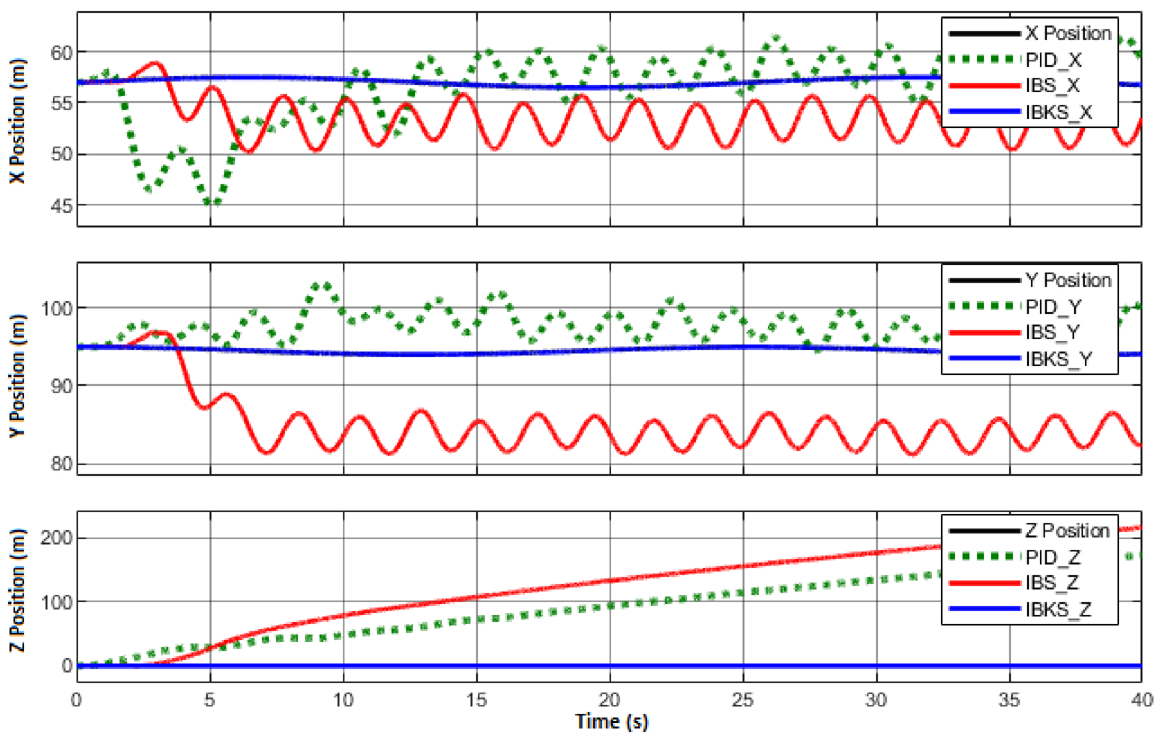

Fig. 11 PID, IBS and IBKS position graph under input delay affect

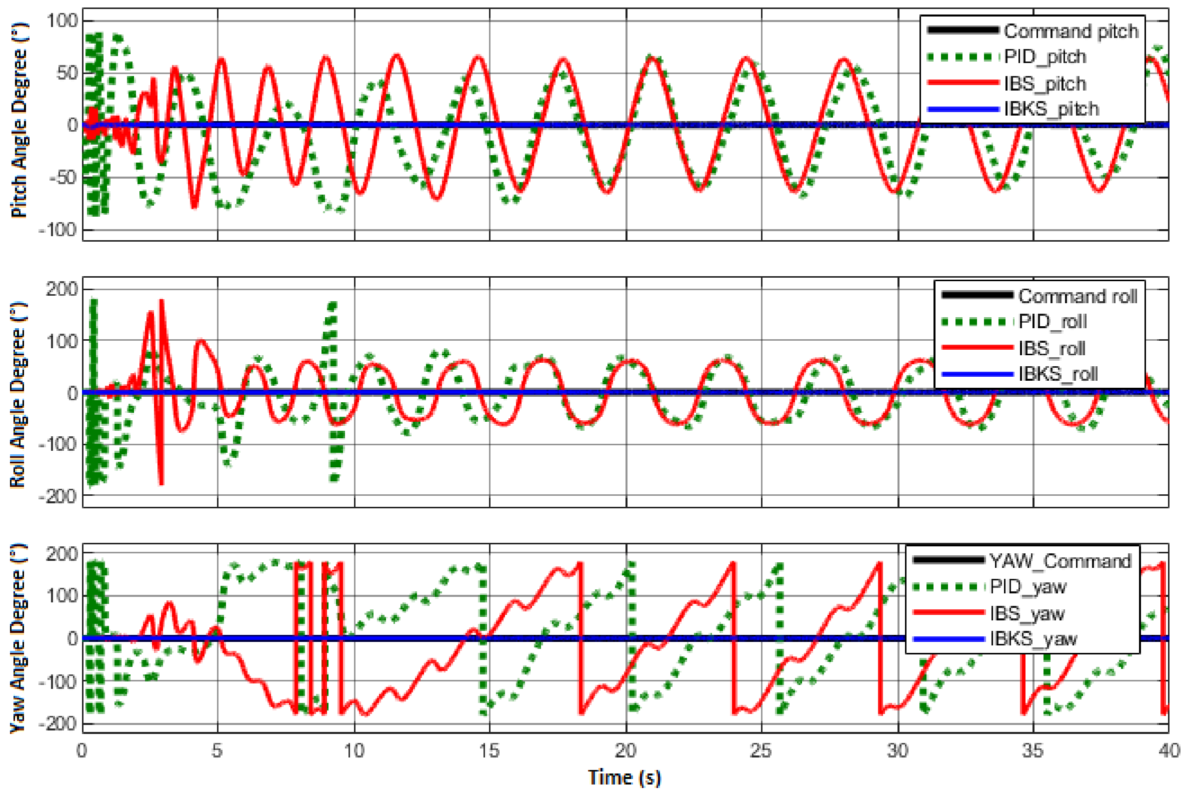

Fig. 12 PID, IBS and IBKS angular state graph under input delay affect

mitigated the disturbance caused by the given delay. Since the position graph sdo not have much distortion, only angular states graph will be inspected as shown in the Fig 13. The behaviour of the distortion is similar to noise sensor affects.

VI. Conclusion

The incremental control paradigm has been applied to design an efficient and robust control system for a quadcopter structure. The controller's performance was compared with PID and IBS controllers, and evaluated through command tracking performance and input delay rejection. The non-linear mathematical model was utilized during simulations to investigate and observe the controller's performance under various conditions, including regular and signal delay related problematic scenarios.

Through this methodology, the capability of the proposed algorithm (IBKS) was tested and demonstrated. In the command tracking performance simulation, all three controllers exhibited similar performances, indicating their competence. However, some abnormal outputs were observed, such as very small yaw oscillations in the IBS-controlled system.

While PID and IBS algorithms failed to reject disturbances caused by input delay, the IBKS algorithm successfully managed the situation using its input signal-based methodology. This promising result highlights the algorithm's capability to manipulate the system as desired. Based on these findings, further improvements will be made to enhance the Incremental Backstepping controlling algorithm.

It is important to note that sensor-based methodologies have their own challenges. Obtaining reliable information from sensors plays a crucial role in the control process. Any disturbances or errors in the feedback data can have a greater impact on the system compared to model-based approaches. Moreover, hardware components have physical limitations and disadvantages due to its mechanical nature. On the other hand sensor-based systems are susceptible to sensor noise. Effectively rejecting systemic faults becomes a critical task in sensor-based methodologies that heavily rely on measurements.

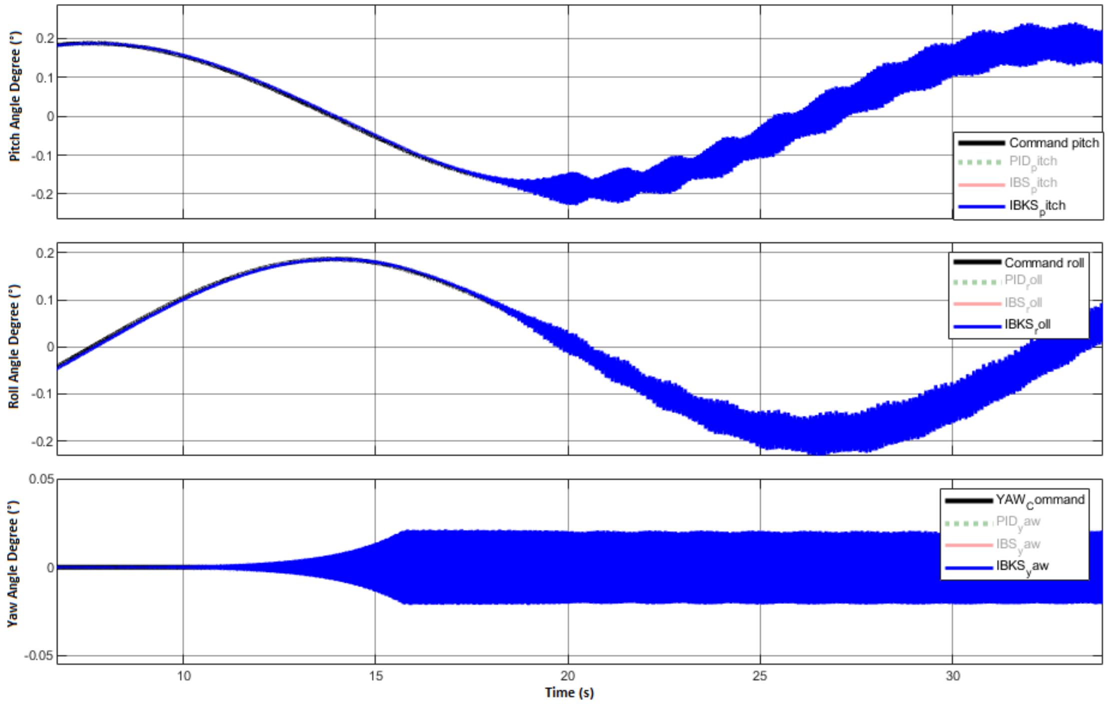

Fig. 13 PID, IBS and IBKS angular state graph under input delay affect

References

[1] Kartal, Muhammed Rasit, Ignatyev, Dmitry, Argyrios Zolotas. "FIXED-WING UAVs CONTROL WITH MODEL UNCERTAINTIES." 33th Congress of the International Council of the Aeronautical Sciences (2022) At: Stockholm, Sweden.

[2] European Union Aviation Safety Agency (EASA), "Study on the societal acceptance of Urban Air Mobility in Europe." [Online]. Available: https://www.easa.europa.eu/downloads/127760/en.

[3] O. Mofid, S. Mobayen, and A. Fekih, "Adaptive Integral-Type Terminal Sliding Mode Control for Unmanned Aerial Vehicle under Model Uncertainties and External Disturbances," IEEE Access, vol. 9, pp. 53255–53265, 2021, doi: 10.1109/ACCESS.2021.3070400.

[4] W. Youn and H. Myung, "Robust Interacting Multiple Model with Modeling Uncertainties for Maneuvering Target Tracking," IEEE Access, vol. 7, pp. 65427–65443, 2019, doi: 10.1109/ACCESS.2019.2915506.

[5] Ignatyev, Dmitry I., Hyo-Sang Shin, and Antonios Tsourdos. "Two-layer adaptive augmentation for incremental backstepping flight control of transport aircraft in uncertain conditions." Aerospace Science and Technology 105 (2020): 106051.

[6] Ignatyev, Dmitry I., Hyo-Sang Shin, and Antonios Tsourdos. "Sparse online Gaussian process adaptation for incremental backstepping flight control." Aerospace Science and Technology (2023): 108157.

[7] Smith P. A simplified approach to nonlinear dynamic inversion based flight control. In23rd Atmospheric Flight Mechanics Conference 1998 Aug (p. 4461).

[8] Bacon B, Ostroff A. Reconfigurable flight control using nonlinear dynamic inversion with a special accelerometer implementation. InAIAA Guidance, Navigation, and Control Conference and Exhibit 2000 Aug (p. 4565).

[9] Sieberling S, Chu QP, Mulder JA. Robust flight control using incremental nonlinear dynamic inversion and angular acceleration prediction. Journal of guidance, control, and dynamics. 2010 Nov;33(6):1732-42.

[10] Acquatella, B. P. J. "Robust nonlinear spacecraft attitude control: An incremental backstepping approach." (2011).

[11] P. van Gils, E. van Kampen, C. C. de Visser, and Q. P. Chu, "Adaptive incremental backstepping flight control for a high-performance aircraft with uncertainties," 2016 AIAA Guid. Navig. Control Conf., no. January, 2016, doi: 10.2514/6.2016-1380.

[12] R. A. Cordeiro, J. R. Azinheira, and A. Moutinho, “Cascaded Incremental Backstepping Controller for the Attitude Tracking of Fixed-Wing Aircraft,” no. Id, [Online]. Available: https://inception-h2020.tekever.com/.

[13] B. J. Jeon, M. G. Seo, H. S. Shin, and A. Tsourdos, “Understandings of Classical and Incremental Backstepping Controllers with Model Uncertainties,” IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 4, pp. 2628–2641, 2020, doi: 10.1109/TAES.2019.2952631.

[14] Pollini L, Metrangolo A. Simulation and robust backstepping control of a quadrotor aircraft. InAIAA modeling and simulation technologies conference and exhibit 2008 Aug 18 (p. 6363).

[15] Jeon, B.J., Seo, M.G., Shin, H.S., and Tsourdos, A. (2021). Understandings of incremental backstepping controller considering measurement delay with model uncertainty. Aerospace Science and Technology, 109, 106408.ss

[16] Farrell, Jay, Marios Polycarpou, and Manu Sharma. "On-line approximation based control of uncertain nonlinear systems with magnitude, rate and bandwidth constraints on the states and actuators." In Proceedings of the 2004 American control conference, vol. 3, pp. 2557-2562. IEEE, 2004.

Acknowledgments

This work has supported by Republic of Turkiye Ministry of National Education and Cranfield University

2024-01-04

Non-linear control of a quadrotor with actuator delay

Kartal, Muhammed R.

AIAA

Kartal MR, Ignatyev D, Zolotas A. (2024) Non-linear control of a quadrotor with actuator delay. In: AIAA SCITECH 2024 Forum, AIAA SCITECH 2024 Forum, 8-12 January 2024, Orlando, USA. Paper number AIAA 2024-0569

https://doi.org/10.2514/6.2024-0569